Chapter 10
The Nature of Reality

In this chapter, we explore how quantum mechanics leads to some rather strange consequences for our notion of what is real and not real. What does quantum mechanics actually say about reality? Why does this theory make predictions that are so radically different from our everyday experience? We must answer these questions if quantum mechanics is to properly explain the world around us.

10.1 The Emergent Classical World

Quantum mechanical objects behave in a completely different way to the physical objects in our everyday experience. Typically, quantum mechanical objects such as photons, electrons or atoms are very small, and it is therefore not really surprising that their behaviour is different from what we naively expect. Our physical intuition evolved in order to exploit our environment for survival and to deal with physical threats, not to study atoms. Why should they behave in the same way as rocks and spears? On the other hand, since we think that quantum mechanics is a more fundamental theory of nature than classical (Newtonian) mechanics, it is certainly valid to ask how the classical world that we experience emerges from quantum mechanics. How do we lose the typical counterintuitive quantum behaviour in the every-day world around us?

In 1935, Erwin Schrödinger (1935) came up with his famous thought experiment about a cat in a box (see Fig. 10.1). This argument pushed our question to the foreground with some urgency, so let us consider it in detail: A cat is put in a sealed box

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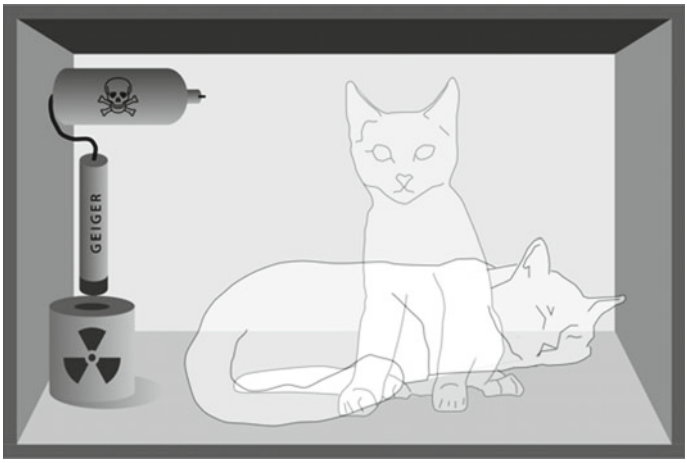

Fig. 10.1 Schrödinger's cat. The interactive figure is available online (see supplementary material 1)

(with enough oxygen so it does not suffocate), together with a devious device consisting of a radioactive atom (for example the alpha particle emitter polonium-210), and a Geiger counter that is linked to a valve that opens a vial of poisonous gas. The device is rigged in such a way that when the polonium decays into a lead atom ($^{210}\text{Po} \rightarrow {}^{206}\text{Pb}$), the Geiger counter registers the emitted alpha particle and releases the valve. This briefly opens the vial with the poisonous gas, and the cat dies.

This all happens inside the sealed, soundproof box, so we do not know whether the atom has decayed or not. After some time, we have about a one in three chance of the atom being decayed (waiting longer increases this probability). Moreover, since the system inside the box is isolated, it will evolve quantum mechanically, turning the state of the cat (including the deadly contraption) into a superposition of alive and dead. At some specific time the superposition will be

$$|\text{cat}\rangle = \sqrt{\frac{2}{3}}|\text{alive}\rangle + \sqrt{\frac{1}{3}}|\text{dead}\rangle\,. \tag{10.1}$$

According to the fifth postulate of quantum mechanics—the "projection postulate"—only when we open the box does the state of the cat collapse to "alive" or "dead".

This is clearly absurd. Classical objects like cats do not appear in quantum superpositions. Somewhere, we must pass from the quantum description to a classical description. Is the cat itself an observer of the valve on the vial, and already capable of triggering the projection postulate? Is the vial opening the classical observer? Or is it the Geiger counter? It is easy to see that we can in principle devise a chain of interacting systems of increasing size from the atom upwards (much like the internal workings of the Geiger counter), and it does not seem to be possible to say exactly at what point the quantum state collapses. This is known as the *measurement problem*.

However, this paradox assumes that the quantum state corresponds directly to the reality of the physical system, something about which we were told to be cautious in Chap. 1. We will return to the precise meaning of the quantum state in the next section.

If we wanted to perform the experiment with Schrödinger's cat in reality, we would quickly find that we made another crucial assumption: the cat and the contraption are completely sealed off from the environment. A moment's thought reveals that this is very difficult to achieve in practice. For example, while you are reading this, you may feel chilly, hot, or just right. This reflects the fact that the molecules in the air constantly interact with your skin. The same is true for the cat. And in the cat's case the air may have traces of the poisonous gas, depending on whether the atom decayed or not.

Before the box is opened the joint quantum state $|\Psi\rangle$ of the air and the cat is given by

$$|\Psi\rangle \equiv \sqrt{\frac{2}{3}}|\text{pure}\rangle|\text{alive}\rangle + \sqrt{\frac{1}{3}}|\text{poison}\rangle|\text{dead}\rangle\,, \tag{10.2}$$

where the different states of the air in the box are given by $|\text{pure}\rangle$ and $|\text{poison}\rangle$ for an undecayed and a decayed atom, respectively. In Chap. 7 we showed that we can remove the air from our description of the system via the partial trace. This leaves us with

$$\rho = \text{Tr}_{\text{air}}\left(|\Psi\rangle\langle\Psi|\right) = \frac{2}{3}|\text{alive}\rangle\langle\text{alive}| + \frac{1}{3}|\text{dead}\rangle\langle\text{dead}| \tag{10.3}$$

We see that the cat is no longer in the superposition of dead or alive. Instead, whether the cat is dead or alive is now subject to classical uncertainty. As we demonstrated in Chap. 7—where we considered a photon in a Mach–Zehnder interferometer interacting with the environment—we see the typical quantum behaviour only when the quantum system is isolated in the right way, and the larger the quantum system, the harder it is to keep it free from interactions with the environment. This transition from a quantum superposition to a state of classical uncertainty is called decoherence. The cat decoheres long before we open the box due to interactions with the environment that are beyond our control.

There are currently many experimental efforts to overcome this decoherence and create ever larger physical systems in a quantum superposition, approaching Schrödinger's cat states. Not with real cats, obviously! For example, people are now performing experiments similar to the double slit experiment in Chap. 8, in which we can see quantum behaviour in the interference pattern (Gerlich et al. 2011). These experiments are not performed with single photons, but with increasingly larger molecules, and interference is indeed observed.

Finally, we can ask why the decoherence of the quantum state of the cat gives us a classical uncertainty over the states "dead" and "alive", rather than some other superposition states such as

$$|+\rangle = \frac{1}{\sqrt{2}}|\text{alive}\rangle + \frac{1}{\sqrt{2}}|\text{dead}\rangle\,, \tag{10.4}$$

or

$$|-\rangle = \frac{1}{\sqrt{2}}|\text{alive}\rangle - \frac{1}{\sqrt{2}}|\text{dead}\rangle\,, \tag{10.5}$$

After all, quantum mechanics does not have any preference for the dead/alive basis over the $\pm$ basis. We saw this already in Chap. 3, where we could create the spin state of a spin-1/2 particle in any direction we want. This corresponds to different quantum states $|\uparrow\rangle$ and $|\downarrow\rangle$, or $|\pm\rangle$. So why does the cat decohere into the state in Eq. (10.3) and not, for example into

$$\frac{2}{3}|+\rangle\langle+| + \frac{1}{3}|-\rangle\langle-|\,? \tag{10.6}$$

Such a state would exhibit interference (caused by the off-diagonal terms in the density matrix) in the "dead"/"alive" basis, which is not what we expect in our classical description of the cat.

The answer to this question is in the *interaction* between the poisonous gas and the cat. The fact that, physically, it is the poisonous gas in the air that kills the cat means that the interaction between the cat and the air must take the form

$$|\text{poison}\rangle|\text{alive}\rangle \xrightarrow[H_I]{} |\text{poison}\rangle|\text{dead}\rangle\,, \tag{10.7}$$

where H_I denotes the interaction Hamiltonian (see Chap. 4). If the interaction between the air and the cat had been different, the final state of the cat would be different, and possibly exhibit nonclassical interference between the states "alive" and "dead". But the interaction is what it is, and we end up finding the cat either dead or alive, with a classical uncertainty.

Therefore, the specific interaction between the quantum system and the environment causes rapid decoherence of large quantum systems, and this ensures that the macroscopic world looks classical, rather than quantum. However, this does not answer the question what the quantum state really means, only what we find in our measurements at the macroscopic, classical level. In the next section we will face the much more difficult question of the meaning of the quantum state head on.

