Chapter 6
Design in the Frequency Domain

A closed loop control system has to meet several prescribed quality specifications. These specifications can be formulated in the time domain and also in the frequency domain. The behaviour of the closed loop control system can be evaluated on the basis of the frequency function of the open loop. The characteristics of the open loop frequency function in the low-, middle- and high frequency ranges determine the quality characteristics of the closed loop system.

- For good reference signal tracking, $|L(j\omega)|$ should be large in the low frequency range.
- For effective rejection of measurement noise, $|L(j\omega)|$ should be small in the high frequency range.
- For faster performance, the cut-off frequency ω_c should be as large as possible.
- To ensure stability, the cut-off frequency should be located at that part of the Bode amplitude diagram where the slope of its asymptote is −20 dB/decade.
- The overshoot of the step response will be within 10%, if the phase margin is about 60°.

These requirements are partly contradictory. Different prescriptions have to be given for different frequency ranges. The requirements can be fulfilled by appropriate shaping of the frequency characteristics. Figure 6.1 shows a typical open loop amplitude-frequency function.

Example 6.1 Let us consider a system whose loop frequency function shows similar performance to Fig. 6.1. Analyse the static response, reference signal tracking, disturbance rejection and transient behaviour of the closed loop system.

$$L(s) = C(s)P(s) = 0.01(20s + 1)/\left[s^2(s + 1)\right]$$

L. Keviczky et al., *Control Engineering: MATLAB Exercises*,
Advanced Textbooks in Control and Signal Processing,
https://doi.org/10.1007/978-981-10-8321-1_6

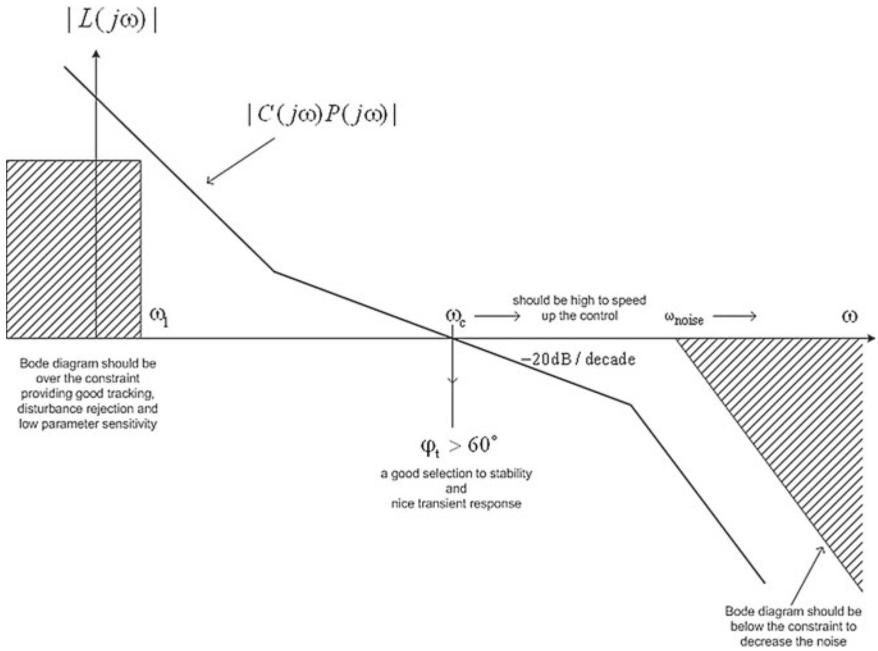

Fig. 6.1 Formulation of the quality specifications in the frequency domain

Plot the Bode diagram of the open loop.

```
s=zpk('s')
L=0.01*(20*s+1)/(s*s*(s+1))
T=L/(1+L),T = minreal(T)
figure(1),bode(L),grid
```

The Bode diagram is shown in Fig. 6.2. Analyse the behaviour of the system for reference signal tracking and disturbance rejection. Let us take a reference signal containing two components: a low and a high frequency sinusoidal signal.

$$r(t) = r_a(t) + r_m(t) = \sin(\omega_a t) + 0.5\sin(\omega_m t)$$

Let ω_a be a low frequency which is located in the first part of the Bode diagram of slope −40 dB/decade, and ω_m be on the second part on the high frequency part with the same slope: $\omega_a = 0.05$, $\omega_m = 2$. Plot the input and the output signal!

```
t=0:0.1:500;
wa=0.05; wm=2;
r=sin(wa*t)+0.5*sin(wm*t);
y=lsim(T,r,t);
figure(2);
subplot(211); plot(t,r);
subplot(212); plot(t,y);
```

