Mathematical Appendix 14

This chapter covers ...

- an introduction into functions with several variables.
- an introduction into linear equations.
- the concept of elasticities.

> Do not worry about your difficulties in Mathematics. I can assure you mine are still greater. (Albert Einstein)

> If I were again beginning my studies, I would follow the advice of Plato and start with mathematics. (Galileo Galilei)

> (1) Use mathematics as shorthand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in 4, burn 3. This I do often. (Alfred Marshall)

The purpose of scientific theories is to develop hypotheses about causal relationships and to test them empirically. This is why the mathematical concept of a *function* is very important in both the natural and the social sciences. A function is a mapping from a set of explanatory variables onto a set of explained variables. One should know simple functions from high school: in order to define a function, it is usually assumed that a variable x, which is an element of some set X, and a variable y, which is an element of some set Y, exist and that y is related to x by some mapping $f : X \rightarrow Y$. Such a function is the easiest representation of a causal mechanism. If one states that $y = f(x)$, one means that some "state" y is caused by some "state" x and the function $f(.)$ represents this causal relationship between x and y. One calls x the *explanatory* and y the *explained* variable, because y is caused or "explained" by x via the function $f(.)$. Look at the following example: an individual demand function $x(p)$ assumes a relationship between a market price p and a quantity x that the consumer is willing to buy at this price. This is a causal relationship that is represented by the function $x(.)$ and for which the price, p, is the explanatory and the quantity, x, is the explained variable.

M. Kolmar, *Principles of Microeconomics*, Springer Texts in Business and Economics,
DOI 10.1007/978-3-319-57589-6_14

The simple, one-explanatory–one-explained-variable function is convenient, but often too simplistic to appropriately cope with economic phenomena. In social systems there are usually several factors that causally determine some outcome. In the case of individual or market demand for some good, i, for example, it is not only the price of this good, p_i, that determines demand, but also the prices for other goods, as well as the income of the individual. With n goods, one would, therefore, have prices, $p_1, \ldots, p_i, \ldots, p_n$, and income, b, that explain demand, x_i, and one has to denote this by means of a demand function that depends on all these variables, $x_i(p_1, \ldots, p_i, \ldots, p_n, b)$. Otherwise, one would not be able to fully understand the causal mechanisms at work.

There are two important fields of application for functions that represent causal mechanisms. First, it might be important to understand how the change in one explanatory variable changes the explained variable because, in empirical tests, it is often possible to measure changes in some variables, but not their absolute values. In order to describe those changes one can use the concept of the partial derivative of a function. The next subchapter introduces and works with partial derivatives.

Second, there are important cases in which a causal system is described by several functions. In markets, for example, both supply, $y(p)$, and demand, $x(p)$, are of importance. Supply and demand are mappings from explanatory to explained variables. In such situations, it is a standard problem to analyze whether it is possible to find values of the explanatory variables that are consistent with some constraints on the explained variables. In the case of supply and demand, such a constraint is the condition that supply equals demand, $x(p) = y(p)$ (equilibrium). If one asks if a price exists such that supply equals demand, one asks, from a mathematical point of view, if a value p exists such that $x(p) - y(p) = 0$. In other words, one is looking for the root of the equation $x(p) - y(p)$. This will be done in the subchapter after the next.

Functions are rather abstract and complicated tools. In order to avoid complications, assume throughout this book that the domain, as well as the codomain, of all functions are the set of real numbers and that all functions are continuous and have no “kinks.” Why this is important, as well as more general properties of functions, will be discussed in math class.

14.1 Functions with Several Explanatory Variables

This subchapter now leaves the demand and supply context behind to talk about functions more generally. Most people are familiar with the $y = f(x)$ notation of functions. (y no longer stands for supply, but for an arbitrary explained variable and x no longer stands for demand, but for an arbitrary explanatory variable, from now on.) For a function with only one explanatory variable, it is possible to use a very lean notation in order to be able to describe a change in the explained variable that is caused by a (small, infinitesimal) change in the explanatory variable: $f'(x)$. For example, the derivative of $f(x) = x^2$ is denoted as $f'(x) = 2 \cdot x$. There is nothing wrong with this notation, but it is not sufficiently precise, if one faces a problem with several explanatory variables. Assume that there are two explanatory variables x_1 and x_2, and denote by $y = f(x_1, x_2)$ the causal relationship. If one denotes derivatives as $f'(x_1, x_2)$ one cannot distinguish between changes in x_1 or x_2. One, therefore, has to introduce a way to denote derivatives that solves this problem. In principle, there are several ways to do so. For example, one could use the notation $f^1(x_1, x_2)$, $f^2(x_1, x_2)$ for the derivatives with respect to x_1 and x_2. However, this is not the usual convention.

