Chapter 7
Control of Stable Continuous Processes, YOULA Parameterization

YOULA parameterization can be used to control stable processes. The block diagram of a common control system is shown in Fig. 7.1. With the appropriate regulator design, a closed loop control based on negative feedback ensures reference signal tracking, rejection of the effects of input and output disturbances, and also attenuation of the measurement noise.

Denote by Q the resulting transfer function between the control signal (manipulated variable) u and the reference signal r.

$$\frac{U(s)}{R(s)} = \frac{C}{1 + CP} = Q$$

The resulting transfer function between the output signal y and the reference signal r, supposing that the other input signals are zero, is $T = QP$. Therefore if P is stable, any stable Q ensures a stable closed loop control circuit.

The parameter Q (which is a transfer function) is called the YOULA parameter.

The series regulator $C(s)$ can be expressed also using the $Q(s)$ parameter:

$$C(s) = \frac{Q}{1 - QP}$$

Let us remark that the resulting transfer function $T = QP$ is linear in the parameter Q, whereas in the regulator C, it is nonlinear. The aim of regulator design is to fulfill the prescribed quality specifications set for the control system. Because of the linearity of the relation, the design of Q is simpler than the design of C.

In a closed loop control system the resulting transfer functions can be expressed also by the YOULA parameter Q.

The <u>servo (tracking) property</u> (reference signal => output signal): $T = QP$ (this is the complementary sensitivity function)

<u>Disturbance rejection</u> (output disturbance => output signal, or reference signal => error signal):

$$S = \frac{1}{1+CP} = \frac{1+CP-CP}{1+CP} = 1 - \frac{CP}{1+CP} = 1 - QP \text{ (sensitivity function)}$$

and $T + S = 1$ is fulfilled.

Relations between the output and the input signals:

The input signals are the reference signal and the output disturbance: r, $y_n = y_{no}$. The output signals are the control signal, the error signal and the output signal: u, e, y.

$$\begin{bmatrix} u \\ e \\ y \end{bmatrix} = \begin{bmatrix} Q & -Q \\ 1-QP & -(1-QP) \\ QP & 1-QP \end{bmatrix} \begin{bmatrix} r \\ y_n \end{bmatrix} = \begin{bmatrix} Q & -Q \\ S & -S \\ T & S \end{bmatrix} \begin{bmatrix} r \\ y_n \end{bmatrix}$$

The aim is always to ensure $T \Rightarrow 1$ (good servo property) over a wide frequency range, and $S \Rightarrow 0$ (good disturbance rejection). In the ideal case $T = 1$ and $S = 0$.

$$y = Tr + Sy_n \Rightarrow r \quad \text{or} \quad e = Sr - Sy_n \Rightarrow 0.$$

Regarding reference signal tracking Fig. 7.2 is equivalent to Fig. 7.1.

The best reference signal tracking is obtained if Q is the inverse of the transfer function of the process, i.e. $Q = P^{-1}$.

But this open loop control structure does not ensure disturbance rejection. Supplement this circuit with the inner model according to Fig. 7.3 (here only the output disturbance is shown). This is the so called *Internal Model Control* (*IMC*) structure.
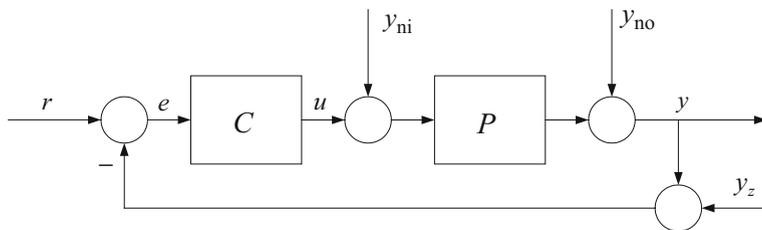

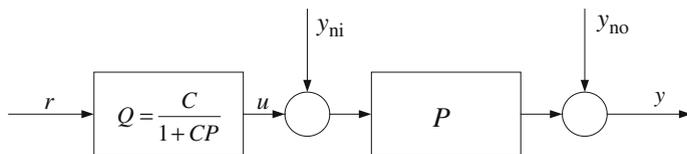Fig. 7.1 Block diagram of a control system

