

Chapter 18

Charting the Universe with Stars

The physics of stars has turned out to be surprisingly important for cosmology. If we understand how stars work, we can use their observed properties to infer their intrinsic luminosities, and combine those with measured fluxes to determine distances. Finding stars at different distances then allows us to map the geometry of the universe. Two types of stars have come to play vital roles in cosmology: pulsating stars called Cepheids, and exploding white dwarfs called type Ia supernovae.

18.1 Stellar Pulsations

As we will see, we do not actually need to understand the physics of Cepheid variable stars in detail to use them as distance indicators. Nevertheless, the general ideas (if not the full details) are interesting and within our reach, so it is worthwhile to take a brief look at stellar pulsations.

18.1.1 Observations

For at least 400 years (and probably longer) people have noted that certain stars vary in brightness. In 1595, David Fabricius saw that α Ceti faded to the point that it became invisible to the naked eye, then returned to visibility, with a period of 11 months (see Fig. 18.1). He named the star Mira, meaning “wonderful.” In 1784, John Goodricke discovered that δ Cephei varies with a period of a little over 5 days. This is the prototype for a class of stars now called **Cepheids**.

In the early twentieth century, Henrietta Swan Leavitt was one of the “computers” working at Harvard (like Williamina Fleming and Annie Jump Cannon, whom we encountered in Chap. 14). Leavitt discovered some 2,400 Cepheids by painstakingly comparing photographs of star fields taken at different times and identifying stars

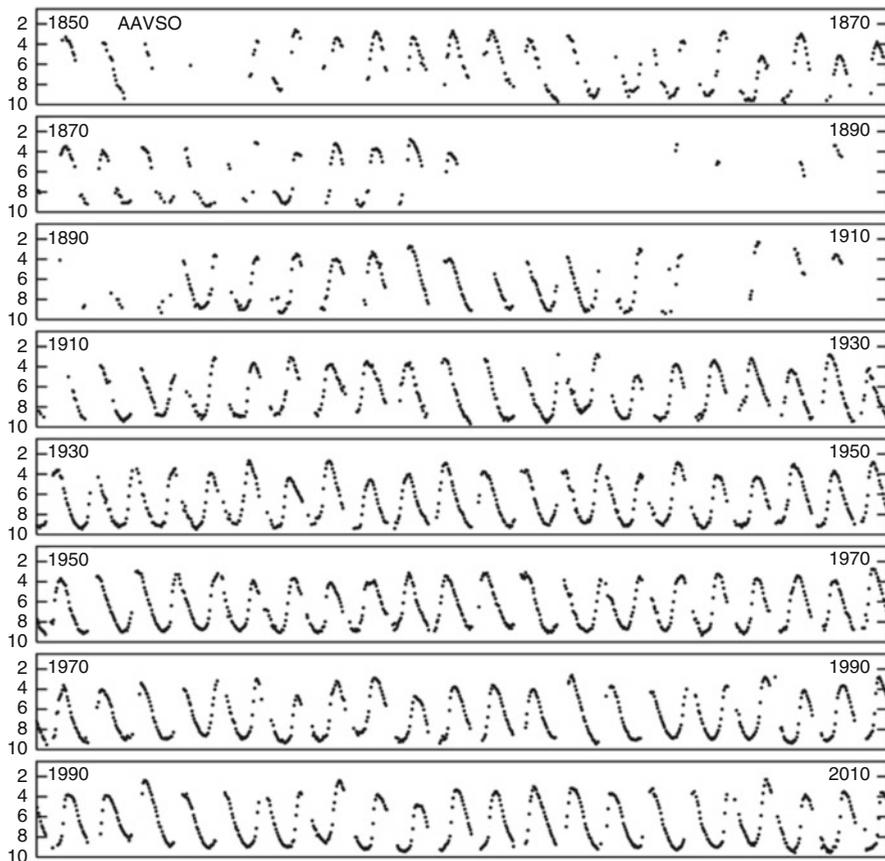
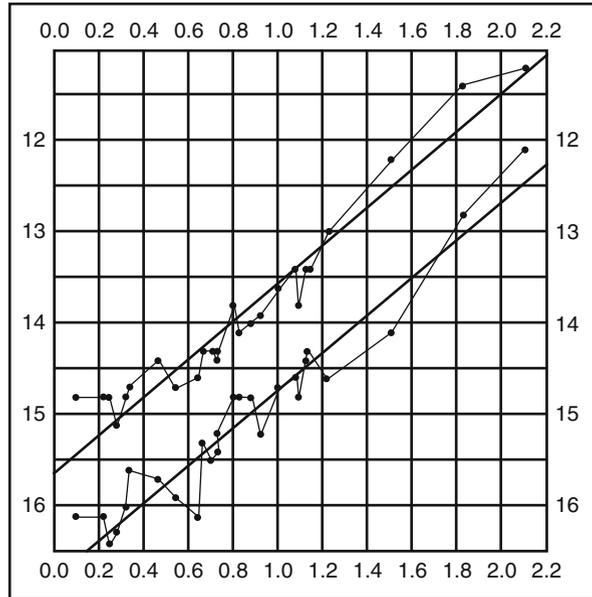


