CHAPTER 5

Linear and Angular Kinematics

Kinematics is the accurate description of motion and is essential to understanding the biomechanics of human motion. Kinematics can range from anatomical descriptions of joint rotations to precise mathematical measurements of musculoskeletal motions. Recall from chapter 2 that kinematics is subdivided according to the kinds of measurements used, either linear or angular. Whatever the form of measurement, biomechanical studies of the kinematics of skilled performers provide valuable information on desirable movement technique. Biomechanics has a long history of kinematic measurements of human motion (Cappozzo, Marchetti, & Tosi, 1990). Accurate kinematic measurements are sometimes used for the calculation of more complex, kinetic variables. This chapter will introduce key kinematic variables in documenting both linear and angular human motions. The principles of biomechanics that apply kinematics to improving human movement are **Optimal Projection** and the **Coordination Continuum**.

LINEAR MOTION

Motion is change in position with respect to some frame of reference. In mathematical terms, linear motion is simple to define: final position minus initial position. The simplest linear motion variable is a scalar called **distance** (l). The use of the symbol l may be easy to remember if you associate it with the length an object travels irrespective of direction. Typical units of distance are meters and feet. Imagine an outdoor adventurer leaves base camp and climbs for 4 hours through rough terrain along the path illustrated in Figure 5.1. If her final position traced a 1.3-km climb measured relative to the base camp (0 km) with a pedometer, the distance she climbed was 1.3 km (final position – initial position). Note that 1.3 km (kilometers) is equal to 1300 meters. The odometer in your car works in a similar fashion, counting the revolutions (angular motion) of the tires to generate a measurement of the distance (linear variable) the car travels. Because distance is a scalar, your odometer does not tell you in what direction you are driving on the one-way street!

The corresponding vector quantity to distance is **displacement** (**d**). Linear displacements are usually defined relative to right-angle directions, which are convenient for the purpose of the analysis. For most two-dimensional (2D) analyses of human movements, like in Figure 5.1, the directions used are horizontal and vertical, so displacements are calculated as final position minus initial position in that particular direction. The usual convention is that motions to the right on the x-axis and upward along the y-axis are positive, with motion in the opposite directions negative. Since displacement is a vector quantity, if motion upward and to the right is defined as positive, motion downward and motion to the left is a negative displacement. Recall that the sign of a number in mechanics refers to direction.

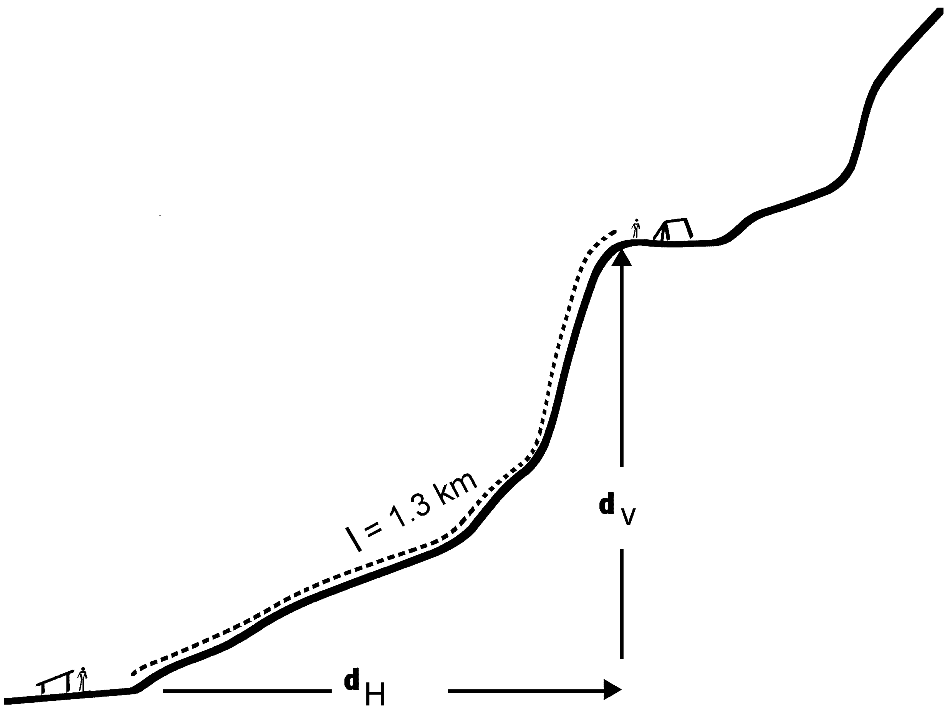

Figure 5.1. An outdoor adventurer climbs from base camp to a camp following the illustrated path. The distance the climber covers is 1.3 km. Her displacement is 0.8 km horizontally and 0.7 km vertically.

Assuming Figure 5.1 is drawn to the scale shown, it looks like the climber had a positive 0.8 km of horizontal displacement and 0.7 km of vertical displacement. This eyeballing of the horizontal and vertical components of the hike will be fairly accurate because displacement is a vector. Vectors can be conveniently represented by combinations of right-angle components, like the horizontal and vertical displacements in this example. If our adventurer were stranded in a blizzard and a helicopter had to lower a rescuer from a height of 0.71 km above base camp, what would be the rescuer's vertical displacement to the climber? The vertical displacement of the rescuer would be –0.01 km or 100 meters (final vertical position minus initial vertical position or 0.7 km – 0.71 km).

Biomechanists most often use measures of displacement rather than distance because they carry directional information that is crucial to calculation of other kinematic and kinetic variables. There are a couple of subtleties to these examples. First, the analysis is a simple 2D model of truly 3D reality. Second, the human body is modeled as a **point mass**. In other words, we know nothing about the orientation of the body or body segment motions; we just confine the analysis to the whole body mass acting at one point in space. Finally, an absolute frame of reference was used, when we are interested in the displacement relative to a moving object—like the helicopter, a relative frame of reference can be used.

In other biomechanical studies of human motion the models and frames of reference can get quite complicated. The analysis might not be focused on whole-body movement but how much a muscle is shortening between two attachments. A three-dimensional (3D) analysis of the small accessory gliding motions of the knee joint motion would likely measure along anatomically relevant axes like proximal-distal, medio-lateral, and antero-posterior. Three-dimensional kinematic measurements in biomechanics require considerable numbers of markers, spatial calibration, and mathematical complexity for completion. **Degrees of freedom** represent the kinematic complexity of a biomechanical model. The degrees of freedom (dof) correspond to the number of kinematic measurements needed to completely describe the position of an object. A 2D point mass model has only 2 dof, so the motion of the object can be described with an x (horizontal) and a y (vertical) coordinate.

The 3D motion of a body segment has 6 dof, because there are three linear coordinates (x, y, z) and three angles (to define the orientation of the segment) that must be specified. For example, physical therapy likes to describe the 6 dof for the lower leg at the knee joint using using the terms arthrokinematics (three anatomical rotations) and osteokinematics (three small gliding or linear motions between the two joint surfaces). The mathematical complexity of 3D kinematics is much greater than the 2D kinematics illustrated in this text. Good sources for a more detailed description of kinematics in biomechanics are available (Allard, Stokes, & Blanchi, 1995; Zatsiorsky, 1998). The field of biomechanics is striving to develop standards for reporting joint kinematics so that data can be exchanged and easily applied in various professional settings (Wu & Cavanagh, 1995).

The concept of **frame of reference** is, in essence, where you are measuring or observing the motion from. Reference frames in biomechanics are either absolute or relative. An absolute or global frame of reference is essentially motionless, like the apparent horizontal and vertical motion we experience relative to the earth and its gravitational field (as in Figure 5.1). A relative frame of reference is measuring from a point that is also free to move, like the motion of the foot relative to the hip or the plant foot relative to the soccer ball. There is no one frame of reference that is best, because the biomechanical description that is most relevant depends on the purpose of the analysis.

This point of motion being relative to your frame of reference is important for several reasons. First, the appearance and amount of motion depends on where the motion is observed or measured from. You could always answer a question about a distance as some arbitrary number from an "unknown point of reference," but the accuracy of that answer may be good for only partial credit. Second, the many ways to describe the motion is much like the different anatomical terms that are sometimes used for the identical motion. Finally, this is a metaphor for an intellectually mature kinesiology professional who knows there is not one single way of seeing or measuring human motion because your frame of reference affects what you see. The next section will examine higher-order kinematic variables that are associated with the rates of change of an object's motion. It will be important to understand that these new variables are also dependent on the model and frame of reference used for their calculation.

Speed and Velocity

Speed is how fast an object is moving without regard to direction. Speed is a scalar quantity like distance, and most people have an accurate intuitive understanding of speed. **Speed** (s) is defined as the rate of change of distance ($s = l/t$), so typical units are m/s, ft/s, km/hr, or miles/hr. It is very important to note that our algebraic shorthand for speed (l/t), and other kinematic variables to come, means "the change in the numerator divided by the change in the denominator." This means that the calculated speed is an *average* value for the time interval used for the calculation. If you went jogging across town (5 miles) and arrived at the turn-around point in 30 minutes, your average speed would be (5 miles / 0.5 hours), or an average speed of 10 miles per hour. You likely had intervals where you ran faster or slower than 10 mph, so we will see how representative or accurate the kinematic calculation is depends on the size of time interval and the accuracy of your linear measurements.

