7 Decisions and Consumer Behavior

This chapter covers ...

- the concepts of preferences and utility functions and how these are related.
- how the assumption that individuals maximize preferences can be used to determine the individual demand functions on a competitive market.
- the strengths and weaknesses of this approach as a foundation of choice and decision-making in general and the structure of demand functions specifically.

7.1 Basic Concepts

> The theory of Economics must begin with a correct theory of consumption. (William Stanley Jevons)

Individual and market demand are the consequences of decisions made by individuals. Until now I have taken a shortcut and skipped a more detailed analysis of the way individuals make decisions, because I wanted to keep the focus on the functioning of markets. For that purpose, it was sufficient to heuristically explain how prices, income and other factors influence demand. However, this shortcut's cost is preventing one from developing a deeper understanding of the structure of individual and market demand. Additionally, the way I related the idea of Pareto efficiency and the demand function was also pretty clumsy.

Reduced to its essential core, economic decision theory is very simple: one assumes that individuals choose the best alternative from a set of admissible alternatives. In a market context, the admissible alternatives are the goods and services a consumer can afford, given prices and income. It is more difficult to model the meaning of individuals choosing the best alternative, though. This chapter is devoted to making these ideas precise and to seeing how they can help us to gain a better understanding of market behavior and of behavior in general.

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7.1.1 Choice Sets and Preferences

✍ In order to develop a decision theory, one needs two conceptual ingredients. First, a set of alternatives from which an individual can choose. Call it a *choice set* and denote it by $X = \{x^1, x^2, \ldots, x^n\}$, in which $x^i, i = 1, \ldots, n$ is one of the possible alternatives and assume, for simplicity, that the total number of alternatives n is finite. The idea of a choice set is very general. If one goes to a café, one's choice set is a subset of all of the items on the menu. This implies that an alternative can be a list of individual items, like "one cup of tea, two scones and one portion of orange jam." Mathematically speaking, this type of list is called a tuple. If x^i is the above-mentioned alternative, it could be denoted as $x^i = \{$quantity of tea, number of scones, quantity of orange jam$\} = \{1, 2, 1\}$. If one goes to vote, one's choice set is the set of all admissible parties or candidates and if one is deciding what to do after high school, one's choice set is the set of all potential professions.

Second, the individual may prefer some alternatives to others, which is an expression of her taste or preferences. Assume that she is able to make pairwise comparisons of all the alternatives in X to make statements like, "I prefer alternative x^i to alternative x^j," or "I am indifferent between alternative x^i and alternative x^j." In order to have a lean notation, economists use the following symbols for these statements: "I prefer alternative x^i to alternative x^j" is denoted by "$x^i \succ x^j$" and "I am indifferent between alternative x^i and alternative x^j" by "$x^i \sim x^j$."

✍ It is important to understand the exact meaning of the terminology. Mathematically speaking, I am taking two arbitrary elements of X, x^i and x^j, and comparing them to each other. This comparison is called a *binary relation* on X. The *strict preference relation*, "$\succ$," and the *indifference relation*, "$\sim$," can therefore be denoted as a subset of the Cartesian product of X, $X \times X$. (I am slightly abusing the notation by using the symbols as names for both the relation and for indicating the binary comparison of alternatives.)

Here is an example: assume that Ann can choose between an apple, x^1, an orange, x^2, and a cherry, x^3. In this case, the choice set is equal to $X = \{x^1, x^2, x^3\}$ and the Cartesian product is the set of all ordered pairs $X \times X = \{(x^1, x^1), (x^1, x^2), (x^1, x^3), (x^2, x^1), (x^2, x^2), (x^2, x^3), (x^3, x^1), (x^3, x^2), (x^3, x^3)\}$. Assume that Ann prefers apples to oranges and is indifferent between oranges and cherries, $x^1 \succ x^2$, $x^2 \sim x^3$. If one reads a pair (x^i, x^j) as "x^i stands in relation R to x^j," one can represent her preferences, "$\succ$," by the subset of pairs $\{(x^1, x^2)\}$ and her preferences, "$\sim$," by the subset of pairs $\{(x^1, x^1), (x^2, x^2), (x^2, x^3), (x^3, x^3)\}$. Note that the pairs (x^i, x^i) are elements of the subset, because Ann is indifferent between an alternative and itself. This property is not self-evident from a purely mathematical point of view and, therefore, sometimes is stated as an assumption of the preference relation that is known as *reflexivity*.

✍ As stated, relation "$\succ$" is called the *strict preference relation* and relation $\sim$ the *indifference relation*. It turns out that it is easier to work with a third type of relation that is called the *weak preference relation*, which is denoted by "$\succsim$." It contains all of the pairs from $X \times X$ that either belong to the strict preference or the indifference relation. In this example, it is the set $\{(x^1, x^1), (x^1, x^2), (x^2, x^2), (x^2, x^3), (x^3, x^3)\}$. The strict preference and indifference relations can easily be reconstructed from the weak preference relation by the following operations:

- $(x^i \succ x^j) \Leftrightarrow (x^i \succsim x^j) \wedge \neg(x^j \succsim x^i)$,
- $(x^i \sim x^j) \Leftrightarrow (x^i \succsim x^j) \wedge (x^j \succsim x^i)$,

in which $\wedge$ and $\neg$ stand for the logical operations "and" and "not."

In order for the concepts to have predictive power, one has to make additional assumptions on the structure of the weak preference relation.

► **Assumption 7.1 (Completeness)** For every $x^i, x^j \in X$, either $x^i \succsim x^j$ or $x^j \succsim x^i$ or both are true.

Assumption 1 implies that the individual can compare any two pairs of alternatives. This assumption may sound innocuous, because it seems obvious that one should either be better off with one alternative or the other. However, critiques point out that, depending on the context, alternatives can exist that cannot be compared in a meaningful way. Think, for example, of the alternative "destruction of human life by means of nuclear weapons" and "destruction of human life by means of a lethal virus." It is argued that there is a meaningful difference between being indifferent between two alternatives and not being able to compare them. If one has to choose between alternatives whose consequences are beyond our imagination, it is not clear that an inability to compare and an indifference are the same.

► **Assumption 7.2 (Transitivity)** For every $x^i, x^j, x^k \in X$, if $x^i \succsim x^j$ and $x^j \succsim x^k$, then $x^i \succsim x^k$.

Transitivity implies that there are no "cycles" in the relation. The main justification for this assumption stems from the so-called "money-pump" argument, which rests on the idea that a person with intransitive preferences can be exploited by some other person. In order to understand this, assume that there is a "cycle" $x^i \succ x^j \succ x^k \succ x^i$ and that the individual is willing to pay at least one cent for the next best alternative. In that case, she would be willing to give up x_i plus a small amount of money in exchange for x_k, x_k plus a small amount of money in exchange for x_j and – attention money pump – x_j plus a small amount of money in exchange for x_i. Now she is back where she started, with the exception that the individual has lost three cents. Continuing this process would, in the end, separate the individual from all her money.

However, a lot of empirical experiments have shown that transitivity cannot be taken for granted. Here is an example. Procrastination describes the tendency to delay uncomfortable duties until later. A tendency to procrastinate may have very adverse consequences and the intransitivity of inter-temporal preferences seems to be playing an important role. This is why: assume that it is Monday and you have a report due on Thursday. Overall, you would like to hand in a high-quality report.

However, starting to work on Monday is less preferable to starting to work on Tuesday ("you know, I had a stressful day anyway"). However, when Tuesday comes, it is preferable to delay and start working on Wednesday ("I need the pressure to get things done"). However, from Wednesday's perspective it seems better to delay another day ("well, I simply cannot do it"). However, on Thursday it is too late to prepare and hand in a report of decent quality.

A weak preference relation that fulfills Assumptions 1 and 2 is also called a *preference ordering*. What do they imply in this little example? One already knows that Ann's preferences are $x^1 \succsim x^2$ (because $x^1 \succ x^2$ and $\succsim$ is a weaker condition than $\succ$) and $x^2 \succsim x^3$ (because $x^2 \sim x^3$ and $\succsim$ is weaker than $\sim$). Completeness is guaranteed by assumption (there are only three alternatives in the example) and transitivity implies that one can infer $x^1 \succsim x^3$ from $x^1 \succsim x^2 \succsim x^3$. Hence, the completed preference ordering is given by $\{(x^1, x^1), (x^1, x^2), (x^1, x^3), (x^2, x^2), (x^2, x^3), (x^3, x^3)\}$.

Completeness and transitivity are usually taken for granted in almost all economic applications. However, depending on the specific context that is analyzed additional assumptions have to be imposed. I list three of them in the following paragraphs.

► **Assumption 7.3 (Continuity)** For any $x^i \in X$ the set of all $x^j \in X$ is such that $x^i \succsim x^j$ and the set of all $x^k \in X$ is such that $x^k \succsim x^i$ are closed sets in X.

Continuity is less obvious from an economic point of view, but it still has some intuitive plausibility. It implies that the preference relation does not "jump" in the following sense. Assume that an individual is comparing two alternatives, x^1 and x^2, and she weakly prefers x^1 to x^2, $x^1 \succsim x^2$. For example, if one modifies x^1 a tiny bit to $x^1 + \epsilon$, in which ϵ is a very small quantity, then the preference ordering does not suddenly reverse, $x^1 \succsim x^2 \Rightarrow x^1 + \epsilon \succsim x^2$.

► **Assumption 7.4 (Monotonicity)** For any $x^i, x^j \in X$, $x^i \geq x^j$ and $x^i \neq x^j$ implies that $x^i \succ x^j$.

Assumption 4 needs a few words of clarification. The specification of X is completely general: elements can be arbitrarily complex or very simple alternatives. However, in some cases the alternatives can be quantitatively measured and compared, for example the quantity of a good like milk. In that case, x^i could be two liters of milk and x^j one liter. In all of these cases, an expression like "$x^i \geq x^j \wedge x^i \neq x^j$" makes sense. It makes no sense, however, to compare smartphones with ice cream. Assumption 4 is, therefore, only applicable for those alternatives that can be measured and quantified on an absolute scale. It then implies that the individual prefers larger quantities to smaller quantities.

► **Assumption 7.5a (Convexity)** For any $x^i, x^j \in X$, such that $x^i \succsim x^j$ and for all $t : 0 \leq t \leq 1$, it follows that $t \cdot x^i + (1 - t) \cdot x^j \succsim x^j$.

► **Assumption 7.5b (Strict Convexity)** For any $x^i, x^j \in X$, such that $x^i \sim x^j$ and for all $t : 0 < t < 1$, it follows that $t \cdot x^i + (1-t) \cdot x^j \succ x^i$.

Assumptions 7.5a and 7.5b are similar in spirit. What they imply is that individuals prefer balanced over extreme alternatives. However, in order to illustrate this idea, one has to restrict one's attention to the alternatives that are quantifiable and measurable in the same way as one has assumed in Assumption 7.4.

Assumptions 7.5a and 7.5b will play an important role in the theory of consumer choice on competitive markets, which is why it makes sense to discuss them in greater detail. Assume that the alternatives from which the individual can choose are quantities of two different goods, like bread and water. Denote two alternatives by $x^1 = \{10, 0\}$ and $x^2 = \{0, 10\}$. In alternative 1 the individual gets 10 units of water and no bread, while in alternative 2 she gets no water and 10 units of bread. In this example, convexity as well as strict convexity imply that an individual would, for example, prefer the more balanced alternative $x^3 = 0.5 \cdot x^1 + 0.5 \cdot x^2 = (5, 5)$ to the extreme ones.

In the example, Assumptions 7.5a and 7.5b seem to make perfect sense, but looking at the alternative example, in which the first good is Miso soup and the second is vanilla ice cream, few people would like to eat them together.

The above assumptions are usually not all imposed simultaneously. As the theory of consumer choice on competitive markets will show, economists try to establish properties of choice behavior with minimal assumptions about preferences, because every additional assumption constrains the admissible behavior of individuals, thus making the theory less general.

One can now define the concept of rationality as used in economics. It has two different aspects. First, if individuals have a preference ordering, a well-defined subset of alternatives $X^o \subset X$ exists that defines the best or optimal alternatives given the preferences. This would not necessarily be the case, if preferences were not complete and transitive. Hence, a preference relation is called *rational*, if it is complete and transitive. Second, it is not sufficient that individuals are able to consistently order the alternatives according to their preferences; they must also *act* according to them. Hence, individual *behavior* is rational, if the individual *chooses* a best alternative given the choice set and the preference ordering. This idea of rationality is at the heart of the concept of *homo oeconomicus*.

