Chapter 4
Negative Feedback

Feedback is the most important structure in control systems. The regulator gets information about the value of the controlled variable through feedback. Feedback significantly modifies the performance of the system.

4.1 Quality Characteristics and the Properties of Negative Feedback

The performance of a system controlled by negative feedback can be characterised numerically by its quality characteristics.

4.1.1 Requirements Set for Control Systems

Generally the following requirements are set for closed-loop control systems.

Stability: Stable operation of the control system is a basic requirement. Stability can be formulated in several ways. *Bounded Input–Bounded Output* (*BIBO*) stability means that the system provides a bounded output as a response to any and all bounded inputs. The system is asymptotically stable if its transients decay.

Robustness: The performance of a closed-loop system should not be sensitive to the inaccuracy of the available information about the process. Stability has to be guaranteed even if the parameters of the system are not known accurately or their values change within a given range around their nominal values.

L. Keviczky et al., *Control Engineering: MATLAB Exercises*,
Advanced Textbooks in Control and Signal Processing,
https://doi.org/10.1007/978-981-10-8321-1_4

Static behaviour: Another important requirement is the static accuracy of the system, i.e. its accuracy in steady state. Static requirements set for the steady state of the control system can include:

– Reference signal tracking. The tracking error should be below the prescribed value.
– Disturbance rejection: In steady state the control system should eliminate the effect of disturbances.

Static accuracy depends on the structure of the system and also on the input signals.

Transient response: The transient response is an important dynamic feature of a control system. The characteristic properties of the transient response can be given in the time domain by the characteristics of the transient response of the system in the case of a step reference signal or disturbance. The prescriptions for the transient response are the following:

– Overshoot of the output signal.
– Settling time: During the settling time the controlled variable approximates its steady value within 1–2%. Generally a small overshoot within 5–10% of the steady state value, can be tolerated, but there are processes where aperiodic transients are required.

Error integrals: In more complex control problems restrictions can be prescribed for the whole course of the output and the control signal. E.g. the quadratic integral values of the error signal should be minimised and the value of the control signal restricted.

Example 4.1 Analyse the effect of negative feedback and determine the quality properties of a closed-loop control circuit. The transfer function of the open-loop is

$$L(s) = \frac{4}{(10s+1)(4s+1)} = \frac{num}{den}$$

Unity negative feedback is applied (Fig. 4.1). The resulting transfer function of the closed loop is:

$$T(s) = \frac{L}{1+L} = \frac{num}{num+den}$$

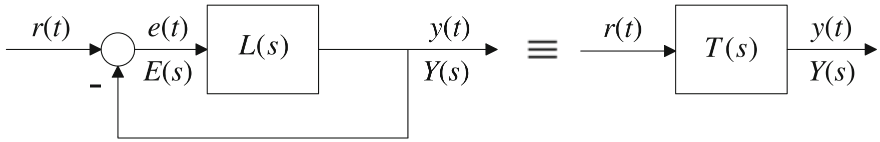


Fig. 4.1 Negative feedback

```
s = zpk('s')
L = 4/((10*s + 1)*(5*s + 1))
T = L/(1 + L)
T = minreal(T)
```

The command minreal is used to cancel the common poles and zeros.

The resulting (overall) transfer function can also be calculated by the command feedback. The second input parameter gives the transfer function in the feedback path.

```
T = feedback(L,1)
```

Let us compare the step responses of the open- and of the closed-loop (Fig. 4.2).

```
step(L,'b',T,'r'),grid on
```

It is evident that the closed-loop behaviour differs from that of the open-loop. The static and transient properties of the system were influenced significantly by the feedback. Let us determine the quality properties of the feedback system. The static value $(t \to \infty)$ can be read from the figure. For the open-loop this value is 4, while in case of the closed-loop this is a bit less than 1. These values can be calculated more accurately by

```
ysL = dcgain(L)
ysT = dcgain(T)
```

```
ysL = 4
ysT = 0.8
```

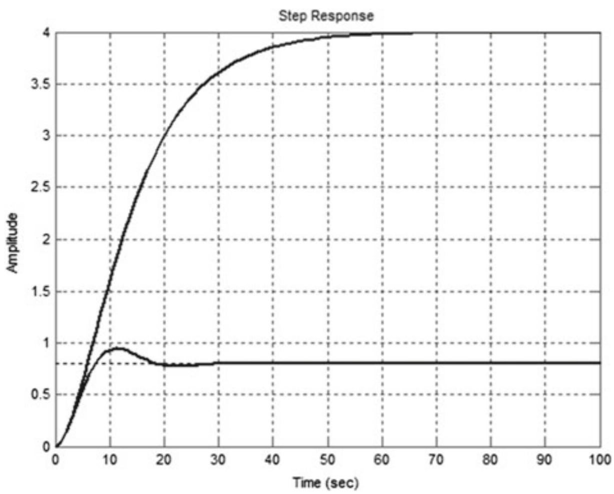


Fig. 4.2 Open- and closed-loop step responses

In control systems, reference signal tracking is one of the most important system characteristics. It is seen that in the case of negative feedback, a closed-loop system in steady state approximates the value of the reference signal. Let us calculate the steady state error.

```
esL = 1-ysL
esT = 1-ysT
    esL = -3
    esT = 0.2
```

In the figure it is also seen that the transient behaviour also has changed. The settling process became faster (the settling time can be read from Fig. 4.2).

$$t_{sL} \cong 45; \quad t_{sT} \cong 20.$$

There is an overshoot in closed-loop response, which can be calculated by

```
y = step(T)
ym = max(y)
yt = (ym-ysT)/ysT
```

The value of the overshoot is 18%.

