4 Photometric Concepts and Magnitudes

Most astronomical observations utilise electromagnetic radiation in one way or another. We can obtain information on the physical nature of a radiation source by studying the energy distribution of its radiation. We shall now introduce some basic concepts that characterise electromagnetic radiation.

4.1 Intensity, Flux Density and Luminosity

Let us assume we have some radiation passing through a surface element dA (Fig. 4.1). Some of the radiation will leave dA within a solid angle dω; the angle between dω and the normal to the surface is denoted by θ. The amount of energy with frequency in the range $[\nu, \nu + \mathrm{d}\nu]$ entering this solid angle in time dt is

$$\mathrm{d}E_\nu = I_\nu \cos\theta \,\mathrm{d}A \,\mathrm{d}\nu \,\mathrm{d}\omega \,\mathrm{d}t. \qquad (4.1)$$

Here, the coefficient I_ν is the *specific intensity* of the radiation at the frequency ν in the direction of the solid angle dω. Its dimension is $\mathrm{W\,m^{-2}\,Hz^{-1}\,sterad^{-1}}$.

The projection of the surface element dA as seen from the direction θ is $\mathrm{d}A_n = \mathrm{d}A\cos\theta$, which explains the factor $\cos\theta$. If the intensity does not depend on direction, the energy dE_ν is directly proportional to the surface element perpendicular to the direction of the radiation.

The intensity including all possible frequencies is called the *total intensity* I, and is obtained by integrating I_ν over all frequencies:

$$I = \int_0^\infty I_\nu \,\mathrm{d}\nu.$$

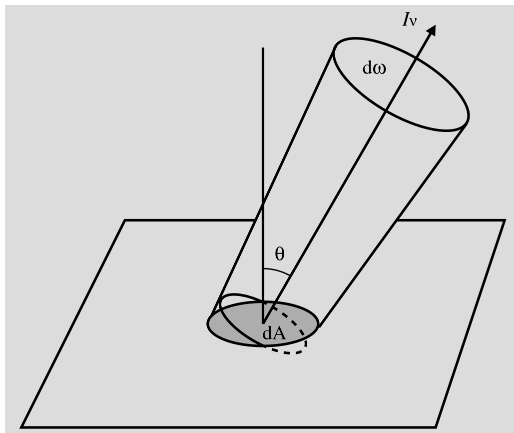

Fig. 4.1 The intensity I_ν of radiation is related to the energy passing through a surface element dA into a solid angle dω, in a direction θ

More important quantities from the observational point of view are the *energy flux* (L_ν, L) or, briefly, the *flux* and the *flux density* (F_ν, F). The flux density gives the power of radiation per unit area; hence its dimension is $\mathrm{W\,m^{-2}\,Hz^{-1}}$ or $\mathrm{W\,m^{-2}}$, depending on whether we are talking about the flux density at a certain frequency or about the total flux density.

Observed flux densities are usually rather small, and $\mathrm{W\,m^{-2}}$ would be an inconveniently large unit. Therefore, especially in radio astron-

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omy, flux densities are often expressed in *Janskys*; one Jansky (Jy) equals $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$.

When we are observing a radiation source, we in fact measure the energy collected by the detector during some period of time, which equals the flux density integrated over the radiation-collecting area of the instrument and the time interval.

The flux density F_ν at a frequency ν can be expressed in terms of the intensity as

$$F_\nu = \frac{1}{\mathrm{d}A\,\mathrm{d}\nu\,\mathrm{d}t}\int_S \mathrm{d}E_\nu = \int_S I_\nu \cos\theta\,\mathrm{d}\omega, \tag{4.2}$$

where the integration is extended over all possible directions. Analogously, the total flux density is

$$F = \int_S I\cos\theta\,\mathrm{d}\omega.$$

For example, if the radiation is *isotropic*, i.e. if I is independent of the direction, we get

$$F = \int_S I\cos\theta\,\mathrm{d}\omega = I\int_S \cos\theta\,\mathrm{d}\omega. \tag{4.3}$$

The solid angle element $\mathrm{d}\omega$ is equal to a surface element on a unit sphere. In spherical coordinates it is (Fig. 4.2; also cf. Appendix A.5):

$$\mathrm{d}\omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi.$$

Substitution into (4.3) gives

$$F = I\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = 0,$$

so there is no net flux of radiation. This means that there are equal amounts of radiation entering and leaving the surface. If we want to know the amount of radiation passing through the surface, we can find, for example, the radiation leaving the surface. For isotropic radiation this is

$$F_\mathrm{l} = I\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi = \pi I. \tag{4.4}$$

In the astronomical literature, terms such as intensity and brightness are used rather vaguely.

