

## Chapter 38

# Neutron Stars

As early as 1934 Baade and Zwicky correctly predicted the birth of the strange objects neutron stars in supernova explosions (Baade and Zwicky 1934). The first models were calculated by Oppenheimer and Volkoff (1939), and the stage was then left for the next 28 years to particle physicists who struggled with the problem of matter at extreme densities (a struggle not yet finished). Radio astronomers accidentally found the first *pulsar* in 1967; it was interpreted soon after as a rapidly rotating neutron star (Gold 1968), emitting synchrotron radiation in a narrow beacon along the magnetic axis. In addition, neutron stars were identified as sources of energetic X-ray emission, resulting from accretion in binary systems. By now, the existence of neutron stars is well established. The number of detected pulsars in the Galaxy already amounts to more than 1,800 (Lorimer 2008). These known neutron stars constitute only a tiny fraction of a population as large as a few hundred million. In some cases, their masses could be determined quite accurately (Fig. 38.2) because they are members of binary systems or from relativistic effects in their extreme gravitational potential. Everything is extreme with neutron stars, their interior state (simulating a huge nucleus), the velocity of sound (not far from  $c$ ), their rotation (frequencies 1 . . . 1,000 Hz), and their magnetic fields (from  $10^9$  to  $10^{15}$  gauss). One is far from really understanding them. So we content ourselves here with a few remarks on the state of matter and the resulting models. For more detailed and complete information about neutron stars, we recommend one of the many existing textbooks on compact objects (e.g., Glendenning 1997; Camenzind 2007; Haensel et al. 2007).

### 38.1 Cold Matter Beyond Neutron Drip

Neutron stars (NS) are born hot ( $T > 10^{10}$  K) in the collapse of a highly evolved star (see Chap. 36). But the interior temperature drops rapidly because of neutrino emission: after a day, temperatures of  $10^9$  K are reached; after 100 years, maybe  $10^8$  K. And this ( $kT \approx 10$  keV) can be considered cold in view of the degenerate

nearly relativistic neutrons ( $E_F \approx 1,000$  MeV). The equation of state is essentially the same as for  $T \approx 0$ . We refer to the descriptions of high-density matter in Sect. 37.2 and of the equation of state in Chap. 16.

With increasing density the rising Fermi energy of the electrons provides an increasing neutronization by electron captures. The neutron-rich equilibrium nuclei (such as  $^{118}\text{Kr}$ ) begin to release free neutrons at  $\varrho_{\text{dr}} \approx 4.3 \times 10^{11} \text{ g cm}^{-3}$ . This is called the *neutron drip*. The matter consists of nuclei (usually arranged in a lattice) plus sufficient electrons for charge neutrality, and free neutrons. Their number  $n_n$  increases with  $\varrho$ , and so does their pressure  $P_n$ . While  $P \approx P_e \gg P_n$  still at  $\varrho = \varrho_{\text{dr}}$ , we have  $P_n = P/2$  at  $\varrho \approx 4 \times 10^{12} \text{ g cm}^{-3}$  (here the Coulomb lattice is dissolving) and  $P_n > 0.8P$  for  $\varrho \gtrsim 1.5 \times 10^{13} \text{ g cm}^{-3}$ , and finally  $P_n \approx P$ . The neutrons are increasingly degenerate, but still non-relativistic, as their Fermi energy is much smaller than their rest-mass energy. Note that all characteristic densities quoted here and in the following depend in general on the model assumed for the particles and their interaction. The higher the values of  $\varrho$ , the more uncertain are the details (see below).

With progressing neutron drip the number of nuclei is diminished by fusion. The nuclei more or less touch each other at the *nuclear density* of  $\varrho_{\text{nuc}} \approx 2.7 \times 10^{14} \text{ g cm}^{-3}$ , and hence they merge and dissolve, leaving a degenerate gas (or liquid) of neutrons plus a small admixture of  $e^-$  and  $p$ . The concentrations of these particles can be calculated as an equilibrium between back and forth exchanges in the reaction  $n \rightleftharpoons p + e^-$  (The neutrinos leave the system immediately and can be left out of the considerations.). The conditions are that the Fermi energies fulfil  $E_F^n = E_F^p + E_F^e$ , and that  $n_e = n_p$  for neutrality. This gives that  $n_p$  is about 1 % (or less) of  $n_n$  for a wide range of  $\varrho$  up to  $\varrho_{\text{nuc}}$ . At  $\varrho \approx 6 \times 10^{15} \text{ g cm}^{-3}$  the neutrons are relativistically degenerate. With increasing relativity of the neutrons the fraction of protons raises slowly, until at an infinite relativity parameter one finds the limiting ratio  $n_n : n_p : n_e = 8 : 1 : 1$ . When  $\varrho$  exceeds  $10^{15} \text{ g cm}^{-3}$ , the Fermi energy of the neutrons,  $E_F = [(p_{\text{FC}})^2 + (m_n c^2)^2]^{1/2}$ , will gradually exceed the rest masses of the hyperons of lowest mass (such as  $\Lambda$ ,  $\Sigma$ ,  $\Delta$ , ...). These particles will then appear, i.e. a “hyperonization” begins. Finally even free quarks can occur. Obviously, at these densities, nuclear forces, the interaction between elementary particles, and the masses of hadron states are determining the exact composition of neutron star matter. Astrophysics meets quantum chromodynamics!

