Chapter 20
Cosmology: Early Universe

In Chap. 18 we saw how Cepheid stars and type Ia supernovae have been used to measure the expansion of the universe. If we run the clock backward, we deduce that in the past the universe was smaller, denser, and hotter than it is today. In this chapter we use gas physics and particle physics to understand the early, hot phase of the universe, and we discuss observations that probe this phase directly (through the cosmic microwave background radiation) and indirectly (through the abundances of elements created in the first few minutes after the big bang).

20.1 Cosmic Microwave Background Radiation

When the universe was young, it was hot enough for gas to be ionized. The many free electrons were very effective at scattering light, so the gas was optically thick and photons were tightly coupled to matter. As the universe expanded and cooled, the electrons and ions were able to come together to form neutral atoms. Suddenly the photons were liberated, free to travel great distances through the universe. Those photons are still around and visible as the **cosmic microwave background (CMB)** radiation.

These ideas were initially developed in the late 1940s by Ralph Alpher, George Gamow, and Robert Herman (see [1]), and later by Robert Dicke and Jim Peebles (see [2]). In the early 1960s, Arno Penzias and Robert Wilson were working with a new microwave antenna in Holmdel, NJ. They measured a low level of noise no matter what direction they pointed the antenna, which persisted despite all efforts to clean the antenna and eliminate sources of noise (including pigeons). They realized the signal was consistent with blackbody radiation at a temperature $\sim$3 K, but did not know how to interpret such a signal. Eventually they heard that Dicke, Peebles, and others at Princeton University had predicted microwave radiation from the early

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universe and even set out to look for it.[1] The Holmdel and Princeton groups got together, and the rest is history [4,5]. Penzias and Wilson won the 1978 Nobel Prize in Physics for their discovery of the CMB.

20.1.1 Hot Big Bang

We said the universe was hotter when it was younger, but to study the CMB we need to quantify that statement. In Chap. 11 we discussed the theoretical framework for describing the expansion of the universe. For our purposes here, there are two key results. First, we characterize the expansion using the dimensionless scale factor $a(t)$ such that distances scale as a and volumes scale as a^3 relative to today. Second, the expansion causes light waves to stretch, creating the cosmological redshift. From Eqs. (11.19) and (11.20), the ratio of observed and emitted wavelengths is

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{1}{a}$$

Suppose the universe today is filled with blackbody radiation with some temperature T_0. In the past, the wavelengths were all smaller by the factor a. What was the corresponding temperature? From the Planck spectrum (Eq. 13.8), or equivalently Wien's law (Eq. 13.13), wavelength and temperature are related by $\lambda T = \text{constant}$, which immediately implies

$$T \propto a^{-1} \quad \Rightarrow \quad T = \frac{T_0}{a} \tag{20.1}$$

This allows us to characterize how the universe has cooled as it has expanded.

We also need to specify how the density has changed. Density times volume is mass, so if total mass is conserved[2] then ρa^3 is constant. In other words, conservation of mass implies

$$\rho \propto a^{-3} \quad \Rightarrow \quad \rho = \frac{\rho_0}{a^3} \tag{20.2}$$

Combining Eqs. (20.1) and (20.2), we can write a relation between density and temperature:

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^3 \tag{20.3}$$

[1] See [3] for more of the story.

[2] While some mass is converted to energy via fusion in stars, it is a tiny fraction of the total.

20.1.2 Theory: Recombination Temperature

As the universe expanded, two effects caused photons to **decouple** from matter. As the density decreased, the photon's mean free path increased; at some point this effect alone would have made the gas optically thin. But another process was also underway: as the universe cooled, electrons and ions were able to combine to form neutral atoms in a process known as **recombination**. The gas effectively became transparent, leaving the universe filled with a bath of photons that are free but carry a memory of the physical conditions of the universe at the time they were released.

We can estimate the temperature of the universe at recombination, which helps us understand the state of the universe that we study when we observe the CMB (also see [6]). For simplicity, let's assume the universe was pure hydrogen. Let n_b and ρ_b be the total number and mass density of baryons, which in a hydrogen universe just means protons.[3] Let X be the ionization fraction, so the number densities of free electrons and ions are $n_e = n_i = X n_b$ while the number density of neutral hydrogen atoms is $n_H = (1 - X) n_b$. If the gas was in equilibrium, the Saha equation (14.3) gives the ratio of ionized to neutral atoms to be

$$\frac{n_i}{n_H} = \frac{X}{1 - X} = \frac{2\, Z_{II}}{X\, n_b\, Z_I} \left(\frac{2\pi\, m_e\, k\, T}{h^2} \right)^{3/2} \mathrm{e}^{-\chi_I / kT} \tag{20.4}$$