Plot the BODE diagrams of the open-loop and the closed-loop in one diagram (Fig. 4.3).

```
bode(L,'b',T,'r'),grid on
```

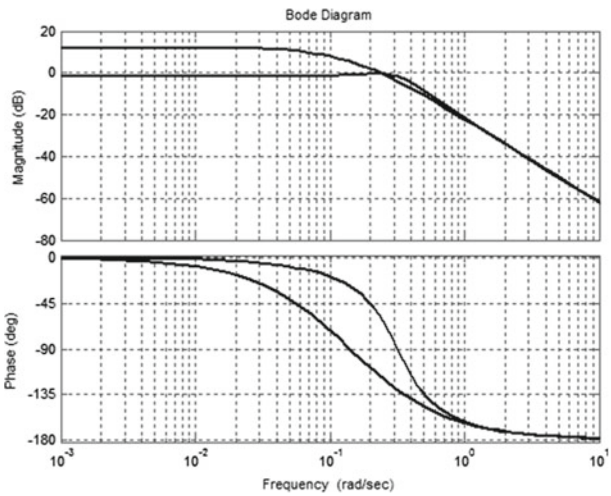


Fig. 4.3 BODE diagrams of the open- and the closed-loop

It can be seen that the Bode amplitude diagram of the closed-loop is approximately 1 in the low frequency domain, while in the high frequency domain it coincides approximately with the diagram of the open-loop.

Example 4.2 The transfer function of the closed-loop is

$$T(s) = \frac{1}{(1+10s)(1+s)}$$

Determine the linear error area from its step response.

The system is given by

```
T = 1/((10*s + 1)*(s + 1))
```

Plot the sampled points of its step response (Fig. 4.4).

The distance between two consecutive points is the sampling time Ts.

```
Ts = 0.5;
t = 0:Ts:60;
y = step(T,t);
plot(t,y,'.');grid on
```

The linear error area can be calculated by evaluating the integral

$$I_1 = \lim_{t\to\infty} \int_0^t e(\tau)\,\mathrm{d}\tau,$$

which can be calculated by using the relation

$$I_1 = A\left(\sum_{j=1}^{n} T_j - \sum_{k=1}^{m} \tau_k\right).$$

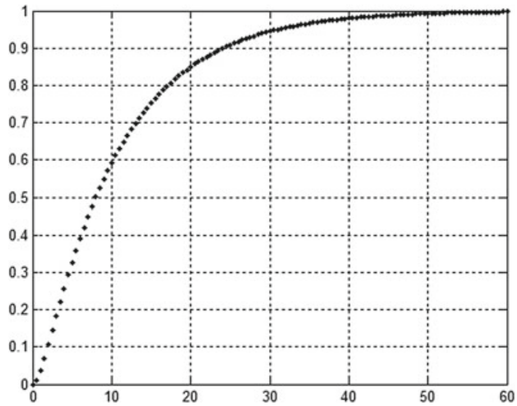


Fig. 4.4 Step response

(See formula (4.51) in the textbook [1]). Here A is the static gain and τ and T are the time constants of the numerator and the denominator, respectively. So $I_1 = 10 + 1 = 11$.

With MATLAB™ the value of the integral can be determined using the rectangle rule: $I_1 = [1 - y(0)]T_s + [1 - y(1)]T_s + \ldots + [1 - y(N)]T_s$.

Summation is executed by the command `sum` for the elements of the vector, which gives a good approximation to I_1.

```
I1 k = sum((1-y)*Ts)
      I1 k  = 11.2232
```

The result will be more accurate if the sampling time is smaller. Repeat the calculation for $T_s = 0.05$. The quadratic error integral can be evaluated similarly.

$$I_2 = \lim_{t\to\infty} \int_0^t e^2(\tau)\,\mathrm{d}\tau$$

```
I2 k = sum((1-y).*(1-y)*Ts)
      I2 k =  6.2045
```

4.1.2 Demonstrating the Basic Properties of Negative Feedback

The effects of feedback can be described by the following properties:

(a) it modifies the transient behaviour;
(b) it improves reference signal tracking;
(c) it may stabilise an unstable process;
(d) it improves disturbance rejection;
(e) it improves the insensitivity to parameter changes;
(f) it has a linearizing effect;
(g) it can be used to approximate the inverse of a transfer function.

4.2 Resulting Transfer Functions

The usual block diagram of a control system is given in Fig. 4.5.

A filter $F(s)$ is not always applied. The disturbance sometimes is taken into consideration only at the output or at the input of the process. The behaviour of the system can be described by 6 different resulting transfer functions.

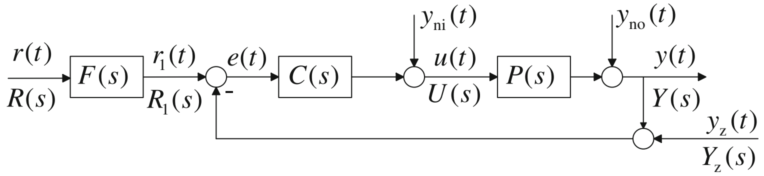


Fig. 4.5 Block diagram of a control system

Example 4.3 In a control system the process is given by the transfer function $P(s) = \frac{1}{(1+10s)(1+s)}$, the transfer function of the regulator is $C(s) = 5\frac{1+10s}{10s}$ and the filter is $F(s) = \frac{1}{1+s}$ (See the topic of regulator design in Sect. 8.2.3.).

Let us determine the following 6 resulting transfer functions:

$$\frac{Y(s)}{R(s)}; \quad \frac{Y(s)}{Y_z(s)}; \quad \frac{Y(s)}{Y_{ni}(s)}$$
$$\frac{U(s)}{R(s)}; \quad \frac{U(s)}{Y_z(s)}; \quad \frac{U(s)}{Y_{ni}(s)}$$

```
P = 1/((10*s + 1)*(1 + s))
C = 0.5*(0.5*s + 1)/s
F = 1/(1 + s)
L = minreal(C*P);
N = 1+L;
HYR = F*L/N;HYR = minreal(HYR)
HUR = F*C/N;HUR = minreal(HUR)
HYYz = -L/N;HYYz = minreal(HYYz)
HUYz = -C/N;HUYz = minreal(HUYz)
HYYni = P/N;HYYni = minreal(HYYni)
HUYni = 1/N;HUYni = minreal(HUYni)
H = [HYR,HYYz,HYYni;HUR,HUYz,HUYni];
```