Let $x_1, \ldots, x_n$ be the explanatory variables. One is interested in the changes of the function f evaluated at some point $a_1, \ldots, a_n$, which is caused by some infinitesimal change in x_i, holding all other explanatory variables constant (comparative statics). The most common notation for these so-called *partial derivatives* is given by:

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i}, i = 1, \ldots, n.$$

The notation $f(a_1, \ldots, a_n)$ reminds one that one is looking for the derivative of the function at a specific point $(a_1, \ldots, a_n)$. The "∂"-sign is pronounced as "del" and is reminiscent of the definition of partial derivatives by means of the difference coefficient,

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \lim_{dx_i \to 0} \frac{\overbrace{f(a_1, \ldots, a_i + dx_i, \ldots, a_n) - f(a_1, \ldots, a_n)}^{=df(a_1, \ldots, a_n)}}{dx_i},$$

$i = 1, \ldots, n$.

The notation "d" represents a discrete change in x_i and $f(.)$, respectively, and ∂ indicates the limit of this change, if dx_i becomes arbitrarily small (converges to zero).

In order to be able to work with partial derivatives, one has to generalize the rules of differentiation. Here are the most important ones:

Additive functions Let $f(x_1, \ldots, x_n) = g(x_1, \ldots, x_n) + h(x_1, \ldots, x_n)$; then

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \frac{\partial g(a_1, \ldots, a_n)}{\partial x_i} + \frac{\partial h(a_1, \ldots, a_n)}{\partial x_i},$$

$i = 1, \ldots, n$.

Product rule Let $f(x_1, \ldots, x_n) = g(x_1, \ldots, x_n) \cdot h(x_1, \ldots, x_n)$; then

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \frac{\partial g(a_1, \ldots, a_n)}{\partial x_i} \cdot h(a_1, \ldots, a_n) + g(a_1, \ldots, a_n) \cdot \frac{\partial h(a_1, \ldots, a_n)}{\partial x_i},$$

$i = 1, \ldots, n$.

Quotient rule Let $f(x_1, \ldots, x_n) = g(x_1, \ldots, x_n)/h(x_1, \ldots, x_n)$; then

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \frac{\dfrac{\partial g(a_1, \ldots, a_n)}{\partial x_i} \cdot h(a_1, \ldots, a_n) - g(a_1, \ldots, a_n) \cdot \dfrac{\partial h(a_1, \ldots, a_n)}{\partial x_i}}{(h(a_1, \ldots, a_n))^2},$$

$i = 1, \ldots, n$.

Chain rule For a number of scientific problems, the causal chain between explanatory and explained variables is more complex, because the effect of some explanatory on the explained variable is mediated by some "intermediate" variable. For example, it could be that some variable, x_i, has an influence on the intermediary variable z, $z = g(x_i)$, and z has an influence on y, $y = \tilde{f}(x_1, \ldots x_{i-1}, z, x_{i+1}, x_n)$. (For simplicity, assume that there is no direct effect of x_i on y, which will generalize the analysis in the next section. One calls this function $\tilde{f}(.)$, because it is a function of z and one has to be able to distinguish it from $f(.)$, which is a function of x_i. One can denote this structure as $y = f(x_1, \ldots x_{i-1}, x_i, x_{i+1}, x_n) = \tilde{f}(x_1, \ldots x_{i-1}, g(x_i), x_{i+1}, x_n)$.

The individual demand function can be used as an example. One has assumed that individual demand is a function of prices and income, b. If one further assumes that income is, itself, determined by some other factors, like qualification, then one gets a chain of causal effects: qualification determines income and income determines demand.

In a situation like this, one gets the following rule for the differentiation of $f(.)$ with respect to x_i:

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \frac{\partial \tilde{f}(a_1, \ldots, a_n)}{\partial z} \cdot \frac{\partial g(a_i)}{\partial x_i}.$$

The above expression is intuitive: x_i has an influence on z. This effect is captured by the second term of the product. The induced change in z, in turn, influences y. This is captured by the first term.

