Chapter 4
Conservation of Energy

Since we do not wish to interrupt the derivation of the energy equation for stars with lengthy formalisms, we first provide a few thermodynamic relations which will be used extensively later on.

4.1 Thermodynamic Relations

The first law of thermodynamics relates the heat dq added per unit mass,

$$dq = du + Pdv, \tag{4.1}$$

to the internal energy u and the specific volume $v = 1/\varrho$ (both also defined per unit mass).

We now assume rather general equations of state, $\varrho = \varrho(P, T)$ and $u = u(\varrho, T)$. Usually they will also depend on the chemical composition, but here this is assumed to be fixed. With the derivatives defined as

$$\alpha := \left(\frac{\partial \ln \varrho}{\partial \ln P}\right)_T = -\frac{P}{v}\left(\frac{\partial v}{\partial P}\right)_T, \tag{4.2}$$

$$\delta := -\left(\frac{\partial \ln \varrho}{\partial \ln T}\right)_P = \frac{T}{v}\left(\frac{\partial v}{\partial T}\right)_P, \tag{4.3}$$

the equation of state can be written in the form $d\varrho/\varrho = \alpha dP/P - \delta dT/T$.

We also need the specific heats:

$$c_P := \left(\frac{dq}{dT}\right)_P = \left(\frac{\partial u}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P, \tag{4.4}$$

$$c_v := \left(\frac{dq}{dT}\right)_v = \left(\frac{\partial u}{\partial T}\right)_v. \tag{4.5}$$

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With

$$du = \left(\frac{\partial u}{\partial v}\right)_T dv + \left(\frac{\partial u}{\partial T}\right)_v dT \tag{4.6}$$

and with (4.1) we find the change $ds = dq/T$ of the specific entropy to be

$$ds = \frac{dq}{T} = \frac{1}{T}\left[\left(\frac{\partial u}{\partial v}\right)_T + P\right] dv + \frac{1}{T}\left(\frac{\partial u}{\partial T}\right)_v dT. \tag{4.7}$$

Since ds is a total differential form, $\partial^2 s/\partial T \partial v = \partial^2 s/\partial v \partial T$ and

$$\frac{\partial}{\partial T}\left[\frac{1}{T}\left(\frac{\partial u}{\partial v}\right)_T + \frac{P}{T}\right] = \frac{1}{T}\frac{\partial^2 u}{\partial T \partial v}, \tag{4.8}$$

which after the differentiation on the left is carried out gives

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P. \tag{4.9}$$

Next we derive an expression for $(\partial u/\partial T)_P$, taking P, T as independent variables. From (4.6) it follows that

$$\frac{du}{dT} = \left(\frac{\partial u}{\partial T}\right)_v + \left(\frac{\partial u}{\partial v}\right)_T \frac{dv}{dT}, \tag{4.10}$$

and therefore

$$\begin{aligned} \left(\frac{\partial u}{\partial T}\right)_P &= \left(\frac{\partial u}{\partial T}\right)_v + \left(\frac{\partial u}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_P \\ &= \left(\frac{\partial u}{\partial T}\right)_v + \left(\frac{\partial v}{\partial T}\right)_P \left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right], \end{aligned} \tag{4.11}$$

where we have made use of (4.9). From the definitions (4.4), (4.5) and from (4.11) we write

$$\begin{aligned} c_P - c_v &= P\left(\frac{\partial v}{\partial T}\right)_P + \left(\frac{\partial u}{\partial T}\right)_P - \left(\frac{\partial u}{\partial T}\right)_v \\ &= \left(\frac{\partial v}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_v T. \end{aligned} \tag{4.12}$$

On the other hand, the definitions (4.2) and (4.3) for α and δ imply that

$$\left(\frac{\partial P}{\partial T}\right)_v = -\frac{\left(\frac{\partial v}{\partial T}\right)_P}{\left(\frac{\partial v}{\partial P}\right)_T} = \frac{P\delta}{T\alpha}, \tag{4.13}$$

and therefore

$$c_P - c_v = T\left(\frac{\partial v}{\partial T}\right)_P \frac{P\delta}{T\alpha} = \frac{P\delta^2}{\varrho T\alpha}\ , \tag{4.14}$$

where we have made use of $T(\partial v/\partial T)_P = v\delta = \delta/\varrho$; hence we arrive at the basic relation

$$c_P - c_v = \frac{P\delta^2}{\varrho T\alpha}\ . \tag{4.15}$$

For a perfect gas this equation reduces to the well-known relation $c_P - c_v = \Re/\mu$ [see (4.33)].