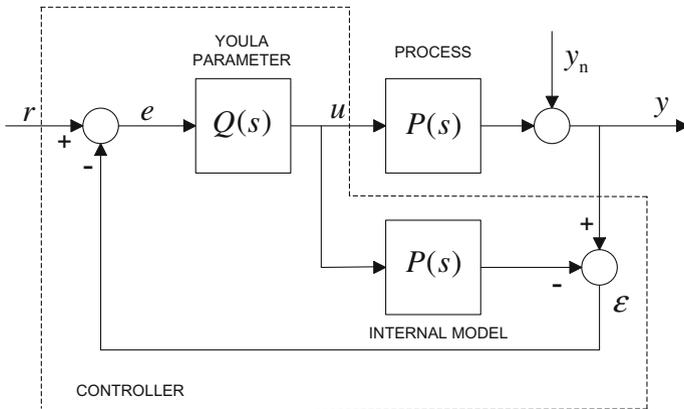Fig. 7.2 Demonstration of the YOULA parameter

Fig. 7.3 Supplementing the control circuit with internal model control

If there is no disturbance and the internal model is exactly the same as the process, then the value of the feedback signal is zero, and reference signal tracking is determined by the forward path of the open loop. Feedback will ensure disturbance rejection and eliminating the effect of plant-model mismatch.

If the model is perfect, then $\varepsilon = y_n$, and then the input of element Q is $r - \varepsilon = r - y_n$. Thus

$$u = Q(r - y_n)$$
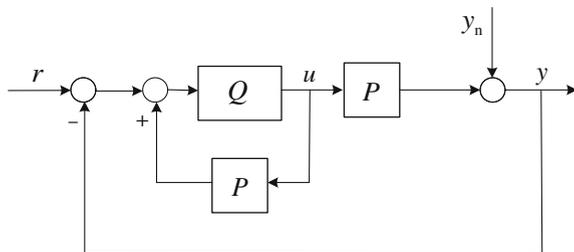$$y = Pu + y_n = PQr - PQy_n + y_n = Tr - Ty_n + y_n = Tr + (1 - T)y_n.$$

The block diagram in Fig. 7.3 can be redrawn according to Fig. 7.4.

Figure 7.4 is equivalent to the usual feedback control system shown in Fig. 7.5 and also in Fig. 7.1.

The series regulator $C(s)$ can be expressed with the YOULA parameter $Q(s)$, as was shown previously:

$$C(s) = \frac{Q}{1 - QP}.$$

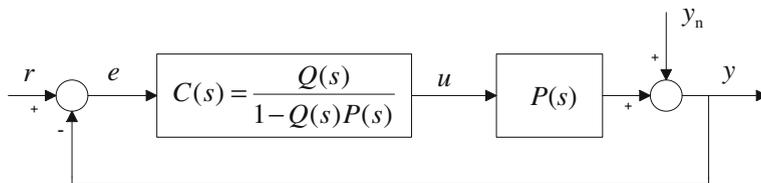**Fig. 7.4** Equivalent control scheme

Fig. 7.5 Equivalent usual feedback control scheme

The two structures (Figs. 7.3 and 7.5)—as a *YP* regulator—give results equivalent to the closed loop circuit.

Generally ideal reference signal tracking can not be realized, since the transfer function of the process can not always be inverted. Dead-time is non-invertible, as its inverse is not realizable. A transfer function can not be inverted if the degree of its denominator polynomial is higher than the degree of its numerator polynomial, as in this case the inverse is not realizable. A non-minimum phase transfer function, which contains zeros in the right half-plane is not invertible either, as its inversion would introduce unstable poles in the transfer function of the regulator.