If x_i has an additional direct effect on y, one gets a function $y = \tilde{f}(x_1, \ldots x_{i-1}, x_i, x_{i+1}, x_n, z)$. The derivative with respect to x_i must, therefore, also include this direct effect:

$$\frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} = \frac{\partial \tilde{f}(a_1, \ldots, a_n, z)}{\partial x_i} + \frac{\partial \tilde{f}(a_1, \ldots, a_n, z)}{\partial z} \cdot \frac{\partial g(a_i)}{\partial x_i}.$$

A frequent application of partial derivatives is to estimate the effect of a discrete change, or simultaneous changes, in the explanatory variables on the explained variable (for example, because only discrete changes can be measured empirically). This can be done by means of the total differential.

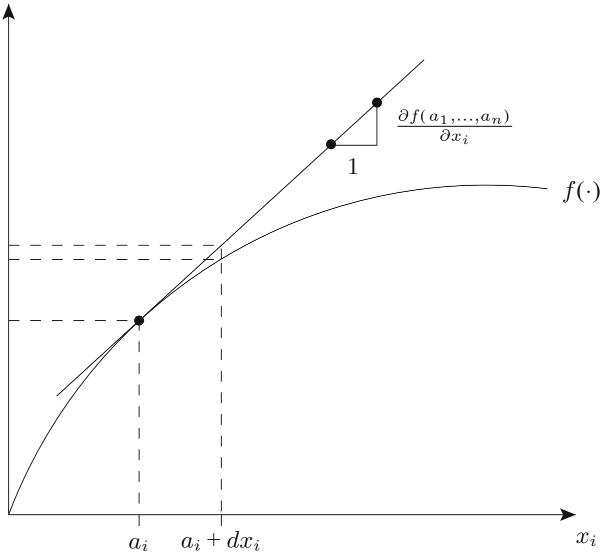

Fig. 14.1 Linear approximation of a function at a point a_i

Total differential Take $f(x_1, \ldots, x_n)$ and consider a simultaneous change in the explanatory variables dx_i. Then, the total effect is given as:

$$df(a_1, \ldots, a_n) = \frac{\partial f(a_1, \ldots, a_n)}{\partial x_1} dx_1 + \ldots + \frac{\partial f(a_1, \ldots, a_n)}{\partial x_n} dx_n.$$

In order to understand this expression, assume that all changes are zero except for x_i. Then, the total differential simplifies to:

$$df(a_1, \ldots, a_n) = \frac{\partial f(a_1, \ldots, a_n)}{\partial x_i} dx_i.$$

The right-hand side is a *linear* function of x_i, because the partial derivative is evaluated at a given point $a_1, \ldots, a_n$. However, this means that one can estimate the effect of an explanatory variable on y by means of a linear approximation, which is sometimes also called the *linear form*. Figure 14.1 illustrates this method.

Graphically speaking, the slope of the tangent line is equal to the partial derivative of the function at a given point. As can be seen, for discrete changes in x_i there is a gap between the true effect on y and the effect that is measured by the linear approximation: the linear approximation overestimates the true effect, in this example. However, if dx_i becomes very small, the "error" becomes arbitrarily small and vanishes in the limit for an infinitesimal change in x_i. One of the reasons why linear approximations are popular is that linear systems can be analyzed by means of linear algebra, which is powerful and simplifies the analysis considerably.

The following will reveal how the above rules can be used to determine derivatives of specific functions.

Example 1 Let $f(x_1, x_2) = x_1^2 + x_2$; then

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 2 \cdot x_1, \quad \frac{\partial f(x_1, x_2)}{\partial x_2} = 1.$$

The additive structure of the function implies that the different variables do not influence each other. As a consequence, the partial derivatives are independent of the other variable.

Example 2 Let $f(x_1, x_2) = x_1^2 \cdot x_2$; then

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 2 \cdot x_1 \cdot x_2, \quad \frac{\partial f(x_1, x_2)}{\partial x_2} = x_1^2.$$

The multiplicative structure implies that the partial derivatives, with respect to one variable, also depend on the other variable. However, in order to determine the derivative, the other variable can be treated as a number, because it is, in fact, a number, given that the partial derivative is an exercise in comparative statics (which means that all other variables are treated as constants).

Example 3 Let $f(x_1, x_2) = x_1^2/x_2$; then

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{2 \cdot x_1 \cdot x_2 - x_1^2 \cdot 0}{(x_2)^2} = \frac{2 \cdot x_1}{x_2},$$

because of the quotient rule.