► **Definition 7.1 (Homo Oeconomicus)** An individual behaves as homo oeconomicus, if (i) she perceives a choice situation as a choice set X, (ii) has a preference ordering over this choice set and (iii) chooses one of the best alternatives from this choice set, given her preferences.

Two statements are helpful to understand this. First, the concept of rationality is purely instrumental. It only requires that the preferences are structured in a manner that makes it possible to talk about better and worse alternatives in a meaningful way and that individuals act according to their preferences. It does not scrutinize the individual's taste or value judgments that caused her preferences. A debate that allows one to distinguish between better and worse preference orderings would build on a different concept of rationality, which is called value-based rationality. Mainstream economists accept a philosophical position called *subjectivism*, a value judgment that leads to an acceptance of all types of preference orderings. Second, note that no such thing such as selfishness enters this definition of homo oeconomicus. Selfishness is not an integral part of what economists consider rational behavior, even though selfish behavior is added as an *additional* assumption in a lot of analyses. The reason is that concepts such as selfish, altruistic, sadistic etc. preferences refer to *motives of action* and, as I have just said, mainstream economists do not scrutinize such motives but take them as given. It would, therefore, be alien to the idea of instrumental rationality, if it required any specific motive to act. (See Digression 3, Chap. 1, for a more detailed analysis of this topic.)

In order to get started with an analysis of decision making, one needs a few more concepts.

▶ **Definition 7.2 (Not-Worse-Than-x Set)** The Not-Worse-Than-x Set, for an alternative $x \in X$, $NW(x)$, is given by the set of all $x^i \in X$, such that $x^i \succsim x$.

▶ **Definition 7.3 (Not-Better-Than-x Set)** The Not-Better-Than-x Set, for an alternative $x \in X$, $NB(x)$, is given by the set of all $x^i \in X$, such that $x \succsim x^i$.

▶ **Definition 7.4 (Indifferent-To-x Set)** The Indifferent-To-x Set, for an alternative $x \in X$, $I(x)$, is given by the intersection $NW(x) \cap NB(x)$.

7.1.2 Indifference Curves

Thus far, I have introduced the concept of a preference ordering in the simplistic case of a finite set of alternatives X. However, the concepts can be readily generalized to allow for infinitely many different alternatives, which is usually done if the theory is applied to market contexts. In this case, if there are n different goods, then the choice set is a subset of the n-dimensional set of positive real numbers, $X \subset R^n_+$. In this case, one can illustrate the indifferent-to-x set by a graph. Assume that there are two goods whose quantities are represented by the two axes of Fig. 7.1.

✍ The downward-sloping graph represents the indifferent-to-x set for an alternative that one calls a consumption bundle $\bar{x} = (\bar{x}_1, \bar{x}_2)$. It is called *indifference curve*. Hence, Ann is indifferent between this consumption bundle and any other consumption bundle on the indifference curve ($\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$ and $\hat{x} = (\hat{x}_1, \hat{x}_2)$ are two examples for such bundles in the figure), $\bar{x} \sim \tilde{x} \sim \hat{x}$. Please note that if the curvature of this curve is representative for the whole preference ordering, "$\succsim$," then the ordering is both convex and strictly convex. The continuity of the curve reveals that preferences are continuous.

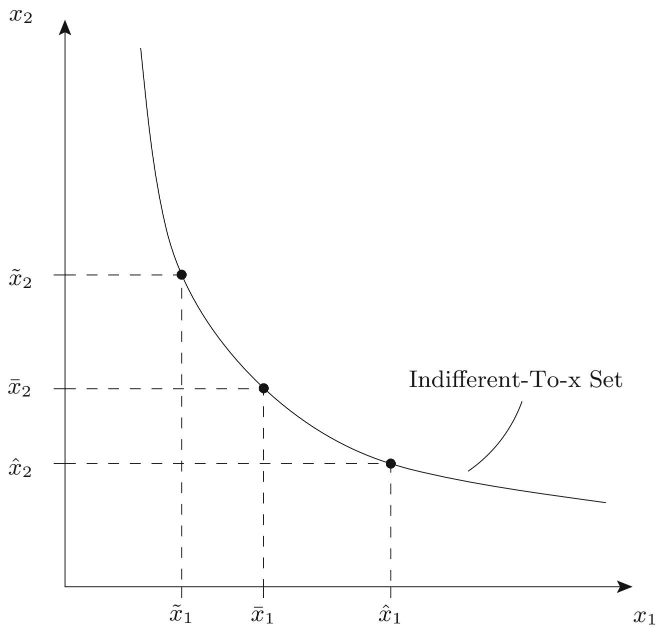

Fig. 7.1 Ann's Indifferent-To-x-Set

The indifference curve in Fig. 7.1, of course, only partially represents the individual's preference ordering. There exists an indifferent-to-x set for every consumption bundle x that can in principle be represented by an indifference curve.

The slope of an indifference curve has an important economic interpretation. Suppose that one not only wants to reallocate the consumption goods but also wants to ensure that the individual is neither better nor worse off. This is only possible if one chooses consumption bundles that lie on the same indifference curve. Now, suppose that at some point x one takes $dx^2 < 0$ away from the individual. Given that the indifference curve is downward sloping, one has to compensate the individual by some extra quantity, $dx^1 > 0$, to ensure that one stays on the indifference curve. See Fig. 7.2 for an illustration of this.

If one looks at infinitesimal changes, $dx^2 \to 0$, then the exchange rate between the two goods is given by the slope of the tangent to the indifference curve at the point $\bar{x}$. The absolute value of this exchange rate, dx^2/dx^1, is called the *marginal rate of substitution* (MRS) between good 2 and good 1. It is an expression of the idea of opportunity costs in the context of the individual's decision problem: if one takes a little bit of one good away, how much of the other good does one have to give the individual to make her indifferent?

Figure 7.3a–d illustrate the shape of indifference curves for different types of preference orderings.

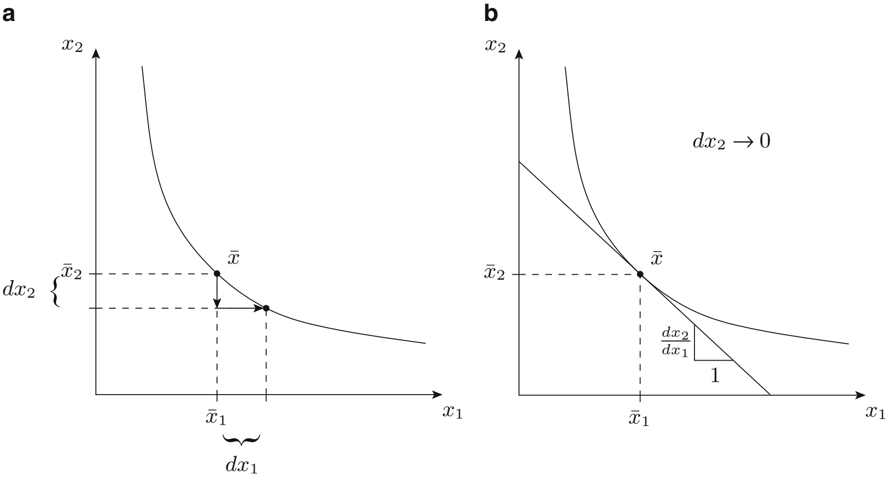

Fig. 7.2 Marginal rate of substitution

Figure 7.3a illustrates so-called *perfect substitutes*. Indifference curves are straight, parallel lines. The outward-pointing arrow indicates that the individual prefers larger quantities to smaller ones (monotonicity). If indifference curves are straight lines, then the MRS is independent of the consumption bundle. This means that the individual is always willing to substitute one good for the same quantity of the other good, hence the name perfect substitutes. Whether two goods are perfect substitutes to each other or not ultimately depends on the perception of the individual, but plausible examples are different brands of toothpaste, yoghurt, shoes, etc. Perfect substitutes are preference orderings that fulfill continuity, monotony and convexity, but not strict convexity.

Figure 7.3b illustrates so-called *perfect complements*. Indifference curves are L-shaped with a kink. L-shaped indifference curves imply that the individual wants to consume the two goods in a fixed ratio. This fixed ratio is given by the slope of the straight line through the origin that connects the kinks. Examples could be left and right shoes (which is why they are sold as pairs), computer hard- and software, coffee and cream, etc. Perfect complements are preference orderings that fulfill continuity, monotony and convexity, but not strict convexity.

Figure 7.3c illustrates strictly convex preferences. Indifference curves bend inwards, but not as extremely as it does for perfect complements. Perfectly convex preferences are somewhere in between perfect substitutes and perfect complements. An individual with such preferences is willing to substitute one good for the other, but has a *ceteris paribus* preference for more balanced bundles.

Finally, Fig. 7.3d illustrates another type of strictly convex preferences, however, with a point of saturation. As the arrows indicate, such preferences are not monotonic, because a globally optimal consumption bundle exists. If consumption falls short of this point, then increasing it makes the individual better off. If consumption

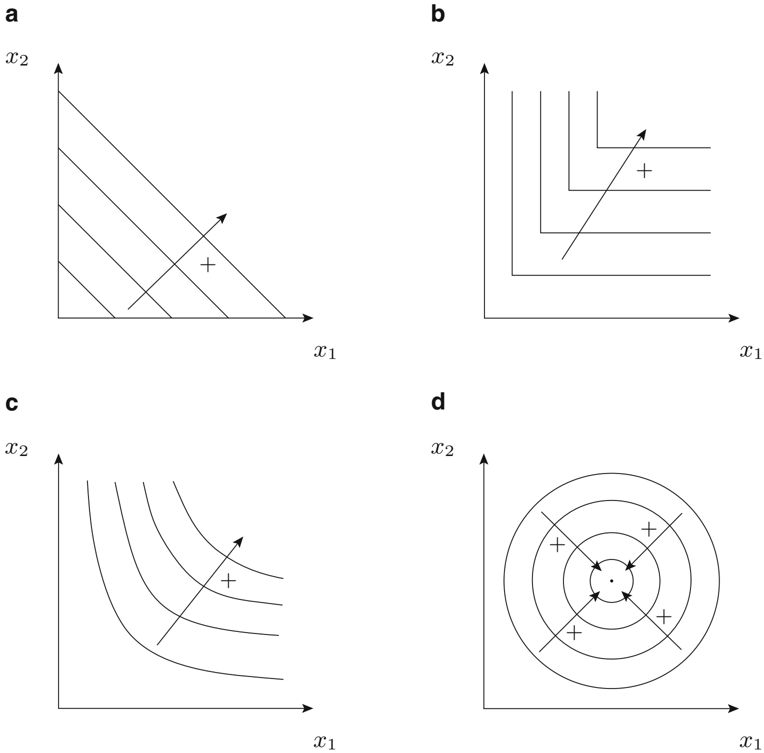

Fig. 7.3 Indifference curves for different preference orderings. **a** Perfect substitutes, **b** perfect complements, **c** strictly convex and **d** strictly convex preferences

exceeds this point, then the individual is better off if she can reduce consumption. Preferences like these are plausible in situations in which goods are not storable and there are physical limits to consumption. Think of ice cream as an example: the first scoop is very good, the second still good, the third is ok, but a fourth, fifth, or sixth scoop makes you sick. It is important to note, however, that if it were possible to produce goods in quantities such that individuals are on or beyond their points of satiation (and the excess can be freely disposed of), society would have overcome scarcity. Hence, the assumption that economics is the science that studies the allocation of scare goods and services implies that one implicitly assumes that one is *not* beyond these points of satiation, either because no such points exist (monotonicity), or because our technological means to production are insufficient to reach these points for all goods in X. In this latter case, however, the indifference curves in Fig. 7.3d look qualitatively similar to the indifference curves in Fig. 7.3c.

7.1.3 Utility Functions

✍ The decision problem of an individual can be completely analyzed by the use of the concept of preference orderings. However, it has turned out that it is sometimes more convenient to represent an ordering by a function, because it allows one to use different and more standard tools from mathematics. This kind of a functional representation of a preference ordering is called a *utility function*. This subchapter will first introduce the concept and then describe some of the potential pitfalls and misunderstandings that come with it.

Economists use the following convention when they represent preference orderings by a function $u(x)$, in which x refers to an arbitrary alternative that can itself be a tuple. They assume that the function assigns a larger number to strictly preferred alternatives, $x^i \succ x^j \Leftrightarrow u(x^i) > u(x^j)$, and the same number to indifferent alternatives, $x^i \sim x^j \Leftrightarrow u(x^i) = u(x^j)$. Any function that meets these requirements qualifies as a utility representation, $u(x)$, of a preference ordering "$\succsim$." More formally, this means:

► **Definition 7.5 (Utility Function)** A function $u : X \to \mathbb{R}$ is called a utility function for a preference ordering "$\succsim$" if and only if $x^i \succ x^j \Leftrightarrow u(x^i) > u(x^j)$ and $x^i \sim x^j \Leftrightarrow u(x^i) = u(x^j)$ for all $x^i, x^j \in X$.