We have now derived all the tools for rewriting (4.1) in terms of T and P. The first step is to write it in the form

$$\begin{aligned} dq = du + Pdv &= \left(\frac{\partial u}{\partial T}\right)_v dT + \left[\left(\frac{\partial u}{\partial v}\right)_T + P\right] dv \\ &= \left(\frac{\partial u}{\partial T}\right)_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv \end{aligned} \tag{4.16}$$

by making use of (4.9), and then with (4.5) and (4.13) we have

$$\begin{aligned} dq &= c_v dT - \frac{T}{\varrho}\left(\frac{\partial P}{\partial T}\right)_v \frac{d\varrho}{\varrho} = c_v dT - \frac{P\delta}{\varrho\alpha}\frac{d\varrho}{\varrho} \\ &= c_v dT - \frac{P\delta}{\varrho\alpha}\left(\alpha\frac{dP}{P} - \delta\frac{dT}{T}\right) = \left(c_v + \frac{P\delta^2}{\varrho T\alpha}\right) dT - \frac{\delta}{\varrho} dP. \end{aligned} \tag{4.17}$$

The terms in parentheses in the last expression are, according to (4.15), simply c_P and therefore

$$dq = c_P dT - \frac{\delta}{\varrho} dP. \tag{4.18}$$

Next we define the adiabatic temperature gradient $\nabla_{\rm ad}$, a quantity often used in astrophysics, by

$$\nabla_{\rm ad} := \left(\frac{\partial \ln T}{\partial \ln P}\right)_s\ , \tag{4.19}$$

where the subscript s indicates that the definition is valid for constant entropy. Since for adiabatic changes the entropy has to remain constant, i.e. $ds = dq/T = 0$, we can easily derive an expression for $\nabla_{\rm ad}$ from (4.18), i.e.

$$0 = dq = c_P dT - \frac{\delta}{\varrho} dP \tag{4.20}$$

or $(dT/dP)_s = \delta/\varrho c_P$ and

$$\nabla_{\rm ad} \equiv \left(\frac{P}{T}\frac{dT}{dP}\right)_s = \frac{P\delta}{T\varrho c_P} \ . \tag{4.21}$$

4.2 The Perfect Gas and the Mean Molecular Weight

For a perfect gas consisting of n particles per unit volume that all have the molecular weight μ, the equation of state is

$$P = nkT = \frac{\Re}{\mu}\varrho T \ , \tag{4.22}$$

with $\varrho = n\mu m_{\rm u}$($k = 1.38 \times 10^{-16}\,{\rm erg\,K^{-1}}$ = Boltzmann constant; $\Re = k/m_{\rm u} = 8.31 \times 10^7\,{\rm erg\,K^{-1}\,g^{-1}}$ = universal gas constant; $m_{\rm u} = 1\,{\rm amu} = 1.66053 \times 10^{-24}\,{\rm g}$ = the atomic mass unit). Note that we here use the gas constant with a dimension (energy per K and per *unit mass*) different from that in thermodynamic text books (energy per K and per mole). This has the consequence that here the molecular weight μ is dimensionless (instead of having the dimension mass per mole); it is simply the particle mass divided by 1 amu.

In the deep interiors of stars the gases are fully ionized, i.e. for each hydrogen nucleus, there also exists a free electron, while for each helium nucleus, there are two free electrons. We therefore have a mixture of two gases, that of the nuclei (which in itself can consist of more than one component) and that of the free electrons. The mixture can be treated similarly to a one-component gas, if all single components obey the perfect gas equation.

We consider a mixture of fully ionized nuclei. The chemical composition can be described by specifying all X_i, the weight fractions of nuclei of type i, which have molecular weight μ_i and charge number Z_i. If we have n_i nuclei per volume and a "partial density" ϱ_i, then obviously $X_i = \varrho_i/\varrho$ and

$$n_i = \frac{\varrho_i}{\mu_i m_{\rm u}} = \frac{\varrho}{m_{\rm u}}\frac{X_i}{\mu_i} \ . \tag{4.23}$$

(Here and in the following, we neglect the mass of the electrons compared to that of the ions.) The total pressure P of the mixture is the sum of the partial pressures

$$P = P_{\rm e} + \sum_i P_i = \left(n_{\rm e} + \sum_i n_i\right) kT \ . \tag{4.24}$$