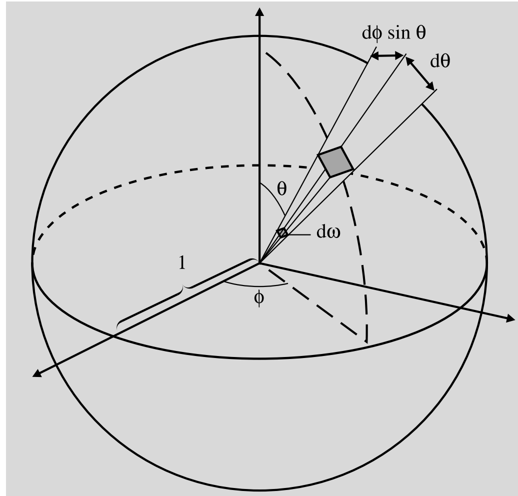


Fig. 4.2 An infinitesimal solid angle $\mathrm{d}\omega$ is equal to the corresponding surface element on a unit sphere: $\mathrm{d}\omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$

Flux density is hardly ever called flux density but intensity or (with luck) flux. Therefore the reader should always carefully check the meaning of these terms.

Flux means the power going through some surface, expressed in watts. The flux emitted by a star into a solid angle ω is $L = \omega r^2 F$, where F is the flux density observed at a distance r. *Total flux* is the flux passing through a closed surface encompassing the source. Astronomers usually call the total flux of a star the *luminosity* L. We can also talk about the luminosity L_ν at a frequency ν ($[L_\nu] = \mathrm{W\,Hz^{-1}}$). (This must not be confused with the luminous flux used in physics; the latter takes into account the sensitivity of the eye.)

If the source (like a typical star) radiates isotropically, its radiation at a distance r is distributed evenly on a spherical surface whose area is $4\pi r^2$ (Fig. 4.3). If the flux density of the radiation passing through this surface is F, the total flux is

$$L = 4\pi r^2 F. \tag{4.5}$$

If we are outside the source, where radiation is not created or destroyed, the luminosity does not depend on distance. The flux density, on the other hand, falls off proportional to $1/r^2$.

For extended objects (as opposed to objects such as stars visible only as points) we can define the *surface brightness* as the flux density per unit solid angle (Fig. 4.4). Now the observer is at the apex of the solid angle. The surface brightness is independent of distance, which can be understood in the following way. The flux density arriving from an area A is inversely proportional to the distance squared. But also the solid angle subtended by the area A is proportional to $1/r^2$ ($\omega = A/r^2$). Thus the surface brightness $B = F/\omega$ remains constant.

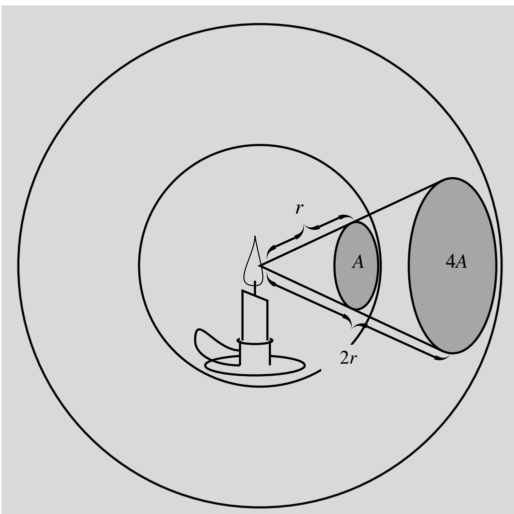


Fig. 4.3 An energy flux which at a distance r from a point source is distributed over an area A is spread over an area $4A$ at a distance $2r$. Thus the flux density decreases inversely proportional to the distance squared

The *energy density* u of radiation is the amount of energy per unit volume ($\mathrm{J\,m^{-3}}$):

$$u = \frac{1}{c}\int_S I\,\mathrm{d}\omega. \tag{4.6}$$

This can be seen as follows. Suppose we have radiation with intensity I arriving from a solid angle $\mathrm{d}\omega$ perpendicular to the surface $\mathrm{d}A$ (Fig. 4.5). In the time $\mathrm{d}t$, the radiation travels a distance $c\,\mathrm{d}t$ and fills a volume $\mathrm{d}V = c\,\mathrm{d}t\,\mathrm{d}A$. Thus the energy in the volume $\mathrm{d}V$ is (now $\cos\theta = 1$)

$$\mathrm{d}E = I\,\mathrm{d}A\,\mathrm{d}\omega\,\mathrm{d}t = \frac{1}{c}I\,\mathrm{d}\omega\,\mathrm{d}V.$$

Hence the energy density $\mathrm{d}u$ of the radiation arriving from the solid angle $\mathrm{d}\omega$ is

$$\mathrm{d}u = \frac{\mathrm{d}E}{\mathrm{d}V} = \frac{1}{c}I\,\mathrm{d}\omega,$$

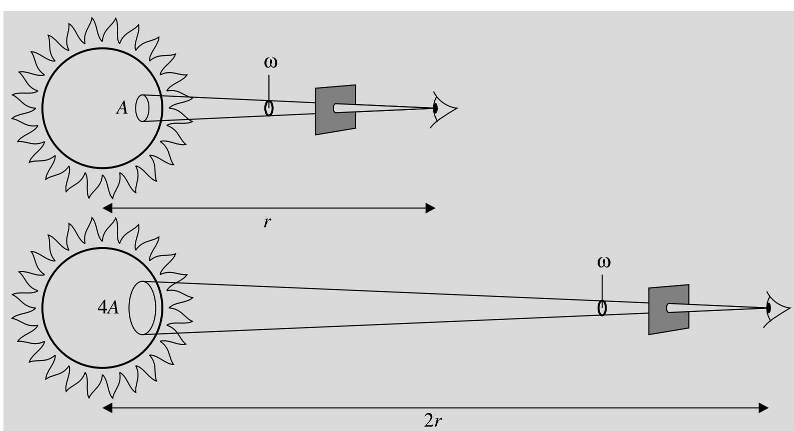
and the total energy density is obtained by integrating this over all directions. For isotropic radiation we get

$$u = \frac{4\pi}{c}I. \tag{4.7}$$

4.2 Apparent Magnitudes

As early as the second century B.C., Hipparchos divided the visible stars into six classes according to their apparent brightness. The first class contained the brightest stars and the sixth the faintest ones still visible to the naked eye.