Fig. 18.1 Light curve (brightness versus time) of the variable star Mira, from 1850 to the present. Each point is the average brightness over 10 days. The vertical axis is $-2.5 \log L + \text{constant}$; because of the minus sign, the number is smaller when the star is brighter. Courtesy of the American Association of Variable Star Observers (AAVSO)

that varied. Many of them were in the Small Magellanic Cloud, a dwarf galaxy orbiting the Milky Way. Leavitt noticed that the brighter Cepheids seemed to have longer periods. Since all the stars were in the SMC, she knew the stars that appeared to be brighter were intrinsically more luminous. To check for a connection between period and luminosity, Leavitt plotted the two quantities as shown in Fig. 18.2. Her discovery of the Cepheid **period/luminosity relation**—now known as the **Leavitt law** [1]—was a breakthrough in our ability to chart the universe (as we will see in Sect. 18.2).

Fig. 18.2 Henrietta Swan Leavitt's measurement of the relation between period and brightness for variable stars in the Small Magellanic Cloud. The horizontal axis is the logarithm of the period in days, while the vertical axis is $-2.5 \log L + \text{constant}$. The two sets of points represent the maximum and minimum brightness for each star (Credit: Leavitt and Pickering [2])



Now many different kinds of variable stars are known, with periods as long as hundreds of days (as for Mira) or as short as hours or even minutes. Several classes of variable stars, including Cepheids, lie in a particular region of the HR diagram. This “instability strip” intersects the horizontal branch of the stellar evolution tracks, implying that the stars are in the helium core burning stage of their lives. We now understand that these stars are pulsating—rhythmically expanding and contracting or undergoing even more complex oscillations.

18.1.2 Theory

Pulsations in Cepheids are thought to be driven by a mechanism known as the “Eddington valve” [3, 4]. Suppose there is region of the star where compressing the gas causes the opacity to rise. If the star contracts, the opacity increases and acts as a closed valve, trapping light and causing heat and pressure to build. Conversely, if the star expands, the opacity decreases and the valve opens. A star has to have a region with a significant amount of partially ionized helium in order for opacity to behave this way, which is why opacity-driven pulsations occur in a specific region of the HR diagram.

