Chapter 14
A Larger, Expanding Universe

14.1 The Milky Way

14.1.1 A Fathomless Disk of Stars

On a clear, moonless night, we can look up and see a hazy, faintly luminous band of light that stretches across the sky from one horizon to the other; it is known as the Milky Way (Fig. 14.1). According to ancient Greek myth, the goddess Hera, Queen of Heaven, spilled milk from her breast into the sky. The Romans called the spilt milk the *Via Lactea*, or the "Milky Way." It also is designated as our Galaxy, derived from the Greek word *galakt*- for "milk," the celestial milk from Hera's breast.

We are immersed within the *Milky Way*, viewing it edgewise from inside. When gazing directly into the band of starlight, we cannot see through to stars at the center or distant edges of the Milky Way, but if we look up and outside the thin disk of stars, we can look beyond them. It is similar to living in a city: We notice buildings all around us, but none when we look up into the sky.

When Galileo Galilei (1564–1642) turned one of the first telescopes toward the Milky Way, he found that it contains many otherwise unseen stars, which are too faint to be seen by the unaided human eye (Galilei 1610). Astronomers subsequently built increasingly larger telescopes, which collect more starlight and enable us to see the dim, golden beacons of fainter stars. They discovered more dim stars located between or beyond the brighter ones, which make the Milky Way look like a continuously distributed band of light when observed by the unaided eye.

The German-born English astronomer William Herschel (1738–1822) spent much of his life trying to determine the shape and size of the Milky Way; he constructed the biggest telescopes at the time, with the largest mirrors and greatest light-gathering power. By counting the number of stars of different observed brightness in various directions in the night sky, he hoped to determine the places at which the stars disappeared, thereby determining the depths of the Milky Way.

K. R. Lang, *Essential Astrophysics*, Undergraduate Lecture Notes in Physics,
DOI: 10.1007/978-3-642-35963-7_14, © Springer-Verlag Berlin Heidelberg 2013

Fig. 14.1 The Milky Way A panoramic telescopic view of the Milky Way, the luminous concentration of bright stars and dark intervening dust clouds that extends in a band across the celestial sphere. We live in this disk and look out through it. Our view is eventually blocked by the buildup of interstellar dust, and the light from more distant regions of the disk cannot get through. The center of the Milky Way is located at the center of the image, in the direction of the constellation Sagittarius. Although the disk appears wider in that direction, the center is not visible through the dust. The large and small magellanic clouds can be seen as bright swirls of light below the plane to the right of center (this map of the Milky Way was hand-drawn from many photographs by Martin and Tatjana Keskula under the direction of Knut Lundmark; courtesy of the Lund Observatory, Sweden)

But the giant telescopes were not big enough to fathom the profundity or depth of the Milky Way. By collecting greater amounts of light than a smaller telescope, Herschel’s biggest telescope brought fainter stars into view and pushed the edge of the known universe further into space. These stars were concentrated in a flattened disk with the Sun at the center, and whose greatest extent is in the plane of the Milky Way (Herschel 1785).

Although Herschel concluded that the Sun is in the center of a flattened disk of stars with a disk diameter five time times its thickness, he had no way to determine its size. Early in the twentieth century, the Dutch astronomer Jacobus C. Kapteyn (1851–1922) and his colleagues resumed the star counts, arriving at a similarly flattened, Sun-centered distribution of stars with the greatest extent in the Milky Way. Measurements of the distance of some of these stars, using their parallax, provided a scale to Kapteyn’s universe of about 1.5 kpc by 12 kpc, where 1 kpc is 1,000 pc or 3.0857×10^{19} m (Kapteyn 1922).

However, astronomers have never succeeded in deciphering the true extent of the Milky Way by observing its stars, even when looking much farther into it using larger telescopes and photographic or electronic techniques that permitted long exposures. This is because the most powerful telescopes can discern only the

Fig. 14.2 Globular star cluster More than a hundred thousand stars are collected together in this globular star cluster designated NGC 362, which is located about 27,700 light-years away in the southern sky. It is one of many star clusters that are located in an extensive, spherical halo around our Milky Way. They formed in the early evolution of the Milky Way, and contain stars that are more than 10 billion years old. (Courtesy of Royal Observatory, Edinburgh.)

visible parts of our stellar system, not its most distant, invisible parts that lie behind an opaque veil of interstellar dust.

This dust blocks our view when we look deep into the plane of the Milky Way. The total amount of dust through which we are looking builds up with distance and eventually makes an impenetrable barrier; it becomes so thick and dense that it blocks the light of distant stars. We can see only that far; more distant objects are hidden from view. New perspectives were required to look outside and eventually beyond this barrier to the heavens.

14.1.2 The Sun is Not at the Center of Our Stellar System

The true enormity of our stellar system was discovered by using Cepheid variable stars to gauge the distances of globular star clusters (Fig. 14.2) located outside the plane of the Milky Way. These yellow supergiant Cepheids periodically brighten and dim with a period that increases with a star's luminosity. Measurements of this period and, therefore, the stellar luminosity can be combined with observations of a star's brightness to determine its distance (Focus 14.1). Because the Cepheid variable stars are very luminous, they are conspicuous and can be seen to exceptional distances, where conventional parallax methods of determining stellar distance do not work. At large distances, the parallax angles are too small to be reliably measured, even from space.

Focus 14.1 Cepheid variable stars

The luminosity of some stars does not remain constant but instead fluctuates over regular periods. These stars do not only turn on and off, like a switched house light, but instead gradually vary from dimmer to brighter and then back to dimmer again with periods ranging from a few days to a few months.

The very luminous variable stars are know as *Cepheid variable stars*, from their prototype, Delta Cephei. The deaf English astronomer John Goodricke (1764–1786) first noticed its variability (Goodricke 1785). Edward Pigott (1753–1825), another Englishman, discovered the Cepheid variable Eta Aquilae a few months earlier in the same year (Pigott 1785). The North Star, Polaris, is also a Cepheid variable star, the closest one known.

The Cepheids have luminosities up to 100,000 times that of the Sun and masses of 4–20 times the solar mass. Because they are so luminous, these stars can be seen over a wide range of distances, from Delta Cephei, located at only about 272 parsecs, or 887 light-years, from the Earth (Benedict et al. 2002), to galaxies 100 million light-years away (Freedman et al. 2001).

The more luminous a Cepheid variable star is, the more slowly it varies and the longer the period of its luminosity change. This period-luminosity relationship was first discovered from observations of variable stars in the Large and Small Magellanic clouds, which are nearby satellites of the Milky Way. These stellar systems, visible from the Earth's Southern Hemisphere, are named for the Portuguese explorer Ferdinand Magellan (1480–1521), who observed them when his ships were circumnavigating the world for the first time.

At the end of the nineteenth century, Harvard College established an observatory at Arequipa, Peru, with a 0.6 m (24 inch) refractor that was used in a photographic survey of the southern sky, including the Magellanic Clouds. Because of their proximity, the clouds could be resolved into stars. From these photographs, Henrietta Swan Leavitt (1868–1921), a researcher in Cambridge, Massachusetts, found an extraordinary total of 1,777 variable stars. She reported that the brighter stars tended to have the longer cycles of variation (Leavitt 1908). Because the extent of the Magellanic Clouds is small compared to their distance, the relationship of period to apparent brightness also implied a real connection with luminosity. Four years later, Leavitt had obtained precise apparent brightness and period data for 25 variable stars in the Small Magellanic Cloud, thereby establishing the important *period-luminosity relation* for the Cepheid variables (Leavitt 1912).

Once this relation is calibrated suitably by the measurement of a precise distance to one Cepheid variable star using independent methods, observation of the variation period leads to determination of the star's luminosity. Then, using the observed brightness, the distance of the star can be calculated. This technique of measuring distances with Cepheid variable stars has been used to demonstrate the vast extent of the Milky Way – as well as the

Sun's place within it – and, subsequently, to discover the extragalactic nature of spiral nebulae.

The early history of the period-luminosity relation and its calibration has been reviewed by Fernie (1969) and Sandage (1972), and includes the initial calibration. The relation has been used to place constraints on the size and shape of the Milky Way, and to determine slightly incorrect distance estimates for nearby spiral nebulae.

There are the brilliant, younger, and more massive classical Cepheids, of stellar Population I, which are found in the arms of spiral galaxies, and the older, fainter Cepheids of Population II, located in globular star clusters.

Leavitt had no idea why the luminous output of these stars varies, but within a few years the inquisitive English astronomer Arthur Stanley Eddington (1882–1944) showed how (Eddington 1918, 1919). The stars are pulsating with a regular beat, expanding out and contracting in, rising and falling back. And since the star's size is changing, the period-luminosity relation can also be expressed as a period-radius relation. The radial oscillations have been observed using the Doppler shift of spectral lines that arise in the stellar atmospheres.

It follows from Eddington's theory that the pulsation period is inversely proportional to the square root of the star's average mass density, so more rarefied stars pulsate more slowly than dense ones. This also means that the more massive stars, which are also the larger and more luminous stars, possess longer pulsation periods.

Once it was realized that stars are primarily composed of hydrogen and helium, Eddington could show that stellar pulsations might originate in an outer convective zone where hydrogen is alternately ionized and neutral, acting as a valve that absorbs and releases heating radiation during the course of the pulsation.

Over a decade later, the Russian astronomer Sergei A. Zhevakin (1916–2001) showed that the convective ionized hydrogen zone could not maintain the pulsations because it does not absorb sufficient energy during the contraction of the star. He found that an outer region of doubly ionized helium acts as a valve for the heat engine that drives the pulsations of Cepheid variable stars (Zhevakin 1963; Cox 1980). Lang (1999) provides references to individual papers on stellar pulsations. The varying star absorbs the outward flow of energy from the star's center during stellar contraction and repeatedly returns it during expansion.

Because the pulsation depends upon a critical stage of ionization, a star can maintain them only for a specific combination of size and temperature in a narrow range of mass densities. This defines a strip of instability in the Hertzsprung-Russell diagram.

At optically visible wavelengths, astronomers establish the zero point, a, and slope, b, in the *period-luminosity relation* for a variable star:

$$M_V = -a - b \log P, \tag{14.1}$$

where M_V is the absolute visual magnitude and P is the period. A precise calibration of the period-luminosity relation involves an accurate determination of the stellar distances by some independent method. Benedict et al. (2007), for example, used the *Hubble Space Telescope* to obtain the parallax and distance of 10 nearby Cepheid variables, obtaining the relation:

$$M_V = -1.62 - 2.43 \log P, \tag{14.2}$$

where P is the period in days and the uncertainty in the absolute visual magnitude M_V is ± 0.10. This is the period-luminosity relation for classical *Cepheid variables*, also known as *Population I Cepheids*, in the Milky Way.

Example: Distance to Delta Cephei
The prototype Cepheid variable star is Delta Cephei, whose variation period is $P = 5.36634$ days. The period-luminosity relation $M_V = -1.62 - 2.43 \log P$ results in an absolute visual magnitude $M_V \approx -3.40 \pm 0.01$. One could infer the star's distance, D, from the apparent magnitude, m_V, and the relation $m_V - M_V = 5 \log D - 5$, where the distance is in parsecs (Sect. 10.1). Unlike most stars, however, Delta Cephei is a variable star with an apparent magnitude that ranges from $m_V = 3.48$ to 4.37, resulting in uncertainties in the distance determination unless a mean apparent magnitude is used. Instead, the *Hubble Space Telescope* has been use to determine the parallax, $\pi_A = (3.66 \pm 0.15) \times 10^{-3}$ s of arc and a distance of $D = 1/\pi_A = 273 \pm 10$ parsecs and about 887 light-years (Benedict et al. 2002). This distance corresponds to an apparent visual magnitude $m_V \approx 3.8 \pm 0.1$.

It was the American astronomer Harlow Shapley (1885–1972) who observed Cepheids in globular star clusters outside the plane of the Milky Way. He showed that they are distributed within a roughly spherical system, which is centered far from the Sun in the direction of the constellation Sagittarius (Shapley 1918, Fig. 14.3).

These dense stellar clusters orbit the core of the Milky Way in great elongated ellipses, passing through the Milky Way every 100 million years or so. In contrast, the stars within the plane of the Milky Way go around its center in roughly circular orbits with comparable periods for stars about as distant from the core as the Sun is.

Most of the stars and interstellar gas in the Milky Way are located within this flattened, plate-shaped, rotating disk. Its center, known as the *galactic center*, is located at a distance $D_{\odot} \approx 8.5$ kpc, or about 2.5×10^{20} m and 27,700 light-years, from the Sun. This distance is 1.7 billion times the distance between the Earth and

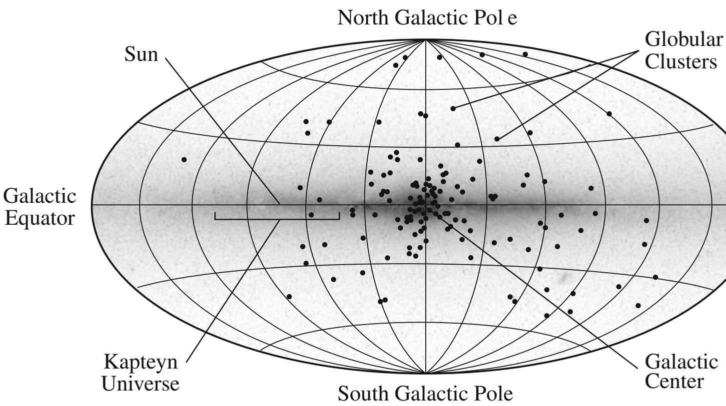

Fig. 14.3 Edge-on view of the Milky Way As shown in 1918 by the American astronomer Harlow Shapley (1885−1972), the globular star clusters are distributed in a roughly spherical system whose center coincides with the core of our Milky Way. The Sun is located in the disk, about 27,700 light-years away from the center. The disk and central bulge are shown edge-on in a negative print of an infrared image taken from the *InfraRed Astronomical Satellite*. The infrared observations can penetrate the obscuring veil of interstellar dust that hides the distant Milky Way from observation at optically visible wavelengths. It is this dust that limited astronomers' view of stars to a much smaller Kapteyn Universe, centered on the Sun

the Sun. Reid (1993) has reviewed determinations of this distance that were available at the time. Even relatively recent estimates indicate that it is quite uncertain with $D_{\odot} = 8.33 \pm 0.35\,\text{kpc}$ (Gillessen et al. 2009).

The disk has a radius of about 50,000 light-years or 15 kpc, and a thickness around 3,000 light-years or 1 kpc. The bright, massive young stars are found in a disk that is only 120 pc or so thick, despite being over 30,000 pc across. Other older types of stars define thicker disks; some of them form disks that are 2,000 pc thick. Assuming a disk thickness of about 1,000 pc and a disk radius of 15,000 pc, the Milky Way has a volume of about 700 billion (7×10^{11}) cubic parsecs.

How many stars are in the Milky Way? The distribution of stars mapped by the *HIPPARCOS* mission indicates that the mass density of the stars near the Sun is $\rho_{disk} \approx 0.515 \times 10^{-20}$ kg m^{-3} $\approx 0.076\, M_{\odot}\,\text{pc}^{-3}$, which is probably accurate to within a factor of two. The local mass density of main-sequence stars has, for example, been estimated at about 0.031 $M_{\odot}$ pc^{-3}, including those of spectral type M that account for the overwhelming majority of stars in the Milky Way (Reid et al. 2002). Assuming a uniform distribution of stars in the Milky Way disk and multiplying the lower mass density estimate by the disk volume of 700 billion cubic parsecs, we obtain a lower limit of at least 20 billion stars with a mass equal to that of the Sun, which is designated $M_{\odot}$. Because the stars are more concentrated toward the central region, the Milky Way most likely contains at least 50 billion to 100 billion stars like the Sun, and we can see only about 5,000 of them with the unaided eye.

The physical parameters of the galactic disk are listed in Table 14.1.