Let us express $P(s)$, the transfer function of the process, as the product of its invertible part $P_+(s)$ and non-invertible $\bar{P}_-(s)$ part. $\bar{P}_-(s) = P_-(s)e^{-sT_d}$ contains the non-invertible part of the process transfer function and the dead-time. The gain of $P_-(s)$ should be 1 to ensure accurate reference tracking in the control system: $P_-(s = 0) = 1$.

$$P(s) = P_+(s)\bar{P}_-(s)$$

When realizing the control system the YOULA parameter performs the inverse of the invertible part only (Fig. 7.6): $Q = P_+^{-1}$.
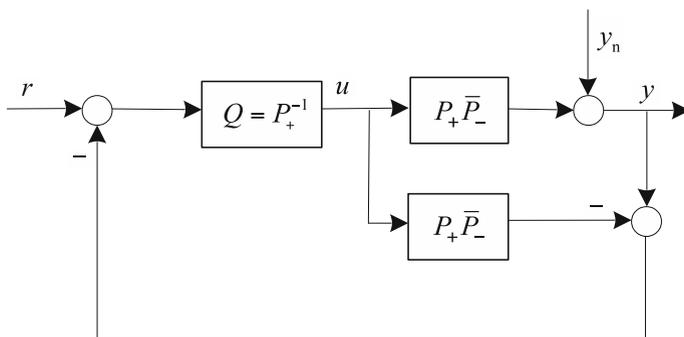

Fig. 7.6 The YOULA parameter is the inverse of the invertible part of the process

The dynamics of reference signal tracking and disturbance rejection are the same in this structure.

$$\left.\frac{Y(s)}{R(s)}\right|_{y_n=0} = \overline{P}_-(s) \quad \text{and} \quad \left.\frac{Y(s)}{Y_n(s)}\right|_{r=0} = 1 - \overline{P}_-(s)$$

It is often required that the dynamics of tracking be different than the dynamics of disturbance rejection, for example, that the disturbance rejection should be faster than the reference signal tracking. This is expressed also as that the *one-degree-of-freedom* (*1DOF*) control should be converted to *two-degree-of-freedom* (*2DOF*) control.

This can be ensured by using filters $R_r(s)$ and $R_n(s)$ according to Fig. 7.7. The gain of the filters should be unity to ensure the correct static values.

In this structure, the dynamics of the reference signal tracking and that of the disturbance rejection are obtained by the following:

$$\left.\frac{Y(s)}{R(s)}\right|_{y_n=0} = R_r(s)\overline{P}_-(s) \quad \text{and} \quad \left.\frac{Y(s)}{Y_n(s)}\right|_{r=0} = 1 - R_n(s)\overline{P}_-(s).$$

A further role of the filters is that they modify the maximum value of the control signal u (the manipulated variable), so it can be ensured that it will not exceed its limit. The filters may have also a robustification effect. With their appropriate choice, the control system could be made less sensitive to plant-model mismatch.

Figure 7.7 can be redrawn according to Fig. 7.8. The YOULA parameter now is $Q = R_n P_+^{-1}$.

With further restructuring, a conventional feedback structure is obtained (Fig. 7.9), where the series regulator is expressed in the form: $C = \frac{R_n P_+^{-1}}{1 - R_n P_-} = \frac{Q}{1 - QP}.$
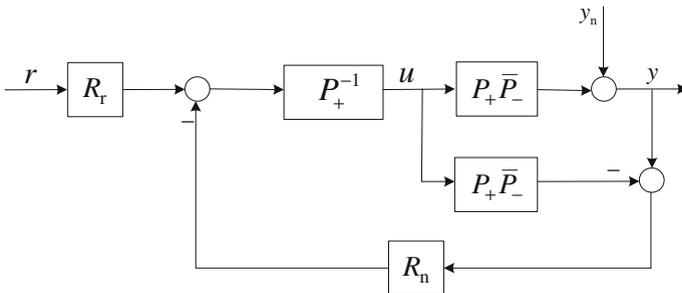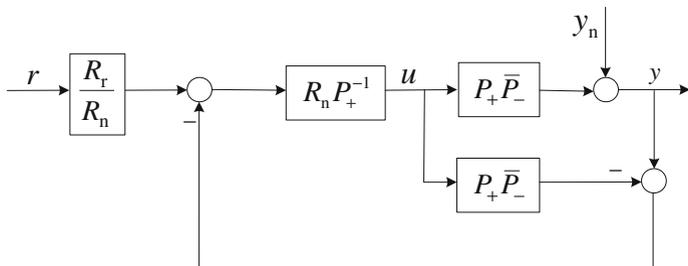