(Note that X appears on both the left- and right-hand sides.) We can relate the baryon number density, n_b, to its value today using Eq. (20.3):

$$n_b = \frac{\rho_b}{m_p} = \frac{\rho_{b0}}{m_p} \frac{T^3}{T_0^3}$$

Since $n_b \propto T^3$, the net temperature scaling on the right-hand side of Eq. (20.4) is $T^{-3/2} \exp(-\chi_I / kT)$. For hydrogen, the relevant numbers are:

$$\begin{aligned} Z_I &= 2 \\ Z_{II} &= 1 \\ \chi_I &= 13.6\,\mathrm{eV} \\ \rho_{b0} &= 4.14 \times 10^{-28}\,\mathrm{kg\,m^{-3}} \\ T_0 &= 2.725\,\mathrm{K} \end{aligned}$$

[3] Electrons are leptons, and they contribute so little mass compared to baryons that we neglect them when characterizing the mass density of the universe. They are important when it comes to charge, though.

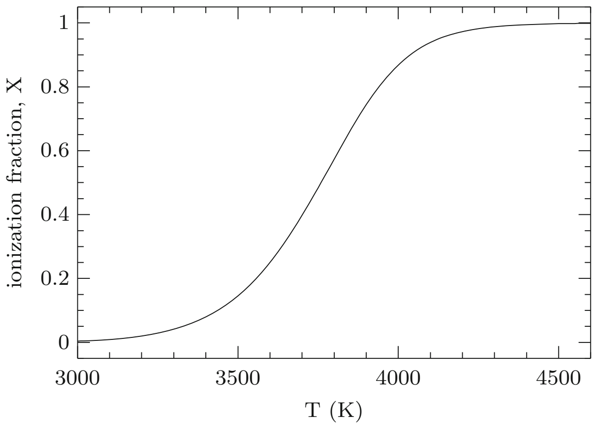

Fig. 20.1 Equilibrium ionization fraction as a function of temperature, for a universe of pure hydrogen

Plugging in the numbers lets us rewrite Eq. (20.4) as

$$\frac{X^2}{1-X} = 1.57 \times 10^{17} \left(\frac{kT}{\text{eV}}\right)^{-3/2} \text{e}^{-13.6\,\text{eV}/kT}$$

This is a quadratic equation that we can solve to plot X as a function of temperature, as shown in Fig. 20.1. According to this simple estimate, recombination should have occurred between about 3,500 and 4,000 K. More detailed analyses account for the fact that a photon released when one atom combined could reionize a nearby neutral atom, and place everything in an expanding universe (e.g., [7]). Those analyses indicate that the temperature had to be a little lower, around 3,000 K, for recombination to be complete.[4]

Incidentally, inverting Eq. (20.1) lets us determine the scale factor at recombination:

$$a_{\text{recomb}} = \frac{T_0}{T_{\text{recomb}}} \approx \frac{2.7\,\text{K}}{3{,}000\,\text{K}} \approx \frac{1}{1{,}100}$$

At the time we observe with the CMB, distances were more than a 1,000 times smaller, and densities more than a billion times larger, than they are today.

20.1.3 Observations

Mapping the CMB is a vital part of cosmology today. The first detailed maps were obtained by the Cosmic Background Explorer (COBE, launched in 1989). The CMB spectrum measured by COBE (Fig. 20.2) matches a theoretical blackbody spectrum

[4]Recall that we made a similar calculation for the Sun's hydrogen in Sect. 14.1.3 and found a higher transition temperature of $\sim 10^4$ K. In the Sun, the higher density facilitates recombination.

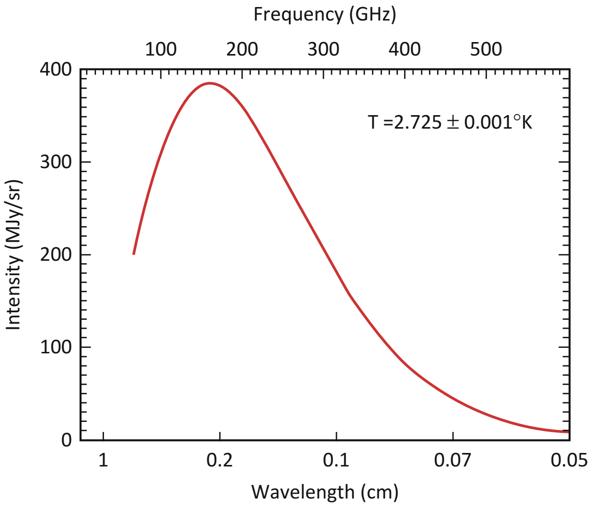

Fig. 20.2 CMB spectrum measured by the FIRAS instrument on COBE. The intensity is plotted as a function of frequency (top axis); the corresponding wavelengths are shown on the bottom axis. The curve shows a theoretical blackbody spectrum with a temperature of 2.725 K. Points with errorbars are shown, but they are actually too small to see (Credit: NASA/WMAP)

phenomenally well; in fact, the CMB is the best blackbody known. This is the first piece of evidence that we truly understand what was happening in the early universe when the CMB was produced. COBE also found that the CMB is not quite uniform: there are small **anisotropies**, or directions where the temperature is slightly warmer or cooler than the average. George Smoot and John Mather won the 2006 Nobel Prize in Physics for their discoveries with COBE.