We now come to the *equation of state*, in particular the dependence of  $P$  on  $\varrho$ . For  $\varrho$  up to  $\varrho_{\text{drip}}$ , the pressure is dominated by the relativistic, degenerate electrons, and  $P \approx P_e \sim \varrho^{4/3}$  [cf. (15.26)].

The onset of the neutron drip ( $\varrho = \varrho_{\text{drip}}$ ) has severe consequences for the equation of state. An increase  $d\varrho$  mainly increases  $n_n$  at the expense of  $n_e$  (which yields the pressure), such that the increment  $dP$  is small (see Fig. 16.2). Therefore the gas becomes more compressible, which is described as a “softening” of the equation of state (in the opposite case one speaks of “stiffening”). In other terms the adiabatic index  $\gamma_{\text{ad}} = (d \ln P / d \ln \varrho)_{\text{ad}}$  drops appreciably below the critical value  $4/3$  (cf. Sect. 25.3.2), and only when  $P_n$  contributes sufficiently to  $P$  will  $\gamma_{\text{ad}}$  again rise above  $4/3$  at  $\varrho \approx 7 \times 10^{12} \text{ g cm}^{-3}$ .

When the neutron pressure  $P_n$  dominates one may tentatively consider the approximation that the gas consists of *ideal* (non-interacting), *fully degenerate* neutrons. These are fermions like the electrons, and they obey the same statistics, so that the same relations hold as derived in Sect. 15.2, if there  $m_e$  is replaced by  $m_n$  and  $\mu_e$  by 1 (since we now have one nucleon per fermion). Instead of (15.23) and (15.26) we can write

$$P_n = K_{\gamma'} \varrho_0^{\gamma'} \quad (38.1)$$

with the non-relativistic and relativistic limit cases (for  $\varrho_0 \ll 6 \times 10^{15}$  and  $\varrho_0 \gg 6 \times 10^{15} \text{ g cm}^{-3}$  respectively)

$$\begin{aligned} \gamma' &= \frac{5}{3}, & K_{5/3} &= \frac{1}{20} \left( \frac{3}{\pi} \right)^{2/3} \frac{h^2}{m_n^{8/3}}, \\ \gamma' &= \frac{4}{3}, & K_{4/3} &= \frac{1}{8} \left( \frac{3}{\pi} \right)^{1/3} \frac{hc}{m_n^{4/3}}, \end{aligned} \quad (38.2)$$

with  $m_u \approx m_n$ . In (38.1) we have used the rest-mass density  $\varrho_0 = n_n m_n$ . For relativistic configurations instead of  $\varrho_0$  one has to use the total mass-energy density  $\varrho = \varrho_0 + u/c^2$ . This distinction was not necessary for the electron gas, where  $\varrho_0$  (coming mainly from the non-degenerate nucleons) was always large compared with the energy density  $u/c^2$  coming from the degenerate electron gas. Now both  $\varrho_0$  and  $u/c^2$  are provided by the degenerate neutrons. For non-relativistic neutrons,  $\varrho_0 \gg u/c^2$  and  $\varrho \approx \varrho_0$ ; for relativistic neutrons,  $\varrho_0 \ll u/c^2$  and  $\varrho \approx u/c^2$ . For relativistic particles, however, we know that  $P = u/3$ , i.e.  $P = \varrho c^2/3$ . So we can write

$$\begin{aligned} P_n &\sim \varrho^\kappa, \\ \kappa &= 5/3 \quad (\text{non-relativistic}), \\ \kappa &= 1 \quad (\text{relativistic}). \end{aligned} \quad (38.3)$$

The distinction between  $\varrho$  and  $\varrho_0$  will be seen to be important for NS models. The relation  $P = \varrho c^2/3$  also yields the velocity of sound directly as  $v_s^2 = (dP/d\varrho)_{\text{ad}} = c^2/3$ , i.e.  $v_s = 0.577c$ .

Of course, with the densities considered here, the *interaction between nucleons* is far from being negligible. It dominates the behaviour long before the limit  $6 \times 10^{15} \text{ g cm}^{-3}$ , where  $p_F = m_n c$ , is reached. In order to calculate its influence on the equation of state, one faces two problems. The first is the determination of a reasonable potential. In the absence of a rigorous theory and of experiments at such high densities, one has to use a model of the interacting particles that meets the results of low-energy scattering, the properties of saturation of nuclear forces, etc. It is not surprising that such models yield large uncertainties when extrapolated and applied to the densities found in NS. The qualitative influence of some effects

on the equation of state is quite obvious. For example, the interaction between two nucleons depends (aside from spin and isospin properties) on their distance. When approaching each other they first feel an attraction, which turns to repulsion below a critical distance (in the extreme: at an inner hard core). Attraction (dominant at not too high  $\rho$ ) reduces  $P$  and gives a softer equation of state. Repulsion (dominant at very high  $\rho$  and small average particle distances) increases  $P$  and thus stiffens the equation of state. Obviously details of the potential can shift the border appreciably between these two regimes.