All other rules that one has learned in school remain applicable to this generalized problem. If, for example, the problem is to determine the derivative of $f(x) = 10 \cdot \ln[x]$, with respect to x, it follows that $f'(x) = 10/x$. One can use this function to generalize the rules in the direction of functions with more than one variable. In order to do so, recognize that the above function has multiple variables already, because it is a function of x as well as 10, $f(x, 10)$, because 10 influences the result. Now, assume that one is not only interested in the partial derivative of this function at 10, but also at $9, 11, \ldots$. In this case, one can either determine the derivative for each case separately, or one can replace the specific number 10 by a dummy variable. If one redefines x by x_1 and calls the dummy variable x_2, one ends up with a new function $f(x_1, x_2) = x_2 \cdot \ln[x_1]$. However, now one has crossed the border from standard to *multivariate analysis*. The partial derivative of this function, with respect to x_1, can now be determined:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{x_2}{x_1}.$$

It is time for a quick plausibility check: if $x_2 = 10$, one gets $10/x_1$, which is reassuring. What one sees, from this example, is that one treats all the explanatory variables that stay constant in the same way as one has always treated numbers and the reason is that they are, in fact, numbers. The only difference is that they are written in an abstract way. In addition, one can, of course, also analyze the effect of a change of x_2 on y:

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = \ln[x_1].$$

14.2 Solution to Systems of Equations

Economists are interested in equilibria, because they tell them something about the logical consistency of the assumptions of a model. As already stated, an equilibrium exists, if there is a price such that supply equals demand. Supply and demand, however, are both functions, which implies that the previous chapters have implicitly talked about a property of mathematical objects (functions). If $x(p)$ and $y(p)$ are the market-demand and market-supply functions, an equilibrium is a price p^*, such that $x(p^*) = y(p^*)$. One can, alternatively, rearrange this condition to get $x(p^*) - y(p^*) = 0$: excess demand has to be equal to zero. If one looks at the problem from this perspective, one can see that the economic problem of the existence of an equilibrium is equivalent to the mathematical problem of the existence of a root of a function, the excess demand function $ED(p) := x(p) - y(p)$.

Most students will have touched the problem of the existence of roots in high school: a function has a maximum or minimum, if its first derivative is zero. The intermediate-value theorem is useful, in this respect, because it specifies sufficient conditions that guarantee the existence of a root of a function $ED(p)$: $ED(.)$'s domain has to be closed, $ED(.)$ has to be continuous and at least two prices, p and p', exist, such that $ED(p) < 0 < ED(p')$.

In order to be able to analyze problems like the one above, one needs a little knowledge about how to solve functions. The above problem is very simple, because it only has one equation in one explanatory variable: $ED(p) = 0$. In a number of more realistic situations, the problem is more complex, however. Assume, for example, that there is not one, but two markets, with goods 1 and 2, and one wants to know if prices exist that equilibrate both markets simultaneously. The mathematical problem becomes:

$$ED_1(p_1, p_2) = 0 \wedge ED_2(p_1, p_2) = 0,$$

with $ED_1(.)$, $ED_2(.)$ being the excess-demand functions for both markets, which are functions of both prices, p_1 and p_2. The mathematical problem is to find a solution to a system of two equations and two unknowns.

In reality, there are many more goods and services that are simultaneously traded in markets, such that one has to specify n markets with excess-demand functions and an equilibrium exists, if the system of n equations in n unknowns has a solution. This is a rather involved problem, which is why I restrict my attention to, at most, two equations and two unknowns and one also restricts one's attention to linear functions, most of the time in this book. Here, I denote the explanatory variables by x_1, x_2, the explained variables by y_1, y_2 and the causal mechanisms by $y_1 = f_1(x_1, x_2), y_2 = f_2(x_1, x_2)$.

Assume that one has to identify a pair of explanatory variables, x_1^* and x_2^*, that set both functions equal to zero, $f_1(x_1^*, x_2^*) = 0 \wedge f_2(x_1^*, x_2^*) = 0$. As can be conjectured from the intermediate-value theorem, it is not guaranteed that such a solution exists for general functions. However, if both equations are linear, one can use methods from *linear algebra* to identify the solution. Let

$$f_1(x_1, x_2) = a_1 + b_1 \cdot x_1 + c_1 \cdot x_2, \quad f_2(x_1, x_2) = a_2 + b_2 \cdot x_1 + c_2 \cdot x_2$$

be a linear system of equation with $a_1, b_1, c_1, a_2, b_2, c_2$ as the exogenous parameters of the equations. (a_1, a_2) are the intercepts and the other parameters measure the respective slopes. The problem of finding a zero is then given as:

$$a_1 + b_1 \cdot x_1^* + c_1 \cdot x_2^* = 0 \wedge a_2 + b_2 \cdot x_1^* + c_2 \cdot x_2^* = 0.$$