10.2 The Quantum State Revisited

What does the quantum state really mean? Some physicists hold that the state vector itself is a real property of the system, while others maintain that it is merely an expression of our knowledge of the system. Still others even argue that we should

altogether drop the question of what $|\psi\rangle$ refers to in the real world. Niels Bohr (Petersen 1963) famously argued that we cannot meaningfully speak about what is going on at the quantum level at all:

> *There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature...*

For many people this is unacceptable. Physical theories should have something to say about what nature really is, and not just be a prescription of what we can say about nature. This is what we identified as the second purpose of physical theories in Chap. 1: they should provide an explanatory framework to understand the natural world.

In this book, we developed quantum mechanics via careful reasoning about the probabilities of measurement outcomes. In particular, we did not ask the troublesome question what it all means at a "deeper" level. However, we cannot avoid this any longer. To attempt an answer to the question what $|\psi\rangle$ stands for, we need to introduce two concepts from philosophy:

- Epistemology: the study of what we can know.
- Ontology: the study of what exists.

We can broadly classify the different meanings of $|\psi\rangle$ into epistemic and ontic interpretations (Harrigan and Spekkens 2010). Epistemic interpretations of $|\psi\rangle$ say that the quantum state is an expression of our knowledge of the system, while ontic interpretations of $|\psi\rangle$ say that the quantum state corresponds to something real.

There are two types of epistemic interpretations:

- Quantum states reflect our knowledge of the system without referring to a deeper underlying reality. Call this ψ-epistemic of the first kind.
- Quantum states reflect our knowledge of the system with respect to a deeper underlying reality. We call this ψ-epistemic of the second kind.

Similarly, there are two types of ontic interpretations:

- Quantum states correspond to an underlying reality without any additional variables. We call this ψ-complete.
- Quantum states are supplemented by additional variables that correspond to an underlying reality. We call this ψ-supplemented.

The additional variables in the ψ-supplemented interpretation are typically hidden from us, and are commonly referred to as "hidden variables". They are not part of the theory of quantum mechanics as we have developed it here.

If you believe, like Niels Bohr, that there is no deeper underlying quantum reality (the epistemic interpretation of the first kind) then you are done. Quantum mechanics does not pose any conceptual problems for you. The phenomena are what they are, and quantum mechanics accurately predicts their probabilities.

However, if you believe that quantum mechanics says something about a microscopic physical reality, then three possibilities for the interpretation of the quantum state remain:

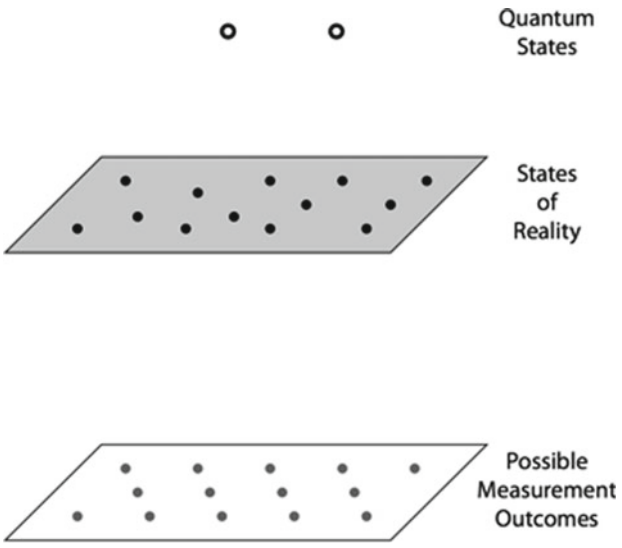

Fig. 10.2 A model of an underlying reality for quantum probabilities. The interactive figure is available online (see supplementary material 2)

- complete,
- supplemented, and
- epistemic (of the second kind).

These three types of interpretations are collected in Fig. 10.2.

What constraints do the predictions of quantum mechanics place on the nature of the underlying reality? This is a question that is currently actively researched, since it allows us to narrow down what $|\psi\rangle$ can mean. In the next two sections we will set up two thought experiments that will limit the way reality can behave, given ψ-complete and ψ-supplemented interpretations. In the remainder of this section, we will deal a blow to the viability of ψ-epistemic interpretations of the second kind.

At first sight, ψ-epistemic interpretations of the second kind are rather attractive. The way we have developed quantum mechanics in this book shows that the entire theory can be constructed as a generalised probability theory. Probability theories are often interpreted as epistemic: the probabilities tell us something about our knowledge of a system. For example, when we say that a flipped coin can come up heads or tails—each with a probability of 1/2—we typically mean that we do not know the details of the movement of the coin and cannot calculate exactly how it is going to fall. If we could do this, we would know more about the coin flip, and we would not assign a 50:50 chance to the outcome. There are people who beat the odds at roulette by calculating how the ball moves, and where it will likely land. Seen in this light, quantum mechanics as a probability theory suggests a ψ-epistemic interpretation of the quantum state.

Another example is quantum teleportation (see Chap. 6): Alice sends an unknown quantum state to Bob by performing a two-qubit Bell measurement on her qubit in the unknown quantum state and one qubit of an entangled quantum system, which she shares with Bob. Before Alice sends the outcome of the measurement to Bob, he describes the state of his qubit as $\rho_B = \mathbb{I}/2$. This means that he is completely uncertain about the state of his qubit. After learning Alice's measurement outcome, he describes the state of his qubit as

$$|\psi'\rangle_B = U^\dagger |\psi\rangle_B \,, \tag{10.8}$$

where U is the corrective operation Bob has to perform in order to retrieve the quantum state $|\psi\rangle$.

Bob has not touched his system at all, but learning something about his qubit allows him to change the state of the qubit. In this situation it is natural to assume that there is no instantaneous physical process that changes the state of the system, Bob is merely updating his knowledge about the qubit. Again, this strongly suggests that the ψ-epistemic interpretation of the quantum state is the natural one.

Finally, the measurement process—where we learn something about the state of a system—is a discontinuous and non-deterministic process, while the unitary evolution determined by the Schrödinger equation is continuous and deterministic. Is it not strange that nature relies on two types of evolution, especially when the mechanism for the collapse of the wave function is not specified at all? On the other hand, wave function collapse as an update of our knowledge is not mysterious, but natural!

We now present a simple argument by Pusey, Barrett, and Rudolph (2012) (PBR), which shows that a straightforward ψ-epistemic interpretation is a problematic view of the quantum state. First, we set up an experiment: we have two sources that can each create a qubit in the state $|0\rangle$ or $|+\rangle$. As usual, we define

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \,. \tag{10.9}$$

The qubits that are sent to the detector can therefore be in the states

$$|0,0\rangle \,, \quad |0,+\rangle \,, \quad |+,0\rangle \,, \quad \text{or} \quad |+,+\rangle \,. \tag{10.10}$$

The detector is set up to measure a special two-qubit observable with eigenstates

$$|\text{not } 00\rangle = \frac{1}{\sqrt{2}}|0,1\rangle + \frac{1}{\sqrt{2}}|1,0\rangle \,, \tag{10.11a}$$

$$|\text{not } 0+\rangle = \frac{1}{\sqrt{2}}|0,-\rangle + \frac{1}{\sqrt{2}}|1,+\rangle \,, \tag{10.11b}$$

$$|\text{not } +0\rangle = \frac{1}{\sqrt{2}}|+,1\rangle + \frac{1}{\sqrt{2}}|-,0\rangle \,, \tag{10.11c}$$

$$|\text{not} + +\rangle = \frac{1}{\sqrt{2}}|+, -\rangle + \frac{1}{\sqrt{2}}|-, +\rangle\,. \tag{10.11d}$$

These states form an orthonormal basis, and can therefore be used to create a valid observable. As the labels of the state indicate, a measurement outcome corresponding to the state in Eq. (10.11a) can happen only when the sources did *not* produce $|0, 0\rangle$, the state in Eq. (10.11b) indicates that the sources did *not* produce $|0, +\rangle$, and so on:

$$\Pr(\text{not } 00|00) = |\langle \text{not } 00|00\rangle|^2 = 0\,, \tag{10.12a}$$

$$\Pr(\text{not } 0+|0+) = |\langle \text{not } 0+|0+\rangle|^2 = 0\,, \tag{10.12b}$$

$$\Pr(\text{not } +0|+0) = |\langle \text{not } +0|+0\rangle|^2 = 0\,, \tag{10.12c}$$

$$\Pr(\text{not } ++|++) = |\langle \text{not } ++|++\rangle|^2 = 0\,. \tag{10.12d}$$

This is what quantum mechanics predicts we would find in the experiment.

