Chapter 26
The Onset of Star Formation

Stars form out of interstellar matter. With modern telescopes and instruments this can nowadays be observed directly and in many phases of the formation process. Indeed a homogeneous cloud of compressible gas can become gravitationally unstable and collapse. In this section we shall deal with gravitational instability and then discuss some of its consequences. But before we do so it may be worth comparing this instability with those discussed in Chap. 25. For gravitational instability the inertia terms are important as well as heat exchange of the collapsing mass with its surroundings. But it is not a vibrational instability, since the classification scheme of Chap. 25 holds only if the free-fall time is much shorter than the timescale of thermal adjustment. As we will see later, just the opposite is the case here, during the earliest phases of star formation.

26.1 The Jeans Criterion

26.1.1 An Infinite Homogeneous Medium

We start with an infinite homogeneous gas at rest. Then density and temperature are constant everywhere. However, we must be aware that this state is not a well-defined equilibrium. For symmetry reasons the gravitational potential Φ must also be constant. But then Poisson's equation $\nabla^2\Phi = 4\pi G\varrho$ demands $\varrho = 0$. Indeed the gravitational stability behaviour should be discussed starting from a better equilibrium state, as we will do later. Nevertheless we first assume a medium of constant non-vanishing density. If we here apply periodic perturbations of sufficiently small wavelength, the single perturbation will behave approximately like one with the same wavelength in an isothermal sphere in hydrostatic equilibrium (which is a well-defined initial state).

R. Kippenhahn et al., *Stellar Structure and Evolution*, Astronomy and Astrophysics Library, DOI 10.1007/978-3-642-30304-3_26,

The gas has to obey the equation of motion of hydrodynamics

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\varrho}\nabla P - \nabla \Phi \tag{26.1}$$

(Euler equation), together with the continuity equation

$$\frac{\partial \varrho}{\partial t} + \boldsymbol{v}\nabla\varrho + \varrho\nabla \cdot \boldsymbol{v} = 0\,. \tag{26.2}$$

In addition we have Poisson's equation

$$\nabla^2 \Phi = 4\pi G\varrho \tag{26.3}$$

and the equation of state for an ideal gas

$$P = \frac{\Re}{\mu}\varrho T = v_s^2 \varrho\,, \tag{26.4}$$

where v_s is the (isothermal) speed of sound. For equilibrium we assume $\varrho = \varrho_0 =$ constant, $T = T_0 =$ constant, and $\boldsymbol{v}_0 = \mathbf{0}$. Φ_0 may be determined by $\nabla^2\Phi_0 = 4\pi G\varrho_0$ and by boundary conditions at infinity.

We now perturb the equilibrium

$$\varrho = \varrho_0 + \varrho_1\,, \quad P = P_0 + P_1\,, \quad \Phi = \Phi_0 + \Phi_1\,, \quad \boldsymbol{v} = \boldsymbol{v}_1\,, \tag{26.5}$$

where the functions with subscript 1 depend on space and time. In (26.5) we have already used that $\boldsymbol{v}_0 = \mathbf{0}$. If we substitute (26.5) in (26.1) and (26.4), assuming that the perturbations are isothermal (v_s is not perturbed), and if we ignore non-linear terms in these quantities, we find

$$\frac{\partial \boldsymbol{v}_1}{\partial t} = -\nabla\left(\Phi_1 + v_s^2\frac{\varrho_1}{\varrho_0}\right)\,, \tag{26.6}$$

$$\frac{\partial \varrho_1}{\partial t} + \varrho_0 \nabla \cdot \boldsymbol{v}_1 = 0\,, \tag{26.7}$$

$$\nabla^2 \Phi_1 = 4\pi G\varrho_1\,. \tag{26.8}$$

The terms with index 0, describing the equilibrium part, have vanished, as usual. This is a linear homogeneous system of differential equations with constant coefficients. We therefore can assume that solutions exist with the space and time dependence proportional to $\exp\,[\mathrm{i}(kx + \omega t)]$ such that

$$\frac{\partial}{\partial x} = \mathrm{i}k\,, \qquad \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0\,, \qquad \frac{\partial}{\partial t} = \mathrm{i}\omega\,. \tag{26.9}$$