Plot the step responses (Fig. 4.6) and the Bode diagrams. It can be seen that the different transfer functions cause different dynamics. The rejection of the effect of the step disturbance is faster than the dynamics of step reference signal tracking. In design, the dynamics of the reference signal tracking and the dynamics of the disturbance rejection can be influenced by the appropriate choice of the regulator $C(s)$ and the prefilter $F(s)$.

```
t = 0:0.1:10;
figure (1),step(H,t),grid on
figure (2),bode(H),grid on
```

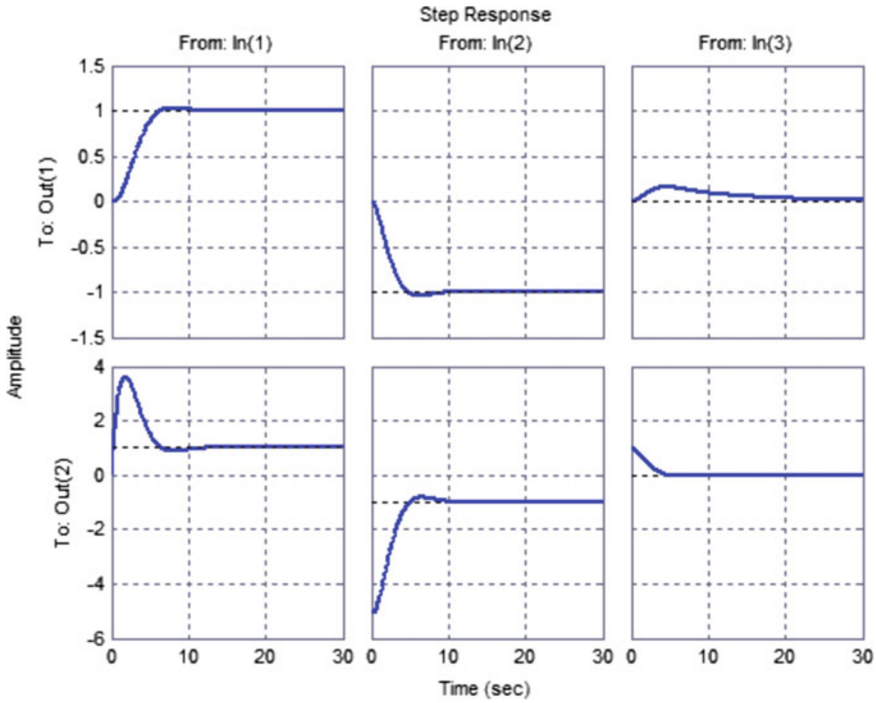


Fig. 4.6 Step responses of different signals in a control system

4.3 The Effect of the Poles of the Excitation Signal and the Effect of the Poles of the Open Loop on Steady State Behaviour

Let us consider an exponentially decreasing input signal with time constant $T_a = 10$, i.e. its pole is $p_a = -1/T_a = -0.1$. The input signal is $r(t) = e^{-t/T_a} = e^{-tp_a}$. Its LAPLACE transform is $R(s) = 1/(s - p_a)$. The open loop can be given by a second-order lag element with gain $K = 5$. Let us analyse the reference signal transfer properties of the closed loop, if the transfer function of the open loop contains the pole of the reference signal and has no smaller pole which would cause a slower response. Analyse the response also for the case when the open loop does not contain the pole of the reference signal.

```
s = zpk('s')
Ta = 10; K = 5
L = K/((Ta*s + 1)*(s + 1))
L1 = K/((2*Ta*s + 1)*(s + 1))
L2 = K/((0.5*Ta*s + 1)*(s + 1))
T = L/(1 + L);
T1 = L1/(1 + L1);
T2 = L2/(1 + L2);
t = 0:0.01:50;
r = exp(-t/Ta);
y = lsim(T,r,t);
y1 = lsim(T1,r,t);
y2 = lsim(T2,r,t);
figure (1); plot(t,r,'b',t,y,'r');
figure (2); plot(t,r,'b',t,y1,'r',t,y2,'g')
```

It can be seen that the output signal of the closed loop tracks accurately the input signal only if the transfer function of the open loop contains the pole of the reference signal (Figs. 4.7 and 4.8). A special case of tracking is when $p = 0$ is the pole of the reference signal, i.e. the input is the step reference signal, whose Laplace transform is $1/s$. In the transfer function of the open loop, a pole at zero represents an integrator. As in our example the open loop does not contain the pole of the step input (there is no integrator in the loop), there will be a steady state error (Fig. 4.9).

```
r = ones(1,length(t));
y = lsim(T,r,t);
figure (3); plot(t,r,'b',t,y,'r');
```