Here $P_{\rm e}$ is the pressure of the free electrons, while P_i is the partial pressure due to the nuclei of type i. The contribution of one completely ionized atom of element i

to the total number of particles (nucleus plus Z_i free electrons) is $1 + Z_i$; therefore

$$n = n_\mathrm{e} + \sum_i n_i = \sum_i (1 + Z_i) n_i \ . \tag{4.25}$$

With this and (4.23), (4.24) becomes

$$P = nkT = \Re \sum_i \frac{X_i(1 + Z_i)}{\mu_i} \varrho T \ , \tag{4.26}$$

which can be written simply in the form (4.22) with the *mean molecular weight*

$$\mu = \left(\sum_i \frac{X_i(1 + Z_i)}{\mu_i} \right)^{-1} \ . \tag{4.27}$$

By introducing the mean molecular weight, we are able to treat a mixture of perfect gases as a uniform perfect gas. We just have to replace the molecular weight in (4.22) by the mean molecular weight. In the case of pure (fully ionized) hydrogen with $X_\mathrm{H} = 1$, $\mu_\mathrm{H} = 1$, $Z_\mathrm{H} = 1$, we have $\mu = 1/2$, while for a fully ionized helium gas ($X_\mathrm{He} = 1$, $\mu_\mathrm{He} = 4$, $Z_\mathrm{He} = 2$), we find $\mu = 4/3$.

Equation (4.27) can be easily modified for the partial gas consisting of the ions only, or equivalently, for the case of a *neutral* gas where all the electrons are still in the atom. In (4.25) we just have to replace $1 + Z_i$ by 1 and we find

$$\mu_0 = \left(\sum_i \frac{X_i}{\mu_i} \right)^{-1} \ . \tag{4.28}$$

Here we have dealt with the cases of full ionization and of no ionization at all. In Chap. 14 we will deal with the case of partial ionization.

At this point we also define the mean molecular weight per free electron μ_e, a quantity which we shall need later. For a fully ionized gas each nucleus i contributes Z_i free electrons and we have

$$\mu_\mathrm{e} = \left(\sum_i X_i Z_i / \mu_i \right)^{-1} \ . \tag{4.29}$$

Since for all (not too rare) elements heavier than helium $\mu_i / Z_i \approx 2$ is a good approximation, we find

$$\mu_\mathrm{e} = \left(X + \frac{1}{2} Y + \frac{1}{2}(1 - X - Y) \right)^{-1} = \frac{2}{1 + X} \ , \tag{4.30}$$

where we have followed the custom of using $X := X_{\mathrm{H}}, Y := X_{\mathrm{He}}$ for the weight fractions of hydrogen and helium. Then $1 - X - Y$ is the mass fraction of the elements heavier than helium.

4.3 Thermodynamic Quantities for the Perfect, Monatomic Gas

If the gas is monatomic, the internal energy per gram is the kinetic energy of the translational motion of the particles only

$$u = \frac{3}{2} kT \frac{n}{\varrho}. \tag{4.31}$$

From (4.2) and (4.3) we find

$$\alpha = \delta = 1, \tag{4.32}$$

and from (4.15)

$$c_P - c_v = \frac{P}{\varrho T} = \frac{\Re}{\mu} \tag{4.33}$$

and therefore with (4.5)

$$c_v = \left(\frac{\partial u}{\partial T}\right)_\varrho = \frac{3}{2} k \frac{n}{\varrho} = \frac{3}{2} \frac{\Re}{\mu} \tag{4.34}$$

and with (4.33)

$$c_P = \frac{5}{2} \frac{\Re}{\mu}. \tag{4.35}$$

Equation (4.21) therefore yields

$$\nabla_{\mathrm{ad}} = \frac{\Re}{\mu c_P} = \frac{c_P - c_v}{c_P} = \frac{2}{5}. \tag{4.36}$$

Sometimes also the quantity

$$\gamma_{\mathrm{ad}} := \left(\frac{\partial \ln P}{\partial \ln \varrho}\right)_s \tag{4.37}$$

for adiabatic changes is needed. If we differentiate the equation of state (4.22), we find

$$\frac{dP}{P} = \frac{d\varrho}{\varrho} + \frac{dT}{T} \tag{4.38}$$

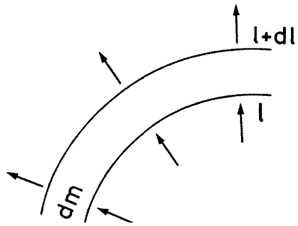

Fig. 4.1 Energy flux through a mass shell

which holds for all variations of the variables in the perfect gas equation, including the adiabatic variation. For these we obtain from (4.36)

$$\frac{dT}{T} = \nabla_{\rm ad}\frac{dP}{P} = \left(1 - \frac{c_v}{c_P}\right)\frac{dP}{P}. \tag{4.39}$$