This definition of a utility representation or function leaves a lot of freedom when assigning numbers to alternatives or, to put it differently, a given preference ordering has not only one utility representation but many. Here is an example: assume that an individual must choose from a choice set $X = \{x^1, x^2, x^3\}$ and has preferences $x^1 \succ x^2 \succ x^3$. In this case, the following three assignments of numbers to alternatives u_A, u_B, u_C are all utility representations of this preference ordering: $u_A : u_A(x^1) = 3, u_A(x^2) = 2, u_A(x^3) = 1$, $u_B : u_B(x^1) = 354, u_B(x^2) = 7.65, u_B(x^3) = 0$, $u_C : u_C(x^1) = -1, u_C(x^2) = -2, u_C(x^3) = -3$. However, Function D does not represent the preference ordering: $u_D : u_D(x^1) = 3, u_D(x^2) = 1, u_D(x^3) = 2$, because it assigns a larger number to the worst alternative x^3 rather than to the second-best alternative x^2 (2 compared to 1).

An implication of this definition of a utility function is that the absolute values that it assigns to alternatives are meaningless. By the same token, the differences in utility levels for different alternatives are meaningless, as well. The only thing that counts is that preferred alternatives are assigned larger numbers. This is why it is called an *ordinal* concept (absolute values and cardinal differences have no economic meaning).

An immediate implication of this concept is summarized with the following result: assume that $u : X \to \mathbb{R}$ is a utility representation of preference ordering "$\succsim$" and assume that $f : \mathbb{R} \to \mathbb{R}$ is a monotonic and increasing function. In that case, the composite function $v = f \circ u$ is also a utility representation of "$\succsim$." In order to show this, I assume that $u : X \to \mathbb{R}$ is a utility representation of "$\succsim$," which implies, by the definition of a utility function, that

$$u(x^i) > u(x^j) \Leftrightarrow x^i \succ x^j \quad \wedge \quad u(x^i) = u(x^j) \Leftrightarrow x^i \sim x^j.$$

If $f(x)$ is a monotonic increasing function, then one knows that

$$f(u(x^i)) > f(u(x^j)) \Leftrightarrow u(x^i) > u(x^j) \quad \wedge$$
$$f(u(x^i)) = f(u(x^j)) \Leftrightarrow u(x^i) = u(x^j).$$

However, this implies that

$$f(u(x^i)) > f(u(x^j)) \Leftrightarrow x^i \succ x^j \quad \wedge \quad f(u(x^i)) = f(u(x^j)) \Leftrightarrow x^i \sim x^j,$$

And, thereby, that

$$v(x^i) > v(x^j) \Leftrightarrow x^i \succ x^j \quad \wedge \quad v(x^i) = v(x^j) \Leftrightarrow x^i \sim x^j.$$

The transfer from preference orderings to utility functions bears some risk of misinterpretation. Because utility functions assign numbers to alternatives, it is tempting to use these numbers and perform all types of operations with them, like calculating differences ($u(x^i) = 10, u(x^j) = 7$, hence $u(x^i) - u(x^j) = 10 - 7 = 3$ and thus the individual must be three units better off) and comparing them between different individuals (individual A has 8 units of utility, whereas individual B only has 3 units, which makes individual A 5 units better off than individual B). These calculations are mathematically well defined, but economically meaningless, because absolute values of utility or differences in utilities have no meaning if the underlying, primary concept is a preference ordering. What remains, however meaningful, is the marginal rate of substitution MRS, because it is independent of the exact utility representation used. To see this, return to the two representations used above, $u(x)$ and $v(x) = f(u(x))$, and use the following notation: alternative x^i consists of the quantities x_1^i and x_2^i of the two goods 1 and 2. One can express the marginal rate of substitution dx_2^i/dx_1^i by the total differential of the utility function. One can start with the representation $u(x)$ to get the total differential

$$du = \frac{\partial u}{\partial x_1^i} dx_1^i + \frac{\partial u}{\partial x_2^i} dx_2^i.$$

If one wants to stay on the same indifference curve, one has to set $du = 0$, which implies that

$$\frac{dx_2^i}{dx_1^i} = -\frac{\partial u/\partial x_1^i}{\partial u/\partial x_2^i}.$$

For infinitesimal changes in the quantities of the goods, the marginal rate of substitution is equal to the inverse ratio of marginal utilities $\partial u/\partial x_i^k, k = 1, 2$. If one does the same exercise with the representation $v(.)$ instead of $u(.)$, one gets

$$\frac{dx_2^i}{dx_1^i} = -\frac{\partial v/\partial x_1^i}{\partial v/\partial x_2^i} = -\frac{(\partial f/\partial u)(\partial u/\partial x_1^i)}{(\partial f/\partial u)(\partial u/\partial x_2^i)} = -\frac{\partial u/\partial x_1^i}{\partial u/\partial x_2^i}.$$

The MRS is independent of the utility representation that is used. It is the same, irrespective of the exact utility function used, as long as it represents the underlying preference ordering. Hence, the MRS is an economically meaningful concept, because it is a property of the preference ordering, which itself is an explanatory element of the theory.

Digression 17. What Do Preferences and Utility Functions Stand for? The Development of the Modern Concept of Preference Orderings
The view on the concept of utility has gone through substantial changes over the past 100 years or so. What unifies all interpretations is the assumption that individual behavior is somehow related to individual well-being. Initially, economists used the term utility as a proxy for what is called *hedonic* well-being. This position was put forward by *utilitarian* philosophers, like Jeremy Bentham or John Stuart Mill. Mill wrote: "The creed which accepts as the foundation of morals, Utility, or the Greatest-Happiness Principle, holds that actions are right in proportion as they tend to promote happiness, wrong as they tend to produce the reverse of happiness. By happiness is intended pleasure, and the absence of pain; by unhappiness, pain, and the privation of pleasure." Therefore, these philosophers had a specific understanding of what is now called the *theory of mind* and a substantive claim as to what promotes happiness: feeling good. Both the brain and the mind were conceptualized as pleasure- and pain-generating machines and these feelings were considered to be the exclusive motivators for behavior. According to this view, a utility function is a measure for hedonic pleasure (higher utility = more (pleasure minus pain), lower utility = less (pleasure minus pain)) and – together with the assumption that pleasure motivates behavior – is therefore a highly stylized theory of mind. This view of utility was pretty much in line with the leading paradigm of psychology of the time. Psychologists like Gustav Theodor Fechner or Wilhelm Wundt were convinced that mental processes could be measured and compared.

At the turn of the century, however, this view was increasingly scrutinized. The idea that mental phenomena could be measured was mocked as "metaphysical *hocus pocus*," the paradigm in psychology shifted towards what is today called behaviorism and economics followed swiftly. One of the main proponents was Vilfredo Pareto, who wrote in a letter in 1897: "It is an empirical fact that the natural sciences have progressed only when they have taken secondary principles as their point of departure, instead of trying to discover the essence of things. [. . .] Pure political economy has therefore a great interest in relying as little as possible on the domain of psychology." He replaced the concept of measurable and comparable utility with the concept of an ordinal preference ordering and even went a step further by suggesting that one should not think of a preference ordering as something that summarizes what is going on in the mind or brain, but as a mere *as-if*-device that allows one to explain behavior without giving it a deeper meaning.

However, Pareto kept a minimal theory of mind by assuming that alternatives that individuals rank higher in their preference ordering are better for them (given their own subjective standard). This assumption led to the idea of what is today called *Pareto efficiency* as a normative criterion (see Chap. 5 for the definition).

This concept of preferences and the associated idea that utility functions have no deeper ontological meaning beyond representing preferences led to the development of economic analysis of individual behavior on the basis of indifference curves by Edgeworth and it was perceived as liberating at the time. The enthusiasm can still be sensed in the following quote (Eugen Slutsky (1915/1952)): "[I]f we wish to place economic science upon a solid basis, we must make it completely independent of psychological assumptions [. . .]." In the wake of this enthusiasm, economics also developed from a rather narrow science of market behavior to a one-size-fits-all tool in an attempt to understand society at large (John Hicks and Douglas Allen 1934): "The methodological implications of [the new] conception of utility [. . .] are far reaching indeed. By transforming the subjective theory of value into a general logic of choice, they extent its applicability over wide fields of human conduct."

There is one issue remaining before I can move on to applying the theory of preference orderings in order to better understand the market behavior of consumers. Up until this point the assumption has been that preference orderings can be represented by a utility function, but this is far from obvious. In fact, there is a counterexample that is not too far off the mark when it comes to human behavior. Assume that a consumer who has the choice between two goods, x_1 and x_2, has the following preferences: she prefers more of good 1 to less of good 1 and the same for good 2, but for every quantity of good 1 and, irrespective of the quantity of good 2 that she could consume, she prefers more of good 1. These preferences are called *lexicographic*, because the individual orders the quantities of the goods in the same way as a lexicon orders entries: it defines a hierarchy that gives priority of the first letter over the second, the second letter over the third and so on. Only in the event of a tie in the first letter, the second letter becomes relevant and so on. Figure 7.4 illustrates this case.

Here is an example: assume the consumer has the choice between three alternatives $x^1 = (1, 1)$, $x^2 = (1, 100)$, $x^3 = (2, 1)$. With lexicographic preferences, the consumer prefers x^2 to x^1 (more of good 2) and x^3 to x^2 (more of good 1, in which good 2 does not matter as soon as there is more of 1).

Lexicographic preferences may seem rather special and they probably are, but one cannot exclude them from consideration without knowing what people really want. However, the problem with these preferences is that they cannot be represented by a utility function. Understanding the deeper reason for this odd result requires some knowledge in measure theory. Intuitively speaking, there are not

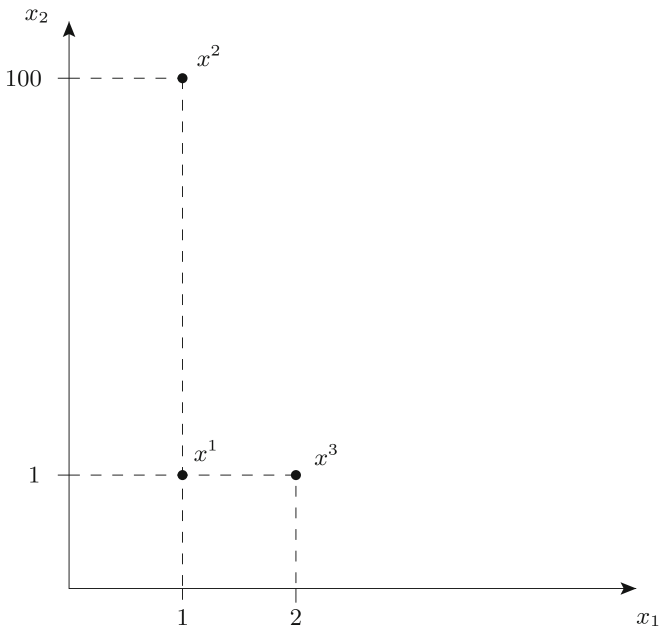

Fig. 7.4 Lexicographic preferences

enough real numbers to represent all the information that is in such a preference ordering. One way to fix the problem is to assume a continuity of preferences, which gives one an explanation for this assumption, and I will henceforth assume that preference orderings are continuous.

7.2 Demand on Competitive Markets

Chapter 4 described several causal factors that explain both individual and market demand on a competitive market. It was argued that demand will most likely depend on the price of the good as well as the prices of other goods, the income of an individual, the individual's tastes and expectations of the future. I am now in a position to replace these intuitive arguments with a sound decision-theoretical analysis using the model of preference or utility maximization introduced before. Remember that economic decision theory comes in two parts: the specification of a choice set and the determination of individual choices from this set for given preferences.