Treating the valve mechanism in detail is beyond the scope of our analysis, but we can make a model that captures the basic physics. The key ingredient is having

pressure vary inversely with size so that squeezing a star causes it to recoil. Let's postulate an equation of state of the form¹

$$P \propto R^{-K} \quad \Rightarrow \quad P = P_0 \left(\frac{R}{R_0} \right)^{-K} \quad (18.1)$$

Furthermore, let's assume the star expands and contracts as a whole, maintaining spherical symmetry. Then we can obtain an equation of motion by considering a spherical shell of mass m at the surface of a star of mass M and radius R . Applying Newton's second law to the shell yields

$$m \frac{d^2 R}{dt^2} = -\frac{GMm}{R^2} + 4\pi R^2 P \quad (18.2)$$

where the first term on the right-hand side represents gravity pulling inward, while the second term represents pressure pushing outward. If the star were in equilibrium it would have some radius R_0 and pressure P_0 . Setting $d^2 R/dt^2 = 0$ in equilibrium, we can solve the equation of motion to find

$$P_0 = \frac{GMm}{4\pi R_0^4} \quad (18.3)$$

To analyze departures from equilibrium, let's introduce scaled variables:

$$\xi = \frac{R}{R_0} \quad \text{and} \quad \tau = \frac{t}{t_0}$$

where

$$t_0 = \left(\frac{R_0^3}{GM} \right)^{1/2} \quad (18.4)$$

is a time scale. Rewriting the equation of motion in terms of the scaled variables, and using Eqs. (18.3) and (18.1), yields

$$\frac{mR_0}{t_0^2} \frac{d^2 \xi}{d\tau^2} = -\frac{GMm}{R_0^2 \xi^2} + \frac{GMm}{R_0^2} \xi^{2-K}$$

which simplifies to

$$\frac{d^2 \xi}{d\tau^2} = -\xi^{-2} + \xi^{2-K} \quad (18.5)$$

¹An adiabatic equation of state would fit the bill: from Eq. (16.4), $P \propto R^{-3\gamma}$ for an adiabatic process. We do not necessarily assume that pulsations are adiabatic, but we do consider the possibility below.

This is the scaled equation of motion for pulsations if the equation of state has the form of Eq. (18.1). As with polytropic stars in Sect. 17.2.2, we have made some progress with the differential equation by identifying the key physical scalings so that the remaining piece is dimensionless.

In Problem 18.1 you can work with numerical solutions to Eq. (18.5). To make further analytic process here, let's imagine that the oscillations have a small amplitude such that

$$\xi = 1 + \delta\xi \quad \text{with} \quad \delta\xi \ll 1$$

Then we can make a Taylor series expansion of Eq. (18.5) to first order in ξ and obtain:

$$\frac{d^2(\delta\xi)}{d\tau^2} \approx -(K - 4)\delta\xi \quad (18.6)$$

If $K > 4$, this is the equation of motion for a simple harmonic oscillator. The angular frequency in scaled units is $(K - 4)^{1/2}$, so when we put in the physical scaling from Eq. (18.4) we obtain the angular frequency and period in physical units as follows:

$$\omega = \left[\frac{GM}{R_0^3} (K - 4) \right]^{1/2} \quad \text{and} \quad \Pi = \frac{2\pi}{\omega} = 2\pi \left[\frac{R_0^3}{GM(K - 4)} \right]^{1/2} \quad (18.7)$$

We see that the period depends on the star properties through the combination M/R_0^3 , which is proportional to the mean density. From the Vogt-Russell theorem of stellar structure (see Sect. 16.2.3), the radius and mean density are determined mainly by the mass, and to a lesser extent by the composition; thus, there should be a reasonably tight relation between period and mass. Since pulsating stars lie on the horizontal branch in the HR diagram, and thus are in a similar phase of evolution, there is a close relation between mass and luminosity. Putting the pieces together, we imagine there to be a reasonably tight relation between period and luminosity—which is exactly what Henrietta Swan Leavitt discovered.

What can we say quantitatively about the scaling of period with mass? If we postulate that stars on the horizontal branch have a mass–radius relation of the form $R \propto M^\alpha$, then the period scales as $\Pi \propto M^{(3\alpha-1)/2}$. If $\alpha > 1/3$ then the period will increase as the mass increases. In the examples below we see that this is indeed the case.