Table 14.1 Physical properties of the Milky Way disk

R_{disk} = radius of disk = 50,000 light-years $\approx$ 15,000 pc $\approx 4.6 \times 10^{20}$ m
L_{disk} = thickness of disk = 3,000 light-years $\approx$ 1,000 pc $\approx 3.0857 \times 10^{19}$ m
$D_{\odot} = R_0$ = Sun's distance from the center = 27,700 light-years = 8.5 kpc = 2.6×10^{20} m
$V_{\odot} = V_0$ = Sun's orbital velocity about center = 220 km s^{-1}
$P_{\odot} = P_0$ = Sun's orbital period about center = 7.6×10^{15} s = 2.4×10^8 years
M_{disk} = mass of disk = $10^{11}\ M_{\odot} \approx 2 \times 10^{41}$ kg
N_{disk} = number of stars in Milky Way = 100 billion or 10^{11} stars like the Sun
L_{Bdisk} = luminosity in blue band = $1.9 \times 10^{10}\ L_{B\odot} \approx 7.2 \times 10^{36}$ J s^{-1}
ρ_{disk} = mass density of disk near the Sun $\approx 0.515 \times 10^{-20}$ kg m^{-3} $= 0.076\,M_{\odot}$ pc^{-3} $= 0.0022\,M_{\odot}$ (light-year)$^{-3}$.
N_{disk} = number density of stars in disk near Sun = 2.59×10^{-51} m^{-3}
S_{disk} = separation of adjacent stars in disk $\approx$ 6.5 light-years $\approx$ 2 pc $\approx 6.2 \times 10^{16}$ m
Age = oldest disk stars = $(6-13.5) \times 10^9$ year = 6 to 13.5 Gyr, young stars are still forming
Oort's constants $A = (+14.82 \pm 0.84)$ km s^{-1} kpc^{-1} and $B = (-12.7 \pm 0.64)$ km s^{-1} kpc^{-1}.
Center of the Milky Way: right ascension α(2,000) = 17 h 45 m 40.04 s, Declination δ(2,000) = $-29°\ 00'\ 28.1''$

Relatively young stars are found in the Milky Way disk; some are seen even in the earliest stages of formation and they are all less than 10 billion years old. These stars are designated *Population I stars*. In addition to their cosmic youth, they contain a relatively high abundance of the heavier elements, commonly called the metals. The Sun is a Population I star; the open star clusters found in the Milky Way contain Population I stars.

The *Population II stars* are found mainly outside the Milky Way, in globular star clusters. They include the oldest known stars of up to almost 14 billion years old, and these stars have a relatively low abundance of elements heavier than hydrogen or helium. A spherical aggregation of Population II, metal-poor stars also is found near the center of the Milky Way. It is mainly closer to the galactic center than the Sun. Ivezic et al. (2012) have reviewed our recent knowledge of galactic stellar populations. The first stars to be formed in the observable universe have been designated *Population III*; Heger (2012) has provided a review of metal enrichment by Population III.

Freeman and Bland-Hawthom (2002) have reviewed the formation of our Galaxy; Putman et al. (2012) summarized our knowledge of the gaseous galactic halo; and Van den Berg et al. (1996) have reviewed estimates for the age of the galactic globular cluster system.

Physical properties of this galactic spheroid are listed in Table 14.2.

Table 14.2 Physical properties of the globular cluster spheroid

R_{gs} = radius of spheroid $\geq$ 130,000 light-years = 40 kpc $\approx 1.2 \times 10^{21}$ m
M_{gs} = mass of spheroid = $(2–10) \times 10^9\ M_{\odot} \approx (4–20) \times 10^{39}$ kg
ρ_{gs} = mass density of spheroid $\approx 0.000\,26\ M_{\odot}$ pc^{-3} $\approx 1.9 \times 10^{-23}$ kg m^{-3}
L_{Bgs} = luminosity of spheroid, blue band = (1–2) x $10^9\ L_{\odot} \approx (4–8) \times 10^{35}$ J s^{-1}

14.1.3 The Rotating Galactic Disk

As suggested by the German philosopher Immanuel Kant (1724–1804) near the end of the eighteenth century, the flattened shape of the Milky Way can be attributed to its formation from a large, collapsing, rotating nebula, much like the origin of our solar system from a considerably smaller nebula (Kant 1755). Observations of the motions of nearby stars and interstellar gas in the galactic disk indicate that the entire stellar system indeed is rotating around a remote axis that pierces the center of the Milky Way. The enormous mass at this central hub steers stars into circular orbits with an orbital speed that decreases with increasing distance from the center – all in accordance with Kepler's third law.

It is rotation that has flattened the Milky Way. Observations of the motions of nearby stars and interstellar gas indicate that the entire system is whirling about a remote, massive center. The enormous mass at this central hub steers the stars into circular orbits with an orbital speed that decreases with increasing distance from the center, at least as far as the Sun, all in accordance with Kepler's third law (Lindblad 1925).

Stars in orbits inside the solar orbit travel faster than the Sun, thereby forging ahead of it, whereas the stars moving in orbits outside the Sun's orbit are falling behind. When viewed from the Earth, nearby stars that are a little closer than the Sun to the center therefore seem to move in one direction, whereas those a little farther away appear to move in the opposite direction, in two star streams (Kapteyn 1905; Joy 1939).

We can measure the radial, or line-of-sight, component of a star's motion by observing the Doppler shift in the wavelength of its spectral lines. Moreover, radio astronomers can use the same Doppler effect with the spectral line of interstellar hydrogen atoms, emitted at a wavelength of 21 cm, to trace out the motions of interstellar gas. Both techniques indicate that the Sun and nearby gas and stars are revolving about the distant center of the Milky Way at a speed of $V_{\odot} \approx 220$ km s^{-1}.

The period $P_{\odot}$ for one rotation of the Sun around the center is $P_{\odot} = 2\pi R_0 / V_{\odot} \approx 7.6 \times 10^{15}$ s $\approx 2.4 \times 10^{8}$ years, where 1 year $= 3.1557 \times 10^{7}$ sec and $R_0 = D_{\odot} = 8.5$ kpc $= 2.6 \times 10^{20}$ m $=$ 27,700 light-years.

The stars are moving with speeds that are greater than those of the planets that orbit them. The orbital speed of the Earth around the Sun, for example, is about 30 km s^{-1} and that of Mercury is just 48 km s^{-1}. The orbiting planets accompany their planet star in its faster motion through space.

Stars and interstellar gas rotate differentially, revolving about the galactic center in independent orbits at speeds that vary with distance. Stars that are nearer to the center of the Milky Way than the Sun revolve about the center at faster speeds and take less time to circle it. Astronomers describe this *differential orbital motion* by specifying Oort's constants (Focus 14.2).

Focus 14.2 Differential rotation of the Milky Way

The Dutch astronomer Jan Oort (1900–1992) provided observational evidence for the differential rotation of stars and interstellar matter in the Milky Way, and described their circular motion about a distant galactic center (Oort 1927, 1928). The Doppler shift velocity along the line-of-sight, or the observed radial velocity, V_r, of a galactic object at the distance, R, from the galactic center is given by:

$$V_r = R_0[\omega(R) - \omega(R_0)]\sin l, \tag{14.3}$$

where R_0 is the distance of the Sun from the galactic center, $\omega(R)$ is the circular velocity of the Milky Way at R, and l denotes the galactic longitude (Fig. 14.4). The observed proper motion μ of the celestial object in galactic longitude and at distance D less than R or R_0 is:

$$\mu = \frac{1}{4.74}\left[-\frac{1}{2}R_0\left(\frac{d\omega}{dR}\right)_{R=R_0}\cos 2l - \frac{1}{2}R_0\left(\frac{d\omega}{dR}\right)_{R=R_0} - \omega(R_0)\right] = \frac{V_\perp}{D}, \tag{14.4}$$

where the velocity $V_\perp$ transverse to the line of sight is in units of km s^{-1}, the distance is in parsecs and the proper motion is in seconds of arc (Sect. 4.2).

Oort's constants, A and B, are given by:

$$A = -\frac{1}{2}R_0\left(\frac{d\omega}{dR}\right)_{R=R_0} = \frac{1}{2}\left[\frac{V_0}{R_0} - \left(\frac{dV}{dR}\right)_{R=R_0}\right] \tag{14.5}$$
$$= 14.82 \pm 0.84 \text{ km s}^{-1}\text{ kpc}^{-1}$$

and

$$B = -\frac{1}{2}R_0\left(\frac{d\omega}{dR}\right)_{R=R_0} - \omega(R_0) = -\frac{1}{2}\left[\frac{V_0}{R_0} + \left(\frac{dV}{dR}\right)_{R=R_0}\right] \tag{14.6}$$
$$= -12.7 \pm 0.64 \text{ km s}^{-1}\text{ kpc}^{-1},$$

where the numerical values are derived from *HIPPARCOS* observations (Feast and Whitelock 1997), which also give the distance of the Sun from the galactic center, R_0, as

$$R_0 = 8.5 \pm 0.5\,\text{kpc} \approx 2.6 \times 10^{20}\text{ m} \approx 27{,}700\,\text{light-years}, \tag{14.7}$$

and a rotation velocity of:

$$V_0 = R_0(A - B) \approx 234 \text{ km s}^{-1}. \tag{14.8}$$

Blauuw et al. (1960) reviewed the IAU system of galactic coordinates. In 1986, the International Astronomical Union adopted the standard values of $R_0 = 8.5$ kpc and a rotational velocity of the Sun about the galactic center, V_0, given by

$$V_0 = R_0(A - B) \approx 220\,\text{km}\,\text{s}^{-1}, \quad (14.9)$$

(Kerr and Lynden-Bell 1986), which implies $A - B = 25.9$ km s^{-1} kpc^{-1}, not quite in accord with the *HIPPARCOS* result that would give $V_0 = 231$ km s^{-1} at $R_0 = 8.5$ kpc.

Oort's constants can be used to determine:

$$V_r = -2A(R - R_0)\sin l \;\; \text{for } R - R_0 \ll R_0 \quad (14.10)$$

and

$$\mu = \frac{1}{4.74}[B + A\cos 2l]. \quad (14.11)$$

For a nearby object in the galactic plane at a distance D of less than one kiloparsec from the Sun, the radial component of velocity due to differential galactic rotation is

$$V_r = A\,D\sin 2l, \quad (14.12)$$

and its transverse velocity due to differential galactic rotation is:

$$V_\perp = D[A\cos(2l) + B]. \quad (14.13)$$

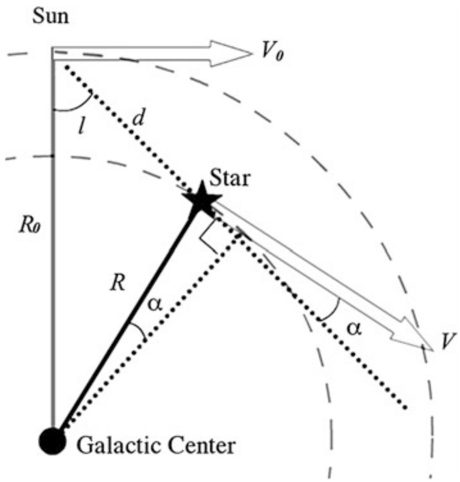

Fig. 14.4 Differential rotation of the Milky Way The Sun rotates about the galactic center, located at a distance R_0 of about 8.5 kpc or 27,700 light-years and a speed of about 220 km s^{-1}. Another star that lies closer to the center revolves about it at a distance R and a different, faster speed. This diagram provides the geometry for deriving Oort's constants $A = 15$ km s^{-1} kpc^{-1} and $B = -10$ km s^{-1} kpc^{-1} that describe this differential rotation about the center of our galaxy

We can use Kepler's third law with measurements of the Sun's distance from and velocity about the remote center of the Milky Way to infer the mass that gravitationally controls solar motion. That is, the galactic disk within the orbit of the Sun has a central mass, M_{cdisk}, of:

$$M_{cdisk} = \frac{4\pi^2 D_\odot^3}{G P_\odot^2} = \frac{V_\odot^2 D_\odot}{G} \approx 1.9 \times 10^{41}\ \mathrm{kg} \tag{14.14}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $P_\odot \approx 7.6 \times 10^{15}$ s, $V_\odot = V_0 \approx 220$ km s^{-1}, and $D_\odot = R_0 \approx 2.6 \times 10^{20}$ m. That is equivalent to about 100 billion solar masses, or 10^{11} $M_\odot$, where the mass of the Sun is $M_\odot = 1.989 \times 10^{30}$ kg. So there is a mass equivalent to about 100 billion stars like the Sun within the Sun's galactic orbit. Fish and Tremaine (1991) have provided a review of the mass of the Galaxy.

The luminosity of our galactic disk, L_{Bdisk}, in blue light is about 25 billion times that of the Sun, or $L_{Bdisk} = 2.5 \times 10^{10}$ $L_{B\odot}$, where the blue luminosity of the Sun is $L_{B\odot} = 3.0 \times 10^{26}$ J s^{-1}. These stars shine with the light of 25 billion Suns. Their combined absolute magnitude is $M_{Bdisk} = -20.5$, and their mass to luminosity ratio, $M_{disk}/L_{disk} \approx 4\ M_\odot/L_\odot$.

14.1.4 Whirling Coils of the Milky Way

The stars do not reside in a uniform whirling disk. They instead are concentrated into arms that coil out from the center of the Milky Way, giving our stellar system a spiral shape. These features are delineated by relatively young, very luminous, and massive stars (Morgan et al. 1952; Georgelin and Georgelin 1976; Paladini et al. 2004), which light up the nearby arms (Fig. 14.5). They coincide with the well-known emission nebulae, or H II regions, which are less than a few million years old and at least a thousand times younger than the oldest stars in the Milky Way. This suggests that recent star formation takes place in the spiral arms of the Milky Way.

Because the Sun is embedded in one of the arms, astronomers must look through that arm to see the rest of the Milky Way. This obscures their distant vision, hiding most of our stellar system from view in optically visible light. However, radio waves pass unimpeded through the obscuring material, permitting the detection of most of the Milky Way. This is because long radio waves are not absorbed by the relatively small particles of interstellar dust.

By observing the radio emission of interstellar hydrogen atoms at a wavelength of 21 cm, radio astronomers constructed a face-on view of the Milky Way, which we might see if we were transported into distant space and looked down on the plane of the Milky Way from above (Fig. 14.6). They delineated extensive, arm-like concentrations that extend out from the short segments defined by young massive stars in the vicinity of the Sun (Oort et al. 1958).

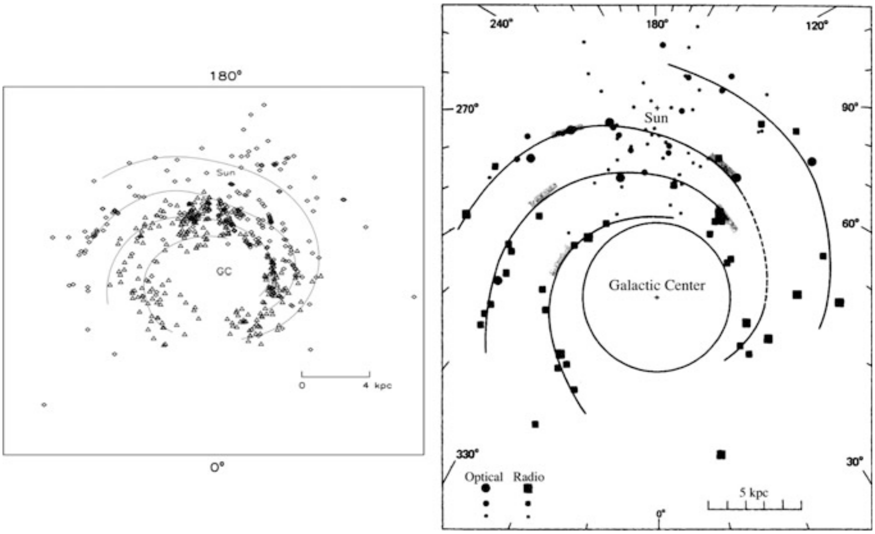

Fig. 14.5 Spiral arms of the Milky Way from H II regions Luminous emission nebulae, known as H II regions, act like beacons that mark out the spiral structure of the Milky Way. The H II regions have lifetimes of just a few million years, which is thousands of times less than the ages of the oldest stars in our Milky Way. This suggests that stars are now formed in the spiral arms of the Milky Way. The center of both diagrams coincides with the center of the Milky Way, labeled as the galactic center or GC, and the galactic longitude is indicated along the figure edges with 180° at center top and 0° at center bottom. The linear scales, shown in the lower right of each diagram, are set at 5 kpc $\approx$ 16,000 light years (*right*) and 4 kpc $\approx$ 13,000 light-years (*left*). The Sun is located at the upper center of both diagrams, and H II regions are denoted by filled circles and squares (*right*) and diamonds and triangles for 550 objects (*left*). [Adapted from (*right*) Y. M. Georgelin and V. P. Georgelin, "The spiral structure of our Galaxy determined from H II regions," *Astronomy and Astrophysics* **49**, 57–69 (1976) and (*left*) R. Paladini R. D. Davies and G. DeZotti, "Spatial Distribution of Galactic H II regions," *Monthly Notices of the Royal Astronomical Society* **347**, 237–245 (2004).]