Fig. 4.4 An observer sees radiation coming from a constant solid angle ω. The area giving off radiation into this solid angle increases when the source moves further away ($A \propto r^2$). Therefore the surface brightness or the observed flux density per unit solid angle remains constant



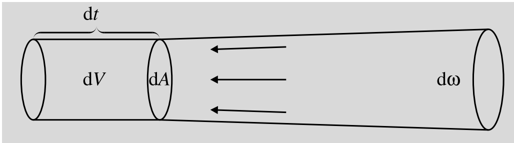


Fig. 4.5 In time dt, the radiation fills a volume $\mathrm{d}V = c\,\mathrm{d}t\,\mathrm{d}A$, where d$A$ is the surface element perpendicular to the propagation direction of the radiation

The response of the human eye to the brightness of light is not linear. If the flux densities of three stars are in the proportion 1:10:100, the brightness difference of the first and second star seems to be equal to the difference of the second and third star. Equal brightness ratios correspond to equal apparent brightness differences: the human perception of brightness is logarithmic.

The rather vague classification of Hipparchos was replaced in 1856 by Norman R. Pogson. The new, more accurate classification followed the old one as closely as possible, resulting in another of those illogical definitions typical of astronomy. Since a star of the first class is about one hundred times brighter than a star of the sixth class, Pogson defined the ratio of the brightnesses of classes n and $n+1$ as $\sqrt[5]{100} = 2.512$.

The brightness class or *magnitude* can be defined accurately in terms of the observed flux density F ($[F] = \mathrm{W\,m^{-2}}$). We decide that the magnitude 0 corresponds to some preselected flux density F_0. All other magnitudes are then defined by the equation

$$m = -2.5 \lg \frac{F}{F_0}. \tag{4.8}$$

Note that the coefficient is exactly 2.5, not 2.512! Magnitudes are dimensionless quantities, but to remind us that a certain value is a magnitude, we can write it, for example, as 5 mag or 5^{m}.

It is easy to see that (4.8) is equivalent to Pogson's definition. If the magnitudes of two stars are m and $m+1$ and their flux densities F_m and F_{m+1}, respectively, we have

$$\begin{aligned} m - (m+1) &= -2.5 \lg \frac{F_m}{F_0} + 2.5 \lg \frac{F_{m+1}}{F_0} \\ &= -2.5 \lg \frac{F_m}{F_{m+1}}, \end{aligned}$$

whence

$$\frac{F_m}{F_{m+1}} = \sqrt[5]{100}.$$

In the same way we can show that the magnitudes m_1 and m_2 of two stars and the corresponding flux densities F_1 and F_2 are related by

$$m_1 - m_2 = -2.5 \lg \frac{F_1}{F_2}. \tag{4.9}$$

Magnitudes extend both ways from the original six values. The magnitude of the brightest star, Sirius, is in fact negative -1.5. The magnitude of the Sun is -26.8 and that of a full moon -12.5. The magnitude of the faintest objects observed depends on the size of the telescope, the sensitivity of the detector and the exposure time. The limit keeps being pushed towards fainter objects; currently the magnitudes of the faintest observed objects are over 30.

4.3 Magnitude Systems

The *apparent magnitude* m, which we have just defined, depends on the instrument we use to measure it. The sensitivity of the detector is different at different wavelengths. Also, different instruments detect different wavelength ranges. Thus the flux measured by the instrument equals not the total flux, but only a fraction of it. Depending on the method of observation, we can define various magnitude systems. Different magnitudes have different zero points, i.e. they have different flux densities F_0 corresponding to the magnitude 0. The zero points are usually defined by a few selected standard stars.

In daylight the human eye is most sensitive to radiation with a wavelength of about 550 nm, the sensitivity decreasing towards red (longer wavelengths) and violet (shorter wavelengths). The magnitude corresponding to the sensitivity of the eye is called the *visual magnitude* m_v.

Photographic plates are usually most sensitive at blue and violet wavelengths, but they are also able to register radiation not visible to the human eye. Thus the *photographic magnitude* m_pg usually differs from the visual magnitude. The sensitivity of the eye can be simulated by using a yellow filter and plates sensitised to yellow and green light. Magnitudes thus observed are called *photovisual magnitudes* m_{pv}.