Assume that an individual (Ann) has the choice between two consumption goods, 1 and 2, whose quantities are denoted by x_1 and x_2, both from the set of positive real numbers (including 0). The individual behaves as a price taker and has a budget or income b that she completely spends on the two goods. (The model is very versatile, if one assumes, for example, that x_1 is the consumption today

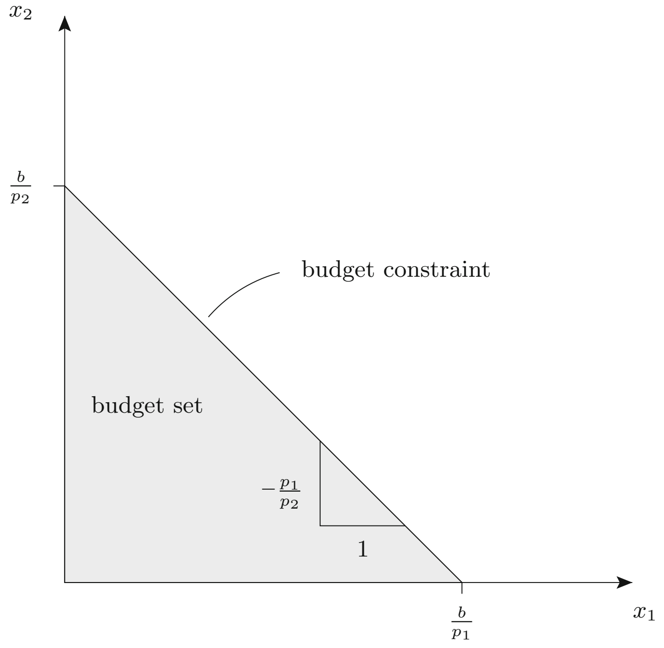

Fig. 7.5 The budget set and the budget constraint on a competitive market

and that x_2 is the consumption tomorrow, it can be interpreted in an intertemporal way to analyze savings behavior.) The prices of the two goods are p_1 and p_2, respectively.

This information can be used to specify Ann's choice set: we know that Ann can spend at most b units of money for the two goods. Expenditures for them are equal to $p_1 \cdot x_1 + p_2 \cdot x_2$. Hence, if expenditures cannot exceed the budget, it must be that:

$$p_1 \cdot x_1 + p_2 \cdot x_2 \leq b.$$

This inequality defines all the pairs x_1, x_2 that Ann can afford to buy, given her income b and prices p_1, p_2. It is her *choice set* that will henceforth also be called her *budget set* and denoted by $B(p_1, p_2, b)$. If Ann completely spends her budget, one will reach a point along the boundary of this set, $p_1 \cdot x_1 + p_2 \cdot x_2 = b$. This equality implicitly defines a function that is called the *budget constraint* or the *budget line*. Figure 7.5 illustrates the budget set.

In this figure, x_1 is drawn along the abscissa and x_2 along the ordinate. Using this convention, one can use the budget constraint to solve for x_2,

$$x_2 = \frac{b}{p_2} - \frac{p_1}{p_2} \cdot x_1.$$

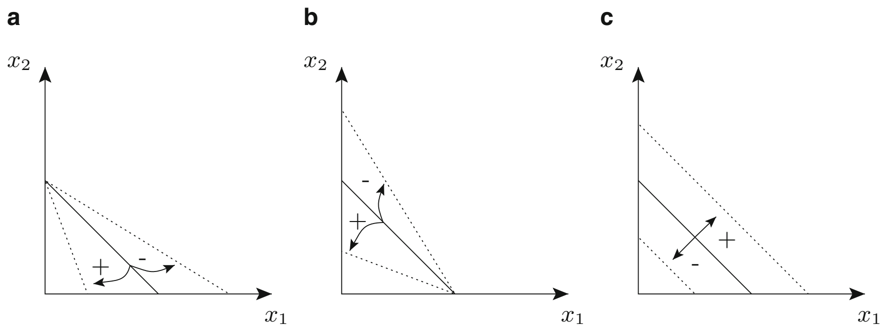

Fig. 7.6 The effects of price and income changes on the budget constraint, **a** Change in p_1, **b** Change in p_2, **c** Change in b

This equation reveals that the budget constraint is a downward sloping straight line that intersects the abscissa at b/p_2, the ordinate at b/p_1 and has a slope $-p_1/p_2$. The set below and to the left of this line is the budget set. It defines the set of all consumption bundles that Ann can afford to buy.

The budget constraint changes with changes in prices or income, as indicated in Fig. 7.6. Note that it shifts outwards (inwards) in a parallel way if the income goes up (down). It rotates outwards (inwards) through the intersection with the ordinate $(0, b/p_2)$ if p_1 goes down (up) and it rotates outwards (inwards) through the intersection with the abscissa $(b/p_1, 0)$ if p_2 goes down (up).

The slope of the budget constraint $-p_1/p_2$ has an important economic interpretation; it measures the rate at which the two goods can be exchanged against each other. Assume that $b = 100$, that $p_1 = 8$ and that $p_2 = 4$. In this example, $-p_1/p_2 = -2$: if one spends one's whole income on the two goods, one has to forfeit two units of good one if one wants to consume an additional unit of good 2, because good 2 is twice as expensive as good 1. The slope $-p_1/p_2$ is, therefore, the *relative price* of good 1 in units of good 2 and measures the opportunity costs of an additional unit of good 2 as defined by market prices.

7.2.1 Graphical Solution

Now one can apply the concept of preference orderings or utility functions in order to analyze choice. The hypotheses that can be derived depend on the assumptions that one makes regarding the structure of the preference ordering. Most of the literature assumes that individual behavior in markets can be described as if individuals would like to maximize a continuous (Assumption 3), monotonic (Assumption 4) and convex (Assumption 5a) or strictly convex (Assumption 5b) preference ordering based on their respective budget sets $B(p_1, p_2, b)$. In order to have an easier diagrammatic representation of the choice problem, it is also assumed that pref-

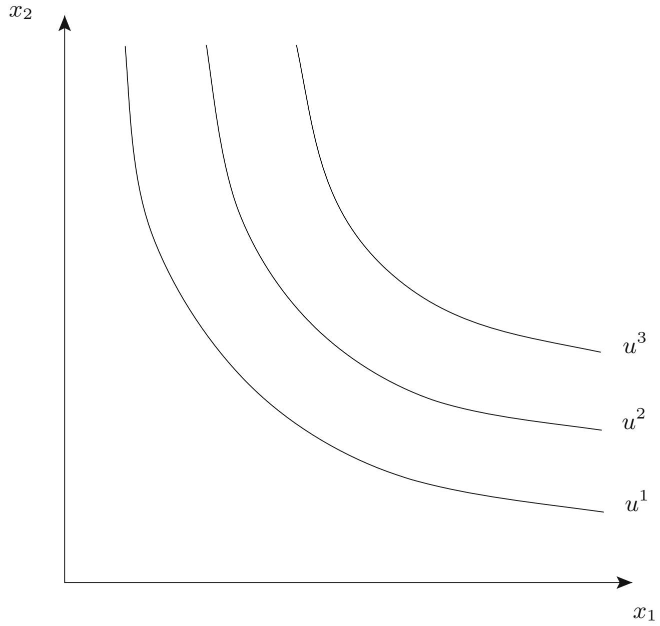

Fig. 7.7 Indifference curves in the context of a competitive market

erences are not only defined on $B(p_1, p_2, b)$ but also on all possible consumption bundles (x_1, x_2), irrespective of whether the individual can afford them or not.

Continuity implies that a preference ordering can be represented by a (utility) function, $u(x_1, x_2)$, and I will henceforth work with this convention. In order to illustrate the choice problem of an individual (Ann) I will assume in the remainder of this subchapter that her preferences are strictly convex and that they can be represented by a continuously differentiable utility function. In that case, her indifference curves for different levels of utility u^j must be inward bending, as illustrated in Fig. 7.7, where I have drawn three indifference curves for utility levels $u^1 < u^2 < u^3$. In order to keep the language simple I will refer to indifference curves that have larger utility indices as "higher" and indifference curves that have smaller utility indices as "lower."

Monotonicity implies that indifference curves that correspond to higher utility levels lie to the upper right of indifference curves that correspond to lower levels of utility. As one can see, the indifference curves provide an ordering of the set of potential consumption bundles. Starting from a given indifference curve, consumption bundles that lie on indifference curves with a larger utility index are preferred and bundles on indifference curves with a smaller utility index make the individual worse off, in comparison.

If one adds the budget set to the picture, one can use the ordering induced by indifference curves to predict behavior.

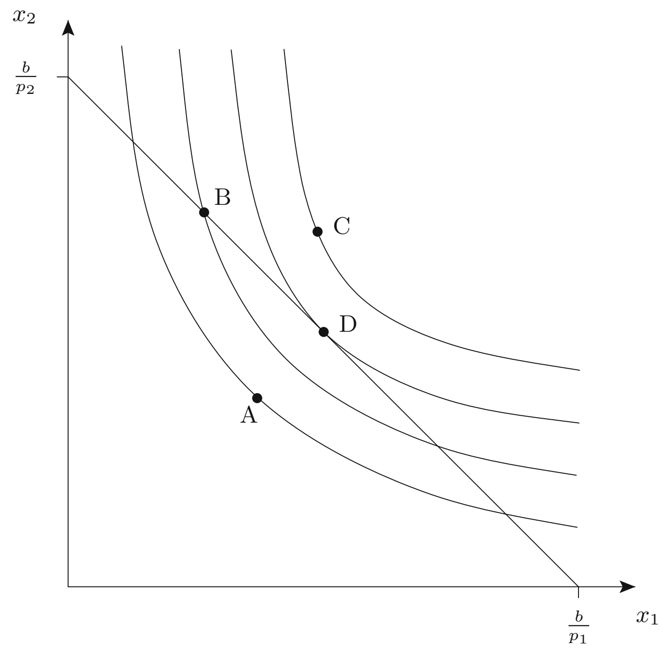

Fig. 7.8 Structure of optimal choices

Figure 7.8 displays a family of indifference curves that is derived from a utility function $u(x_1, x_2)$, and a budget set $B(p_1, p_2, b)$. Qualitatively there are four potential situations that can occur. These are denoted by consumption bundles A, B, C and D. Consumption bundle A is affordable for Ann because it is within her budget set. However, it is not Ann's best choice. If one compares A and B, one can see that B is on a higher indifference curve than A, but still within Ann's budget set. Hence, she would prefer B to A. Is B optimal for Ann? One could argue that C is even better, because it is on an even higher indifference curve. However, note that a consumption bundle like C is outside of Ann's budget set: she would prefer C to B but cannot afford it. Hence, C cannot be her optimal choice either. What one therefore has to do in order to determine Ann's best choice is to look for the highest indifference curve that still belongs to her budget set. Consumption bundle D fulfills this requirement. D is associated with the highest indifference curve that still belongs to budget set $B(p_1, p_2, b)$.

A situation like D has a straightforward economic meaning that is important for understanding the concept of opportunity costs as well as the mechanics of the utility-maximization model. Note that the slope of the budget constraint and the slope of the indifference curve are identical at a point like D. The slope of the budget constraint measures the relative price of the two goods and the ratio at which they can be exchanged on the market. The slope of the indifference curve is the marginal rate of substitution (MRS) and thus the exchange rate between the two goods that makes Ann indifferent between two bundles. At a point like D, both exchange rates coincide and the marginal rate of substitution is equal to the relative price. Why is this condition economically meaningful? Look at the following example: assume that the relative price of good 1 in terms of good 2 is -2 and that the marginal rate of substitution of good 1 in terms of good 2 is -4 at point B. (The budget constraint is less steep than the indifference curve.) Hence, Ann would be willing to give away four units of good 2 for an additional unit of good 1 to stay indifferent. However, given the market rate of exchange, she only has to give away two units. Hence, she can be better off by consuming more of good 1 at the expense of good 2. This logic applies to all consumption bundles for which the "internal" rate of exchange (the MRS) differs from the "external" rate of exchange (the relative price). Hence, only consumption bundles for which the marginal rate of substitution equals the relative price are consistent with the assumption of utility maximization.

The fact that the utility-maximizing consumption bundle is on the budget constraint and not in the interior of the budget set is a consequence of the assumption of monotonicity of preferences. With non-monotonic preferences, it could be that Ann is satiated without fully spending her income. Monotonicity, in this sense, can therefore be thought of as an expression of the underlying assumption of scarcity: with non-monotonic preferences, there could be situations with high incomes b such that all of Ann's desires are fulfilled. This would be the point at which scarcity – at least for Ann – ceases to exist. It is, ultimately, an empirical question as to whether such a point can ever be reached or not. One should, therefore, take the assumption with caution because its unscrutinized acceptance implies that one has implicitly commuted to the idea of quantitative growth in terms of some measure, like gross national product.

In addition, the fact that the MRS equals the relative price of the two goods at the optimum is a consequence of the assumption of the strict convexity of preferences (in fact, indifference curves additionally have to be continuously differentiable in order to guarantee this, otherwise they could have "kinks").