Other uncertainties are connected with the appearance of new particles when  $\rho$  increases. For example, if hyperons of some type occur in sufficient number, they contribute to  $\rho$ , but scarcely to  $P$ , since their creation lowers the Fermi sea of the neutrons. Therefore “hyperonization” makes the gas more compressible. At ultra-high densities (say  $\approx 10\rho_{\text{nuc}}$ ) so many new resonances appear that, in the extreme, attempts have been made to describe their number in a certain energy range only by statistics (which leads, e.g., to the rather soft Hagedorn equation of state). But if the nucleons almost touch each other, one might have to consider something like quark interaction. The question was even discussed whether this might lead to quark matter and possibly to *quark stars*.<sup>1</sup> Finally, in case that the absolute ground state of strong interactions is that of quark matter in a deconfined state, in which up, down, and strange quarks are present in about equal number, neutron stars will consist of this so-called *strange matter* and would be *strange stars* (For a discussion of quark and strange stars, see, e.g., the corresponding chapters in the book by Glendenning.).

As early as  $\rho \lesssim 2\rho_{\text{nuc}}$  the possibility of the reaction  $n \rightarrow p + \pi^-$  (if  $E_n \geq E_p + E_{\pi^-}$ ) gives the possibility of having a Bose–Einstein condensate of the cold  $\pi^-$  bosons in momentum space with zero momentum, i.e. no contribution to  $P$  but to  $\rho$ .

The second quite general problem for determining the equation of state is that, even if the potential were known exactly, one would not know how to solve convincingly the many-body problem. Several attempts use different assumptions and yield different results.

To resume, we must stress that the equation of state is highly uncertain for at least two independent reasons (concerning the potential and the many-body problem), but there are still more open questions concerning possible effects of superfluidity and superconductivity, which might influence the evolution of neutron stars, in particular their rotation and magnetic fields. In fact particle physics cannot yet decide which of the available equations of state is correct, but the softest ones now seem to be ruled out by observation of neutron stars (see below). In Fig. 16.2 just one of them is plotted, which should resemble the general properties, but will not be exact in the details.

---

<sup>1</sup>The full beauty of this term can be savoured only in German, where the term “quark” means either a popular, soft white cheese or, in slang, complete nonsense.

## 38.2 Models of Neutron Stars

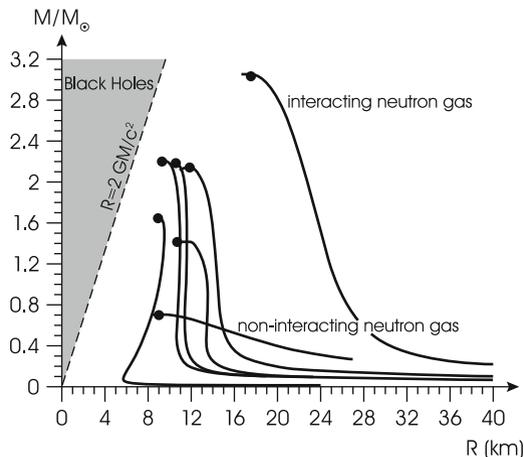
For a given equation of state of the form  $P = P(\varrho)$  it is easy to obtain the corresponding hydrostatic models of NS. One has only to integrate the relativistic equation of hydrostatic equilibrium (2.31) (the Tolman–Oppenheimer–Volkoff equation) together with (2.30), starting at  $r = 0$  with a chosen central density  $\varrho_c$ . Since the equation of state is independent of  $T$ , these two equations suffice for obtaining the mechanical structure. This is seen after replacing  $P$  by  $\varrho$  in (2.31), so that there are two equations for the variables  $\varrho$  and  $m$ . When the integration comes to  $\varrho = P = 0$ , the surface is reached, i.e. we have found  $R = r$  and  $M = m(R)$  (We do not have to worry about the obvious failure of the equation of state for  $P \rightarrow 0$ . The transition region to the non-degenerate atmosphere, and even the whole atmosphere, are negligibly thin so that the error made is small.)

Repeating this integration for a variety of starting values  $\varrho_c$ , one can produce a sequence of models for the chosen equation of state. They give, in particular, the relations  $M = M(\varrho_c)$ ,  $R = R(\varrho_c)$ , and by elimination of  $\varrho_c$  also  $R = R(M)$  (cf. Fig. 38.1).

