Stellar Evolution

12

In the preceding chapter we have seen how one can compute the evolution of a star by starting from a homogeneous model representing a newly formed system. When the chemical composition of the star changes with time, a new model is computed each time. In this chapter we shall consider the theoretical evolutionary paths of systems with various masses and see how the computed evolution explains the observational data. The following discussion is rather qualitative, since the details of the theoretical calculations are too involved for the present book. Also, the different evolutionary tracks would become prohibitively complicated if more than the very basic stellar properties were included.

12.1 Evolutionary Time Scales

Changes in a star may take place on quite different time scales at different evolutionary phases. There are three important basic time scales: the nuclear time scale t_{n}, the thermal time scale t_{t} and the dynamical or freefall time scale t_{d}.

The Nuclear Time Scale The time in which a star radiates away all the energy that can be released by nuclear reactions is called the nuclear time scale. An estimate of this time can be obtained if one calculates the time in which all available hydrogen is turned into helium. On the basis of theoretical considerations and evolutionary computations it is known that only just over 10 % of the total mass of hydrogen in the star can be consumed before other, more rapid evolutionary mechanisms set in. Since 0.7 % of the rest mass is turned into energy in hydrogen burning, the nuclear time scale will be

$$t_{\mathrm{n}} \approx \frac{0.007 \times 0.1\, M c^2}{L}. \tag{12.1}$$

For the Sun one obtains the nuclear time scale 10^{10} years, and thus

$$t_{\mathrm{n}} \approx \frac{M/M_{\odot}}{L/L_{\odot}} \times 10^{10}\ \mathrm{a}. \tag{12.2}$$

This gives the nuclear time scale as a function of the mass M and luminosity L of a given star. For example, if the mass is $30\, M_{\odot}$, one obtains t_{n} about 2 million years. The reason for the shorter time scale is that the stellar luminosity strongly increases for higher masses (Table 12.1).

The Thermal Time Scale The time in which a star would radiate away all its thermal energy if the nuclear energy production were suddenly turned off is called the thermal time scale. This is also the time it takes for radiation from the centre to reach the surface. According to the virial theorem (6.51) the kinetic energy of the thermal motion of the gas particles equals half of the potential energy. Thus he thermal time scale is roughly may be estimated as

$$\begin{aligned} t_{\mathrm{t}} &\approx \frac{0.5\, G M^2/R}{L} \\ &\approx \frac{(M/M_{\odot})^2}{(R/R_{\odot})(L/L_{\odot})} \times 2 \times 10^7\ \mathrm{a}, \end{aligned} \tag{12.3}$$

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Table 12.1 Stellar lifetimes (unit 10^6 years)

Mass [$M_\odot$]	Spectral type on the main sequence	Contraction to main sequence	Main sequence	Main sequence to red giant	Red giant
30	O5	0.02	4.9	0.55	0.3
15	B0	0.06	10	1.7	2
9	B2	0.2	22	0.2	5
5	B5	0.6	68	2	20
3	A0	3	240	9	80
1.5	F2	20	2000	280	
1.0	G2	50	10,000	680	
0.5	M0	200	30,000		
0.1	M7	500	10^7		

where G is the constant of gravity and R the stellar radius. For the Sun the thermal time scale is about 20 million years or $1/500$ of the nuclear time scale.

The Dynamical Time Scale The third and shortest time scale is the time it would take a star to collapse if the pressure supporting it against gravity were suddenly removed. It can be estimated from the time it would take for a particle to fall freely from the stellar surface to the centre. This is half of the period given by Kepler's third law, where the semimajor axis of the orbit corresponds to half the stellar radius R:

$$t_\mathrm{d} = \frac{2\pi}{2}\sqrt{\frac{(R/2)^3}{GM}} \approx \sqrt{\frac{R^3}{GM}}. \tag{12.4}$$

The dynamical time scale of the Sun is about half an hour.

The ordering of the time scales is normally like that in the Sun, i.e. $t_\mathrm{d} \ll t_\mathrm{t} \ll t_\mathrm{n}$.

12.2 The Contraction of Stars Towards the Main Sequence

The formation and subsequent gravitational collapse of condensations in the interstellar medium will be considered in a later chapter. Here we shall follow the behaviour of such a *protostar*, when it is already in the process of contraction.

When a cloud contracts, gravitational potential energy is released and transformed into thermal energy of the gas and into radiation. Initially the radiation can propagate freely through the material, because the density is low and the opacity small. Therefore most of the liberated energy is radiated away and the temperature does not increase. The contraction takes place on the dynamical time scale; the gas is falling freely inwards.

The density and the pressure increase most rapidly near the centre of the cloud. As the density increases, so does the opacity. A larger fraction of the released energy is then turned into heat, and the temperature begins to rise. This leads to a further increase in the pressure that is resisting the free fall. The contraction of the central part of the cloud slows down. The outer parts, however, are still falling freely.

At this stage, the cloud may already be considered a protostar. It consists mainly of hydrogen in molecular form. When the temperature reaches 1800 K, the hydrogen molecules are dissociated into atoms. The dissociation consumes energy, and the rise in temperature is slowed down. The pressure then also grows more slowly and this in turn means that the rate of contraction increases. The same sequence of events is repeated, first when hydrogen is ionised at 10^4 K, and then when helium is ionised. When the temperature has reached about 10^5 K, the gas is essentially completely ionised.

The contraction of a protostar only stops when a large fraction of the gas is fully ionised in the form of *plasma*. The star then settles into hydrostatic equilibrium. Its further evolution takes

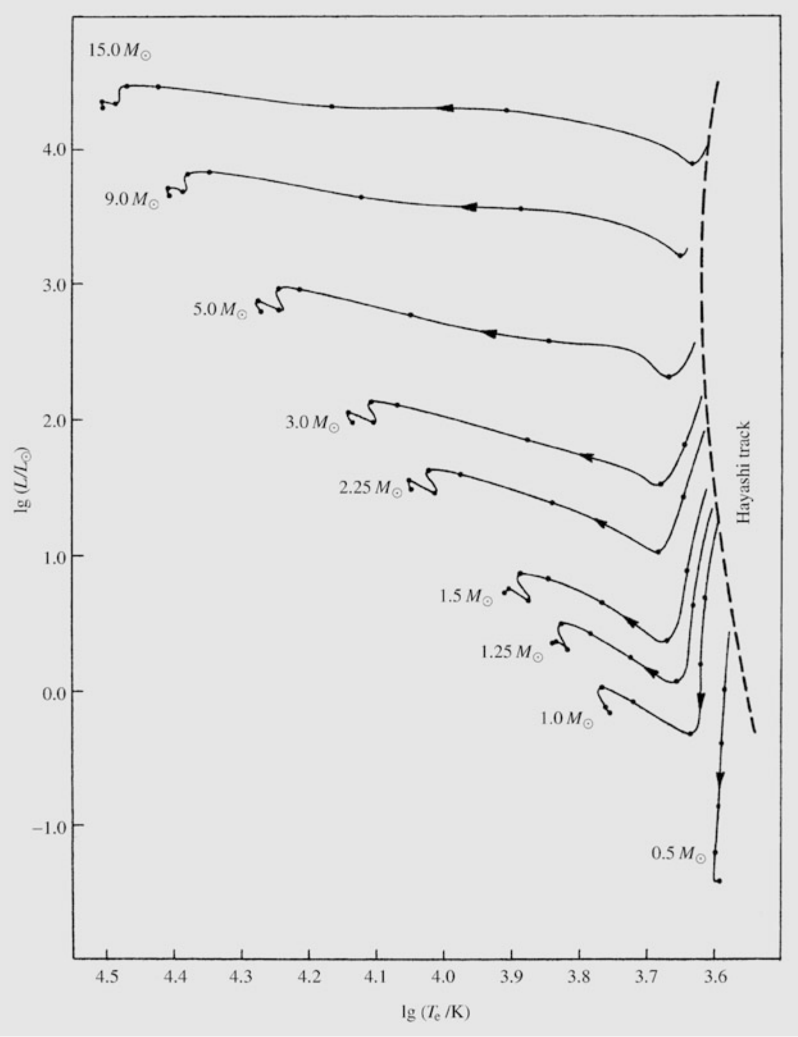

Fig. 12.1 The paths in the HR diagram of stars contracting to the main sequence on the thermal time scale. After a rapid dynamical collapse the stars settle on the Hayashi track and evolve towards the main sequence on the thermal time scale. (Models by Iben, I. (1965): Astrophys. J. **141**, 993)

place on the thermal time scale, i.e. much more slowly. The radius of the protostar has shrunk from its original value of about 100 au to about 1/4 au. It will usually be located inside a larger gas cloud and will be accreting material from its surroundings. Its mass therefore grows, and the central temperature and density increase.

The temperature of a star that has just reached equilibrium is still low and its opacity correspondingly large. Thus it will be convective in its centre. The convective energy transfer is quite efficient and the surface of the protostar will therefore be relatively bright.