The Sun has circled the center of the Milky Way more than 19 times during the Sun's 4.6-billion-year lifetime. So, the spiral arms should have wrapped around the massive center many times during the lifetime of the Sun. A persistent dilemma has been why they haven't wound up forming a featureless ball of gas, dust and stars. The explanation seems to be density waves that control the concentrations of stellar and interstellar material (Lin and Shu 1967).

The wave pattern orbits the galactic center at a steady rate and does not wind up; it moves independently of the motions of individual stars, which follow their own orbit around the center. The spiral arms are places where the interstellar material and stars linger – like traffic at a stoplight – and they mark the locations where new stars tend to form and hot, massive, luminous, young stars are found.

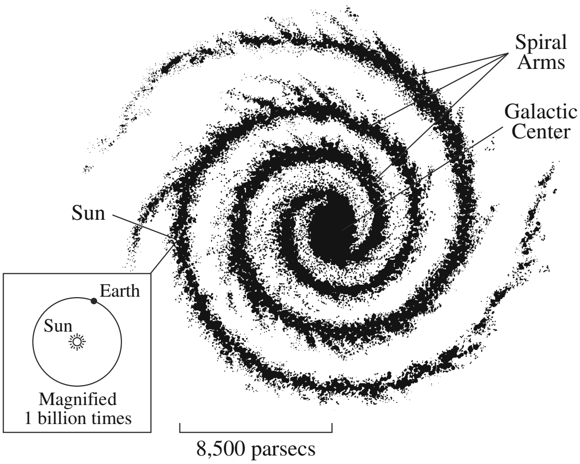

Fig. 14.6 Structure of our stellar system This drawing depicts our Milky Way as viewed from above its plane. The stars and interstellar material are concentrated within spiral arms. The Sun lies within one of these spiral arms at a distance of 27,700 light-years, from the center, designated here as 8,500 pc, or 8.5 kpc. This distance is 1.75 billion times the distance between the Earth and the Sun

14.1.5 A Central Super-Massive Black Hole

Radio astronomers have looked right through interstellar dust and detected an exceptionally powerful and compact radio source at the center of the Milky Way. It is in the direction of the constellation Sagittarius, and therefore been named *Sagittarius A** (abbreviated Sgr A* and pronounced "Sag A" star).

Radio interferometer measurements with very long baselines (VLBI) show that Sgr A* is smaller than our planetary system, with a radius of about half the distance between the Earth and the Sun, which is 1 AU $= 1.496 \times 10^{11}$ m (Doeleman et al. 2008; Reynolds 2008). It seems likely that an exceptionally massive black hole is energizing the extremely bright, compact radio source.

Because such a black hole is very massive, dense, and compact on a cosmic scale, its formidable gravity can dominate a star's motion if it is close enough. The *super-massive black hole* guides nearby stars into rapid orbital motion, betraying its presence. These stars can be seen at infrared wavelengths that also penetrate interstellar dust.

Astronomers have used large visible-light telescopes in Chile and Hawaii to detect individual stars in the infrared, although they shine too faintly at radio wavelengths to be observed with radio telescopes. By watching the motions of infrared stars that are near the center of the Milky Way and orbit it, a central super-massive black hole has been found.

Melia and Falcke (2001) reviewed evidence, available at the time, for a super-massive black hole at the galactic center, whereas Genzel and Townes (1987) provided an earlier review of the center of our Galaxy.

After an unprecedented study lasting more than a decade, researchers were able to track the full revolution of one infrared star, designated S2, around the invisible black hole. This star moves within 17 light-hours of the unseen center, with speeds of up to 5,000 km s^{-1} (Ghez et al. 2005, 2008). These orbital parameters imply that the mass of the central black hole is a colossal 4 million times the mass of the Sun – that is, 4 million invisible solar masses not shining but rather gravitationally confining the observed stellar orbit.

Example: Super-massive black hole at the galactic center
After more than a decade of observations, Andrea Ghez (1965–) and her colleagues reported measurements from the W.M. Keck 10 m telescopes describing the elliptical orbit of the star S0-2 about the galactic center (Ghez et al. 2005, 2008). Its orbit passes within a distance of $D = 17$ light-hours $= 1.83 \times 10^{13}$ m of the black hole, at which time it accelerates to a speed of $V = 5 \times 10^6$ m s^{-1}. If we assume that this velocity is equal to the escape velocity, V_{esc}, of the black hole, of mass, M_{BH}, controlling the orbit at that time, then we can estimate the mass from

$$M_{BH} = \frac{DV_{esc}^2}{2G}, \tag{14.15}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. We obtain $M_{BH} \approx 3.4 \times 10^{36}$ kg $\approx 1.7 \times 10^6$ $M_\odot$ where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg.

Detailed comparison with the stellar orbit indicate that $M_{BH} = (4.1 \pm 0.6) \times 10^6$ $M_\odot$. Using this mass, we can determine the Schwarzschild radius, R_S, of the black hole from:

$$R_S = \frac{2GM_{BH}}{c^2} \approx 1.21 \times 10^{10}\,\text{m} \approx 0.08\ \text{AU}, \tag{14.16}$$

where the speed of light $c = 2.9979 \times 10^8$ m s^{-1} and the mean distance between the Earth and the Sun is 1 AU $= 1.496 \times 10^{11}$ m. VLBI observations at 1.3 mm wavelength indicate that the central radio source Sagittarius A* subtends an angle, θ, of $\theta \approx 4 \times 10^{-5}$ s of arc $\approx 2 \times 10^{-10}$ radians, where 1 radian $= 2.063 \times 10^5$ s of arc (Doeleman et al. 2008). The linear radius R corresponding to this angular size is $R = \theta D \approx 1.7 \times 10^{-6}$ pc $\approx 5 \times 10^{10}$ m ≈ 0.34 AU at the $D \approx 8.5$ kpc distance of the galactic center. That size is about 4 times the gravitational radius of 4 million solar masses.

Where did this super-massive black hole come from and why is it located at the center of the Milky Way? Its formation probably coincided with the origin of our stellar system, by the collapse of a huge rotating mass with a nucleus at the center and the flattened Milky Way spinning around it. The globular star clusters probably date back to the beginning of the collapse, about 14 billion years ago.

The super-massive black hole may have originated at the central nucleus, perhaps as the result of the gravitational collapse of an exceptionally massive cloud of gas located there. Alternatively, it may have grown by the coalescence of smaller stellar black holes, each formed at the end of the lifetime of the first massive stars. The central black hole would have continued to gather in nearby, smaller black holes and surrounding stars and gas, with an active youth and a more sedate old age.

14.1.6 Dark Matter Envelops the Milky Way

As the result of differential rotation, stars and gas near the Sun should revolve about the galactic center at faster speeds than those at greater distances from the center. However, the stars and gas observed near the apparent edges of the Milky Way rotate at speeds that do not decrease with distance. This means that the Milky Way does not end where the light does, and that there are appreciable amounts of dark unseen matter well outside the boundary of the visible Milky Way. That dark invisible matter keeps the fast-spinning visible material connected to our stellar system.

The mass of the Milky Way within the Sun's orbit around the center of the Milky Way is roughly 100 billion, or 10^{11}, Suns. However, the rapid motions of dwarf satellite collections of stars, which revolve about the Milky Way at distances of up to 1 million light-years, indicate that a great reservoir of unseen matter envelops the observed disk of stars. A total mass of roughly 1 trillion, or 10^{12}, times the mass of the Sun and about 10 times the mass of its visible stars, is required to hold onto these dwarf systems. This invisible, massive, outer region is known as the *dark halo*. It surrounds the Milky Way and outweighs it by a factor of about 10.

Example: Mass of the dark halo

We can infer the total mass, M_G, of our stellar system under the assumption that distant, small companions are gravitationally bound to it. (Here we use the subscript G to denote our Galaxy, which is a term that is introduced later in the book.) The dwarf spheroid Leo I is, for example, located at a distance of $D = 230$ kpc, where 1 kpc $= 3.0857 \times 10^{19}$ m, and moves with a radial velocity $V_r = 177$ km s^{-1} $= 1.77 \times 10^5$ m s^{-1}. This velocity must be less than or equal to the escape velocity, V_{esc}, of our stellar system at this distance, so

$$V_r \leq V_{esc} = \left(\frac{2GM_G}{D}\right)^{1/2}, \tag{14.17}$$

or

$$M_G \geq \frac{DV_r^2}{2G}, \tag{14.18}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Inserting the distance and radial velocity of Leo I into this equation, we obtain $M_G \geq 1.67 \times 10^{42}$ kg $\geq 0.8 \times 10^{12}$ $M_\odot$ where $M_\odot = 1.989 \times 10^{30}$ kg. That is about 20 times the total mass of stars in the Milky Way. The mass-to-luminosity ratio is about one hundred times that of stars like the Sun, or $M_G/L_G \approx 100\ M_\odot/L_\odot$, indicating that most of the mass is dark and unseen (Zaritsky et al. 1989; Kulessa and Lynden-Bell 1992; Peebles 1995).

Thus, our stellar system is held together by the gravity of *dark matter*, which is beyond the range of our vision. A similar darkness pervades and controls much of the universe, giving off neither light nor any other radiation to let us know it is there. Dark matter is studied by its gravitational influence on the motions of the stars that we can see.

Moreover, even the observable universe, the part we can see directly, is not limited to our stellar system, the Milky Way, but instead is populated by more than 100 billion galaxies, each composed of about 100 billion stars and perhaps containing 10 times as much mass in unseen dark matter. These galaxies stretch as far as the largest telescope can see – and perhaps beyond.

14.2 The Discovery of Galaxies

Long before the discovery of dark matter, Edwin Hubble (1889–1953), showed that the Milky Way does not contain everything there is, and settled an ongoing controversy about the nature of spiral nebulae. The issue was presented during the now-famous Shapley-Curtis debate over "The Scale of the Universe" during a meeting of the National Academy of Sciences on 26 April 1920 at the Smithsonian Institution in Washington, DC (Shapley and Curtis 1921). Harlow Shapley (1885–1972), of the Harvard College Observatory, defended his novel conception of a much larger Milky Way than previously had been supposed, with a distant center and the Sun at its periphery, but he supposed that the spiral nebulae are embedded in the Milky Way. In contrast, Heber D. Curtis (1872–1942), of the Lick Observatory, attempted to defend a smaller Sun-centered stellar system but provided cogent arguments that the spiral nebulae are distant stellar systems located far beyond the Milky Way (Curtis 1919). Shapley was correct about the shape and size of the Milky Way, and Curtis was correct in supposing that the spiral nebulae are distant "island universes" composed of numerous stars.

The argument over the location of the spiral nebulae was finally and definitely resolved when Hubble used the 2.5 m (100 inch) Hooker telescope on Mount Wilson to photograph the spiral nebula Andromeda, or M 31 (Fig. 14.7), night after night. He compared hundreds of photographs to find Cepheid variable stars whose brightness waxed and waned like clockwork in a period of several days.

Fig. 14.7 The Andromeda Nebula The nearest spiral galaxy, the Andromeda Nebula, also known as M 31 and NGC 224, is located at a distance of about 800 kpc or 2.6 million light-years, so its light takes about 2.6 million years to reach us. Both the Andromeda Nebula and our Galaxy are spiral galaxies with total masses of about 1 million million, or 10^{12}, solar masses, and roughly 100 billion, or 10^{11}, optically visible stars. The several distinct stars surrounding the diffuse light from Andromeda are stars within our own Galaxy; these stars lie well in front of Andromeda. Two smaller galaxies also are shown in this image: M 32, also designated NGC 221, at the edge of the Andromeda Nebula, and NGC 205, that is located somewhat farther away. These are elliptical systems at about the same distance as M 31 but with only about 1/100th of its mass. (Courtesy of Karl-Schwarzschild Observatorium, Tautenburg.)

On New Year's Day 1925, Hubble's results were read *in absentia* at a meeting of the American Astronomical Society in Washington, DC and caused an overwhelming sensation. The landmark paper, titled *Cepheids in Spiral Nebulae* (Hubble 1925), combined the known period-luminosity relation of Cepheids with observations of the variable stars in M 31 and M 33, another spiral nebula, to derive a distance of 0.275 Mpc = 275 kpc for the two spiral nebulae, where 1 Mpc = 3,260,000 light-years = 1,000 kpc = 3.09857×10^{22} m. Their size or linear extent, determined from this distance and their angular extents, was roughly comparable to that of our Milky Way. They had to be remote objects ablaze with stars, galaxies in their own right and separated from the Milky Way by wide gulfs of apparently empty space.

It took so long to establish the true nature of the spiral nebulae because the method of establishing the distances of the remote, luminous Cepheid variable stars needed to be developed, and a large powerful telescope was needed to detect the stars in spiral nebulae and collect enough of their faint starlight for a reliable measurement of their periodic brightness variation and, therefore, distance.

The notable Estonian astronomer Ernst Öpik (1893–1985) had already contributed to the notion that spiral nebulae lie outside the confines of the Milky Way (Öpik 1922). He used F. G. Pease's (1881–1938) measurements of the rotation velocities of M 31 to show that its distance has to be about 0.480 Mpc if its mass to luminosity ratio is comparable to that of stars in our the Milky Way. Öpik was much more nearly correct than Hubble in estimating the distance to Andromeda, whose current distance estimate is about 0.788 Mpc. As Walter Baade (1893–1960) showed nearly three decades later, the period-luminosity relation that Hubble used had been incorrectly calibrated.

Example: Öpik's calculation of the distance to M 31

If a star in the Andromeda spiral nebula is located at a distance R from the center of the nebula, with a rotational velocity V_{rot}, then

$$V_{rot}^2 = \frac{2GM}{R} = \frac{4GM}{\theta D} = \frac{4GlD^2}{\theta D}\left(\frac{M}{L}\right), \tag{14.19}$$

where M is the mass of the galaxy within radial distance R, the angular diameter of the nebula is θ, the distance of the Andromeda nebula from the Earth is D, so $\theta = 2R/D$, and its apparent luminosity is $l = L/D^2$ for an absolute luminosity L. Collecting terms, we can specify the distance to Andromeda by

$$D = \frac{\theta V_{rot}^2}{4Gl}\left(\frac{M}{L}\right)^{-1}. \tag{14.20}$$

By assuming the mass to luminosity ratio M/L was the same as that of the Sun, Öpik (1922) was able to derive a distance of $D = 0.480$ Mpc using the angular diameter and rotational velocity of the Andromeda Nebula measured by F. G. Pease (1918), and the apparent magnitude of Andromeda, at $m = 3.44$ to infer the apparent luminosity.

Once the observed periodic variations of the Cepheid variable stars was correctly calibrated, the distance to Andromeda turned out to be 2.54 million light-years, about three times farther away than Hubble initially supposed – but his dramatic conclusion remained unchanged. Hubble broke through the stars, and moved the outer boundaries of the universe far out into space, enlarging our horizons. The universe was no longer limited to the objects our unaided eyes perceive, and our stellar system had become just one of myriad galaxies located far beyond the Milky Way – which became our Galaxy, written with an uppercase G to show that it is special. All of the other galaxies were shown to be extragalactic, or outside of our Galaxy.

