Chapter 2
Energy

Energy is a quantity that not only plays a dominant role in the most diverse areas of the sciences, technology, and economy, but is omnipresent in the everyday world around us. For example, we pay for it with every bill for electricity, gas, and heating oil that arrives at our homes. But we are also confronted more and more with questions about how we can save energy in order to cover our current and future demands. At the beginning of the chapter, the conventional indirect way of defining energy is briefly presented. A much simpler way to introduce this quantity is characterizing it by its typical and easily observable properties using everyday experiences. This phenomenological description may be supported by a direct measuring procedure, a method normally used for the quantification of basic concepts such as length, time, or mass. Subsequently, the *law of conservation of energy* and different manifestations of energy like that in a stretched spring, a body in motion, etc., are discussed. In this context, important quantities such as *pressure* and *momentum* are introduced via the concept of energy.

2.1 Introducing Energy Indirectly

Energy is a quantity that not only plays a dominant role in the most diverse areas of the sciences, technology, and economy, but is omnipresent in the everyday world around us. We buy it in large amounts and pay for it with every bill for electricity, gas, and heating oil that arrives at our homes. There is also information on every food package about the energy content of the food inside. We are confronted more and more with questions about how we can save energy in order to cover our current and future demands.

Ironically, this everyday quantity is defined and explained in a very complicated way. To start with, it is dealt with as a special concept within the subject of mechanics and then gradually expanded and generalized. The quantity called energy is almost always introduced *indirectly* through mechanical work. The relation

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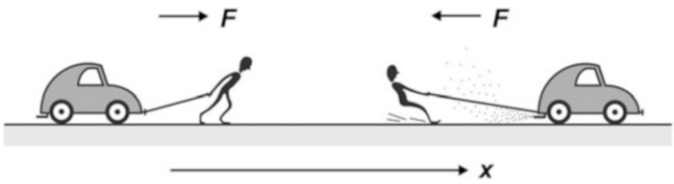

Fig. 2.1 Interaction of force and distance when doing work (here, as seen by the person involved).

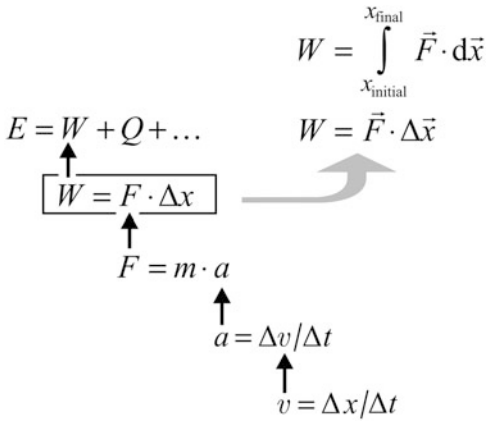

Fig. 2.2 The usual indirect way over many steps to energy (a simplified image here). The formulas above on the right clarify the framed equation at step 4. a acceleration, E energy, F force, m mass, Q heat, t time, v velocity, W work, x position.

"work = force times distance" is our access route. It tells us that a lot of work must be *done* if, for instance, one wishes to cover a distance using great force against a strong resistance. Examples of this might be riding a bicycle against a strong wind or towing a car (Fig. 2.1) across a sandy surface. The figure shows the interaction of force and distance from the perspective of the person performing the work. The force being applied by him in the x-direction is positive for the example on the left: $F_x = F$; on the right, it is negative: $F_x = -F$. Correspondingly, the work $W = F_x \cdot \Delta x$ done by the person in the figure is positive on the left and negative on the right. Seen from the car being pulled, all the forces and work have the opposite algebraic signs.

The unit for work is the *Joule* (J), named for the British beer brewer and private scholar James Prescott Joule, who lived in Manchester, England, in the nineteenth century. One Joule corresponds to the product of the units of force (Newton, N) and length, $\mathrm{N\ m = kg\ m^2\ s^{-2}}$.

The path to the concept of work leads through many steps (Fig. 2.2). The quantity called force is also defined *indirectly* (force = mass times acceleration). The same holds for acceleration (= change of velocity divided by time interval) as well as velocity (= distance covered divided by time needed to cover it). Mechanical work is only one form of energy input. There are other forms as well, the most

important of which is *heat*. The name *energy* is an umbrella term for these various forms. This new quantity has its own symbol, mostly E, with the unit Joule. Furthermore, each of the numerous variants has its own name and symbol. Along with work W and heat Q, there are internal energy U, enthalpy H, Gibbs (free) energy G, exergy B, etc.

In order to avoid this convoluted derivation, we will introduce energy directly by metricization. As a first step, we will characterize the concept using typical and easily observable properties. We will see that we can do without the large number of energy terms because a single one is basically enough to do what is necessary to describe all processes we deal with in physical chemistry.

2.2 Direct Metricization of Energy

Basic Idea Almost everything that we do requires *effort* and *strenuousness*. We notice this especially when doing something that is so strenuous, it makes us sweat and gasp. So let us imagine all devices and things we use to be so large and heavy that we feel the consequences of dealing with them. We will take a look at a few *strenuous* activities that we accomplish *without* tools or *with* the help of levers, ropes, pulleys, winches, etc. (Fig. 2.3). All the activities in the figure belong to the subject of mechanics.