With $v_{1x} = v_1, v_{1y} = v_{1z} = 0$ we find from (26.6)–(26.8) that

$$\omega v_1 + \frac{k v_s^2}{\varrho_0}\varrho_1 + k\Phi_1 = 0, \tag{26.10}$$

$$k\varrho_0 v_1 + \omega\varrho_1 = 0, \tag{26.11}$$

$$4\pi G\varrho_1 + k^2\Phi_1 = 0. \tag{26.12}$$

This homogeneous linear set of three equations for v_1, ϱ_1, Φ_1 can only have nontrivial solutions if the determinant

$$\begin{vmatrix} \omega & \dfrac{kvs^2}{\varrho 0} & k \\ k\varrho_0 & \omega & 0 \\ 0 & 4\pi G & k^2 \end{vmatrix}$$

is zero. Assuming a non-vanishing wave number k we obtain

$$\omega^2 = k^2 v_s^2 - 4\pi G\varrho_0. \tag{26.13}$$

For sufficiently large wave numbers the right-hand side is positive, i.e. ω is real. The perturbations vary periodically in time. Since the amplitude does not increase, the equilibrium is stable with respect to perturbations of such short wavelengths.

In the limit $k \to \infty$, (26.13) gives $\omega^2 = k^2 v_s^2$, which corresponds to isothermal sound waves. Indeed for very short waves gravity is not important, any compression is restored by the increased pressure, and the perturbations travel with the speed of sound through space.

If $k^2 < 4\pi G\varrho_0/v_s^2$, the eigenvalue ω is of the form $\pm i\xi$, where ξ is real. Therefore there exist perturbations $\sim\exp(\pm\xi t)$ which grow exponentially with time, so that the equilibrium is unstable. If we define a characteristic wave number k_J by

$$k_J^2 := \frac{4\pi G\varrho_0}{v_s^2}, \tag{26.14}$$

or a corresponding characteristic wavelength

$$\lambda_J := \frac{2\pi}{k_J}, \tag{26.15}$$

then perturbations with a wave number $k < k_J$ (or a wavelength $\lambda > \lambda_J$) are unstable; otherwise, they are stable with respect to the perturbations applied here. The condition for instability $\lambda > \lambda_J$, where

$$\lambda_J = \left(\frac{\pi}{G\varrho_0}\right)^{1/2} v_s, \tag{26.16}$$

is called the *Jeans criterion* after James Jeans, who derived it in 1902. Depending on the detailed geometrical properties of equilibrium and perturbation, the factors on the right-hand side of (26.16) can differ.

For our special choice of perturbations the case of instability can be described as follows: after a slight compression of a set of plane-parallel slabs, gravity overcomes pressure and the slabs collapse to thin sheets. If we estimate ω for the collapsing sheets only from the gravitational term in (26.13) (which indeed is larger than the pressure term), we have $\mathrm{i}\omega \approx (G\varrho_0)^{1/2}$ and the corresponding timescale is $\tau \approx (G\varrho_0)^{-1/2}$, which corresponds to the free-fall time, as defined in Sect. 2.4.

26.1.2 A Plane-Parallel Layer in Hydrostatic Equilibrium

We have already mentioned the contradictions connected with the assumption of an infinite homogeneous gas as initial condition. One way out of this difficulty is to investigate the equilibrium of an isothermal plane-parallel layer stratified according to hydrostatic equilibrium in the z direction. Perpendicular to the z direction all functions are constant, the layer extending to infinity. This defines a one-dimensional problem: ϱ_0, P_0, T_0 depend only on one coordinate, say z. Poisson's equation then is

$$\frac{d^2\Phi_0}{dz^2} = 4\pi G\varrho_0, \tag{26.17}$$

while hydrostatic equilibrium, $dP_0/dz = -\varrho_0 d\Phi_0/dz$, can be written with (26.4) as

$$v_\mathrm{s}^2 \frac{d \ \ln \varrho_0}{dz} = -\frac{d\Phi_0}{dz}\,. \tag{26.18}$$

After differentiation of (26.18) one obtains from (26.17)

$$\frac{d^2 \ln \varrho_0}{dz^2} = -\frac{4\pi G}{v_\mathrm{s}^2}\varrho_0\,. \tag{26.19}$$