To relate this experiment to what "really" happens in nature, we assume that the quantum states $|0\rangle$ and $|+\rangle$ merely describe our knowledge about the underlying states of reality λ. In this description, the physical system is "really" in a state λ, independent of what we know about it or which observable we decide to measure. Each preparation procedure that creates a qubit in the state $|0\rangle$ will then result in some probability distribution $\mu_0(\lambda)$ over the states of reality λ. Similarly, the preparation procedure for $|+\rangle$ will result in a probability distribution $\mu_+(\lambda)$ over the same states of reality λ. These probabilities may overlap such that for some states of reality λ both $\mu_0(\lambda)$ and $\mu_+(\lambda)$ are nonzero. Alternatively the distributions may not overlap, such that any λ can give rise only to a nonzero $\mu_0(\lambda)$ or a non-zero $\mu_+(\lambda)$, but not both. Each λ then uniquely determines the state $|0\rangle$ or $|+\rangle$. However, the difference between the quantum state and the probability distribution over the states of reality is then merely a matter of words; the quantum state is just a shorthand for the possible real states λ, and can be considered a complete description of reality, leading to a ψ-complete interpretation. Therefore, ψ-epistemic interpretations imply distributions $\mu_0(\lambda)$ and $\mu_+(\lambda)$ with some overlap (see Fig. 10.3).

In this light, let us consider the above experiment again with a ψ-epistemic interpretation. When we create the state $|0\rangle$, there is a probability $p > 0$ that the actual state of reality λ lies in the overlap region where both $\mu_0(\lambda) > 0$ and $\mu_+(\lambda) > 0$. This implies directly that such a state could have been produced in either the $|0\rangle$ or the $|+\rangle$ preparation procedure (for keeping a record of the preparation procedure in the states of reality would effectively put λ outside the region of overlap).

Next, there is a probability $p^2 > 0$ that the state preparation $|0, 0\rangle$ by the two sources results in two states of reality (one for each qubit) that both lie in the overlap region where $\mu_0(\lambda) > 0$ and $\mu_+(\lambda) > 0$. However, if both states of reality lie in the overlap region, it is impossible to know whether the state preparation procedure was in fact $|0, 0\rangle$, $|0, +\rangle$, $|+, 0\rangle$, or $|+, +\rangle$. When we measure the observable with the eigenvectors from Eqs. (10.11a)–(10.11d) the state of reality in the overlap region must occasionally produce measurement outcomes "not 00", even though the state preparation was in fact $|0, 0\rangle$. This contradicts the predictions of quantum mechanics

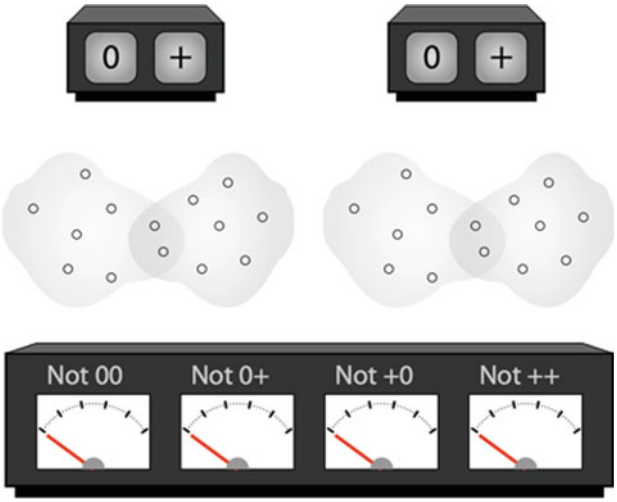

Fig. 10.3 The PBR theorem. The interactive figure is available online (see supplementary material 3)

unless $p = 0$, in which case the probability distributions $\mu_0(\lambda)$ and $\mu_+(\lambda)$ do not overlap. We argued above that non-overlapping distributions over the real states is not compatible with a ψ-epistemic interpretation of the quantum state, and we therefore have to seriously question its viability.

We should not be too hasty, though. This argument rests on two key assumptions. First, a physical system needs to have an objective real state ψ, independent of the observer. The argument depends crucially on it. Second, independently prepared systems have independent real states: we must assume that there is no grand conspiracy in the preparation of the two qubits, such that a preparation of $|0, 0\rangle$ will never trigger the outcome "not 00". These both seem natural assumption, and it is hard to see how we can save the ψ-epistemic interpretation without adding severe complications in the objective states of reality and how they interact with the rest of the universe.

10.3 Nonlocality

One of the problems in coming up with a realistic interpretation of the quantum state is how to deal with the probabilistic character of measurement outcomes. Any attempt at removing the probabilistic nature of quantum mechanics by introducing hidden variables (in a ψ-supplemented interpretation) inevitably leads to these hidden variables being nonlocal. To see how this comes about, we give a simple example, originally devised by Lucien Hardy (1993).

Alice and Bob, who are far apart from each other, each hold one qubit. They both have the choice of measuring their qubit either in the basis $\{|0\rangle, |1\rangle\}$ or in the basis

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad \text{and} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}\,, \tag{10.13}$$

corresponding to the eigenstates of the Pauli matrices σ_z and σ_x, respectively. For example, the qubits can be electrons with spin $1/2$, and Alice and Bob each have a Stern-Gerlach apparatus that they can orient in the z- or the x-direction.

We set up the state of the two qubits held by Alice and Bob in such a way that they never find the following joint measurement outcomes

$$\begin{aligned} &\text{Alice measures } \mathsf{1} \quad \text{and} \quad \text{Bob measures } +\,, \\ &\text{Alice measures } + \quad \text{and} \quad \text{Bob measures } \mathsf{1}\,, \\ &\text{Alice measures } - \quad \text{and} \quad \text{Bob measures } -\,. \end{aligned}$$

We can express these outcomes with the probabilities

$$\Pr(\mathsf{1}, +) = 0\,, \tag{10.14a}$$
$$\Pr(+, \mathsf{1}) = 0\,, \tag{10.14b}$$
$$\Pr(-, -) = 0\,. \tag{10.14c}$$

Later, we will see which specific quantum state of the two qubits will give these probabilities.

Before that, however, we expect that these probabilities allow us to infer that a measurement outcome "$\mathsf{1}$" by Alice implies that if Bob measures σ_z, he must find measurement outcome "$\mathsf{0}$". To see this, we note first that according to Eq. (10.14a), if Alice finds outcome "$\mathsf{1}$", Bob must find outcome "$-$" in a measurement of σ_x:

$$\mathsf{1}_A \underset{(10.14a)}{\longrightarrow} -_B\,. \tag{10.15}$$

There are only two measurement outcomes for Bob's σ_x measurement, namely "$+$" and "$-$", and Eq. (10.14a) rules out the outcome "$+$".

Second, if Bob finds outcome "$-$", then according to Eq. (10.14c), Alice should always find outcome "$+$" in a σ_x measurement:

$$-_B \underset{(10.14c)}{\longrightarrow} +_A\,. \tag{10.16}$$

Finally, if Alice finds outcome "$+$", then according to Eq. (10.14b), Bob should always find outcome "$\mathsf{0}$" in a σ_z measurement:

$$+_A \underset{(10.14b)}{\longrightarrow} \mathsf{0}_B\,. \tag{10.17}$$

Therefore, we have the chain of inference

$$1_A \underset{(10.14a)}{\longrightarrow} -_B \underset{(10.14c)}{\longrightarrow} +_A \underset{(10.14b)}{\longrightarrow} 0_B\,, \tag{10.18}$$

In other words, the probability that Alice and Bob both find the measurement outcome "1" is zero:

$$\Pr(1,1) = 0\,. \tag{10.19}$$

This is a perfectly reasonable conclusion when we assume that the properties of each qubit completely determine the outcomes of the measurements that are performed on it.

Next, we show how we can achieve the probabilities given in Eqs. (10.14a)–(10.14c) in quantum mechanics. We prepare the two qubits held by Alice and Bob in the quantum state

$$|\psi\rangle = \frac{3|00\rangle + |01\rangle + |10\rangle - |11\rangle}{\sqrt{12}}\,. \tag{10.20}$$

You can verify that the probabilities are indeed

$$\begin{aligned}
\Pr(1,+) &= |\langle 1+|\psi\rangle|^2 = 0\,,\\
\Pr(+,1) &= |\langle +1|\psi\rangle|^2 = 0\,,\\
\Pr(-,-) &= |\langle --|\psi\rangle|^2 = 0\,,
\end{aligned} \tag{10.21}$$

as required. However, when we calculate the probability that Alice and Bob both find the measurement outcome "1", we are in for a surprise:

$$\Pr(1,1) = |\langle 11|\psi\rangle|^2 = \frac{1}{12}\,. \tag{10.22}$$

This directly contradicts our previous argument in Eq. (10.19) that $\Pr(1,1) = 0$, which assumed that each qubit has properties that locally determine the outcomes of the measurements. Therefore, this assumption must be false!