Since biomechanical studies have used both the English and metric systems of measurements, students need to be able to convert speeds from one system to the other. Speeds reported in m/s can be converted to speeds that make sense to American drivers (mph) by essentially doubling them (mph = m/s · 2.23). Speeds in ft/s can be converted to m/s by multiplying by 0.30, and km/hour can be converted to m/s by multiplying by 0.278. Other conversion factors can be found in Appendix B. Table 5.1 lists some typical speeds in sports and other human movements that have been reported in the biomechanics literature. Examine Table 5.1 to get a feel for some of the typical peak speeds of human movement activities.

Be sure to remember that speed is also relative to frame of reference. The motion of

Table 5.1
TYPICAL PEAK SPEEDS IN HUMAN MOVEMENT

	Speed	
	m/s	mph
Bar in a bench press	0.25	0.6
Muscle shortening	0.5	1.1
Walking	1.1–1.8	2.5–4.0
Vertical jump	2.3	5.1
Free throw	7.0	15.7
Sprinting	12.0	26.8
Tennis forehand	20.0	44.6
Batting	31.3	70.0
Soccer kick	35.0	78.0
Baseball pitch	45.1	101
Tennis serve	62.6	140
Golf drive	66.0	148

Application: Speed

One of the most important athletic abilities in many sports is speed. Coaches often quip that "luck follows speed" because a fast athlete can arrive in a crucial situation before their opponent. Coaches that often have a good understanding of speed are in cross-country and track. The careful timing of various intervals of a race, commonly referred to as pace, can be easily converted average speeds over that interval. Pace (time to run a specific distance) can be a good kinematic variable to know, but its value depends on the duration of the interval, how the athlete's speed changes over that interval, and the accuracy of the timing. How accurate do you think stopwatch measures of time and, consequently, speed are in track? What would a two-tenths-of-a-second error mean in walking (200 s), jogging (100 s), or sprinting (30 s) a lap on a 220 m track? Average speeds for these events are 1.1, 2.2, and 7.3 m/s, with potential errors of 0.1, 0.2, and 0.7%. Less than 1%! That sounds good, but what about shorter events like a 100-m dash or the hang time of a punt in football? American football has long used the 40-yard dash as a measure of speed, ability, and potential for athletes despite little proof of its value (see Maisel, 1998). If you measured time by freezing and counting frames of video (30 Hz), how much more accurate would a 40-meter dash timing be? If the current world record for 100 m is 9.79 seconds and elite runners cover 40 m (43.7 yards) out of the starting blocks in about 4.7 seconds, should you believe media guides that say that a certain freshman recruit at Biomechanical State University ran the forty (40-yard dash) in 4.3 seconds?

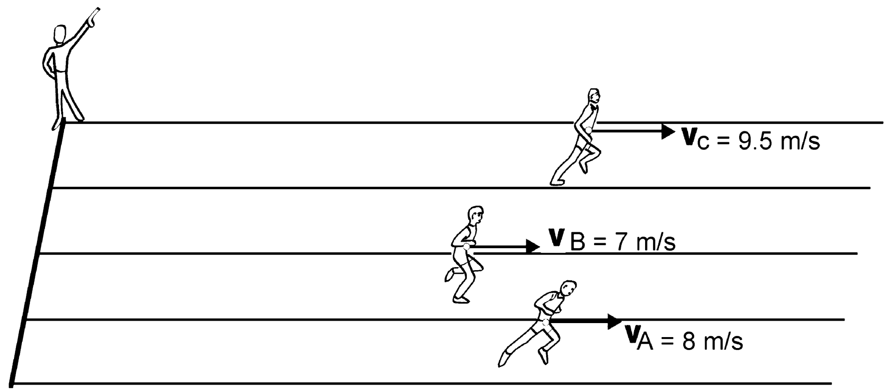

Figure 5.2. The speed of runner A depends on the frame of reference of the measurement. Runner A can be described as moving at 8 m/s relative to the track or –1.5 m/s relative to runner C.

the runner in Figure 5.2 can correctly be described as 8 meters per second (m/s), 1 m/s, or –1.5 m/s. Runner A is moving 8 m/s relative to the starting line, 1 m/s faster than runner B, and 1.5 m/s slower than runner C. All are correct kinematic descriptions of the speed of runner A.

Velocity is the vector corresponding to speed. The vector nature of velocity makes it more complicated than speed, so many people incorrectly use the words interchangeably and have incorrect notions about velocity. Velocity is essentially the speed of an object, *in a particular direction*. Velocity is the rate of change of displacement ($\mathbf{V} = \mathbf{d}/t$), so its units are the same as speed, and are usually qualified by a directional adjective (i.e., horizontal, vertical, resultant). Note that when the adjective "angular" is not used, the term *velocity* refers to linear velocity. If you hear a coach say a pitcher has "good velocity," the coach is not using biomechanical terminology correctly. A good question to ask in this situation is: "That's interesting. When and in what direction was the pitch velocity so good?"

The phrase "rate of change" is very important because velocity defines how quickly position is *changing* in the specified direction (displacement). Most students might recognize this phrase as the same one used to describe the derivative or the slope of a graph (like the hand dynamometer example in Figure 2.4).

Remember to think about velocity as a speed, but in a particular direction. A simple example of the velocity of human movement is illustrated by the path (dotted line in Figure 5.3) of a physical education student in a horizontal plane as he changes exercise stations in a circuit-training program. The directions used in this analysis are a fixed reference frame that is relevant to young students: the equipment axis and water axis.

The student's movement from his initial position (I) to the final position (F) can be vectorially represented by displacements along the equipment axis ($\mathbf{d}_E$) and along the water axis ($\mathbf{d}_W$). Note that the definition of these axes is arbitrary since the student must combine displacements in both directions to arrive at the basketballs or a drink. The net displacements for this student's movement are positive, because the final position measurements are larger than the initial positions. Let's assume that $\mathbf{d}_E$ = 8 m and $\mathbf{d}_W$ = 2 m and that the time it took this student to change stations was 10 seconds. The average velocity along the

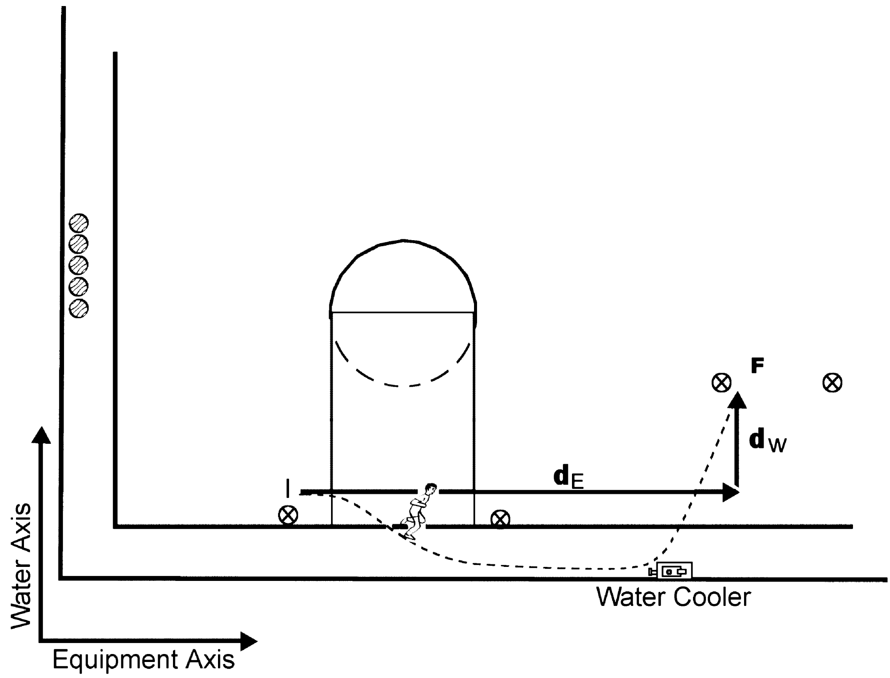

Figure 5.3. The horizontal plane path (dashed line) of a physical education student changing practice stations. The linear displacements can be measured along a water fountain axis and equipment axis.

water axis would be $\mathbf{V}_W = \mathbf{d}_W/t = 2/10 = 0.2$ m/s. Note that the motion of most interest to the student is the negative displacement ($-\mathbf{d}_W$), which permits a quick trip to the water fountain. The average velocity along the equipment axis would be 0.8 m/s ($\mathbf{V}_E = \mathbf{d}_L/t = 8/10$). Right-angle trigonometry can then be used to calculate the magnitude and direction of the resultant displacement ($\mathbf{d}_R$) and then the average velocity of the student. We will use right-angle trigonometry in chapter 6 to analyze the effect of force vectors. By the way, if your right-angle trigonometry is a little rusty, check out appendix D for a refresher.

Calculations of speed and velocity using algebra are *average* velocities over the time interval used. It is important to realize that the smaller the time interval the greater the potential accuracy of kinematic calculations. In the previous example, for instance, smaller time intervals of measurement would have detected the negative velocity (to get a drink) and positive velocity of the student in the water direction. Biomechanics research often uses high-speed film or video imaging (Gruen, 1997) to make kinematic measurements over very small time intervals (200 or thousands of pictures per second). The use of calculus allows for kinematic calculations ($\mathbf{v} = d_d/d_t$) to be made to *instantaneous* values for any point in time of interest. If kinematic calculations are based over a too large time interval, you may not be getting information much better than the time or pace of a whole race, or you may even get the unusual result of zero velocity because the race finished where it started.