If, in ideal case, we were able to measure the radiation at all wavelengths, we would get the *bolometric magnitude* m_{bol}. In practice this is very difficult, since part of the radiation is absorbed by the atmosphere; also, different wavelengths require different detectors. (In fact there is a gadget called the bolometer, which, however, is not a real bolometer but an infrared detector.) The bolometric magnitude can be derived from the visual magnitude if we know the *bolometric correction* BC:

$$m_{bol} = m_V - BC. \tag{4.10}$$

By definition, the bolometric correction is zero for radiation of solar type stars (or, more precisely, stars of the spectral class F5). Although the visual and bolometric magnitudes can be equal, the flux density corresponding to the bolometric magnitude must always be higher. The reason of this apparent contradiction is in the different values of F_0.

The more the radiation distribution differs from that of the Sun, the higher the bolometric correction is. The correction is positive for stars both cooler or hotter than the Sun. Sometimes the correction is defined as $m_{bol} = m_V + BC$ in which case $BC \leq 0$ always. The chance for errors is, however, very small, since we must have $m_{bol} \leq m_V$.

The most accurate magnitude measurements are made using photoelectric photometers or CCD cameras. Usually filters are used to allow only a certain wavelength band to enter the detector. One of the multicolour magnitude systems used widely in photoelectric photometry is the UBV system developed in the early 1950's by *Harold L. Johnson* and *William W. Morgan*. Magnitudes are measured through three filters, U = ultraviolet, B = blue and V = visual. Figure 4.6 and Table 4.1 give the wavelength bands of these filters. The magnitudes observed through these filters are called U, B and V magnitudes, respectively.

The UBV system was later augmented by adding more bands. One commonly used system is the five colour UBVRI system, which includes R = red and I = infrared filters.

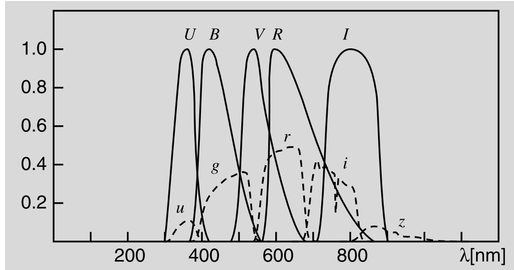


Fig. 4.6 Relative transmission profiles of filters used in the UBVRI magnitude system. The maxima of the bands are normalised to unity. The R and I bands are based on the system of Johnson, Cousins and Glass, which includes also infrared bands J, H, K, L and M. Previously used R and I bands differ considerably from these. The curves of the ugriz magnitudes (*dashed lines*) give quantum efficiencies. They include the atmospheric extinction for airmass 1.3 (Sect. 4.5)

Table 4.1 Wavelength bands of the UBVRI and uvby filters and their effective (≈ average) wavelengths

	Magnitude	Band width [nm]	Effective wavelength [nm]
U	ultraviolet	66	367
B	blue	94	436
V	visual	88	545
R	red	138	638
I	infrared	149	797
u	ultraviolet	30	349
v	violet	19	411
b	blue	18	467
y	yellow	23	547

There are also other broad band systems, but they are not as well standardised as the UBV, which has been defined moderately well using a great number of standard stars all over the sky. The magnitude of an object is obtained by comparing it to the magnitudes of standard stars.

In *Strömgren's* four-colour or *uvby system*, the bands passed by the filters are much narrower than in the UBV system. The uvby system is also well standardised, but it is not quite as common as the UBV. Other narrow band systems exist as well. By adding more filters, more information on the radiation distribution can be obtained.

In any multicolour system, we can define *colour indices*; a colour index is the difference of two magnitudes. By subtracting the B magnitude from U we get the colour index $U - B$, and so on. If the UBV system is used, it is common to give only the V magnitude and the colour indices $U - B$ and $B - V$.

The constants F_0 in (4.8) for U, B and V magnitudes have been selected in such a way that the colour indices $B - V$ and $U - B$ are zero for stars of spectral type A0 (for spectral types, see Chap. 8). The surface temperature of such a star is about 10,000 K. For example, Vega (α Lyr, spectral class A0V) has $V = 0.04$, $B - V = U - B = 0.00$. The Sun has $V = -26.8$, $B - V = 0.64$ and $U - B = 0.12$.

Before the UBV system was developed, a colour index C.I., defined as C.I. $= m_{\mathrm{pg}} - m_{\mathrm{v}}$ was used. The definition shows that C.I. corresponds to the colour index $B - V$. In fact, C.I. $= (B - V) - 0.11$.

Nowadays it is becoming more customary to use the AB system (ABsolute), in which F_0 is the same, 3631 Jy, for all wavelength bands. For instance the *ugriz* magnitudes used by the Sloan Digital Sky Survey (SDSS) are based on this system. There are several different transformation equations between the UBV and ugriz systems for different kinds of objects. For ordinary stars, we can use the following:

$$\begin{aligned} V &= g - 0.2906\,(u - g) + 0.0885, \\ V &= g - 0.5784\,(g - r) - 0.0038, \\ R &= r - 0.1837\,(g - r) - 0.0971, \\ R &= r - 0.2936\,(r - i) - 0.1439, \\ I &= r - 1.2444\,(r - i) - 0.3820, \\ I &= i - 0.3780\,(i - z) - 0.3974. \end{aligned} \tag{4.11}$$

4.4 Absolute Magnitudes

Thus far we have discussed only apparent magnitudes. They do not tell us anything about the true brightness of stars, since the distances differ. A quantity measuring the intrinsic brightness of a star is the *absolute magnitude*. It is defined as the apparent magnitude at a distance of 10 parsecs from the star (Fig. 4.7). Officially this definition was accepted in the general meeting of the IAU in 1922.