The implications of this are profound: You may believe that the measurement outcomes for a quantum mechanical system are determined by extra hidden variables in order to "save" determinism in nature. To keep the predictions of quantum mechanics for our example—in particular Eq. (10.22)—the hidden variables for Alice's qubit must be influenced by the hidden variables of Bob's qubit, and vice versa. However, Alice and Bob can be arbitrary far apart, and can make their measurements at any time they like. This means that the influence of the hidden variables must travel faster than light! In this sense, quantum mechanics is often said to be nonlocal. Any hidden variable model must be nonlocal in order to reproduce the predictions of quantum mechanics. Moreover, these predictions are in fact verified in many experiments, and it therefore seems that nature really is nonlocal!

A note of caution: These results are often presented as contradicting Einstein's theory of relativity, which says that nothing can travel faster than light. However, quantum mechanics does not allow you to send signals faster than light. Any attempt to do so will ultimately fail due to the probabilistic nature of the measurement outcomes. The faster-than-light behaviour is restricted to the hidden variables, which are an addition to quantum mechanics, and not directly accessible to experimenters.

In 1964, John Bell proposed a very general proof that any hidden variable theory that reproduces the predictions of quantum mechanics *must* be nonlocal (Bell 1987). That is, even though it is still impossible to come up with schemes that allow faster than light communication, the hidden variables themselves must influence each other instantaneously. Bell derived a series of inequalities—now called Bell's inequalities—that relate the statistics of the possible measurement outcomes of an experiment in such a way that their violation demonstrates the nonlocal character of hidden variable models. To demonstrate a violation of Bell's inequalities, one has to make measurements on two entangled particles that lie outside of each other's light cone (to ensure no slower than light communication takes place). The first demonstration of such a violation for spatially separated measurements was carried out by Alain Aspect (1982) and his co-workers in 1982. They measured the polarisation of two photons coming from a central caesium source. A possible loophole could still be found in the detection efficiencies of the photodetectors. This loophole was closed by David Wineland's group (Rowe et al. 2001) in 2001, but without the spatial separation. In 2015, Ronald Hanson's group (Hensen et al. 2015) demonstrated the violation of Bell's inequality with spatially separated measurements, while simultaneously closing the detection loophole. These experiments conclusively show that nature cannot be accurately described by local realistic theories. This is one of the most profound insights into the nature of reality that science has delivered in its long history.

10.4 Contextuality

We have seen that quantum mechanics is nonlocal, even though it does not allow us to exploit this for sending messages faster than light. Another way in which quantum mechanics is strange already shows up in a single system, and is called *contextuality*.

Before we give an example of contextuality in quantum mechanics, we consider a classical form of contextuality that was proposed by John Wheeler (1990). The group game of Twenty Questions is played as follows: one person—the player, let's call him Patrick—temporarily leaves the group, while the rest determine a secret word. When Patrick returns, he may ask up to twenty questions that can be answered with either "yes" or "no". Patrick will address the questions to individual members of the group. For example, the group chose the word "steamboat", and Patrick asks the first group member, Alice: "Is it a person"? Alice will answer "no". Patrick then

addresses Bob with the next question, and so on. If Patrick can guess the secret word using twenty questions or less, he wins. Otherwise the group wins.

At the next instance of the game, while a new player—Patricia—is waiting outside the room, the group feels mischievous and decides not to choose a word. Instead, each group member silently chooses a private word and will answer "yes" or "no" based on the word they have chosen. Every group member will have a different word in mind! Patricia first asks Alice: "Is it a person?", to which the answer is "yes". Bob had chosen the word "tree", which is clearly not a person. He therefore has to update his secret word such that it conforms to Alice's answer. Each successive group member will have to make sure that their private word conforms to all previous answers by the other members. Of course, this quickly becomes fiendishly difficult for the group members! Assume that Patricia wins with her final guessed word. It is unlikely that this word was thought of by any of the group members at the start, since everybody probably had to update their secret word at least once during the questions stage. So the final word was determined not only by the group members (as in the traditional way the game is played), but also by Patricia's choice of questions. The final outcome therefore depends not only on the choice of the group alone, but also on the context provided by the questions.

To see how this relates to quantum mechanics, we first set up a mathematical puzzle similar to Twenty Questions. We have five statements about a situation, $A_1, \ldots, A_5$, and use them to construct the following four joint questions, which all happen to have the answer "no":

$$\text{Are } A_1 \text{ and } A_2 \text{ both true?} \rightarrow \text{ No}, \tag{10.23a}$$

$$\text{Are } A_2 \text{ and } A_3 \text{ both false?} \rightarrow \text{ No}, \tag{10.23b}$$

$$\text{Are } A_3 \text{ and } A_4 \text{ both true?} \rightarrow \text{ No}, \tag{10.23c}$$

$$\text{Are } A_4 \text{ and } A_5 \text{ both false?} \rightarrow \text{ No}. \tag{10.23d}$$

We can naturally assume that the statements $A_1, \ldots, A_5$ each have pre-determined truth values (true or false, or "yes" or "no"), just like in the original game of Twenty Questions, and we construct the following chain of reasoning:

$$\text{If } A_1 \text{ is true} \underset{(10.23a)}{\longrightarrow} A_2 \text{ is false}, \tag{10.24a}$$

$$\text{If } A_2 \text{ is false} \underset{(10.23b)}{\longrightarrow} A_3 \text{ is true}, \tag{10.24b}$$

$$\text{If } A_3 \text{ is true} \underset{(10.23c)}{\longrightarrow} A_4 \text{ is false}, \tag{10.24c}$$

$$\text{If } A_4 \text{ is false} \underset{(10.23d)}{\longrightarrow} A_5 \text{ is true}, \tag{10.24d}$$

Therefore, we know the answer to the following question with certainty: If A_1 is true then A_5 is true, so

$$\text{Is } A_1 \text{ true and } A_5 \text{ false?} \rightarrow \text{ No}. \tag{10.25}$$

This is what we expect if the statements $A_1, \ldots, A_5$ are individually true or false. If this game was played by the Twenty Questions group (the non-evil version), all members would agree on the truth values of the statements $A_1, \ldots, A_5$, and the answer to the player's question (10.25) must indeed be "no."

Next, we implement this experiment with a quantum mechanical system. Each experimental question is of the form "Are A_1 and A_2 both true?" or "Are A_4 and A_5 both false?", and corresponds to a different observable with a yes or no answer. To calculate the probability that A_1 is true given the quantum state $|\psi\rangle$, we first identify $|A_1\rangle$ as the state of the quantum system with the property that A_1 is true. The probability that A_1 is true is then given by the Born rule:

$$\Pr(A_1 \text{ is true}) = |\langle A_1|\psi\rangle|^2 = \langle\psi|P_1|\psi\rangle\,, \tag{10.26}$$

where $P_1 = |A_1\rangle\langle A_1|$ is the projector onto the state $|A_1\rangle$ (see Sect. 5.5). Similarly, we can calculate the probability that A_1 is false via

$$\begin{aligned}\Pr(A_1 \text{ is false}) &= 1 - \Pr(A_1 \text{ is true}) \\ &= 1 - \langle\psi|P_1|\psi\rangle \\ &= \langle\psi|(\mathbb{I} - P_1)|\psi\rangle\,,\end{aligned} \tag{10.27}$$

since A_1 is either true or false whenever we ask. Here, $\mathbb{I}$ is the identity operator. Measuring A_1 (that is, asking if A_1 is true or false) then amounts to the quantum state jumping into the eigenstate $|A_1\rangle$—indicating that A_1 is true—or in a state orthogonal to $|A_1\rangle$, indicating that A_1 is false. Finding that A_1 is true is therefore equivalent to saying that the quantum state is projected onto $|A_1\rangle$, by virtue of the projection postulate.

We set up a similar reasoning for A_2, and we associate the state "A_2 is true" with a quantum state $|A_2\rangle$ such that

$$\Pr(A_2 \text{ is true}) = |\langle A_2|\psi\rangle|^2 = \langle\psi|P_2|\psi\rangle \tag{10.28}$$

and

$$\Pr(A_2 \text{ is false}) = \langle\psi|(\mathbb{I} - P_2)|\psi\rangle\,. \tag{10.29}$$

To calculate the probability that both A_1 and A_2 are true, the observables must be compatible. This means that we must be able to ask the question "is A_1 true" and "is A_2 true" simultaneously in an experimental setting. Since a measurement of an observable leads to a state vector collapse into one of the eigenstates of the observable, the eigenstates for the observable A_1 and A_2 must be the same.

Therefore the projectors P_1 and P_2 must commute:

$$[P_1, P_2] = P_1P_2 - P_2P_1 = 0\,. \tag{10.30}$$

The probability that A_1 and A_2 are *both* true can then be calculated as

$$\Pr(A_1, A_2 \text{ both true}) = \langle\psi|P_1 P_2|\psi\rangle\,. \tag{10.31a}$$

When $|A_2\rangle$ is perpendicular to $|A_1\rangle$ we find that $P_1 P_2 = 0$, and the statement "A_1 and A_2 are both true" is always false, regardless of the quantum state $|\psi\rangle$.