With the boundary condition $\varrho_0 = 0$ for $z = \pm\infty$, (26.19) has the solution

$$\varrho_0(z) = \frac{\varrho_0(0)}{\cosh^2(z/H)}\,, \tag{26.20}$$

with

$$H = \left(\frac{\Re T}{2\pi\mu G\varrho_0(0)}\right)^{1/2} = \frac{v_\mathrm{s}}{[2\pi G\varrho_0(0)]^{1/2}}\,, \tag{26.21}$$

which can be seen if (26.20) and (26.21) are inserted into (26.19). The (stratified) disc does not cause problems similar to those encountered in the case of the infinite homogeneous gas.

In order to investigate the stability of this disc one defines cartesian coordinates x, y in the plane perpendicular to the z-axis and considers perturbations of the form $\varrho_1 \sim f(z) \exp[\mathrm{i}(kx + \omega t)]$. Since the perturbations do not depend on y the layer collapses in the x-direction to a set of plane-parallel "sticks" in y-direction in the case of instability. We shall not go into the details of the stability analysis, which has been described by Spitzer (1968). The result is that again there is a critical wave number

$$k_\mathrm{J} = \frac{1}{H} = \frac{[2\pi G \varrho_0(0)]^{1/2}}{v_\mathrm{s}} \tag{26.22}$$

and that instability occurs for wave numbers $k < k_\mathrm{J}$, while perturbations with $k > k_\mathrm{J}$ remain finite. This is very similar to what we have obtained in the homogeneous case, as can be seen by comparing (26.22) and (26.14). The difference in the numerical factors is due to the different geometry.

The two cases discussed above have in common that for smaller wave numbers (larger wavelengths and therefore larger amounts of mass involved in the resulting collapse) the equilibrium is unstable, while for larger wave numbers, it is stable. In hydrostatic equilibrium the force due to the pressure gradient and the gravitational force cancel each other. In general this balance is disturbed after a slight compression. If only a small amount of mass is compressed, the pressure increases more than the force due to gravity, and the gas is pushed back towards the equilibrium state. This is the case if a toy balloon is slightly compressed. Only the increase of pressure counts, since the gravity of the trapped gas is negligible. The same is true for the compressions which occur in sound waves where gravity plays no role. But if a sufficient amount of gas is compressed simultaneously, the increase of gravity overcomes that of pressure and makes the compressed gas contract even more.

26.2 Instability in the Spherical Case

In order to investigate the Jeans instability for interstellar gas in a configuration more realistic than the two examples of Sect. 26.1, we now consider an isothermal sphere of finite radius imbedded in a medium of pressure $P^* > 0$. The sphere is supposed to consist of an ideal gas. The structure of the sphere can be obtained from a solution of the Lane–Emden equation (19.35) for an isothermal polytrope. The solution is cut off at a certain radius where P has dropped to the surface pressure $P = P^*$. The stratification outside the sphere is not relevant as long as it is spherically symmetric with respect to the centre, since then there is no gravitational influence of the outside on the inside. Its only influence will be via the surface pressure, which we assume to be constant during the perturbation.

The essential points of this problem can be easily seen if one discusses the virial theorem for the sphere, as described in Sect. 3.4. Since our sphere of mass M and radius R is isothermal, its internal energy is $E_i = c_v MT$. For the gravitational energy we write $E_\mathrm{g} = -\Theta GM^2/R$, where Θ is a factor of order one. It can be obtained by numerical integration of the Lane–Emden equation and is related to the polytropic index n by $\Theta = 3/(5-n)$. For a fully convective sphere ($n = 3/2$), for example, its value would be 6/7; for the homogenous sphere with $n = 0$, $\Theta = 3/5$, and for an $n = 3$ polytrope (see Sect. 19.4) it is 1.5. Here, however, we use it as a general factor that depends on the actual density distribution within the sphere. With these expressions and with $\zeta = 2$ [ideal monatomic gas; (3.8)] the virial theorem (3.21) can be solved for the surface pressure P_0 giving

$$P_0 = \frac{c_v M T}{2\pi R^3} - \frac{\Theta G M^2}{4\pi R^4}\,. \tag{26.23}$$

The first term on the right is due to the internal gas pressure, which tries to expand the sphere. It is proportional to the mean density. The second term is due to the self-gravity of the sphere, which tries to bring all matter to the centre.