Fig. 7.7 Introducing filters in the control scheme
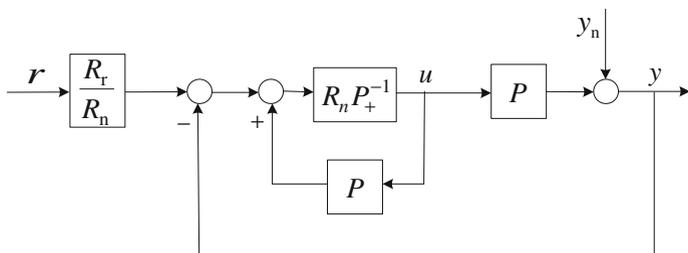
Fig. 7.8 Equivalent control scheme

Fig. 7.9 Equivalent control scheme

With these filters, the relations between the signals can be expressed as

$$
\begin{bmatrix} u \\ e \\ y \end{bmatrix} = \begin{bmatrix} \frac{R_r}{R_n}Q & -Q \\ \frac{R_r}{R_n}(1-QP) & -(1-QP) \\ \frac{R_r}{R_n}QP & 1-QP \end{bmatrix} \begin{bmatrix} r \\ y_n \end{bmatrix} = \begin{bmatrix} \frac{R_r}{R_n}Q & -Q \\ \frac{R_r}{R_n}S & -S \\ \frac{R_r}{R_n}T & S \end{bmatrix} \begin{bmatrix} r \\ y_n \end{bmatrix}
$$

Summarizing: Youla parameterization gives a method for regulator design (determines the Q parameter in IMC structure, or regulator C in feedback structure). For the design the transfer function of the process has to be separated into the invertible and non-invertible components and furthermore the transfer functions of the reference and disturbance filters have to be given.

Examples First let us consider processes without dead-time.

Write a MATLAB™ program, where the inputs are P_+ and P_-, the invertible and non-invertible components of the process, respectively, and the filters R_r and

R_n. The program calculates and plots the output and control signals for unit step reference signal and zero output disturbance, and then the output and the control signals for zero reference signal and unit step output disturbance.

Let us save the program with the name Youla_cont.

```
% Youla_cont: Youla continuous basic program
display('..... Q='),Q=minreal(Rn/Pp,0.0001)
display('..... C='),C=minreal(Q/(1-Q*P),0.0001)
display('..... Tr='),Tr=minreal((Rr/Rn)*Q*P,0.0001)
display('..... Ur='),Ur=minreal((Rr/Rn)*Q,0.0001)
pause;
t=0:0.1:50;
figure(1)
y=step(Tr,t); subplot(211),plot(t,y),grid;
u=step(Ur,t); subplot(212),plot(t,u),grid;
pause;
display('.....Sn='),Sn=minreal((1-Q*P),0.0001);
display('.....Un'),Un=-Q;
figure(2)
y=step(Sn,t); subplot(211),plot(t,y),grid;
u=step(Un,t); subplot(212),plot(t,u),grid;
```

Example 7.1 Consider *Example 7.2* of the textbook [1].

The process to be controlled is a second-order proportional system given by the transfer function

$$P = \frac{(1+5s)(1+6s)}{(1+10s)(1+8s)} = P_+$$

This system is invertible, so $P_- = 1$.

The filters are $R_r = \frac{1}{1+4s}$ and $R_n = \frac{1}{1+2s}$.