We can follow this procedure also for the statements A_3, A_4 and A_5 with respective quantum states $|A_3\rangle$, $|A_4\rangle$, $|A_5\rangle$ and projectors P_3, P_4 and P_5. We then find that

$$\Pr(A_2, A_3 \text{ both false}) = \langle\psi|(\mathbb{I} - P_2)(\mathbb{I} - P_3)|\psi\rangle \tag{10.31b}$$

$$\Pr(A_3, A_4 \text{ both true}) = \langle\psi|P_3 P_4|\psi\rangle\,, \tag{10.31c}$$

and

$$\Pr(A_4, A_5 \text{ both false}) = \langle\psi|(\mathbb{I} - P_4)(\mathbb{I} - P_5)|\psi\rangle\,. \tag{10.31d}$$

This means that A_2 and A_3 must also be compatible observables, since we establish a joint truth value of A_2 and A_3 in Eq. (10.31b). Therefore P_2 and P_3 must commute: $[P_2, P_3] = 0$. Similarly, Eq. (10.31c) tells us that $[P_3, P_4] = 0$, and Eq. (10.31d) gives $[P_4, P_5] = 0$. However, just because $[P_1, P_2] = 0$ and $[P_2, P_3] = 0$ does not mean that $[P_1, P_3] = 0$, since we are not making joint statements about A_1 and A_3. The same is true for any other pair of observables that we do not wish to measure simultaneously. We will construct projectors with these commutation relations shortly.

First, let's summarise the probabilities that we have so far:

$$\Pr(A_1, A_2 \text{ both true}) = 0\,, \tag{10.32a}$$

$$\Pr(A_2, A_3 \text{ both false}) = 0\,, \tag{10.32b}$$

$$\Pr(A_3, A_4 \text{ both true}) = 0\,, \tag{10.32c}$$

$$\Pr(A_4, A_5 \text{ both false}) = 0\,. \tag{10.32d}$$

We saw in the classical case that this leads us to believe that

$$\Pr(A_1 \text{ true and } A_5 \text{ false}) = 0\,, \tag{10.32e}$$

as long as $A_1, \ldots, A_5$ have predetermined values. These predetermined values are the physical properties of the system that exist independent of any measurements of the system. Equation (10.32e) implies that $[P_1, P_5] = 0$.

Next, let's see what quantum mechanics makes of this. An explicit example of the game was proposed by Adàn Cabello (2013) and co-workers, in which we consider a three-level quantum system in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\,. \tag{10.33}$$

For example, the quantum system may be an atom with three energy levels, a particle with spin 1, or a photon that has the choice of three paths. It is convenient to solve this problem in vector form instead of kets, because the vectors are real and three-dimensional, so they are easy to visualise.

The eigenstates corresponding to the eigenvalue "true" for the five statements $A_1, \ldots, A_5$ are chosen as

$$|A_1\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad |A_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \tag{10.34}$$

$$|A_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad |A_4\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |A_5\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \tag{10.35}$$

with projectors $P_1 = |A_1\rangle\langle A_1|$, $P_2 = |A_2\rangle\langle A_2|$, and so on. Using matrix multiplication, you can verify that

$$P_1 = |A_1\rangle\langle A_1| = \frac{1}{3} \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \tag{10.36a}$$

$$P_2 = |A_2\rangle\langle A_2| = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{10.36b}$$

$$P_3 = |A_3\rangle\langle A_3| = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{10.36c}$$

$$P_4 = |A_4\rangle\langle A_4| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{10.36d}$$

$$P_5 = |A_5\rangle\langle A_5| = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \tag{10.36e}$$

You should also check that the projectors pairwise commute according to our requirements above:

$$[P_1, P_2] = 0\,, \qquad [P_2, P_3] = 0\,, \qquad [P_3, P_4] = 0\,, \tag{10.37}$$

$$[P_4, P_5] = 0\,, \qquad \text{and} \qquad [P_5, P_1] = 0\,. \tag{10.38}$$

You can calculate the projectors $\mathbb{I} - P_2$ and $\mathbb{I} - P_3$ by using the matrix form of $\mathbb{I}$:

$$\mathbb{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{10.39}$$

Now we are finally ready to calculate the probabilities in Eqs. (10.31a)–(10.31d), and indeed we find that they are zero. For example,

$$P_1 P_2 = \frac{1}{6} \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{10.40}$$

and therefore $\langle \psi | P_1 P_2 | \psi \rangle = 0$. Similarly, $(\mathbb{I} - P_2)(\mathbb{I} - P_3)$ can be calculated as

$$(\mathbb{I} - P_2)(\mathbb{I} - P_3) = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{10.41}$$

and the probability $\langle \psi | (\mathbb{I} - P_2)(\mathbb{I} - P_3) | \psi \rangle = 0$. You can calculate the other probabilities in a similar way.

Finally, we calculate the probability of A_1 being true and A_5 being false: $\langle \psi | P_1 (\mathbb{I} - P_5) | \psi \rangle$. This should be zero according to the classical thinking about physical properties that lead to Eq. (10.25). First, we determine

$$P_1 (\mathbb{I} - P_5) = P_1 - P_1 P_5 = P_1 , \tag{10.42}$$

since $P_1 P_5 = 0$. The probability $\Pr(A_1 \text{ true and } A_5 \text{ false})$ is then given by the expectation value

$$\Pr(A_1 \text{ true and } A_5 \text{ false}) = \langle \psi | P_1 (\mathbb{I} - P_5) | \psi \rangle = \langle \psi | P_1 | \psi \rangle = \frac{1}{9}. \tag{10.43}$$

This is in contradiction to the earlier expected result that $\Pr(A_1 \text{ true and } A_5 \text{ false}) = 0$ if every proposition $A_1, \ldots, A_5$ is separately true or false. In other words, the propositions do not have separate truth values, and much like Wheeler's Twenty Questions, the outcome of the measurement is determined by the questions that you ask. In other words, whether $A_1, \ldots, A_5$ are real properties of the system depends on the context of the measurements that are being made! The geometric construction of this argument is shown in Fig. 10.4.

The contextuality of quantum mechanics was proved in 1967 by Simon Kochen and Ernst Specker, and it is now called the Kochen–Specker theorem (1967). It is an extremely important result in the foundations of quantum mechanics. It proves that it is impossible to assign definite values to all physical properties of a quantum

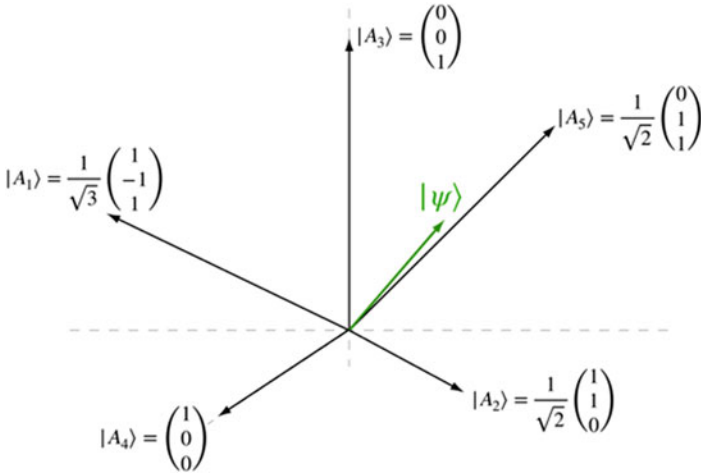

Fig. 10.4 Contextuality in three dimensions. The interactive figure is available online (see supplementary material 4)

system prior to a measurement without changing the experimental predictions. Since experiments have come out in favour of quantum mechanics, this means that you cannot say that quantum systems have properties in and of themselves. We can now appreciate Wheeler's original quote from Chap. 1 at an even deeper level:

No phenomenon is a physical phenomenon until it is an observed *phenomenon*

Our act of making a measurement—asking a question—brings physical properties into reality.

10.5 A Compendium of Interpretations

So how do we interpret the state of a physical system in quantum mechanics? Over the past century a great many attempts have been made to answer this question, and physicists and philosophers of physics have come up with all sorts of clever, crazy, outlandish and plausible solutions. At this point, no single interpretation of quantum mechanics clearly stands out as the generally accepted interpretation (although there are some front runners), and in the remaining pages of this book we will have a look at some of the most important ones at the time of writing. Detailed descriptions of these interpretations are given in the online Stanford Encyclopedia of Philosophy, and I have included footnotes to the various articles throughout.