In the years since COBE, a number of instruments have been used to map the CMB (see Fig. 20.3). The Wilkinson Microwave Anisotropy Probe (WMAP, launched in 2001 [8]) observed the full sky with better resolution than COBE. The ground-based Atacama Cosmology Telescope (ACT [9]) and South Pole Telescope (SPT [10]) have produced maps that cover only a portion of the sky but have even higher resolution than WMAP. The Planck spacecraft (launched in 2009 [11]) recently mapped the full sky at a resolution higher than WMAP but not quite as high as SPT and ACT.

The anisotropies are small—less than 1 part in 10,000—but measurable. In a map like Fig. 20.3, it is apparent that warmer and colder regions tend to have a characteristic angular size. We quantify this effect in terms of the **angular power spectrum**, shown in Fig. 20.4. Roughly speaking, you can think of creating a circle with some particular angular size, computing the average temperature within that circle, and then measuring the variations as you move the circle around the sky. Repeating the process for circles with different sizes yields the power spectrum. The CMB power spectrum shows a prominent peak at about 1°, which is the main scale visible in Fig. 20.3, but there are significant features on other scales as well.

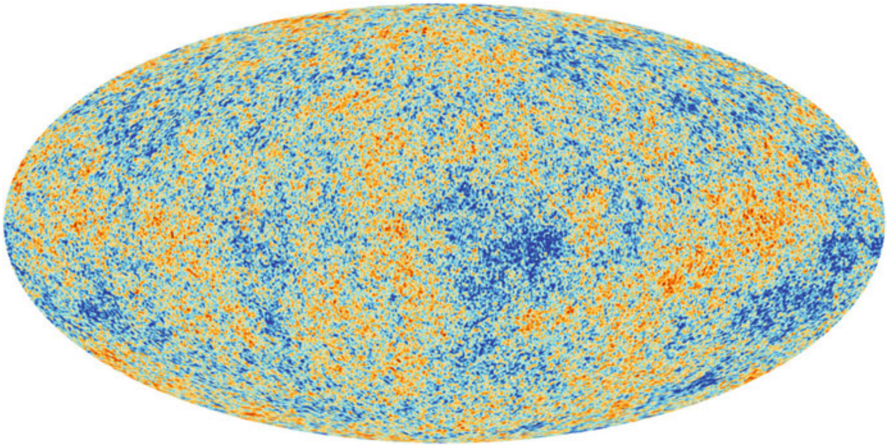

Fig. 20.3 Map of temperature fluctuations in the CMB, from the Planck spacecraft. In this projection, the plane of the Milky Way galaxy runs through the middle of the map from left to right (© ESA and the Planck Collaboration [11])

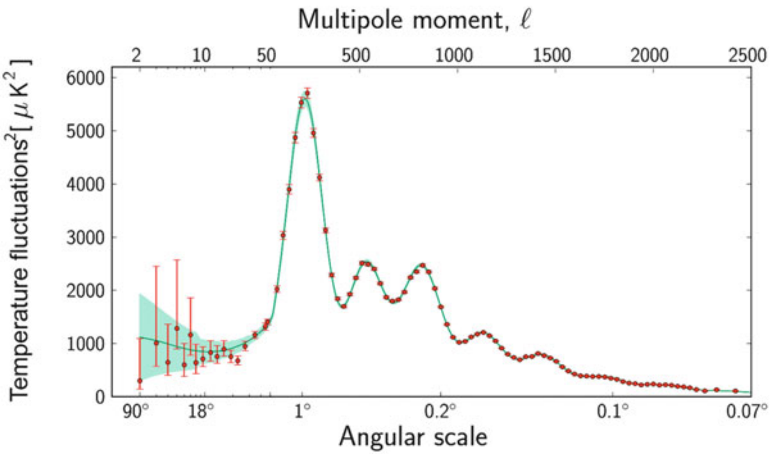

Fig. 20.4 Angular power spectrum of temperature fluctuations in the CMB. The horizontal axis shows the angular scale running from large angles on the left to small angles on the right. (Formally, the power spectrum is computed in terms of the multipole moment ℓ, indicated on the top axis.) The vertical axis quantifies the amplitude of temperature fluctuations. Points with errorbars are Planck measurements; the curve indicates a ΛCDM model fit to the data (© ESA and the Planck Collaboration [11])