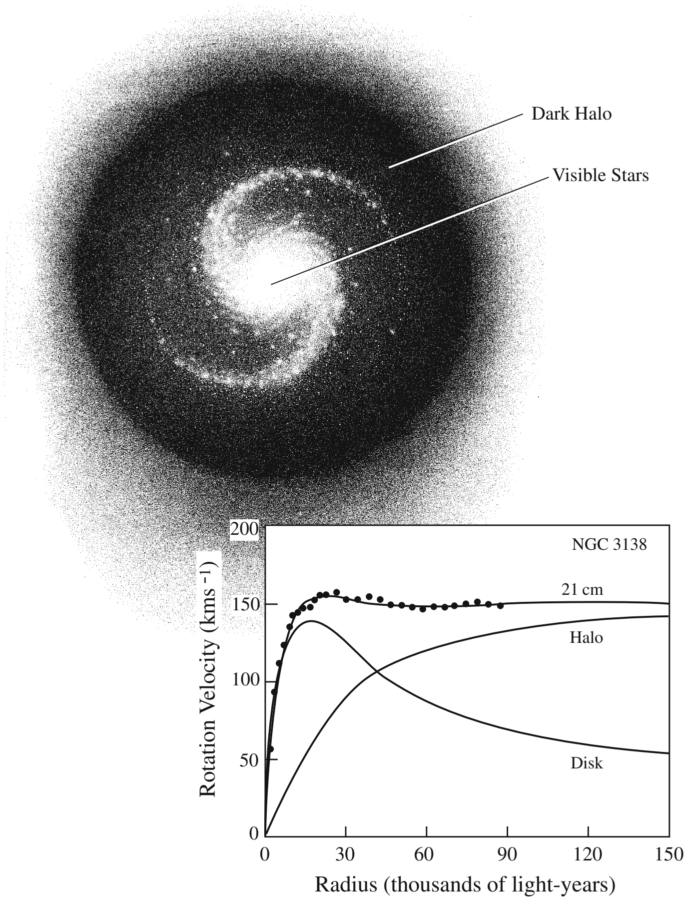

Fig. 14.8 Dark matter envelops a spiral galaxy The rotation velocity of the spinning spiral galaxy NGC 3198 plotted as a function of radius from its center (*bottom*). The observed neutral hydrogen 21 cm data are attributed to an optically luminous disk, containing all the visible stars, and a dark halo that contributes most of the mass at distant regions from the center. The visible stars and surrounding halo of dark matter are illustrated in a hypothetical drawing of a spiral galaxy seen from above (*top*). The fact that the rotational speed of the cool hydrogen gas remains high even at the largest distances indicates that the outermost gas must be constrained and held in by the gravitational pull of dark matter far outside the visible part of the galaxy. [Adapted from T. S. Van Albada et al. "Distribution of dark matter in the spiral galaxy NGC 3198," *Astrophysical Journal* **295**, 305 (1985).]

Hubble called the spirals extragalactic nebulae, but they are not large gaseous nebulae. Hubble (1926, 1936) has described our early knowledge of extra-galactic nebulae, now known as the galaxies.

The spectral characteristics of the light from the spirals are similar to that of our Sun, indicating stellar temperatures of thousands of K. If a spiral were filled with gas at this temperature throughout its enormous dimensions, the nebula would be more than 1 billion, billion times more luminous than the Sun, rather than the much smaller luminosity of Andromeda at about 100 billion times the solar luminosity. This means that light is not coming from the entire surface of a spiral nebula but instead from individual stars, separated by vast spaces without any stars.

Therefore, we now have dropped the nebula designation and use the term *spiral galaxy* or *elliptical galaxy*, depending on their shape. Each galaxy contains about 100 billion stars, just like our Galaxy. The designation *nebula* now is reserved for cloudy, gaseous material enveloping bright stars. An exception is the famous Andromeda Nebula, the closest spiral galaxy.

Like our Milky Way Galaxy, the other galaxies contain about ten times more dark matter then optically visible matter (Fig. 14.8). Sofue and Rubin (2001) have reviewed rotation curves of spiral galaxies; references to individual papers about dark matter that envelops galaxies are given in Lang (1999).

When looking up at the night sky, we see only stars and the black spaces between them. The galaxies are out there, but we cannot see them without a telescope. They are so far away that their brightness is below the detection threshold of the human eye. This meant that the universe was no longer limited to the things our unaided eyes can focus on, and our stellar system became just one of myriads of galaxies.

Blanton and Moustakas (2009) have summarized our knowledge of the physical properties and environments of nearby galaxies. Faber and Gallagher (1979) provided an earlier review of the masses and mass-to-light ratios of galaxies, while Binggeli et al. (1988) reviewed their luminosity function. Binney (1982) has reviewed the dynamics of elliptical galaxies. Mateo (1998) provided a review of dwarf galaxies in the Local Group of galaxies. Van Der Kruit and Freeman (2011) have reviewed galaxy disks.

Some properties of these galaxies are listed in Table 14.3.

The light we receive from the most distant galaxies was emitted before the Earth and the Sun were formed. Even more fantastic, they are all in flight, rushing away from us at speeds that increase with their distance. Nearly a decade before Hubble's determination of the distance of Andromeda, Vesto M. Slipher (1875–1969) already had helped us move beyond the stars in an entirely unsuspected way.

14.3 The Galaxies are Moving Away from us and from Each Other

At the time of their discovery in enormous numbers, most astronomers thought that the spiral nebulae were nascent planetary systems, not galaxies. The bright center was supposed to be a newborn star, and the spiral arms surrounding it were thought to be developing planets, whirling and rotating around the central star just as the Earth revolves around the Sun.

Table 14.3 Physical properties of galaxies[a]

R_g = radius of a spiral or elliptical galaxy ≈ 33 light-years to 326 light-years = 10 kpc to 100 kpc ≈ (3–30) × 10^{20} m
M_S = mass of a spiral galaxy ≈ (10^{11}–10^{12}) $M_\odot$ ≈ (2–20) × 10^{41} kg
M_E = mass of elliptical galaxy ≈ (10^{12}–10^{13}) $M_\odot$ ≈ (2–20) × 10^{42} kg
M_S/L_B = mass to light ratio of spiral galaxy within radius R ≈ 60 h (R/0.1 Mpc) ≈ 42 (R/0.1 Mpc)
M_E/L_B = mass to light ratio of elliptical galaxy within radius R ≈ 200 h (R/0.1 Mpc) ≈ 140 (R/0.1 Mpc)
L_B = mean galaxy luminosity density in blue band = 1.93 × 10^8 h $L_{B\odot}$ Mpc^{-3} ≈ 3 × 10^{34} J s^{-1} Mpc^{-3}
L_g = mean galaxy luminosity density = (2–3) × 10^8 h $L_\odot$ Mpc^{-3} ≈ (5–8) × 10^{34} J s^{-1} Mpc^{-3}
L_x = x-ray luminosity of spiral or elliptical galaxy = 10^{31}–10^{35} J s^{-1}
M_{Sgas} = mass of cool gas of hydrogen atoms and molecules in spiral galaxy ≈ 10^9–10^{10} $M_\odot$ ≈ (2–20) × 10^{39} kg
M_{Egas} = mass of hot gas in elliptical galaxy ≈ 10^9–10^{10} $M_\odot$ ≈ (2–20) × 10^{30} kg
N_g = average volume density of galaxies = 5.52 × 10^{-2} h^3 Mpc^{-3} ≈ 1.89 × 10^{-2} Mpc^{-3}
ρ_g = mass density of galaxies ≈ 0.37 × 10^{-26} h^3 kg m^{-3} ≈ 1.5 × 10^{-27} kg m^{-3} (for visible stars and dark matter with a galaxy mass of about 10^{12} solar masses.)

[a] For Hubble constant H_0 = 100 h km s^{-1} Mpc^{-1} ≈ 75 km s^{-1} Mpc^{-1} with h ≈ 0.75. The mass of the Sun is $M_\odot$ = 1.989 × 10^{30} kg and the total luminosity of the Sun is $L_\odot$ = 3.828 × 10^{26} J s^{-1} and its luminosity in the blue region of the spectrum is $L_{B\odot}$ = 1.9 × 10^{26} J s^{-1}

Percival Lowell (1855–1916), a wealthy Bostonian, had built an observatory in Tucson, Arizona, primarily to detect canals on Mars supposedly built by parched, industrious Martians. Lowell also believed that the spiral nebulae resemble our solar system in its early formative stages; he therefore instructed his staff astronomer, Vesto M. Slipher (1895–1916), to measure their rotations, hoping to gather insight about our own planetary system.

When using the 0.6 m (24 inch) refractor at the Lowell Observatory in Flagstaff, Arizona, to record the spectra of bright spiral nebulae and measure their rotations, he found that they almost unanimously are moving away from us at high velocities. They were also rotating, and a few were approaching – but at modest speeds in comparison to the outward motion of most spirals. The Andromeda Nebula, for example, was moving toward the Earth at an apparent velocity of 300 km s^{-1} (Slipher 1914). However, the other bright spirals were moving in the opposite direction, usually with higher velocities of up to 1,100 km s^{-1}, much faster than any star in the Milky Way (Slipher 1917).

By 1917, Slipher had accumulated spectra of 25 spiral nebulae, using the Doppler effect to measure their radial velocities, and he showed that none of them are at rest (Slipher 1917). All but three were rushing away from us and from each other, dispersing, moving apart, and occupying an ever-increasing volume.

According to Slipher, the observed motions of the majority of spirals indicated a general fleeing from the Milky Way or us. It certainly was difficult to believe that objects with such enormous speeds could long remain a part of our stellar system. The combined gravitational pull of the entire 100 billion stars in the Milky Way is not enough to retain any spiral nebula moving at speeds in excess of 1,000 km s^{-1}.

Example: Escape velocity of the Milky Way
The escape velocity $V_{esc} = (2GM/R)^{1/2}$ of the stellar Milky Way can be determined by using $R = R_0 = 8.5$ kpc $\approx 2.6 \times 10^{20}$ m, the distance of the Sun from the center of the Milky Way, and $M = 10^{11} M_{\odot}$ where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, as inferred from the Sun's orbital velocity around the center. The gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The inferred escape velocity of the stellar Milky Way is about $V_{esc} \approx 3.2 \times 10^5$ m s^{-1} ≈ 320 km s^{-1}. The recession velocities measured by Vesto Slipher for spiral nebulae exceeded this escape velocity by up to four times. When the dark matter that envelops the Milky Way is taken into account, the total mass increases to $M \approx 10^{12} M_{\odot}$ and the escape velocity to about 1,000 km s^{-1}, but most spiral nebulae were soon found to have recessional velocities exceeding this amount.

By 1929, Hubble showed that the measured distances of spirals, which he had established using the superb light-gathering power of the 2.5 m (100 inch) Hooker telescope, were roughly correlated with Slipher's velocities (Hubble 1929). The comparison indicated that the farther a spiral is the faster it is moving away from us. This relationship now is attributed to the expanding universe, which no one had anticipated at the time Slipher made his measurements.

In his publication of these results, titled *A Relation between Distance and Radial Velocity Among Extra-Galactic Nebulae*, Hubble drew a straight line through a plot of the observed data. However, there was a wide dispersion between the plotted points and only a mild tendency for velocity to increase with distance (Fig. 14.9). Nevertheless, his conclusion subsequently was confirmed by more comprehensive observations of a much greater number of galaxies.

The discovery of the expanding universe also explained why the night sky is dark, resolving Olbers' paradox, named for the German astronomer Heinrich Wilhelm Olbers (1758–1840). He realized that the night sky in an infinite, uniform, non-expanding universe should be covered with stars shining as bright as the Sun (Jaki 1969). The expansion of the universe redshifts the most intense light of distant galaxies, and their stars, out of the visible part of the spectrum; therefore, we do not see them and the paradox is resolved.

The connection between velocity and distance is known now as the Hubble law, and the ratio of the velocity of recession of any galaxy and its distance from us is called now the Hubble constant, a fundamental measure of the universe. It is designated by the symbol H_0, in which the H is for Hubble and the zero subscript denotes its current value.

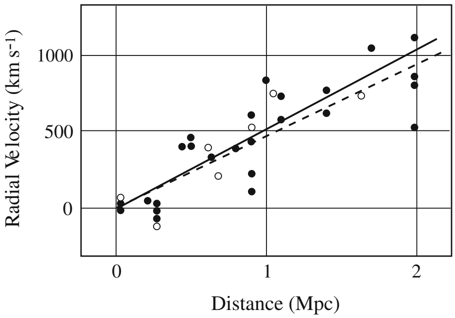

Fig. 14.9 Discovery diagram of the expanding universe A plot of the distance of extragalactic nebulae, or galaxies, versus the radial velocity at which each galaxy is receding from the Earth, published in 1929 by the American astronomer Edwin Hubble (1889−1953). The linear relationship between the distance and radial velocity indicates that the universe is expanding. Vesto M. Slipher (1875−1969) determined most of these velocities more than a decade before this diagram was drawn. Here, the velocity is in units of kilometers per second, abbreviated km s^{-1}, and the distance is in units of millions of parsecs, or Mpc, where 1 Mpc is equivalent to 3.26 million light-years. Hubble underestimated the distances of the spiral nebulae; therefore, the distance scale for modern versions of this diagram is about seven times larger. The filled circles and solid line represent the solution for individual nebulae; the open circles and dashed line are for groups of them

The units of the Hubble constant are given in kilometers per second per Megaparsec, abbreviated km s^{-1} Mpc^{-1}, and Hubble's initial estimate was pegged at 530 in these units. The radial velocity is in units of km s^{-1}, and the distance is in Megaparsecs, abbreviated Mpc, in which 1 Mpc $= 10^6$ pc $= 3.08568 \times 10^{22}$ m is equivalent to 3.26 million light-years. Galaxies typically are separated by a few Mpc, or about 10 million light-years, which is about 100 galaxy diameters. So, the universe is largely empty space relative to galaxies.

According to the now-famous Hubble law, the radial velocity, V_r, of a galaxy, as measured by the Doppler effect, is given by the linear relation:

$$\text{Hubble's Law} = cz = H_0 \times D, \tag{14.21}$$

where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, and z is the redshift. The Hubble constant, H_0, is the ratio of the speed with which distant galaxies are receding from us to their distance, D. It is as though the expanding universe started with a gigantic explosion, with the fastest-moving parts having traversed the greatest distances. The redshift is an observational parameter defined by:

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}, \tag{14.22}$$

which means that

$$1 + z = \frac{\lambda_{observed}}{\lambda_{emitted}} \tag{14.23}$$

for a spectral line emitted at the wavelength $\lambda_{emitted}$ and observed at $\lambda_{observed}$. For relatively low velocities

$$V_r = cz = H_0 D \text{ for } V_r \ll c, \tag{14.24}$$

and for velocities comparable to the speed of light,

$$\frac{V_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \text{ for } V_r \approx c, \tag{14.25}$$

or equivalently:

$$1 + z = \left[\frac{1 + \left(\frac{V_r}{c}\right)}{1 - \left(\frac{V_r}{c}\right)}\right]^{1/2}. \tag{14.26}$$

In just 2 years, Hubble extended the velocity-distance relationship to substantially greater distances, with the help of Milton Humason (1891–1972), who made the velocity measurements (Hubble and Humason 1931, 1934). Because of the impossibility of measuring distances to such faint objects, Hubble and Humason simply assumed that all galaxies have the same intrinsic luminosity. They inferred a velocity-distance relationship by comparing the observed velocities of the galaxies to their apparent brightness. Thus, they reformulated Hubble's law and showed that a linear relationship between distance and recession velocity is valid, within the observational uncertainties, to distances as far as 100 Mpc and radial velocities of nearly 20,000 km s^{-1}.

Since distances can only be measured for relatively nearby extragalactic objects, Hubble's law for remote objects is sometimes expressed by the redshift-magnitude relation in which the apparent magnitude, m, is given by:

$$m = 5 \log\left(\frac{cz}{H_0}\right) + M + 25, \tag{14.27}$$

where M is the absolute magnitude, the Hubble constant H_0 is given in units of km s^{-1} Mpc^{-1} and the factor of $+25$ arises because the distance unit is in Megaparsecs, abbreviated Mpc.

Humason then teamed up with Nicholas Mayall (1906–1993) at the Lick Observatory in an ambitious 25 year project of painstakingly measuring the Doppler-effect redshifts of nearly a thousand galaxies visible from the northern hemisphere. Mayall used the venerable 0.9 m (36 inch) Crossley reflector on Mount Hamilton to observe the brighter galaxies, while Humason observed the fainter ones using the 2.5 m (100 inch) Hooker telescope on Mount Wilson.

Although Hubble had initiated the project, he died before the work was finished, and the analysis was left in the hands of his young protégé Allan Sandage (1926–2010). He obtained a value for the Hubble constant of $H_0 =$ 180 km s^{-1} Mpc^{-1}, or about one third the value previous found by Hubble, whose distance scale was in error. His mistake was not discovered until the early 1950s when red-sensitive photographic plates, developed for military reconnaissance in World War II (1939–1945), became routinely available. Sandage then used them to discover that the brightest stars, which Hubble used to infer distances, are much more luminous emission nebulae.

The resulting redshift-apparent brightness diagram of 474 extra-galactic nebulae, as they preferred to call them, had a large scatter. But a straight line could be drawn through it, out to a radial velocity of 100,000 km s^{-1}, or one-third the speed of light (Humason et al. 1956). So Hubble's law connecting radial velocity and distance still held in every direction as far as one could see. This meant that the entire universe is expanding swiftly and evenly in all directions, with the fastest-moving parts having traversed the greatest distance.