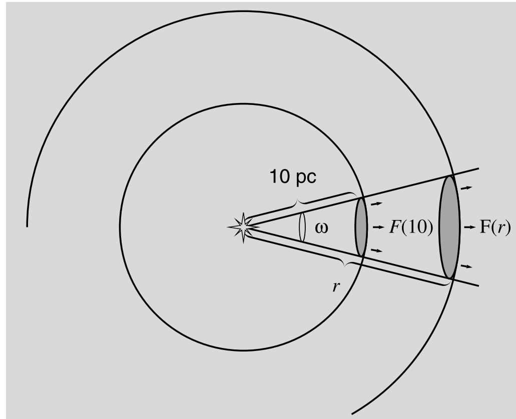


Fig. 4.7 The apparent magnitude at a distance r depends on the flux density $F(r)$. The absolute magnitude is defined as the apparent magnitude at a distance of 10 parsecs from the star depending only on the flux density $F(10)$ 10 parsecs away from the star

We shall now derive an equation which relates the apparent magnitude m, the absolute magnitude M and the distance r. Because the flux emanating from a star into a solid angle ω has, at a distance r, spread over an area ωr^2, the flux density is inversely proportional to the distance squared. Therefore the ratio of the flux density at a distance r, $F(r)$, to the flux density at a distance of 10 parsecs, $F(10)$, is

$$\frac{F(r)}{F(10)} = \left(\frac{10\ \mathrm{pc}}{r}\right)^2.$$

Thus the difference of magnitudes at r and 10 pc, or the *distance modulus* $m - M$, is

$$m - M = -2.5 \lg \frac{F(r)}{F(10)} = -2.5 \lg \left(\frac{10\ \mathrm{pc}}{r}\right)^2$$

or

$$m - M = 5 \lg \frac{r}{10\ \mathrm{pc}}. \tag{4.12}$$

For historical reasons, this equation is almost always written as

$$m - M = 5 \lg r - 5, \tag{4.13}$$

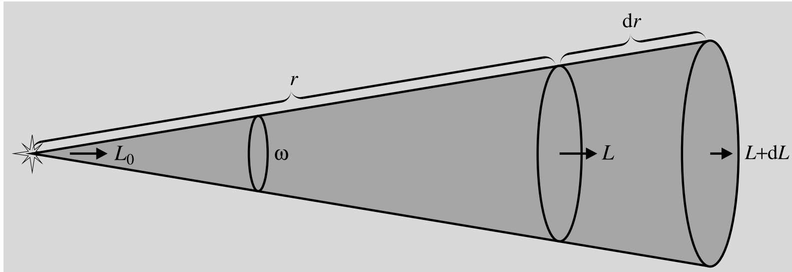


Fig. 4.8 The interstellar medium absorbs and scatters radiation; this usually reduces the energy flux L in the solid angle ω ($\mathrm{d}L \le 0$)

which is valid *only* if the distance is expressed in parsecs. (The logarithm of a dimensional quantity is, in fact, physically absurd.) Sometimes the distance is given in kiloparsecs or megaparsecs, which require different constant terms in (4.13). To avoid confusion, we highly recommend the form (4.12).

Absolute magnitudes are usually denoted by capital letters. Note, however, that the U, B and V magnitudes are apparent magnitudes. The corresponding absolute magnitudes are M_U, M_B and M_V.

The absolute bolometric magnitude can be expressed in terms of the luminosity. Let the total flux density at a distance $r = 10$ pc be F and let $F_\odot$ be the equivalent quantity for the Sun. Since the luminosity is $L = 4\pi r^2 F$, we get

$$M_\mathrm{bol} - M_{\mathrm{bol},\odot} = -2.5 \lg \frac{F}{F_\odot} = -2.5 \lg \frac{L/4\pi r^2}{L_\odot/4\pi r^2},$$

or

$$M_\mathrm{bol} - M_{\mathrm{bol},\odot} = -2.5 \lg \frac{L}{L_\odot}. \tag{4.14}$$

The absolute bolometric magnitude $M_\mathrm{bol} = 0$ corresponds to a luminosity $L_0 = 3.0 \times 10^{28}$ W.

4.5 Extinction and Optical Thickness

Equation (4.12) shows how the apparent magnitude increases (and brightness decreases!) with increasing distance. If the space between the radiation source and the observer is not completely empty, but contains some interstellar medium, (4.12) no longer holds, because part of the radiation is absorbed by the medium (and usually re-emitted at a different wavelength, which may be outside the band defining the magnitude), or scattered away from the line of sight. All these radiation losses are called the *extinction*.