We now describe the evolution in the HR diagram. Initially the protostar will be faint and cool, and it will reside at the lower far right in the HR diagram (outside Fig. 12.1). During the collapse its surface rapidly heats up and brightens and it moves to the upper right of Fig. 12.1. At the end of the collapse the star will settle at a point corresponding to its mass on the *Hayashi track*. The Hayashi track (Fig. 12.1) gives the location in the HR diagram of completely convective stars. Stars to its right cannot be in equilibrium and will collapse on the dynamic time scale.

The star will now evolve almost along the Hayashi track on the thermal time scale. In the HR diagram it moves almost vertically downwards, its radius decreases and its luminosity

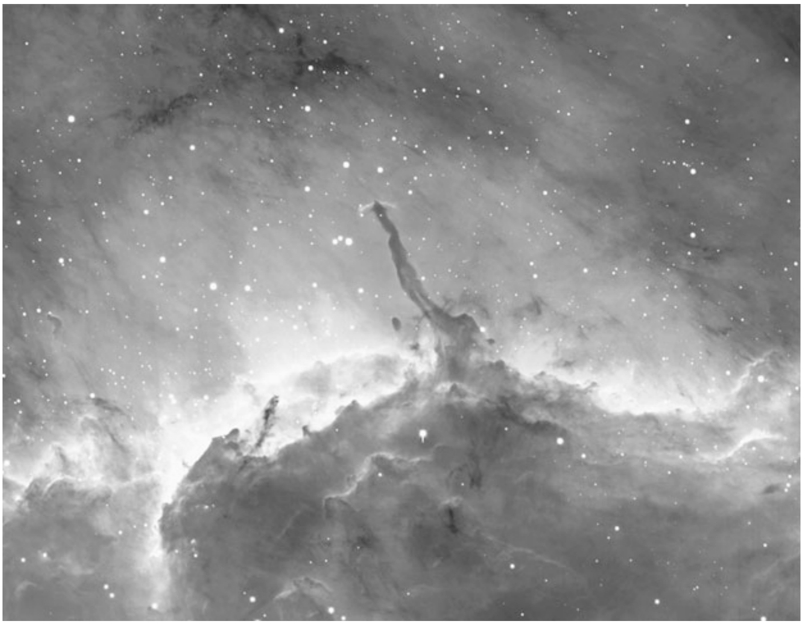

Fig. 12.2 Herbig–Haro object number 555 lies at the end of the "elephant's trunk" in Pelican Nebula in Cygnus. The small wings are shockwaves, which give evidence for powerful outflows from newly formed stars embedded within the clouds. (Photo University of Colorado, University of Hawaii and NOAO/AURA/NSF)

drops (Fig. 12.1). As the temperature goes on increasing in its centre, the opacity diminishes and energy begins to be transported by radiation. The mass of the radiative region will gradually grow until finally most of the star is radiative. By then the central temperature will have become so large that nuclear reactions begin. Previously all the stellar energy had been released potential energy, but now the nuclear reactions make a growing contribution and the luminosity increases. The stellar surface temperature will also increase and the star will move slightly upwards to the left in the HR diagram. In massive stars, this turn to the left occurs much earlier, because their central temperatures are higher and the nuclear reactions are initiated earlier.

For solar mass stars, the rapid collapse of the protostellar cloud only lasts for a few hundred years. The final stage of condensation is much slower, lasting several tens of millions of years. This length of time strongly depends on the stellar mass because of the luminosity dependence of the thermal time scale. A 15 $M_\odot$ star condenses to the main sequence in 60,000 years, whereas for a 0.1 $M_\odot$ star, the time is hundreds of millions of years.

Some of the hydrogen burning reactions start already at a few million degrees. For example, lithium, beryllium and boron burn to helium in the ppII and ppIII branches of the pp chain long before the complete chain has turned on. Because the star is convective and thus well mixed during the early stages, even its surface material will have been processed in the centre. Although the abundances of the above-mentioned elements are small, they give important information on the central temperature.

The beginning of the main sequence phase is marked by the start of hydrogen burning in the pp chain at a temperature of about 4 million degrees. The new form of energy production completely supersedes the energy release due to contraction. As the contraction is halted, the star makes a few oscillations in the HR diagram, but soon settles in an equilibrium and the long, quiet main sequence phase begins.

It is difficult to observe stars during contraction, because the new-born stars are usually hidden among dense clouds of dust and gas. However, some condensations in interstellar clouds have been discovered and near them, very young stars. One example are the *T Tauri stars*. Their lithium abundance is relatively high, which indicates that they are newly formed stars in which the central temperature has not yet become large enough to destroy lithium. Near the T Tauri stars, small, bright, star-like nebulae, *Herbig–Haro objects*, have been discovered. These are thought to be produced in the interaction between a stellar wind and the surrounding interstellar medium.

12.3 The Main Sequence Phase

The *main sequence phase* is that evolutionary stage in which the energy released by the burning of hydrogen in the core is the only source of stellar energy. During this stage, the star is in stable equilibrium, and its structure changes only because its chemical composition is gradually altered by the nuclear reactions. Thus the evolution takes place on a nuclear time scale, which means that the main sequence phase is the longest part of the life of a star. For example, for a solar mass star, the main sequence phase lasts for about 10,000 million years. More massive stars evolve more rapidly, because they radiate much more power. Thus the main sequence phase of a 15 solar mass star is only about 10 million years. On the other hand, less massive stars have a longer main sequence lifetime: a $0.25\,M_\odot$ star spends about 70,000 million years on the main sequence.

Since stars are most likely to be found in the stage of steady hydrogen burning, the main sequence in the HR diagram is richly populated, in particular at its low-mass end. The more massive upper main sequence stars are less abundant because of their shorter main sequence lifetimes.

If the mass of a star becomes too large, the force of gravity can no longer resist the radiation pressure. Stars more massive than this upper limit cannot form, because they cannot accrete additional mass during the contraction phase. Theoretical computations give a limiting mass of about $120\,M_\odot$; the most massive stars observed are claimed to be about $150\,M_\odot$. These values are, however, uncertain.

There is also a lower-mass limit of the main sequence. Stars below $0.08\,M_\odot$ never become hot enough for hydrogen burning to begin. They can still generate some luminosity from the burning of deuterium, but this energy source is rapidly exhausted. These *brown dwarfs* have surface temperatures in the range of 1000–2000 K. Hundreds of brown dwarfs have now been found in dedicated surveys. The lower limit for brown dwarf mass is sometimes taken to be about $0.015\,M_\odot$, corresponding to the minimum mass for deuterium burning.

If the mass is even lower there are no nuclear sources of energy. The smallest protostars therefore contract to planet-like dwarfs. During the contraction phase they radiate because potential energy is released, but eventually they begin to cool. In the HR diagram such stars first move almost vertically downwards and then further downwards to the right.

Is there a difference between the lowest-mass brown dwarfs and the most massive planets? If brown dwarfs have formed by gravitational collapse and fragmentation as described in the previous section and in Sect. 15.4, there is no reason not to count them as stars, although they are not producing energy by nuclear reactions. Planets in contrast are thought to form much more slowly by the clumping of solids and accretion of gas in a protoplanetary disk. The objects formed by this mechanism start out with a quite different structure. Whether such a clear-cut distinction between the formation mechanisms of dark stars and planets really can be made still remains an open question.

The Upper Main Sequence The stars on the *upper main sequence* are so massive and their central temperature so high that the CNO cycle can operate. On the *lower main sequence* the energy is produced by the pp chain. The pp chain and the CNO cycle are equally efficient at a temperature of 18 million degrees, corresponding to the central temperature of a $1.5\,M_\odot$ star. The boundary between the upper and the lower main sequence corresponds roughly to this mass.

The energy production in the CNO cycle is very strongly concentrated at the core. The outward energy flux will then become very large, and can no longer be maintained by radiative transport. Thus the upper main sequence stars have a *convective core*, i.e. the energy is transported by material motions. These keep the material well mixed, and thus the hydrogen abundance decreases uniformly with time within the entire convective region.