Give the data in MATLAB™.

```
s=zpk('s')
P=((1+5*s)*(1+6*s))/((1+10*s)*(1+8*s))
Pp=P
Rr=1/(1+4*s)
Rn=1/(1+2*s)
```

Then call the program Youla_cont.

```
Youla_cont
```

The calculated Q and C are

$$Q = \frac{1.3333\,(s+0.1)\,(s+0.125)}{(s+0.5)\,(s+0.2)\,(s+0.1667)}$$

$$C = \frac{1.3333\,(s+0.1)\,(s+0.125)}{s\,(s+0.2)\,(s+0.1667)}$$

It can be seen that the regulator C contains an integrator, so the control system will track the reference signal without steady error.

The left side of Fig. 7.10 gives the output and the control signals for reference signal tracking, while the right side of the figure shows these signals for the output disturbance. It is seen that the disturbance rejection is faster than the reference signal tracking.

Example 7.2 The process is of non-minimum phase with one zero in the right half-plane. Its transfer function is
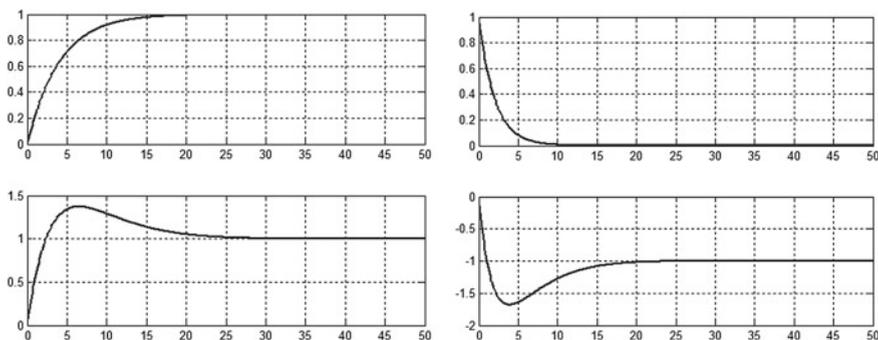
$$P = \frac{1 - s}{(1 + s)(1 + 2s)}$$

Its step response is shown in Fig. 7.11. It can be seen that the response starts downwards, then turns and reaches the steady state value determined by unit gain.

Let us separate the transfer function of the process into its invertible and non-invertible components: $P_- = 1 - s$ (its gain, which is its value at, $s = 0$ is 1)

$$P_+ = \frac{1}{(1 + s)(1 + 2s)}$$

The filters have to be of second-order to ensure the realizability of Q and C, and also of ratio R_r/R_n.
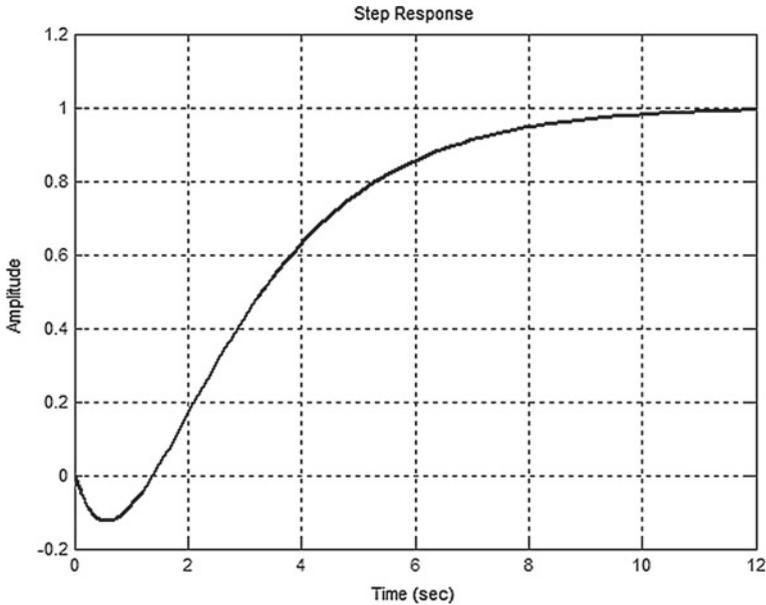

Fig. 7.10 Output and control signals

Fig. 7.11 Step response of a non-minimumphase process

Put $R_r = \frac{1}{(1+s)^2}$ and $R_n = \frac{1}{(1+0.5s)(1+s)}$.
Give the data in MATLAB™.

```
s=zpk('s')
P=(1-s)/((1+s)*(1+2*s))
Pp=1/((1+s)*(1+2*s))
Rr=1/(1+s)^2
Rn=1/((1+0.5*s)*(1+s))
```

Then call the Youla_cont program.