Now we want to find out how the extinction depends on the distance. Assume we have a star radiating a flux L_0 into a solid angle ω in some wavelength range. Since the medium absorbs and scatters radiation, the flux L will now decrease with increasing distance r (Fig. 4.8). In a short distance interval $[r, r + \mathrm{d}r]$, the extinction $\mathrm{d}L$ is proportional to the flux L and the distance travelled in the medium:

$$\mathrm{d}L = -\alpha L \,\mathrm{d}r. \tag{4.15}$$

The factor α tells how effectively the medium can obscure radiation. It is called the *opacity*. From (4.15) we see that its dimension is $[\alpha] = \mathrm{m}^{-1}$. The opacity is zero for a perfect vacuum and approaches infinity when the substance becomes really murky. We can now define a dimensionless quantity, the *optical thickness* τ by

$$\mathrm{d}\tau = \alpha \,\mathrm{d}r. \tag{4.16}$$

Substituting this into (4.15) we get

$$\mathrm{d}L = -L \,\mathrm{d}\tau.$$

Next we integrate this from the source (where $L = L_0$ and $r = 0$) to the observer:

$$\int_{L_0}^{L} \frac{\mathrm{d}L}{L} = -\int_0^\tau \mathrm{d}\tau,$$

which gives

$$L = L_0 \mathrm{e}^{-\tau}. \tag{4.17}$$

Here, τ is the optical thickness of the material between the source and the observer and L, the observed flux. Now, the flux L falls off exponentially with increasing optical thickness. Empty space is perfectly transparent, i.e. its opacity is $\alpha = 0$; thus the optical thickness does not increase in empty space, and the flux remains constant.

Let F_0 be the flux density on the surface of a star and $F(r)$, the flux density at a distance r. We can express the fluxes as

$$L = \omega r^2 F(r), \qquad L_0 = \omega R^2 F_0,$$

where R is the radius of the star. Substitution into (4.16) gives

$$F(r) = F_0 \frac{R^2}{r^2} \mathrm{e}^{-\tau}.$$

For the absolute magnitude we need the flux density at a distance of 10 parsecs, $F(10)$, which is still evaluated without extinction:

$$F(10) = F_0 \frac{R^2}{(10\ \mathrm{pc})^2}.$$

The distance modulus $m - M$ is now

$$\begin{aligned} m - M &= -2.5 \lg \frac{F(r)}{F(10)} \\ &= 5 \lg \frac{r}{10\ \mathrm{pc}} - 2.5 \lg \mathrm{e}^{-\tau} \\ &= 5 \lg \frac{r}{10\ \mathrm{pc}} + (2.5 \lg \mathrm{e})\tau \end{aligned}$$

or

$$m - M = 5 \lg \frac{r}{10\ \mathrm{pc}} + A, \tag{4.18}$$

where $A \geq 0$ is the extinction in magnitudes due to the entire medium between the star and the observer. If the opacity is constant along the line of sight, we have

$$\tau = \alpha \int_0^r \mathrm{d}r = \alpha r,$$

and (4.18) becomes

$$m - M = 5 \lg \frac{r}{10\ \mathrm{pc}} + ar, \tag{4.19}$$

where the constant $a = 2.5\alpha \lg \mathrm{e}$ gives the extinction in magnitudes per unit distance.

Colour Excess Another effect caused by the interstellar medium is the *reddening of light*: blue light is scattered and absorbed more than red. Therefore the colour index $B - V$ increases. The visual magnitude of a star is, from (4.18),

$$V = M_\mathrm{V} + 5 \lg \frac{r}{10\ \mathrm{pc}} + A_\mathrm{V}, \tag{4.20}$$

where M_V is the absolute visual magnitude and A_V is the extinction in the V passband. Similarly, we get for the blue magnitudes

$$B = M_\mathrm{B} + 5 \lg \frac{r}{10\ \mathrm{pc}} + A_\mathrm{B}.$$

The observed colour index is now

$$B - V = M_\mathrm{B} - M_\mathrm{V} + A_\mathrm{B} - A_\mathrm{V},$$

or

$$B - V = (B - V)_0 + E_{\mathrm{B-V}}, \tag{4.21}$$

where $(B - V)_0 = M_\mathrm{B} - M_\mathrm{V}$ is the *intrinsic colour* of the star and $E_{\mathrm{B-V}} = (B - V) - (B - V)_0$ is the *colour excess*. Studies of the interstellar medium show that the ratio of the visual extinction A_V to the colour excess $E_{\mathrm{B-V}}$ is almost constant for all stars:

$$R = \frac{A_\mathrm{V}}{E_{\mathrm{B-V}}} \approx 3.0.$$

This makes it possible to find the visual extinction if the colour excess is known:

$$A_\mathrm{V} \approx 3.0\, E_{\mathrm{B-V}}. \tag{4.22}$$

When A_V is obtained, the distance can be solved directly from (4.19), when V and M_V are known.

We shall study interstellar extinction in more detail in Sect. 15.1 ("Interstellar Dust").

Atmospheric Extinction As we mentioned in Sect. 3.1, the Earth's atmosphere also causes extinction. The observed magnitude m depends on the location of the observer and the zenith distance of the object, since these factors determine the distance the light has to travel in the

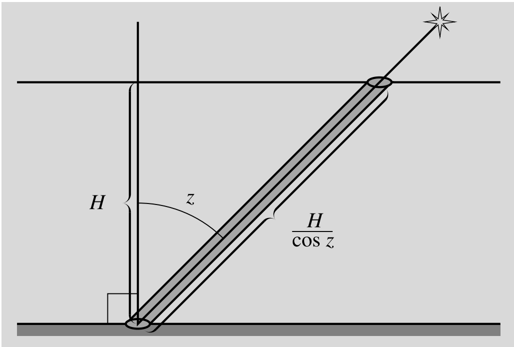


Fig. 4.9 If the zenith distance of a star is z, the light of the star travels a distance $H/\cos z$ in the atmosphere; H is the height of the atmosphere

atmosphere. To compare different observations, we must first *reduce* them, i.e. remove the atmospheric effects somehow. The magnitude m_0 thus obtained can then be compared with other observations.