7.2.2 Analytical Solution

The utility-maximization problem can also be studied analytically. In order to do so, one starts by formally stating Ann's choice situation. One is confronted with an optimization problem that has the following structure: p_1, p_2 and b are the explanatory variables of the model, which means that the variables that determine Ann's consumption decisions are x_1 and x_2. They are, therefore, the explained variables. Hence, one needs to determine the functions $x_1(p_1, p_2, b), x_2(p_1, p_2, b)$. How is this possible? By assuming that Ann maximizes her utility function, $u(x_1, x_2)$, under the constraint that she does not spend more on consumption than her income is, $p_1 \cdot x_1 + p_2 \cdot x_2 \leq b$, one assumes that her preferences are monotonic, one knows that Ann will spend her whole income, and one can write:

$$\max_{x_1,x_2} u(x_1, x_2) \quad s.t. \quad p_1 \cdot x_1 + p_2 \cdot x_2 = b.$$

This notation needs some explanation. The $\max_{x_1,x_2}$-term indicates that one is looking for the maximum of the objective (utility) function with respect to the endogenous variables. The term s.t. abbreviates "such that," which indicates that Ann has to respect her budget constraint.

Formally, this is a constrained optimization problem and there are several ways to solve it. As long as one restricts one's attention to two endogenous variables, the solution does not require advanced mathematical techniques; instead one can simply use the constraint to eliminate one of the endogenous variables in the objective function. For more general (and realistic) problems in which Ann can choose between more than two goods, however, one needs a more general procedure that uses so-called *Lagrange* multipliers. I will not get any deeper into the field of optimization problems under constraints, because the results that I can derive with the more elementary approach for the two-goods case gives me sufficient mileage in explaining individual behavior on competitive markets for an introductory course in microeconomics.

In order to solve the problem, one can convert the budget constraint in the same way as I have shown before, $x_2 = b/p_2 - (p_1/p_2) \cdot x_1$, and denote the function that relates x_1 and x_2 by $X_2(x_1) = b/p_2 - (p_1/p_2) \cdot x_1$. This equation can be used to eliminate x_2 in the utility function. One, therefore, ends up with a modified, unconstrained optimization problem:

$$\max_{x_1} u(x_1, X_2(x_1)) = \max_{x_1} u(x_1, b/p_2 - (p_1/p_2) \cdot x_1).$$

In order to illustrate how this problem can be solved, assume that $u(x_1, x_2)$ is twice continuously differentiable and that the underlying preference ordering is strictly convex. If these assumptions are fulfilled, then an interior maximum is characterized by a value of x_1, such that the first derivative is equal to zero (first-order condition):

$$\frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2} \cdot \frac{\partial X_2}{\partial x_1} = \frac{\partial u}{\partial x_1} - \frac{\partial u}{\partial x_2} \cdot \frac{p_1}{p_2} = 0.$$

This condition can be simplified to:

$$\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{p_1}{p_2},$$

which is the optimality condition for the consumer-choice problem. In order to be able to interpret this condition, one has to understand the term on the left-hand side. In order to do so, one can use the total differential of the utility function

$$du = \frac{\partial u}{\partial x_1} \cdot dx_1 + \frac{\partial u}{\partial x_2} \cdot dx_2.$$

The total differential measures the total effect on utility with a change in the explanatory variable of dx_1 and dx_2, respectively. One is not interested in arbitrary changes but in changes that leave total utility constant, $du = 0$, because this keeps one on the same indifference curve. In other words, the set of all (x_1, x_2) that lead to the same level of utility constitute the marginal rate of substitution:

$$du = \frac{\partial u}{\partial x_1} \cdot dx_1 + \frac{\partial u}{\partial x_2} \cdot dx_2 = 0$$
$$\Leftrightarrow MRS(x_1, x_2) = \frac{dx_1}{dx_2} = \frac{\partial u / \partial x_1}{\partial u / \partial x_2}.$$

However, this is exactly the left-hand side of the optimality condition. One can therefore conclude that a preference- or utility-maximizing individual chooses consumption in a way that the marginal rate of substitution equals the relative price of the goods.

Consumption bundles (x_1^*, x_2^*) that fulfill the first-order condition are the individual's utility-maximizing choices. Formally, they are functions of the explanatory variables $x_1(p_1, p_2, b)$ and $x_2(p_1, p_2, b)$ and are named *Marshallian demand functions* after Alfred Marshall. What is interesting, from the point of view of the structure of individual demand, is whether the Marshallian demand functions have any particular properties that allow one to better understand the structure of individual and, thereby ultimately, market-demand behavior.

However, here comes the challenge: the ultimate test for the usefulness of a theory is – according to critical rationalism (see Chap. 1 for a description of this position in the philosophy of science) – its empirical validity. Hence, one has to formulate a theory in a way that makes it empirically testable. The theory of consumer choice has two building blocks: preferences and choice sets that are determined by prices and income. It is relatively straightforward to empirically measure the latter elements of the theory, but it is not possible to determine individual preferences directly. This is bad news for empirical tests: behavior is determined by both, choice sets and preferences. If one cannot observe preferences, one cannot test the theory. Hence, one can only measure the properties of the theory that are *independent* of the specific preference ordering underlying consumer choices. However, given that any choice of consumption can be rational for some preference ordering, the only hope that one has is that the theory is testable when one looks at *changes* in observable behavior that are caused by *changes* in prices or income. It may be that a change in prices or income induces stable and predictable reactions that can, in principle, be falsified by confronting them with empirical data. In order to be able to do so, however, one has to impose the (dogmatic, see Chap. 1) assumption that preferences remain stable over a period of time. This is why *comparative statics* plays such an important role in economics: if there is any hope for empirically testing the theory, it is because it produces refutable hypotheses regarding the change in Marshallian demand functions when prices or income change. Whether the theory can live up to these standards or not will be the subject of this investigation.

One important property is that Marshallian demand functions are homogeneous of degree zero, i.e. that a proportional change in all prices and income has no influence on individual behavior. Formally, this means that $x_i(p_1, p_2, b) = x_i(\lambda \cdot p_1, \lambda \cdot p_2, \lambda \cdot b)$ for $i = 1, 2$ and $\lambda > 0$. Intuitively this means that it does not matter whether prices and income are measured in Swiss Francs or in Rappen, Euro or Cent; as long as the relative price of both goods and the purchasing power of income remains the same, the individual will not change her behavior. In order to see that this must be the case return to the budget constraint $x_2 = b/p_2 - (p_1/p_2) \cdot x_1$. If all prices and income are multiplied by the same factor λ, one gets

$$x_2 = \frac{\lambda b}{\lambda p_2} - \frac{\lambda p_1}{\lambda p_2} \cdot x_1 = \frac{b}{p_2} - \frac{p_1}{p_2} \cdot x_1.$$

The effect of λ cancels out and, therefore, leaves the location of the budget constraint unaltered. However, with an unaltered budget constraint, the optimal behavior of the individual must be unaltered as well, hence the Marshallian demand functions are homogeneous of degree zero in prices and income.

Digression 18. Money Illusion and the Debate Between Keynesian and Neoclassical Economics

The homogeneity of degree zero in prices and income of the Marshallian demand function may sound like an innocuous mathematical property, but, in fact, it marks a very important watershed in the history of economic thinking. Keynesian and neoclassical economists have profoundly disagreed on the role of economic policy to stabilize the economy. One important field of disagreement is monetary policy. Neoclassical economists are usually sceptic on the role that monetary policy can or should play, with the implication that price stability is usually the primary focus of neoclassical monetary policy. On the contrary, Keynesian economists usually see a much more active role for monetary policy in stimulating and stabilizing the economy (Keynes 1936).

There are several reasons why these schools disagree, but at least one can be traced back to the homogeneity of degree zero of the Marshallian demand functions. If this property holds, the possibility to influence the economy by means of monetary policy are severely limited. Increasing or reducing money supply is like multiplying all prices and income by λ. However, if this is the case and if the model of utility- or preference-maximizing individuals is correct, then the real effects of these changes on the economy are zero: general inflation or deflation is like measuring prices in different currencies without changing the purchasing power of income or the relative prices of goods. This property is sometimes also called the *absence of money illusion*. Without a money illusion, monetary policy has no impact on the economy, because people will not change their behavior and, if people's behavior does not change, then everything remains the same. The only way monetary policy can influence behavior, according to this view, is if inflation or deflation change different prices and incomes differently, hence either changing relative prices, purchasing power or both. This can happen if some prices are nominally fixed, while other prices can adjust to changes in money supply. A Keynesian economist, who sees an active role for monetary policy, therefore, either has to think that some prices or incomes are nominally fixed or that the model of preference or utility maximization is flawed to begin with.

The first-order condition is only a necessary condition for a utility maximum, and one does not know yet if it characterizes a local maximum, a local minimum or a point of inflection. In order to say more, one has to check the second-order condition. The first-order condition is the function:

$$\frac{\partial u(x_1, x_2)}{\partial x_1} - \frac{\partial u(x_1, x_2)}{\partial x_2} \cdot \frac{\partial X_2}{\partial x_1}.$$

It characterizes a local maximum if its derivative, with respect to x_1, is smaller or equal to zero,

$$\frac{\partial^2 u(x_1, x_2)}{\partial x_1^2} + \frac{\partial^2 u(x_1, x_2)}{\partial x_1 \partial x_2}\frac{\partial X_2}{\partial x_1} + \frac{\partial X_2}{\partial x_1}\frac{\partial^2 u(x_1, x_2)}{\partial x_2 \partial x_1} - \left(\frac{\partial X_2}{\partial x_1}\right)^2 \frac{\partial^2 u(x_1, x_2)}{\partial x_2^2} \leq 0.$$

This condition can be simplified, if one remembers that:

$$\frac{\partial X_2}{\partial x_1} = -\frac{p_1}{p_2} = -\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}}$$

and notes that:

$$\frac{\partial^2 u(x_1, x_2)}{\partial x_1 \partial x_2} = \frac{\partial^2 u(x_1, x_2)}{\partial x_2 \partial x_1}.$$

This leads one to:

$$\left(\frac{\partial u(x_1, x_2)}{\partial x_2}\right)^2 \cdot \frac{\partial^2 u(x_1, x_2)}{\partial x_1^2} + \left(\frac{\partial u(x_1, x_2)}{\partial x_1}\right)^2 \frac{\partial^2 u(x_1, x_2)}{\partial x_2^2}$$
$$-2\frac{\partial u(x_1, x_2)}{\partial x_1}\frac{\partial u(x_1, x_2)}{\partial x_2}\frac{\partial^2 u(x_1, x_2)}{\partial x_1 \partial x_2} \leq 0.$$

If the condition holds at (x_1^*, x_2^*), then the indifference curve is locally convex at that point, hence it characterizes a local maximum. If the condition holds for every (x_1, x_2), then the indifference curve is globally convex. This condition is fulfilled only if the underlying preference ordering is convex. Hence, the assumption that the preference ordering is convex guarantees that the first-order condition characterizes a maximum. If the inequality is strict, then the preference ordering is strictly convex and the solution is unique.

A utility function with this property is called (strictly) *quasi-concave*. Quasi-concavity is weaker than concavity of functions, because it only guarantees that the not-worse-than-x sets (whose boundaries are the respective indifference curves) are convex sets, but makes no assumptions about the concavity of the rest of the function.

7.2.3 Three Examples

There are three utility functions that represent typical preference orderings and that play an important role in a lot of economic applications of the model. This sub-chapter will analyze these examples both graphically and analytically.

7.2.3.1 Homothetic Strictly Convex Preferences

An example of a so-called *homothetic* utility function is given by $u(x_1, x_2) = a \cdot (x_1)^{\alpha} \cdot (x_2)^{\beta}$, where a, α, β are positive real numbers. It is an example for a strictly quasi-concave utility function that has the additional property that the MRS is constant for proportional changes of the two goods ($x_1/x_2 = c$, with $c > 0$ being constant).

The following paragraphs focus on a special case in which $a = 1$ and $\alpha = \beta = 1/2$, because it is more convenient to solve mathematically. These assumptions imply that $u(x_1, x_2) = \sqrt{x_1} \cdot \sqrt{x_2}$. Before one derives the Marshallian demand functions, it makes sense to familiarize oneself with the structure of this function. One can, for example, derive the indifference curve for some arbitrary level of utility $\bar{u}$, $u(x_1, x_2) = \sqrt{x_1} \cdot \sqrt{x_2} = \bar{u}$. In order to derive the function, $X_2(x_1)$, that describes the indifference curve, one solves for x_2, $\sqrt{x_2} = \bar{u}/\sqrt{x_1} \Leftrightarrow x_2 = (\bar{u})^2/x_1$. This is a family of hyperbolic functions, one for each value of $\bar{u}$, which implies that the underlying preference ordering is strictly convex.