Youla_cont

It can be seen that the control system operates appropriately (Fig. 7.12). The left side of the figure gives the output and the control signals for reference signal tracking, while the right side shows these signals for the output disturbance. (Without the separation of the transfer function of the process with the original algorithm the control has unstable behaviour.) The settling time of the control system is about the half of that of the process itself.

Example 7.3 Let us consider now a process with dead-time, where the value of the dead-time is bigger than the values of the time constants in the lag elements. In such cases a YOULA parameterized regulator results in a significantly faster control system than *PID* control (see Chap. 8).
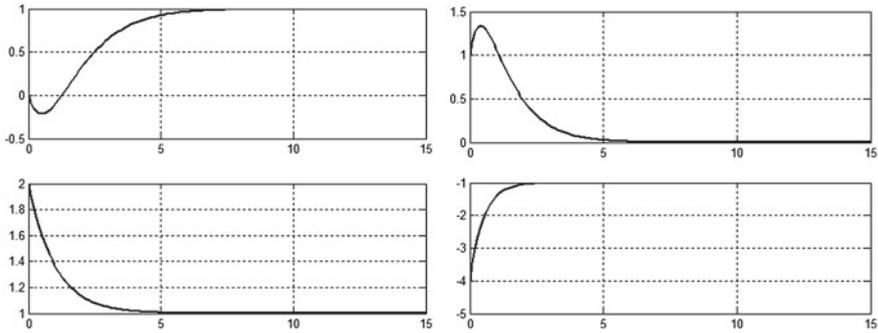
Fig. 7.12 Output and control signals

The transfer function of the process is: $P(s) = \frac{1}{(1+5s)(1+10s)} e^{-30s}$.

Design a Youla parameterized regulator for this process. Separate the transfer function into invertible and non-invertible parts:

$$P_+ = \frac{1}{(1+5s)(1+10s)} \quad \text{and} \quad \bar{P}_- = P_- e^{-30s} = 1 \cdot e^{-30s}$$

The filters have to be of second order to ensure realizable Q and R_r/R_n transfer functions.

Let us first choose two identical filters, $R_r = R_n = \frac{1}{(1+s)^2}$, then put $R_r = \frac{1}{(1+s)^2}$ and $R_n = \frac{1}{(1+25s)(1+s)}$.

Analyse the reference signal tracking and disturbance rejection of the control system for unit step reference signal and disturbance, first when the system and its model are the same, then when there is plant-model mismatch: the dead-time of the process is 40 s, and the dead-time in the model, which is the basis of regulator design is 30 s.

The simulation is executed in SIMULINK™, as in MATLAB™ the dead-time could be considered only approximately.

Give the data in MATLAB™ and the commands calculating the Youla parameter and the filters.

```
s=zpk('s')
Pp=1/((1+5*s)*(1+10*s))
Rr1=1/(1+s)^2
Rn1=1/(1+s)^2
Rr2=Rr1;
Rn2=1/((1+25*s)*(1+s))
Q1=minreal(Rn1/Pp,0.0001)
Q2=minreal(Rn2/Pp,0.0001)
F1=1;
```

```
F2=minreal(Rr2/Rn2,0.0001)
```

Build the SIMULINK™ block diagram according to the *IMC* structure (Fig. 7.13).

SIMULINK™ takes over the referred variables given in MATLAB™.

The reference signal is a unit step acting at $t = 0$, the output disturbance is a step with amplitude 0.5 which acts at time $t = 100$. Figure 7.14 shows the output signals of the two filter choices. The control signals could be also plotted.