If the zenith distance z is not too large, we can approximate the atmosphere by a plane layer of constant thickness (Fig. 4.9). If the thickness of the atmosphere is used as a unit, the light must travel a distance

$$X = 1/\cos z = \sec z \tag{4.23}$$

in the atmosphere. The quantity X is the *air mass*. According to (4.18), the magnitude increases linearly with the distance X:

$$m = m_0 + kX, \tag{4.24}$$

where k is the *extinction coefficient*.

The extinction coefficient can be determined by observing the same source several times during a night with as wide a zenith distance range as possible. The observed magnitudes are plotted in a diagram as a function of the air mass X. The points lie on a straight line the slope of which gives the extinction coefficient k. When this line is extrapolated to $X = 0$, we get the magnitude m_0, which is the apparent magnitude outside the atmosphere.

In practice, observations with zenith distances higher than 70° (or altitudes less than 20°) are not used to determine k and m_0, since at low altitudes the curvature of the atmosphere begins to complicate matters. The value of the extinction coefficient k depends on the observation site and time and also on the wavelength, since extinction increases strongly towards short wavelengths.

4.6 Examples

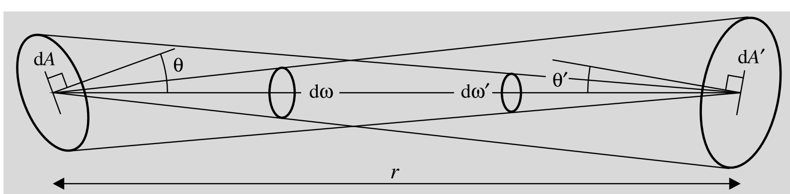
Example 4.1 Show that intensity is independent of distance.

Suppose we have some radiation leaving the surface element dA in the direction θ. The energy entering the solid angle $d\omega$ in time dt is

$$dE = I \cos\theta \, dA \, d\omega \, dt,$$

where I is the intensity. If we have another surface dA' at a distance r receiving this radiation from direction θ', we have

$$d\omega = dA' \cos\theta'/r^2.$$



The definition of the intensity gives

$$dE = I' \cos\theta' \, dA' \, d\omega' \, dt,$$

where I' is the intensity at dA' and

$$d\omega' = dA \cos\theta/r^2.$$

Substitution of $d\omega$ and $d\omega'$ into the expressions of dE gives

$$I\cos\theta\,d\theta\,dA\frac{dA'\cos\theta'}{r^2}\,dt$$
$$= I'\cos\theta'\,dA'\frac{dA\cos\theta}{r^2}\,dt \quad\Rightarrow\quad I' = I.$$

Thus the intensity remains constant in empty space.

Example 4.2 (Surface Brightness of the Sun) Assume that the Sun radiates isotropically. Let R be the radius of the Sun, $F_\odot$ the flux density on the surface of the Sun and F the flux density at a distance r. Since the luminosity is constant,

$$L = 4\pi R^2 F_\odot = 4\pi r^2 F,$$

the flux density equals

$$F = F_\odot \frac{R^2}{r^2}.$$

At a distance $r \gg R$, the Sun subtends a solid angle

$$\omega = \frac{A}{r^2} = \frac{\pi R^2}{r^2},$$

where $A = \pi R^2$ is the cross section of the Sun. The surface brightness B is

$$B = \frac{F}{\omega} = \frac{F_\odot}{\pi}.$$

Applying (4.4) we get

$$B = I_\odot.$$

Thus the surface brightness is independent of distance and equals the intensity. We have found a simple interpretation for the somewhat abstract concept of intensity.

The flux density of the Sun on the Earth, the *solar constant*, is $S_\odot \approx 1370\ \mathrm{W\,m^{-2}}$. The angular diameter of the Sun is $\alpha = 32'$, whence

$$\frac{R}{r} = \frac{\alpha}{2} = \frac{1}{2} \times \frac{32}{60} \times \frac{\pi}{180} = 0.00465\ \text{rad}.$$

The solid angle subtended by the Sun is

$$\omega = \pi\left(\frac{R}{r}\right)^2 = \pi \times 0.00465^2$$
$$= 6.81 \times 10^{-5}\ \text{sterad},$$

and the surface brightness

$$B = \frac{S_\odot}{\omega} = 2.01 \times 10^7\ \mathrm{W\,m^{-2}\,sterad^{-1}}.$$

Example 4.3 (Magnitude of a Binary Star) Since magnitudes are logarithmic quantities, they can be a little awkward for some purposes. For example, we cannot add magnitudes like flux densities. If the magnitudes of the components of a binary star are 1 and 2, the total magnitude is certainly not 3. To find the total magnitude, we must first solve the flux densities from

$$1 = -2.5\lg\frac{F_1}{F_0}, \qquad 2 = -2.5\lg\frac{F_2}{F_0},$$

which give

$$F_1 = F_0 \times 10^{-0.4}, \qquad F_2 = F_0 \times 10^{-0.8}.$$

Thus the total flux density is

$$F = F_1 + F_2 = F_0\left(10^{-0.4} + 10^{-0.8}\right)$$

and the total magnitude,

$$m = -2.5\lg\frac{F_0(10^{-0.4} + 10^{-0.8})}{F_0}$$
$$= -2.5\lg 0.5566 = 0.64.$$

Example 4.4 The distance of a star is $r = 100$ pc and its apparent magnitude $m = 6$. What is its absolute magnitude?