20.1.4 Implications

The anisotropies in the CMB are important for two reasons. First, they represent seeds from which galaxies grew. What we see as temperature fluctuations actually correspond to variations in density. Overdensities create cold spots in the temperature map because photons lose energy as they climb out of a gravitational potential well; this is known as the **Sachs-Wolfe effect** [12]. In places where the density was a little higher than average, gravity was a little stronger, which tended to pull in more matter from nearby, causing the overdensity to grow, making gravity even stronger, drawing even more matter, and ... well, you get the picture. There was a runaway process that turned tiny overdensities into the large structures we see today as galaxies and even clusters of galaxies. We know those all-important seeds were there because we observe them in the CMB.

Second, the anisotropies constrain the geometry and composition of the universe. Before recombination, the material in the universe was governed by a fairly straightforward combination of gas physics and gravity. Over- and underdensities acted as sound waves propagating through a nearly-uniform medium in an expanding universe. The sound waves had a characteristic physical scale, which we can compare with the measured angular scale to determine the geometry of the universe. Also, the power spectrum of fluctuations depended on the relative abundances of normal matter, dark matter, and dark energy in the universe. Although we will not delve into the details, it is possible to predict the CMB power spectrum for different assumptions about the composition of the universe. Adjusting the predictions to match the data (as shown by the curve in Fig. 20.4) yields strong constraints on cosmological parameters. (See [13] for a full discussion of cosmological constraints from Planck.) Constraints from the CMB and other datasets are shown in Fig. 11.6. It is striking is that three very different ways of probing the universe yield consistent results. Even if we do not yet know what dark matter and dark energy are, we think we know how much of each substance the universe contains.

20.2 Big Bang Nucleosynthesis

The CMB provides direct access to the physical state of the universe when it was about 380,000 years old. We can reach back even further—to when the universe was only a few minutes old—by using the idea that the very young universe was a nuclear reactor.

20.2.1 Theory: "The First Three Minutes"

When the universe was young and hot, it was filled with a sea of elementary particles: protons, neutrons, electrons, positrons, neutrinos, antineutrinos, and photons. Why not anti-protons and anti-neutrons? This is the unsolved question of **baryogenesis**: why is there a (slight) asymmetry favoring matter over antimatter in our universe? If there were exact symmetry between matter and antimatter, there would have been equal numbers of baryons and anti-baryons in the early universe, and they would have annihilated to leave a universe filled only with photons. Particle physicists are still trying to understand the origin of the asymmetry, but for our purposes we simply accept that there is matter in the universe and attempt to understand how the density of matter affects things we can measure.

Early on, the temperature and density were high enough to allow reactions among the particles[5]:

$$n + e^{+} \rightleftharpoons p + \bar{\nu}_e \tag{20.5a}$$

$$n + \nu_e \rightleftharpoons p + e^{-} \tag{20.5b}$$

$$n \rightleftharpoons p + e^{-} + \bar{\nu}_e \tag{20.5c}$$

How high did the temperature have to be for such reactions to occur? The neutron weighs a little more than the proton, so there is a Boltzmann factor that describes their relative numbers (in thermodynamic equilibrium):

$$\frac{N_n}{N_p} \sim \mathrm{e}^{-(m_n - m_p)c^2/kT}$$

The mass difference corresponds to energy difference

$$\Delta E_{np} = (m_n - m_p)c^2 = 1.29\,\mathrm{MeV}$$

Roughly speaking, then, protons and neutrons could be in thermodynamic equilibrium only when

$$kT \gtrsim 1.29\,\mathrm{MeV} \quad \Rightarrow \quad T \gtrsim 1.5 \times 10^{10}\,\mathrm{K}$$

As the universe cooled, two things happened. First, the "inverse" reactions slowed down and protons began to outnumber neutrons. Second, while the protons and neutrons were doing their thing, electron/positron pairs were forming and annihilating:

$$e^{-} + e^{+} \rightleftharpoons 2\gamma$$

Electrons and positrons each have a mass of 0.51 MeV, so this reaction could occur only when

$$kT \gtrsim 1.02\,\mathrm{MeV} \quad \Rightarrow \quad T \gtrsim 1.2 \times 10^{10}\,\mathrm{K}$$

Once the temperature fell below about 1 MeV, it was no longer possible to form new electron/positron pairs. As existing pairs annihilated, the decreasing number of electrons made it harder to form neutrons (see Eqs. 20.5b and 20.5c). When all the details are taken into account, it turns out that the above reactions ceased when