We can also include thermal processes (such as a "heat pump"), electric processes (an "electrostatic machine," for instance), or chemical processes (such as a water electrolysis apparatus) in this (Fig. 2.4). However, because these kinds of processes are less familiar, we will come back to them later on in more detail. For now, we will limit ourselves to mechanical processes.

It is noteworthy that the effort we have expended for these activities does not just disappear, but can be used to accomplish other strenuous activities. For example,

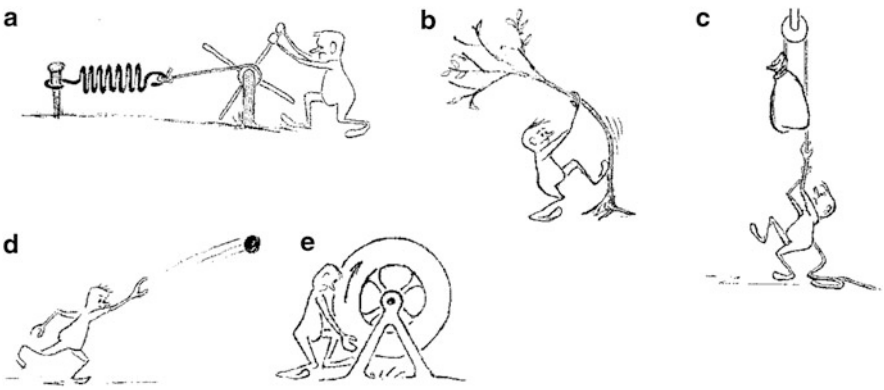

Fig. 2.3 Strenuous activities: (**a**) stretching, (**b**) bending, (**c**) lifting, (**d**) throwing, (**e**) starting a wheel turning.

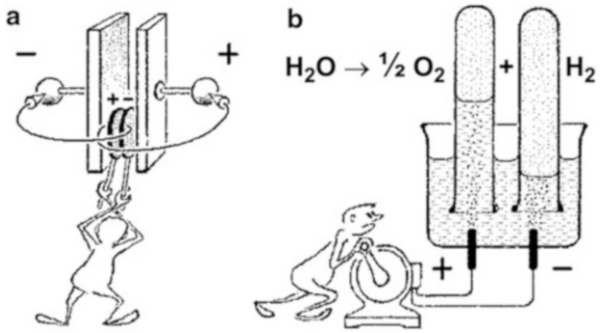

Fig. 2.4 (**a**) A primitive "electrostatic machine": progressive charging of a capacitor by separating charge in the already existing field and transporting the charge to the corresponding opposite plate, (**b**) A "water electrolysis apparatus": forcing a spontaneous reaction in the opposite direction is strenuous, here using the example of decomposing water into its elements.

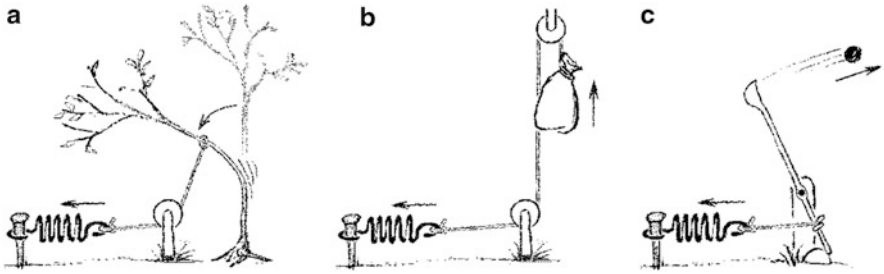

Fig. 2.5 Utilizing the effort needed for stretching the spring in order to (**a**) bend a tree, (**b**) lift a sack, (**c**) hurl a stone.

we can directly or indirectly use a stretched spring that is trying to contract, in order to bend a tree, to lift a sack, or to hurl a stone, etc. (Fig. 2.5).

Conversely, we can stretch a spring by using a bent tree, a lifted sack, or even a hurling stone (if it can be caught correctly). Any combination can be accomplished, if the right tools are put to use. What is important here is that the effort put into something can be used to perform other activities.

It appears that the effort expended to change something is *stored* within the changed objects. We can imagine it to be contained in the stretched spring, the bent tree, the lifted sack, the flying stone, etc. It can be taken out again by reversing the change and then reused to change other things.

Lost Effort All of us experience activities where all effort apparently seems to disappear (Fig. 2.6). We rub our hands together and they get warm, but we cannot lift a sack with this heat. It is hard work to pull a heavy cart across a sandy surface. Not only do we start to sweat, but the sand also becomes warm even if we do not notice it. Even when we do not accomplish anything, for example, when we try to hold back an attacking dog or try to pull a firmly rooted bush out of the ground, we get hot.

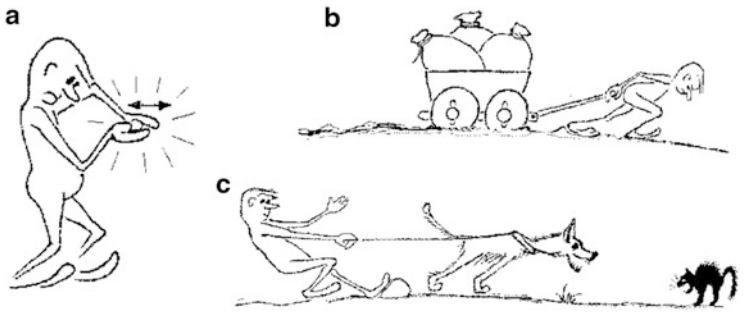

Fig. 2.6 Activities where all effort seems lost: (**a**) rubbing, (**b**) pulling, (**c**) restraining.