Substitution into (4.12)

$$m - M = 5\lg\frac{r}{10\ \text{pc}}$$

gives

$$M = 6 - 5\lg\frac{100}{10} = 1.$$

Example 4.5 The absolute magnitude of a star is $M = -2$ and the apparent magnitude $m = 8$. What is the distance of the star?

We can solve the distance r from (4.12):

$$r = 10\ \text{pc} \times 10^{(m-M)/5} = 10 \times 10^{10/5}\ \text{pc}$$
$$= 1000\ \text{pc} = 1\ \text{kpc}.$$

Example 4.6 Although the amount of interstellar extinction varies considerably from place to place, we can use an average value of 2 mag/kpc near the galactic plane. Find the distance of the star in Example 4.5, assuming such extinction.

Now the distance must be solved from (4.19):

$$8 - (-2) = 5 \lg \frac{r}{10} + 0.002\, r,$$

where r is in parsecs. This equation cannot be solved analytically, but we can always use a numerical method. We try a simple iteration (Appendix A.7), rewriting the equation as

$$r = 10 \times 10^{2-0.0004\, r}.$$

The value $r = 1000$ pc found previously is a good initial guess:

$$r_0 = 1000,$$
$$r_1 = 10 \times 10^{2-0.0004 \times 1000} = 398,$$
$$r_2 = 693,$$
$$\vdots$$
$$r_{12} = r_{13} = 584.$$

The distance is $r \approx 580$ pc, which is much less than our earlier value 1000 pc. This should be quite obvious, since due to extinction, radiation is now reduced much faster than in empty space.

Example 4.7 What is the optical thickness of a layer of fog, if the Sun seen through the fog seems as bright as a full moon in a cloudless sky?

The apparent magnitudes of the Sun and the Moon are −26.8 and −12.5, respectively. Thus the total extinction in the cloud must be $A = 14.3$. Since

$$A = (2.5 \lg e)\tau,$$

we get

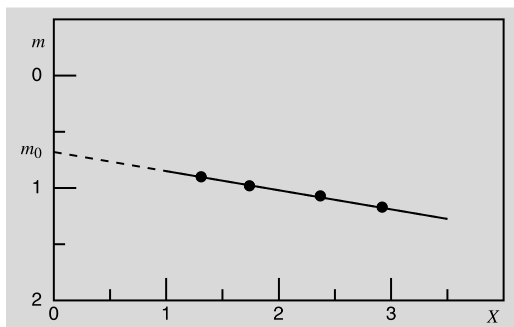
$$\tau = A/(2.5 \lg e) = 14.3/1.086 = 13.2.$$

The optical thickness of the fog is 13.2. In reality, a fraction of the light scatters several times, and a few of the multiply scattered photons leave the cloud along the line of sight, reducing the total extinction. Therefore the optical thickness must be slightly higher than our value.

Example 4.8 (Reduction of Observations) The altitude and magnitude of a star were measured several times during a night. The results are given in the following table.

Altitude	Zenith distance	Air mass	Magnitude
50°	40°	1.31	0.90
35°	55°	1.74	0.98
25°	65°	2.37	1.07
20°	70°	2.92	1.17

By plotting the observations as in the following figure, we can determine the extinction coefficient k and the magnitude m_0 outside the atmosphere. This can be done graphically (as here) or using a least-squares fit.



Extrapolation to the air mass $X = 0$ gives $m_0 = 0.68$. The slope of the line gives $k = 0.17$.

4.7 Exercises

Exercise 4.1 The total magnitude of a triple star is 0.0. Two of its components have magnitudes 1.0 and 2.0. What is the magnitude of the third component?

Exercise 4.2 The absolute magnitude of a star in the Andromeda galaxy (distance 690 kpc) is $M = 5$. It explodes as a supernova, becoming one billion (10^9) times brighter. What is its apparent magnitude?

Exercise 4.3 Assume that all stars have the same absolute magnitude and stars are evenly distributed in space. Let $N(m)$ be the number of stars brighter than m magnitudes. Find the ratio $N(m+1)/N(m)$.

Exercise 4.4 The V magnitude of a star is 15.1, $B - V = 1.6$, and absolute magnitude $M_V = 1.3$. The extinction in the direction of the star in the visual band is $a_V = 1$ mag kpc^{-1}. What is the intrinsic colour of the star?

Exercise 4.5 Stars are observed through a triple window. Each surface reflects away 15 % of the incident light.

(a) What is the magnitude of Regulus ($M_V = 1.36$) seen through the window?
(b) What is the optical thickness of the window?