Graphs of kinematic variables versus time are extremely useful in showing a pattern within the data. Because human movement occurs across time, biokinematic variables like displacement, velocity, and acceleration are usually plotted versus time, although there are other graphs that are of value. Figure 5.4 illustrates the horizontal displacement and velocity graphs for an elite male sprinter in a 100-m dash. Graphs of the speed over a longer race precisely document how the athlete runs the race. Notice that the athlete first approaches top speed at about the 40- to 50-meter mark. You can compare your velocity profile to Figure 5.4 and to those of other sprinters in Lab Activity 5.

Acceleration

The second derivative with respect to time, or the rate of change of velocity, is **acceleration**. Acceleration is how quickly velocity is changing. Remember that velocity changes when speed or direction change. This vector nature of velocity and acceleration means that it is important to think of acceleration as an unbalanced force in a particular direction. The acceleration of an object can speed it up, slow it down, or change its direction. It is incorrect to assume that "acceleration" means an object is speeding up. The use of the term "deceleration" should be avoided because it implies that the object is slowing down and does not take into account changes in direction.

Let's look at an example that illustrates why it is not good to assume the direction of motion when studying acceleration. Imagine a person is swimming laps, as illustrated in Figure 5.5. Motion to the right is designated positive, and the swimmer has a relatively constant velocity (zero horizontal acceleration) in the middle of the pool and as she approaches the wall. As her hand touches the wall there is a negative acceleration that first slows her down and then speeds her up in the negative direction to begin swimming again. Thinking of the acceleration at the wall as a push in the negative direction is correct throughout the

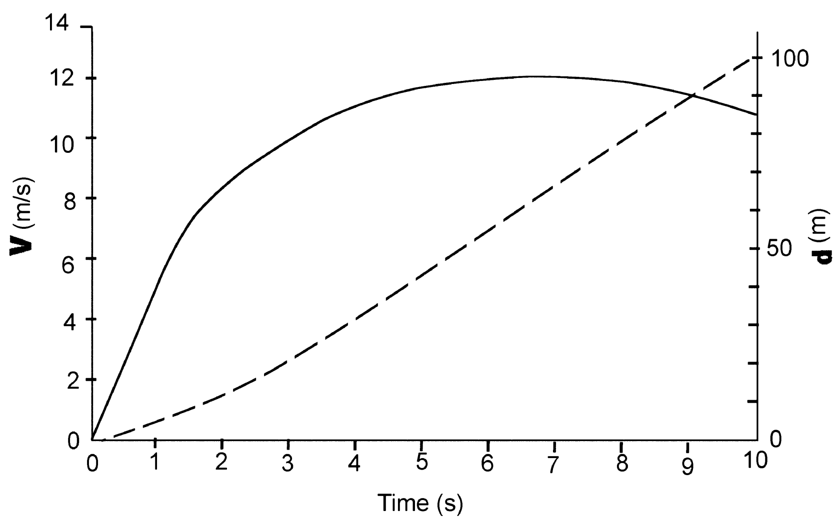

Figure 5.4. The displacement–time (dashed curve) and velocity–time (solid curve) graphs for the 100-m dash of an elite male sprinter.

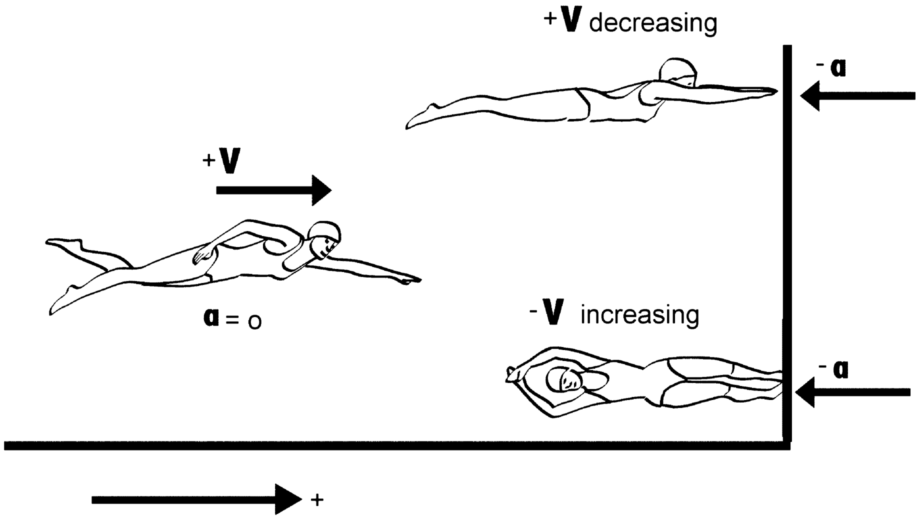

Figure 5.5. The motion and accelerations of swimmers as they change direction in lap swimming. If motion to the right is designated positive, the swimmer experiences a negative acceleration as they make the turn at the pool wall. The negative acceleration first slows positive velocity, and then begins to build negative velocity to start swimming in the negative direction. It is important to associate signs and accelerations with directions.

turn. As the swimmer touches the other wall there is a positive acceleration that decreases her negative velocity, and if she keeps pushing (hasn't had enough exercise) will increase her velocity in the positive direction back into the pool.

The algebraic definition of acceleration (**a**) is **V**/t, so typical units of acceleration are m/s^2 and ft/s^2. Another convenient way to express acceleration is in units of gravitational acceleration (*g*'s). When you jump off a box you experience (in flight) one *g* of acceleration, which is about –9.81 m/s/s or –32.2 ft/s/s. This means that, in the absence of significant air resistance, your vertical velocity will change 9.81 m/s every second in the negative direction. Note that this means you slow down 9.81 m/s every second on the way up and speed up 9.81 m/s every second on the way down. *G*'s are used for large acceleration events like a big change of direction on a roller coaster (4 *g*'s), the shockwaves in the lower leg following heel strike in running (5 *g*'s), a tennis shot (50 *g*'s), or head acceleration in a football tackle (40–200 *g*'s). When a person is put under sustained (several seconds instead of an instant, like the previous examples) high-level acceleration like in jet fighters (5–9 *g*'s), pilots must a wear pressure suit and perform whole-body isometric muscle actions to prevent blacking out from the blood shifting in their body.

Acceleration due to gravity always acts in the same direction (toward the center of the earth) and may cause speeding up or slowing down depending on the direction of motion. Remember to think of acceleration as a push in a direction or a tendency to change velocity, not as speed or velocity. The vertical acceleration of a ball at peak flight in the toss of a tennis serve is 1 *g*, not zero. The vertical velocity may be instantaneously zero, but the constant pull of gravity is what prevents it from staying up there.

Let's see how big the horizontal acceleration of a sprinter is in getting out of the blocks. This is an easy example because the rules require that the sprinter have an initial horizontal velocity of zero. If video measurements of the sprinter showed that they passed the 10-m point at 1.9 seconds with a horizontal velocity of 7 m/s, what would be the runner's acceleration? The sprinter's change in velocity was 7 m/s (7 – 0), so the sprinter's acceleration was: **a** = **V**/t = 7/1.9 = 3.7 m/s/s. If the sprinter could maintain this acceleration for three seconds, how fast would he be running?

Close examination of the displacement, velocity, and acceleration graphs of an object's motion is an excellent exercise in qualitative understanding of linear kinematics. Examine the pattern of horizontal acceleration in the 100-m sprint mentioned earlier (see Figure 5.4). Note that there are essentially three phases of acceleration in this race that roughly correspond to the slope of the velocity graph. There is a positive acceleration phase, a phase of near zero acceleration, and a negative acceleration phase. Most sprinters struggle to prevent running speed from declining at the end of a race. Elite female sprinters have similar velocity graphs in 100-m races. What physiological factors might account for the inability of people to maintain peak speed in sprinting?

Note that the largest accelerations (largest rates of change of velocity) do not occur at the largest or peak velocities. Peak velocity must occur when acceleration is zero. Coaches often refer to quickness as the ability to react and move fast over short distances, while speed is the ability to cover moderate distances in a very short time. Based on the velocity graph in Figure 5.4, how might you design running tests to differentiate speed and quickness?

Acceleration is the kinematic (motion description) variable that is closest to a kinetic variable (explanation of motion). Kinesiology professionals need to remember that the pushes (forces) that create accelerations precede the peak speeds they eventually create. This delay in the development of motion is beyond the Fore–Time Principle mentioned earlier. Coaches observing movement cannot see acceleration, but they can perceive changes in speed or direction that can be interpreted as acceleration. Just remember that by the time the coach perceives the acceleration the muscular and body actions which created those forces occurred just before the motion changes you are able to see.

Uniformly Accelerated Motion

In rare instances the forces acting on an object are constant and therefore create a constant acceleration in the direction of the resultant force. The best example of this special condition is the force of earth's gravity acting on projectiles. A projectile is an object launched into the air that has no self-propelled propelling force capability (Figure 5.6). Many human projectile movements have vertical velocities that are sufficiently small so that the effects of air resistance in the vertical direction can be ignored (see chapter 8). Without fluid forces in the vertical direction, projectile motion is uniformly accelerated by one force, the force of gravity. There are exceptions, of course (e.g., skydiver, badminton shuttle), but for the majority of human projectiles we can take advantage of the special conditions of vertical motion to simplify kinematic description of the motion. The Italian Galileo Galilei is often credited with discovering the nearly constant nature of gravitational acceleration using some of the first accurate of measurements of objects falling and rolling down inclines. This section will briefly summarize these mathematical descriptions, but will emphasize several important facts about this kind of motion, and how this can help determine optimal angles of projection in sports.