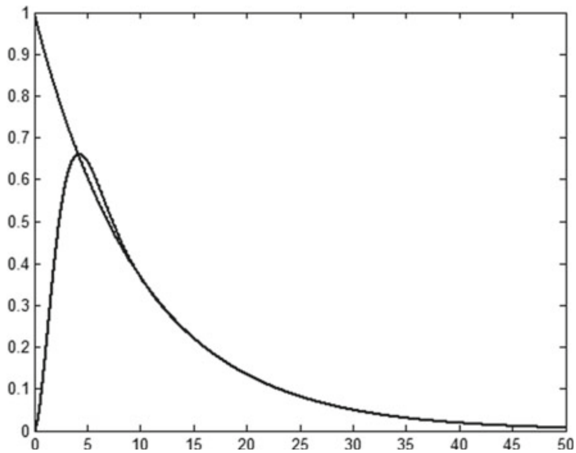


Fig. 4.7 Tracking of an exponential signal

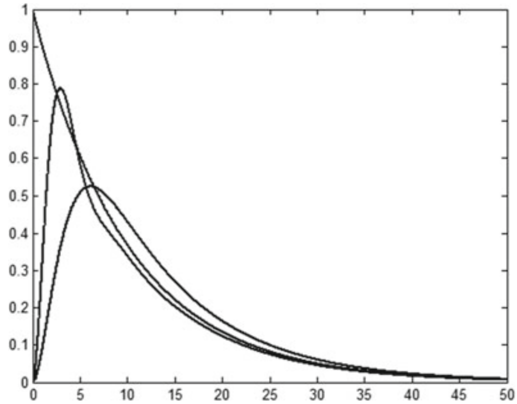


Fig. 4.8 Tracking of an exponential signal

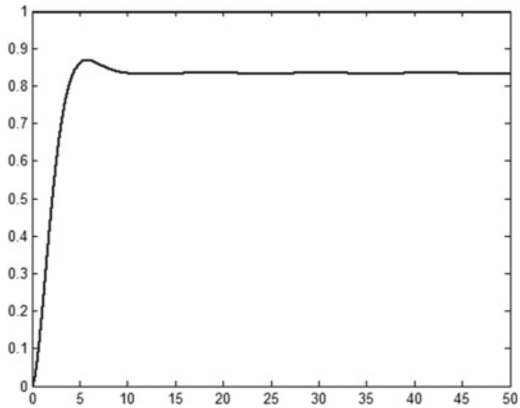


Fig. 4.9 Steady state error in step response

4.4 Properties of the Static Response

The steady state response of a feedback system (closed loop) depends on the type number of the system. 'Type number' means the number of integrators in the open loop. The table below gives the steady state error for different reference signals and different type numbers.

Type number	0	1	2
Unit step reference signal $j = 1$	$\frac{1}{1+K}$	0	0
Ramp reference signal $j = 2$	∞	$1/K$	0
Quadratic reference signal $j = 3$	∞	∞	$1/K$

Example 4.4 The loop transfer function of a system is

$$L(s) = \frac{K}{(1+s)(1+5s)} = \frac{10}{(1+s)(1+5s)}$$

Analyse the behaviour of the open- and the closed-loop.

```
s = zpk('s')
L = 10/((1 + s)*(5*s + 1))
T = feedback(L,1)
```

Determine the steady state values of the step responses of the open- and of the closed loop. According to the finite value theorem of the LAPLACE transformation for unit step input

$$r(t) = 1(t); \quad R(\mathrm{s}) = \frac{1}{s}$$

$$y(t \to \infty) = \lim_{s \to 0} s\, R(\mathrm{s})\, H(\mathrm{s}) = \lim_{s \to 0} s \frac{1}{s} H(\mathrm{s}) = \lim_{s \to 0} H(\mathrm{s})$$

The steady state values of the step responses of the open- and of the closed-loop are then

$$y_{\text{open-loop}}(t \to \infty) = \lim_{s \to 0} L(s) = K = 10$$

$$y_{\text{closed-loop}}(t \to \infty) = \lim_{s \to 0} T(s) = \frac{y_{\text{open-loop}}(t \to \infty)}{1 + y_{\text{open-loop}}(t \to \infty)} = \frac{K}{1+K} = \frac{10}{1+10}$$

The value of the steady state error is: $e(t \to \infty) = 1 - y_{\text{open-loop}}(t \to \infty) = \frac{1}{1+K} = \frac{1}{11}$

Plot the step responses of the open- and the closed-loop in the same diagram:

```
step(L,'r',T,'b')
```

The steady values can be read from the diagrams or can be calculated with the command dcgain.

```
yos = dcgain(L)
ycs = dcgain(T)
```

Also plot the BODE diagrams of the open and the closed loop in the same diagram.

```
bode(L,'r',T,'b')
```

Determine the value of the steady state error for $K = 1, 20, 100$.

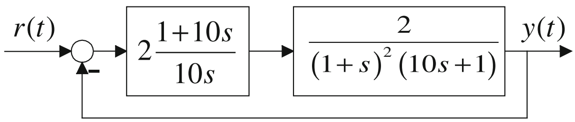


Fig. 4.10 Control system of type number 1

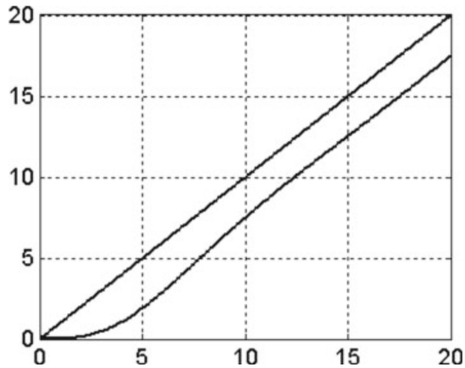


Fig. 4.11 The system tracks the ramp signal with steady error

Example 4.5 Determine the steady state error of the system given in Fig. 4.10 for unit step, ramp and parabolic reference signals.

The system contains one integrator, therefore its type number is 1. On the basis of the table above the steady error is

for step reference signal: $e(\infty) = 0$
for ramp reference signal: $e(\infty) = 1/\mathrm{K} = 1/(2 \cdot 2/10) = 2.5$
for parabolic reference signal: $e(\infty) = \infty$

Check the steady error value for a ramp reference signal by simulation (Fig. 4.11)!

```
s = zpk('s')
C = 2*(1 + 10*s)/(10*s)
P = 2/(s + 1)^2/(10*s + 1)
L = minreal(C*P)
T = L/(1 + L); T = minreal(T)
t = 0:0.1:20;
r = t;
y = lsim(T,r,t);
plot(t,r,t,y); grid
```

4.5 Relation Between the Frequency Functions of the Open- and Closed-Loop

The nonlinear relation $T(s) = L(s)/[1 + L(s)]$ describing the resulting transfer function of a control system based on negative feedback determines the behaviour of the control system. Let us analyse how this relation maps the complex plane L to the complex plane T. Plot the absolute value of the frequency function of the closed loop: $M = |L(j\omega)/[1 + L(j\omega)]|$.