This problem has a unique solution, if the two equations are not parallel:

$$x_1^* = \frac{a_1 \cdot c_2 - a_2 \cdot c_1}{b_2 \cdot c_1 - b_1 \cdot c_2}, \quad x_2^* = \frac{a_1 \cdot b_2 - a_2 \cdot b_1}{b_2 \cdot c_1 - b_1 \cdot c_2}.$$

These formulas give one the general solution to the problem. In order to make sure that the denominator does not become zero, one has to, in addition, assume that $b_2 \cdot c_1 - b_1 \cdot c_2 = 0$ is excluded. If one inserts specific numbers, one can see what the general solution implies.

One can calculate the above solution with a little effort by, for example, solving the first equation for x_1, which yields $x_1 = -a_1/b_1 - c_1/b_1 x_2$. This equation is an intermediate step that can be used to eliminate x_1 in the second equation, $a_2 + b_2 \cdot (-a_1/b_1 - c_1/b_1 x_2) + c_2 \cdot x_2 = 0$. Now, one is left with only one equation with one unknown variable that can be solved for x_2.

This approach comes to an end, if one is confronted with a problem with more than two variables and unknowns. In such a case, one can use techniques from matrix algebra to characterize a solution.

Another problem may exist, if the equations are not linear. It would be far beyond the scope of this textbook to dig deeper into the solution of systems of nonlinear equations.

14.3 Elasticities

The measurement and comparison of changes is very important in economics and market research. So-called *elasticities* are a bread-and-butter concept with which everyone should be familiar. This subchapter will introduce the problems to which elasticities provide an answer and introduce the concept formally.

Assume one wants to know how demand $x(p)$ reacts to price changes. To be more specific, I will analyze the demand for bread and will assume that the demand function is linear, $x(p) = 100 - p$. Additionally, the price is in Swiss Francs and the quantity is in kilos.

An obvious candidate for the measurement of the effect of price changes is the partial derivative of the demand function:

$$\frac{dx}{dp} = x'(p) = -1.$$

This finding has a very straightforward interpretation: an increase in the price of bread by one Swiss Franc reduces the demand by one kilo.

This is a perfectly reasonable and informative statement and one could leave it at that. However, it has one disadvantage that limits its usefulness in practice: the instrument depends on the units in which one measures the dependent, as well as the independent, variable. Why is this a problem? Assume that one measures bread in grams instead of kilos. In this case, the demand function would be $x(p) = 100,000 - 1,000 \cdot p$ and the partial derivative becomes:

$$\frac{dx}{dp} = x'(p) = -1,000.$$

This is, again, a perfectly reasonable number: an increase in the price of bread by one Swiss Franc reduces the demand by 1,000 grams. However, without knowing the units of measurement, one cannot compare the two numbers and, at first glance, one could conclude that they are referring to completely different markets.

The same thing happens if one measures the price in Rappen instead of Franks. The demand function becomes $x(p) = 100 - 0.01 \cdot p$, and the first derivative is

$$\frac{dx}{dp} = x'(p) = -0.01 :$$

an increase in the price of bread by 1 Rappen reduces the demand for bread by 0.01 kilos (or 10 grams).

This dependence on the units of measurement also limits the usefulness of the instrument, because it makes it difficult to compare changes between countries that use different currencies. However, it is a potentially interesting question to ask if Swiss customers react more or less strongly to price changes than, for example, the French customers. Nevertheless, even within a country, it may be interesting to understand if the demand for bread reacts more or less strongly to price changes than does the demand for smartphones and it is very hard to make the units of measurement for these two products commensurable.

This is why economists use a measure that is independent of the units of measurement. The basic idea is to focus on relative instead of absolute changes. The absolute change in demand is given by dx and the relative change can be constructed by dividing the absolute change by some reference level x^r:

$$\text{relative change in demand} = \frac{\text{absolute change in demand}}{\text{reference level of demand}} = \frac{dx}{x^r} = \frac{x - x^r}{x^r}.$$

The same can be done for price changes. Let dp be the price change and p^r the reference price, one gets:

$$\text{relative change in price} = \frac{\text{absolute change in price}}{\text{reference level of price}} = \frac{dp}{p^r} = \frac{p - p^r}{p^r}.$$

The relative changes are independent of the units of measurement, because they cancel out: if the numerator is measured in, for example, kilos or Swiss Francs, the denominator is measured in kilos or Swiss Francs, as well. Relative changes can be transformed into percentage changes, by multiplying them by 100.