Eliminating dT/T from (4.38) and (4.39) gives

$$\left(\frac{d\varrho}{\varrho}\right)_{\rm ad} = \frac{c_v}{c_P}\left(\frac{dP}{P}\right)_{\rm ad} \tag{4.40}$$

or

$$\gamma_{\rm ad} = \left(\frac{d\ln P}{d\ln\varrho}\right)_s = \frac{c_P}{c_v}. \tag{4.41}$$

4.4 Energy Conservation in Stars

By $l(r)$ we define[1] the net energy per second passing outward through a sphere of radius r. The function l is zero at $r = 0$, since there can be no infinite energy source at the centre, while l reaches the total luminosity L of the star at the surface. In between, l can be a complicated function, depending on the distribution of the sources and sinks of energy.

The function l comprises the energies transported by radiation, conduction, and convection, transport mechanisms with which we shall deal in Chaps. 5 and 7. Not included is a possible energy flux by neutrinos, which normally have negligible interaction with the stellar matter (see below). Included in l are only those fluxes which require a temperature gradient.

Consider a spherical mass shell of radius r, thickness dr, and mass dm, as indicated in Fig. 4.1. The energy per second entering the shell at the inner surface is l, while $l + dl$ is the energy per second leaving it through the outer surface. The surplus power dl can be provided by nuclear reactions, by cooling, or by compression or expansion of the mass shell.

[1] In many textbooks our function l is denoted by L_r.

We first consider a *stationary* case in which dl is due to the release of energy from nuclear reactions only. Let ε be the nuclear energy released per unit mass per second; then

$$dl = 4\pi r^2 \varrho\varepsilon\, dr = \varepsilon\, dm\,, \qquad \text{or} \tag{4.42}$$

$$\frac{\partial l}{\partial m} = \varepsilon. \tag{4.43}$$

In general ε depends on temperature and density and on the abundance of the different nuclear species that react, described in detail in Chap. 18.

If we relax the condition of time independence, then dl can become non-zero even if there are no nuclear reactions. A *non-stationary* shell can change its internal energy, and it can exchange mechanical work ($P\ dV$) with the neighbouring shells. Instead of (4.43) we write

$$dq = \left(\varepsilon - \frac{\partial l}{\partial m}\right) dt, \tag{4.44}$$

where dq is the heat per unit mass added to the shell in the time interval dt. Replacing dq by the first law of thermodynamics (4.1) we obtain

$$\begin{aligned}\frac{\partial l}{\partial m} &= \varepsilon - \frac{\partial u}{\partial t} - P\frac{\partial v}{\partial t}\\ &= \varepsilon - \frac{\partial u}{\partial t} + \frac{P}{\varrho^2}\frac{\partial \varrho}{\partial t}\end{aligned} \tag{4.45}$$

This can be rewritten in terms of P and T, with the help of (4.18), as

$$\frac{\partial l}{\partial m} = \varepsilon - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\varrho}\frac{\partial P}{\partial t}\,, \tag{4.46}$$

where δ is defined in (4.3). This is the third of the basic equations of stellar structure. One often combines the terms containing the time derivatives in a source function

$$\begin{aligned}\varepsilon_{\rm g} &:= -T\frac{\partial s}{\partial t}\\ &= -c_P\frac{\partial T}{\partial t} + \frac{\delta}{\varrho}\frac{\partial P}{\partial t}\\ &= -c_P T\left(\frac{1}{T}\frac{\partial T}{\partial t} - \frac{\nabla_{\rm ad}}{P}\frac{\partial P}{\partial t}\right)\,,\end{aligned} \tag{4.47}$$

where use is made of the fact that $ds = dq/T$ and of (4.21).