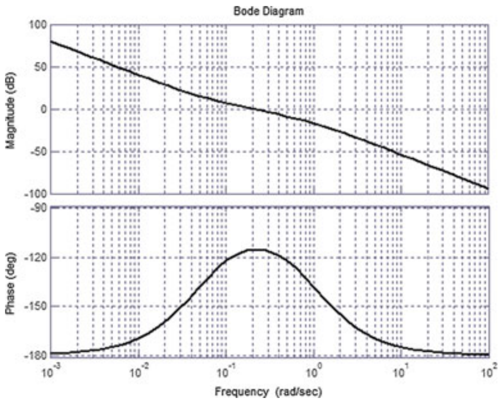

Fig. 6.2 Open-loop frequency function

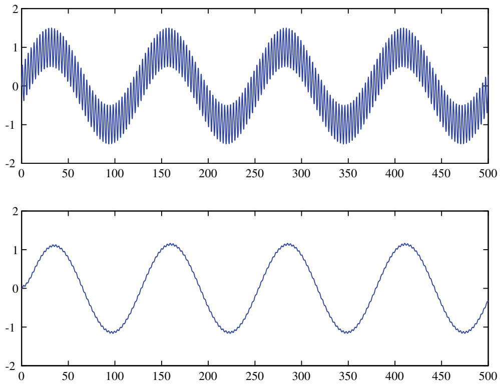

Fig. 6.3 The system attenuates the high frequency component

In Fig. 6.3 it can be seen that the system with such frequency characteristics tracks the low frequency component of the input signal and attenuates the high frequency component (the upper curve in the figure is the input signal, and the lower curve is the output signal).

The transient behaviour can be analysed on the step response of the closed loop system.

```
figure(3),step(T),grid
```

The overshoot depends on the phase margin, which can read from the Bode diagram ($\sim 60°$). The settling time depends on the cut-off frequency ω_c.

Example 6.2 Let us analyse the relationship between the cut-off frequency and the settling time. Choose now a faster L_1 system than the system L in Example 6.1. The cut-off frequency of system L_1 is higher than that of system L.

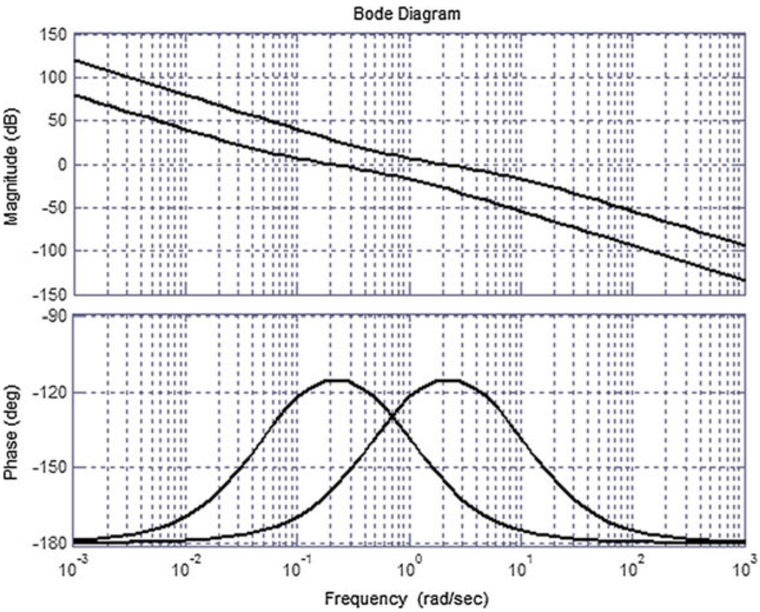

Fig. 6.4 BODE diagrams of two systems

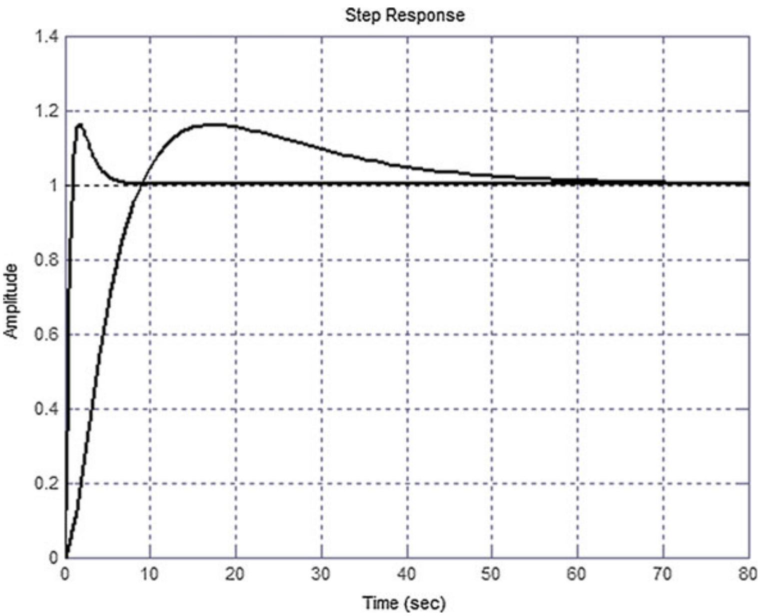

Fig. 6.5 Higher cut-off frequency means faster step response

$$L_1(s) = \frac{2s+1}{s^2(0.1s+1)}$$

```
L1=(2*s+1)/(s*s*(0.1*s+1))
T1=L1/(1+L1),T1=minreal(T1)
figure(1),bode(L,'b',L1,'r'),grid
figure(3),step(T,'b',T1,'r'),grid
```

It can be seen (Figs. 6.4 and 6.5) that higher cut-off frequency means a faster time response.

The cut-off frequency of the first system is $\omega_c \approx 0.2$ and that of the second system is $\omega_c \approx 2$. The settling time for the first system is $t_s \approx 50$ s, while for the second system it is $t_s \approx 5$ s (The settling time can be estimated by the relation $3/\omega_c < t_s < 10/\omega_c$).

As the open loop contains two integrators, the control system tracks the step and also the ramp input accurately, without steady error. Show this by simulation!