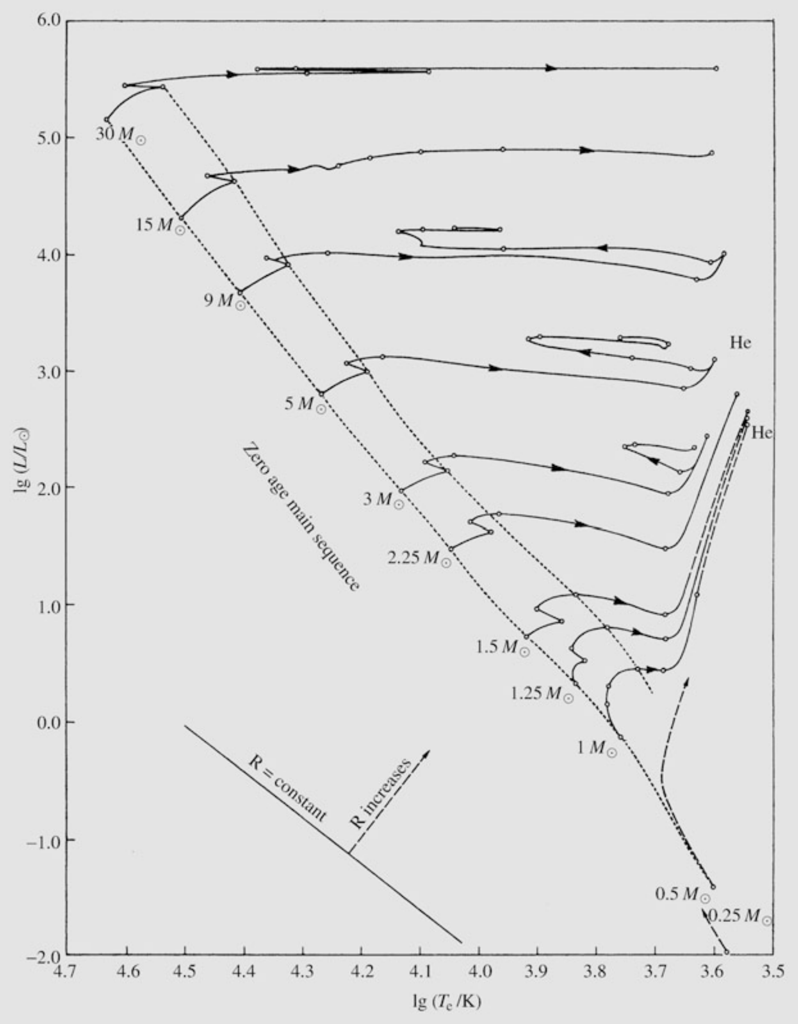

Fig. 12.3 Stellar evolutionary paths in the HR diagram at the main sequence phase and later. On the main sequence, bounded by *dashed curves*, the evolution is on the nuclear time scale. The post-main sequence evolution to the red giant phase is on the thermal time scale. *The point* marked He corresponds to helium ignition and in low-mass stars the helium flash. *The straight line* shows the location of stars with the same radius. (Iben, I. (1967): Annual Rev. Astron. Astrophys. **5**, 571; data for 30 $M_\odot$ from Stothers, R. (1966): Astrophys. J. **143**, 91)

Outside the core, there is *radiative equilibrium*, i.e. the energy is carried by radiation and there are no nuclear reactions. Between the core and the envelope, there is a transition region where the hydrogen abundance decreases inwards.

The mass of the convective core will gradually diminish as the hydrogen is consumed. In the HR diagram the star will slowly shift to the upper right as its luminosity grows and its surface temperature decreases (Fig. 12.2). When the central hydrogen supply becomes exhausted, the core of the star will begin to shrink rapidly. The surface temperature will increase and the star will quickly move to the upper left. Because of the contraction of the core, the temperature in the hydrogen shell just outside the core will increase. It rapidly becomes high enough for hydrogen burning to set in again.

The Lower Main Sequence On the lower main sequence, the central temperature is lower than for massive stars, and the energy is generated by the pp chain. Since the rate of the pp chain is not as sensitive to temperature as that of the CNO cycle, the energy production is spread over a larger region than in the more massive stars (Fig. 12.3). In consequence, the core never becomes convectively unstable, but remains radiative.

In the outer layers of lower main sequence stars, the opacity is high because of the low temperature. Radiation can then no longer carry all the energy, and convection will set in. The structure of lower main sequence stars is thus opposite to that of the upper main sequence: the centre is radiative and the envelope is convective. Since there is no mixing of material in the core, the hydrogen is most rapidly consumed at the very centre, and the hydrogen abundance increases outwards.

As the amount of hydrogen in the core decreases, the star will slowly move upwards in the HR diagram, almost along the main sequence (Fig. 12.2). It becomes slightly brighter and hotter, but its radius will not change by much. The evolutionary track of the star will then bend to the right, as hydrogen in the core nears its end. Eventually the core is almost pure helium. Hydrogen will continue to burn in a thick shell around the core.

Stars with masses between $0.08\,M_\odot$ and $0.26\,M_\odot$ have a very simple evolution. During their whole main sequence phase they are fully convective, which means that their entire hydrogen content is available as fuel. These stars evolve very slowly toward the upper left in the HR diagram. Finally, when all their hydrogen has burned to helium, they contract to become white dwarfs.

12.4 The Giant Phase

The main-sequence phase of stellar evolution ends when hydrogen is exhausted at the centre. The star then settles in a state in which hydrogen is burning in a shell surrounding a helium core. As we have seen, the transition takes place gradually in lower main-sequence stars, giving rise to the *Subgiant Branch* in the HR diagram, while the upper main-sequence stars make a rapid jump at this point.

The mass of the helium core is increased by the hydrogen burning in the shell. This leads to the expansion of the envelope of the star, which moves almost horizontally to the right in the HR diagram. As the convective envelope becomes more extensive, the star approaches the Hayashi track. Since it cannot pass further to the right, and since its radius continues to grow, the star has to move upwards along the Hayashi track towards larger luminosities (Fig. 12.2). The star has become a red giant.

In low-mass stars ($M \leq 2.3\,M_\odot$), as the mass of the core grows, its density will eventually become so high that it becomes degenerate. The central temperature will continue to rise. The whole helium core will have a uniform temperature because of the high conductivity of the degenerate gas. If the mass of the star is larger than $0.26\,M_\odot$ the central temperature will eventually reach about 100 million degrees, which is enough for helium to burn to carbon in the triple alpha process.

Helium burning will set in simultaneously in the whole central region and will suddenly raise its temperature. Unlike a normal gas, the degenerate core cannot expand, although the temperature increases (cf. (11.16)), and therefore the increase in temperature will only lead to a further acceleration of the rate of the nuclear reactions. When the temperature increases further, the degeneracy of the gas is removed and the core will begin to expand violently. Only a few seconds after the ignition of helium, there is an explosion, the *helium flash*.

The energy from the helium flash is absorbed by the outer layers, and thus it does not lead to the complete disruption of the star. In fact the luminosity of the star drops in the flash, because when the centre expands, the outer layers contract. The energy released in the flash is turned into potential energy of the expanded core. Thus after the helium flash, the star settles into a new state, where helium is steadily burning to carbon in a nondegenerate core.

After the helium flash the star finds itself on the horizontal giant branch in the HR diagram. The exact position of a star on the horizontal branch after the helium flash is a sensitive function of its envelope mass. This in turn depends on the amount of mass lost by the star in the helium flash, which can vary randomly from star to star. While the luminosity does not vary much along the horizontal branch, the effective temperatures are higher for stars with less mass in the envelope. The horizontal branch is divided into a blue and a red part by a gap corresponding to the pulsational instability leading to RR Lyrae variables (see Sect. 14.2). The form of the horizontal branch for a collection of stars depends on their metal-abundance, in the sense that a lower metal abundance is related to a more prominent blue horizontal branch. Thus the blue horizontal branch in globular clusters with low metal-abundances is strong and prominent (Sect. 16.3). For stars with solar element abundances the horizontal branch is reduced to a short stump, the *red clump*, where it joins the red giant branch.

In intermediate-mass stars ($2.3\,M_\odot \leq M \leq 8\,M_\odot$), the central temperature is higher and the central density lower, and the core will therefore not be degenerate. Thus helium burning can set in non-catastrophically as the central regions contract. As the importance of the helium burning core increases, the star first moves away from the red giant branch towards bluer colours, but then loops back towards the Hayashi track again. An important consequence of these *blue loops* is that they bring the star into the strip in the HR diagram corresponding to the cepheid instability (Sect. 14.2). This gives rise to the classical cepheid variables, which are of central importance for determining distances in the Milky Way and to the nearest galaxies.

In the most massive stars helium burning starts before the star has had time to reach the red giant branch. Some stars will continue moving to the right in the HR diagram. For others this will produce a massive stellar wind and a large mass loss. Stars in this evolutionary phase, such as P Cygni and η Carinae, are known as *luminous blue variables, LBV*, and are among the brightest in the Milky Way. If the star can retain its envelope it will become a red supergiant. Otherwise it will turn back towards the blue side of the HR diagram, ending up as a Wolf–Rayet star.

The Asymptotic Giant Branch The evolution that follows core helium burning depends strongly on the stellar mass. The mass determines how high the central temperature can become and the degree of degeneracy when heavier nuclear fuels are ignited.

When the central helium supply is exhausted, helium will continue to burn in a shell, while the hydrogen burning shell is extinguished. In the HR diagram the star will move towards lower effective temperature and higher luminosity. This phase is quite similar to the previous red giant phase of low-mass stars, although the temperatures are slightly hotter. For this reason it is known as the *asymptotic giant branch, AGB*.

After the early phase, when the helium shell catches up with the extinguished hydrogen shell, the AGB star enters what is known as the *thermally pulsing phase*, where hydrogen and helium shell burning alternate. A configuration with two burning shells is unstable, and in this phase the stellar material may become mixed or matter may be ejected into space in a shell, like that of a planetary nebula.

