Chapter 16
Suggestions for Further Reading

Needles to say, there is a vast literature on model theory of first-order logic and its applications. Some references have already been given throughout the text. I will repeat some of them and will add other recommendations.

Much can be learned about any subject by studying its history. Mathematical logic has a well documented and quite intriguing past. For a very comprehensive account, I recommend Paolo Mancosu's *The Adventure of Reason. Interplay between philosophy of mathematics and mathematical logic: 1900–1940* [20]. For a shorter discussion of issues around the origin and the history of definitions by abstraction and the concept of infinity, see Mancosu's *Abstraction and Infinity* [21].

Stanford Encyclopedia of Philosophy has informative entries on most topics that have been discussed here. In particular, see the entries on Zermelo's axioms of set theory [9] and first-order model theory [15]. For more on recent developments in set theory and model theory see [2] and [14].

From its inception, set theory has always been a tantalizing subject not only for the philosophy of mathematics, but also for philosophy in general. In his review of Alain Badiou's *Being and Event* [1], Peter Dews writes:

> All we can say ontologically about the world, Badiou contends, is that it consists of multiplicities of multiplicities, which can themselves be further decomposed, without end. The only process which endows any structure on this world is the process which Badiou calls 'counting-as-one', or—in more explicitly mathematical terms—the treatment of elements of whatever kind, and however disparate, as belonging to a set. We must not imagine, prior to this counting, any ultimate ground of reality [12].

In contrast, for many years model theory stayed in a shadow. This seems to be changing now. Recently published books by John Baldwin [3], Tim Button and Sean Walsh [4], and Fernando Zalamea [39], take the philosophy of model theory as their subjects.

For more about more recent mathematical advances in model theory, I recommend Boris Zilber's chapters in [25]. That book is also highly recommended for its unique vision of mathematical logic.

R. Kossak, *Mathematical Logic*, Springer Graduate Texts in Philosophy 3,
https://doi.org/10.1007/978-3-319-97298-5_16

A full history of model theory still waits to be written, but a very good introduction to it is Wilfrid Hodges' appendix in [4].

It is interesting to compare the contemporary view of the discipline with its descriptions in the past. Andrzej Mostowski's *Thirty years of foundational studies: lectures on the development of mathematical logic and the study of the foundations of mathematics in 1930–1964* [26], reprinted in [27], gives a very thorough outline of mathematical logic up to the 1960s, including a chapter on model theory.

In Part I in this book, much attention was given to the development of the basic number systems in the logical framework. I roughly followed a much more detailed presentation given by Craig Smoryński in [32], and the reader is referred to this excellent book for all details that were skipped here. For a briefer, but also very informative description of how large parts of mathematics, including portions of analysis and algebra, can be formally developed starting from the standard model of arithmetic $(\mathbb{N}, +, \cdot)$, see introductory chapters in John Stillwell's [33]. Stillwell's book is devoted to a relatively new, very attractive direction in foundational studies known as *Reverse Mathematics*.

Finally, I recommend two biographies by Constance Reid: *Hilbert* [30] (that book has had many editions), and *Julia, a Life in Mathematics* [29] about the life and work of her sister Julia Robinson and the history of the solution of Hilbert's 10th problem.