Let us now turn to the problem of *neutrino losses.* These can be formed in appreciable amounts in a star either as a by-product of nuclear energy generation or by other reactions. Stellar material is normally transparent to neutrinos and therefore they can easily "tunnel" the energy they have to the surface. This is the reason we have excluded the energy flux due to neutrinos from l. The only mass elements affected by the neutrinos are at the place of their creation, where they act as an energy sink; hence ε_ν is used to represent the energy taken per unit mass per second from the stellar material in the form of neutrinos. In general, the energy lost by neutrinos in nuclear reactions is already taken into account in the net energy Q released in each reaction (see Sect. 18.3). By definition, $\varepsilon_\nu > 0$. Obviously the complete energy equation is then

$$\frac{\partial l}{\partial m} = \varepsilon - \varepsilon_\nu + \varepsilon_g \, . \tag{4.48}$$

As mentioned at the beginning of Sect. 4.4, the boundary values of l are $l = 0$ at the centre and $l = L$ at the surface. In between, l is not necessarily monotonic, since the right-hand side of (4.48) may be positive or negative; l can even become larger than L, or negative. For instance, the surface luminosity L of an expanding star can be smaller than the energy produced in the central core by nuclear reactions ($\varepsilon > 0$), since part of it is used to expand the star ($\varepsilon_g < 0$); and strong neutrino losses can make $l < 0$ in certain parts of the stellar interior (see Sect. 33.5).

The energy per second carried away from the star by neutrinos is often called the *neutrino luminosity:*

$$L_\nu := \int_0^M \varepsilon_\nu \, dm. \tag{4.49}$$

4.5 Global and Local Energy Conservation

In Chap. 3 we considered gravitational energy (E_g) and internal energy (E_i), but ignored nuclear and neutrino energies, as well as the kinetic energy E_{kin} of radial motion. We now define the total energy of the star as $W = E_{kin} + E_g + E_i + E_n$, where E_n is the nuclear energy content of the whole star. Obviously the energy equation is

$$\frac{d}{dt}(E_{kin} + E_g + E_i + E_n) + L + L_\nu = 0 \, , \tag{4.50}$$

and, of course, this must also be obtained from the *local* energy equation (4.48) by integration over m. Clearly, the integration of $\partial l/\partial m$ gives L, the integration of $-\varepsilon_\nu$ gives $-L_\nu$, while the integral over ε gives $-dE_n/dt$. Integration over ε_g, however, needs some consideration.

Let us write ε_g as in (4.45):

$$\varepsilon_g = -\frac{\partial u}{\partial t} + \frac{P}{\varrho^2}\frac{\partial \varrho}{\partial t} \, . \tag{4.51}$$

Then integration over $-\partial u/\partial t$ gives $-dE_{\mathrm{i}}/dt$. In order to deal with the last term in (4.51) we use (3.2, 3.3) and find that

$$E_{\mathrm{g}} = -3 \int_0^M \frac{P}{\varrho} dm \;, \tag{4.52}$$

which we differentiate with respect to time (indicated by dots):

$$\dot{E}_{\mathrm{g}} = -3 \int_0^M \frac{\dot{P}}{\varrho} dm + 3 \int_0^M \frac{P}{\varrho^2} \dot{\varrho}\, dm \;. \tag{4.53}$$

We first treat hydrostatic equilibrium ($dE_{\mathrm{kin}}/dt = 0$). Then differentiation of (2.5) gives

$$\frac{\partial \dot{P}}{\partial m} = 4 \frac{Gm}{4\pi r^4} \frac{\dot{r}}{r} \;. \tag{4.54}$$

We multiply this by $4\pi r^3$ and integrate over m:

$$\int_0^M 4\pi r^3 \frac{\partial \dot{P}}{\partial m} dm = 4 \int_0^M \frac{Gm}{r} \frac{\dot{r}}{r} dm = 4\, \dot{E}_{\mathrm{g}} \;. \tag{4.55}$$

Partial integration of the left-hand side gives

$$[4\pi r^3 \dot{P}]_0^M - 3 \int_0^M 4\pi r^2 \frac{\partial r}{\partial m} \dot{P}\; dm \;, \tag{4.56}$$

where the term in brackets vanishes at both ends of the interval, since either $r = 0$ or $P = 0$ independent of time. If we replace $\partial r/\partial m$ by $1/4\pi r^2 \varrho$ we find from (4.55) that

$$-3 \int_0^M \frac{\dot{P}}{\varrho} dm = 4 \dot{E}_{\mathrm{g}} \;. \tag{4.57}$$

Introducing this into the right-hand side of (4.53) gives

$$\dot{E}_{\mathrm{g}} = -\int_0^M \frac{P}{\varrho^2} \dot{\varrho}\, dm \;, \tag{4.58}$$

and therefore the integration of the last term of (4.51) gives $\dot{E}_{\mathrm{g}}$ so that the equation (4.50) without $\dot{E}_{\mathrm{kin}}$ is now recovered.