The period clearly depends on K . Without getting into details of the Eddington valve, we might consider a simple case in which the fluctuations are adiabatic. From Eq. (16.4), the adiabatic equation of state is $P \propto R^{-3\gamma}$ where the adiabatic index γ is the ratio of specific heats. An ideal, non-relativistic, monatomic gas has $\gamma = 5/3$ and hence $K = 5$. We use this in Eq. (18.7) and plug in numbers to work in reference to the Sun:

$$\begin{aligned}
\Pi &= 2\pi \left(\frac{R_{\odot}^3}{G M_{\odot}} \right)^{1/2} \times \frac{(R/R_{\odot})^{3/2}}{(M/M_{\odot})^{1/2}} \\
&= 2\pi \left[\frac{(9.6 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}) \times (1.99 \times 10^{30} \text{ kg})} \right]^{1/2} \times \frac{(R/R_{\odot})^{3/2}}{(M/M_{\odot})^{1/2}} \\
&= (1.0 \times 10^4 \text{ s}) \times \frac{(R/R_{\odot})^{3/2}}{(M/M_{\odot})^{1/2}} \\
&= 0.12 \text{ day} \times \frac{(R/R_{\odot})^{3/2}}{(M/M_{\odot})^{1/2}}
\end{aligned}$$

This leads to the following quantitative examples:

- If the Sun were to pulsate in the way we have described, it would have a period of about 0.12 day. (It would not actually be a Cepheid, though, because it is not in the helium burning phase.)
- The star δ Cephei is the original Cepheid variable. It has $M \approx 5 M_{\odot}$ and $R \approx 45 R_{\odot}$ [5, 6], so our model predicts $\Pi = 15.6$ day. For comparison, the observed period is 5.4 days.
- Well-studied Cepheids in our galaxy have masses in the range 4–11 M_{\odot} and radii in the range 30–120 R_{\odot} [6, 7]. The corresponding range of periods, according to our model, is then:

$$\begin{aligned}
\text{low mass : } \Pi_{\text{lo}} &= 0.12 \text{ day} \times \frac{30^{3/2}}{4^{1/2}} = 9.5 \text{ day} \\
\text{high mass : } \Pi_{\text{hi}} &= 0.12 \text{ day} \times \frac{120^{3/2}}{11^{1/2}} = 46 \text{ day}
\end{aligned}$$

For comparison, the observed range of periods is about 3–40 day.

Our model is not highly accurate because it is based on a simplified treatment of the pulsation physics. Nevertheless, it gives useful estimates, and it reveals in a general way how the pulsation period depends on the properties (mass and radius) of a star.

18.2 Standard Candles

The practical value of Cepheids comes from their use as distance indicators. Since we cannot lay down a ruler to another star, let alone another galaxy, we need to find indirect ways to measure distances. Distance appears, of course, in the inverse square law relating flux and luminosity,²

²In an expanding universe, the d in (18.8) is the “luminosity distance” (see Sect. 11.3.2).

$$F = \frac{L}{4\pi d^2} \quad (18.8)$$

We measure F , so if we somehow knew L then we could use Eq. (18.8) to infer d . Put another way, if we had a set of **standard candles** with known luminosities, we could use their fluxes to determine their distances.

The Leavitt law made that possible by letting astronomers determine a Cepheid's luminosity from a direct measurement of its pulsation period. In fact, it was by combining Cepheid-based distance measurements with Doppler-based velocity measurements that Edwin Hubble discovered the expanding universe (see Sect. 11.1). Astronomers have continued to refine and extend the use of Cepheids as standard candles, even making it a Key Project for the Hubble Space Telescope [8].