When an object is thrown or kicked without significant air resistance in the ver-

Figure 5.6. Softballs (left) and soccer balls (right) are projectiles because they are not self-propelled when thrown or kicked.

tical direction, the path or **trajectory** will be some form of a parabola. The uniform nature of the vertical force of gravity creates a linear change in vertical velocity and a second-order change in vertical displacement. The constant force of gravity also assures that the time it takes to reach peak vertical displacement (where vertical velocity is equal to zero) will be equal to the time it takes for the object to fall to the same height that it was released from. The magnitude of the vertical velocity when the object falls back to the same position of release will be the same as the velocity of release. A golf ball tossed vertically at shoulder height at 10 m/s (to kill time while waiting to play through) will be caught at the same shoulder level at a vertical velocity of –10 m/s. The velocity is negative because the motion is opposite of the toss, but is the same magnitude as the velocity of release. Think about the 1 *g* of acceleration acting on this golf ball and these facts about uniformly accelerated motion to estimate how many seconds the ball will be in flight.

This uniformly changing vertical motion of a projectile can be determined at any given instant in time using three formulas and the kinematic variables of displacement, velocity, acceleration, and time. My physics classmates and I memorized these by calling them VAT, SAT, and VAS. The various kinematic variables are obvious, except for "S," which is another common symbol for displacement. The final vertical velocity of a projectile can be uniquely determined if you know the initial velocity (V_i) and the time of flight of interest (VAT: $V_f^2 = V_i + at$). Vertical displacement is also uniquely determined by initial velocity and time of flight (SAT: $d = V_i t + 0.5at^2$). Finally, final velocity can be determined from initial velocity and a known displacement (VAS: $V_f^2 = V_i^2 + 2ad$).

Let's consider a quick example of using these facts before we examine the implications for the best angles of projecting objects. Great jumpers in the National Basketball Association like Michael Jordan or David Thompson are credited with standing vertical jumps about twice as high (1.02 m or 40 inches) as typical college males. This outstanding jumping ability is not an exaggeration (Krug & LeVeau, 1999). Given that the vertical velocity is zero at the peak of the jump and the jump height, we can calculate the takeoff velocity of our elite jumper by applying VAS. Solving for V_i in the equation:

$$V_f^2 = V_i^2 + 2ad$$

$$0 = V_i^2 + 2(-9.81)(1.002)$$

$$V_i = 4.47 \text{ m/s or } 9.99 \text{ mph}$$

We select the velocity to be positive when taking the square root because the initial velocity is opposite to gravity, which acts in the negative direction. If we wanted to calculate his hang time, we could calculate the time of the fall with SAT and double it because the time up and time down are equal:

$$d = V_i t + 0.5at^2$$

$$-1.02 = 0 + 0.5(-9.81)t^2$$

$$t = 0.456 \text{ s}$$

So the total fight time is 0.912 seconds. If you know what your vertical jump is, you can repeat this process and compare your takeoff velocity and hang time to that of elite jumpers. The power of these empirical relationships is that you can use the mathematics as models for simulations of projectiles. If you substitute in reasonable values for two variables, you get good predictions of kinematics for any instant in time. If you wanted to know when a particular height was reached, what two equations could you use? Could you calculate how much higher you could jump if you increased your takeoff velocity by 10%?

So we can see that uniformly accelerated motion equations can be quite useful in modeling the vertical kinematics of projectiles. The final important point about uniformly accelerated motion, which reinforces the directional nature of vectors, is that, once the object is released, the vertical component of a projectile's velocity is independent of its horizontal velocity. The extreme example given in many physics books is that a bullet dropped the same instant another is fired horizontally would strike level ground at the same time. Given constant gravitational conditions, the height of release and initial vertical velocity uniquely determine the time of flight of the projectile. The range or horizontal distance the object will travel depends on this time of flight and the horizontal velocity. Athletes may increase the distance they can throw by increasing the height of release (buying time against gravity), increasing vertical velocity, and horizontal velocity. The optimal combination of these depends on the biomechanics of the movement, not just the kinematics or trajectory of uniformly accelerated motion. The next section will summarize a few general rules that come from the integration of biomechanical models and kinematic studies of projectile activities. These rules are the basis for the Optimal Projection Principle of biomechanics.

OPTIMAL PROJECTION PRINCIPLE

For most sports and human movements involving projectiles, there is a range of angles that results in best performance. The **Optimal Projection Principle** refers to the angle(s) that an object is projected to achieve a particular goal. This section will outline some general rules for optimal projections that can be easily applied by coaches and teachers. These optimal angles are "rules of thumb" that are consistent with the biomechanical research on projectiles. Finding true optimal angles of projection requires integration of descriptive studies of athletes at all ability levels (e.g., Bartlett, Muller, Lindinger, Brunner, & Morriss, 1996), the laws of physics (like uniformly accelerated motion and the effects of air resistance; see chapter 8), and modeling studies that incorporate the biomechanical effects of various release parameters. Determining an exact optimal angle of projection for the unique characteristics of a particular athlete and in a particular environment has been documented using a combination of experimental data and modeling (Hubbard, de Mestre, & Scott, 2001). There will be some general trends or rules for teaching and coaching projectile events where biomechanical research has shown that certain factors dominate the response of the situation and favor certain release angles.

In most instances, a two-dimensional point-mass model of a projectile is used to describe the compromise between the height of release and the vertical and horizontal components of release velocity (Figure 5.7). If a ball was kicked and then landed at the same height, and the air resistance was negligible, the optimal angle of projection for producing maximum horizontal displacement would be 45°. Forty-five degrees above the horizontal is the perfect mix of horizontal and vertical velocity to maximize horizontal displacement. Angles above 45° create shorter ranges because the extra flight time from larger vertical velocity cannot overcome the loss in horizontal velocity. Angles smaller than 45° cause loss of flight time (lower vertical velocity) that cannot be overcome by the larger horizontal velocity. Try the activity below to explore optimal angles of projection.

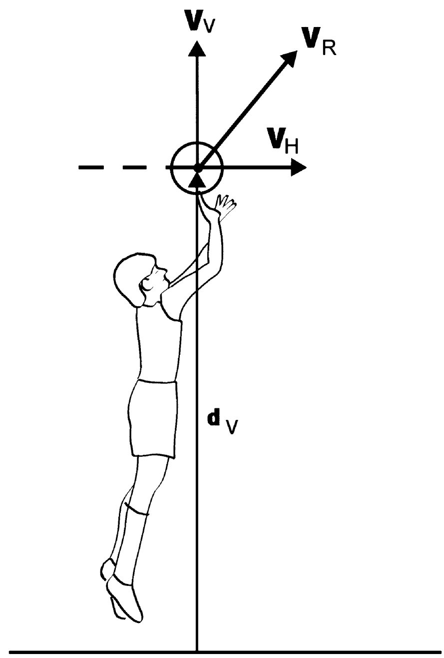

Figure 5.7. The three variables that determine the release parameters of a projectile in two-dimensions: height of release, and the horizontal and vertical velocities of release.

Activity: Angles of Projection

Use a garden hose to water the grass and try various angles of projection of the water. The air resistance on the water should be small if you do not try to project the water too far. Experiment and find the angle that maximizes the distance the water is thrown. First see if the optimal angle is about 45°, when the water falls back to the height that it comes out of the hose. What happens to the optimal angle during long-distance sprinkling as the height of release increases?

Note how the optimal angle of projection changes from 45° when the height of release is above and below the target.

Before we can look at generalizations about how these factors interact to apply the Optimal Projection Principle, the various goals of projections must be analyzed. The mechanical objectives of projectiles are displacement, speed, and a combination of displacement and speed. The goal of an archer is accuracy in displacing an arrow to the target. The basketball shooter in Figure 5.7 strives for the right mix of ball speed and displacement to score. A soccer goalie punting the ball out of trouble in his end of the field focuses on ball speed rather than kicking the ball to a particular location.

When projectile displacement or accuracy is the most important factor, the range of optimal angles of projection is small. In tennis, for example, Brody (1987) has shown that the vertical angle of projection (angular "window" for a serve going in) depends on many factors but is usually less than 4º. The goal of a tennis serve is the right combination of displacement and ball speed, but traditionally the sport and its statistics have emphasized the importance of consistency (accuracy) so as to keep the opponent guessing. In a tennis serve the height of projection above the target, the net barrier, the spin on the ball, the objective of serving deep into the service box, and other factors favor angles of projection at or above the horizontal (Elliott, 1983). Elite servers can hit high-speed serves 3º below the horizontal, but the optimal serving angle for the majority of players is between 0 and 15º above the horizontal (Elliott, 1983; Owens & Lee, 1969).

This leads us to our first generalization of the Optimal Projection Principle. *In most throwing or striking events, when a mix of maximum horizontal speed and displacement are of interest, the optimal angle of projection tends to be below 45º.* The higher point of release and dramatic effect of air resistance on most sport balls makes lower angles of release more effective. Coaches observing softball or baseball players throwing should look for initial angles of release between 28 and 40º above the horizontal (Dowell, 1978). Coaches should be able to detect the initial angle of a throw by comparing the initial flight of the ball with a visual estimate of 45º angle (Figure 5.8). Note that there is a larger range of optimal or desirable angles that must accommodate differences in the performer and the situation. Increasing the height of release (a tall player) will tend to shift the optimal angle downward in the range of angles, while higher speeds of release (gifted players) will allow higher angles in the range to be effectively used. What do you think would happen to the optimal angles of release of a javelin given the height of release and speed of approach differences of an L5-disabled athlete compared to an able-bodied athlete?