[5]For more discussion of particle physics in the early universe, see [6, 14, 15].

the temperature cooled to about 0.8 MeV. This is called **neutron freezeout**, and it (momentarily) fixed the relative abundance of protons and neutrons to be

$$\frac{N_n}{N_p} \approx \mathrm{e}^{-1.3\,\mathrm{MeV}/0.8\,\mathrm{MeV}} \approx 0.20$$

I say "momentarily" because free neutrons spontaneously decay by the process in Eq. (20.5c), with a half-life of just 615 s. The remaining neutrons did begin to decay, and the neutron/proton ratio fell to

$$\frac{N_n}{N_p} \approx \frac{1}{7} \tag{20.6}$$

Another process kicked in once the temperature cooled to about 0.1 MeV: neutrons could combine with protons to form deuterium,

$$n + p \rightarrow {}^2_1\mathrm{H} + \gamma$$

Neutrons that are bound into nuclei are stable, so this was a key step that locked in the (primordial) abundance of neutrons. Once deuterium formed, it could go through various reactions to create helium-4 (see, e.g., [16]). One channel involves hydrogen-3,

$$\begin{aligned} {}^2_1\mathrm{H} + {}^2_1\mathrm{H} &\rightarrow {}^3_1\mathrm{H} + p \\ {}^3_1\mathrm{H} + {}^2_1\mathrm{H} &\rightarrow {}^4_2\mathrm{He} + n \end{aligned}$$

while another involves helium-3,

$$\begin{aligned} {}^2_1\mathrm{H} + p &\rightarrow {}^3_2\mathrm{He} + \gamma \\ {}^2_1\mathrm{H} + {}^2_1\mathrm{H} &\rightarrow {}^3_2\mathrm{He} + n \\ {}^3_2\mathrm{He} + {}^2_1\mathrm{H} &\rightarrow {}^4_2\mathrm{He} + p \end{aligned}$$

The process could even produce a little lithium and beryllium:

$$\begin{aligned} {}^4_2\mathrm{He} + {}^3_1\mathrm{H} &\rightarrow {}^7_3\mathrm{Li} + \gamma \\ {}^4_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^7_4\mathrm{Be} + \gamma \end{aligned}$$

It is possible to work out the reaction rates in detail, taking into account the expansion and cooling, and thus predict the abundances of different elements as a function of time. Figure 20.5 shows the results and reveals two important points. First, all of the action occurred when the universe was just a few hundred seconds old. This is reflected in the title of a famous book by Steven Weinberg about the formation of the elements: *The First Three Minutes* [18].

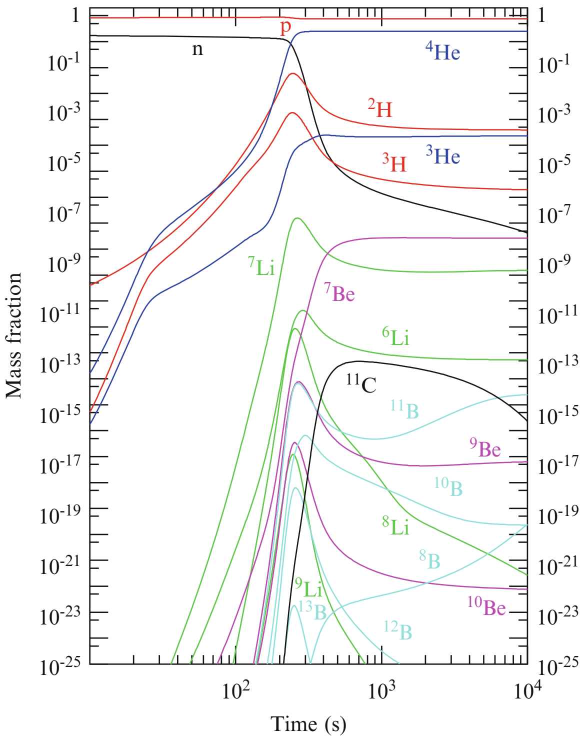

Fig. 20.5 Predicted abundances of light elements as a function of time in the early universe ("p" and "n" indicate protons and neutrons, respectively). The vertical axis is mass fraction; note the enormous range reflected in the logarithmic scale (Credit: Coc et al. [17]. Reproduced by permission of the AAS)

Second, the binding energy of helium-4 makes it very stable (recall our discussion of fusion in Chap. 15), so the vast majority of neutrons wound up in helium-4. We can estimate the mass fraction of helium-4 in the primordial gas as follows. Each helium-4 nucleus has 2 neutrons. From (20.6), there must be about 14 protons for those 2 neutrons. Two of the protons are in the helium-4 with the neutrons, leaving 12 protons left over. Thus, there are about 12 hydrogens for every helium. If we consider the *mass* fraction that is in helium, we have

$$\frac{M_{\mathrm{He}}}{M_{\mathrm{H}} + M_{\mathrm{He}}} \approx \frac{1 \times 4m_p}{12 \times m_p + 1 \times 4m_p} \approx \frac{4}{16} \approx 0.25$$

In other words, we predict that about 25 % of the mass of the primordial gas was helium (and almost all the rest was hydrogen). This follows directly from the neutron/proton ratio and is not very sensitive to other details of the nuclear processes.