At this point we introduce two scaling factors for radius and pressure which allow us to write (26.23) in dimensionless form

$$\widetilde{R} = \frac{\Theta G M}{2 c_v T}, \qquad \widetilde{P} = \frac{c_v M T}{2\pi \widetilde{R}^3}, \tag{26.24}$$

and write

$$R = x\widetilde{R}, \qquad P_0 = y\widetilde{P}. \tag{26.25}$$

We then obtain instead of (26.23)

$$y = \frac{1}{x^3}\left(1 - \frac{1}{x}\right). \tag{26.26}$$

We now discuss how P_0 varies with R for fixed values of M, T, and Θ (Fig. 26.1). For small x the value of y is negative. It changes sign with increasing x at $x = 1$ (or $R = \widetilde{R}$), and approaches zero from positive values for x (or R) $\to \infty$. x has a (positive) maximum at 4/3 (or P_0 at $R = R_\mathrm{m}$), a value which can be obtained by differentiation of (26.26) or (26.23). After replacing c_v by $3\Re/(2\mu)$ we find that dP_0/dR vanishes at

$$R_\mathrm{m} = \frac{4\Theta}{9}\frac{G\mu M}{\Re T} = \frac{4}{3}\widetilde{R}\,. \tag{26.27}$$

Suppose the sphere to be in equilibrium with the surroundings: $P_0 = P^*$. For $R < R_\mathrm{m}$, the surface pressure P_0 decreases with decreasing R. Therefore, after a slight compression, $P_0 < P^*$ and the sphere will be compressed even more; it is unstable. For $R > R_\mathrm{m}$, the pressure P_0 increases during a slight compression and the sphere will expand back to equilibrium; it is stable (These simple plausibility arguments are supported by the results of a decent stability analysis.). We have obviously recovered the Jeans instability discussed in Sect. 26.1. This can be seen if in (26.27) M is replaced by $4\pi R_\mathrm{m}^3 \bar{\varrho}/3$, where $\bar{\varrho}$ is the mean density of the sphere. We then obtain

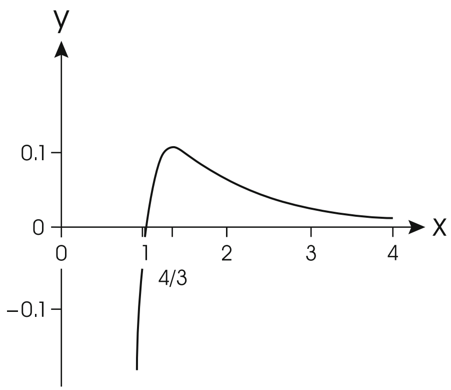

Fig. 26.1 The function given in (26.23) in dimensionless form (26.26). The variable y and therefore also P_0 change sign at $x = 1$ (or at $R = \tilde{R}$). It has a positive maximum at $x = 4/3$ (or at $R = R_m = 4\tilde{R}/3$)

$$R_{\mathrm{m}}^2 = \frac{27}{16\pi\Theta}\frac{\Re T}{G\mu\bar{\varrho}}\,. \tag{26.28}$$

Here R_{m} is the critical radius of a gaseous mass of mean density $\bar{\varrho}$ and temperature T which is marginally stable. We compare it with the critical Jeans wavelength obtained in (26.16), which with $v_{\mathrm{s}}^2 = \Re T/\mu$ becomes

$$\lambda_{\mathrm{J}}^2 = \frac{\Re T\pi}{G\mu\bar{\varrho}}\,. \tag{26.29}$$

Clearly λ_{J} and R_{m} are of the same order of magnitude.