The resulting relations  $M(\varrho_c)$  and  $R(M)$  change considerably if we replace the equation of state by another one, as can be seen in Fig. 38.1 for  $M(R)$ , where the results are plotted for several equations of state. The persisting common feature is that all relations  $M(\varrho_c)$  show a minimum and a maximum of  $M$ , although at quite different values. One can easily understand the qualitative changes which occur when a soft equation of state is replaced by a stiffer one. The matter is then less compressible; for given  $M$  one expects a larger  $R$  and a smaller  $\varrho_c$ . For given  $\varrho_c$  one can put more mass on top until reaching the surface with  $\varrho = 0$ . This lowers the gravity inside the model, and  $M_{\max}$  is higher. A particularly soft equation of state is that for the ideal degenerate neutron gas in (38.3), since the repulsive forces at small particle distances are completely neglected. Correspondingly Oppenheimer and Volkoff (1939) obtained for this equation of state a maximum mass of only  $M_{\max} \approx 0.72M_{\odot}$ . Normally the maxima range roughly between  $1M_{\odot}$  and  $3M_{\odot}$ , but Fig. 38.1 also demonstrates that a particularly stiff equation of state, obtained by including interactions into the Oppenheimer–Volkoff equation, may lead to maximum masses above  $3M_{\odot}$ . We have stressed in Sect. 38.1 that particle physics cannot yet supply the correct equation of state. All the more interesting are objects like the binary pulsar PSR B1913+16 (also called *the Hulse–Taylor pulsar*), for which the masses could be determined very accurately when details of the orbital motion were interpreted as general relativistic effects.<sup>2</sup> The result for the NS is  $M = 1.442M_{\odot}$  with a vanishing small uncertainty, which rules out all equations of state

---

<sup>2</sup>These effects include a shrinking of the orbit—and therefore a decrease of the orbital period (of the order of 60 ms)—due to the loss of gravitational waves. The observations agree perfectly with the predictions of Einstein’s theory of general relativity and are considered as indirect proof for the existence of gravitational waves. J.H. Taylor and R. Hulse were awarded with the Nobel Prize for this in 1993.

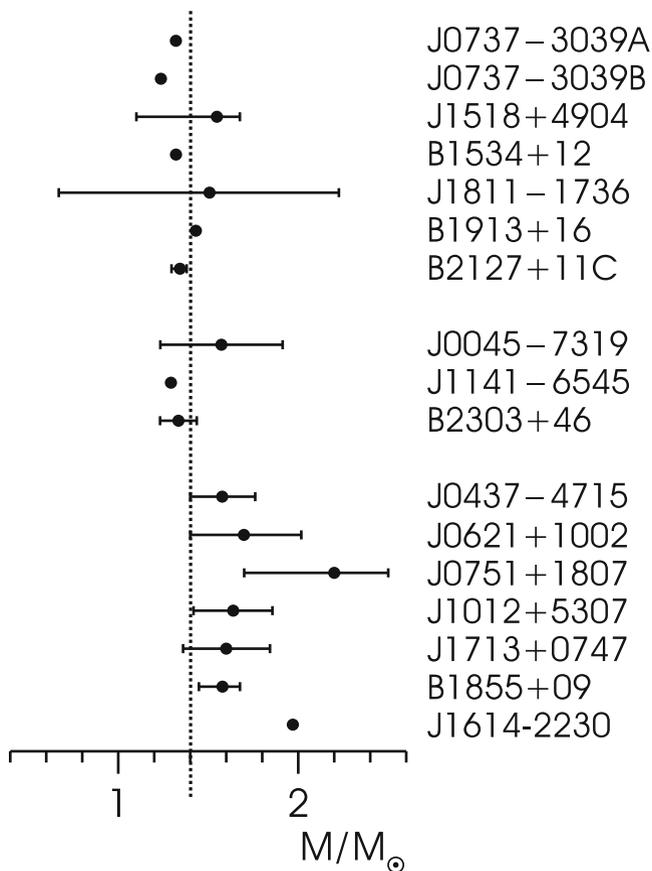


**Fig. 38.1** The relation  $M$  against  $R$  of neutron-star models calculated using seven different equations of state. The maximum mass is indicated in each case by the *solid dot*. Two extreme assumptions are specifically indicated: a non-interacting neutron gas (Oppenheimer and Volkoff 1939), which leads to an extremely soft equation of state and a very low maximum mass, and one where interactions are included and which leads to a maximum neutron star mass above  $3M_{\odot}$  (After Fig. 10.3 in Weber et al. 2009, where also details about the other five equations of state can be found)

so soft that their  $M_{\max}$  is below  $1.44M_{\odot}$ . Very recently, another binary millisecond pulsar—J1614-2230—was analysed by Demorest et al. (2010), who determine a pulsar mass of  $1.97 \pm 0.04M_{\odot}$ , using the so-called *Shapiro delay* of the pulsar signal, which is caused by the fact that light signals do not travel a straight line, but follow null-geodesics which are bent, and therefore longer, by the gravitational potential. This result rules out at least two more equations of state of Fig. 38.1, among them one for “strange stars” (in the figure, this corresponds to the left-most line). Here seems to be one of the cases where astrophysical measurements set a discriminating limit to particle physics. A collection of accurately determined neutron-star masses is given in Fig. 38.2. They were obtained by different methods, on which we do not comment further, but refer to the respective textbooks.