The thermally pulsing AGB continues until radiation pressure has led to the complete expulsion of the outer layers into a planetary nebula. Low- and intermediate-mass giants ($M \leq 8\,M_\odot$) never become hot enough to ignite carbon burning in the core, which remains as a carbon–oxygen white dwarf (Fig. 12.6).

The End of the Giant Phase After the end of helium burning the evolution of a star changes character. This is because the nuclear time scale at the centre becomes short compared to the thermal time scale of the outer layers. Secondly, the energy released in nuclear reactions will be carried away by neutrinos, instead of being deposited at the centre. In consequence, while the thermonuclear burning follows the same pattern as hydrogen and helium burning, the star as a whole does not have time to react immediately.

In stars with masses around $10\,M_\odot$ either carbon or oxygen may be ignited explosively just like helium in low-mass stars: there is a *carbon* or *oxygen flash*. This is much more powerful than the helium flash, and may make the star explode as a supernova (Sects. 12.5 and 13.3).

For even larger masses the core remains non-degenerate and burning will start non-catastrophically as the core goes on contracting and becoming hotter. First carbon burning and subsequently oxygen and silicon burning (see Sect. 10.3) will be ignited. As each nuclear fuel

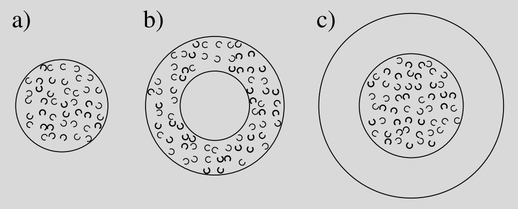

Fig. 12.4 Energy transport in the main sequence phase. (**a**) The least massive stars ($M < 0.26\,M_\odot$) are convective throughout. (**b**) For $0.26\,M_\odot < M < 1.5\,M_\odot$ the core is radiative and the envelope convective. (**c**) Massive stars ($M > 1.5\,M_\odot$) have a convective core and a radiative envelope

is exhausted in the centre, the burning will continue in a shell. The star will thus contain several nuclear burning shells. At the end the star will consist of a sequence of layers differing in composition, in massive stars (more massive than $15\,M_\odot$) all the way up to iron.

The central parts of the most massive stars with masses larger than $15\,M_\odot$ burn all the way to iron ^{56}Fe. All nuclear sources of energy will then be completely exhausted. The structure of a 30 solar mass star at this stage is schematically shown in Fig. 12.4. The star is made up of a nested sequence of zones bounded by shells burning silicon ^{28}Si, oxygen ^{16}O and carbon ^{12}C, helium ^{4}He and hydrogen ^{1}H. However, this is not a stable state, since the end of nuclear reactions in the core means that the central pressure will fall, and the core will collapse. Some of the energy released in the collapse goes into dissociating the iron nuclei first to helium and then to protons and neutrons. This will further speed up the collapse, just like the dissociation of molecules speeds up the collapse of a protostar. The collapse takes place on a dynamical time scale, which, in the dense stellar core, is only a fraction of a second. The outer parts will also collapse, but more slowly. In consequence, the temperature will increase in layers containing unburnt nuclear fuel. This will burn explosively, releasing immense amounts of energy in a few seconds, principally in the form of neutrinos.

The final stages of stellar evolution may be described as an implosion of the core, which is briefly halted every time a new source of nuclear fuel becomes available for burning It is still an open problem how exactly the energy released in this collapse is transformed into the disruption of the entire star and the ejection of its outer layers. It is also still unclear whether in a given case the remnant will be a neutron star or a black hole.

Although the exact mechanism is not yet understood, the end-point of the evolution of stars more massive the about $8\,M_\odot$ is that the outer layers explode as a supernova. In the dense central core, the protons and electrons combine to form neutrons. The core will finally consist almost entirely of neutrons, which become degenerate because of the high density. The degeneracy pressure of the neutrons will stop the collapse of a small mass core. However, if the mass of the core is large enough, a black hole will probably be formed.

12.5 The Final Stages of Evolution

The endpoints of stellar evolution can be seen from Fig. 12.7. This shows the relation between mass and central density for a body at zero temperature, i.e. the final equilibrium when a massive body has cooled. There are two maxima on the curve. The mass corresponding to the left-hand maximum is called the *Chandrasekhar mass*, $M_{\mathrm{Ch}} \approx 1.2\text{–}1.4\,M_\odot$, and that corresponding to the right-hand one, the *Oppenheimer–Volkoff mass*, $M_{\mathrm{OV}} \approx 1.5\text{–}2\,M_\odot$.

Let us first consider a star with mass less than M_{Ch}. Suppose the mass does not change. When the nuclear fuel is exhausted, the star will become a white dwarf, which will gradually cool down and contract. In Fig. 12.5 it moves horizontally to the right. Finally it will reach zero temperature and end up on the left-hand rising part of the equilibrium curve. Its final equilibrium is a completely degenerate black dwarf.

If the mass of the star is larger than M_{Ch} but smaller than M_{OV}, it can continue cooling until it reaches the right-hand rising section of the curve. Again there is a stable final state, this time corresponding to a completely degenerate neutron star.

An even more massive star with mass larger than M_{OV} will go on contracting past the density corresponding to a neutron star. There is then no longer any known possible stable equilibrium, and the star must go on contracting to form a black hole.

The only endpoints of stellar evolution predicted by theory are the two stable states of Fig. 12.7 and the two extreme possibilities, collapse to a black hole or explosive disruption.

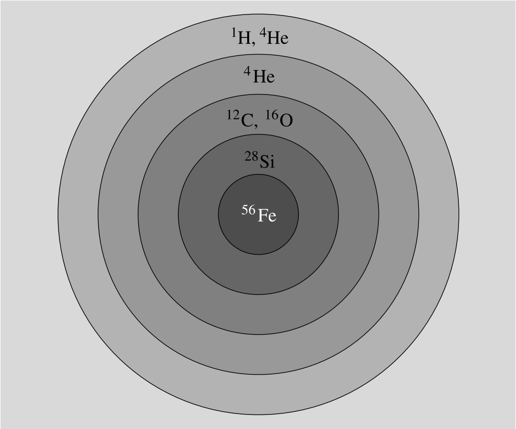

Fig. 12.5 The structure of a massive star (30 $M_\odot$) at a late evolutionary stage. The star consists of layers with different composition separated by nuclear burning shells

The preceding considerations are purely theoretical. The final evolutionary stages of real stars involve many imperfectly known factors, which may affect the final equilibrium. Perhaps most important is the question of mass loss, which is very difficult to settle either observationally or theoretically. For example, in a supernova explosion the whole star may be disrupted and it is very uncertain whether what remains will be a neutron star, a black hole or nothing at all. (The structure of compact stars will be discussed in Chap. 15.)

A summary of the various evolutionary paths is given in Fig. 12.8.

12.6 The Evolution of Close Binary Stars

If the components of a binary star are well separated, they do not significantly perturb one another. When studying their evolution, one can regard them as two single stars evolving independently, as described above. However, in close binary pairs, this will no longer be the case.

Close binary stars are divided into three classes, as shown in Fig. 12.9: *detached*, *semidetached* and *contact binaries*. The figure-eight

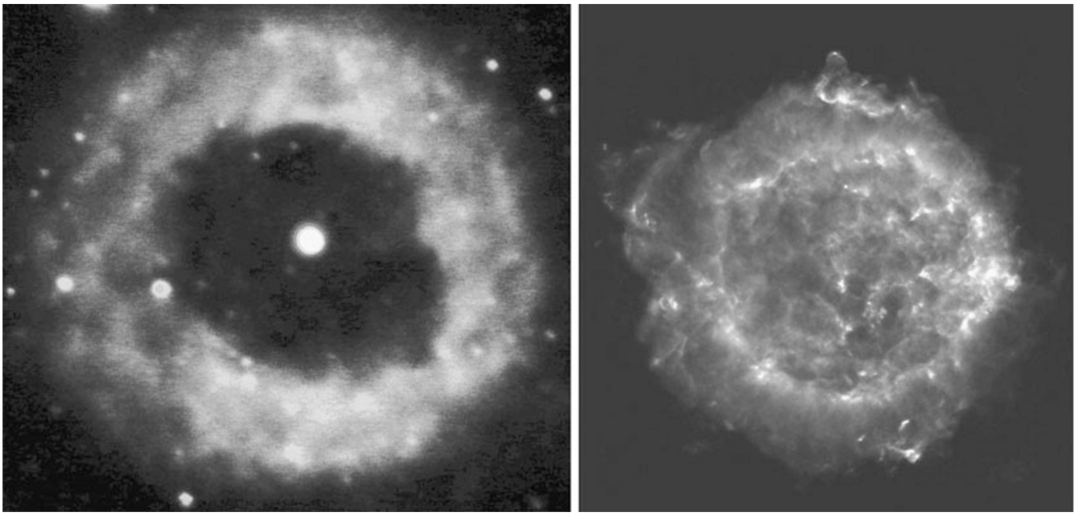

Fig. 12.6 The usual endpoint for the development of a star with a mass of less than three solar masses, is a white dwarf, with an expanding planetary nebula around it. *On the left*, the planetary nebula NGC 6369, photographed with the 8-meter Gemini South telescope. For a massive star, the life ends with a supernova explosion. *On the right*, the supernova remnant Cassiopeia A on radio wavelengths. The image was created by the VLA telescope. (Images Gemini Observatory/Abu Team/NOAO/AURA/NSF and NRAO/AUI)