10.5.1 The Copenhagen Interpretation

In the years immediately following the development of quantum mechanics, Niels Bohr and Werner Heisenberg (among many others) grappled with the meaning of the new theory. Their thinking resulted in a body of writing that we now call the Copenhagen interpretation.[1] At various points in time, the proponents of this interpretation have given contradictory statements as to what the interpretation entails, and as a result there is no clear unambiguous definition of the Copenhagen position on the quantum state.

Nevertheless, the key features of the Copenhagen interpretation seem to be that the wave function, or the state of a quantum system, is not a real property of the system, but a theoretical construction that we use to calculate measurement outcomes. The quantum state can therefore be classified as ψ-epistemic. There is a fundamental separation between the quantum and the classical world, where our direct experience and our measurement devices are described by the classical theories of physics. The microscopic quantum world is described by the quantum state, and somewhere between the quantum system and the measurement device the collapse of the wave function occurs. Historically, this prompted the thought experiment about Schrödinger's cat.

In addition to the separation between the classical and the quantum world, Bohr introduced the Complementarity Principle (Bohr 1928) that must be invoked in the Copenhagen interpretation to explain how quantum objects can sometimes behave as particles, and other times as waves. In Bohr's own words:

> *Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects.*

This principle provides an explanation of both de Broglie's wave-particle duality and Heisenberg's Uncertainty Principle, and also hints at the discovery of contextuality forty years later.

The Correspondence Principle says that for large systems the quantum description approaches the classical description. It is somewhat doubtful that this principle holds generally. For example, the aim of building a quantum computer is to create large systems that behave entirely differently from classical systems, thus violating the Correspondence Principle.

The Copenhagen interpretation is difficult to make philosophically precise while at the same time keeping to the key aspects that Bohr and Heisenberg put forward. One such attempt is the consistent histories interpretation, which specifies sets of questions about a quantum system that have consistent answers that do not lead to logical contradictions. In this interpretation, decoherence in the measuring device is enough to account for wave function collapse.

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[1] See *Copenhagen Interpretation of Quantum Mechanics*, in: Stanford Encyclopedia of Philosophy, Fall 2008 edition.

10.5.2 Quantum Bayesianism

Another refinement of the Copenhagen interpretation is Quantum Bayesianism (Mermin 2014). To understand this philosophical position, we must first get to grips with regular Bayesianism. In this book we have talked about probabilities as if we know exactly what they mean. Indeed, we can measure probabilities by repeating an experiment a large number of times, and record the relative frequencies of the measurement outcomes. For example, if we measure the spin of an electron in the z-direction for N identically prepared spin states $a|\uparrow\rangle + b|\downarrow\rangle$, we expect to obtain the relative frequencies $f_\uparrow$ and $f_\downarrow$ defined by

$$f_\uparrow = \frac{N_\uparrow}{N} \simeq |a|^2 \quad \text{and} \quad f_\downarrow = \frac{N_\downarrow}{N} \simeq |b|^2 \,, \tag{10.44}$$

when N becomes very large. Here $N_\uparrow$ and $N_\downarrow$ are the number of times we measure $\uparrow$ and $\downarrow$, respectively. However, $f_\uparrow$ and $f_\downarrow$ merely approach the probabilities $|a|^2$ and $|b|^2$ as $N \to \infty$, they are not the same. As a result, we cannot define the probabilities as the relative frequencies. The question then becomes: what is a probability?

Bayesianism answers this question in the following way: A probability is a number assigned to a particular event by an agent (e.g., you or me), which expresses the degree of belief of the agent that such an event will happen. For example, I can say that the probability of rain today is 60%. This is an expression of my belief about the weather, and weather forecasters make statements like this all the time. This notion of probability does not require that an experiment is repeatable a large number of times. If we require that the rules for assigning and updating the values of the probabilities must be internally consistent, we recover the standard rules of probability theory!

Quantum Bayesianism adopts this view of probabilities, and interprets quantum mechanics as the theory that describes the degree of belief in the measurement outcomes according to an agent, such as an experimentalist. It does not say anything about the ontology of quantum systems, and is therefore a ψ-epistemic theory of the first kind. As we saw before, such a theory does not suffer from the interpretational problems in quantum mechanics, but at the same time lacks the explanatory power that we traditionally require from our theories about nature.

10.5.3 Quantum Logic

The problematic aspects of quantum mechanics such as nonlocality and contextuality (as described in the previous two sections) rely on specific chains of inference in Eqs. (10.18) and (10.25). But what if logic itself is not valid in is ordinary form in quantum mechanics?

This notion is perhaps not as ridiculous as it may seem at first. Remember that geometry started as a branch of mathematics when Euclid formulated its fundamental

rules in the third century BC. It turned out much later that there are many different geometries that are all mathematically consistent. The question is then: what is the geometry of the space around us? Consequently, geometry ceases to be a purely mathematical exercise, and becomes branch of physics with experimentally testable predictions. Not long after this realisation in the 19th century, general relativity showed that the geometry of space can change due to physical processes involving mass and energy.

We can apply the same argumentation to logic. Perhaps quantum mechanics can be understood if we use different logical rules, i.e., different rules of inference. Our everyday logic is then a bit like Euclid's geometry, while quantum logic is analogous to non-Euclidean geometry. We call this new type of logic quantum logic. It must replace some rules of classical logic with quantum versions and still form a consistent mathematical structure. On top of that, it must solve the conceptual puzzles of quantum mechanics. In most versions of quantum logic, the mathematical rule that is abandoned is the distributive law[2]:

$$A \text{ AND } (B \text{ OR } C) = (A \text{ AND } B) \text{ OR } (A \text{ AND } C)\,. \tag{10.45}$$

The standard example goes as follows: we make three statements A, B, and C about a particle,

$$\begin{aligned} A &: \text{the momentum is in the interval } [0, \hbar/4\ell]\,, \\ B &: \text{the position is in the interval } [-\ell, 0]\,, \\ C &: \text{the position is in the interval } [0, \ell]\,, \end{aligned}$$

where ℓ is some length. From statement A we see that the uncertainty in momentum is $\Delta p = \hbar/4\ell$, and from the statement B OR C we deduce that the uncertainty in position is $\Delta x = 2\ell$. Their product is therefore

$$\Delta p \Delta x = \frac{\hbar}{2}\,, \tag{10.46}$$

which obeys the uncertainty relation for position and momentum, and is therefore allowed by quantum mechanics. Consequently, A AND (B OR C) can indeed be a true statement. However, the right-hand side of Eq. (10.45) can never be true. To see this, note that statements B and C both imply $\Delta x = \ell$, and the clauses A AND B and A AND C both lead to

$$\Delta p \Delta x = \frac{\hbar}{4}\,. \tag{10.47}$$

[2]If you are not very familiar with formal logic, you can think of this as the logical equivalent of the distributive property of numbers: $x(y+z) = xy + xz$, where multiplication plays the role of AND, and addition plays the role of OR.

This is not allowed by the uncertainty relation. Therefore, the left-hand side of Eq. (10.45) can be true, while the right-hand side must be false. Hence, the distributive law fails in quantum logic (Birkhoff and von Neumann 1936), and has to be abandoned.

What exactly we should put into place depends on the particular version of quantum logic you wish to consider. It is safe to say that quantum logic is not an intuitive interpretation of quantum mechanics, and it is currently studied mostly for its mathematical structure, and less for its philosophical resolution of the conceptual problems in quantum mechanics.

10.5.4 Objective Collapse Theories

Objective collapse theories[3] embrace the projection postulate, which says that the quantum state collapses after a measurement. These theories are ψ-complete, in that the quantum state is considered real, and add to quantum mechanics a physical mechanism that describes how the collapse happens.

One of the most well-known objective collapse theories is the version by Ghirardi, Rimini, and Weber (1986) (GRW). It says that in addition to the standard evolution in quantum mechanics, particles very occasionally collapse their quantum state spontaneously. This happens on average once every hundred million years, so for individual particles this behaviour is never observed in practice. However, when we consider a large object that consists of 10^{23} particles, the spontaneous collapse of a few particles in the object triggers a cascade that causes the quantum state of the entire object to behave classically. The interaction of a small quantum system with a naturally collapsing, large measurement device is then enough to explain Schrödinger's cat: The cat is a macroscopic system, and will naturally decohere due to the spontaneous collapse mechanism of the GRW theory. In fact, the decoherence will already take place in the Geiger counter, since that is also a macroscopic object. The uneasy conclusion of the Schrödinger's cat paradox about counterintuitive macroscopic quantum superpositions is then circumvented.