At this point, one can further illustrate that every monotonic transformation of a utility function represents the same preference ordering. If one squares the utility function (this is a monotonic transformation, because the underlying utility function only has positive values) one gets $v(x_1, x_2) = (u(x_1, x_2))^2 = (\sqrt{x_1} \cdot \sqrt{x_2})^2 = x_1 \cdot x_2$. It follows that the indifference curves of this function are also hyperbolic: $x_2 = \bar{u}/x_1$. The only difference between the two indifference curves is the *absolute* value of utility, but remember that this number has no meaningful interpretation.

With these prerequisites one can move on to analyze Ann's demand if her preferences have a utility representation of $u(x_1, x_2) = \sqrt{x_1} \cdot \sqrt{x_2}$. Figure 7.9 displays a family of indifference curves for different utility levels. Given that they are hyperbolic, the MRS remains constant along a ray through the origin (points A, B, and C).

✍ Figure 7.10a shows the same family of indifference curves and adds different budget constraints for different income levels $b^1 < b^2 < b^3$. The above-mentioned property that the MRS remains constant along a ray through the origin implies that the utility-maximizing choices for different income levels must be on a ray through the origin, as well. This path of optimal choices for different income levels is called

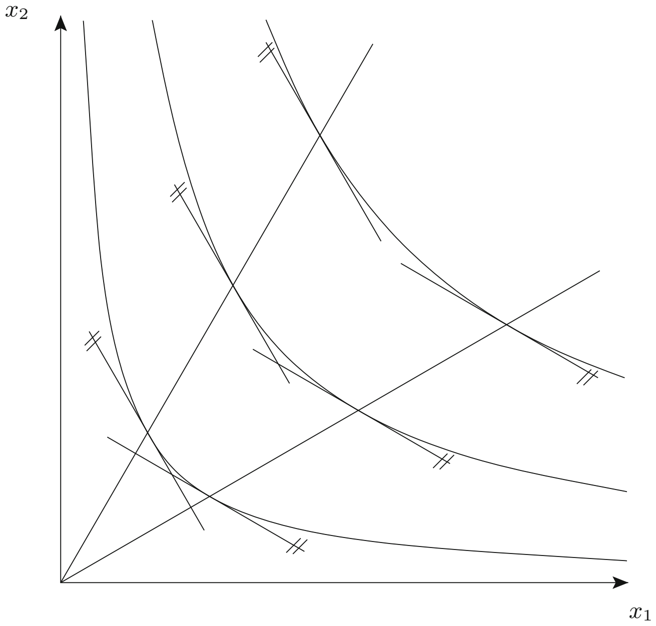

Fig. 7.9 MRS for homothetic strictly convex preferences preferences

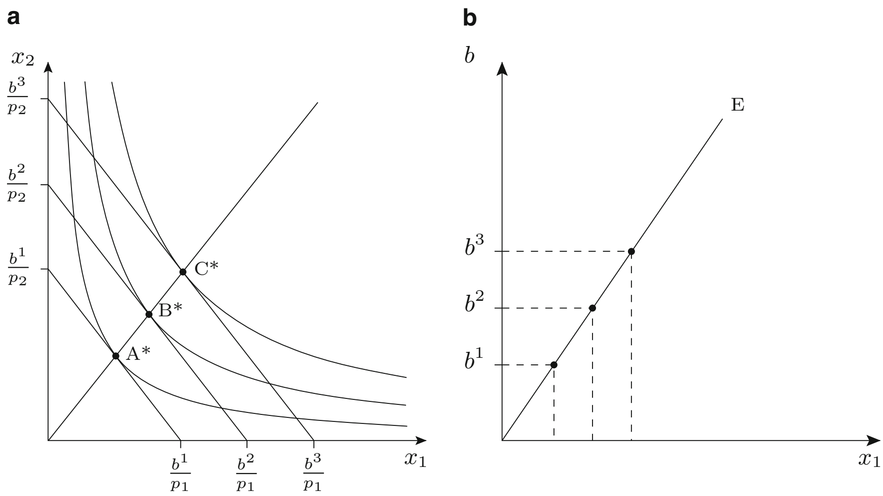

Fig. 7.10 Income-consumption path and Engel curve for homothetic strictly convex preferences

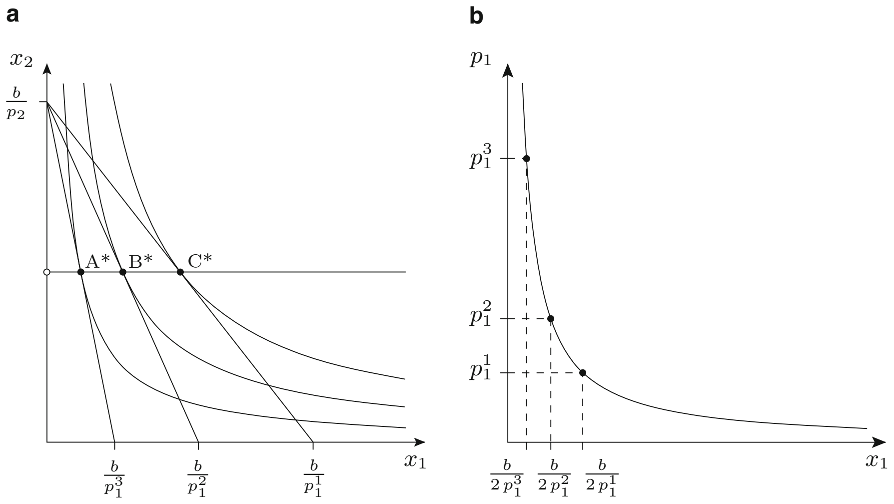

Fig. 7.11 The price-consumption path and demand function for homothetic strictly convex preferences

the *income-consumption path*. This is depicted by A^*, B^*, and C^*. Figure 7.10b displays the demand of one of the two goods (say 1) as a function of income levels. The argument above implies that the relationship between income b and demand x_1^* must be linear. The straight line E reflects this fact. The relationship between income and utility- or preference-maximizing consumption is called an *Engel curve*. In this case, it is upward sloping, which means that the good is normal (see Chapt. 4 for the definition of this term).

✍ Figure 7.11a displays the same family of indifference curves and adds different budget constraints for different price levels of good 1, $p_1^1 < p_1^2 < p_1^3$. An increase in p_1 rotates the budget constraint inwards around the point $0, b/p_2$. The utility-maximizing consumption bundles are, again, depicted by A^*, B^* and C^*. They lie on the horizontal line displayed in the figure that is called the *price-consumption path*. Figure 7.11b displays the demand for good 1 as a function of its price p_1. This is the *individual demand function* that is already known from Chap. 4. It is downward sloping, which means that it is ordinary (see Chap. 4 for the definition of the term).

Alternatively, one can derive Ann's demand function analytically. In order to do so, one can either use the information that MRS has to be equal to the relative price directly, or start with the utility-maximization problem. In order to practice, I will follow the second road in the following paragraphs. Additionally, in order to simplify the mathematics I will use a utility representation $v(x_1, x_2) = x_1 \cdot x_2$ (try the other formulation to see if it leads to the same result):

$$\max_{x_1} \quad (x_1) \cdot (b/p_2 - (p_1/p_2) \cdot x_1).$$

To get the first-order condition one can apply the product rule:

$$(b/p_2 - (p_1/p_2)\cdot x_1) - (p_1/p_2)\cdot x_1 = 0.$$

One can solve this condition for x_1 to get the Marshallian demand function for good 1:

$$x_1(p_1, p_2, b) = \frac{b}{2\cdot p_1}.$$

Knowing that $x_2 = b/p_2 - (p_1/p_2)\cdot x_1$, one can also derive the Marshallian demand function for good 2:

$$x_2(p_1, p_2, b) = \frac{b}{2\cdot p_2}.$$

The demand functions have three remarkable properties. First, they are linear in income, which is what one would have expected from the linearity of the Engel curve. Second, they are downward sloping and hyperbolic in their own prices, which is also what one would have expected from the graphic analysis. Third, they do not depend on the price of the other good.

These utility functions play an important role in economic applications, because of their simplicity. They can be generalized by assuming that the relative importance of the two different goods can be measured by some parameter $\alpha \in [0, 1]$ that yields $u(x_1, x_2) = x_1^{\alpha}\cdot x_2^{1-\alpha}$. It is, however, not clear if individuals behave as if they maximize preferences of this type. It is more useful as a thought experiment than an empirically supported claim about actual behavior.

7.2.3.2 Perfect Substitutes

The above utility function represents a case in which individuals prefer to consume the goods in relatively balanced bundles, but react to price changes by increasing the relative demand of the good that gets relatively cheaper. This need not be the case. There may be goods for which the individual has more extreme preferences: Ann consumes either one or the other, depending on which one is cheaper. If this is the case, the two goods are called *perfect substitutes* and such a taste can be represented by the following utility function:

$$u(x_1, x_2) = \alpha\cdot x_1 + \beta\cdot x_2,$$

in which α/β measures the relative importance of good 1 compared to good 2. This function is homothetic as well. In the following paragraphs, assume that they are of equal importance to Ann and normalize them to $\alpha = \beta = 1$. In this case, Ann's indifference curves are downward sloping straight lines with a slope of -1. A family of indifference curves is denoted by the dotted lines in Fig. 7.12. I have also drawn a budget constraint in this figure. It is already known that its graph is a straight line with slope $-p_1/p_2$, so the optimal consumption bundle depends on the relative slope of the indifference curves and the budget constraint: if the former is steeper than the latter, then Ann only consumes good 2 (as in the Figure) and *vice*

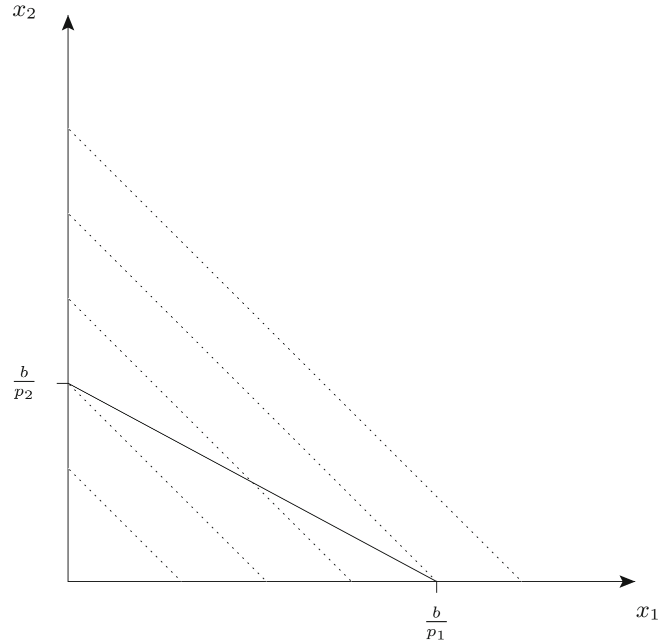

Fig. 7.12 Indifference curves and a budget constraint for perfect substitutes

versa. If both curves have an equal slope, then Ann would be indifferent between both goods.

Given that Ann will only buy the relatively cheaper good, the Engel curves are easy to derive. They are a straight line with slope 0 for the relatively more expensive and a straight line with slope $1/p_i$ for the relatively cheaper good. Price changes can be analyzed as before, but have a somewhat different effect on demand. Figure 7.13a shows a family of indifference curves and – as in the first example of a utility function analyzed before – adds different budget constraints to different price levels for good 1, $p_1^1 < p_1^2 = p_2 < p_1^3$. An increase in p_1 lets the budget constraint rotate inwards around the point $(0, b/p_2)$. The utility-maximizing consumption bundles are, again, depicted by A^*, B^* and C^*. If the price for good 1 is smaller than the price for good 2, then Ann spends all her income on good 1, which is indicated by point A^*. If both prices are equal ($p_1^2 = p_2$), then Ann is indifferent between both goods and we use the convention that she buys equal quantities, in this case. If p_1 rises further, then demand is zero, because Ann prefers the cheaper good. This behavior is illustrated by the demand function in Fig. 7.13b. It is discontinuous at $p_1 = p_2$ and hyperbolic for smaller prices of good 1.