There are a few exceptions to this generalization, which usually occur due to the special environmental or biomechanical conditions of an event. In long jumping, for example, the short duration of takeoff on the board limits the development of vertical velocity, so that takeoff angles are usually between 18 and 23º (Hay, Miller, & Canterna, 1986; Linthorne *et al.*, 2005). In the standing long jump, jumpers prefer slightly higher takeoff angles with relatively small decreases in performance (Wakai & Linthorne, 2005). We will see in chapter 8 that the effect of air resistance can quickly become dominant on the optimal release parameters for many activities. In football place-kicking, the lower-than-45º generalization applies (optimal angles are usually between 25 and 35º), but the efficient way the ball can be punted and the tactical importance of time during a punt make the optimal angle of release about 50º. With the wind at the punter's back he might kick above 50º, while using a flatter kick against a wind. The backspin put on various golf shots is another example of variations in the angle of release because of the desirable effects of spin on fluid forces and the

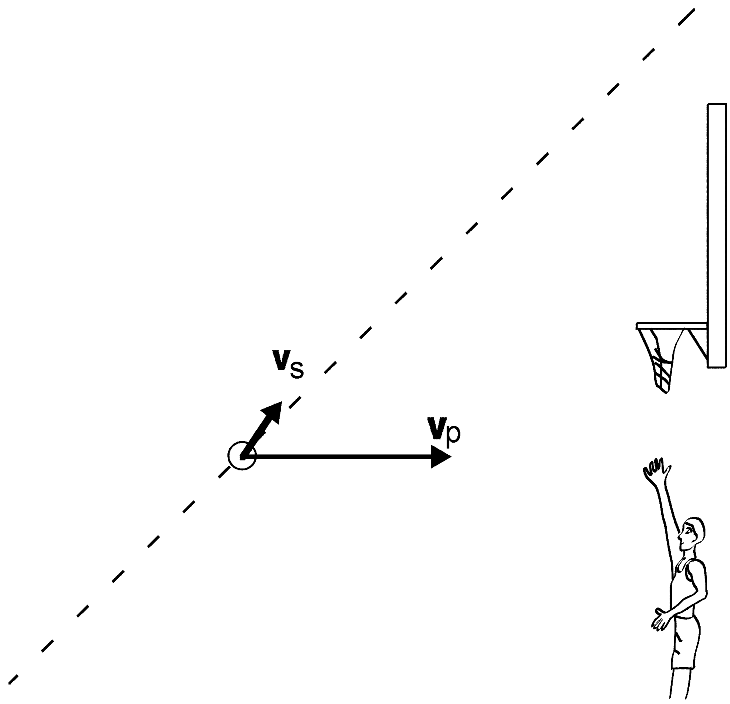

Figure 5.8. Coaches can visually estimate the initial path of thrown balls to check for optimal projection. The initial path of the ball can be estimated relative to an imaginary 45° angle. When throwing for distance, many small children select very high angles of release that do not maximize the distance of the throw.

bounce of the ball. Most long-distance clubs have low pitches, which agrees with our principle of a low angle of release, but a golfer might choose a club with more loft in situations where he wants higher trajectory and spin rate to keep a ball on the green.

The next generalization relates to projectiles with the goal of upward displacement from the height of release. *The optimal angle of projection for tasks emphasizing displacement or a mix of vertical displacement and speed tends to be above 45°*. Examples of these movements are the high jump and basketball shooting. Most basketball players (not the giants of the NBA) release a jump shot below the position of the basket. Considerable research has shown that the optimal angle of projection for basketball shots is between 49 and 55° (see Knudson, 1993). This angle generally corresponds to the arc where the minimum speed may be put on the ball to reach the goal, which is consistent with a high-accuracy task. Ironically, a common error of beginning shooters is to use a very flat trajectory that requires greater ball speed and may not even permit an angle of entry so that the ball can pass cleanly through the hoop! Coaches that can identify appropriate shot trajectories can help players improve more quickly (Figure 5.9). The optimal angles of release in basketball are clearly not "high-arc" shots, but are slightly greater than 45° and match the typical shooting conditions in recreational basketball.

The optimal angle of projection principle involves several generalizations about

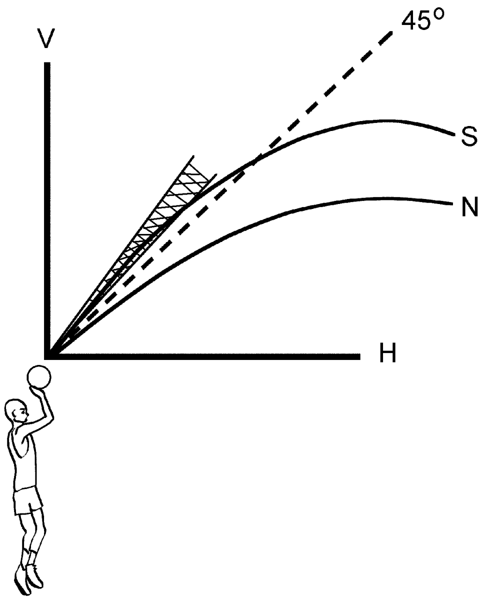

Figure 5.9. The optimal projection angles for most basketball jump shots are between 49 and 55° above the horizontal (hatched). These initial trajectories represent the right mix of low ball speed and a good angle of entry into the hoop. Novice shooters (N) often choose a low angle of release. Skilled shooters (S) really do not shoot with high arcs, but with initial trajectories that are in the optimal range and tailored to the conditions of the particular shot.

desirable initial angles of projection. These general rules are likely to be effective for most performers. Care must be taken in applying these principles in special populations. The biomechanical characteristics of elite (international caliber) athletes or wheelchair athletes are likely to affect the optimal angle of projection. Kinesiology professionals should be aware that biomechanical and environmental factors interact to affect the optimal angle of projection. For example, a stronger athlete might use an angle of release slightly lower than expected but which is close to optimal for her. Her extra strength allows her to release the implement at a higher point without losing projectile speed so that she is able to use a slightly lower angle of release. Professionals coaching projectile sports must keep up on the biomechanical research related to optimal conditions for their athletes.

ANGULAR MOTION

Angular kinematics is the description of angular motion. Angular kinematics is particularly appropriate for the study of human movement because the motion of most human joints can be described using one, two, or three rotations. Angular kinematics should also be easy for biomechanics students because for every linear kinematic variable there is a corresponding angular kinematic variable. It will even be easy to distinguish angular from linear kinematics because the adjective "angular" or a Greek letter symbol is used instead of the Arabic letters used for linear kinematics.

Angular displacement (**θ**: theta) is the vector quantity representing the change in angular position of an object. Angular displacements are measured in degrees, radians (dimensionless unit equal to 57.3°), and revolutions (360°). The usual convention to keep directions straight and be consistent with our 2D linear kinematic calculations is to consider counterclockwise rotations as positive. Angular displacement measured with a **goniometer** is one way to measure **static flexibility**. As in linear kinematics, the frames of reference for these angular measurements are different. Some tests define complete joint extension as 0° while other test refer to that position as 180°. For a review of several physical therapy static flexibility tests, see Norkin & White (1995).

In analyzing the curl-up exercise shown in Figure 5.10, the angle between the thoracic spine and the floor is often used. This exercise is usually limited to the first 30 to 40° above the horizontal to limit the involvement of the hip flexors (Knudson, 1999a). The angular displacement of the thoracic spine in the eccen-

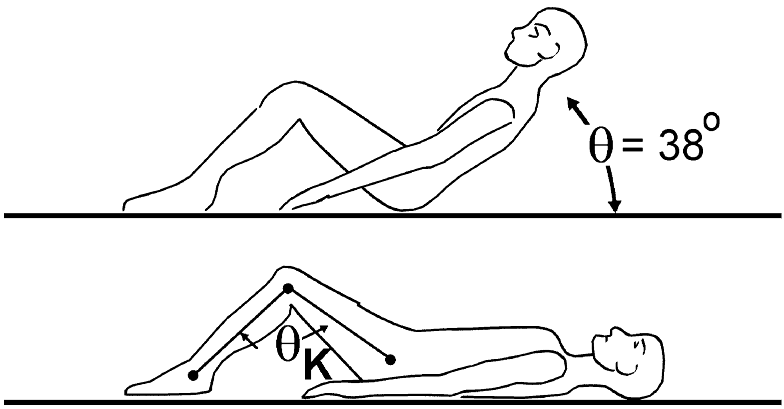

Figure 5.10. The angular kinematics of a curl-up exercise can be measured as the angle between the horizontal and a thoracic spine segment. This is an example of an absolute angle because it defines the angle of an object relative to external space. The knee joint angle (θ_K) is a relative angle because both the leg and thigh can move.

tric phase would be –38° (final angle minus initial angle: 0 – 38 = –38°). This trunk angle is often called an **absolute angle** because it is measured relative to an "unmoving" earth frame of reference. **Relative angles** are defined between two segments that can both move. Examples of relative angles in biomechanics are joint angles. The knee angle ($\boldsymbol{\theta_K}$) in Figure 5.10 is a relative angle that would tell if the person is changing the positioning of their legs in the exercise.