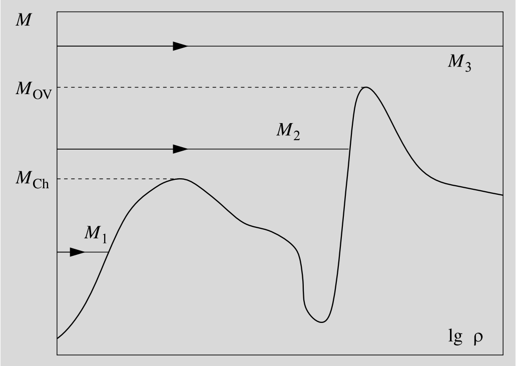

Fig. 12.7 The evolutionary end points of stars with different masses shown as a function of central density. *The curve* shows the behaviour of the central density of completely degenerate ($T = 0$ K) bodies. The Chandrasekhar mass M_{Ch} and the Oppenheimer–Volkoff mass M_{OV} correspond to maxima on this curve

curve drawn in the figure is an equipotential surface called the *Roche surface*. If the star becomes larger than this surface, it begins to lose mass to its companion through the waist of the Roche surface.

During the main sequence phase the stellar radius does not change much, and each component will remain within its own Roche lobe. When the hydrogen is exhausted, the stellar core will rapidly shrink and the outer layers expand, as we have seen. At this stage a star may exceed its Roche lobe and mass transfer may set in.

Close binary stars are usually seen as eclipsing binaries. One example is Algol in the constellation Perseus. The components in this binary system are a normal main sequence star and a subgiant, which is much less massive than the main sequence star. The subgiant has a high luminosity and thus has apparently already left the main sequence. This is unexpected, since the components were presumably formed at the same time, and the more massive star should evolve more rapidly. The situation is known as the *Algol paradox*: for some reason, the less massive star has evolved more rapidly.

In the 1950's a solution to the paradox proposed that the subgiant was originally more massive, but that it had lost mass to its companion during its evolution. Since the 1960's mass transfer in close binary systems has been much studied, and has turned out be a very significant factor in the evolution of close binaries.

As an example, let us consider a close binary, where the initial masses of the components are 1 and 2 solar masses and the initial orbital period 1.4 days (Fig. 12.10). After evolving away from the main sequence the more massive component will exceed the Roche limit and begin to lose mass to its companion. Initially the mass will be transferred on the thermal time scale, and after a few million years the roles of the components will be changed: the initially more massive component has become less massive than its companion.

The binary is now semidetached and can be observed as an Algol-type eclipsing binary. The two components are a more massive main sequence star and a less massive subgiant filling its Roche surface. The mass transfer will continue, but on the much slower nuclear time scale. Finally, mass transfer will cease and the less massive component will contract to a $0.6\,M_\odot$ white dwarf.

The more massive $2.4\,M_\odot$ star now evolves and begins to lose mass, which will accumulate on the surface of the white dwarf. The accumulated mass may give rise to *nova outbursts*, where material is ejected into space by large explosions. Despite this, the mass of the white dwarf will gradually grow and may eventually exceed the Chandrasekhar mass. The white dwarf will then collapse and explode as a type I supernova.

As a second example, we can take a massive binary with the initial masses 20 and $8\,M_\odot$ and the initial period 4.7 days (Fig. 12.11). The more massive component evolves rapidly, and at the end of the main sequence phase, it will transfer more than $15\,M_\odot$ of its material to the secondary. The mass transfer will occur on the thermal time scale, which, in this case, is only a few ten thousand years. The end result is a *helium star*, having as a companion an unevolved main sequence star. The properties of the helium star are like those of a *Wolf–Rayet star* (Fig. 12.12).

Helium continues to burn to carbon in the core of the helium star, and the mass of the carbon core will grow. Eventually the carbon will be explosively ignited, and the star will explode as a su-

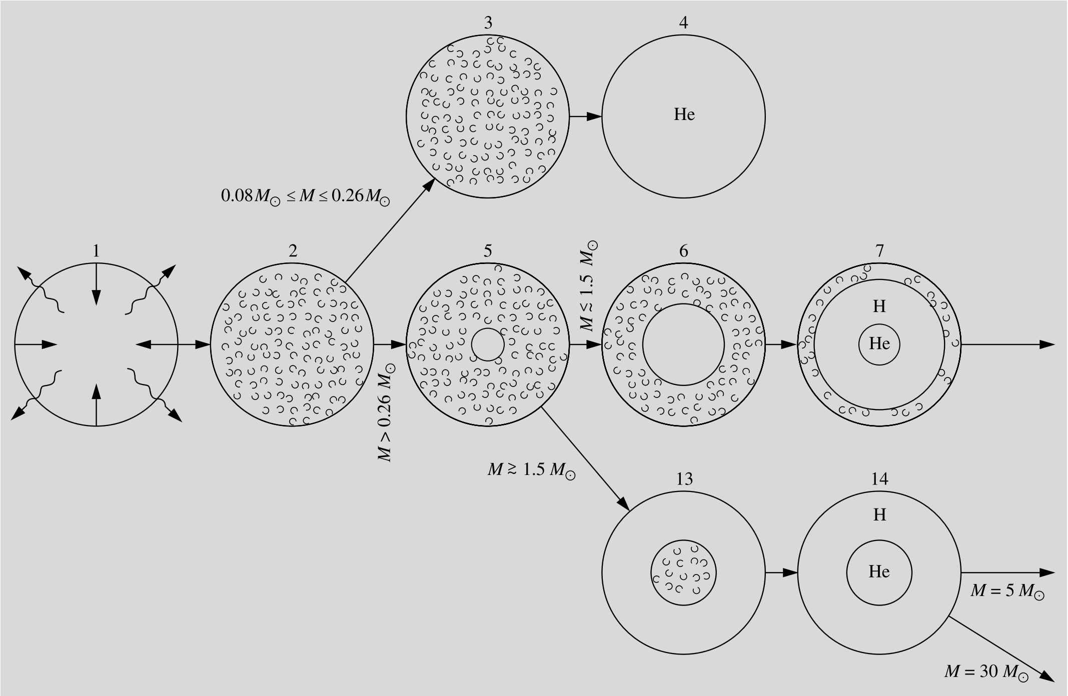

Fig. 12.8 Evolution schemes for stars with different masses. The radius is scaled to be the same in all drawings. In reality, there are vast differences in the sizes of different stars and different phases of evolution. In the beginning (*1*) a gas cloud is contracting rapidly in free fall. Because the gas is quite rarefied, radiation escapes easily from the cloud. As the density increases, radiation transport becomes more difficult, and the released energy tends to warm up the gas. The contraction lasts until the gas is completely ionised, and the star, which has become a protostar, is in hydrostatic equilibrium (*2*). The star is convective throughout its interior.
Now evolution continues on a thermal time scale. The contraction is much slower than in the free-fall phase. The phases of further evolution are determined by the mass M of the star. For $M < 0.08\,M_\odot$ the temperature in the centre does not rise high enough for hydrogen burning, and these stars contract to planetlike brown dwarfs. Stars with $M \geq 0.08\,M_\odot$ start hydrogen burning when the temperature has reached about 4×10^6 K. This is the beginning of the main sequence phase. In the main sequence, the lowest-mass stars with $0.08\,M_\odot \leq M \leq 0.26\,M_\odot$ are entirely convective, and thus they remain homogeneous (*3*). Their evolution is very slow, and after all the hydrogen has been burnt to helium, they contract to white dwarfs (*4*).
The increasing temperature makes the stars with $M > 0.26\,M_\odot$ radiative in the centre as the opacity decreases (*5*). The low-mass stars with $0.26\,M_\odot \leq M \leq 1.5\,M_\odot$ remain radiative in the centre during the main sequence phase (*6*) as they burn their hydrogen through the pp chain. The outer part is convective. At the end of the main sequence phase, hydrogen burning continues in a shell surrounding the helium core (*7*). The outer part expands, and the giant phase begins. The contracting helium core is degenerate and warms up. At about 10^8 K, the triple alpha process begins and leads immediately to the helium flash (*8*). The explosion is damped by the outer parts, and helium burning goes on in the core (*9*). Hydrogen is still burning in an outer shell. As the central helium is exhausted, helium burning moves over to a shell (*10*). At the same time, the outer part expands and the star loses some of its mass. The expanding envelope forms a planetary nebula (*11*). The star in the centre of the nebula becomes a white dwarf (*12*).
In the upper main sequence with $M \geq 1.5\,M_\odot$ energy is released through the CNO cycle, and the core becomes convective, while the outer part is radiative (*13*). The main sequence phase ends as the hydrogen in the core is exhausted, and shell burning begins (*14*). The helium core remains convective and nondegenerate, and helium burning begins without perturbations (*15* and *19*). Afterwards, helium burning moves over to a shell (*16* and *20*). For stars with $3\,M_\odot \leq M \leq 15\,M_\odot$ the carbon in the core is degenerate, and a carbon flash occurs (*17*). This leads to a supernova explosion (*18*) and possibly to the complete destruction of the star.
For the most massive stars with $M \geq 15\,M_\odot$ the carbon core remains convective, and carbon burns to oxygen and magnesium. Finally, the star consists of an iron core surrounded by shells with silicon, oxygen, carbon, helium and hydrogen (*21*). The nuclear fuel is now exhausted, and the star collapses on a dynamical time scale. The result is a supernova (*22*). The outer parts explode, but the remaining core continues to contract to a neutron star or a black hole