As -1 is a singularity of the mapping, the neighbourhood of this point is investigated. Write the following program as an *m*-file.

```
res = 0.01; Mlimit = 5;
x = -3:res:1;
y = -2:res:2;
Mm = zeros(length(y),length(x));
for kx = 1:length(x)
  for ky = 1:length(y)
    L = x(kx) + y(ky)*i;
    T = L/(1 + L);
    M = abs(T);
    if M > Mlimit M = Mlimit;
    end
    Mm(ky,kx) = M;
  end
end
surf(x,y, Mm), shading INTERP, colormap('jet'), view(0,90)
```

In window *figure* in menu '*tools*' with option '*rotate 3D*' the 3D surface can be visualised from an arbitrary viewpoint. The value of M should be restricted to ensure the visualisability of the picture. For fixed values of M, the contour lines are circles (Fig. 4.12).

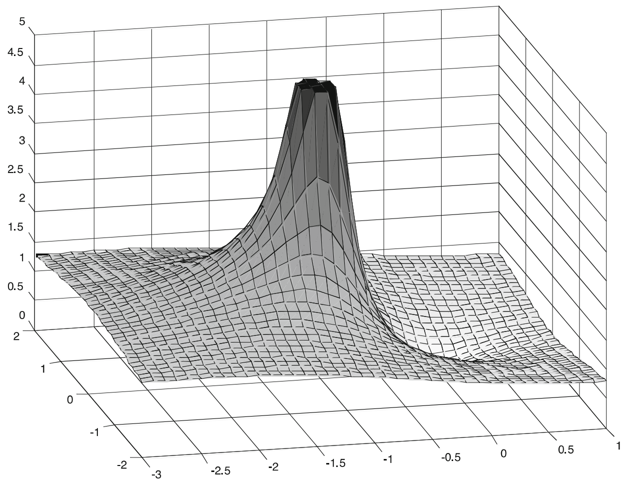


Fig. 4.12 Conform mapping of space L into space T

4.6 Relation Between the Overshoot of the Step Response and the Amplification of the Frequency Function

The maximum of the amplitude-frequency function of the closed-loop depends on how closely the Nyquist curve of the open-loop approaches the point −1 of the complex plane. On the Bode diagram this maximum means the amplification of the absolute value. Big amplification in the frequency domain means high overshoot in the time domain in the step response. The relationship between these values is not simple, as in the time domain the output signal is calculated by convolution, which means that the maximum overshoot in the time domain depends not only on the maximum value of the amplification in the frequency domain, but also on the amplifications on the other frequencies.

Example 4.6 Let us analyse, in the case of a second order oscillating element, how a change of the damping factor influences the overshoot in the time domain and the amplification of the amplitude in the frequency domain. Plot the values of the overshoot of the step response ym and the maximum amplitude of the frequency response Mm versus the damping factor (Fig. 4.13).

```
s = zpk('s')
T0 = 1;
kszi = [0.2:0.1:1];
t = 0:0.01:30;
w = logspace(-1,1,500);
for k = 1:length(kszi),
    T = 1/(s*s*T0*T0 + 2*T0*s*kszi(k) + 1);
    y = step(T,t);
    ym(k) = max(y);
    M = bode(T,w);
    Mm(k) = max(M);
    figure (2); hold on; plot(t,y)
    figure (3); loglog(w,M(:)); hold on;
end
figure (1);plot(kszi,ym,'r',kszi,Mm,'b'),
grid on
```

It can be seen that if the amplification in the frequency domain is higher, the overshoot in the time domain is also higher (Figs. 4.14 and 4.15). This holds exactly only for the given system and input signal, but shows well the important relations.

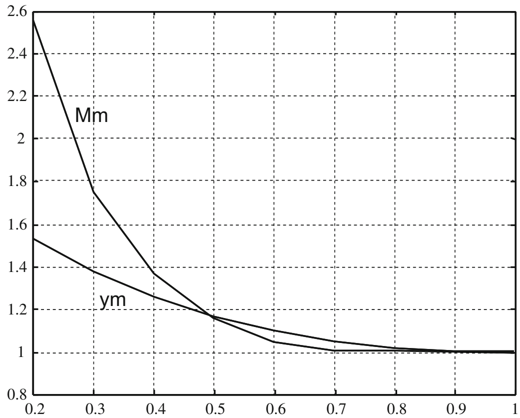


Fig. 4.13 The damping factor influences the overshoot and the Bode amplification

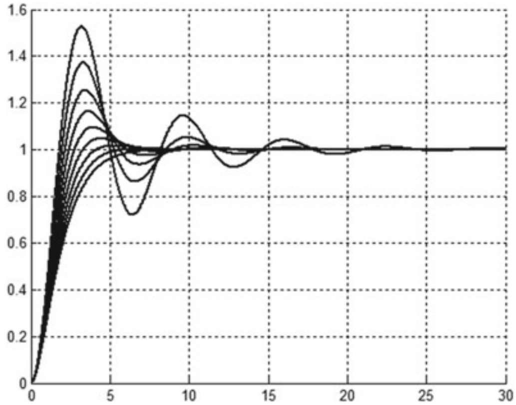


Fig. 4.14 Step responses of a second order oscillating element

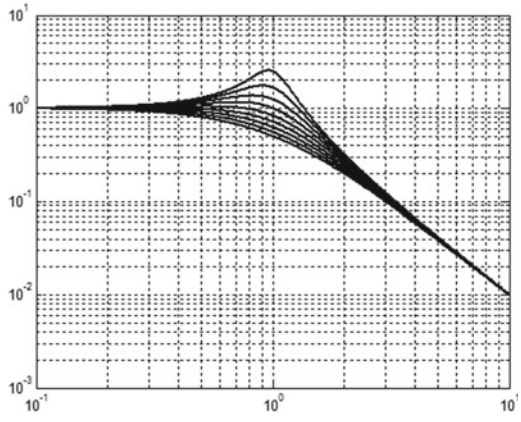


Fig. 4.15 Bode amplitude diagrams of a second order oscillating element

4.7 The Sensitivity Function

An important goal of a control system is to ensure acceptable behaviour also in case of changes in the parameters of the process model. In practical circumstances parameter changes can be the consequences of several effects. Warming of the system, ageing of its elements, change of humidity of the environment, etc., may influence significantly the behaviour of the system.