Angular Velocity

Angular velocity (**ω**: omega) is the rate of change of angular position and is usually expressed in degrees per second or radians per second. The formula for angular velocity is $\boldsymbol{\omega} = \boldsymbol{\theta}/t$, and calculations would be the same as for a linear velocity, except the displacements are angular measurements. Angular velocities are vectors are drawn by the right-hand rule, where the flexed fingers of your right hand represent the rotation of interest, and the extended thumb would be along the axis of rotation and would indicate the direction of the angular velocity vector. This book does not give detailed examples of this technique, but will employ a curved arrow just to illustrate angular velocities and torques (Figure 5.11).

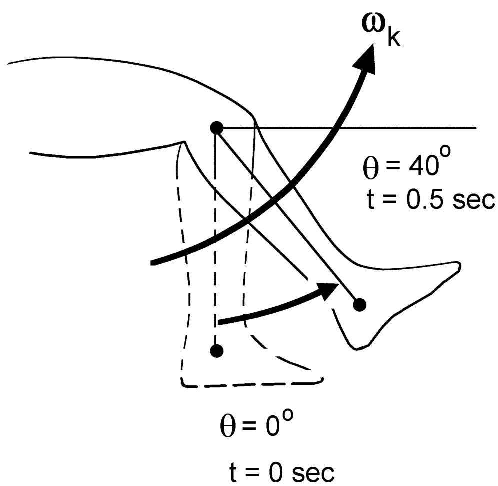

Figure 5.11. The average angular velocity of the first half of a knee extension exercise can be calculated from the change in angular displacement divided by the change in time.

The angular velocities of joints are particularly relevant in biomechanics, because they represent the angular speed of anatomical motions. If relative angles are calculated between anatomical segments, the angular velocities calculated can represent the speed of flexion/extension and other anatomical rotations. Biomechanical research often indirectly calculates joint angles from the linear coordinates (measurements) derived from film or video images, or directly from electrogoniometers attached to subjects in motion. It is also useful for kinesiology professionals to be knowledgeable about the typical angular velocities of joint movements. This allows professionals to understand the similarity between skills and determine appropriate training exercises. Table 5.2 lists typical peak joint angular speeds for a variety of human movements.

Table 5.2
TYPICAL PEAK ANGULAR SPEEDS IN HUMAN MOVEMENT

	Speed deg/s	Speed rad/s
Knee extension: sit-to-stand	150	2.6
Trunk extension: vertical jump	170	3.0
Elbow flexion: arm curl	200	3.5
Knee extension: vertical jump	800	14.0
Ankle extension: vertical jump	860	15.0
Wrist flexion: baseball pitching	1000	17.5
Radio/ulnar pronation: tennis serve	1400	24.4
Knee extension: soccer kick	2000	34.9
Shoulder flexion: softball pitch	5000	87.3
Shoulder internal rotation: pitching	7400	129.1

Let's calculate the angular velocity of a typical knee extension exercise and compare it to the peak angular velocity in the table. Figure 5.11 illustrates the exercise and the data for the example. The subject extends a knee, taking their leg from a vertical orientation to the middle of the range of motion. If we measure the angle of the lower leg from the vertical, the exerciser has moved their leg 40° in a 0.5-second period of time. The average knee extension angular velocity can be calculated as follows: $\boldsymbol{\omega}_K = \boldsymbol{\theta}/t = 40/0.5 = 80$ deg/s. The angular velocity is positive because the rotation is counterclockwise. So the exercise averages 80° per second of knee extension velocity over the half-second time interval, but the instantaneous angular velocity at the position shown in the figure is likely larger than that. The peak knee extension angular velocity in this exercise likely occurs in the midrange of the movement, and the knee extension velocity must then slow to zero at the end of the range of motion. This illustrates some limitations of free-weight exercises. There is a range of angular velocities (which have an affect on the linear Force–Velocity Relationship of the muscles), and there must be a decrease in the angular velocity of the movement at the end of the range of motion. This negative acceleration (if the direction of motion is positive) protects the joints and ligaments, but is not specific to many events where peak speed is achieved near the release of an object and other movements can gradually slow the body in the follow-through.

Angular Acceleration

The rate of change of angular velocity is **angular acceleration** ($\boldsymbol{\alpha} = \boldsymbol{\omega}/t$). Angular acceleration is symbolized by the Greek letter alpha ($\boldsymbol{\alpha}$). The typical units of angular acceleration are deg/s/s and radians/s/s. Like linear acceleration, it is best to think about

Interdisciplinary Issue: Specificity

One of the most significant principles discovered by early kinesiology research is the principle of specificity. Specificity applies to the various components of fitness, training response, and motor skills. Motor learning research suggests that there is specificity of motor skills, but there is potential transfer of ability between similar skills. In strength and conditioning the principle of specificity states that the exercises prescribed should be specific, as close as possible to the movement that is to be improved. Biomechanics research that measures the angular kinematics of various sports and activities can be used to assess the similarity and potential specificity of training exercises. Given the peak angular velocities in Table 5.2, how specific are most weight training or isokinetic exercise movements that are limited to 500° per second or slower? The peak speed of joint rotations is just one kinematic aspect of movement specificity. Could the peek speeds of joint rotation in different skills occur in different parts of the range of motion? What other control, learning, psychological, or other factors affect the specificity of an exercise for a particular movement?

angular acceleration as an unbalanced rotary effect. An angular acceleration of –200 rad/s/s means that there is an unbalanced clockwise effect tending to rotate the object being studied. The angular acceleration of an isokinetic dynamometer in the middle of the range of motion should be zero because the machine is designed to match or balance the torque created by the person, so the arm of the machine should be rotating at a constant angular velocity.

Angular kinematics graphs are particularly useful for providing precise descriptions of how joint movements occurred. Figure 5.12 illustrates the angular displacement and angular velocity of a simple elbow extension and flexion movement in the sagittal plane. Imagine that the data repre-

Interdisciplinary Issue: Isokinetic Dynamometers

Isokinetic (iso = constant or uniform, kinetic = motion) dynamometers were developed by J. Perrine in the 1960s. His Cybex machine could be set at different angular velocities and would accommodate the resistance to the torque applied by a subject to prevent angular acceleration beyond the set speed. Since that time, isokinetic testing of virtually every muscle group has become a widely accepted measure of muscular strength in clinical and research settings. Isokinetic dynamometers have been influential in documenting the balance of strength between opposing muscle groups (Grace, 1985). There is a journal (*Isokinetics and Exercise Science*) and several books (e.g., Brown, 2000; Perrin, 1993) that focus on the many uses of isokinetic testing. Isokinetic machines, however, are not truly isokinetic throughout the range of motion, because there has to be an acceleration to the set speed at the beginning of a movement that often results in a torque overshoot as the machine negatively accelerates the limb (Winter, Wells, & Orr, 1981) as well as another negative acceleration at the end of the range of motion. The effects of inertia (Iossifidou & Baltzopoulos, 2000), shifting of the limb in the seat/restraints (Arampatzis *et al.,* 2004), and muscular co-contraction (Kellis & Baltzopoulos, 1998) are other recent issues being investigated that affect the validity of isokinetic testing. It is important to note that the muscle group is not truly shortening or lengthening in an isokinetic fashion. Muscle fascicle-shortening velocity is not constant (Ichinose, Kawakami, Ito, Kanehisa, & Fukunaga, 2000) in isokinetic dynamometry even when the arm of the machine is rotating at a constant angular velocity. This is because linear motion of points on rotating segments do not directly correspond to angular motion in isokinetic (Hinson, Smith, & Funk, 1979) or other joint motions.