In such cases, it seems there is nothing left of the effort that was expended that can then be put to use. At least under the given circumstances, this appears to be the case.

Although the effort needed for this does not get reused, it does not simply disappear without a trace. *We* sweat, and not only do we get hot, but also the things around us: the rubbed hands, the sand under the wheels, the squeaking wheel bearings. The more effort wasted, the more pronounced the warming. This is a significant trace left by the lost effort. However, even this trace gradually fades so that eventually it seems that all the effort disappears into nothing.

We must take into account that with everything we do, a part of our effort does not accomplish anything but will get lost in some unintended secondary activity. Friction almost always hinders our activities. It takes a lot of effort to overcome friction and it is something we seek to avoid, if possible. To give another example, we ourselves are affected by the activity of our muscles where—just like with rubbing—heat is produced. This is an unwanted but unavoidable secondary activity.

Measuring Effort The question now arises of whether it is possible (unencumbered by what we feel) to determine how much effort a certain activity entails—stretching a spring, lifting a sack, or charging a capacitor? We expect that an objective measure of the effort expended for the same activity will always have the same value, no matter who does the work, where it is done, and when. If something is done twenty times in exactly the same way, it should mean, all in all, twenty times the amount of effort.

The unit used (as for example, the unit of length) can in principle be chosen arbitrarily. One could for instance select a coil spring. The position of the end of the spring in its relaxed state receives a value of 0, and at an arbitrary but defined stretched state, a value of 1 (Fig. 2.7). In this manner, an "amount" of effort is defined that can serve as our private unit. Naturally, we can also choose the unit to be equivalent to the SI unit. Such springs can be made so that the initial and final states are easily recognizable, comparable to a spring balance whose scale is limited to values of 0 and 1 (compare Fig. 2.7a, lower spring).

We can set the flywheel in motion using the stretched "unit-spring." The spring then returns to its rest position and the flywheel rotates. The effort stored in the

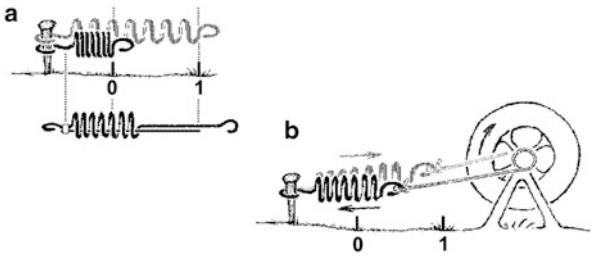

Fig. 2.7 (**a**) Showing the "unit of effort" by a spring stretched between the marks 0 and 1, (**b**) Transferring the effort to a flywheel (front, *black*) and back to the spring again (back, *gray*).

spring is now in the turning wheel and can be given back to the spring by using the spin of the wheel to stretch the spring (Fig. 2.7b). This interplay can in principle be repeated indefinitely and in various ways. Unfortunately, air and bearing friction gradually consume the stored effort, or in other words, friction continuously takes some of it for other purposes.

If we were able to prevent such unwanted losses, it would be easy to measure the effort necessary for our activity. We will therefore assume that by using appropriate measures, losses can be avoided. Ball bearings help against axle friction, a vacuum helps against air friction, thicker wires help against line resistance, and the friction of wheels on a surface can be compensated for by a harder surface, or better yet, air cushions. Later on we will see how we can deal with things when such compensating measures are unavailable or insufficient.

Another type of error arises when a part of the effort gets "caught" in the storage. This is the case where the tree is being bent and the sack is being lifted by springs (Fig. 2.5a, b). The process comes to a halt when the pull of the spring upon the rope reduces to the point where it can no longer overcome the counter-pull of the tree or the sack. The rope and deflection pulleys alone are not enough to make the best use of the effort stored in a spring during stretching. To do so, it would be necessary to make use of a somewhat more complicated construction which we will not do at this time. We will just assume that it is, in principle, possible to utilize the stored effort entirely.

Measuring effort simply means counting how many unit portions it can be divided into. One counts the number of unit-springs that can be stretched or one counts the number of already stretched springs needed to obtain a desired change (compare Fig. 2.8).

Energy We call the quantity introduced by the process described above, *energy*. Of course there are more accurate and more easily reproducible means to represent the energy unit (or an arbitrary multiple of it) than our spring. As an example, consider the energy of a photon emitted by a hydrogen atom when its electron drops from the 2p state to the 1s state. This sounds a little bit strange at the beginning, too. But, also the meter, the fundamental unit of length in the SI system, is meanwhile no longer defined by the international prototype meter bar composed of an alloy of platinum and iridium. In 1960, the meter was fixed as equal to 1,650,763.73 times the wavelength in a vacuum of the radiation corresponding to the transition between the levels $2p^{10}$ and $5d^5$ of the krypton-86 atom. In order to enable traceability

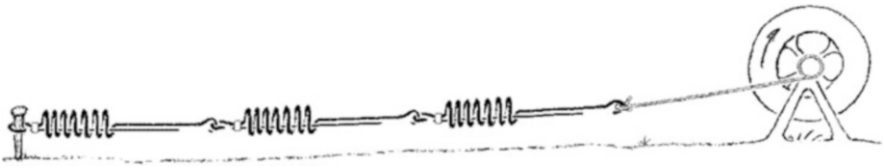

Fig. 2.8 Creating a multiple of the "unit of effort" by connecting unit-springs in series.

between this new definition and the old prototype, the numeric value (1,650,763.73) was chosen (according to the measurement accuracy of that time). To further reduce uncertainty, the length of the meter was currently fixed indirectly in terms of the second and the speed of light. However, precision of measurement is not what is actually important when first learning about a quantity, here the quantity called energy.