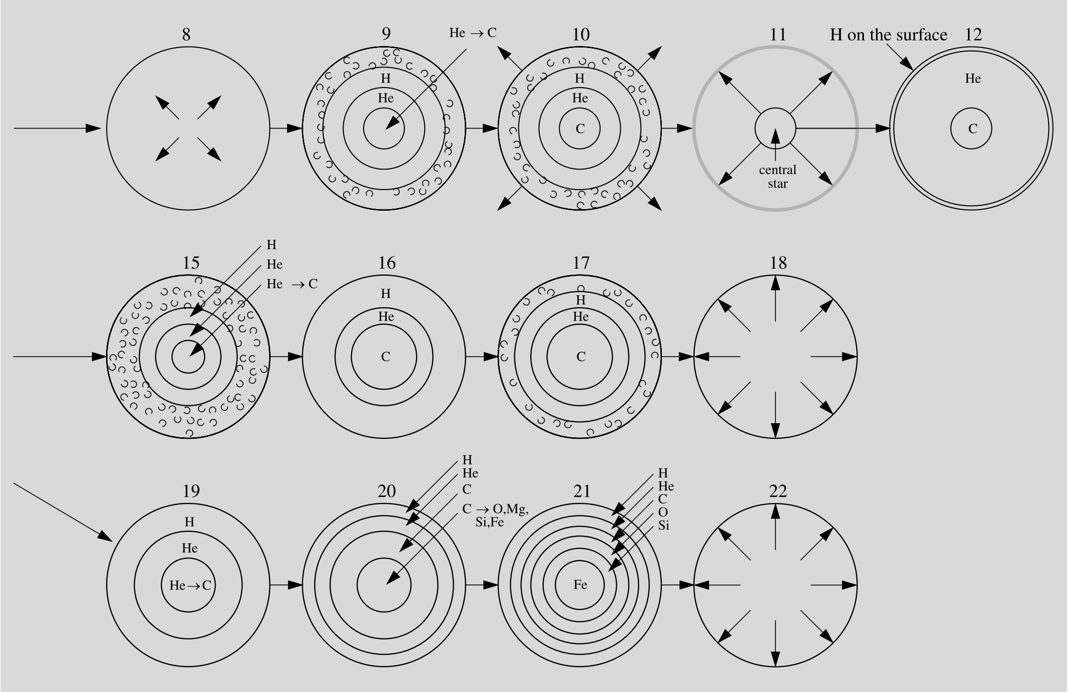

Fig. 12.8 (Continued)

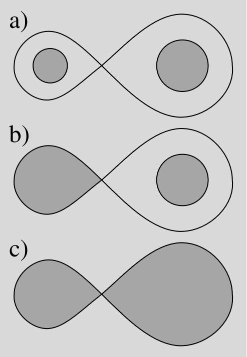

Fig. 12.9 The types of close binary systems: (**a**) detached, (**b**) semidetached and (**c**) contact binary

pernova. The consequences of this explosion are not known, but let us suppose that a $2\,M_\odot$ compact remnant is left. As the more massive star expands, its stellar wind will become stronger, giving rise to strong X-ray emission as it hits the compact star. This X-ray emission will only cease when the more massive star exceeds its Roche surface.

The system will now rapidly lose mass and angular momentum. A steady state is finally reached when the system contains a $6\,M_\odot$ helium star in addition to the $2\,M_\odot$ compact star. The helium star is seen as a Wolf–Rayet star, which, after about a million years, explodes as a supernova. This will probably lead to the breakup of the binary system. However, for certain values of the mass, the binary may remain bound. Thus a binary neutron star may be formed.

12.7 Comparison with Observations

The most important direct support for the theoretical evolutionary models is obtained from the properties of observed HR diagrams. If the theoretical models are correct, the observed number of stars should reflect the duration of the various evolutionary phases. These are given for stars of different masses in Table 12.1. The stars are most numerous along the main sequence. Giants are also common and, in addition to these, there are white dwarfs, subgiants, etc. The sparsely populated region to the right of the main sequence, the

Hertzsprung gap, is explained by the rapid transition from the main sequence to the giant phase.

The cepheids provide an important test for the evolutionary models. The pulsations and the relation between period and luminosity for the cepheids can be understood on the basis of theoretical stellar models.

The evolutionary models can also explain the HR diagrams of star clusters. Let us assume that all the stars in a cluster were formed at the same time. In the youngest systems, the associations, the stars will mainly be found on the upper main sequence, since the most massive stars evolve most rapidly. To the right of the main sequence, there will be less massive T Tauri stars, which are

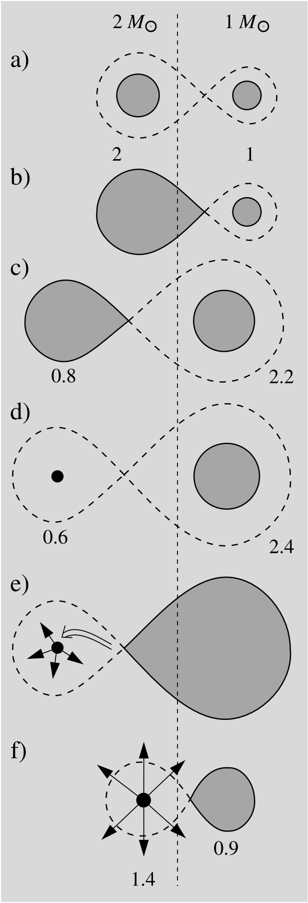
2 $M_\odot$
1 $M_\odot$
a)
2
1
b)
c)
0.8
2.2
d)
0.6
2.4
e)
f)
0.9
1.4

Fig. 12.10 Evolution of a low-mass binary: (**a**) both components on the main sequence; (**b**) mass transfer from the more massive component; (**c**) light subgiant and massive main sequence star; (**d**) white dwarf and main sequence star; (**e**) mass transferred to the white dwarf from the more massive component leads to nova outbursts; (**f**) the white dwarf mass exceeds the Chandrasekhar mass and explodes as a type I supernova

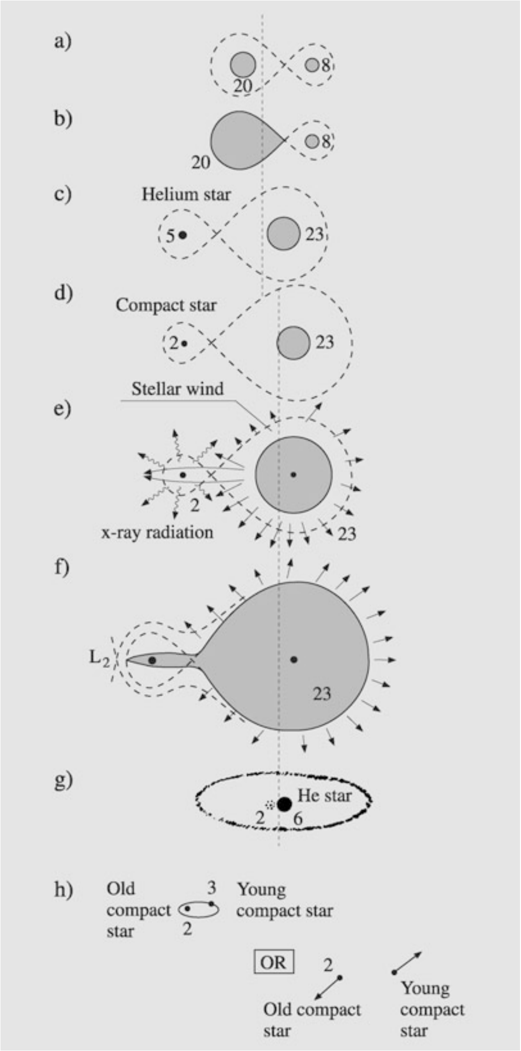
a)
20
8
b)
20
8
c)
Helium star
5
23
d)
Compact star
2
23
Stellar wind
e)
2
x-ray radiation
23
f)
L_2
23
g)
He star
2
6
h)
Old compact star
3
2
Young compact star
OR
2
Old compact star
Young compact star