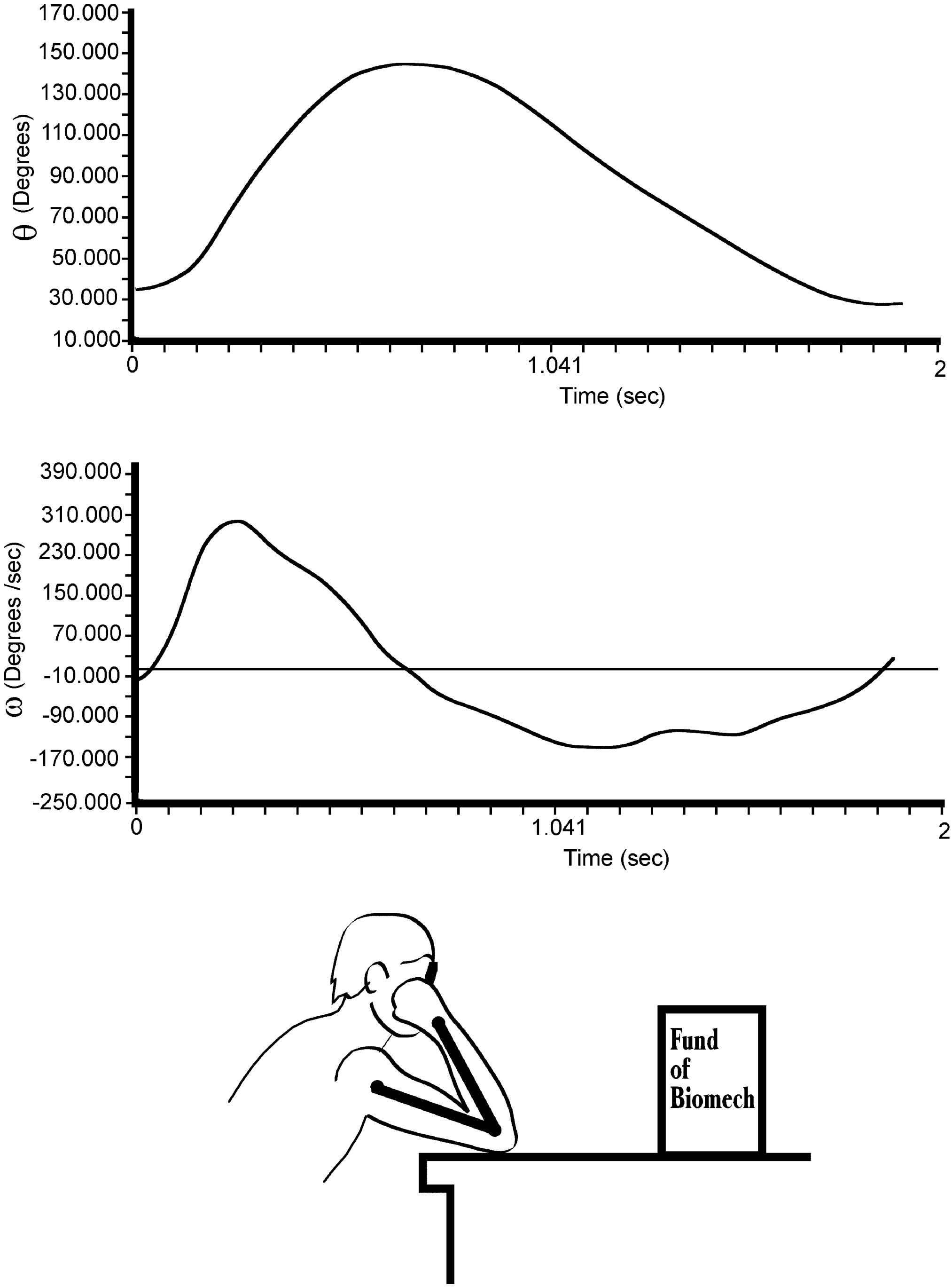

Figure 5.12. The angular displacement and angular velocity of a simple elbow extension/flexion movement to grab a book. See the text for an explanation of the increasing complexity of the higher-order kinematic variables.

sented a student tired of studying exercise physiology, who reached forward to grab a refreshing, 48-ounce *Fundamentals of Biomechanics* text. Note as we look at the kinematic information in these graphs that the complexity of a very simple movement grows as we look at the higher-order derivatives (velocity).

The elbow angular displacement data show an elbow extended (positive angular displacement) from about a 37º to about an 146º elbow angle to grasp the book. The extension movement took about 0.6 seconds, but flexion with the book occurred more slowly. Since the elbow angle is defined on the anterior aspect of a subject's arm, larger numbers mean elbow extension. The corresponding angular velocity–time graph represents the speed of extension (positive **ω**) or the speed of flexion (negative **ω**). The elbow extension angular velocity peaks at about 300 deg/s (0.27 sec) and gradually slows. The velocity of elbow flexion increases and decreases more gradually than the elbow extension.

The elbow angular acceleration would be the slope of the angular velocity graph. Think of the elbow angular acceleration as an unbalanced push toward extension or flexion. Examine the angular velocity graph and note the general phases of acceleration. When are there general upward or downward trends or changes in the angular velocity graph? Movements like this often have three major phases. The extension movement was initiated by a phase of positive acceleration, indicated by an increasing angular velocity. The second phase is a negative acceleration (downward movement of the angular velocity graph) that first slows elbow extension and then initiates elbow flexion. The third phase is a small positive angular acceleration that slows elbow flexion as the book nears the person's head. These three phases of angular acceleration correspond to typical muscle activation in this movement. This movement would usually be created by a triphasic pattern of bursts from the elbow extensors, flexors, and extensors. Accelerations (linear and angular) are the kinematic variables closest to the causes (kinetics) of the motion, and are more complex than lower-order kinematic variables like angular displacements.

Figure 5.13 plots the ankle angle, angular velocity, plantar flexor torque, and REMG for the gastrocnemius muscle in a concentric-only and an SSC hop. Notice how only the SSC has a negative angular velocity (describing essentially the speed of the eccentric stretch of the calf muscles) and the dramatic difference in the pattern and size of the plantar flexor torque created.

Angular and linear kinematics give scientists important tools to describe and understand exactly how movement occur. Remember to treat the linear and angular measurements separately: like the old saying goes, "don't mix apples and oranges." A good example is your CD player. As the CD spins, a point near the edge travels a larger distance compared to a point near the center. How can two points make the same revolutions per minute and travel at different speeds? Easy, if you notice the last sentence mixes or compares angular and linear kinematic variables. In linear kinetics we will look at the trigonometric functions that allow linear measurements to be mapped to angular.

Biomechanists usually calculate angular kinematic variables from linear coordinates of body segments with trigonometry. There is another simple formula that converts linear to angular kinematics in special conditions. It is useful to illustrate why the body tends to extend segments prior to release events. The linear velocity of a point on a rotating object, *relative to its axis of rotation*, can be calculated as the product of its angular velocity and the distance from the axis to the point (called the radius): $\mathbf{V} = \boldsymbol{\omega} \cdot \mathbf{r}$. The special condition for using this

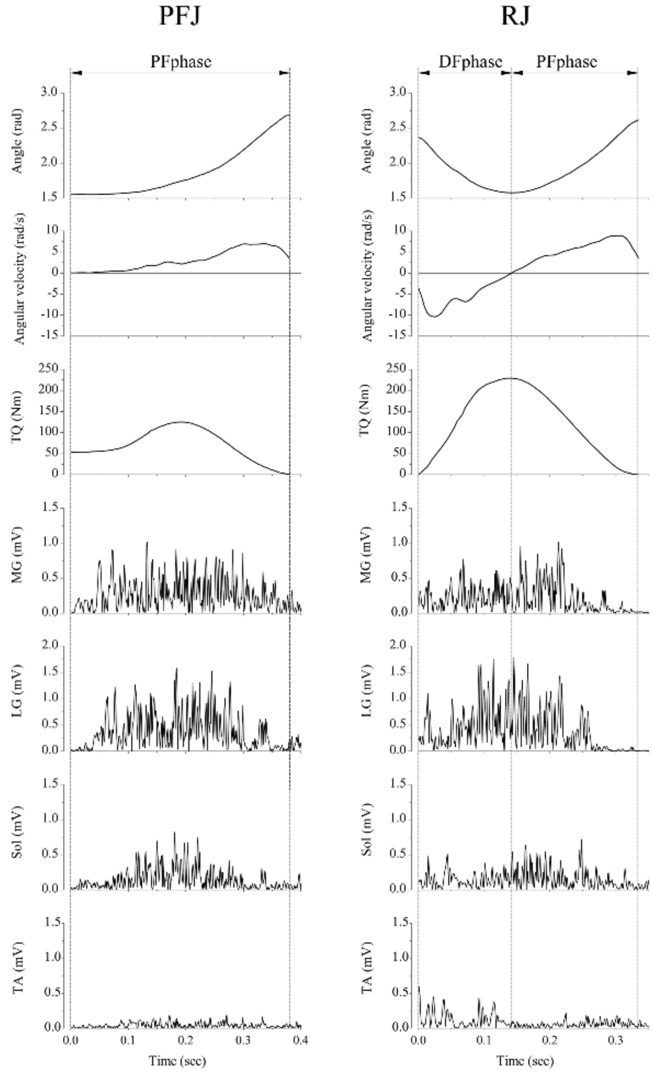

Figure 5.13. Ankle angle, angular velocity, torque, and rectified EMG in a concentric-only (PFJ: plantar flexion jump) and SSC hop exercise (RJ: rebound jump). Figure reprinted permission of Sugisaki et al. (2005).

formula is to use angular velocity in radians/second. Using a dimensionless unit like rad/s, you can multiply a radius measured in meters and get a linear velocity in meters/second.

The most important point is to notice that the angular velocity and the radius are equally important in creating linear velocity. To hit a golf ball harder you can either use a longer club or rotate the club faster. We will see in chapter 7 that angular kinetic analysis can help us decide which of these two options is best for a particular situation. In most throwing events the arm is extended late in the throw to increase the linear velocity of a projectile. Angular kinetics is necessary to understand why this extension or increase in the radius of segments is delayed to just before release.

COORDINATION CONTINUUM PRINCIPLE

Many kinesiology professionals are interested in the coordination of movement. Coordination is commonly defined as the sequence and timing of body actions used in a movement. Unfortunately there is no universally agreed-upon definition or way to study coordination in the kinesiology literature. A wide variety of approaches has been proposed to describe the coordination of movement. Some approaches focus on the kinematics of the joint or segmental actions (Hudson, 1986; Kreighbaum & Bartels, 1996), while others are based on the joint forces and torques (kinetics) that create the movement (Chapman & Sanderson, 1990; Prilutsky, 2000; Putnam, 1991, 1993; Roberts, 1991; Zajac, 1991). This section presents the **Coordination Continuum Principle**, which is adapted from two kinematic approaches to defining coordination (Hudson, 1986; Kreighbaum & Bartels, 1996), because teachers and coaches most often modify the spatial and temporal aspects of movement. While teaching cues that focus on muscular effort may be used occasionally, much of the and coaching of movement remains in the positioning and motions of the body.

Kinematic coordination of movements can be pictured as a continuum ranging from simultaneous body actions to sequential actions. The **Coordination Continuum Principle** suggests that movements requiring the generation of high forces tend to utilize simultaneous segmental movements, while lower-force and high-speed movements are more effective with more sequential movement coordination. A person lifting a heavy box simultaneously extends the hips, knees, and ankles (Figure 5.14). In overarm throwing, people usually use a more sequential action of the whole kinematic chain, beginning with the legs, followed by trunk and arm motions.