Depending upon the purpose, different symbols such as $E, W, U, Q, H, G, \ldots$ are in use for the quantity called energy. We will use only the symbol W because E will be used for electric field strength. Moreover, there is no good reason to give different symbols and names to stored and transported energy.

The quantity W has been introduced by *direct metricization* of the everyday concept of *effort*. Although we have relied on our senses, the quantity W is ultimately independent of subjective feelings. This fact is essential for dealing with it objectively because the same activity can, for example, seem more strenuous to one person than to another or more exhausting to a tired person than to a well-rested one. The quantity called energy defines and quantifies what we call *effort* in everyday life, a term relating to an activity. On the other hand, it also denotes what is *stored* in an object that is deformed, moved, lifted, charged ... and that can be retrieved when needed. That means energy also quantifies the ability to do something, what in everyday language could be circumscribed vaguely by "power to do something."

We should, though, be wary of taking this comparison of energy (in physics) and "power to do something" (in everyday life) too literally. In the economy, the more money you have, the more you can accomplish, but coins and notes do not have any intrinsic power. Almost everything we do involves some kind of turnover of energy. Energy might be considered the *price* paid for an activity or, conversely, what can be gained. If we know the price of an activity, we also know whether or not it is possible to pay for it.

It is a matter of preference whether one wishes to describe processes as *dynamic*, meaning as a result of forces working with and against each other or simply as a form of *accounting* by considering credits and debits on a balance sheet. The former takes into account everyday images and what we sense and feel. The latter makes use of our experiences dealing with cash and non-cash money values.

2.3 Energy Conservation

One of the most important insights of nineteenth century physics was that energy—or "force" or "power," as it was called then—never gets lost, i.e., cannot disappear into nothingness. A hundred years before that people knew that no matter how ingenious you are, energy cannot be created from nothing. In Sect. 2.2, we saw several examples of effort seeming to get lost. This loss was always accompanied by the evolution of heat. In the so-called "Joule's apparatus" (Fig. 2.9), a falling weight was used to spin a paddle wheel. The cold water in the insulated vessel (a kind of calorimeter) was warmed up by the rotating paddles and the corresponding temperature was measured.

An ice calorimeter (Fig. 3.23b and Experiment 3.5) can also be used to determine how much heat has evolved in a process by showing us how much ice it melts. The amount of melt water is proportional to the amount of energy expended, independent of where the energy comes from or how it gets into the ice. Naturally, this is assuming that nothing comes into it from other sources or drains off to some sink—through a leak in the heat insulation, for example.

It was therefore concluded already at that time that a certain amount of energy is needed to heat a body, whether or not this happens intentionally. If the amount of energy used for this is included, one finds that the entire "stock" of energy remains unchanged. Energy can be moved from one storage to another, but the amount remains the same. This finding is called the "*law of conservation of energy*" or short *energy principle*. One result of the law of conservation of energy is that any energy expended must be independent of the path or the tools used. Otherwise, in contradiction to this law, it would be possible to create energy from nothing (or let energy disappear into nothing) by delivering it by one path and releasing it via another.

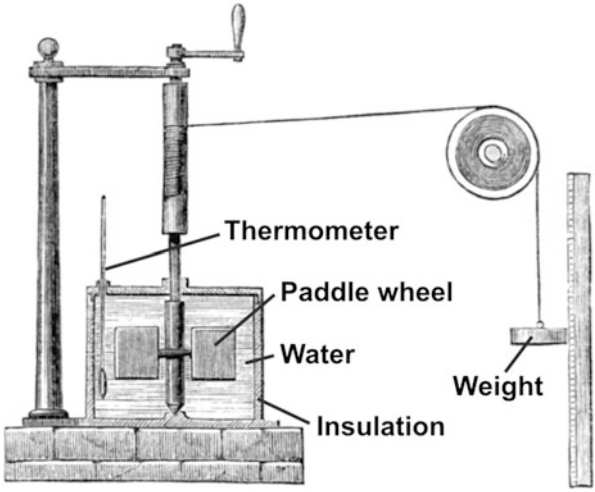

Fig. 2.9 Joule's apparatus for demonstrating the equivalency of energy and heat (from: Abbott (1869) The new theory of heat. Harper's New Monthly Magazine 39:322–329).

Until now, we have assumed that the energy consumed producing a change can be retrieved if the process involved is reversed. However, we run into a problem with processes associated with the evolution of heat: they cannot be reversed, or at least not entirely so. The energy has not actually disappeared, but is somehow not quite accessible any longer. This situation caused a lot of headaches in the nineteenth century and still does today. We will look more closely at this subject in Chap. 3.