Now that the units of measurement have been eliminated, one can come back to the initial question of how to measure changes in demand that are caused by changes in prices. An *elasticity* relates the relative change of one variable (demand) to the relative change in another variable (price):

$$\text{price elasticity of demand} = \frac{\text{relative change in demand}}{\text{relative change in price}}$$

or, more formally:

$$\epsilon_p^x = \frac{dx/x}{dp/p} = \frac{dx}{dp}\frac{p}{x}.$$

This elasticity is called the *price elasticity of demand* and it measures the percentage change in demand that is caused by a 1% change in the price.

If one allows for infinitesimal changes in prices, one can use partial derivatives to characterize elasticities:

$$\epsilon_p^x = \frac{dx/x}{dp/p} = \frac{dx}{dp}\frac{p}{x} = \frac{\partial x}{\partial p}\frac{p}{x}.$$

The elasticity one gets for infinitesimal changes is also called *point elasticity*.

This determines one important elasticity, but the concept can also be used to determine changes in demand that are caused by changes in other explanatory variables, as well: for example, income levels or prices of other goods. Definitions 14.1–14.3 cover the most commonly used elasticities of demand. The following notation is used: the demand for good i is a function of the price of good i, p_i, as well as of the prices of other goods j, p_j, as well as income b.

▶ **Definition 14.1, Price elasticity of demand** The price elasticity of demand measures the percentage change in the demand for good i that is caused by a 1% change in the price of good i:

$$\epsilon_{p_i}^{x_i} = \frac{dx_i/x_i}{dp_i/p_i} = \frac{dx_i}{dp_i}\frac{p_i}{x_i} = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}.$$

▶ **Definition 14.2, Cross-price elasticity of demand** The cross-price elasticity of demand measures the percentage change in the demand for good i that is caused by a 1% change in the price of good j:

$$\epsilon_{p_j}^{x_i} = \frac{dx_i/x_i}{dp_j/p_j} = \frac{dx_i}{dp_j}\frac{p_j}{x_i} = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i}.$$

▶ **Definition 14.3, Income elasticity of demand** The income elasticity of demand measures the percentage change in the demand for good i that is caused by a 1% change in income:

$$\epsilon_{b}^{x_i} = \frac{dx_i/x_i}{db/b} = \frac{dx_i}{db}\frac{b}{x_i} = \frac{\partial x_i}{\partial b}\frac{b}{x_i}.$$

The same type of question can also be asked for changes in supply. I will focus on the most commonly used elasticities in the following definitions. Assume that supply y_i is a function of the price of the good p_i and of wages w and interest rates r.

▶ **Definition 14.4, Price elasticity of supply** The price elasticity of supply measures the percentage change in the supply of good i that is caused by a 1% change in its price:

$$\epsilon_{p_i}^{y_i} = \frac{dy_i/y_i}{dp_i/p_i} = \frac{dy_i}{dp_i}\frac{p_i}{y_i} = \frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i}.$$

▶ **Definition 14.5, Wage elasticity of supply** The wage elasticity of supply measures the percentage change in the supply of good i that is caused by a 1% change in the wage level:

$$\epsilon_{w}^{y_i} = \frac{dy_i/y_i}{dw/w} = \frac{dy_i}{dw}\frac{w}{y_i} = \frac{\partial y_i}{\partial w}\frac{w}{y_i}.$$

▶ **Definition 14.6, Interest elasticity of supply** The interest elasticity of supply measures the percentage change in the supply of good i that is caused by a 1% change in the interest rate:

$$\epsilon_{r}^{y_i} = \frac{dy_i/y_i}{dr/r} = \frac{dy_i}{dr}\frac{r}{y_i} = \frac{\partial y_i}{\partial r}\frac{r}{y_i}.$$

Elasticities can be positive or negative. Economists usually use the convention to talk about elasticities in absolute values (i.e. the modulus of the function), unless this is misleading. This convention allows them to use the following qualitative categories (expressed in absolute terms):

► **Definition 14.7, Elastic reaction** A variable reacts elastically to a change in some other variable, if the elasticity is larger than 1.

► **Definition 14.8, Inelastic reaction** A variable reacts inelastically to a change in some other variable, if the elasticity is smaller than 1.

► **Definition 14.9, Isoelastic reaction** A variable reacts isoelastically to a change in some other variable, if the elasticity is equal to 1.

Note that these properties are local measures. A function can be elastic at one point and inelastic at some other point.