If, instead of hydrostatic equilibrium, we had used the full equation of motion (2.16), after multiplication with $4\pi r^2 \dot{r}$ and integration over m, we would have obtained the full equation (4.50) *with* the term $\dot{E}_{\mathrm{kin}}$.

4.6 Timescales

Consider a star balancing its energy loss L essentially by release of nuclear energy. If L remains constant this can go on for a *nuclear timescale* τ_n defined by

$$\tau_n := \frac{E_n}{L} \,. \tag{4.59}$$

Note that E_n means the nuclear energy reservoir from which energy can be released under the given circumstances, i.e. the corresponding reactions must be possible. The most important reaction is the fusion of ^1H into ^4He. This "hydrogen burning" releases $Q = 6.3 \times 10^{18}\,\mathrm{erg\,g^{-1}}$, and, if the Sun consisted completely of hydrogen, E_n would be $QM_\odot = 1.25 \times 10^{52}$ erg. With $L_\odot = 4 \times 10^{33}$ erg/s, (4.59) gives $\tau_n = 3 \times 10^{18}$ s, or 10^{11} years. A comparison with the earlier estimates of τ_{hydr} (Sect. 2.4) and τ_{KH} (Sect. 3.3) shows that

$$\tau_n \gg \tau_{KH} \gg \tau_{hydr} \,, \tag{4.60}$$

which is not only true for the Sun, but for all stars that survive by hydrogen and helium burning. We emphasize this point, since under these circumstances the equation of energy conservation (4.46) can be simplified. As an illustration, we assume that the star changes its properties considerably within the timescale τ (which may be either small or large compared to τ_{KH}). This change may, for instance, be due to exhaustion of nuclear fuel or artificial "squeezing" of the star from the exterior. We now give rough estimates for the four terms in (4.46), assuming a perfect gas:

$$\left|\frac{\partial l}{\partial m}\right| \approx \frac{L}{M} \approx \frac{E_i}{\tau_{KH} M} \,, \tag{4.61}$$

$$\varepsilon \approx \frac{L}{M} = \frac{E_n}{M\tau_n} \approx \frac{E_i}{\tau_{KH} M} \,, \tag{4.62}$$

$$\left|c_P \frac{\partial T}{\partial t}\right| \approx \frac{c_P T}{\tau} \approx \frac{E_i}{\tau M} \,, \tag{4.63}$$

$$\left|\frac{\delta}{\varrho}\frac{\partial P}{\partial t}\right| \approx \frac{\Re}{\mu}\frac{T}{\tau} \approx \frac{c_P T}{\tau} \approx \frac{E_i}{\tau M} \,. \tag{4.64}$$

In the case $\tau \gg \tau_{KH}$, the terms in (4.63) and (4.64) are small compared to those in (4.61) and (4.62); therefore the time derivatives in the energy equation (4.46) can be neglected ($|\varepsilon_g| \ll \varepsilon$), and the energy equation is $\partial l/\partial m = \varepsilon$, as in (4.43). This occurs if, for instance, the consumption of hydrogen and helium steers the evolution, i.e. $\tau = \tau_n$ ($\gg \tau_{KH}$), and represents a considerable simplification for calculating models which are said to be in *complete equilibrium* (i.e. mechanical and thermal equilibrium).

In the case $\tau \ll \tau_{KH}$, the right-hand sides of (4.63) and (4.64) are large compared to those of (4.61) and (4.62). Therefore in (4.46) the last two terms containing the time derivatives must (at least very nearly) cancel each other, which means that $dq/dt \approx 0$, or the change is nearly adiabatic. Note that a relatively small deviation from the strict adiabatic change can still be of the order ε, and therefore ε_g cannot be neglected in the energy equation. An example for this case is a star pulsating with the timescale $\tau = \tau_{hydr} \ll \tau_{KH}$ (see Chaps. 40 and 41). The variable luminosity of a pulsating star, for instance, is not due to changes of ε but of ε_g.

Here we have assumed the simplest case, namely that the star changes more or less uniformly. The situation can be much more complicated if, for example, only parts of the star are affected and local timescales have to be considered which may be quite different.