While most of the deuterium and helium-3 went into helium-4, small amounts stuck around. It is possible to analyze the reactions and predict the relative abundances of all the different elements. The results depend, not surprisingly, on the total density of baryons in the universe, specifically in the combination $\Omega_b h^2$ where $h = H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$ is a dimensionless version of the Hubble constant.

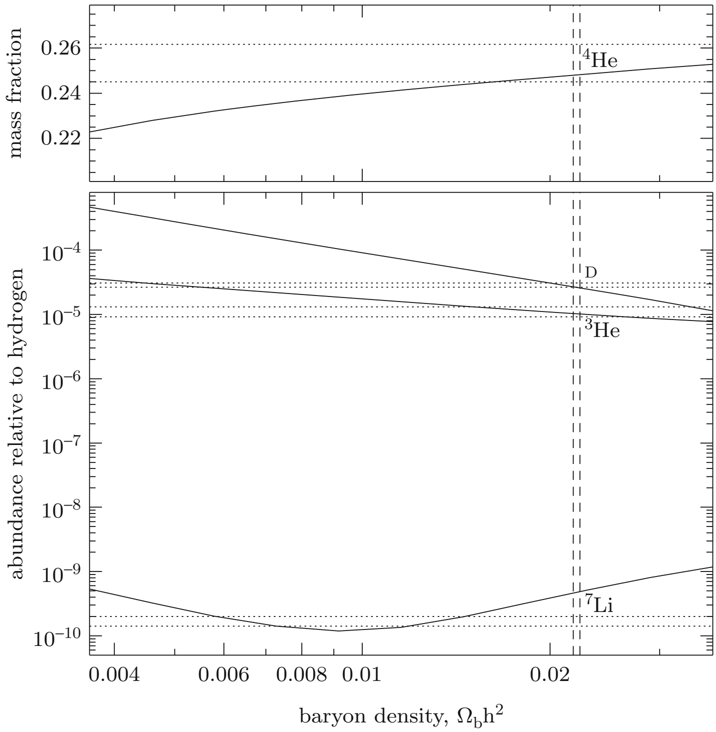

Fig. 20.6 The *solid curves* show theoretical predictions from Burles et al. [19] for the abundance of light elements as a function of the cosmic density of baryons; the natural density parameter is $\Omega_B h^2$ where $h = H_0/(100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1})$ is a dimensionless version of the Hubble constant. For the *top panel* the vertical axis is mass fraction, while for the *bottom panel* the vertical axis is the number ratio relative to hydrogen. The horizontal bands with *dotted lines* indicate measurements of primordial abundances (with the band thickness indicating uncertainties) for $^4_2$He [20], D [21], $^3_2$He [22], and $^7_3$Li [23]. The vertical band with *dashed lines* indicates the constraint on $\Omega_b h^2$ from Planck [13]

Figure 20.6 shows the predicted abundances of light elements as a function of this density parameter. The formation of elements in the early universe is known as **big bang nucleosynthesis (BBN)**, and Fig. 20.6 encapsulates the key theoretical results.

20.2.2 Observations: Primordial Abundances

With predictions in hand, we would like to test them observationally. The challenge is figuring out how to uncover the *primordial* abundances of elements, because much of the gas in the universe has been “polluted” by the lives and deaths of stars in the 13.8 billion years since big bang nucleosynthesis.