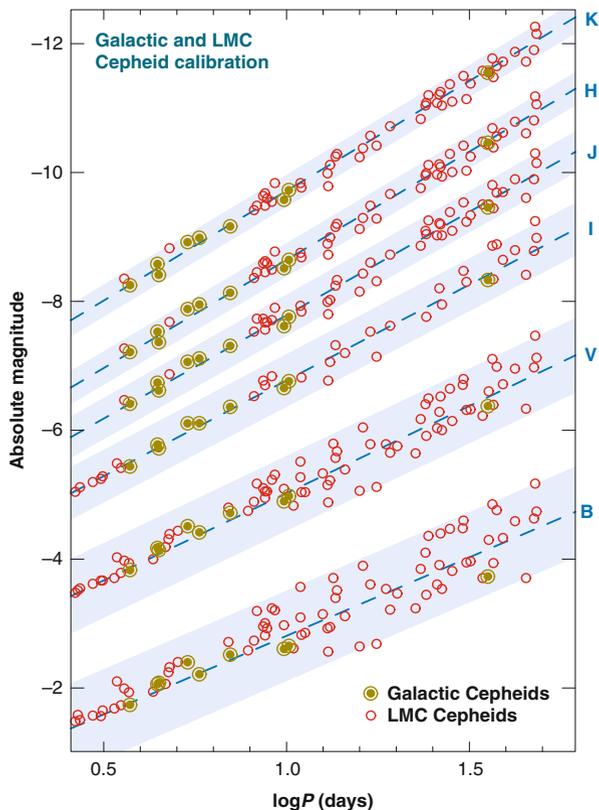
There are some challenges in using standard candles to chart the universe. As a matter of principle, we might be concerned that we lack a complete theoretical interpretation of the Leavitt law. While we can understand the general physics of stellar pulsations (as in Sect. 18.1), that is not the same as predicting Cepheid properties from first principles. The period/luminosity relation that we use is primarily empirical, and we cannot be certain that it captures all of the important physics. For example, luminosity may depend on variables beyond period, such as color and metallicity (see [1]). If additional parameters are important, neglecting them could create scatter in the period/luminosity relation; that would be inconvenient but not terrible. More troublesome is the possibility that there could be a systematic shift in the period/luminosity relation between different galaxies that have, say, different distributions of metallicity. Fortunately, such a possibility can be tested empirically by comparing the period/luminosity relation in many different galaxies.

As a matter of practice, we need to calibrate Cepheids as standard candles before we can use them to measure distances. Strictly speaking, what Henrietta Swan Leavitt discovered was a relation between period and flux for a set of Cepheids in the Small Magellanic Cloud (SMC). Since the stars were all at (essentially) the same distance from Earth, there was a direct proportionality between flux and luminosity, but in order to determine the proportionality constant—and thereby calibrate the period/luminosity relation—Leavitt needed an independent measurement of the distance to the SMC. Today, astronomers use two main techniques to establish the calibration. One is to measure distances to Cepheids in our own Milky Way galaxy using parallax.³ The other is to observe Cepheids in the Large Magellanic Cloud (LMC), and then measure the distance to the LMC independently (which can be done several different ways; see [1] for a review). Figure 18.3 shows that the two techniques yield consistent results for the relation between period and luminosity.

Last but not least, Cepheids can be observed (even with the Hubble Space Telescope) only in galaxies that are relatively nearby in cosmic terms. If we want to chart galaxies that are more distant, we need to find a different standard candle.

³Parallax is a kind of triangulation that uses Earth's motion around the Sun to provide a different perspective on nearby stars relative to background objects (see Sect. 2.1).

Fig. 18.3 Modern version of Leavitt law for Cepheids in the Milky Way (*filled symbols*) and the Large Magellanic Cloud (LMC, *open symbols*). The vertical axis is $-2.5 \log L + \text{constant}$. The different shaded bands indicate brightnesses measured in different filters, from *blue* (bottom) to *infrared* (top) wavelengths (Credit: Freedman and Madore [1], reproduced by permission)



This is where type Ia supernovae became important tools for cosmology. The light curves of these exploding stars exhibit a characteristic rise and fall, and the duration and shape of the curve turn out to be related to the peak luminosity (as shown in the top panel of Fig. 18.4). Correcting for the “stretch” makes it possible to place supernovae on a common curve so they serve as standard candles (bottom panel of Fig. 18.4).