The Hubble constant, H_0, quantifies the current rate of expansion of the universe, and it has often been quantified in the form:

$$H_0 = 100\,h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{14.28}$$

There is an ongoing controversy about the exact value of this important constant, with estimates ranging between 50 and 100 km s^{-1} Mpc^{-1}, and current observational constraints of h lying between 0.50 and 0.85, with a favored value of about 0.75 (Fig. 14.10). The systematic uncertainties in H_0 have decreased as a result of observations with the *Hubble Space Telescope* and the *Spitzer Space Telescope* with recent determinations of $H_0 = 73.8 \pm 2.4$ km s^{-1} Mpc^{-1} (Reiss et al. 2011) and $H_0 = 74.3 \pm 2.1$ km s^{-1} Mpc^{-1} (Freedman et al. 2012). Lang (1999) and Freedman and Madore (2010) have reviewed determinations of the Hubble constant, and Feast and Walker (1987) have discussed Cepheids as distance indicators.

The Hubble constant sets the physical scale of the universe, with the distance to any galaxy given by:

$$D = \frac{cz}{H_0} = 2997.9\left(\frac{z}{h}\right)\,\mathrm{Mpc}. \tag{14.29}$$

The most distant objects exhibit larger redshifts, denoted by the lowercase letter z. The largest observed redshift corresponds to the greatest distance and looks the farthest back in time. Astronomers currently are detecting galaxies out to a redshift as great as $z = 8.6$, the radiation of which was emitted about 13.1 billion years ago (Lehnert et al. 2010).

Once the distance to a galaxy is determined from its redshift and Hubble's constant, one can infer the galaxy's luminosity, L, from it apparent brightness or equivalently its absolute magnitude, M, from its apparent magnitude m. When this is done, a wide range of luminosities is determined. The number of galaxies at different luminosities peaks at a blue luminosity, L_*, given by:

$$L_* \approx 10^{10} L_{B\odot} h^{-2} \approx 2 \times 10^{10} L_{B\odot}, \tag{14.30}$$

where $h \approx 0.75$ and the luminosity of the Sun in the blue spectral region is $L_{B\odot} = 1.9 \times 10^{26}$ J s^{-1} corresponding to an absolute blue magnitude for the Sun

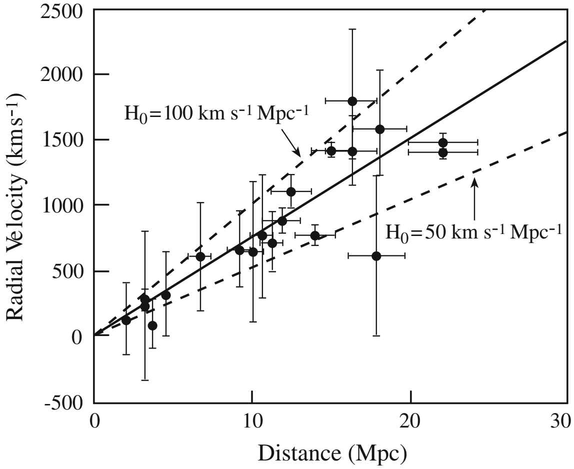

Fig. 14.10 Hubble diagram for Cepheid variable stars This plot of galaxy distance versus recession velocity is analogous to that obtained by Edwin Hubble in his 1929 discovery of the expansion of the universe (see Fig. 14.9). The slope of the linear fit (*solid line*) to the data (*dots*) measures the expansion rate of the universe, a quantity called the Hubble constant, designated H_0. The data shown here summarize 11 years of effort to measure this constant by using the *Hubble Space Telescope* to measure the distances and velocities of Cepheid variable stars in nearby galaxies. The distance is in units of 1 million parsecs, or Mpc, where 1 Mpc is equivalent to 3.26 million light-years; the radial velocity is given in units of kilometers per second, denoted as km s^{-1}. The fit to these data indicate that $H_0 = 75 \pm 10$ km s^{-1} Mpc^{-1} and that this constant lies well within the limits of 50 and 100 in the same units (*dashed lines*). [Adapted from Wendy L. Freedman et al. "Final results from the Hubble Space Telescope Key Project to measure the Hubble constant," *Astrophysical Journal* **553**, 47–72 (2001).]

of $M_{B\odot} = 5.48$. At this luminosity, each galaxy contains at least 20 billion stars like the Sun.

But galaxies can be more or less luminous than L_*, and the range in luminosity is described by a luminosity function $\phi(L)$ that specifies the number of galaxies with luminosities between L and $L + dL$. It is also called the Schechter luminosity function, and it given by (Schechter 1976):

$$\phi(L)dL = \phi_* \left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right)\frac{dL}{L_*}, \tag{14.31}$$

where ϕ_* is a normalization constant for $L = L_*$ and given by:

$$\phi_* \approx 0.02\, h^3\, \mathrm{Mpc}^{-3} \approx 0.0084\, \mathrm{Mpc}^{-3} \tag{14.32}$$

for the total galaxy population, α is the slope of the luminosity function at low luminosity $L < L_*$ where $\phi(L) \propto L^{\alpha}$ and $\alpha = -0.8$ to -1.3. At high luminosity $L > L_*$ there is an exponential cutoff with $\phi(L) \propto \exp(-L)$. In other words, the number of galaxies falls off on the low side of L_* and on the high side of it, in ways described by the two terms in the luminosity distribution.

The wide range in galaxy luminosity corresponds to a wide range in absolute blue magnitude M_B with $M_B = -7.5$ to -22.5, and a peak value M_{B*} given by:

$$M_{B*} \approx -19.7 + 5 \log(h) \approx -20.3, \tag{14.33}$$

where Hubble's constant $H_0 = 100\ h$ km s^{-1} Mpc^{-1} and $h \approx 0.75$.

The space, or volume, density of galaxies, N_g, their blue luminosity density, L_{Bg}, and their total number N_T out to redshift z can be determined by integrating the observed luminosity distribution function obtained from redshift surveys (Focus 14.3). The results indicate that:

$$N_g \approx 0.02\ \mathrm{Mpc}^{-3} \tag{14.34}$$

$$L_{Bg} \approx 3 \times 10^{34}\ \mathrm{J\,s^{-1}\,Mpc^{-3}} \tag{14.35}$$

and

$$N_T \approx 6 \times 10^9 z^3. \tag{14.36}$$

Altogether, there are least 10 billion galaxies in the volume of space that modern telescopes can detect and there is no end in sight.

Focus 14.3 Density and total number of galaxies

The space density of galaxies, N_g, or their density per unit volume, can be determined by adding up, or integrating, the contribution to the luminosity function $\phi(L)$ at different luminosities, L. Expressed mathematically:

$$N_g = \int_0^\infty \phi(L)dL \approx \phi_* \Gamma(\alpha + 1) \tag{14.37}$$

where the normalization constant $\phi_* \approx 0.02\ h^3\ \mathrm{Mpc}^{-1} \approx 0.0084\ \mathrm{Mpc}^{-3}$ at $L_* = 10^{10}\ L_{B_\odot} h^{-2} \approx 2 \times 10^{10}\ L_{B_\odot}$, Γ is a gamma function, and α is low-luminosity slope of the luminosity function.

The blue luminosity density, L_{Bg}, can similarly be defined by:

$$L_{Bg} = \int_0^\infty L\phi(L)dl \approx \phi^* L_* \Gamma(\alpha + 2). \tag{14.38}$$

Redshift surveys can be used to determine these parameters for field galaxies that lie beyond the local concentration of galaxies, known as the Local Group, with the results (Loveday et al. 1992):

$$N_g = 0.0552\, h^3\, \mathrm{Mpc}^{-3} \approx 0.019\, \mathrm{Mpc}^{-3} \tag{14.39}$$

$$L_g \approx 2 \times 10^8\, h L_{B\odot}\ \text{Mpc}^{-3} \approx 3 \times 10^{34}\ \text{J s}^{-1}\ \text{Mpc}^{-3}, \tag{14.40}$$

where Hubble's constant $H_0 = 100\ h$ km s^{-1} Mpc^{-1} and $h \approx 0.75$.

With a total galaxy mass of $10^{12}\ M_\odot$, in both visible and unseen dark matter, this corresponds to a galaxy mass density ρ_g of:

$$\rho_g = 10^{12} N_g M_\odot \approx 10^{-27} \text{kg m}^{-3}. \tag{14.41}$$

From the Hubble law, the distance of a galaxy at redshift, z, is $D = cz/H_0$, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. So the total number of galaxies, N_T, out to redshift z is:

$$N_T = \frac{4\pi}{3}\left(\frac{cz}{H_0}\right)^3 N_g \approx 6 \times 10^9 z^3, \tag{14.42}$$

which is independent of h and the exact value of the Hubble constant.

Since galaxies have been observed out to redshifts greater than one, there are at least 10 billion, or 10^{10}, galaxies in the observable universe. The exact shape and form of the universe complicates the precise calculations, since the curvature of space changes the distances at large redshifts. Still, our estimate should be correct to within an order of magnitude, or a factor of ten.

As Hubble realized, astronomers see only as far as their telescopes permit, eventually reaching a limit – a dim boundary to the observable universe where they measure the shadows. Even now, there is no telescope powerful enough to detect the edge where the galaxies might end. There is no edge to the observable universe and it has no detectable center.

Example: Can visible or invisible matter stop the expansion of the universe?
Throughout most of the past decades, it has been assumed that it is the mass of the universe that curves its shape, establishes its geometry, and determines its fate. Under this assumption, which ignores the more recent discovery of dark energy, the mass density of galaxies, ρ_g, determines the ultimate destiny of the universe. If this mass density exceeds a certain critical value, ρ_c, then gravity eventually overcomes expansion. Imagine the most distant galaxy with mass, m_G, distance, D_G, and velocity, $V_G = H_0 D_G$. Gravity will just balance the expansion of this galaxy if its kinetic energy of expansion is equal to the gravitational potential energy of all of the rest of the universe, or if:

$$\frac{m_G V_G^2}{2} = \frac{m_G H_0^2 D_G^2}{2} = \frac{G m_G M_U}{D_G}, \tag{14.43}$$

where M_U is the total mass of all the rest of the universe inside distance D_G and the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. In other words, the velocity, V_G, of the most distant galaxy is just equal to the escape velocity, V_{esc}, of the entire universe, $V_{esc} = (2GM_U/D_G)^{1/2}$.

Collecting terms we obtain a critical mass density of:

$$\rho_c = \frac{3M_U}{4\pi D_G^3} = \frac{3H_0^2}{8\pi G} \approx 1.88 \times 10^{-26}\, h^2\, \mathrm{m}^{-3} \approx 1.06 \times 10^{-26}\, \mathrm{kg\, m}^{-3}, \quad (14.44)$$

where $H_0 = 100\ h$ km s^{-1} Mpc^{-1} $= 3.24 \times 10^{-18}\ h$ s^{-1} $\approx 2.43 \times 10^{-18}$ s^{-1} for $h = 0.75$.

The space density of galaxies is $N_g = 0.055\ h^3$ Mpc^{-3} (see Focus 14.3), where 1 Mpc = 3.0857×10^{22} m. Assuming that each galaxy has a mass in both visible and unseen dark matter of $M_G = 10^{12}\ M_\odot = 1.989 \times 10^{42}$ kg and $h = H_0/100 = 0.75$, then the mass density of visible galaxies, $\rho_G = N_g M_G \approx 1.57 \times 10^{-27}$ kg m^{-3}. This is about a factor of 10 less than the critical mass density needed to stop the expansion of the universe, even when we have assumed there is ten times more dark matter than visible matter in galaxies.

14.4 Galaxies Gather and Stream Together

14.4.1 Clusters of Galaxies

Astronomers have been mapping the distribution of galaxies for about a century, determining the shape and form of the larger universe. The first cosmic maps were two-dimensional, constructed from catalogues giving the celestial positions of the brightest nebulae. Although a foreground and background galaxy might sometimes coincide, the concentrations were too pronounced to be solely due to such a superposition (Charlier 1922).

The galaxies are not placed randomly throughout expanding space. They are not uniformly strewn here and there or isolated from one another but instead knot together in great clusters that are millions of light-years across (Fig. 14.11). They also contain large additional quantities of unseen matter. A rich cluster of galaxies typically spans 10 million light-years to 20 million light-years and contains hundreds and even thousands of individual galaxies. They move within the cluster at velocities of about 1,000 km s^{-1}, on average, and the amount of time, T_C, required for a galaxy to cross the cluster moving at this speed is about a two billion years.

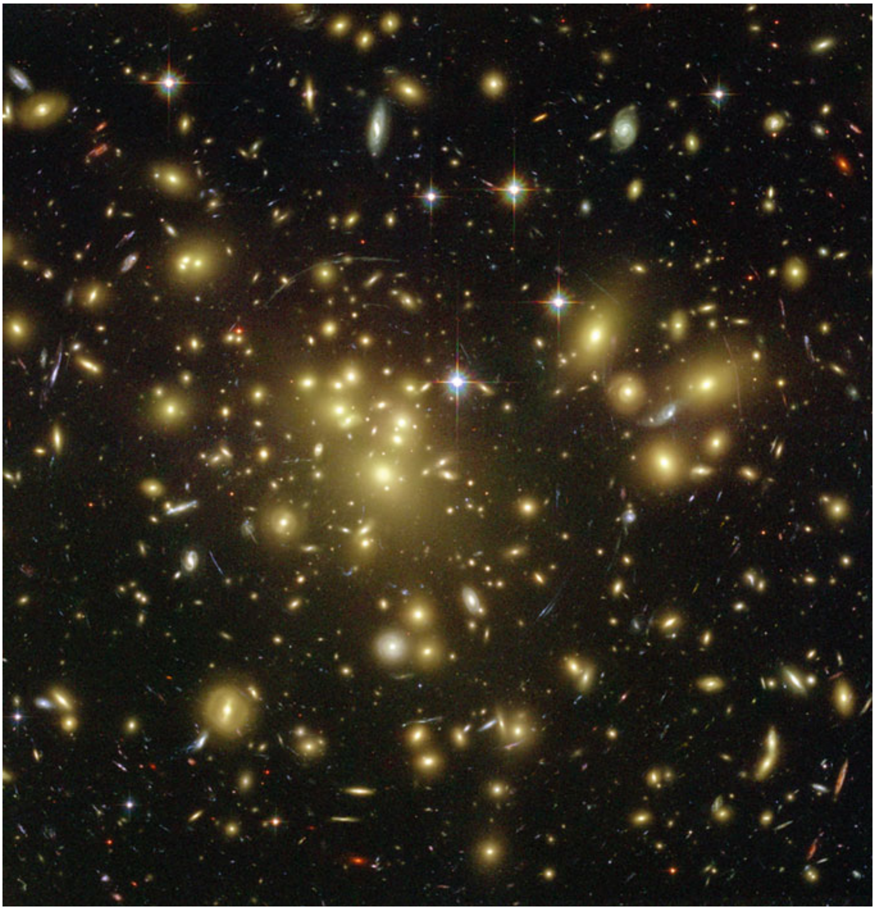

Fig. 14.11 Cluster of galaxies The *Hubble Space Telescope* provided this dramatic view of the center of a massive cluster of galaxies known as Abell 1689, located 2.2 billion light-years away. The gravity of the cluster's million million, or trillion, stars, plus any unseen matter, acts as a gravitational lens in space, bending and magnifying the light of galaxies located far behind it into radiant arcs. (Courtesy of NASA, the ACS Science team of the HST, STScI, and ESA.)

In 1958, George Abell (1927–1983), then a graduate student at Caltech, was able to describe nearly 2,712 rich clusters of galaxies (Table 14.4), using photographic surveys that included both faint and bright galaxies (Abell 1958). Even today, these dense concentrations of galaxies are referred to simply as "Abell clusters" – designated by the word "Abell" or the letter "A" followed by the number in his catalogue.