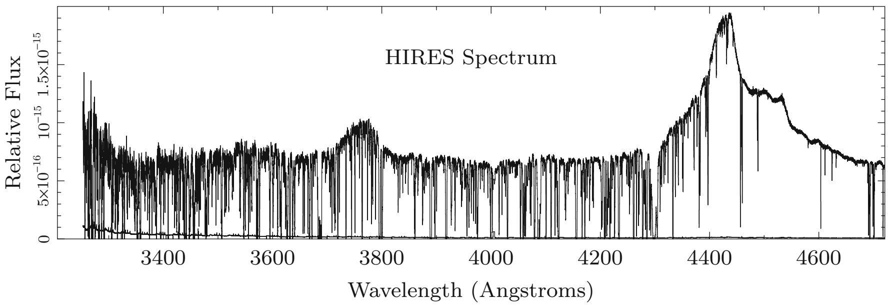

Fig. 20.7 Spectrum of the quasar HS 0105 + 1619 at redshift $z_Q = 2.64$ (when the universe was about 2.5 Gyr old). Lyman-α light from the quasar is emitted with wavelength 1,216 Å but redshifted to the observed wavelength $(1 + z_Q)1{,}216$ Å $=$ 4,425 Å. The "forest" of absorption lines at shorter wavelengths correspond to gas clouds between us and the quasar. A cloud at redshift z produces a Ly-α absorption line at wavelength $(1 + z)1{,}216$ Å (Credit: O'Meara et al. [25]. Reproduced by permission of the AAS)

With a primordial mass fraction of about 25 %, helium-4 would seem to be the easiest element to measure. But helium-4 is also created by fusion in stars. Most stars produce oxygen before they die and release their helium-4 back into the interstellar medium (see Sect. 16.3), so one strategy has been to measure the abundance of helium-4 in gas clouds that contain very little oxygen. Such measurements imply that primordial gas does have an abundance of helium-4 that matches predictions from big bang nucleosynthesis (see [24] for a review).

To probe elements that are more rare, one trick is to look far away, and therefore back in time. If we are hunting for gas that does not contain stars, we cannot rely on *emission* of light from the gas; instead, we search for *absorption* of light that passes through the gas. Quasars provide ideal "flashlights" for this experiment, because they are distant—creating a good chance that the light passed through a gas cloud when the universe was younger—yet bright enough to observe. (The most distant quasars date from when the universe was only ~1 Gyr old.) High-resolution spectra of quasars indeed reveal a "forest" of absorption lines from gas clouds that lie along the intervening line of sight, as shown in Fig. 20.7. Zooming in on different portions of the spectrum, we can identify all the different lines in the Lyman series that are produced by a *single* gas cloud, as shown in Fig. 20.8.

We can use such spectra to search for deuterium. The electron shell structure is similar to that of hydrogen, but the presence of a neutron in the nucleus perturbs the energy levels and shifts the absorption to slightly shorter wavelengths than in hydrogen. With a high-quality spectrum, it is possible to distinguish the deuterium lines from the hydrogen lines (see Fig. 20.8), and thus to measure the abundance of deuterium relative to hydrogen in the distant gas cloud. If we find similar deuterium abundances in a variety of systems, we can infer that it reflects the primordial abundance.