The *maximum mass* for NS is very important, not only in connection with evolutionary considerations, but also in the attempt to identify compact objects with  $M > M_{\max}$  as black holes. If our ignorance of the equation of state does not yet allow the determination of  $M_{\max}$  to better than the interval  $2 \dots 3M_{\odot}$ , we should at least understand that such a maximum mass (well below  $5M_{\odot}$ ) must exist.

In order to make this plausible, we neglect effects of *general* relativity, i.e. consider the usual equation of hydrostatic equilibrium but keep those of *special* relativity as allowed for in (38.3). Let us consider some averages of  $P$  and  $\varrho$  over the whole star. As in (37.15) the normal hydrostatic equation then yields the estimate  $P \sim M^2/R^4$ . Here we eliminate  $R$  by  $\varrho \sim M/R^3$  and obtain  $P \sim M^{2/3}\varrho^{4/3}$ ,



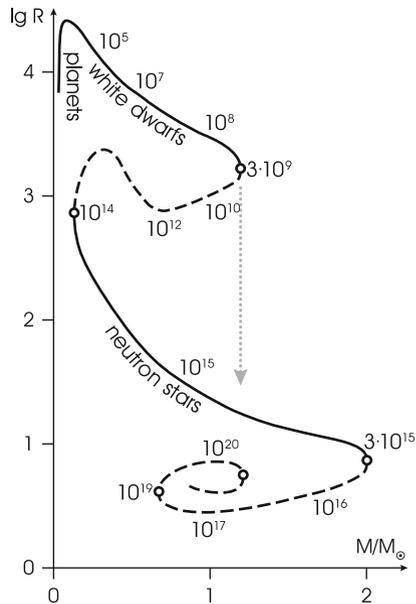
**Fig. 38.2** An overview of empirically determined masses and their errors of neutron stars (identifier to the right of the data point). Except for the last object, J1614-2230 (Demorest et al. 2010), the data are taken from Fig. 6.31 in Camenzind (2007). The vertical line corresponds to the generic mass of  $1.4M_{\odot}$

introduce  $\varrho \sim P^{1/\kappa}$  from the equation of state (38.3), and then solve for  $M$  and find

$$M \sim \varrho^{3(\kappa-4/3)/2}. \tag{38.4}$$

In the non-relativistic limit,  $\kappa = 5/3$ , giving  $M \sim \varrho^{1/2}$  and  $dM/d\varrho > 0$ . The extreme relativistic case requires  $\kappa = 1$ , which gives  $M \sim \varrho^{-1/2}$  and  $dM/d\varrho < 0$ . Somewhere on the border between the two regimes we expect  $dM/d\varrho = 0$ , i.e. the maximum mass (The average  $\varrho$  treated here will be a sufficient measure for  $\varrho_c$  too.). Therefore the maximum of  $M$  must occur when the neutrons start to become relativistic and the energy density  $u/c^2$  begins to overtake the rest-mass density  $\varrho_0$ . Only by neglecting  $u/c^2$  in  $\varrho$  [taking (38.1) instead of (38.3)] could

**Fig. 38.3** Schematic mass–radius relation ( $R$  in km) for configurations of cold catalysed matter, from the planetary regime to ultra-dense neutron stars. Some values of  $\rho_c$  (in  $\text{g cm}^{-3}$ ) are indicated along the curve. At the extrema of  $M$  (open circles) the stability problem has a zero eigenvalue. Solid branches are stable, dashed branches are unstable. The grey, vertical arrow indicates the collapse of a white dwarf exceeding the maximum stable mass to a neutron star



we obtain the Chandrasekhar mass of  $M_{\text{Ch}} = 5.73 M_\odot$  as the mass limit for an *infinite* relativity parameter ( $\gamma' = 4/3$ ). Clearly, therefore,  $M_{\text{max}} < M_{\text{Ch}}$ . The here neglected influence of general relativity [i.e. the description of hydrostatic equilibrium by the TOV equation equation (2.31)] tends to decrease  $M_{\text{max}}$  even more (see below).