Fig. 12.11 Evolution of a massive binary. It has been assumed that the supernova explosion of a 5 $M_\odot$ helium star leaves a 2 $M_\odot$ compact remnant (neutron star or black hole). (**a**) Main sequence phase; (**b**) beginning of the first mass transfer phase; (**c**) end of the first mass transfer phase; the first Wolf–Rayet phase begins; (**d**) the helium star (Wolf–Rayet star) has exploded as a supernova; (**e**) the 23 $M_\odot$ component becomes a supergiant; the compact component is a strong X-ray source; (**f**) beginning of the second mass transfer phase; the X-ray source is throttled and large-scale mass loss begins; (**g**) second Wolf–Rayet phase; (**h**) the 6 $M_\odot$ helium star has exploded as a supernova; the binary may or may not be disrupted, depending on the remaining mass

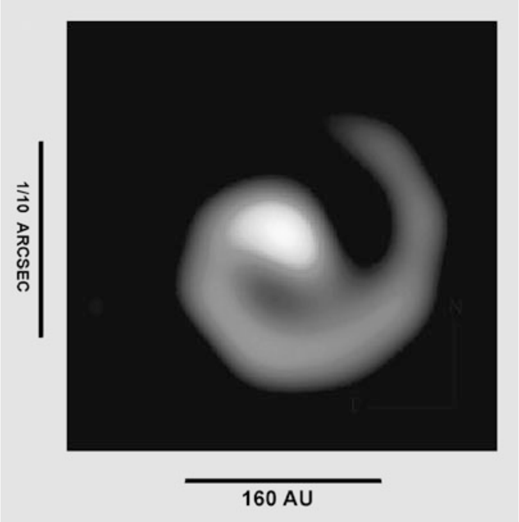

Fig. 12.12 The Wolf–Rayet star WR 104 photographed by the 10-m Keck telescope. The spiral is dust and gas which is thrown out from the rotating binary system. The spiral "pinwheel" is seen to make a full revolution in about 220 days. (Photo U.C. Berkeley Space Sciences Laboratory/W.M. Keck Observatory)

still contracting. In intermediate age open clusters, the main sequence will be well developed and its upper end should bend to the right, since the most massive stars will already have begun to evolve off the main sequence. In the old globular clusters, the giant branch should increase in importance in the older clusters. These predictions are confirmed by the observations, which will be further discussed in Chap. 16 on star clusters.

Of course, the most detailed observations can be made of the Sun, which is therefore a crucial point of comparison for the theoretical models. If a star of one solar mass with an initial composition of 71 % hydrogen, 27 % helium and 2 % heavier elements is allowed to evolve for 5000 million years, it will be very similar to our present Sun. In particular, it will have the same radius, surface temperature and luminosity. According to calculations, about half of the Sun's supply of hydrogen fuel has been consumed. The Sun will go on shining like a normal main sequence star for another 5000 million years, before there will be any dramatic change.

Some problems remain in regard to the observations. One is the solar neutrino problem. The neutrinos produced by solar nuclear reactions have been observed since the beginning of the 1970's by the techniques described in Sect. 3.7. Only the neutrinos formed in the relatively rare ppIII reaction are energetic enough to be observed in this way. Their observed number is too small: whereas the models predict about 5 units, the observations have consistently only registered 1–2.

The discrepancy may be due to a fault in the observational technique or to some unknown properties of the neutrinos. However, if the solar models are really in error, the central temperature of the Sun would have to be about 20 % lower than thought, which would be in serious conflict with the observed solar luminosity. One possibility is that some of the electron neutrinos change to other, unobservable particles during their passage to Earth. (See also Sect. 12.1.)

A second problem is the observed abundance of lithium and beryllium. The solar surface contains a normal abundance of beryllium, but very little lithium. This should mean that during its contraction, the Sun was still fully convective when the central temperature was high enough to destroy lithium (3×10^6 K), but not beryllium (4×10^6 K). However, according to the standard solar evolution models, convection ceased in the centre already at a temperature of 2×10^6 K. One suggested explanation is that the convection has later carried down lithium to layers where the temperature is high enough to destroy it.

12.8 The Origin of the Elements

There are just under a hundred naturally occurring elements, and about 300 isotopes in the solar system (Fig. 12.13). In Sect. 12.4, we have seen how the elements up to iron are produced when hydrogen burns to helium and helium further to carbon, oxygen and heavier elements.

Almost all nuclei heavier than helium were produced in nuclear reactions in stellar interiors. In the oldest stars, the mass fraction of heavy elements is only about 0.02 %, whereas in the youngest stars it is a few per cent. Nevertheless, most of the stellar material is hydrogen and

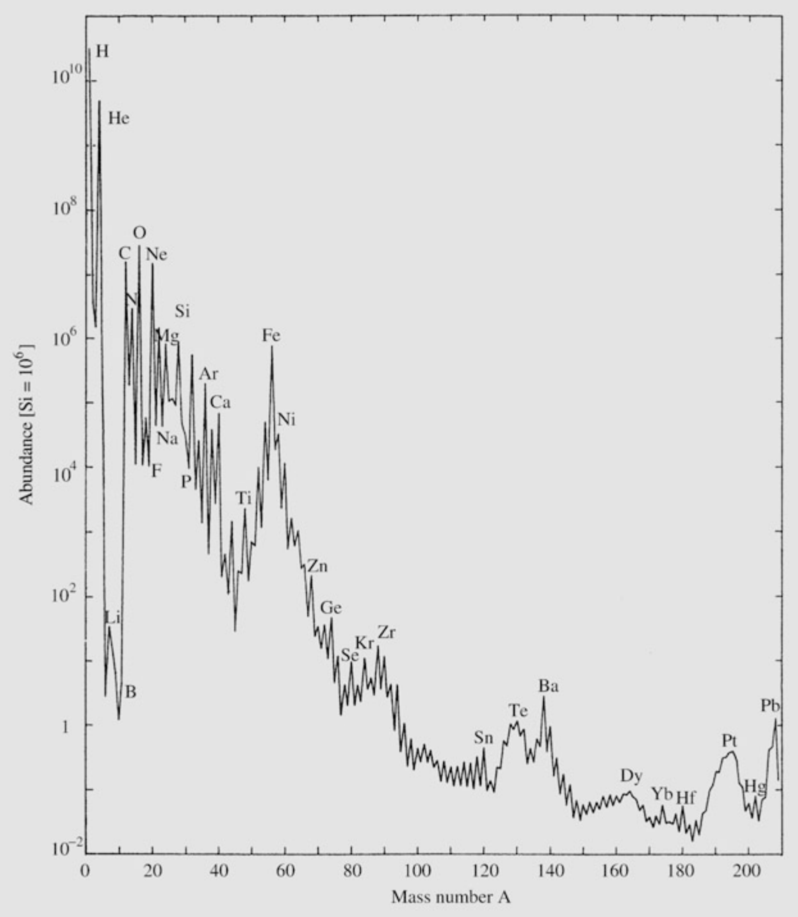

Fig. 12.13 Element abundances in the solar system as a function of the nuclear mass number. The abundance of Si has been normalised as 10^6

helium. According to the standard cosmological model, those were formed in the early stages of the Universe, when the temperature and density were suitable for nuclear reactions. (This will be discussed in Chap. 20.) Although helium is produced during the main sequence stellar evolution, very little of it is actually returned into space to be incorporated into later stellar generations. Most of it is either transformed into heavier elements by further reactions, or else remains locked up inside white dwarf remnants. Therefore the helium abundance does not increase by much due to stellar processes.

The most important nuclear reactions leading to the build-up of the heavy nuclei up to iron were presented in Sect. 11.3. The probabilities of the various reactions are determined either by experiments or by theoretical calculations. When they are known, the relative abundances of the various nuclei produced can be calculated.

The formation of elements heavier than iron requires an input of energy, and thus they cannot be explained in the same manner. Still heavy nuclei are continually produced. In 1952 technetium was discovered in the atmosphere of a red giant. The half-life of the most longlived isotope ^{98}Tc is about 1.5×10^6 years, so that the observed technetium must have been produced in the star.