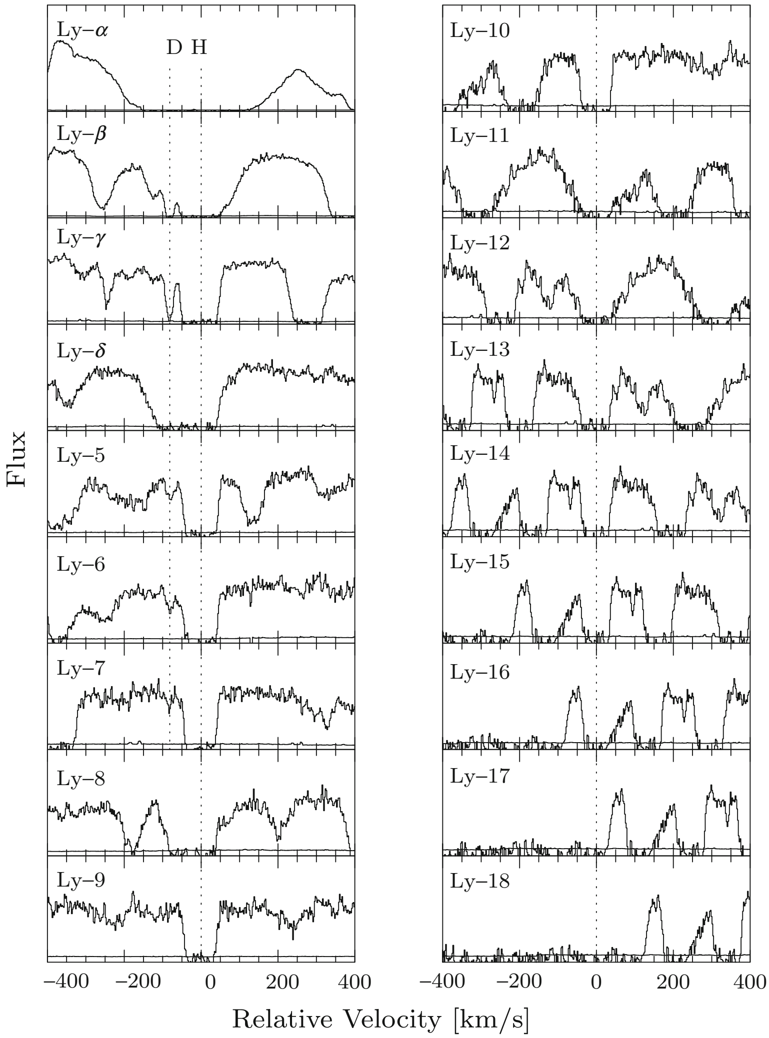

Fig. 20.8 Close-up of various absorption lines in the spectrum from Fig. 20.7. To put the panels on a common scale, wavelength offsets have been converted to velocity offsets using $\Delta v = c\,\Delta\lambda$. The absorption lines shown here are all part of the Lyman series produced by a single gas cloud. In several lines (notably Ly-β through Ly-6) there is clear evidence of absorption by deuterium just blueward of the absorption by hydrogen (Credit: O'Meara et al. [25]. Reproduced by permission of the AAS)

There are ways to recover the primordial abundances of helium-3 and lithium-7 as well, as discussed by Weiss [24]. The measurements are indicated by the horizontal bands in Fig. 20.6. For deuterium, helium-3, and helium-4, the measurements agree well with the theoretical predictions for the value of $\Omega_b h^2$ inferred from Planck observations of the CMB. In other words, for these three elements at least, we seem to have a consistent picture of what the universe was doing when it was just a few minutes old. Curiously, the measured abundance of lithium-7 is a little lower than expected for the relevant value of $\Omega_b h^2$, and we do not yet know what to make of that result. It is thought that lithium-7 might be easier to destroy than other elements, which could explain the low measured abundance. But it is also possible that some "exotic" processes in the early universe could have modified the production of lithium-7 [26]. Either way, we should learn something interesting from the discrepancy between theory and observations.

Notwithstanding the lithium problem, the overall picture suggests that we have a remarkably good understanding of what the universe was like in the first few minutes after the big bang. Together, the CMB and BBN provide exceedingly strong evidence that normal matter makes up a small fraction of the universe, and that the universe is dominated by two exotic substances about which we still have much to learn.

20.3 How Did We Get Here?

This brings to a close our story of how the elements were created: inside stars, and in the early universe. Combining that with our understanding of gas (for the CMB) and stars (especially type Ia supernovae), we can assemble a fairly detailed census of the contents of the universe: today the universe contains about 5 % normal matter, 27 % dark matter, and 68 % dark energy. And with our knowledge of (astro)physics we can reconstruct the history of the universe. To recap: the composition of the primordial gas was determined when the universe was about 3 min old. The gas went through recombination when the universe was around 380,000 years old, and we can observe this with the CMB. In the intervening 13.8 billion years, large gas clouds collapsed under the influence of gravity to become galaxies. Within galaxies, smaller clouds cooled and collapsed to form stars. Inside stars, the density, temperature, and pressure rose to the point that nuclear fusion could begin, releasing energy and creating all the elements heavier than hydrogen, helium, and lithium. At the end of their lives, those stars released heavy elements to the interstellar medium, where they could be incorporated into new stars and planets. Our planet and our bodies are made from elements forged by nuclear fusion in earlier generations of stars; and our lives today are powered by fusion in the Sun. What is even more remarkable than the story itself, I think, is that we can use physical principles to figure it all out. I hope you have enjoyed the journey!

Problems

20.1. Consider the transition from ionized to neutral gas in the early universe. Before recombination, the interaction between photons and ionized hydrogen was dominated by electron scattering characterized by the Thomson cross section, $\sigma_T = 6.65 \times 10^{-29}\,\text{m}^2$.

(a) What was the density of baryons ρ_b just before recombination at redshift $z \approx 1{,}100$? How far (in pc) could an average photon travel before scattering?
(b) The early universe was opaque because of the ionized gas, and then became transparent after recombination. Later on (sometime in the redshift range $z \sim 6$–20), the universe was *reionized* by radiation from stars and quasars. If ionized hydrogen gas pervades the universe today, why isn't the universe opaque now? Be specific and quantitative.

20.2. If the universe were filled with helium rather than hydrogen, when would recombination have occurred? See Problem 14.1 for the ionization stages of helium. Assume the density and temperature today are the same as in our actual universe.

20.3. In Sect. 13.1.4 we thought about blackbody radiation as a gas of photons.

(a) Use dimensional analysis to estimate the energy density in blackbody radiation with temperature T. Hint: photons are both relativistic and quantum entities.
(b) What is the energy density in CMB radiation today? Estimate the corresponding number density of CMB photons, and the ratio of CMB photons to baryons in the universe today.

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