Closely connected with the extrema of  $M$  are the *stability properties*. The relation  $M = M(\rho_c)$  can be considered to represent a sequence of equilibrium models with the parameter  $\rho_c$ . Figure 38.3 shows a schematic overview of the resulting  $M - R$  relation for cold catalysed matter from the regime of planets to that of ultra-dense NS. Starting from planets,  $\rho_c$  increases monotonically along the curve (compare with typical values of  $\rho_c$  indicated in Fig. 38.3). There are extrema of  $R$  which may be interesting in other connections but are not important for the sequence  $M(\rho_c)$ . However, one also encounters extrema of  $M$  (open circles). The most important are  $M_{\text{min}}$  and  $M_{\text{max}}$  for NS, as well as the maximum  $M$  for white dwarfs. These are critical points at which a detailed stability analysis shows that the stability of the equilibrium models changes. The stable parts of the curve are those with  $dM/d\rho_c > 0$ , i.e. the branch of NS with  $M_{\text{min}} < M < M_{\text{max}}$  (and the white-dwarf and planetary branch with  $M < \text{maximum mass for white dwarfs}$ ). When further increasing  $\rho_c$  beyond the point at which  $M = M_{\text{max}}$  there follows an infinite number of maxima and minima of  $M$ . Correspondingly the curve  $R = R(M)$  spirals into a limiting point, which is reached for  $\rho_c \rightarrow \infty$ . All of these branches are in fact unstable. The stability analysis can also be made for general relativistic configurations. In the Newtonian limit one has the well-known result that an average of the exponent  $\gamma_{\text{ad}}$  of  $\gamma_{\text{cr}} = 4/3$  is equivalent to marginal stability (see Sect. 25.3.2),

and in addition it can be shown (see Shapiro and Teukolsky 1983) that small effects of general relativity ( $GM/Rc^2 \ll 1$ ) change the critical value from  $4/3$  to

$$\gamma_{\text{cr}} = \frac{4}{3} + \Lambda \frac{GM}{Rc^2}, \quad (38.5)$$

where  $\Lambda$  is a positive quantity of the order of unity. Therefore general relativity increases  $\gamma_{\text{cr}}$ , making the star more unstable, since stability requires  $\bar{\gamma}_{\text{ad}} > \gamma_{\text{cr}}$ . For  $M = 1M_{\odot}$ ,  $R = 10$  km the correction term in (38.5) is about 0.15, i.e. far from being negligible.  $\gamma_{\text{cr}}$  can be raised well above  $5/3$  (even above 2 for certain models near  $M_{\text{max}}$ ) such that all but the stiffest equations of state would give instability. This increase of  $\gamma_{\text{cr}}$  is an important factor in determining the value of  $M_{\text{max}}$  (together with the lowering of  $\bar{\gamma}_{\text{ad}}$ ).

A very stiff equation of state, for example, gives  $M_{\text{max}} = 2.7M_{\odot}$ , with  $R = 13.5$  km and  $\rho_{\text{c}} = 1.5 \times 10^{15}$  g cm $^{-3}$ , while a softer one yields  $M_{\text{max}} = 2M_{\odot}$ , with  $R = 9$  km and  $\rho_{\text{c}} = 3.3 \times 10^{15}$  g cm $^{-3}$ . At present there is no equation of state that can be considered realistic and that would give  $M_{\text{max}}$  well above  $3M_{\odot}$ . This includes calculations that take into account general relativity.

The model is also marginally stable at the minimum mass  $M_{\text{min}}$ , where the curve in Fig. 38.3 begins leading to the white dwarfs. This instability is essentially caused by the lowering of  $\gamma'$  in connection with the neutron drip (see Sect. 38.1). We have seen that the release of free neutrons from nuclei results in  $\gamma' \lesssim 4/3$  in the range  $\rho \approx 4 \times 10^{11} \dots 7 \times 10^{12}$  g cm $^{-3}$ . Typical models for the minimum mass of stable neutron stars give  $M_{\text{min}} \approx 0.09M_{\odot}$ ,  $R \approx 160$  km,  $\rho_{\text{c}} \approx 1.5 \times 10^{14}$  g cm $^{-3}$ . The average density is, of course, much smaller ( $\approx 10^{10}$  g cm $^{-3}$ ), and the averaged  $\gamma_{\text{ad}}$  becomes just equal to  $\gamma_{\text{cr}}$  (which is here close to  $4/3$ ).

Let us dwell briefly on the meaning of the mass values quoted for NS. The stellar mass  $M$  is here always the “gravitational mass”, which is the value measurable for an outside observer [cf. the comments in Sect. 2.6 after (2.29)].  $M$  differs from the proper mass  $M_0 = Nm_0$ , given by the total number  $N$  of nucleons with a rest mass  $m_0$ , since in relativity, the total binding energy  $W$  of the configuration appears as a mass  $\Delta M = W/c^2$ , such that

$$M = M_0 + \frac{W}{c^2} = M_0 + \Delta M. \quad (38.6)$$

In the Newtonian limit (for weak fields) we were used to identifying particularly the internal energy  $E_i$  (from motion and interaction of particles) and the potential energy  $E_g$  in the gravitational field. Then for a static, stable configuration,  $W = E_i + E_g < 0$ , since  $E_g < 0$  and  $-E_g > E_i$  (In the Newtonian limit  $E_g$  and  $E_i$  were related by the virial theorem, cf. Chap. 3.). Correspondingly we may now say that the mass of a NS is increased by the internal energy and decreased by the (negative) potential energy, and the latter term wins. Therefore  $W < 0$ , and we have a mass defect  $\Delta M < 0$ . Depending on the precise model,  $|\Delta M|$  can go up to  $10 \dots 25$  % of  $M$  near  $M_{\text{max}}$ . Formally  $M$  is given as an integral over  $4\pi r^2 \rho dr$ , where  $\rho$  is the total mass-energy density ( $\rho_0 + u/c^2$ ) and  $4\pi r^2 dr$  is *not* the volume