Type Ia supernovae are bright enough to be seen at great distances (out to cosmological redshifts of $z \sim 1$ and beyond, corresponding to billions of light years). As with Cepheids, though, we lack a theoretical understanding of the all-important relation—in this case, between light curve shape and luminosity. We know that type Ia supernovae occur when a white dwarf accretes enough mass from a binary companion to cross the Chandrasekhar limit and explode (see Sect. 17.2.2), but we do not know whether the companion is another white dwarf or a non-degenerate star. While we do not necessarily need to understand all of the details in order to use type Ia supernovae to measure distances, we do need to consider the possibility that the explosions might not always be the same. This, again, is an issue that can be tested empirically by observing a large sample of supernovae in different galaxies.

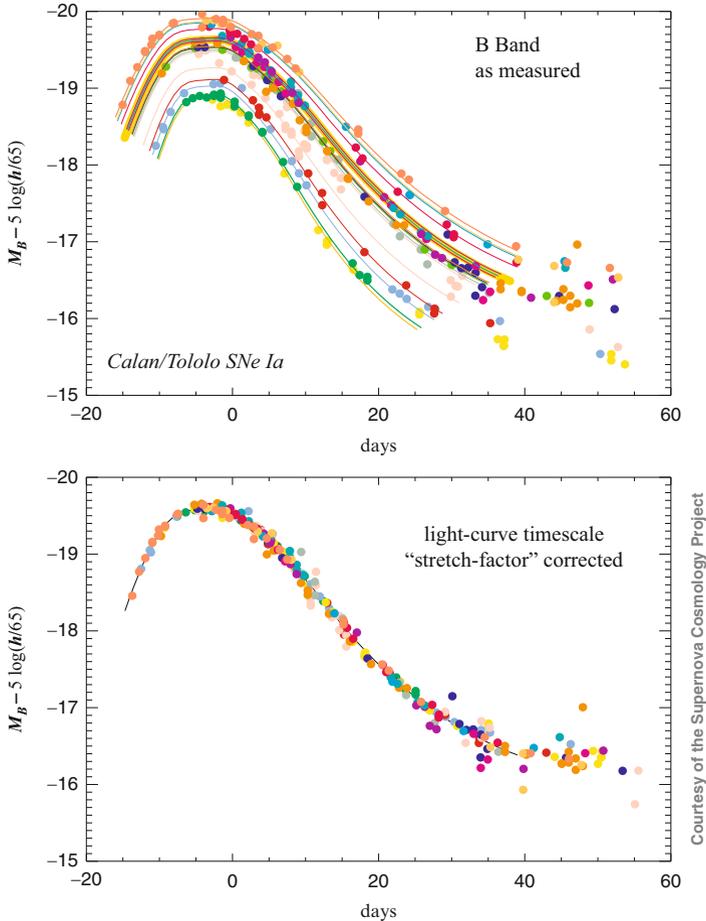


Fig. 18.4 Light curves for type Ia supernovae; different colors denote different observed explosions. For the vertical axis, $M_B = -2.5 \log L + \text{constant}$. (The term $5 \log(h/65)$ is a way astronomers have dealt with imperfect knowledge of the Hubble constant.) For the horizontal axis, time is measured from the peak brightness; the explosion occurs a few weeks before the peak. The *top panel* shows that brighter supernovae tend to have wider light curves. In the *bottom panel*, the observed relation has been used to derive a “stretch” factor that places all of the supernovae on a common curve (Reprinted with permission from Perlmutter [9]. Copyright 2003, American Institute of Physics)

To calibrate type Ia supernovae as standard candles, we can piggyback off of Cepheids: if a supernova is observed in a galaxy whose distance has been measured with Cepheids, we can use that supernova and its distance as an anchor for measuring other supernova distances. The idea of using different but overlapping distance indicators is known as building a **distance ladder**. While modern distance ladders have a number of parallel and interlocking rungs, stellar pulsations and

explosions still play a central role in charting the universe. In fact, it was Cepheid-calibrated type Ia supernovae that led to the discovery of the accelerated expansion of the universe (see Fig. 11.4).