Another well-known collapse theory is due to Roger Penrose (1996), who also views the quantum state as a real physical quantity. He notes that any massive particle in a superposition state of two or more locations creates a superposition of spacetime curvature, since Einstein taught us that mass causes curvature in space. Penrose argues that there is an energy cost associated with creating a superposition of spacetime, and if this energy becomes too big the system will spontaneously collapse into a lower energy system. This is a natural mechanism for the collapse of the wave function to a localised state. It explains how very light particles can be in a superposition, following the predictions of quantum mechanics, and why much heavier objects behave in a classical way.

[3] See *Collapse Theories*, in: Stanford Encyclopedia of Philosophy, Fall 2008 edition.

These interpretations make predictions that are different from standard quantum mechanics. They put a limit on the size of the systems that are put in quantum superpositions, and this opens the way to experimental tests of the interpretation. Such experiments have been proposed, and are currently being implemented.

10.5.5 The de Broglie-Bohm Interpretation

The de Broglie-Bohm interpretation (Bohm 1952) is also known as the pilot-wave theory, or Bohmian mechanics (after David Bohm), and it is a true hidden-variable theory (the only one in our list). As such, it is a ψ-supplemented interpretation of quantum mechanics.

In the de Broglie-Bohm interpretation, the wave function $\psi(x,t)$ is considered a "pilot wave" that follows Schrödinger's equation, and which guides the trajectory of the particle. The hidden variable is the actual trajectory $x(t)$ of the particle, which obeys a separate equation from the Schrödinger equation. By claiming that a particle has an actual trajectory, the problem of the indeterminism of position and momentum in quantum mechanics goes away. However, the price we pay for this is two-fold: First, we must solve an extra, often complicated equation for $x(t)$, such as

$$m\frac{d}{dt}x(t) = \hbar\frac{d}{dx}\mathrm{Im}\left[\ln\psi(x,t)\right] . \tag{10.48}$$

This means that we need to do extra work in Bohmian mechanics, compared to standard quantum mechanics. Second, the trajectory $x(t)$ at time t in Eq. (10.48) depends on the pilot wave $\psi(x,t)$ at the same time, and there is therefore an instantaneous influence of ψ from all positions x at time t at the actual trajectory. This makes the theory highly nonlocal. The de Broglie-Bohm interpretation is historically important, because it prompted Bell to investigate the relationship between locality and hidden variables.

10.5.6 Modal Interpretations

We know from the previous section on contextuality that we cannot assign actual properties to a quantum mechanical system without running into serious contradictions, known as the Kochen-Specker theorem. Modal interpretations (Dieks and Vermaas 1998) circumvent this theorem by assigning only part of all possible properties to the system. In other words, modal interpretations say that certain physical quantities have real values (they are properties of the system), while other physical quantities do not. It is easy to see that the chain of reasoning in Eq. (10.43), which leads to the contradiction with quantum mechanics, breaks down if we deny that one or more statements $A_1, \ldots, A_5$ do not have a definite truth value. The question then

becomes: how do we choose which statements have definite truth values and which do not?

Different modal interpretations give different answers to this question. They have in common that they all accept the standard formalism of quantum mechanics as we have developed it here, with the exception of the projection postulate, and with the addition of a property ascription rule. Modal interpretations therefore assign a certain reality to the quantum state without introducing hidden variables, and they can be classified as ψ-complete interpretations.

An example of a property ascription rule (in the Kochen-Dieks-Healey interpretation) is to look for the joint quantum state of the system S and the measurement device M. We can write this state in a unique way as a superposition of system states and measurement device states:

$$|\Psi\rangle = \sum_j d_j |\psi_j\rangle_S |M_j\rangle_M \,, \tag{10.49}$$

where the index j runs from 1 to the dimension of the system. This is called the Schmidt decomposition. A theorem in linear algebra ensures that we can always find such a state. Next, we choose the states $|\psi_j\rangle$ as the eigenstates associated with the statements A_j that have real property values, along with statements that follow logically from these statements. For example, if A_1 and A_2 have definite property values, then (A_1 AND A_2) and (A_1 OR A_2) also have definite values. The interaction between the measurement device and the system therefore picks out the physical quantities that are the actual properties of the system. This construction ensures that the Kochen-Specker theorem can no longer be derived, while every measurement reveals a real property of the system.

In modal interpretations, quantum mechanics applies to all physical systems, large and small, and all measurements are ordinary physical interactions. In particular, there is no need for a "collapse of the wave function".

10.5.7 The Many Worlds Interpretation

In 1957, Hugh Everett (1957) proposed a new interpretation of quantum mechanics that takes the quantum state at face value and assigns reality to each term in a quantum superposition. This means that an electron with spin in the quantum state $a|\uparrow\rangle + b|\downarrow\rangle$ has two real branches, or "worlds", one where the spin is $\uparrow$, and one where the spin is $\downarrow$. There are therefore as many real worlds as there are terms in a superposition (these are sometimes called parallel universes). No extra rules are required in this interpretation of quantum mechanics, and it is therefore ψ-complete.

The many worlds interpretation (also known as the Everett interpretation) traditionally elicits strong reactions, so it is important to give a good argument why this interpretation is to be taken seriously. Imagine that you want to send a signal to both Alice and Bob. You can achieve this by encoding the signal in the amplitude of a

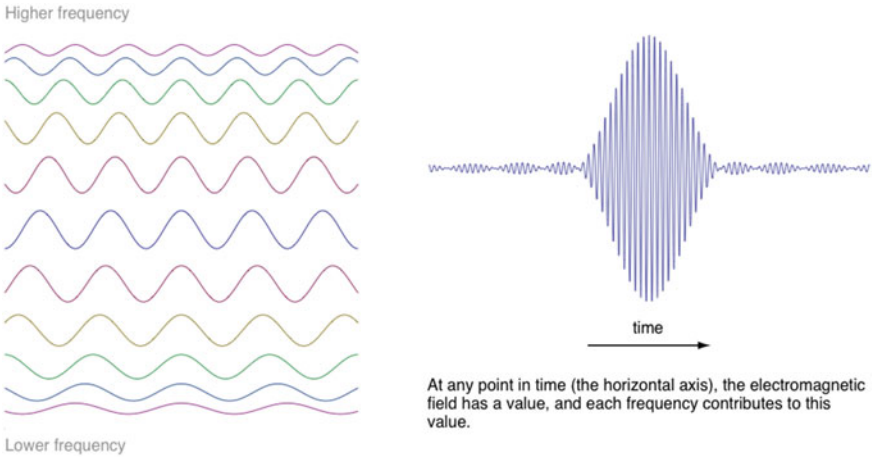

Fig. 10.5 A localised wave packet as a superposition of plane waves. The interactive figure is available online (see supplementary material 5)

laser beam, and divide the beam with a 50:50 beam splitter. The two outgoing beams are directed to Alice and Bob. At this point it is uncontroversial to say that both beams are real physical objects. Next, we slowly attenuate the intensity of the beam, making the signal weaker. Alice and Bob keep receiving the weaker signals, and we continue to regard them as real. Finally, we attenuate the laser so much that to a very good approximation there will be at most one photon in each pulse. The state of the photon can be written as

$$|\psi\rangle = \frac{|1,0\rangle + |0,1\rangle}{\sqrt{2}}\,, \tag{10.50}$$

where $|1,0\rangle$ denotes a photon moving towards Alice, and $|0,1\rangle$ denotes a photon moving towards Bob. If in the classical limit of a bright laser beam both signals to Alice and Bob are real, then in the quantum limit the two terms $|1,0\rangle$ and $|0,1\rangle$ should also both be real, since attenuation is a gradual process. Otherwise, at what point does the beam stop being real? However, Alice and Bob only ever see a single photon in either Alice's or Bob's detector. This means that the there are two "worlds" that are in some sense equally real, but in which different things happen.

The many worlds interpretation embraces the reality of each term in the quantum state, but it immediately raises an important question: we can always decompose a quantum state into a different basis, so which decomposition leads to the real branches? To answer this, consider a similar (but unproblematic) situation in classical electrodynamics. We can construct a wave packet of light by superposing waves of different frequencies (shown in Fig. 10.5). Depending on the choice of measurement (frequency or time-of-arrival measurements) we can decompose the wave packet in different ways. The real underlying electromagnetic field does not exhibit any

ambiguity. Moreover, every frequency component is part of the wave packet and is therefore "real" (you can make them visible using a prism), but so is the amplitude of the wave packet at every point in time. It is in this sense that the different branches of a superposition of frequencies are considered real.

The same happens with quantum mechanics in the many worlds interpretation: the quantum state $|\psi\rangle$ is unique and unambiguous and real just like the optical wave packet, but only when we wish to describe it in terms of measurements do we need to resolve the ambiguity of the measurement and the corresponding basis. This is resolved by the interaction Hamiltonian with the corresponding decoherence. Just like all the frequencies exist in the wave packet of the electromagnetic field, all terms in the superposition of a quantum state exist. Each branch is called a different "world", but you see that there is actually only one universe with a multiplicity of instantiations.