Before we do that, though, we will discuss some simple cases showing how to find values for energy. Many quantities derived from energy are easier to measure than the energy itself, so we usually calculate energy indirectly through these quantities.

2.4 Energy of a Stretched Spring

A stretched spring has the tendency to contract, and the more strongly stretched it is, meaning the more the length l exceeds the value l_0 in its relaxed state, the more strongly it tends to contract. Depending upon how much the spring is already stretched, it becomes increasingly strenuous to continue stretching the spring by the small amount Δl (Fig. 2.10). In other words, the energy ΔW needed for this increases by l proportionally to $l - l_0$ (at least within certain limits) as long as the changes ΔW and Δl are small enough. This condition can be expressed by replacing the differences with differentials:

$$\frac{\Delta W}{\Delta l} = D \cdot (l - l_0) \quad \text{or rather} \quad \frac{\mathrm{d}W}{\mathrm{d}l} = D \cdot (l - l_0). \tag{2.1}$$

The graphic shows that, when greatly magnified, the curve around the point l seems to have become straight. We then calculate the slope of this extremely small section of the curve (see the magnified section in Fig. 2.10). There is a more detailed description of this in Sect. A.1.2 in the Appendix. When we now plot the values gained in this manner as a function of the corresponding l values, we obtain a linear function as we would expect from Eq. (2.1).

The proportionality factor D quantifies the characteristic called *spring stiffness*. When the factor D is great, a spring is *hard* or *stiff*; when it is small, the spring is *soft*. $\mathrm{d}W/\mathrm{d}l$ is a measure of the "force" with which the spring resists stretching. We take W and l as measurable quantities so we can express this force—as usual symbolized by F—as follows:

$$F = \frac{\mathrm{d}W}{\mathrm{d}l} \quad \text{or in more detail} \quad F(l) = \frac{\mathrm{d}W(l)}{\mathrm{d}l}. \tag{2.2}$$

The corresponding SI unit is $\mathrm{J\,m^{-1}} = \mathrm{N}$ (Joule/Meter = Newton). If we insert F into Eq. (2.1), we obtain the usual form of a familiar law,

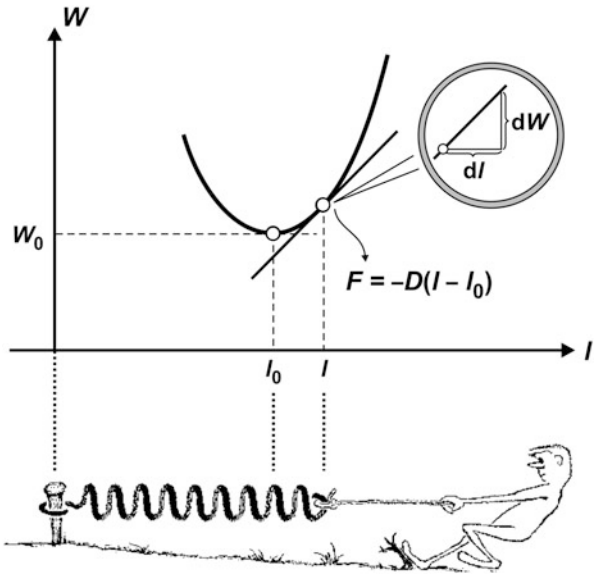

Fig. 2.10 The energy $W(l)$ of a spring as a function of its length l. The graph is close to a parabola for the range around the rest position l_0. The force by which the spring resists stretching at position l corresponds to the slope $\mathrm{d}W/\mathrm{d}l$ of the graph at this point. This is illustrated by the inset "magnifying glass".

$$F(l) = D \cdot (l - l_0) \quad \text{Hooke's law.} \tag{2.3}$$

$F(l)$ then describes the slope of the graphical representation of the function $W(l)$ at point l. In order to find $W(l)$, we need only to find the antiderivative of $F(l)$. In this case, it is the function whose derivative with respect to l results in $D \cdot (l - l_0)$. Antiderivatives are gone into in more detail in Sect. A.1.3 in the Appendix. Now we see that:

$$W(l) = \tfrac{1}{2} D \cdot (l - l_0)^2 + W_0. \tag{2.4}$$

To start with we have assumed that D is constant and that $F(l)$ is a linear function, meaning that the graph (the *characteristic curve*) of the spring is a straight line. However, this does not need to be the case. In Eq. (2.1) a different derivative $\mathrm{d}W/\mathrm{d}l$ would appear on the right, which can be measured like the one before. Now the spring stiffness D that still corresponds to the slope of graph $F(l)$ is itself dependent upon l. Although it can be mathematically more challenging to find the antiderivative of $F(l)$, the method remains the same.