In addition, as Abell noticed, the clusters gather and congregate together into larger superclusters, which he called second-order clustering. Our Galaxy lies in the outskirts of one, known as the Local Supercluster (de Vaucouleurs 1953),

Table 14.4 Physical properties of rich clusters of galaxies[a]

N_T = total number of galaxies in a rich cluster of galaxies = 30–300
R_{cl} = central radius of galaxy cluster $\approx$ (1–2) h^{-1} Mpc $\approx$ (4.4–8.8) $\times$ 10^{22} m
N_{cl} = volume density of galaxies in galaxy cluster $\approx$ 100 Mpc^{-3}
σ_V = velocity dispersion of galaxy motions $\approx$ 100–1,400 km s^{-1} = 10^5–1.4 $\times$ 10^6 m s^{-1}
M_C = virial mass of galaxy cluster = $\sigma_v^2 R_{cl}/G$ $\approx$ (10^{14}–2 $\times$ 10^{15}) h^{-1} $M_\odot$ $\approx$ (2.8–57) $\times$ 10^{44} kg
T_C = cluster crossing time $\approx$ 2 R_{cl}/σ_v $\approx$ 6 $\times$ 10^{16} s $\approx$ 2 $\times$ 10^9 year
L_B = luminosity of galaxy cluster in blue band = (6 $\times$ 10^{11} to 6 $\times$ 10^{12}) h^{-2} $L_\odot$ $\approx$ (4.6–46) $\times$ 10^{38} J s^{-1}
M_C/L_B = mass to light ratio of galaxy cluster $\approx$ 300 h $M_\odot/L_\odot$ $\approx$ 210 $M_\odot/L_\odot$
L_X = x-ray luminosity of galaxy cluster = ($10^{35.5}$–10^{38}) h^{-2} J s^{-1} $\approx$ 2.0 ($10^{35.5}$–10^{38}) J s^{-1}
n_{cl} = cluster number density $\approx$ (10^{-5}–10^{-6}) h^3 Mpc^{-3} $\approx$ 0.34 (10^{-5}–10^{-6}) Mpc^{-3}

[a] For Hubble constant H_0 = 100 h km s^{-1} Mpc^{-1}. The Sun's mass $M_\odot$ = 1.989 $\times$ 10^{30} kg and the Sun's luminosity $L_\odot$ = 3.828 $\times$ 10^{26} J s^{-1}

which is oriented perpendicular to the Milky Way and extends all the way to the rich Virgo cluster of galaxies.

Bahcall (1977) has reviewed clusters of galaxies, and Bahcall (1988, 1993) has reviewed the large-scale structure in the universe indicated by galaxy clusters. Oort (1983) reviewed superclusters of galaxies, and Rood (1981, 1988) has discussed clusters of galaxies and voids.

14.4.2 Dark Matter in Clusters of Galaxies

Clusters of galaxies are bound together by gravity, even though the expansion of the universe is pulling the galaxies away from one another. We could think that the combined gravitational pull of the numerous galaxies might be sufficient to hold them together but, in 1937, Fritz Zwicky (1898–1974) showed that there must be substantial amounts of unseen material that is keeping the clusters of galaxies from dispersing (Zwicky 1937). In his extraordinarily prescient paper, titled *On the Masses of Nebulae and of Clusters of Nebulae*, he concluded that there must be noticeable quantities of invisible intergalactic matter in clusters of galaxies or else they would be unstable dynamically. In a German language article discussing many of the same topics years earlier, Zwicky introduced the term dunkle materie, or "dark matter" for the invisible stuff, and he concluded that it might be present with a greater density than luminous matter (Van den Bergh 1999).

Zwicky measured the amount of mass required to keep the Coma cluster of galaxies stable, assuming that the motions of its constituent visible galaxies are balanced by the gravitational pull of their combined mass. He found that the total mass of the Coma cluster must be about 10 times the sum of the masses of the individual galaxies it contains. That is, he inferred the total cluster mass, M_C, from the velocity dispersion, σ_V, of the galaxy motions, above that due to the expansion of the universe. For a cluster of radius, R_C, and angular diameter θ, the binding mass required to hold the cluster together is:

$$M_C = \frac{R_C \sigma_V^2}{G} = \frac{\theta c z \sigma_V^2}{2GH_0}, \tag{14.45}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, the mean redshift of the cluster due to the expansion of the universe is z, and the Hubble constant $H_0 = 100\ h$ km s^{-1} Mpc^{-1} with $h \approx 0.75$. Zwicky concluded that this binding mass M_C is about 20 times the mass of the visible galaxies in the Coma cluster, and that if this dark matter were not present the Coma cluster would be flying apart.

Example: Binding mass of the Coma cluster of galaxies
The Coma cluster of galaxies has a redshift $z = 0.0231$, which corresponds to a distance of $D = cz/H_0 \approx 92$ Mpc, using the speed of light $c = 2.9979 \times 10^5$ km s^{-1} and the Hubble constant $H_0 = 75$ km s^{-1} Mpc^{-1}. The galaxies within this cluster have a velocity dispersion, σ_V, of $\sigma_V \approx 1{,}000$ km s^{-1} $= 10^6$ m s^{-1}. There are about 800 identified galaxies within an area of $100' \times 100'$ centered on this cluster, where $'$ denotes a minute of arc. For an angular diameter $\theta = 100' = 6{,}000'' = 0.029$ radians, where 1 radian $= 2.06265 \times 10^5$ $''$ and $''$ denotes a second of arc, the cluster radius R_C is $R_C = \theta D \approx 2.7$ Mpc, and the binding mass required to hold the cluster together is $M_C = R_C \sigma_V^2/G \approx 10^{45}$ kg $\approx 5 \times 10^{14}$ $M_\odot$ where 1 Mpc $= 3.0857 \times 10^{22}$ m, the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg. If each galaxy contains about 10^{11} visible stars like the Sun, then the total visible stellar mass of the 800 galaxies in this part of the cluster is about $0.8 \times 10^{14} M_\odot$. This means that roughly 4.2×10^{14} or about 85 % of the total mass of the cluster has to be in unseen dark matter to bind the Coma cluster of galaxies together.

Hot, x-ray emitting gas has also been found permeating the space between galaxies in massive clusters. Any hydrogen atoms immersed in these clusters and moving at a similar speed to the galaxies would have to be very hot, with a temperature of about 100 million K. At this temperature, the gas is an intense emitter of x-rays.

Example: How hot is the intergalactic gas in a cluster of galaxies?
The thermal velocity, $V_{thermal}$, of a hydrogen atom of mass m_H at temperature T is (Sect. 5.2), $V_{thermal} = (3kT/m_H)^{1/2}$, where the mass of the hydrogen atoms is $m_H = 1.673 \times 10^{-27}$ kg and the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. If the thermal velocity is comparable to the galaxy velocity

dispersion $\sigma_V = 1{,}000$ km s^{-1} = 10^6 m s^{-1} in the Coma cluster of galaxies, then the temperature $T = m_H\sigma_v^2/(3k) \approx 4.04 \times 10^7$ K. Such a hot gas emits intense x-rays. To put it another way, the hot gas with a temperature of about 10^8 K that is observed through the x-ray emission of clusters of galaxies indicates that their intergalactic hydrogen atoms are moving at a velocity comparable to the cluster galaxies.

Although most of the observable mass of the clusters of galaxies is in the form of hot, x-ray emitting gas, outweighing all the visible-light stars within all the galaxies by a factor of about seven, both the stars and the intergalactic x-ray emitting gas constitute only about 15 % of the total mass of the galaxy clusters. The remaining 85 % of the gravitating material is some kind of mysterious dark matter, emitting no detectable radio, visible-light or x-ray radiation, and no one knows what it is.

In his 1937 paper, Zwicky also proposed that the formidable gravity of dark matter in clusters of galaxies would act as a powerful lens, diverting, and focusing the light of more distant galaxies. Arthur Stanley Eddington (1822–1944) had previously noted that the gravitational effect of a star would produce multiple images and magnification of images of a more remote background star located behind it (Eddington 1920). Later, Albert Einstein (1879–1955) discussed the effect for stars, but stated that there was no hope of observing such a stellar lens (Einstein 1936; Renn et al. 1977). However, it was not until the early 1980s that astronomers began to observe that effect for the much more massive clusters of galaxies. They spread the light of a distant galaxy that lies directly behind them into an array of faint, tangentially stretched arcs.

Hubble Space Telescope (*HST*) images of rich clusters of galaxies reveal the highly stretched, distorted, and magnified images of faint galaxies lying far behind them (Figs. 14.12 and 14.13). As Zwicky proposed, the gravitational-lens effect provides information on both the visible and unseen matter. Moreover, the dark matter can act like a zoom lens, magnifying distant galaxies too faint to be seen and bringing them into view.

The presence of two galaxies along the same line of sight, one more distant than the other, was suggested from the spectroscopic database of the Sloan Digital Sky Survey, and these gravitational-lens candidates were confirmed using the high-resolution imaging capability of the *HST*. Hundreds of these cosmic gravitational lenses have been found. Some exhibit partial or complete "Einstein" rings, which indicate near-perfect alignment of the foreground and background galaxies (Figs. 14.14 and 14.15). In 1924, Orest Chwolson (1852–1934) described the production of such a ring by a gravitational lens (Chwolson 1924); the first observed Einstein ring was a radio source. Even a double ring arising from the light of three aligned visible-light galaxies has been found.

Treu (2010) has reviewed strong lensing by galaxies, and Blandford and Narayan (1992) reviewed cosmological applications of gravitational lensing.

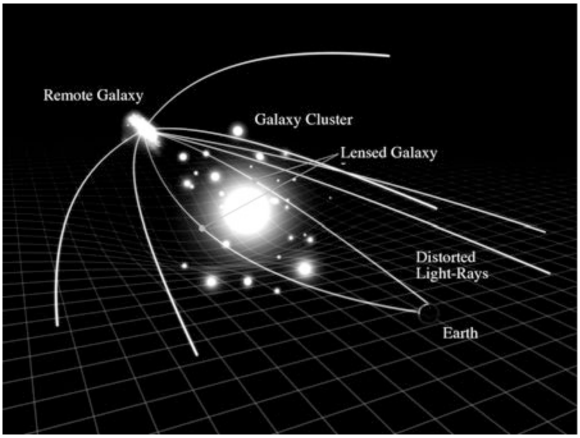

Fig. 14.12 Galaxy cluster lens Very distant and faint galaxies can be investigated by observing them through a cluster of galaxies. The powerful gravitation of the cluster acts like a lens, bending, focusing, and magnifying the light of more distant galaxies that lie behind it (see Fig. 14.14). The gravitational lens action can distort the light from the background galaxies into faint arcs or produce magnified images of individual galaxies that otherwise would remain invisible. (Courtesy of NASA/JPL-Caltech.)

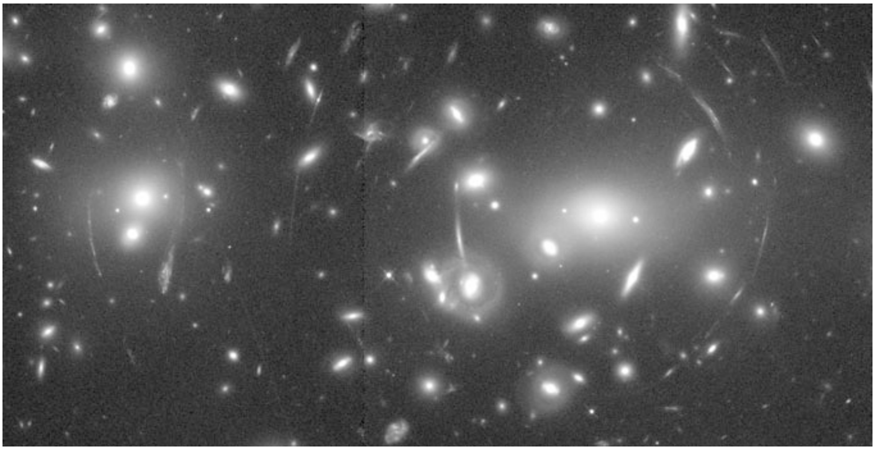

Fig. 14.13 Cluster of galaxies and gravitational lens A *Hubble Space Telescope* image of a rich cluster of galaxies designated Abell 2218. It is about 1,000 Mpc or 3 billion light-years away from the Earth. A typical rich cluster contains hundreds and even thousands of galaxies, each composed of hundreds of billions of stars and possibly up to 10 times more mass within invisible dark matter. The galaxy cluster Abell 2218 is so massive and so compact that its gravity bends and focuses the light from galaxies that lie behind it. Multiple images of these background galaxies are distorted into long faint arcs. Magnified or ring images of individual background galaxies also can be observed. (Courtesy of NASA/STScI/Andrew Fruchter/the ERO team, Sylvia Baggett/STScI, Richard Hook/ST-ECF/Zoltan Levay, STScI.)

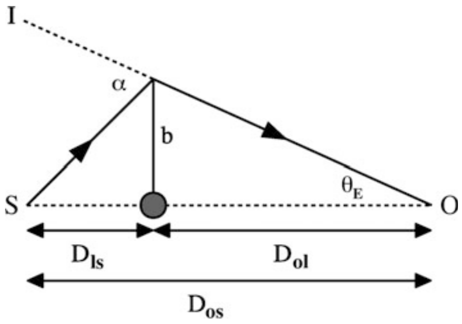

Fig. 14.14 Gravity lens geometry A remote galaxy, labeled S, and an intervening lens galaxy, denoted by L and represented by the *dark filled circle*, lie in a straight line along the observer's line of sight at distances denoted by D and its subscripts. Only rays passing by the lens at the distance b will reach the observer, at O, forming a ring image with an angular radius denoted by θ_E (see Fig. 14.15)

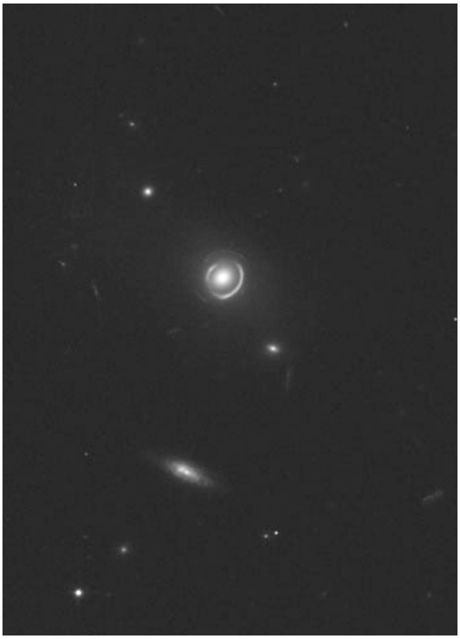

Fig. 14.15 Einstein ring When a background and foreground galaxy are aligned perfectly, the closer galaxy acts as a gravitational lens, bending and magnifying the light of the more distant galaxy and forming a glowing "Einstein" ring. A double ring is captured in this *Hubble Space Telescope* image, indicating an exceptionally rare alignment of a massive foreground galaxy with two background galaxies; the distances of the three galaxies are estimated at 3 billion, 6 billion, and 11 billion light-years from the Earth. (Courtesy of NASA/ESA/Raphael Gavazzi and Tommaso Trea, University of California at Santa Barbara and the SLACS team.)

Example: When a galaxy acts as a gravitational lens

When a light ray from a distant source passes within a distance, b, of an object of mass M, then the bending angle α (see Fig. 14.4), is given by

$$\alpha = \frac{4GM}{c^2 b} \text{ radians} \tag{14.46}$$

where the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

When a light ray passes the limb, or edge, of the Sun's visible disk, the mass is $M = M_\odot = 1.989 \times 10^{30}$ kg, the Sun's mass, and $b = R_\odot = 6.955 \times 10^8$ m, the radius of the Sun. In this case:

$$\alpha = 0.849 \times 10^{-5} \text{ radians } = 1.75'', \tag{14.47}$$

where 1 radian $= 2.06265 \times 10^5\,''$ and the symbol $''$ denotes seconds of arc.

When a lens object, denoted by subscript l and with mass, M, is perfectly aligned along the line of sight to a more distant source, denoted by the subscript s, the light from the distant source will be deformed into a ring (Chwolson 1924) called the Einstein ring. The angular diameter, θ_E, of the ring is given by (Chwolson 1924; Einstein 1936; Lang 1999):

$$\theta_E = \left(\frac{4GM}{c^2 D}\right)^{1/2} = \left(\frac{4GMD_{ls}}{c^2 D_{ol} D_{os}}\right)^{1/2} \text{ radians,} \tag{14.48}$$

where the effective distance $D = D_{ol}D_{os}/D_{ls}$, the distance from the observer to the lens mass is D_{ol}, the distance from the observer to the background source is D_{os}, and the distance from the lens to that source is D_{ls} (also see Fig. 14.4).