An often-heard objection to the many worlds interpretation is that introducing all those worlds violates energy conservation. The standard response is that energy conservation applies only within each world, and not across what may be an infinite number of worlds. Similarly, it is said that the many worlds interpretation is immune to Ockham's razor, since the number of assumptions in the interpretation is actually very small, even if the number of worlds aren't. For example, it does not require any hidden variables or collapse mechanisms that are not already present in standard quantum mechanics.

10.5.8 Relational Quantum Mechanics

In relational quantum mechanics (Rovelli 1996), the quantum state is an expression of the correlations between the quantum system and an observer. This observer may be alive and conscious, or it may be another physical system such as a macroscopic measurement device or even another microscopic quantum system. In this interpretation, all physical systems are quantum systems, and there is no fundamental distinction between the classical and the quantum world. A measurement is a physical interaction like any other, and there are no hidden variables in this interpretation.

In relational quantum mechanics, the basic elements of reality are the so-called "actualisations" of properties as they come about in the measurement by an observer. We can see that this is in some sense plausible when we look at Schrödinger's cat: the classical states of the cat, $|\text{dead}\rangle$ and $|\text{alive}\rangle$, are singled out by the interaction Hamiltonian in Eq. (10.7). An interaction Hamiltonian always requires at least two parties: the system of interest (the cat) and the system it interacts with (for example us, when we open the box). The interaction Hamiltonian describes the physical process, and it is unique for any given interaction. Since the actualisations are defined in the interaction between the system and an observer, there are no absolute, observer-independent properties of quantum systems.

An immediate consequence of relational quantum mechanics is that two observers, Alice and Bob, can give different but equivalent descriptions. For example, Alice may measure the spin of an electron and find the measurement outcome $\uparrow$:

$$a|\uparrow\rangle + b|\downarrow\rangle \;\rightarrow\; |\uparrow\rangle\,, \tag{10.51}$$

while Bob, who describes the joint system of the electron spin and Alice, gives the following account:

$$(a|\uparrow\rangle + b|\downarrow\rangle)|A_0\rangle \;\rightarrow\; a|\uparrow\rangle|A_\uparrow\rangle + b|\downarrow\rangle|A_\downarrow\rangle\,. \tag{10.52}$$

Here, $|A_0\rangle$ is the initial state of Alice, and $|A_\uparrow\rangle$, $|A_\downarrow\rangle$ the state of Alice when she measured spin $\uparrow$ or $\downarrow$, respectively. These two descriptions are equally correct in relational quantum mechanics, and we just need to remember who the observer is when we choose which description we should use. Note that this is exactly what we did when we described the state of the teleported qubit according to Alice and Bob, earlier in Sect. 10.2.

In relational quantum mechanics it is meaningless to refer to the absolute, observer-independent state of any system. This echoes the situation in special relativity, where it is meaningless to call two events simultaneous without any reference to an observer. It also immediately solves the puzzle of Schrödinger's cat, since there is never any doubt how each "observer" (which includes us, the cat, the Geiger counter, etc.) describes the system they observe.

Alice cannot say anything about her own state, because that state is defined only relative to another observer, such as Bob or Charlie. They may interact differently with Alice, which leads to different quantum states of Alice. This impossibility of describing ones own state leads to the perceived non-unitary evolution in a measurement, since the measurement process involves both the states of the system and the observer. The quantum state of the universe is therefore a meaningless concept in this interpretation (unless you can find an observer that does not belong to this universe). Relational quantum mechanics is often compared with Everett's relative state description (see the previous section on the many worlds interpretation), but differs in a crucial way: where Everett assigns an element of reality to each term $|\uparrow\rangle$ and $|\downarrow\rangle$ in Eq. (10.51), thus positing parallel worlds in which an electron has incompatible spin values $\uparrow$ and $\downarrow$, relational quantum mechanics assigns either $\uparrow$ or $\downarrow$ as the real property of the electron *as seen be the observer who measures the spin observable* S_z.

10.5.9 Other Interpretations

The above-mentioned interpretations tend to have strong defenders and detractors. In addition to this short list, there are numerous other interpretations of quantum mechanics, such as cosmological interpretations, statistical interpretations, time-

symmetric theories, transactional interpretations, and so on. Often, these interpretations share key aspects of the ones described here, and differ in subtle but important parts.

All good interpretations of quantum mechanics must stand up to detailed philosophical scrutiny regarding plausibility and internal consistency. It is currently not known if these considerations single out a unique interpretation, or leave open a family of possible interpretations. One thing to be wary of is quantum mysticism. There is no factual reason to believe that quantum mechanics has anything to do with religion, Eastern or otherwise. Likewise, interpretations that rely on some form of consciousness are inherently problematic, since we do not have a good understanding of what consciousness really is in the first place. Invoking one unknown to explain another is bad philosophy and bad physics.

If all these interpretations have left your head spinning, you are not alone. The philosophy of quantum mechanics is notorious for its many subtleties. The good news is that you do not need an interpretation of quantum mechanics in order to use it successfully. If you lack the patience to keep track of the various ways in which interpretations of quantum mechanics make statements about reality, you can always heed the advice of David Mermin (1989), and *just shut up and calculate!*

Exercises

1. Going back to your answer to the first exercise of Chap. 1 (What is the purpose of physical theories?), does quantum mechanics fulfil the requirements implied by your answer there?
2. The Pauli matrices σ_x, σ_y and σ_z can be considered observables, since they have real eigenvalues $+1$ and -1. Consider two qubits in some quantum state, and the following array of possible observables:

$$\begin{array}{lll} \sigma_x^{(1)}\sigma_x^{(2)} & \sigma_x^{(1)}\mathbb{I}^{(2)} & \mathbb{I}^{(1)}\sigma_x^{(2)} \\ \sigma_z^{(1)}\sigma_z^{(2)} & \mathbb{I}^{(1)}\sigma_z^{(2)} & \sigma_z^{(1)}\mathbb{I}^{(2)} \\ \sigma_y^{(1)}\sigma_y^{(2)} & \sigma_x^{(1)}\sigma_z^{(2)} & \sigma_z^{(1)}\sigma_x^{(2)} \end{array}$$

   where the superscript indicates the qubit the matrix operates on.

   (a) Show that the three operators in every column and every row commute, and can therefore be measured simultaneously.
   (b) Show that each operator has eigenvalues ± 1.
   (c) Using the matrix form of the Pauli matrices, show that the operators in each row and column multiply to $+\mathbb{I}$, except the first column, which multiplies to $-\mathbb{I}$.
   (d) Assuming that each entry in the array has a pre-determined value of ± 1, try to populate the array such that the conditions in part (c) are satisfied. What do you conclude?

3. Alice and Bob, positioned far away from each other, each receive an electron that is part of the entangled spin state

$$|\Psi^-\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\,,$$

where $\uparrow$ and $\downarrow$ define the positive and negative z-axis. Alice can freely choose to measure her electron spin in one of two directions: in the z-direction (denoted by $\mathbf{a}$) and in the direction an angle θ_A away from the z-axis towards the x-axis (denoted by $\mathbf{a}'$). Similarly Bob can freely choose to measure his electron spin in the z-direction (denoted by $\mathbf{b}$) and in the direction an angle θ_B away from the z-axis towards the x-axis (denoted by $\mathbf{b}'$).

(a) Calculate the probabilities $\Pr(a = \pm, b = \pm)$ of finding the $\pm\hbar/2$ eigenvalues in all possible measurement directions $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$ and $\mathbf{b}'$.

(b) Define $P(\mathbf{a}, \mathbf{b})$ as the expectation value over the ± 1 values of $\mathbf{a} \cdot \mathbf{b}$:

$$\begin{aligned} P(\mathbf{a}, \mathbf{b}) = &\Pr(a = +, b = +) - \Pr(a = +, b = -) \\ &- \Pr(a = -, b = +) + \Pr(a = -, b = -)\,. \end{aligned}$$

What are the minimum and maximum values that $P(\mathbf{a}, \mathbf{b})$ can take?

(c) Without superluminal signalling between Alice and Bob, the expectation values obey the so-called CHSH inequality

$$|P(\mathbf{a}, \mathbf{b})| + |P(\mathbf{a}, \mathbf{b}')| + |P(\mathbf{a}', \mathbf{b})| - |P(\mathbf{a}', \mathbf{b}')| \leq 2\,,$$

which is a special case of a *Bell inequality*. Show that the probabilities you calculated in part (a) violate this expression when $\theta_{\mathbf{a}} = 0$, $\theta_{\mathbf{a}'} = \pi/2$, $\theta_{\mathbf{b}} = \pi/4$, and $\theta_{\mathbf{b}'} = 3\pi/4$. What do you conclude?

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