Obviously for a given equilibrium state, defined by a radius R and a surface pressure P_0, there exists a critical mass M_{J}, the so-called *Jeans mass*, where $R = R_{\mathrm{m}}$. Masses larger than M_{J} are gravitationally unstable because R would be smaller than the corresponding R_{m}, which grows linearly with M according to (26.27). If slightly compressed they fall together. According to (26.28)

$$M_{\mathrm{J}} = \frac{4\pi}{3}\bar{\varrho}R_{\mathrm{m}}^3 = \frac{27}{16}\left(\frac{3}{\pi}\right)^{1/2}\left(\frac{\Re}{\Theta G}\right)^{3/2}\left(\frac{T}{\mu}\right)^{3/2}\left(\frac{1}{\bar{\varrho}}\right)^{1/2}. \tag{26.30}$$

Depending on the treatment of the perturbation problem and its geometry, one finds slightly differing pre-factors in the expression for M_{J}, but they all give the same order of magnitude.

We can rewrite (26.30) into a more convenient form (setting $\Theta = 1$):

$$\begin{aligned} M_{\mathrm{J}} &= \frac{27}{16}\left(\frac{3}{\pi}\right)^{1/2}\left(\frac{\Re}{G\mu}\right)^{3/2} T^{3/2}\varrho^{-1/2} \\ &= 1.1\,M_\odot\left(\frac{T}{10\mathrm{K}}\right)^{3/2}\left(\frac{\varrho}{10^{-19}\,\mathrm{g\,cm^{-3}}}\right)^{-1/2}\left(\frac{\mu}{2.3}\right)^{-3/2}. \end{aligned} \tag{26.31}$$

The scaling values $\varrho = 10^{-19}\,\mathrm{g\,cm^{-3}}$ and $T = 10\,\mathrm{K}$ are typical for the conditions in star-forming clumps within interstellar clouds. We assumed that all hydrogen is in molecular form and helium is neutral, and therefore $\mu \approx 2.3$. We thus obtain $M_\mathrm{J} \approx 1.1\,M_\odot$ as the typical mass of a clump of molecular gas from which stars form because of the Jeans instability. The typical Jeans mass for the molecular cloud as a whole, with $\varrho \approx 10^{-24}\,\mathrm{g\,cm^{-3}}$ and $T \approx 100\,\mathrm{K}$, would be around $10^5\,M_\odot$, which indeed is in the range of molecular cloud masses. However, it is believed that not the cloud as a whole collapses, but rather that turbulence within the cloud leads to overdense condensations with the conditions outlined above, which then collapse due to the Jeans instability and which may fragment further to form stars of even lower mass.

Equation (26.31) exists in various forms, which differ in the numerical factors. This can be the result of different assumptions about Θ, or ζ not being equal to 2. For example, for a bimolecular gas, it would be 6/5. Sometimes also half of the Jeans wavelength λ_J is used instead of R_m. All this can amount to a variation of the typical Jeans mass by a factor of a few.

We have already shown, following (26.16), that the timescale for the growth of the instability is $\tau \approx (G\varrho)^{-1/2}$, the free-fall time. This is of course also valid for the present spherical case. For a density of $\varrho \approx 10^{-19}\,\mathrm{g\,cm^{-3}}$, the collapse takes place on a timescale of some 10^5 years. During collapse, τ becomes shorter, since the density increases.

This timescale τ is long compared to that for thermal adjustment τ_adj. Since the cloud is optically thin, τ_adj is the internal energy per unit mass divided by the rate of energy losses owing to radiation. For typical neutral hydrogen clouds, Spitzer (1968) and Low and Lynden-Bell (1976) estimate a loss Λ of the order $1\,\mathrm{erg\,g^{-1}\,s^{-1}}$. With $T = 10\,\mathrm{K}$ we find $\tau_\mathrm{adj} \approx c_v T/\Lambda \approx 10$ years. Comparison with the free-fall time of some 10^5 years shows that the collapse proceeds in thermal adjustment (which turns out to mean that it is almost isothermal). In Sect. 26.3 we will show where this breaks down. As a rough estimate a molecular cloud is optically thin for particle densities below $10^{-10}\mathrm{cm^{-3}}$, or mass densities below $10^{-14}\,\mathrm{g\,cm^{-3}}$, and optically thick, if the density is higher.