element. This is rather given by  $dV = 4\pi r^2 e^{\lambda/2} dr$  with  $e^{\lambda/2}$  being a component of the metric tensor (cf. Sect. 2.6). Then simply

$$\begin{aligned}\Delta M \equiv M - M_0 &= \int_0^R (4\pi r^2 \varrho dr - \varrho_0 dV) \\ &= \int_0^R 4\pi r^2 \varrho \left(1 - e^{\lambda/2} \frac{\varrho_0}{\varrho}\right) dr.\end{aligned}\quad (38.7)$$

Here  $\varrho_0/\varrho < 1$ , but  $e^{\lambda/2} > 1$ , and the product of both is  $> 1$ , such that  $\Delta M < 0$ . So if we find an NS with mass  $M$ , we know that it started off as a more massive configuration. The mass defect  $|\Delta M|$  was radiated away in the course of evolution by photons, neutrinos, or gravitational radiation. In that sense the original Kelvin–Helmholtz hypothesis that contraction supplies the radiated energy has turned out to be correct. The mass defect reaches a maximum at  $M = M_{\max}$  and then decreases again towards models with still larger  $\varrho_c$ .

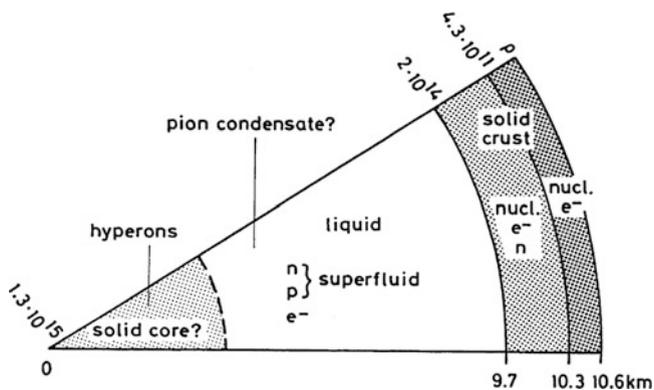
The maximum mass for NS is scarcely influenced by rotation. Except for the very few most rapidly spinning pulsars, centrifugal forces play practically no role in NS, since the overwhelming gravitational forces dominate completely. This is at least true for rigidly rotating NS stars. However, *differential rotation* may stabilize neutron stars and will lead to higher maximum masses. In the case of simple polytrope models, differential rotation can raise  $M_{\max}$  by up to 50 % (see Baumgarte and Shapiro 2010, Chap. 14).

Now we turn to describe the *stratification of matter inside an NS model*. At the very outer part there must be an atmosphere of “normal” non-degenerate matter. Going inwards, we come to gradually larger densities and encounter all characteristic changes of high-density matter as described in Sect. 38.1.

The *atmosphere* of an NS is very hot and incredibly compressed. Typical temperatures are of the order of  $10^6$  K (see below). The extension is very small owing to the high surface gravity  $g_0 \approx 1.3 \times 10^{14} \text{ cm s}^{-2}$  (For comparison,  $g_0 = 2.7 \times 10^4 \text{ cm s}^{-2}$  for the Sun and  $\approx 10^8 \text{ cm s}^{-2}$  for white dwarfs.). This gives a pressure scale height of the order of 1 cm only. In the surface layers (say  $\varrho \lesssim 10^6 \text{ g cm}^{-3}$ ) the behaviour of the matter is still influenced by the temperature and also by strong magnetic fields.

Not far below the surface, the densities will be in and above the range typical for the interior of white dwarfs ( $\gtrsim 10^6 \text{ g cm}^{-3}$ ). As an example we discuss the model for an NS of  $M = 1.4M_\odot$  (see Fig. 38.4), calculated by using an equation of state of moderate stiffness which gives  $M_{\max} \approx 2M_\odot$ . The radius of the  $1.4M_\odot$  model is 10.6 km. Although there are newer models, the present one is still a good representation of the typical structure of neutron stars.