Let us now imagine two different springs stretched and connected in series (Fig. 2.11). The one on the right can only contract as it causes the one on the left to stretch. The process can take place as long as the spring supplies more energy $-\Delta W'$ when shortened by the small length $-\Delta l'$ than the other spring consumes (ΔW) when expanding by the same length $\Delta l = -\Delta l'$. If the spring supplies less than this, the process reverses. The process will come to a standstill or to an *equilibrium of forces* when the energy supplied by a small shift of one spring is compensated by the energy consumption of the other:

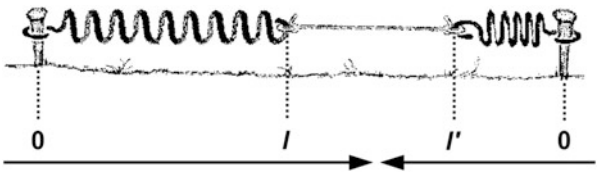

Fig. 2.11 Equilibrium between two stretched springs connected in series.

$$\frac{\Delta W}{\Delta l} = \frac{\Delta W'}{\Delta l'} \text{ or better } \frac{\mathrm{d}W}{\mathrm{d}l} = \frac{\mathrm{d}W'}{\mathrm{d}l'} \text{ this means } F = F'.$$

Springs can therefore be easily used as force meters. In order to find the force acting upon a spring by a rope or a rod, for example, it would be sufficient to use a scale that shows the stretching $l - l_0$. If the characteristic curve of the spring is linear, the scale will be equidistant and can, for the sake of simplicity, be labeled with the unit Newton (N). We are only mentioning this well-known method because other quantities can be measured in a very similar way, among them pressure, temperature, and even the chemical potential.

2.5 Pressure

The same paradigm used for force F can be used to introduce pressure p. Pressing water into a pressure vessel costs energy (Fig. 2.12). The container resists a change ΔV of the volume V of the water in it which manifests itself in some kind of counter pressure. The expenditure of energy ΔW related to the same increase of volume ΔV can be considered the measure for pressure p:

$$\frac{\Delta W}{\Delta V} = p \text{ or more exactly } \frac{\mathrm{d}W}{\mathrm{d}V} = p. \tag{2.5}$$

As we know, when two such vessels with pressures p and p' and water volumes V and V' are connected by a hose, the pressures will equalize when the water flows from the vessel with higher pressure into the one at lower pressure. Energy considerations can be used to explain this. Energy is transported along with the water flowing in and out. This will take place in the mentioned direction as long as more energy is released on the side with water flowing out at higher pressure than is consumed on the other side; otherwise it will reverse. It will only come to a standstill when supply and consumption reach equilibrium, $\mathrm{d}W + \mathrm{d}W' = 0$. If we divide $\mathrm{d}W = -\mathrm{d}W'$ by $\mathrm{d}V = -\mathrm{d}V'$, this leads to

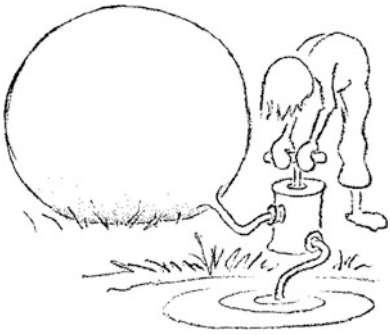

Fig. 2.12 Pumping water into a vessel, in this case a rubber bladder, against the pressure within it. Containers of this kind, mostly of steel with a rubber membrane inside, serve as equalizing vessels in heating systems.

$$\frac{\mathrm{d}W}{\mathrm{d}V} = \frac{\mathrm{d}W'}{\mathrm{d}V'}, \quad \text{i.e.} \quad p = p',$$

which means that we have *pressure equilibrium*. The rather simple case discussed here belongs to the subject of hydraulics where water is considered *incompressible*. Volume takes the role of a substance-like quantity, as a substitute for the amount of water being discussed.

We encounter the quantity called pressure also in another more complex relationship where *compressibility* is concerned. It costs energy to compress an elastic body. The more an object is pressed from all sides, the more strongly volume V decreases. The effort $\mathrm{d}W$ needed to cause a small change of volume $-\mathrm{d}V$ increases according to how compressed the body is to begin with. More precisely: the quotient $\mathrm{d}W/(-\mathrm{d}V)$ increases with a decrease of V, at first linearly (proportionally to $V - V_0$) and then more and more steeply. We might say that the body increasingly resists compression, which is expressed by the growing counter pressure p felt when compressing the body. Similar to the case of hydraulics, the quotient $p = \mathrm{d}W/(-\mathrm{d}V)$ lends itself well as a measure of this kind of pressure:

$$p = -\frac{\mathrm{d}W}{\mathrm{d}V}. \tag{2.6}$$

The expended energy $\mathrm{d}W$ can be retrieved in the course of expansion. One might consider the energy to be contained in the body and that it can be recalled from there when needed. However, the change of energy is not necessarily a measure of the part needed for changing the volume—the only part we are interested in at this point. In order to get this part, it would be necessary to block all the other pathways the energy might use to flow in or out. If an energy exchange similar to that resulting from changes of V is possible for other quantities q, r, ..., then $W(V, q, r, \ldots)$, and the latter must be kept constant:

$$p = -\left(\frac{\partial W}{\partial V}\right)_{q,r\ldots}. \tag{2.7}$$

We were introduced to this method with the indirect metricization of weight in Sect. 1.3. However, we are still missing a crucial quantity. This missing quantity, which we call entropy, will be the subject of the next chapter.

Although we have chosen a totally different way to find the pressure p, it is nevertheless identical to the quantity usually introduced as the force per area, $p = F/A$. The SI unit of the pressure is $\mathrm{J\,m^{-3}} = \mathrm{N\,m^{-2}} = \mathrm{Pa}$ (Joule/Meter^3 = Newton/Meter^2 = Pascal). A non-SI unit still widely used is the bar, where 1 bar = 10^5 Pa. The isotropic pressure p is a quantity that only describes one of the possible stress states of bodies. It describes a very simple but especially important one, which is just about the only one we will be dealing with.