So far, the external pressure P^* has not entered our discussion, because we have asked for the maximum pressure for given mass M at the cut-off radius R. If $P(R)$ is given by the external pressure P^*, one can turn around the question and ask for the maximum mass an isothermal sphere of given T and M can have before it has to collapse. Such a sphere is called *Bonnor-Ebert sphere*, and the critical mass, the Bonnor-Ebert mass M_BE, is given here without derivation (Ebert 1955; Bonner 1956):

$$M_\mathrm{BE} = 1.18 \frac{\Re^2}{\mu^2 G^{3/2}} T^2 (P^*)^{-1/2}\, M_\odot. \tag{26.32}$$

As expected, M_BE is increasing with its temperature because the thermal pressure of the sphere can better balance the outer pressure and is decreasing with increasing external pressure.

26.3 Fragmentation

For a long time it was believed that large molecular clouds of $10^4 \cdots 10^5 \, M_\odot$ were collapsing because they exceeded their Jeans mass. To actually form stars of much lower mass from such clouds, fragmentation into smaller clumps, which are collapsing faster than the cloud as a whole, is required. Due to progress of theories, numerical simulations, and observations of molecular clouds the picture has changed. Molecular clouds are highly turbulent, with supersonic motions of gas streams depositing kinetic energy into the cloud, stabilizing it against gravitational collapse. The same shock waves, on smaller scales, result in a local compression of gas. This process is called gravoturbulent cloud fragmentation (Mac Low and Klessen 2004) and leads to overdense gas filaments and clumps. Some of them remain gravitationally bound and may collapse if they exceed their Jeans mass.

Even then, the question remains whether out of clumps of several solar masses many stars of lower mass can form, or how stars with masses below $1 \, M_\odot$ are formed. Under what circumstances can fragments of a collapsing cloud become unstable and collapse faster than the cloud?

At first glance it seems to be a natural mechanism for producing collapsing objects with masses smaller than the initial M_J. Indeed, if a clump collapses isothermally, then M_J decreases as $\varrho^{-1/2}$. If, however, the gas were to change adiabatically, then for a monatomic ideal gas, $\nabla_{ad} = (d \ln T/d \ln P)_{ad} = 2/5$ or $T \sim P^{2/5}$, and from $P \sim \varrho T$, the temperature would change as $T \sim \varrho^{2/3}$, and therefore $M_J \sim T^{3/2}\varrho^{-1/2} \sim \varrho^{1/2}$. So the Jeans mass would *grow* during an adiabatic collapse. But already in Sect. 26.2 we have seen that under interstellar conditions the thermal adjustment timescale is much shorter than the free-fall time, which is of the order $(G\varrho)^{-1/2}$, and this also holds when the density increases during collapse. One can therefore assume the collapse to be isothermal rather than adiabatic. Then the Jeans mass becomes smaller than the mass of the originally collapsing cloud. If it has dropped, say, to one half its original value, the clump can split into two independently collapsing parts. This kind of fragmentation can go on as long as the collapse remains roughly isothermal. It will stop as soon as matter becomes opaque and the heat gained by gravothermal contraction can no longer be radiated away (Note that in principle it is not justified to apply the concept of the Jeans mass to an already collapsing medium, since it has been derived for an equilibrium state. But we may do it for order-of-magnitude estimates.).

What are the final products of this fragmentation process? Will the collapsing clump finally fall apart into a swarm of clumplets of planetary masses or even smaller? Even detailed multidimensional simulations of the hydrodynamics and thermodynamics of this complicated process cannot follow it in all details. But we may just estimate when the thermal adjustment time of the fragments becomes comparable with the free-fall time. Then the collapse can certainly not be isothermal anymore and must approach an adiabatic one. As we have seen, then the Jeans mass no longer decreases with increasing ϱ. This means that subregions of the fragments do not fall together on their own and fragmentation stops.

For a detailed estimate, one has to know the radiation processes that cool the gas during collapse. One can then find how long the gained work $-P\,dv$ can be radiated away, as is done in modern radiation-hydrodynamical simulations of star formation. Instead, we give a rough estimate of the mass limit of fragmentation based on simple physical arguments, following Rees (1976), without specifying the detailed radiation processes.