If one remembers the analysis of competitive behavior in Chap. 4, this discontinuity can be problematic, because it may be that there is no intersection between market demand and market supply, in this case. This is why the continuity of both

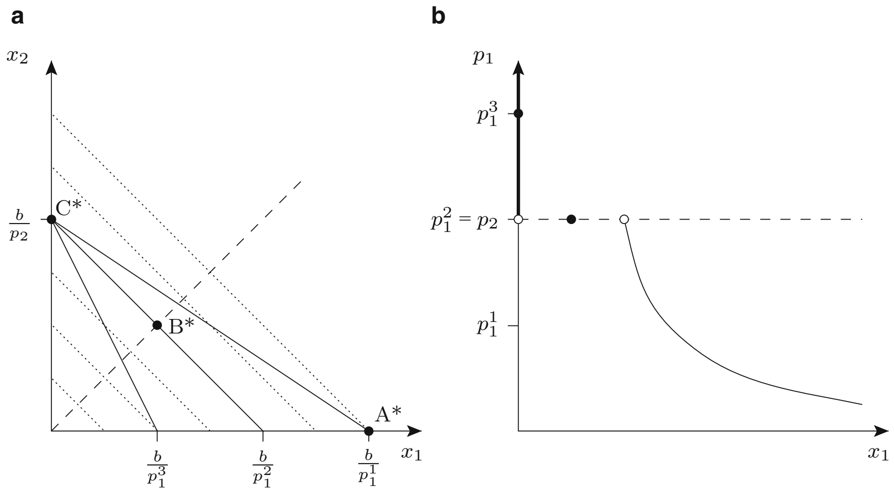

Fig. 7.13 Optimal choices and a demand function for perfect substitutes

demand and supply functions are important to guarantee the existence of a competitive equilibrium and one has now seen a case in which this is not the case. This reveals an advantage of the behavioral foundation of the demand function: it leads to a better understanding of the deeper reasons behind the continuity or discontinuity of demand functions by linking them to individual preferences. As one can see, the potential non-existence of an equilibrium can be a result of preferences that are not entirely absurd. The deeper reason for the discontinuity is that the preference ordering is only convex, not strictly convex. With strictly convex preferences, "small" changes in prices will lead to "small" changes in demand, but this is not the case if goods are, for example, perfect substitutes.

In order to derive the Marshallian demand functions analytically, one has to be careful. Given that both the budget constraint and the indifference curves are linear, the utility maximum cannot be derived from the first-order condition. Fortunately, one has already collected almost all the information that is necessary to determine Marshallian demand. One knows that Ann only buys the cheaper good and one has introduced the convention that she splits her income equally if both goods have the same price. The only step left is to formalize this information:

$$x_1(p_1, p_2, b) = \begin{cases} \frac{b}{p_1}, & p_1 < p_2 \\ \frac{b}{2 \cdot p_1}, & p_1 = p_2 \\ 0, & p_1 > p_2 \end{cases}, \qquad x_2(p_1, p_2, b) = \begin{cases} \frac{b}{p_2}, & p_2 < p_1 \\ \frac{b}{2 \cdot p_2}, & p_2 = p_1 \\ 0, & p_2 > p_1 \end{cases}.$$

7.2.3.3 Perfect Complements

The last example that I discuss is, in a sense, the opposite extreme from the case of perfect substitutes. There are some goods that Ann wants to consume together in fixed proportions, like left and right shoes, printer and toner, or hardware and

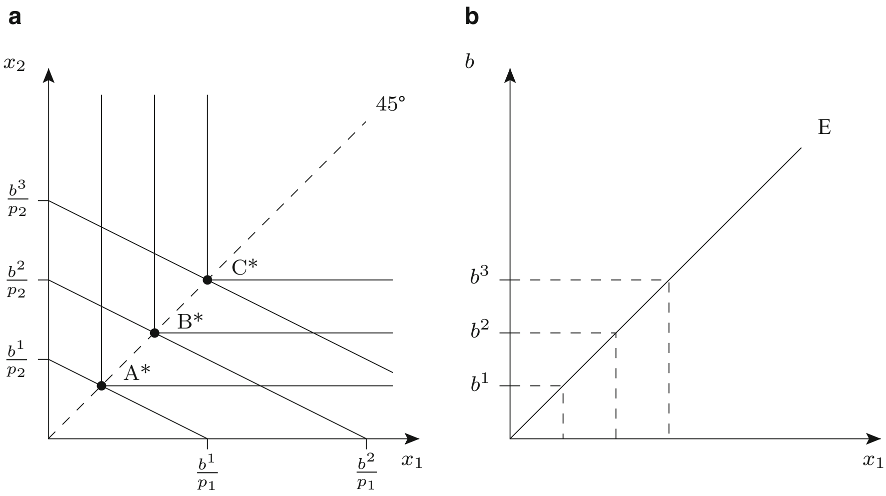

Fig. 7.14 Indifference curves, budget constraints and an Engel curve for perfect complements

software. If this is the case, she wants to spend her income on the two goods in way that makes sure that she buys both goods in fixed proportions. A utility function that expresses such preferences is:

$$u(x_1, x_2) = \min\{\alpha \cdot x_1, \beta \cdot x_2\},$$

in which α/β measures the number of units of good 2 that Ann needs to make use of an additional unit of good 1. This is also a homothetic function. To see this, assume that x_1 is the number of car bodies and x_2 is the number of wheels. It takes four wheels and a car body to assemble a useful car, so if $\alpha = 4$ and $\beta = 1$ one gets $\alpha/\beta = 4$, the number of units of good 2 (wheels) that is needed for one unit of good 1 (car bodies). The following paragraphs will focus on the easiest case in which $\alpha = \beta = 1$, i.e., Ann needs one unit of good 1 together with one unit of good 2, $u(x_1, x_2) = \min\{x_1, x_2\}$.

How do the indifference curves look like? I have drawn a family of them in Fig. 7.14a. They are L-shaped with a kink at the 45-degree line, which is where both goods are consumed in equal quantities. Increasing the quantity of one good while keeping the quantity of the other good constant is useless for Ann, which is why points on the vertical and horizontal lines are on the same indifference curve.

I have also added budget constraints for different income levels $b^1 < b^2 < b^3$. As one can see, the utility-maximizing consumption bundle is always at the kink, which is why they are along the 45-degree line through the origin. This is depicted by A^*, B^* and C^*. It follows immediately that the Engel curve must also be a straight line as in Fig. 7.14b.

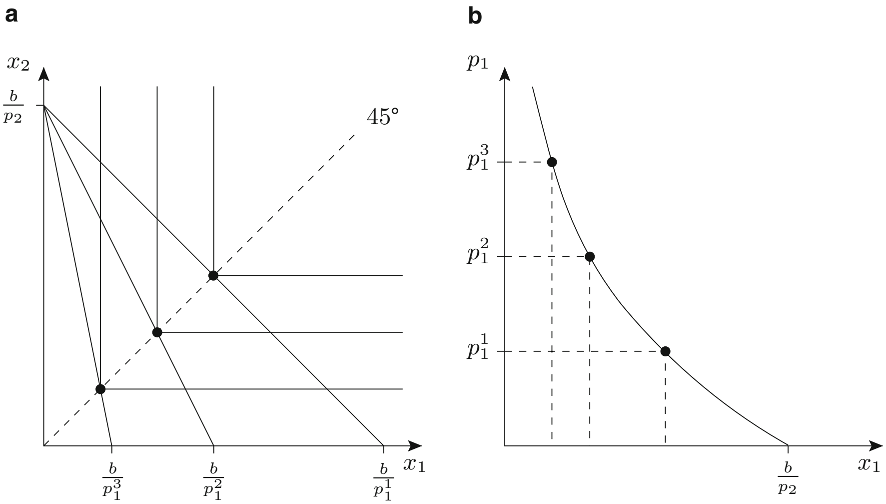

Fig. 7.15 Optimal choices and demand function for perfect complements

How about changes in prices? As before, an increase in the price of good 1 rotates the budget constraint inwards. Once again, I focus on three such prices $p_1^1 < p_1^2 < p_1^3$ and illustrate them in Fig. 7.15a.

One already knows that Ann will always buy both goods in equal quantities, i.e., stay on the 45-degree line. However, this implies that both the demand for good 1 and for good 2 is falling, if the price for good 1 goes up. Hence, the demand function for good 1 is given by the downward sloping graph in Fig. 7.15b. Note that it intersects the abscissa at $x_1 = b/p_2$, because at $p_1 = 0$ Ann can afford b/p_2 units of both goods. It is illustrative to also look at x_2 as a function of p_1, which is done in Fig. 7.16.

One knows that Ann will always buy both goods in equal quantities. However, this implies that the demand of good 2, as a function of p_1, is *identical* to the demand of good 1, as a function of p_1.

In order to derive the Marshallian demand functions analytically, first note that one cannot use first-order conditions in this case either, because the indifference curves have a kink, which implies that they cannot be continuously differentiated. Fortunately, the problem is very intuitive to solve. One knows that Ann is constrained by her budget, $p_1 \cdot x_1 + p_2 \cdot x_2 = b$, and wants to consume both goods in equal quantities, $x_1 = x_2 = x$. This information can be used in the budget constraint to get $p_1 \cdot x + p_2 \cdot x = b$. However, this is a linear function in one endogenous variable, so one can solve it. The solution is:

$$x_1(p_1, p_2, b) = \frac{b}{p_1 + p_2}, \qquad x_2(p_1, p_2, b) = \frac{b}{p_1 + p_2}.$$

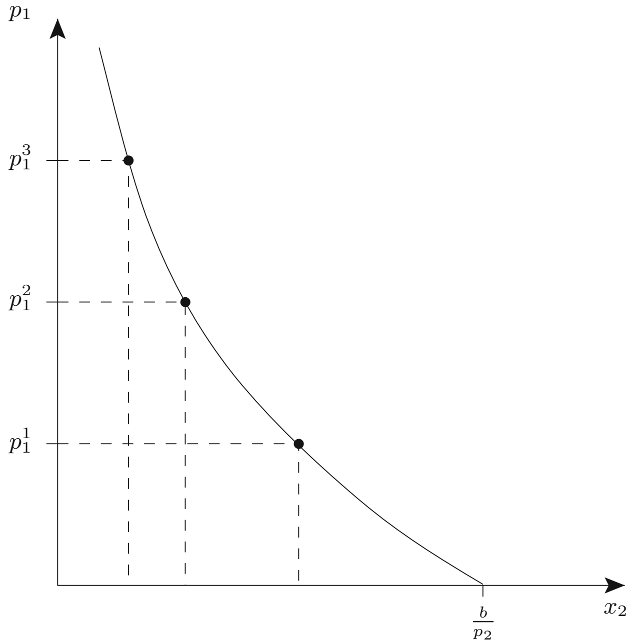

Fig. 7.16 The demand function for good 2 for perfect complements

7.2.4 Comparative Statics and the Structure of Market Demand

The three examples for potential preference orderings with associated utility functions have revealed that there is a stable relationship between the structure of preferences on the one and the structure of market demand on the other hand. In all three cases, one has seen that individual demand is decreasing in the price of the good and (weakly) increasing in income. The cross-price effects, however, seem to be more complex. They do not exist in the strictly convex and homothetic case, are (weakly) positive but extreme in the case of perfect substitutes and negative in the case of perfect complements. One has also seen that the strict convexity of the preference ordering seems to be important in order to guarantee that an equilibrium exists, because individual demand can otherwise be discontinuous.

Now one can find out if these findings can be generalized. Preferences are not directly observable and individuals seem to differ substantially with respect to their tastes. Hence, it would be nice if one did not have to make too many assumptions on the structure of preferences, as every assumption reduces the explanatory power of the theory, because they rule out certain preferences of which one does not know if they accurately describe real-life individuals. Thus, one can see how far one gets if one imposes monotonicity and strict convexity of a preference ordering with respect to the structure of individual demand. In order to do this, one can focus on two comparative-static experiments: a change in income and a change in the price of a good.

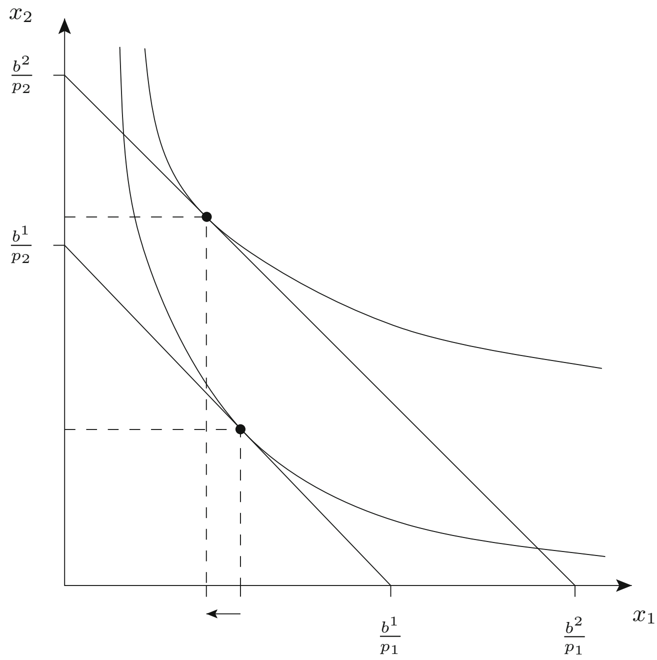

Fig. 7.17 Optimal choices for a change in income: an inferior good

7.2.5 Changes in Income

One has already seen that a change in income leads to a parallel shift of the budget constraint. Furthermore, one has already seen in the above examples that goods can be normal (demand increases if income increases). The remaining question is if this property is an artifact of the specific preference orderings or whether it is a general property of demand functions that are derived from preferences. Figure 7.17 shows that this is unfortunately not the case.