Problems

18.1. We can study pulsations numerically to complement the analysis in Sect. 18.1.2.⁴ Section A.6 discusses a computational algorithm to solve a second-order differential equation such as (18.5). Implement the algorithm in a spreadsheet or computer program. To obtain a specific solution you need to give initial conditions ξ_{init} and v_{init} ; one reasonable approach is to set $v_{\text{init}} = 0$.

- Briefly explain how you solve the differential equation.
- Assume an adiabatic equation of state with $\gamma = 5/3$. Plot the size of the star and the velocity of the surface (in scaled units) assuming $\xi_{\text{init}} = 1.05$. Determine the oscillation period of your numerical solution, and explain how you did it. Calculate the expected period of small-amplitude oscillations from Eq. (18.7). How well do the period values agree?
- Pick a different value of K , find the numerical solution, and compare the oscillation period with the prediction from Eq. (18.7).
- Now go back to the case with $\gamma = 5/3$ and try $\xi_{\text{init}} = 0.65$. What is the oscillation period? Why don't these plots look like your previous ones? Why doesn't the analytic prediction do a good job here?

18.2. In this problem you can get a sense of how to use Cepheids as standard candles to measure distances.

- Imagine you observe 5 Cepheids in a galaxy 1 Mpc away, with the following periods and fluxes:

Star	P (day)	F/\hat{F}
A	10.75	1.17
B	46.53	6.80
C	6.19	0.48
D	36.07	4.15
E	7.89	0.86

Here \hat{F} is a reference flux value, taken to be the flux of a Cepheid with a period of $\hat{P} = 10$ day when seen from a distance of 1 Mpc. Make a plot of $\log P$

⁴This problem is inspired by Problem 14.13 in the book by Carroll and Ostlie [10], but we work with the dimensionless version of the equation of motion (18.5).

and $\log F$ (similar to Fig. 18.3) and use it to calibrate a relation of the form $F/\hat{F} = (P/\hat{P})^\alpha$ by determining α . Hints: a power law relation appears as a straight line in a log-log plot; there is some “noise” in the data, so the plot will not be a perfect line.

- (b) Now suppose you observe a second galaxy and discover a Cepheid with period $P_2 = 18.34$ day and flux $F_2/\hat{F} = 0.22$. How far away is galaxy #2?

18.3. In this problem you can see how to construct a distance ladder. Suppose you observe 10 galaxies: in #1 you measure a well-calibrated Cepheid; in #2–3 you see both Cepheids and type Ia supernovae; and in #4–10 you find SN Ia. Here are the data, including distance and Doppler velocity where known. (The flux units are arbitrary.) For this problem, assume Cepheids have a simple period/luminosity

	Galaxy data		Cepheid data		SN Ia data
	v (km s ⁻¹)	D (Mpc)	P (day)	Flux	Flux
#1		10	6.85	5.45×10^{-4}	
#2			15.57	2.07×10^{-4}	1.33×10^2
#3			18.97	9.98×10^{-5}	5.26×10^1
#4	4,675				1.99×10^1
#5	5,734				1.33×10^1
#6	7,056				8.78
#7	7,081				8.69
#8	12,361				2.85
#9	13,224				2.49
#10	13,472				2.40

relation of the form $L \propto P$, and assume the SN Ia fluxes have been corrected for light curve shape.

- (a) Use the Cepheids to determine the distances to galaxies #2–3.
 (b) Use the supernovae in galaxies #2–3 to calibrate the SN flux/distance relation. This amounts to finding the intrinsic luminosity of a type Ia supernova. Verify that both galaxies give consistent results.
 (c) Use the supernovae to determine distances to at least two of the remaining galaxies. Then determine the Hubble constant H_0 in the relation $v = H_0 D$.

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