The effect of parameter changes can be investigated using the sensitivity function. The transfer function of the process can be given as the sum of the nominal transfer function and its change: $P(s) = P_o(s) + \Delta P(s)$. The sensitivity function S gives the ratio of the relative change of the resulting (overall) transfer function and the relative change of the transfer function of the process. So it characterises how much change is caused in the resulting transfer function if the process parameters change. The resulting transfer function T of the control system between the output and the reference signal is also called the complementary sensitivity function.

$$S = \frac{\Delta T/T}{\Delta P/P} = \frac{1}{1+CP}; \quad T = \frac{CP}{1+CP}; \quad S + T = 1.$$

Example 4.7 The system is a second-order oscillating element with transfer function $P(s) = \frac{1}{1+2\xi T_1 s + s^2 T_1^2}$. Its time constant is $T_1 = 5$ and the damping factor is $\xi = 0.7$.

The transfer function of the regulator is $C(s) = \frac{1}{10s}$. Unity negative feedback is applied. Let us analyse how sensitive is the behaviour of the control system to the changes of the time constant and the damping factor. Let us analyse the dynamics of the control system if the time constant changes to $T_1 = 10$ and the damping factor to $\xi = 0.2$.

For both cases plot the Bode amplitude diagrams of the sensitivity function and of the relative change of the process in one diagram.

```
s = zpk('s')
w = logspace(-3,1,500);
T0 = 5;kszi0 = 0.7;
T1 = 10;kszi1 = 0.2;
P0 = 1/(1 + 2*kszi0*T0*s + T0^2*s^2)
C = 0.1/s
L0 = C*P0
S = 1/(1 + L0)
P1 = 1/(1 + 2*kszi0*T1*s + T1^2*s^2)
P2 = 1/(1 + 2*kszi1*T0*s + T0^2*s^2)
deltaP1 = minreal((P1-P0)/P0,0.001)
deltaP2 = minreal((P2-P0)/P0,0.001)
M = bode(S,w);
```

```
M1 = bode(deltaP1,w);
M2 = bode(deltaP2,w);
figure (1)
loglog(w,M(:),w,M1(:))
figure (2)
loglog(w,M(:),w,M2(:))
t = 0:0.1:100;
figure (3)
step(P0,t,P1,t)
figure (4)
step(P0,t,P2,t)
```

Figure 4.16 shows that decreasing the damping factor, the relative change of the process is significant in the frequency range where the sensitivity function shows also amplification. A closed-loop control system will react strongly to this change.

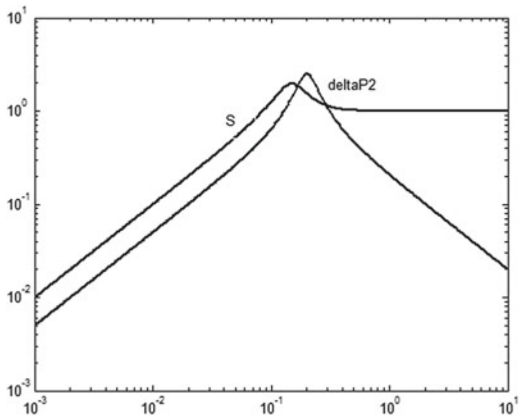


Fig. 4.16 Sensitivity function and the relative change in the damping factor

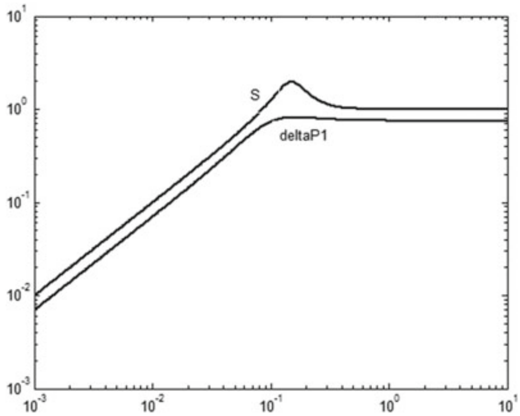


Fig. 4.17 Sensitivity function and the relative change in the time constant

The step response is shown in Fig. 4.18. Figure 4.17 shows the frequency function of the relative change of the process in the case of a change of the time constant. This curve is below the frequency function of the sensitivity function. The control system will be less sensitive to this parameter change (Fig. 4.19).

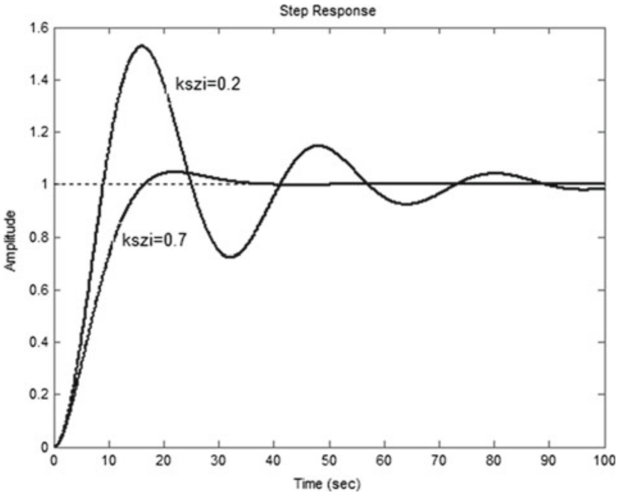


Fig. 4.18 Step responses in cases of two damping factors

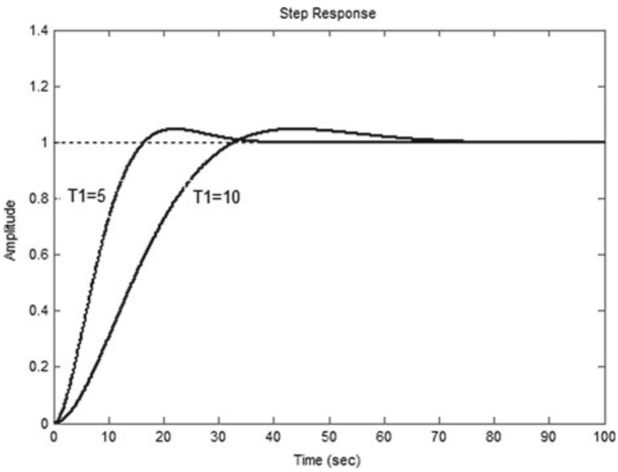


Fig. 4.19 Step responses in cases of two time constants

4.8 Control Structures

The most used control structure is realized by negative feedback where the regulator and the process are serially connected. This structure can be modified, supplemented with further elements to meet more sophisticated control aims (e.g. improvement of disturbance rejection).