Figure 5.14. Coordination to move a heavy load usually involves simultaneous joint motions like in this squat lift.

Because coordination falls on a continuum and the speed and forces of movement vary widely, it is not always easy to determine what coordination pattern is best. In vertical jumping, resistance is moderate and the objective is to maximize height of takeoff and vertical velocity. While a vertical jump looks like a simultaneous movement, biomechanical studies show that the kinematics and kinetics of different jumpers have simultaneous and sequential characteristics (Aragon-Vargas & Gross, 1997a; Bobbert & van Ingen Schenau, 1988; Hudson, 1986). Kinesiology professionals need to remember that coordination is not an either/or situation in many activities. Until there is more research determining the most effective technique, there will be quite a bit of art to the coaching of movements not at the extremes of the continuum.

The motor development of high-speed throwing and striking skills tends to begin with restricted degrees of freedom and simultaneous actions. Children throwing, striking, or kicking tend to make initial attempts with simultaneous actions of only a few joints. Skill develops with the use of more segments and greater sequential action. In high-speed throwing, for example, the sequential or "differential" rotation of the pelvis and upper trunk is a late-developing milestone of high-skill throwing (Roberton & Halverson, 1984). It is critical that physical educators know the proper sequential actions in these low-force and high-speed movements. Kinematic studies help identify these patterns of motion in movement skills. Unfortunately, the youth of biomechanics means that kinematic documentation of coordination in the wide variety of human movements is not complete. Early biomechanics research techniques emphasized elite male performers, leaving little information on gender, special populations, lower skill levels, or age.

Suppose a junior high volleyball coach is working with a tall athlete on spiking. The potential attacker lacks a strong overarm pattern and cannot get much speed on the ball (Figure 5.15). The kinematics of the preparatory action lacks intensity, stretch, and timing. At impact the player's elbow and upper arm are well forward of her shoulder. The coach suspects that her overarm throwing pattern is still immature and must be developed before skilled spiking is possible. This coach has integrated biomechanical and motor development information to determine the best course of action to help this player improve. The lack of ball speed (kinematics), and muscle stretch-shortening cycles within a sequential coordination are biomechanical factors missing in this athlete. The forward elbow position at impact is a motor development indicator of an immature trunk and arm action within an overarm pattern. How coaches work on this problem may vary, but one good strategy would be to simplify the movement and work on throwing the volleyball. Sequential rotation of the trunk, arm, forearm, and wrist is the focus of training.

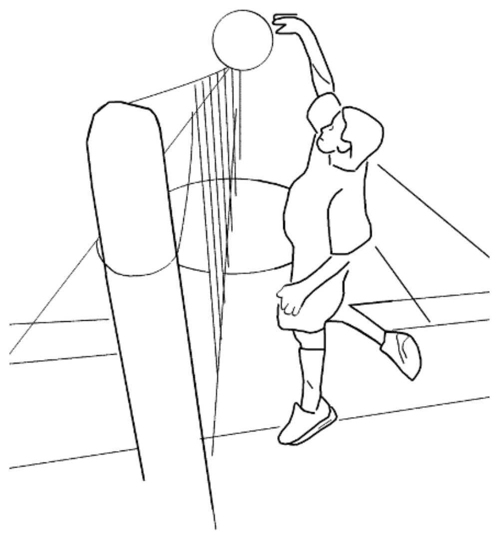

Figure 5.15. Poor sequential coordination in throwing and striking results in slow segment speeds at impact that can be visually identified by slow ball speeds, lack of eccentric loading of distal segments, or limited movement in the follow-through (like this volleyball spike).

Strength and conditioning professionals closely monitor training technique, because body position and motion in exercises dramatically affect muscular actions and risk of injury. In strength training, resistances are near maximal, so coordination in most exercises tends to be simultaneous. Imagine someone performing a squat exercise with a heavy weight. Is the safest technique to simultaneously flex the hips and knees in the eccentric phase and then simultaneously extend in the concentric phase? If the resistance is lighter (bodyweight), like in standing up out of a chair, after a person leans forward to put their upper bodyweight over their feet, do the major joints of the body simultaneously act to stand? In the next chapter we will examine variations in conditioning for high-power and high-speed movements that are different than high-force (strength) movements. Do you think high-power movements will also have simultaneous coordination, or will the coordination shift a little toward sequential? Why?

SUMMARY

A key branch of biomechanics is kinematics, the precise description or measurement of human motion. Human motion is measured relative to some frame of reference and is usually expressed in linear (meters, feet) or angular (radians, degrees) units. Angular kinematics are particularly appropriate in biomechanics because these can be easily adapted to document joint rotations. There are many kinematic variables that can be used to document the human motion. Simple kinematic variables are scalars, while others are vector quantities that take into account the direction of motion. The time derivatives (rates of change) of position measurements are velocity and acceleration. The Optimal Projection Principle states that sporting events involving projectiles have a range of desirable initial angles of projection appropriate for most performers. The kinematic timing of segment motions falls on a Coordination Continuum from simultaneous to sequential movement. High-force movements use more simultaneous joint rotations while high-speed movements use more sequential joint rotations.

REVIEW QUESTIONS

1. What is a frame of reference and why is it important in kinematic measurements?
2. Compare and contrast the scalar and vector linear kinematic variables.
3. Explain the difference between calculation of average and instantaneous velocities, and how does the length of the time interval used affect the accuracy of a velocity calculation?
4. Use the velocity graph in Figure 5.4 to calculate the average acceleration of the sprinter in the first and the last 10-m intervals.
5. A patient lifts a dumbbell 1.2 m in 1.5 s and lowers it back to the original position in 2.0 s. Calculate the average vertical velocity of the concentric and eccentric phases of the lift.
6. Explain why linear and angular accelerations should be thought of as pushes in a particular direction rather than speeding up or slowing down.
7. Why are angular kinematics particularly well suited for the analysis of human movement?
8. From the anatomical position a person abducts their shoulder to 30º above the horizontal. What is the angular displacement of this movement with the usual directional (sign) convention?
9. A soccer player attempting to steal the ball from an opponent was extending her knee at 50 deg/s when her foot struck the opponent's shin pads. If the player's knee was stopped (0 deg/s) within 0.2 seconds, what angular acceleration did the knee experience?
10. A golfer drops a ball to replace a lost ball. If the ball had an initial vertical velocity of 0 m/s and had a vertical velocity before impact of –15.7 m/s exactly 1.6 seconds later, what was the vertical acceleration of the ball?

11. A softball coach is concerned that her team is not throwing at less than 70% speed in warm-up drills. How could she estimate or measure the speeds of the warm-up throws to make sure her players are not throwing too hard?

12. A biomechanist uses video images to measure the position of a box in the sagittal plane relative to a worker's toes during lifting. Which coordinate (x or y) usually corresponds to the height of the box and the horizontal position of the box relative to the foot?

13. Use the formula for calculating linear velocity from angular velocity ($\mathbf{V} = \boldsymbol{\omega} \cdot \mathbf{r}$) to calculate the velocity of a golf club relative to the player's hands (axis of rotation). Assume the radius is 1.5 m and the angular velocity of the club was 2000 deg/s. Hint: remember to use the correct units.

14. Give an example of a fixed and a relative frame of reference for defining joint angular kinematics. Which frame of reference is better for defining anatomical rotations versus rotations in space?

15. What is the vertical acceleration of a volleyball at the peak of its flight after the ball is tossed upward in a jump serve?

KEY TERMS

absolute angle
acceleration
coordination continuum
degrees of freedom
displacement
distance
goniometer
isokinetic
point mass
relative angle
speed
static flexibility
trajectory
velocity

SUGGESTED READING

Cappozzo, A., Marchetti, M., & Tosi, V. (Eds.) (1990). *Biolocomotion: A century of research using moving pictures*. Rome: Promograph.

Hudson, J. L. (1986). Coordination of segments in the vertical jump. *Medicine and Science in Sports and Exercise*, **18**, 242–251.

Kreighbaum, E., & Barthels, K. M. (1996). *Biomechanics: A qualitative approach to studying human movement*. Boston: Allyn & Bacon.

Lafortune, M. A., & Hennig, E. M. (1991). Contribution of angular motion and gravity to tibial acceleration. *Medicine and Science in Sports and Exercise*, **23**, 360–363.

Mero, A., Komi, P. V., & Gregor, R. J. (1992). Biomechanics of sprint running: A review. *Sports Medicine*, **13**, 376–392.

Plagenhoef, S. (1971). *Patterns of human motion: A cinematographic analysis*. Englewood Cliffs, NJ: Prentice-Hall.

Zatsiorsky, V. M. (1998). *Kinematics of human motion*. Champaign, IL: Human Kinetics.

WEB LINKS

Projectiles—page on the path of the center of gravity of a skater by Debra King and others from Montana State University.
http://btc.montana.edu/olympics/physbio/default.htm

Free kinematic analysis software by Bob Schleihauf at San Francisco State.
http://www.kavideo.sfsu.edu/

Human Movement Analysis software by Tom Duck of York University.
http://www.hma-tech.com/

Kinematics of Vectors and Projectiles—Tutorials on vectors and projectiles from The Physics Classroom.
http://www.physicsclassroom.com/mmedia/vectors/vectorsTOC.html

Kinematics of Gait—Introduction to kinematic variables, their use in the analysis of human walking, and some of the determinants of gait. Teach-in feature of the Clinical Gait Analysis website.
http://guardian.curtin.edu.au/cga/teach-in/kinematics.html