Most of the nuclei more massive than iron are formed by *neutron capture* (Fig. 12.14). Since the neutron does not have an electric charge, it can easily penetrate into the nucleus. The probability for neutron capture depends both on the kinetic energy of the incoming neutron and on the mass number of the nucleus. For example, in the solar system the abundances of isotopes show maxima at the mass numbers $A = 70–90,\ 130,$

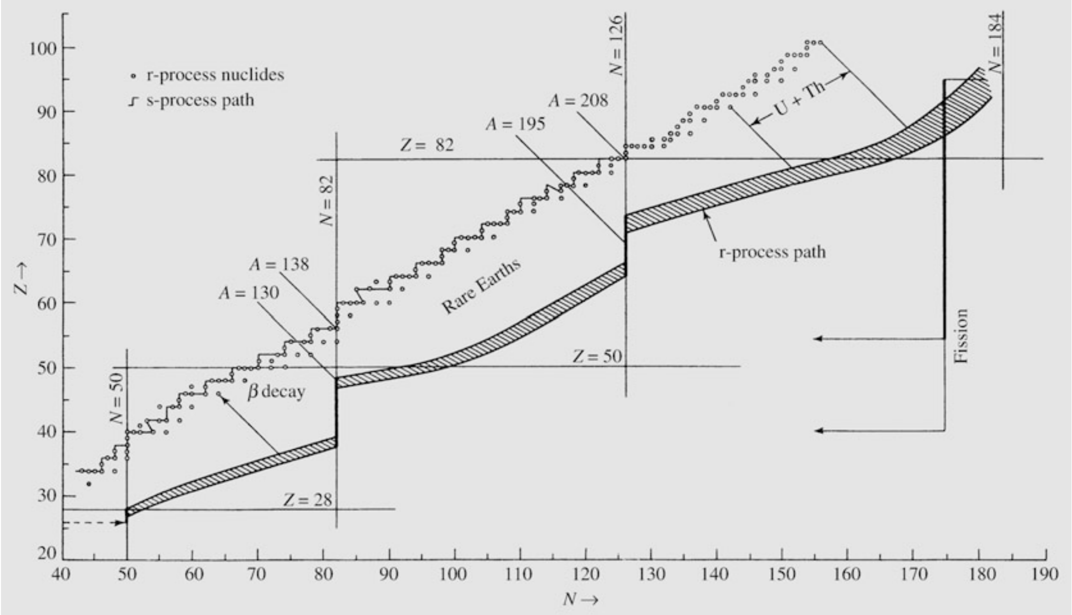

Fig. 12.14 Neutron capture paths for the s-process and r-process (*from left to right*). The s-process follows a path along the line of beta stability. The stable r-process nuclei (*small circles*) result from beta decay of their neutron rich progenitors on the shaded path shown lower. Beta decay occurs along straight lines $A = \text{const}$. The closed neutron shells in nuclei at $N = 50,\ 82$ and 126 correspond to abundance peaks in s-process nuclei at $A = 88,\ 138$ and 208, and in r-process nuclei at $A = 80,\ 130$ and 195. (Seeger, P.A., Fowler, W.A., Clayton, D.D. (1965): Astrophys. J. Suppl. **11**, 121)

138, 195 and 208. These mass numbers correspond to nuclei with closed neutron shells at the neutron numbers $N = 50,\ 82,$ and 126. The neutron capture probability for these nuclei is very small. The closed shell nuclei thus react more slowly and are accumulated in greater abundances.

In a neutron capture, a nucleus with mass number A is transformed into a more massive nucleus:

$$(Z, A) + \mathrm{n} \rightarrow (Z, A + 1) + \gamma .$$

The newly formed nucleus may be unstable to *β decay*, where one neutron is transformed into a proton:

$$(Z, A + 1) \rightarrow (Z + 1, A + 1) + \mathrm{e}^- + \overline{\nu}_\mathrm{e} .$$

Two kinds of neutron capture processes are encountered, depending on the value of the neutron flux. In the slow *s-process*, the neutron flux is so small that any β decays have had time to occur before the next neutron capture reaction takes place. The most stable nuclei up to mass number 210 are formed by the s-process. These nuclei are said to correspond to the β stability valley. The s-process explains the abundance peaks at the mass numbers 88, 138 and 208.

When the neutron flux is large, β decays do not have time to happen before the next neutron capture. One then speaks of the rapid *r-process*, which gives rise to more neutron-rich isotopes. The abundance maxima produced by the r-process lie at mass numbers about ten units smaller than those of the s-process.

A neutron flux sufficient for the s-process is obtained in the course of normal stellar evolution. For example, some of the carbon and oxygen burning reactions produce free neutrons. If there is convection between the hydrogen and helium burning shells, free protons may be carried into the carbon-rich layers. Then the following neutron-producing reaction chain becomes important:

$$\begin{aligned} {}^{12}\mathrm{C}+\mathrm{p} &\rightarrow {}^{13}\mathrm{N}+\gamma, \\ {}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C}+\mathrm{e}^{+}+\nu_{\mathrm{e}}, \\ {}^{13}\mathrm{C}+{}^{4}\mathrm{He} &\rightarrow {}^{16}\mathrm{O}+\mathrm{n}. \end{aligned}$$

The convection can also carry the reaction products nearer to the surface.

The neutron flux required for the r-process is about 10^{22} cm^{-3}, which is too large to be produced during normal stellar evolution. The only presently known site where a large enough neutron flux is expected is near a neutron star forming in a supernova explosion. In this case, the rapid neutron capture leads to nuclei that cannot capture more neutrons without becoming strongly unstable. After one or more rapid β decays, the process continues.

The r-process stops when the neutron flux decreases. The nuclei produced then gradually decay by the β-process towards more stable isotopes. Since the path of the r-process goes about ten mass units below the stability valley, the abundance peaks produced will fall about ten units below those of the s-process. This is shown in Fig. 12.11. The most massive naturally occurring elements, such as uranium, thorium and plutonium, are formed by the r-process.

There are about 40 isotopes on the proton-rich side of the β stability valley that cannot be produced by neutron capture processes. Their abundances are very small, relative to the neighbouring isotopes. They are formed in supernova explosions at temperatures higher than 10^9 K by reactions known as the *p-process*. At this temperature, pair formation can take place:

$$\gamma \rightarrow \mathrm{e}^{+}+\mathrm{e}^{-}.$$

The positron may either be annihilated immediately or be consumed in the reaction

$$\mathrm{e}^{+}+(Z,A) \rightarrow (Z+1,A)+\overline{\nu}_{\mathrm{e}}.$$

Another reaction in the p-process is

$$(Z,A)+\mathrm{p} \rightarrow (Z+1,A+1)+\gamma.$$

Finally, the *fission* of some heavier isotopes may give rise to p-process nuclei. Examples of this are the isotopes ^{184}W, ^{190}Pt and ^{196}Hg formed by the fission of lead.

All the preceding reaction products are ejected into the interstellar medium in the supernova explosion. Collisions between cosmic rays and heavy nuclei then finally give rise to the light elements lithium, beryllium and boron. Thus the abundances of essentially all naturally occurring isotopes can be explained.

During succeeding generations of stars the relative abundance of heavy elements increases in the interstellar medium. They can then be incorporated into new stars, planets—and living beings.

12.9 Example

Example 12.1 An interstellar cloud has a mass of one solar mass and density of 10^{10} hydrogen atoms per cm^3. Its rotation period is 1000 years. What is the rotation period after the cloud has condensed into a star of solar size?

The angular momentum is $L = I\omega$, where ω is the angular velocity and I is the moment of inertia. For a homogeneous sphere

$$I = \frac{2}{5}MR^2,$$

where M is the mass and R the radius. From the conservation of the angular momentum we get

$$\begin{aligned} &L = I_1\omega_1 = I_2\omega_2 \\ &\Rightarrow \quad \frac{I_1 2\pi}{P_1} = \frac{I_2 2\pi}{P_2} \\ &\Rightarrow \quad P_2 = P_1\frac{I_2}{I_1} = P_1\frac{\frac{2}{5}MR_2^2}{\frac{2}{5}MR_1^2} = P_1\left(\frac{R_2}{R_1}\right)^2, \end{aligned}$$

where P_1 and P_2 are the rotation periods before and after the collapse. The mass of the cloud is

$$\begin{aligned} M &= \frac{4}{3}\pi R^3 \rho \\ &= \frac{4}{3}\pi R^3 \times 10^{16} \times 1.6734 \times 10^{-27}\ \mathrm{kg} \\ &= 1\,M_\odot = 1.989 \times 10^{30}\ \mathrm{kg}. \end{aligned}$$

Solving for the radius we get $R = 3 \times 10^{13}$ m. The rotation period after the collapse is

$$\begin{aligned} P_2 &= 1000 \text{ a} \times \left(\frac{6.96 \times 10^8 \text{ m}}{3 \times 10^{13} \text{ m}} \right)^2 \\ &= 5.4 \times 10^{-7} \text{ a} = 17 \text{ s}. \end{aligned}$$

This is several orders of magnitude shorter than the actual period. Somehow the star has to get rid of most of its angular momentum during the process.

12.10 Exercises

Exercise 12.1 Find the free fall time scale for a hydrogen cloud, if the density of H_2 molecules is 3000 cm^{-3}. Assume that stars condense from such clouds, there are 100 clouds in the Galaxy, the mass of each cloud is $5 \times 10^4 M_\odot$, and 10 % of the mass is converted into stars. Also assume that the average mass of a star is $1\,M_\odot$. How many stars are born in one year?

Exercise 12.2 The mass of Vega (spectral class A0 V) is $2\,M_\odot$, radius $3\,R_\odot$, and luminosity $60\,L_\odot$. Find its thermal and nuclear time scales.

Exercise 12.3 Assume that a star remains 10^9 years in the main sequence and burns 10 % of its hydrogen. Then the star will expand into a red giant, and its luminosity will increase by a factor of 100. How long is the red giant stage, if we assume that the energy is produced only by burning the remaining hydrogen?