4.8.1 Feedforward

Disturbance rejection can be improved if not only the output signal is used for control, but intermediate measurable signals are also employed to influence the control process. In these intermediate signals, the effect of the disturbance may show up earlier than in the output signal. In feedforward control, a measurable disturbance signal is measured and fed forward to influence the actuating signal.

Let us build a SIMULINK™ block-diagram to demonstrate feedforward control (Fig. 4.20).

Example 4.8 Let us simulate the behaviour of the system if $P = \frac{1}{(2s+1)(0.5s+1)}$, $C = \frac{2s+1}{s}$, $P_\mathrm{n} = \frac{1}{s+1}$.

```
P = 1/((2*s + 1)*(0.5*s + 1))
C = (2*s + 1)/s
Pn = 1/(s + 1)
```

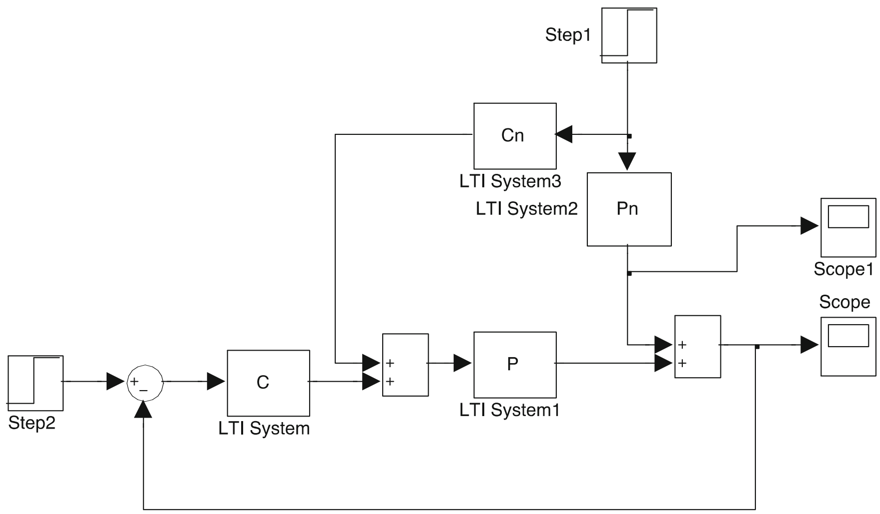


Fig. 4.20 SIMULINK™ block diagram of feedforward control

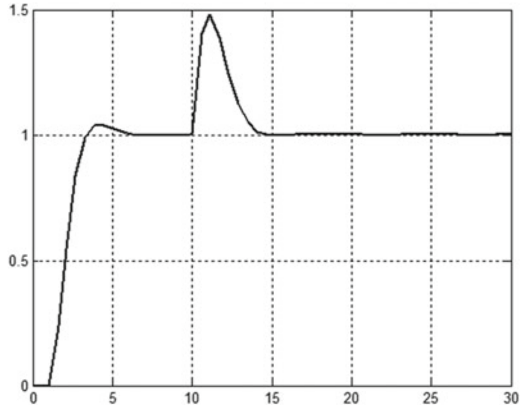


Fig. 4.21 Tracking and disturbance rejection without feedforward

The input signal is: $r(t) = 1(t)$.

The disturbance is: $y_n(t) = 1(t - 10)$.

Let us compare the behaviour of the control system without and with feedforward.

Without feedforward (Fig. 4.21):

```
Cn = 0
```

Feedforward is perfect if the effect of the disturbance through the feedforward regulator $C_n(s)$ cancels the effect of the disturbance, i.e. $P_n(s) + C_n(s)P(s) = 0$, hence $C_n(s) = -\dfrac{P_n(s)}{P(s)}$.

```
Cn = -Pn/P
- (s + 0.5) (s + 2)
-------------------
      (s + 1)
```

This transfer function is non-realizable as the degree of its numerator is higher than the degree of its denominator. Therefore an additional high frequency pole (a small time constant) is added to this transfer function.

$$C_n = -\frac{10(s + 0.5)(s + 2)}{(s + 1)(s + 10)}$$

```
Cn = -Pn/P/(0.1*s + 1)
```

Disturbance elimination will not be perfect, but the effect of the disturbance is decreased significantly (Fig. 4.22). Feedforward can be applied if the disturbance is measurable.

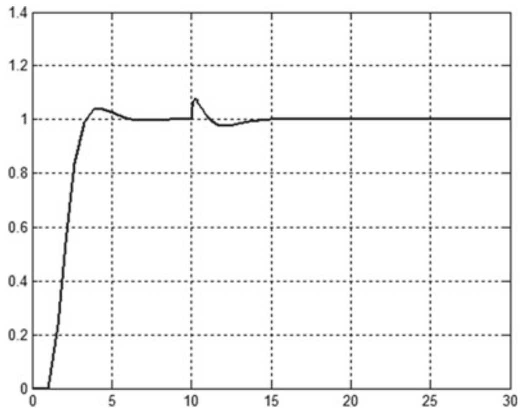


Fig. 4.22 Tracking and disturbance rejection with feedforward

4.8.2 Cascade Control

Cascade control can be applied if the process can be separated into several serially connected parts and the output signals of each part can be measured (Fig. 4.23).

For the first element of the system an inner control system can be built. For this inner circuit connected serially to the second part of the system an outer controller is designed (Fig. 4.24). The inner circuit can be fast, ensuring also fast disturbance rejection of the inner disturbance acting between the two parts of the system. The controller in the outer circuit can be designed for good reference signal tracking and rejection of the effect of the outer disturbance.