It displays two income levels, b_1 and b_2, and the associated indifference curves that Ann can reach, if she maximizes utility. As can be seen, the demand for good 2 goes up if income goes up, but the demand for good 1 does not. Hence, good 2 is normal and good 1 is inferior for this change in income. (Note that these properties are local and that they can hold for some changes in income, but not for others.) Hence, strict convexity and monotonicity do not rule out the inferiority of one of the goods. Besides, they should not, because there are a lot of goods that are, in fact, inferior, like low-quality products that are replaced by higher-quality substitutes, if the individual gets richer.

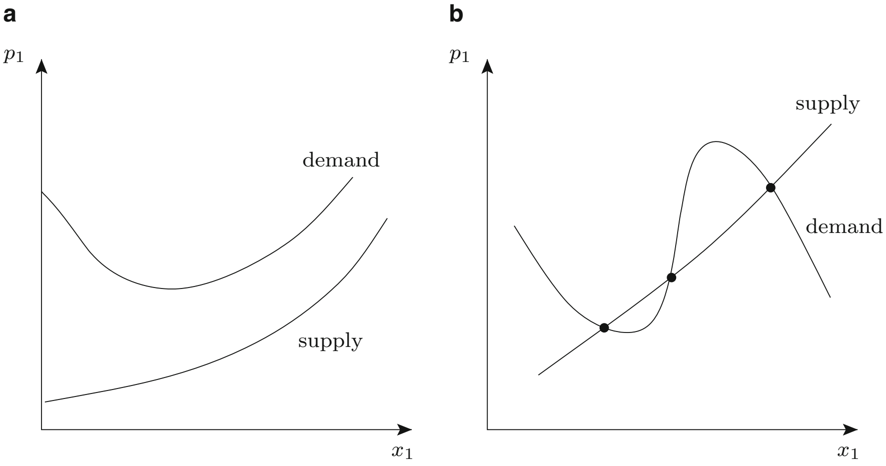

Fig. 7.18 Market equilibrium and non-monotonic demand functions

7.2.6 Changes in Price

In addition to continuity, individual and market demand should be decreasing as the price of the good increases in order to guarantee the existence and uniqueness of a competitive equilibrium. If demand is increasing as its price increases, there may be cases in which an equilibrium does not exist at all or in which multiple equilibria exist. Figures 7.18a and 7.18b illustrate both cases.

✍ Hence, it would be nice if one could show that goods are ordinary if individuals maximize a monotonic and strictly convex preference ordering. Unfortunately, this is not the case. Figure 7.19 gives an example for the so-called *Giffen paradox*, which is a situation in which the demand of a good decreases despite the fact that its price decreases.

Figure 7.19 focuses on a decrease in the price of good 1 from p_1^1 to p_1^2. The utility-maximizing consumption bundle changes from $A = (x_1(p_1^1, p_2, b), x_2(p_1^1, p_2, b))$ to $B = (x_1(p_1^2, p_2, b), x_2(p_1^2, p_2, b))$ and the highest indifference curves that can be reached are denoted by $I(p_1^1, p_2, b)$ and $I(p_1^2, p_2, b)$. As can be seen, the demand for good 1 goes down (and the demand for good 2 goes up) and this follows necessarily from the strict convexity of the preference ordering, because one is moving *along* the indifference curve.

What is going on here? One gets closer to understanding this phenomenon if one focuses on the curvature of the indifference curves. If the change in p_1 did not induce a rotation around $(0, b/p_2)$ but instead induce a rotation *along* the indifference curve (see Fig. 7.20), then the effect of an decrease in the price of good 1 would have the expected negative sign: good 1 gets relatively cheaper compared to good 2 and this isolated effect motivates Ann to buy more of good 1 and less of good 2. However, the reduction of p_1 not only has the effect that good 1 gets relatively

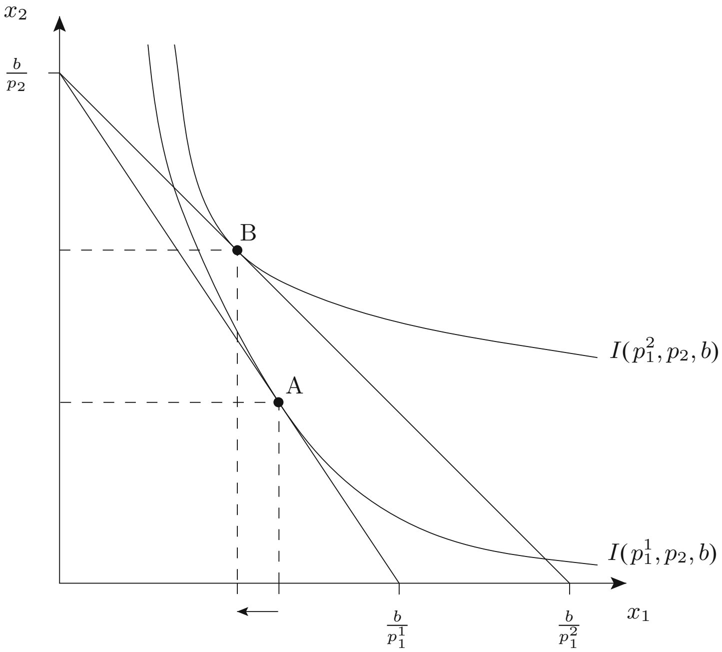

Fig. 7.19 Optimal choices for a change in prices: the Giffen paradox

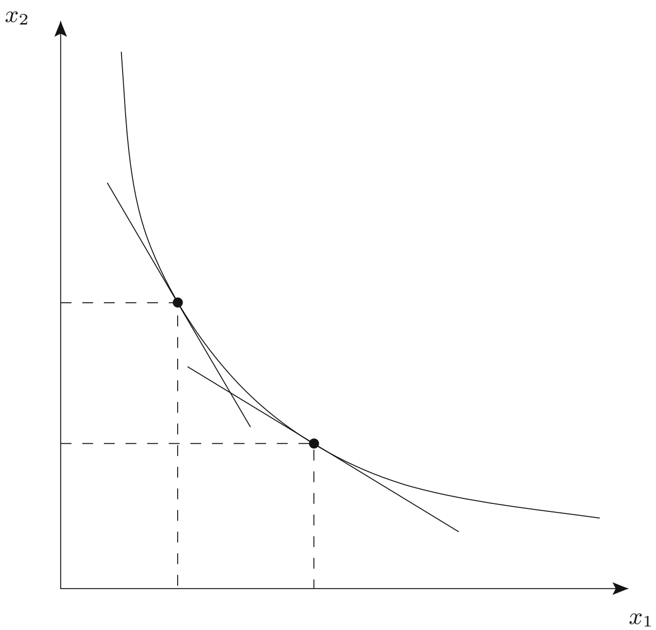

Fig. 7.20 The optimal choice for a compensated change in relative prices

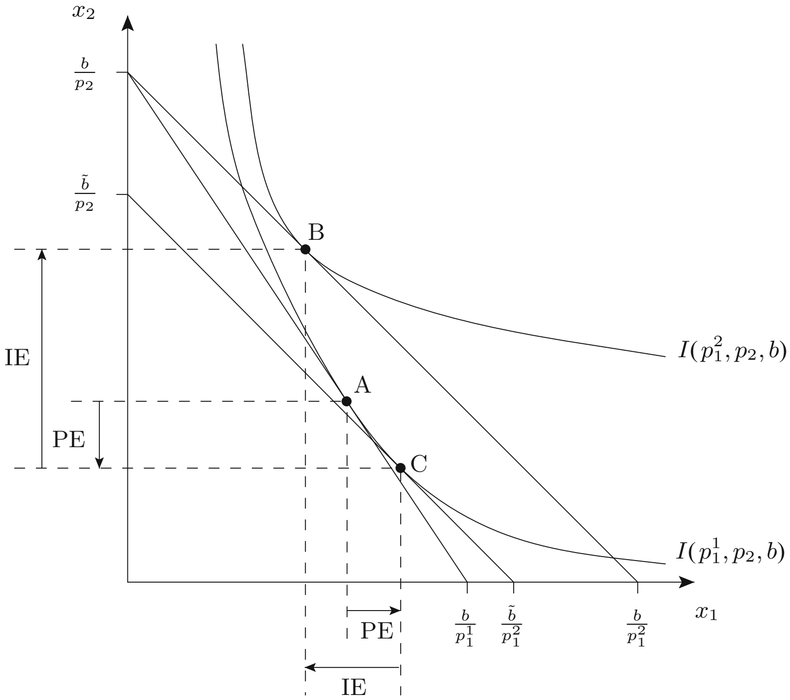

Fig. 7.21 Price- (PE) and Income effect (IE) and the Giffen paradox

cheaper, it also makes Ann richer, because her new budget set contains the old one as a subset. As a matter of fact, it is this latter effect that may cause the Giffen paradox.

In order to understand this, one has to disentangle the two effects in Fig. 7.21.

✍ Figure 7.21 is identical to Fig. 7.19, with the exception that I have introduced an artificial budget constraint for a hypothetical income level $\tilde{b}$. This hypothetical constraint is constructed to allow Ann to reach the same maximum indifference curve as before the price change, $I(p_1^1, p_2, b)$. In order to guarantee this, one has to change her income from b to some hypothetical income level $\tilde{b}$, such that $I(p_1^1, p_2, b) = I(p_1^2, p_2, \tilde{b})$. The utility-maximizing consumption bundle that results from this hypothetical budget constraint $p_1^2 \cdot x_1 + p_2 \cdot x_2 = \tilde{b}$ is denoted by $C = (x_1(p_1^2, p_2, \tilde{b}), x_2(p_1^2, p_2, \tilde{b}))$. One calls it the *compensated demand*. This compensated demand for good 1 is larger than before, $x_1(p_1^1, p_2, b) < x_1(p_1^2, p_2, \tilde{b})$, i.e., the isolated effect of a change in the relative price is negative (a smaller price and a larger quantity). This compensated effect is called the *price effect*, and it brings us from A to C in the figure.

✍ However, the compensation in income from b to $\tilde{b}$ is only the first step in the thought experiment. Therefore, in the next step, one will see what happens if one moves from $(p_1^2, p_2, \tilde{b})$ to (p_1^2, p_2, b). This change holds the relative price constant, but changes Ann's income and one has already seen what can happen. One already knows that this change brings one from C to B, but the additional insight is that this is only possible if the good is *inferior*; comparing C with B reveals that $x_1(p_1^2, p_2, b) < x_1(p_1^2, p_2, \tilde{b})$. This is called the *income effect*.

This thought experiment is important, because it allows one to better understand why individual demand may not fall as its own price falls. Any change in the price of one good has a price as well as an income effect and it is the income effect that may cause the Giffen paradox: if the good is inferior for Ann, then it is possible that her demand is (locally) increasing as the price increases.

This result is perhaps intellectually fascinating and it allows one to understand the mechanics of the preference-maximization model more profoundly, but it is, at the same time, highly unsatisfactory. In the end, the whole exercise to develop a choice-theoretic foundation of market behavior was motivated to better understand the structure of demand functions on competitive markets. These demand functions have to fulfill certain properties, like continuity and ordinary goods, to ensure that a market equilibrium exists. What one can learn from the Giffen paradox and its deeper reasons is that the assumption of preference maximization alone (even with the further restrictive assumptions of strict convexity and monotonicity) is insufficient to guarantee that a unique equilibrium exists. As Chap. 4 described, existence and uniqueness are important for positive economics, because sound economic prognoses depend on them. As one has seen with the three examples above on economic preferences, one can guarantee existence and uniqueness by imposing additional assumptions regarding preferences (like the assumption that all individuals have strictly quasi-concave utility functions), but this comes at the cost of sacrificing generality. Additionally, this cost is substantial indeed, because one does not have epistemic access to individual preferences, so one cannot know if any specific assumptions regarding their structure are empirically justified. However, this is the situation: if one wants a general theory of consumer choice that allows all types of preferences, then one cannot be sure that an equilibrium exists or if it is unique and if one want a unique existing equilibrium, then one has to start from specific assumptions regarding preferences.

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