2.6 Energy of a Body in Motion

Energy is needed to accelerate a body, for instance a car or a projectile, and the faster the body is already moving, the more energy (relative to Δv) will be necessary to accelerate it. The effort ΔW is proportional to the velocity v (= distance covered Δx/time needed Δt):

$$\frac{\Delta W}{\Delta v} = m \cdot v \quad \text{or better} \quad \frac{\mathrm{d}W}{\mathrm{d}v} = m \cdot v. \tag{2.8}$$

The proportionality factor m quantifies a characteristic called *inertia*, *inertial mass*, or simply the *mass* of the body. We can consider m to be invariable in all the cases that will interest us. It is easy to see how W depends upon v. We only have to find the antiderivative for the derivative $\mathrm{d}W/\mathrm{d}v$ in Eq. (2.8):

$$W(v) = \tfrac{1}{2} m v^2 + W_0. \tag{2.9}$$

W_0 is the energy contained by a body at rest, i.e., at $v = 0$. The energy in a moving body $W(v) - W_0$ is called *kinetic energy*. If the body moves uniformly (when v is constant), $W(v)$ also remains constant. When this is not the case, then v and therefore indirectly also W is dependent upon position x: $W(v(x))$. By applying the chain rule (compare Sect. A.1.2 in the Appendix) we easily find the force F with which a body resists change of position:

$$F = \frac{\mathrm{d}W(v(x))}{\mathrm{d}x} = \frac{\mathrm{d}W(v)}{\mathrm{d}v} \cdot \frac{\mathrm{d}v(x)}{\mathrm{d}x} = mv \cdot \frac{a}{v} \quad \text{or} \quad F = m \cdot a, \tag{2.10}$$

where $a = \mathrm{d}v/\mathrm{d}t$ represents acceleration. One can prove that $\mathrm{d}v(x)/\mathrm{d}x = a/v$ by taking the derivative of $v(x(t))$ with respect to t and then solving the equation obtained for $\mathrm{d}v(x)/\mathrm{d}x$:

$$a = \frac{\mathrm{d}v(x(t))}{\mathrm{d}t} = \frac{\mathrm{d}v(x)}{\mathrm{d}x} \cdot \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{\mathrm{d}v(x)}{\mathrm{d}x} \cdot v.$$

There is a shorter way to the same result by using the rules for differentials [expanding and inverting a derivative, Sect. 9.4 (transformation of differential quotients)]:

$$F = \frac{\mathrm{d}W}{\mathrm{d}x} = \frac{\mathrm{d}W}{\mathrm{d}v} \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\mathrm{d}W}{\mathrm{d}v} \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \Big/ \frac{\mathrm{d}x}{\mathrm{d}t} = mv \cdot a/v = m \cdot a.$$

The equation $F = m \cdot a$ is usually used to define the force F, which is then used to introduce the concept of work and, as a generalization, energy (compare Sect. 2.1).

2.7 Momentum

Equation (2.8) can be read another way when *momentum* $\mathfrak{p}$ is introduced in place of velocity. Instead of the usual symbol p that is already being used for pressure, we will use the symbol $\mathfrak{p}$ (Thorn) which is similar to p and comes from the Icelandic language. Momentum plays a decisive role in modern physics, in quantum mechanics and in relativity, for example. Therefore, this would be a good moment to familiarize ourselves with this quantity. This concept is essential to describing interactions between moving bodies such as collisions in kinetic gas theory (Chap. 10) or in the kinetics of elementary chemical reactions.

Momentum is a substance-like quantity. The total momentum of an assembly of moving bodies or of parts of a body is the sum of the momenta of all the parts. It can be transferred from one moving body to another. The total momentum is conserved in such processes. If the momentum of a part has decreased, the momentum of another part must have increased irrespective of how the transfer took place. Knowing this can save us a lot of detailed work. In everyday life, one has to learn to recognize conservation of momentum. If we push a car and it is gaining momentum, or if it is losing momentum when coasting, it is not quite clear where the momentum comes from or goes to (Fig. 2.13). It turns out that the momentum comes from or goes into the Earth. Our planet is so big that we do not notice if it loses or gains a little momentum—just as we do not notice a change in the ocean if we take a bucket of water from it.

Momentum is a vector quantity, which makes it somewhat difficult to deal with. However, it is not more complicated than other vector quantities such as velocity, acceleration, force, etc. In fact, it is less difficult because of its substance-like character. Firstly, it is enough to simply observe motion in one direction, for

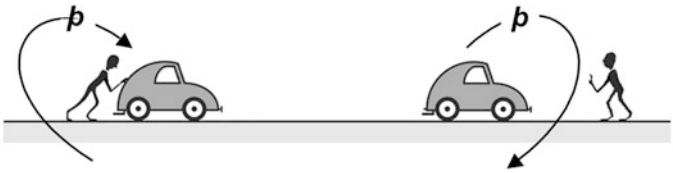

Fig. 2.13 Momentum p when a car is pushed, and when coasting.

example, along the x-axis. The momentum of a body moving in the direction of increasing x values is counted as positive. Motion in the opposite direction is considered negative. This way of seeing things must be learned, though, because we usually speak in absolute values of these quantities. Who would ever say that a vehicle moving down a street is doing so at negative speed?