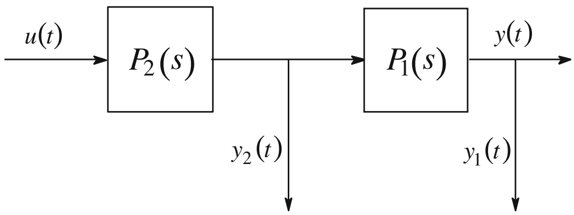


Fig. 4.23 A process separated to two serially connected parts with measurable inner signal

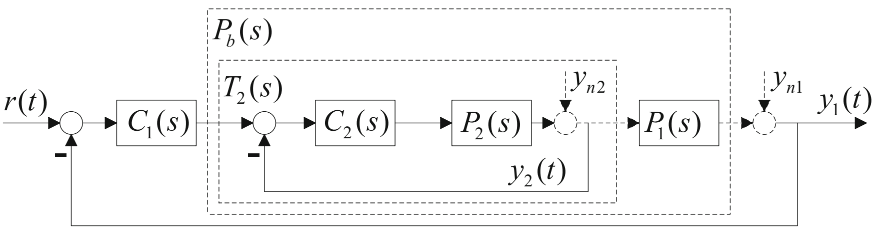


Fig. 4.24 Block diagram of cascade control

Example 4.9 The system to be controlled consists of two serially connected parts with transfer functions

$$P_1 = \frac{1}{1+10s} \quad \text{and} \quad P_2 = \frac{1}{1+s}.$$

The advantage of cascade control is significant if the system consists of a faster and a slower part and the slower part with the bigger time constant is in the outer circuit. Here this condition is fulfilled.

Let us design a fast control in the inner circuit, which ensures fast rejection of the effect of the inner disturbance. Then design a regulator for the outer circuit which ensures tracking of the step reference signal without steady error and rejection of the outer disturbance.

In the inner control circuit, let us choose a proportional regulator (C2 = 10).

```
P2 = 1/(s + 1)
C2 = 10
T2 = C2*P2/(1 + C2*P2);T2 = minreal(T2)
step(P2,'b',T2,'r'),grid
```

On the left side of Fig. 4.25 it can be seen that the behaviour of the inner circuit has become fast, but there is a static error.

In the outer control circuit an integrator has to be used in the regulator to decrease the steady state error to zero. With a regulator C1 the big time constant of the *PI* part of the system is cancelled and instead an integrating effect is introduced. Let $C_1(s) = \frac{5(1+10s)}{s}$. (Regulator design using considerations in the frequency domain, the so called *PID* compensation is discussed in Chap. 8.)

```
P1 = 1/(10*s + 1)
Pb = T2*P1
C1 = 5*(10*s + 1)/s
T = C1*Pb/(1 + C1*Pb); T = minreal(T)
figure (1),step(T),grid
```

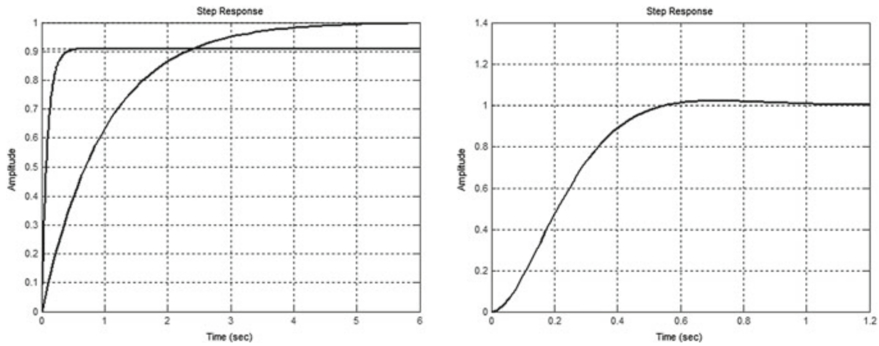


Fig. 4.25 Behaviour of the inner loop and of the output signal in cascade control

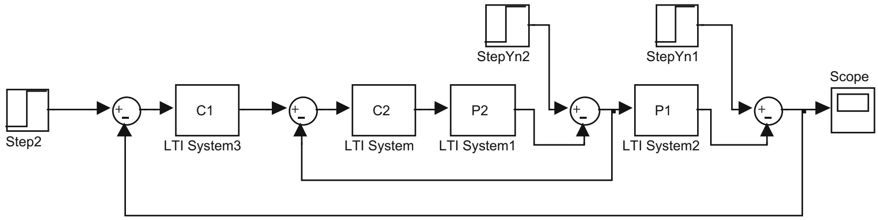
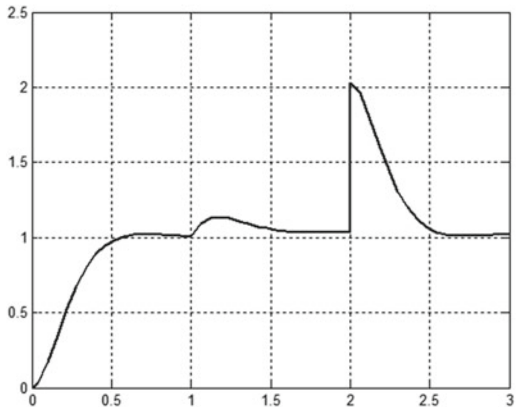


Fig. 4.26 SIMULINK™ block diagram of cascade control

Fig. 4.27 Simulation of cascade control for tracking and disturbance rejection



On the right side of Fig. 4.25 it can be seen that the output response is fast and the static error is zero.

Let us investigate the behaviour of the cascade control circuit in the SIMULINK™ environment (Fig. 4.26).

Set the reference signal and the disturbances to the following values:

$$r(t) = 1(t);\ \ y_{n2} = 1(t-1);\ y_{n1} = 1(t-2)$$

Set the simulation time to 3 s. In Fig. 4.27 it can be seen that the control system tracks the reference signal and eliminates the effects of both the inner and the outer disturbances.