If the lens mass is a galaxy containing 100 billion stars like the Sun, with a mass $M = 10^{11}\, M_\odot$, that is located halfway between the observer and the source, with $D_{ls} = D_{ol} = D_{os}/2$, then:

$$\theta_E = \left(\frac{2GM}{c^2 D_{ol}}\right)^{1/2} \text{ radians} \approx 3.55 \times 10^{12} D_{ol}^{-1/2}\,''. \tag{14.49}$$

For a lens galaxy of redshift $z = 0.5$, the radial velocity $V_r = cz$ and the distance $D_{ol} = cz/H_0 \approx 2.0 \times 10^3$ Mpc $\approx 6.17 \times 10^{25}$ m, where the speed of light $c = 2.9989 \times 10^5$ km s^{-1}, the Hubble constant $H_0 = 100\, h$ km s^{-1} Mpc^{-1}, the $h \approx 0.75$, and 1 Mpc $= 3.0856 \times 10^{22}$ m. For this value of D_{ol} we obtain $\theta_E = 0.45''$, and such rings have been imaged from the *Hubble Space Telescope* (see Fig. 14.15).

14.4.3 Cosmic Streams

The galaxies are not simply flying outward with the expansion of the universe in a smooth and regular manner. Entire groups are streaming together over vast distances in various directions. These so-called peculiar motions are caused by the gravitational pull of huge assemblages of galaxies, and they are not related to the uniform expansion of the galaxies, known as the Hubble flow.

Davis and Peebles (1983) reviewed evidence for local anisotropy in the Hubble flow, and Burstein (1990) reviewed large-scale motions in the universe.

For instance, the nearest large galaxy, Andromeda (M 31), is moving toward the Milky Way at a velocity of about 300 km s^{-1}. A distance of about 778 kpc currently separates the two galaxies, but they are set on an irrevocable collision course. They will meet in a possibly destructive encounter in a few billion years.

Example: When will Andromeda enter the Milky Way?
The distance of the Andromeda nebula from the Earth is $D = 778$ kpc $= 2.40 \times 10^{22}$ m, where 1 kpc $= 3.0857 \times 10^{19}$ m. The observed radial velocity of Andromeda is $V_r = -301$ km s^{-1}, with the negative sign indicating approaching motion. Andromeda will collide with the Milky Way in a time $T = D/V_r \approx 8.0 \times 10^{16}$ s $\approx 2.5 \times 10^{9}$ years $\approx$ 2.5 billion years, where 1 year $= 3.1557 \times 10^{7}$ s.

In addition, an entire swarm of galaxies can set off on a trajectory that is independent of the expansion. Because the galaxies are moving together over vast distances of hundreds of millions of light-years, their collective behavior is known as a large-scale streaming motion, which is large in space but not so big in velocity – generally no more than 1,000 km s^{-1}.

All of the galaxies in our region of space, within a volume of 30 Mpc across, are rushing en masse toward the same remote point in space. All of these galaxies are being pulled through space, forced into mass migration by the gravitational pull of “The Great Attractor,” located at a distance of about 5 Mpc away. Its mass is equivalent to about 50 million billion (5×10^{16}) stars like the Sun and at least 500,000 galaxies like the Milky Way. The Great Attractor most likely is a rich and massive cluster of galaxies, part of an even larger super-cluster.

Occasionally we observe other galaxies colliding, merging, or passing through each other (Fig. 14.16) when they should be moving farther apart. This mingling also is due to the gravitational attraction of neighboring galaxies, drawing them together and producing local eddies within an outward Hubble flow.

Barnes and Hernquist (1992) have reviewed the dynamics of interacting galaxies.

It is the gravitational interaction of galaxies with one another that distorts the smooth cosmic expansion, producing the peculiar motions superposed on the expanding universe. However, the uniform Hubble flow gathers speed with

Fig. 14.16 Colliding galaxies Gravitational interaction of the Antennae galaxies, catalogued as NGC 4038 and NGC 4039, produces long arms of young stars in their wake. The colliding galaxies are located about 62 million light-years from the Earth and have been merging for the past 800 million years. As the two galaxies continue to churn together, clouds of interstellar gas and dust are shocked and compressed, triggering the birth of new stars. This composite image is from the *Chandra X-ray Observatory* (*blue*), the *Hubble Space Telescope* (*gold and brown*), and the *Spitzer Space Telescope* (*red*). The blue x-rays show huge clouds of hot interstellar gas, the red data show infrared radiation from warm dust clouds that have been heated by newborn stars, and the gold and brown data reveal both star-forming regions and older stars. (Courtesy of NASA/ESA/SAO/CXC/JPL-Caltech/STScI.)

distance. Because the localized streaming motions are limited in velocity, they are relatively slow when compared with the expansion speed of remote galaxies. The very existence of the large-scale streaming motions nevertheless indicates a decidedly uneven and lumpy distribution of galaxies, which eventually was mapped across billions of light-years.

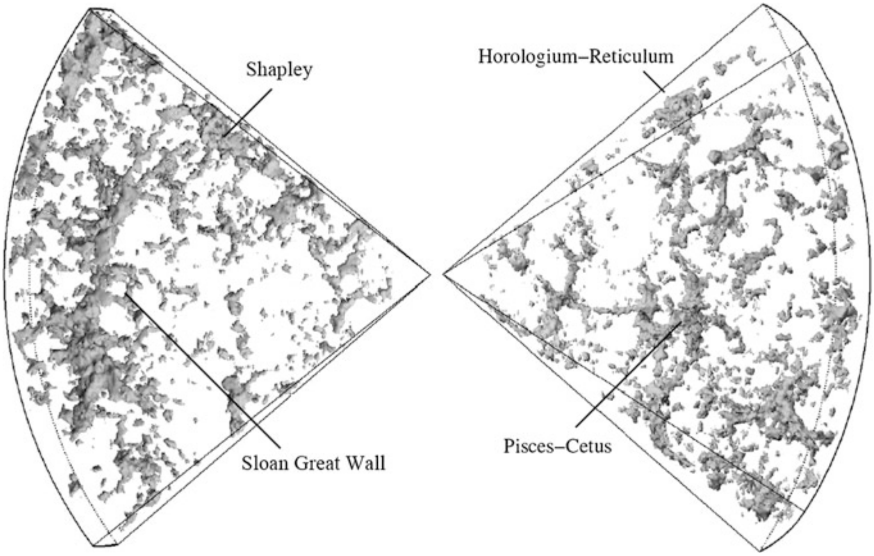

Fig. 14.17 Great walls and voids By measuring the recession velocity, or redshift, of galaxies, astronomers determined their distance and combined it with their location in the sky to obtain the three-dimensional distribution of galaxies. The map shown here is for galaxies within 1 billion light-years (*far left or far right*) from the Earth (*center*). Because galaxies started to form about 12 billion years ago, this is a relatively nearby part of the universe. It includes recession velocities of up to 30,000 km s^{-1}, at a redshift of about $z = 0.1$. The galaxies are concentrated in long, narrow sheet-like walls encircling large empty places known as voids, about 100 million light-years across. The Sloan Great Wall (*left*) spans about 1.4 billion light-years. It may be gravitationally unbound, perhaps beginning to fall apart, but it includes superclusters of galaxies that may stay bound together by their mutual gravitational pull. The Sloan Great Wall was discovered using data from the Sloan Digital Sky Survey in 2004. Other superclusters, or clusters of galaxy clusters, are labeled in the diagram, which is from the Two Degree Field Galaxy Survey. (Courtesy of Willem Schaap, Kapteyn Institute, University of Groningen et al., 2dF Galaxy Redshift Survey.)

14.4.4 Galaxy Walls and Voids

By determining the concentrations of galaxies in different directions and at various redshifts, or depths, astronomers located places where the collective force of gravity pulled galaxies together and locally reversed the uniform expanding motion. The three-dimensional maps reveal fascinating lace-like patterns that connect the galaxies and curving filaments that enclose dark, seemingly vacant places (Lapparent et al. 1986). The galaxies apparently are distributed along the peripheries of gigantic hollow bubbles.

The early redshift surveys also delineated an enormous sheet of galaxies, dubbed "The Great Wall," at distances ranging from 350 million to 500 million light-years away (Geller and Huchra 1989). Giovanelli and Haynes (1991) provided a review of the redshift surveys of galaxies available at that time.

Subsequent three-dimensional maps were obtained using electronic technology that permits the simultaneous measurement of hundreds of galaxy redshifts in a single exposure at a large telescope. The Sloan Digital Sky Survey, for example, revealed the longest sheet of galaxies yet seen (Gott et al. 2005; Einasto et al. 2011). Dubbed the "Sloan Great Wall," it measures 1.37 billion light-years across (Fig. 14.17); and it is the largest observed structure in the universe – at least so far.

Thus, everywhere they look, in whatever direction and near or far, modern telescopes are finding a complex and richly textured universe, filled with luminous concentrations of matter. There is no perceptible end to the lumps and clumps and vacant places, and no one knows where the unevenness will end. Even when looking across 10 % of the observable universe, astronomers continue to find galaxy structures crossing their maps from edge to edge, as well as smaller bubbles, walls, and voids that are nestled together.

All parts of the observable universe are bound within this all-encompassing fabric, glued together by the invisible forces of gravity, suspended in space by motion, and linked by radiation. This all-embracing cosmic web extends throughout the observable universe (Fig. 14.18).

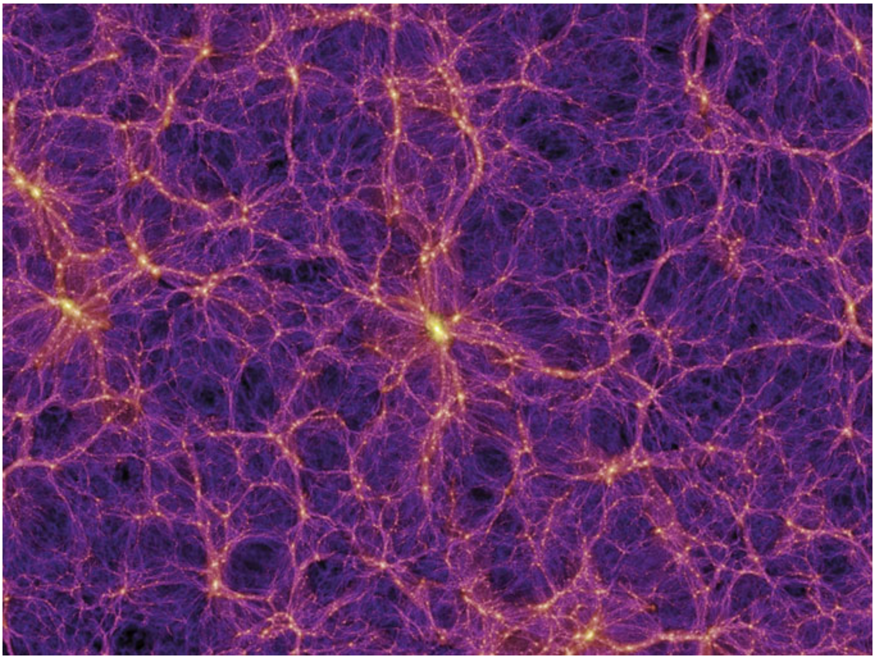

Fig. 14.18 Cosmic web One moment in the ever-changing distribution of galaxies studied using a supercomputer to trace out their formation, evolution, and clustering. The width of this image is about 10 million light-years. (Courtesy of Volker Springel, the Millennium Simulation Project/ Max Planck Institute for Astrophysics, Garching, Germany.)

14.5 Looking Back into Time

Because light travels at a finite speed, to look far into space is to look back into time. When we look farther out into space, we travel back more in time. Large telescopes that detect the faint light of distant objects therefore can be used as time machines to see objects as they were in the past, when their light was emitted, as they were then and not as they are now. In effect, astronomers watch cosmic history race toward us at the speed of light, 299,792 km s^{-1}. The look-back time is simply the amount of time it takes for light to travel from the object to us at that speed.

Moving at the speed of light, it takes 2.3 million years for light to travel from the nearest spiral galaxy, Andromeda, to the Earth. Astronomers have observed radiation from distant galaxies whose light was emitted 13 billion years ago, long before the Sun was formed about 4.6 billion years ago. Therefore, the look-back times for galaxies range from millions to billions of years, spanning an enormous period in which we can watch them evolve.

Some of the most distant galaxies may no longer exist, but they were embryonic galaxies when the light now reaching the Earth began its journey. These galaxies may have perished over time, but their light can survive unchanged, helping us trace out the history of the observable universe from the big bang – about 13.7 billion years ago – to now. As long as a ray of light passes through empty space and encounters no atoms or electrons it will persist forever.

For an object at redshift z, we can specify the look-back time, t_L, at which the radiation was emitted. Looking at objects at larger and larger redshift is looking further and further into the past. The look-back time can be expressed in terms of the Hubble time, t_H, or the reciprocal of the Hubble constant, H_0. For small redshifts $z \ll 1$, we have

$$t_L = zt_H = \frac{z}{H_0} = 9.778 \times 10^9 \frac{z}{h} \text{years} \tag{14.50}$$

where the Hubble constant $H_0 = 100\, h$ km s^{-1} Mpc^{-1}, 1 Mpc $= 3.0857 \times 10^{19}$ km, and 1 year $= 3.1557 \times 10^7$ s. The Hubble time is

$$t_H = \frac{1}{H_0} \approx 3.0857 \times 10^{17} \frac{1}{h} \text{s} \approx 4.1 \times 10^{17} \text{s} \approx 13 \times 10^9 \text{years}. \tag{14.51}$$

At large redshift, the look-back time is given by:

$$t_L = \frac{2}{3H_0\Omega_0^{1/2}(1+z)^{3/2}}, \tag{14.52}$$

where the density parameter $\Omega_0 = \rho_0/\rho_C$, the ratio of the present mass density of the universe, ρ_0, to the critical mass density, $\rho_C = 3H_0^2/(8\pi G) = 1.879 \times 10^{-26}\, h^2$ kg m^{-3}, for a Hubble constant $H_0 = 100\, h$ km s^{-1} Mpc^{-1}, needed to close the universe in the future.

Example: Viewing a distant galaxy

Suppose a galaxy is located at a distance of $D = 100$ Mpc. Assuming a Hubble constant of $H_0 = 75$ km s^{-1} Mpc^{-1}, the radial velocity of the galaxy is $V_r = H_0 \times D = 7{,}500$ km s^{-1} and its redshift is $z = V_r/c = 0.025$. The Hubble time $t_H = 1/H_0 = 4.11 \times 10^{17}$ s $\approx$ 13 billion years, where we use 1 Mpc $= 3.0857 \times 10^{19}$ km and 1 year $= 3.1557 \times 10^7$ s. The look back time $t_L = zt_H = 0.325$ billion years.

If the redshift of a galaxy is $z = 3.0$, then the radial velocity is obtained from $\frac{V_r}{c} = \frac{(z+1)^2-1}{(z+1)^2+1}$, or $V_r = 0.88\,c \approx 2.64 \times 10^5$ km s^{-1}, where the speed of light $c = 2.9979 \times 10^5$ km s^{-1}. Assuming that the density parameter $\Omega_0 \approx 10^{-2}$, then the look-back time for this galaxy is $t_L = 0.83\ t_H = 10.8$ billion years, and about 3 billion years after the big bang that occurred about 13.7 billion years ago.

The Hubble time is the approximate time when the expansion of the universe began. Gravity can slow the expansion to a lesser age, while dark energy can accelerate it, giving a greater age. But the Hubble time provides a pretty good estimate of the age of the expanding universe. Its beginning was not always known with such precision. For a while, it looked as if the Earth was older than the universe itself, leading to a new steady state theory for the universe (Focus 14.3). Kragh (1997) discusses the historical development of the steady state and expanding universe theories.

Focus 14.4 How old is the observable universe?

If the observable universe had a beginning, it must have a finite age, and estimates of that age have lengthened as our astronomical knowledge improved. When the known universe was confined largely to the Earth, an age of 10 million to 100 million years was inferred from the time it would take for the planet to cool from an initially molten state (Kelvin 1862, 1899; Burchfield 1975). The discovery of radioactivity then provided a new source of energy to keep the Earth hot inside (Rutherford 1905), and radioactive elements were used to determine an age of the oldest rocks on the Earth, first at 2 to 3 billion years (Rayleigh 1905; Boltwood 1907; Rutherford 1929; Holmes 1949) and eventually at 4.6 billion years old (Patterson 1956).

When it was realized that subatomic nuclear energy keeps the stars shining, an evaluation of the their nuclear-reaction rates indicated stellar lifetimes of up to 10 billion years (Schönberg and Chandrasekhar 1942). Ages between 10 billion and 20 billion years were inferred from studies of the chemical evolution of our Galaxy (Fowler and Hoyle 1960; Tinsley 1975) as well as the thermonuclear evolution of stars in globular star clusters (Sandage 1970; Hesser et al. 1987; Chaboyer 1995).