The momentum contained in a moving body increases with its mass m and its velocity v. In fact, it is proportional to both, $p \sim mv$. The proportionality factor is set equal to 1:

$$p = mv \quad \text{SI unit: kg ms}^{-1} = \text{Ns}. \tag{2.11}$$

If $v = p/m$ is inserted in $W(v)$ and the derivative of the function $W(v(p))$ is taken with respect to p, one obtains

$$\frac{\mathrm{d}W(v(p))}{\mathrm{d}p} = \frac{\mathrm{d}W(v)}{\mathrm{d}v} \cdot \frac{\mathrm{d}v(p)}{\mathrm{d}p} = mv \cdot \frac{1}{m} \text{ or } \frac{\mathrm{d}W}{\mathrm{d}p} = v. \tag{2.12}$$

This equation agrees with Eq. (2.8), except that the factor m in the denominator is moved from the right side to the left and combined with $\mathrm{d}v$ to make $\mathrm{d}p$. It can also be explained using a similar pattern to the one we used for discussing the force of a stretched spring or pressure in hydraulics: The faster a body is already moving, the more it opposes increasing its momentum. The energy $\mathrm{d}W$ needed for this, relative to the same amount $\mathrm{d}p$, grows proportionally to the velocity v. In this case, v appears in a role similar to that of force or pressure.

For more than a hundred years, quantities that appear in this role have been called *intensive factors*, *intensive quantities*, or simply *intensive*. Unfortunately, this description does not agree completely with the definition in Sect. 1.6. In order to avoid misunderstandings, we have no choice but to look for a new name. The German physicist and physician Hermann von Helmholtz came up with one that would be helpful to us. Using Joseph Louis De Lagrange's concept of "forces" in the field of mechanics, he generalized it. We refer to this and call the quantities "force-like."

Each one of these quantities has a counterpart. These are called *extensive factors*, *extensive quantities*, or simply, *extensive* and appear in the form of differentials. x belongs to F, V or $-V$ belongs to p (as the case may be), and p belongs to v, just to name the ones we have already discussed. Each pair describes a path over which energy can be exchanged:

$$\begin{array}{llll} \mathrm{d}W = F\mathrm{d}x, & \mathrm{d}W = p\mathrm{d}V, & \mathrm{d}W = v\mathrm{d}p, & \text{etc.} \\ \text{Spring} & \text{Hydraulics} & \text{Motion} & \end{array}$$

This term does not agree with the earlier definition either, so a new name must be found here as well. Using the concept of position coordinates to give position and orientation of one or more bodies in a space, Helmholtz expanded this concept analogously to quantities outside of mechanics (electric, chemical, etc.). These are the quantities called "extensive factors" above. For a rough characterization of the role of these quantities, the term "*position-like*" would fit nicely to "*force-like*," as a counterpart.

Let us return for a moment to the equation $F = m \cdot a$. If we take the derivative of the expression for momentum $p = mv$ with respect to time t, and because of $\mathrm{d}v/\mathrm{d}t = a$, we obtain a relation that the famous English physician and mathematician Sir Isaac Newton already used to start his description of classical mechanics in the seventeenth century:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = ma = F.$$

Because p is a conserved quantity, its amount can only increase in the body when it decreases somewhere else. In other words, this quantity must flow in from there. Therefore, F describes the momentum flowing in from the surroundings. This idea may be a bit unusual, but it can be very useful.

2.8 Energy of a Raised Body

We will take another look at the body being hoisted upward by a rope and pulley (Fig. 1.5). In the following, we will ignore buoyancy by imagining the surroundings to be void of air. When we release the body, it falls, as we know, at constant acceleration $a = -g$ (g *gravitational acceleration*), independent of its size, weight, or composition. After a time t, it will have reached a velocity of $v = at = -gt$ and will have fallen the distance $h_0 - h = \frac{1}{2}gt^2$, where h is the height above ground at time t and h_0 is the initial height. The energy necessary for the body with a mass m to accelerate from $0 \rightarrow v$,

$$W = \tfrac{1}{2}\, mv^2 = \tfrac{1}{2}m(-gt)^2 = mg(h_0 - h), \tag{2.13}$$

comes from the gravitational field of the Earth. However, the energy W, which is released by falling and is used here to accelerate the body, is usually attributed to the raised body itself. The energy stored in a raised body (or in the gravitational field, respectively) is called *potential energy*, W_{pot}; the one in the moving body is called *kinetic energy*, W_{kin}, as mentioned. During free fall, energy is transferred from one storage to the other. According to the law of conservation of energy, the

sum of both of these contributions remains constant as long as no energy is diverted (such as during impact or falling through air):

$$W_{\text{kin}} + W_{\text{pot}} = \tfrac{1}{2}mv^2 + mgh = \text{const.} \tag{2.14}$$

The term *potential* energy is often transferred to similar cases. For example, the potential energy W_{pot} of a charged body increases by ΔW when it is moved in a static electric field against the field forces by expending the energy ΔW. The stored energy in a stretched spring at rest (Sect. 2.4) is called potential to distinguish it, if necessary, from the contributions from movements of the spring or other parts.