The characteristic time of the free-fall of a fragment is $(G\varrho)^{-1/2}$, and the total energy to be radiated away during collapse is of the order of the gravitational energy $E_g \approx GM^2/R$ (see Sect. 3.1), where M and R are the mass and radius of the fragment. Therefore the rate A of energy to be radiated away in order to keep the fragment always at the same temperature is of the order

$$A \approx \frac{GM^2}{R}(G\varrho)^{1/2} = \left(\frac{3}{4\pi}\right)^{1/2} \frac{G^{3/2}M^{5/2}}{R^{5/2}}. \tag{26.33}$$

But the fragment at temperature T cannot radiate more than a black body of that temperature (This implies approximate thermal equilibrium, which is not too bad an assumption for the final stage of fragmentation, where matter starts to become opaque.). Therefore the rate of radiation loss of the fragment is

$$B = 4\pi f \sigma T^4 R^2, \tag{26.34}$$

where $\sigma = 2\pi^5 k^4/(15c^2h^3)$ is the Stefan–Boltzmann constant, while f is a factor less than 1 taking into account that the fragment radiates less than the corresponding black body. For isothermal collapse it is necessary that $B \gg A$. The transition to adiabatic collapse will occur if $A \approx B$. From (26.33) and (26.34) we find that this is the case when

$$M^5 = \frac{64\pi^3}{3}\frac{\sigma^2 f^2 T^8 R^9}{G^3}. \tag{26.35}$$

We assume that fragmentation has reached its limit when M_J is equal to this M. We therefore replace M in (26.35) by M_J, R by

$$R = \left(\frac{3}{4\pi}\right)^{1/3} \frac{M_J^{1/3}}{\varrho^{1/3}}, \tag{26.36}$$

and eliminate ϱ with the help of (26.31). The Jeans mass at the end of fragmentation is then obtained as

$$\begin{aligned} M_J &= \frac{81}{64}\left(\frac{3}{\pi}\right)^{3/4} \frac{1}{(\sigma G^3)^{1/2}} \left(\frac{\Re}{\mu}\right)^{9/4} f^{-1/2}T^{1/4} \\ &= 6.2\times 10^{30}\mathrm{g}\, f^{-1/2}T^{1/4} = 0.003 M_\odot \frac{T^{1/4}}{f^{1/2}}, \end{aligned} \tag{26.37}$$

where T is in K and where we have set $\mu = 1$.

Let us assume that the temperature T of the smallest elements is 10 K and, further, that appreciable deviations from isothermal collapse occur when the radiation losses have to exceed 10 % of the maximal possible (black-body) radiation losses ($f = 0.1$). We then find from (26.37) that $M \approx 0.001\, M_\odot$. This rough estimate is surprisingly close to the mass of the smallest optically thick, pressure-supported protostellar cores that were found in numerical simulations. These objects in fact grow in mass by accretion from the surrounding clump.

It should be noted that our result is dependent on the chemical composition because the efficiency of cooling is higher the more heavy elements with rich spectral line systems are present. In particular for stars of the first generation, which are formed shortly after the Big Bang (also called Population III stars), cooling is very inefficient and proceeds mainly via hydrogen molecules, which are even dissociated easily at temperatures around 2,000 K. As a consequence, the smallest condensations in a collapsing cloud of primordial material is of the order of 100 $M_\odot$ (see Bromm and Larson 2004 for a review on first stars).

In the above considerations a number of complicating effects have been ignored. The role of magnetic fields is manifold. They may stabilize clouds against collapse, as long as there are ions in the gas, but are usually found to be too weak to do so. However, they may help to mediate the *angular momentum problem*, which is due to the fact that the initially present angular momentum in the cloud works against gravitational collapse. Magnetic fields may allow the transport of angular momentum away from collapsing clumps. Nevertheless, matter does not accrete spherically onto the smallest condensation objects, but accumulates in an accretion disc around it. Disc, protostellar object, and surrounding matter interact in a complicated way through matter in- and outflow, where magnetic fields and angular momentum influence the geometrical shape and efficiency. Magnetic fields also appear to help in keeping clumps together, after the effects of turbulence, which has created them in the first place, have faded. Zinnecker and Yorke (2007) as well as McKee and Ostriker (2007) give comprehensive reviews of star formation theory.