Below the surface there is a solid *crust* ( $10^6 \lesssim \varrho \lesssim 2.4 \times 10^{14} \text{ g cm}^{-3}$ ) of thickness  $\Delta r \approx 0.9$  km. The matter in the crust contains nuclei, which are mainly Fe near the surface (cf. the equilibrium composition as a function of  $\varrho$  described in Sect. 37.2). These nuclei will form a lattice, thus minimizing the energy of Coulomb interaction as in crystallized white dwarfs. The *outer crust* consists only of



**Fig. 38.4** Illustration of the interior structure of a neutron-star model with  $M = 1.4M_{\odot}$  calculated with an equation of state similar to the intermediate ones in Fig. 38.1. A few characteristic values of the density (in  $\text{g cm}^{-3}$ ) are indicated along the upper radius (After Pines 1980)

these nuclei plus a relativistically degenerate electron gas, though this changes over a depth of  $\Delta r \approx 0.3$  km to where the neutron drip density  $\rho_{\text{dr}} \approx 4 \times 10^{11} \text{ g cm}^{-3}$  is reached. In the subsequent *inner crust* ( $4 \times 10^{11} \lesssim \rho \lesssim 2 \times 10^{14} \text{ g cm}^{-3}$ ), a liquid of free neutrons exists in addition to the nuclei (still arranged in a lattice, and becoming increasingly neutron-rich) and the electrons. With decreasing  $r$  the free neutrons become more and more abundant at the expense of the nuclei, and the lattice disappears with the nuclei, until all nuclei are dissolved into homogeneous, neutron-rich nuclear matter at  $\rho = \rho_{\text{nuc}} \approx 2.4 \times 10^{14} \text{ g cm}^{-3}$ , which therefore defines the lower boundary of the solid crust, at a depth of 0.9 km. The equation of state throughout the crust is relatively well known; this is the reason why our aged neutron-star model is still valid.

Below the crust there is the *interior neutron liquid* ( $\rho \gtrsim 2.4 \times 10^{14} \text{ g cm}^{-3}$ ) consisting mainly of neutrons in equilibrium with a few protons, electrons, and muons. All constituents are strongly degenerate and the hadrons are interacting by nuclear forces. The neutrons will be superfluid, the protons superconductive. The equation of state begins not to be well-known in this density regime, and from here on the structure and composition of the inner core depends on the equation of state used.

It is unclear whether there is finally a central *solid core* in which the neutrons form a solid owing to their repulsive forces at small particle distances. The central density of our model is  $\rho_c \approx 1.3 \times 10^{15} \text{ g cm}^{-3}$ . The inner core may also consist of baryon condensates ( $\pi^-$ ,  $K^-$ ), of a mixture hyperons and baryon resonances ( $\Sigma$ ,  $\Lambda$ ,  $\Xi$ ,  $\Delta$ ), or deconfined quark matter. We refer the reader to Fig. 10.1 in Weber et al. (2009) for a graphical representation of various possibilities. In that figure, our model would be one of the *traditional neutron stars*.

The superfluidity of the neutron and proton liquids and the solid parts (crust and possible core) play a role in the attempts to explain the observed “glitches”

of pulsars. These are sudden spin-ups, interrupting from time to time the normal, regular spin-down (decrease of the rotation frequency  $\Omega$ ). There is a hypothesis according to which a glitch is due originally to a “starquake”, decreasing suddenly the moment of inertia  $I_c$  of the crust. Conservation of angular momentum requires a corresponding increase of  $\Omega$ . The relaxation to the normal state depends critically on the coupling of the rotating crust and the rotating interior liquid (and possible solid core). The charged components could be coupled magnetically, while the superfluid matter may couple via vortices. This coupling is the basis of another model of the glitches: the superfluid neutron liquid in the interior and in the inner crust is considered to rotate with an angular velocity slightly different from that of the lattice of nuclei in the crust. The coupling is provided by vortices in the liquid and is thought to break down suddenly when the crust has been decelerated sufficiently by the pulsar mechanism on the outside. The vortices can contain an appreciable fraction of the star’s angular momentum, and their distortion induces immediate changes of the observed rotation.

The thermal properties (except for the earliest stages) in principle follow once the mechanical models are given. One can then calculate the thermal conductivity, which, together with a given outward flux of energy, determines the  $T$  gradient at any point. It turns out that like white dwarfs (Sect. 37.3) the NS have a nearly isothermal interior because of the high thermal conductivity. Only in the outermost layers does  $T$  drop, by typically a factor of  $10^2$ , to the surface temperature. Particularly in the first, hot phases, the cooling will be very rapid because of strong neutrino losses.

In this chapter we have completely ignored the strong magnetic fields of neutron stars. While they are of only minor importance for the structure and the maximum mass, they are crucial for many phenomena which allow the observation of neutron stars. Most notably this is the pulsar phenomenon, which is due to the emission of synchrotron radiation along the axis of the magnetic dipole, being inclined with respect to the rotation axis. The typical field strength of pulsar magnetic fields are of order  $10^{11}$ – $10^{13}$  G. Some NS possess even stronger magnetic fields, up to  $10^{15}$  G, which is the highest known level in the universe. They are called *magnetars* and are the source for the *soft gamma repeaters* (Thompson and Duncan 1995), a class of gamma-ray bursts that exceed the Eddington luminosity by far, but are characterized by a comparably soft gamma spectrum. In a magnetar, the decaying magnetic field is the source of free energy (rather than rotation, as in pulsars). As we said at the beginning, everything is extreme in neutron stars.