Figure 7.15 shows the output signals in the case of plant-model mismatch. With the filters of smaller time constants the control system becomes unstable, but with the bigger time constants in the filters the control remains stable.

Let us remark that in case of a sampled-data system, the realization of a regulator containing dead-time does not present a problem, and so the simulation can be executed easily also in MATLAB™ (see Sect. 12.1).
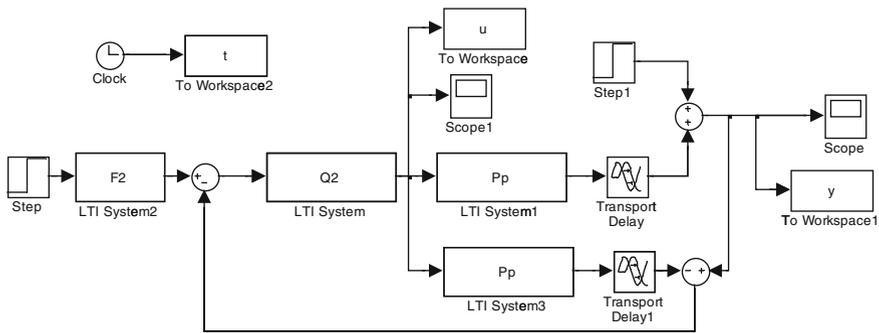

Fig. 7.13 SIMULINK™ block diagram of the YOULA parameterized control system
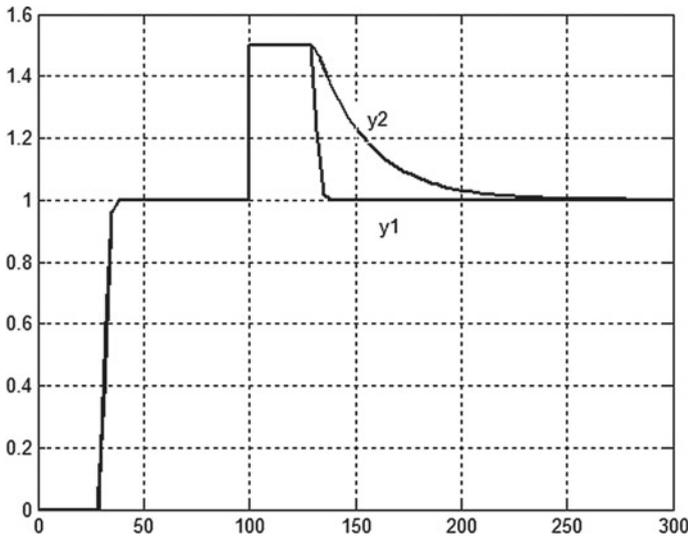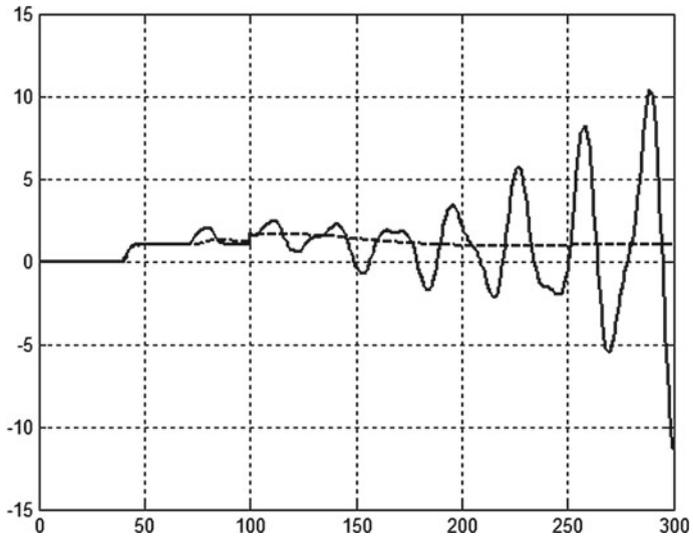

Fig. 7.14 The output signal for step reference signal and step disturbance

Fig. 7.15 In case of dead time mismatch a well selected filter stabilizes the behavior