Another way of estimating the age of the known universe is to watch the expanding galaxies. They are moving away from a beginning, a cosmic horizon, which is as far away as we ever can see. The distance between there and now, as well as the age of the observable universe, is estimated by calculating how long it has taken for the galaxies to move this far at their observed speeds.

If the expanding universe has been moving along at a steady pace, then its age can be determined from the present rate of expansion. That rate is quantified by the current value of the Hubble constant, the reciprocal of which provides an expansion age. The currently accepted value of 75 km s^{-1} Mpc^{-1} corresponds to an expansion age of 13.0 billion years. A lower value of the Hubble constant would imply that the universe is expanding now at a slower rate and has a greater age. A higher value of the constant implies a more rapid expansion and that less time has elapsed since the expansion started.

The universe may not have been moving apart with an unchanging speed. Its expansion would be slowed by the mutual gravitational pull of all of the dark matter in the universe or accelerated by anti-gravitational forces. Moreover, today's value of the Hubble constant has always been imprecise due to the streaming motions of nearby galaxies, which are not part of their expansion, and by uncertain distances for remote galaxies.

Hubble's measurements of the constant that now bears his name, pegged at 530 in the typical units, caused quite a problem. It corresponds to an expansion age of only 1.8 billion years. At about the same time that Hubble was making his observations, Ernest Rutherford (1871–1937) and his colleagues at the Cavendish Laboratory in England used radioactive dating to estimate an age of at least 3.4 billion years for the oldest terrestrial rocks (Rutherford 1929). How could the Earth be older than the expanding universe?

Arthur Stanley Eddington (1882–1944) was not at all troubled by the discrepancy. He simply extended the beginning of the universe farther back in time than the start of its expansion (Eddington 1933). In this interpretation, the world once existed in an unmoving state described by the cosmological constant that Albert Einstein (1879–1955) previously introduced to stop the eventual collapse of a static, nonexpanding universe (Einstein 1917a, b). An initial nonmoving universe of an indeterminate age could be sent into unrestrained expansion whenever we choose by the slightest disturbance, which upsets the balance between gravitational attraction and cosmological repulsion. For Eddington, a disrupting disturbance about 2 billion years ago caused a slight expansion that thinned out the universe, making it less able to resist the cosmic repulsion, and the runaway expansion of the galaxies began.

Three young scientists at Cambridge University – Hermann Bondi (1919–2005), Thomas Gold (1920–2004), and Fred Hoyle (1915–2001) – noticed that we can adjust the cosmological constant to accommodate almost any related observation and the theory thereby lost its simplicity and uniqueness. So the trio proposed a universe that had no beginning (Bondi and Gold 1948; Hoyle 1948). The cosmos, they proclaimed, may have always existed, presenting an unchanging steady state on the largest scales of space and time. We would no longer have to attribute the observable universe to a past creation that was inaccessible to scientific scrutiny or understanding.

They acknowledged the inescapable fact that the galaxies are moving apart but supposed that they have always been doing so, while new matter is being created continuously at an average rate of just one hydrogen atom per cubic meter of space per year. That is just sufficient to counteract the dispersal and thinning out of the expanding universe and keep the overall universe unchanged with time.

The age problem that led to serious consideration of the Steady State Theory was partially resolved by Walter Baade (1893–1960) during World War II (1939–1945). As luck would have it, Eastman Kodak had just developed a red-sensitive emulsion for wartime reconnaissance. Pushing the 2.5 m (100 inch) telescope to its very limits, Baade used the red-sensitive plates to resolve the nucleus of the nearby Andromeda galaxy, distinguishing individual red giant stars in the crowded center. His measurements indicated that these red-colored stars were similar to those found in the globular clusters of our own Galaxy, but different from the highly luminous blue-colored stars found in the outer arms of both the Milky Way and Andromeda.

When the 200 inch telescope on nearby Palomar Mountain began operation, in 1948, Baade continued his investigations of the red and blue kinds of stars, and concluded that they obey different period-luminosity relationships. This meant that Hubble had been confused when applying the distance calibration of Cepheid variable stars in globular clusters to the other types of variables in the arms of nearby spiral nebulae. So Baade made the necessary corrections, obtaining a distance of about 2 million light-years for Andromeda, which reduced the Hubble constant by about half and enlarged both the scale and age of the expanding universe by a factor of about two (Baade 1952).

Although a consummate observer, Baade tended to avoid detailed analysis and written accounts of his discoveries. So it was fortunate that Henrietta H. Swope (1902–1980), the daughter of the wealthy president of the General Electric Company, joined Baade to assist with the analysis of his excellent photographs. They used the results to propose a downward revision of the Hubble constant to a value of 100, in the usual units (Baade and Swope 1955). Allan Sandage (1926–2010) then continued to correct for Hubble's mistaken identification of the brightest stars in Andromeda; in 1958, he announced that the elusive constant had a value of 75 (Sandage 1958). Using Sandage's measurements, the expansion age is about 13 billion years. Therefore, the universe was considerably older than the oldest terrestrial rocks, and scientists were reassured that the observable universe had a definite beginning.

Estimates of the age and size of the observable universe haven't changed all that much over the past decades, but the uncertainties have become smaller. The *Hubble Space Telescope* and the *Spitzer Space Telescope* have been used to refine estimates of Hubble's constant by observing Cepheid variable stars, obtaining values of 72 ± 8 (Freedman et al. 2001), $H_0 = 73.8 \pm 2.4$ km s^{-1} Mpc^{-1} (Reiss et al. 2011) and $H_0 = 74.3 \pm 2.1$ km s^{-1} Mpc^{-1} (Freedman et al. 2012).

Recent estimates indicate that everything we know about, the whole observable universe, is not older than 13.7 billion years. All that we can observe has a history that can be traced back to that beginning, but most of what we see is significantly younger. Perhaps the most interesting consequences are that the observable universe is about three times older than the Earth, and that the light we see from the most distant galaxies was emitted before our solar system existed.

The exact mathematical equations for the look-back time can be complicated, and the uncertainties increase at larger redshifts. Nevertheless, the uncertainties in the distances and look-back times are no larger than those caused by our imprecise knowledge of the Hubble constant. Even at a redshift $z = 5.0$, amongst the largest ones observed, the look-back time ranges from 10 to 15 billion years depending on the choice of the Hubble constant and the mass density of the universe. So we can ignore the effects of space curvature in most practical computations.

Throughout most of the past decades, it has been assumed that it is the mass of the universe that curves its shape, establishes its geometry and determines its fate. Under this assumption, which ignores the more recent discovery of dark energy, the mass density of galaxies, ρ_{G}, determines the ultimate destiny of the universe. If this mass density exceeds a certain critical value, ρ_{c}, then gravity will eventually overcome expansion. Imagine the most distant galaxy with mass, m_G, distance D_G, and velocity $V_G = H_0 D_G$. Gravity will just balance the expansion of this galaxy if its kinetic energy of expansion is equal to the gravitational potential energy of all of the rest of the universe, or if:

$$\frac{m_G V_G^2}{2} = \frac{m_G H_0^2 D_G^2}{2} = \frac{G m_G M_U}{D_G}, \tag{14.53}$$

where M_U is the total mass of all the rest of the universe inside distance D_G and the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. In other words, the velocity, V_G, of the most distant galaxy is just equal to the escape velocity, V_{esc}, of the entire universe, $V_{esc} = (2GM_U/D_G)^{1/2}$.

Collecting terms we obtain a critical mass density of:

$$\rho_c = \frac{3M_U}{4\pi D_G^3} = \frac{3H_0^2}{8\pi G} = 1.879 \times 10^{-26} h^2 \,\mathrm{kg\,m^{-3}} \approx 1.0 \times 10^{-26} \,\mathrm{kg\,m^{-3}} \quad (14.54)$$

where the Hubble constant $H_0 = 100\,h$ km s^{-1} Mpc^{-1}, one parsec = 1 pc = 3.0857×10^{16} m, 1 Mpc = 10^6 pc, and the Newtonian gravitational constant $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}. For $h = 0.75$, we have $\rho_c \approx 10^{-26}$ kg m^{-3}.

14.6 Using Einstein's *General Theory of Relativity* to Explain the Expansion

Edwin Hubble (1889–1953) relied solely on the power of observation, preferring to avoid what he called "the dreamy realms of speculation" (Hubble 1936), most likely referring to theoretical physicists. Although an expanding universe was a possible consequence of Albert Einstein's (1879–1955) *General Theory of Relativity* (Einstein 1917a, b), Hubble thought that such models were a forced interpretation of the observational results. Even as late as 1953, shortly before his death, Hubble insisted that his law should be formulated as an empirical relation between observed data (Hubble 1953). The Belgian astrophysicist and Catholic priest, Georges Lemaître (1894–1966) had nevertheless already interpreted the radial velocities of spiral nebulae in terms of Einstein's theory in 1927.

After being ordained a priest in 1923, Abbé Lemaître spent a year at Cambridge University as a student of Arthur Stanley Eddington (1882–1944), reviewing Einstein's *General Theory of Relativity*, and during the next 2 years, Lemaître studied at the Massachusetts Institute of Technology and worked with Harlow Shapley at the nearby Harvard College Observatory.

The Belgian cleric also toured the country, meeting Vesto Slipher (1875–1969) at the Lowell Observatory in Flagstaff, Arizona, and Edwin Hubble at the Mount Wilson Observatory, California. As a result, he learned all about the latest measurements of the redshifts, or radial velocities, of spiral nebulae, interpreting them as a cosmic effect of the expansion of the universe, all in accordance with the relativity theory, and additionally derived a theoretical expression for the linear increase of their velocities with distance (Lemaître 1927).

But we now see this in hindsight, and in 1927 most scientists were not even aware of the observational support for an expanding universe. Lemaître also published his interpretation in a fairly obscure journal, the *Annales de la Societé Scientifique de Bruxelles*. So practically no one was aware of his findings, which either went unnoticed or were ignored. At this time, no other astronomer or physicist had used the *General Theory of Relativity* to explain the observed universe, and it wasn't until a few years later that Eddington sponsored an English translation of Lemaître's paper in the *Monthly Notices of the Royal Astronomical Society* (Lemaître 1931, b, c). In English, the title read *A Homogeneous Universe of Constant Mass and Increasing Radius Accounting for the Radial Velocity of*

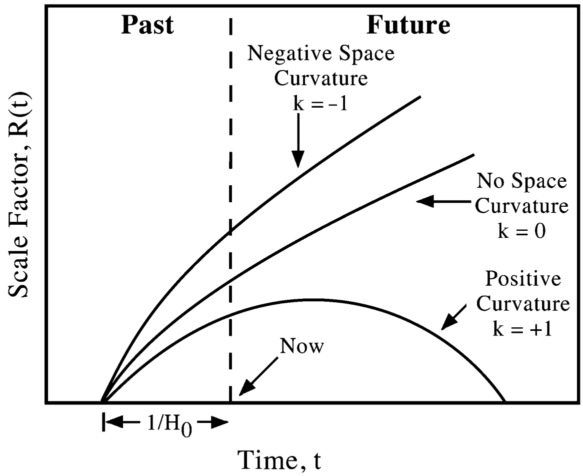

Fig. 14.19 Size of the expanding universe Schematic representation showing the size, or scale factor $R(t)$, of the expanding universe as a function of time, t. The approximate age since the expansion began is given by $1/H_0$ where H_0 is the Hubble constant. Three models describe future possibilities for the universe with no dark energy or cosmological constant. It can become closed with positive space curvature, denoted by space curvature constant $k = +1$, forever open with no space curvature and described by Euclidean space with $k = 0$, or always open with negative space curvature constant $k = -1$

Extra-Galactic Nebulae. Since he was the first to relate the observed galaxy motions to the theory, Lemaître's solution was widely heralded for its novelty and it led to greater appreciation of the significance of Hubble's discovery.

We cannot tell exactly how far away the distant galaxies are, or precisely how long ago their radiation began its journey, until we understand the curvature of space. If space is curved, the path through space might be noticeably longer than that expected in flat, un-curved space, with larger distances and greater look-back times. The detailed expressions for the distance and look-back time therefore depend upon the amount of space curvature, or the mass density of the universe, as well as the redshift.

Schneider (2006) provides a nice textbook of extragalactic astronomy and cosmology, whereas Sandage (1988) and Lang (1999) have reviewed observational tests of world models.

The line element or metric, ds, for a homogenous, isotropic expanding universe is the Robertson-Walker metric (Robertson 1935, 1936; Walker 1936):

$$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \quad (14.55)$$

where the space curvature constant $k = -1$, 0 and +1, and $R(t)$ is called the radius of curvature or the scale factor of the universe (Fig. 14.19). The r coordinate has zero value for some arbitrary fundamental observer, the surface $r =$ constant has the geometry of the surface of the sphere, θ, ϕ are polar coordinates, t is the time coordinate and c is the speed of light.

If light was emitted from an extragalactic object at time t_e, then its redshift z is given by:

$$1 + z = \frac{R(t_0)}{R(t_e)}, \tag{14.56}$$

where the subscript zero is used to denote the present epoch, and $R(t_0)$ is the present value of $R(t)$ at time t_0. As previously mentioned, the observed wavelength, λ_0, of a spectral line emitted by an extragalactic object is longer than the emitted wavelength, λ_e, with a redshift z defined by:

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e}. \tag{14.57}$$

At any cosmic time t, we can define a Hubble expansion parameter, $H(t)$, by:

$$H(t) = \frac{\dot{R}(t)}{R(t)} = \frac{dR(t)/dt}{R(t)}, \tag{14.58}$$

where the $\cdot$ denotes differentiation with respect to time, a deceleration parameter, $q(t)$:

$$q(t) = -\frac{\ddot{R}(t)R(t)}{\dot{R}^2(t)} = -\frac{1}{H_0}\frac{\ddot{R}(t)}{\dot{R}(t)}, \tag{14.59}$$

and a density parameter, $\Omega(t)$, by:

$$\Omega(t) = \frac{8\pi G\rho(t)}{3H^2(t)} \tag{14.60}$$

for mass density $\rho(t)$.

For a homogeneous, isotropic expanding universe of mass-energy density ρ and zero cosmological constant $\Lambda = 0$ (the cosmological constant is discussed in Sect. 15.6.2), we obtain:

$$H^2(t) = \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2(t)} \quad \text{for } \Lambda = 0, \tag{14.61}$$

where G is the gravitational constant, and:

$$\frac{\ddot{R}(t)}{R(t)} = \frac{-4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \quad \text{for } \Lambda = 0, \tag{14.62}$$

where P is the pressure.

These two equations are sometimes called the Friedmann equations, since the Russian mathematician Aleksandr Friedmann (1922–1924) first derived them, but he had no idea that they might apply to the observable universe (Friedmann 1922, 1924). They can be rearranged to imply a conservation of energy relation:

$$\dot{\rho}(t)c^2 = -3\frac{\ddot{R}(t)}{R(t)}\left(\rho c^2 + P\right). \tag{14.63}$$

At the present time, denoted by t_0, the expansion parameter is the Hubble constant $H(t_0) = H_0 = 100\ h$ km s^{-1} Mpc $^{-1}$, with a value of $h \approx 0.75$. Moreover, in the present matter-dominated era the pressure $P_0 = 0$, the radiation energy density $\rho_r(t_0)$ can be omitted when compared to the mass density $\rho_m(t_0) = \rho_0$, and the deceleration parameter is given by:

$$q_0 = q(t_0) = \frac{4\pi G}{3H_0^2}\rho_0 = \frac{\rho_0}{2\rho_C} \text{ for } \Lambda = 0, \tag{14.64}$$

the density parameter Ω_0 is given by:

$$\Omega_0 = \Omega(t_0) = \frac{\rho_0}{\rho_C} \text{ for } \Lambda = 0, \tag{14.65}$$

and ρ_C, the critical mass density needed to close the universe is given by:

$$\rho_C = \frac{3H_0^2}{8\pi G} = 1.879 \times 10^{-26}\ h^2 \text{ kg m}^{-3} \approx 1.0 \times 10^{-26} \text{ kg m}^{-3}, \tag{14.66}$$

where the Hubble constant $H_0 = 100\ h$ km s^{-1} Mpc^{-1} $= 3.24 \times 10^{-18}\ h$ s^{-1}, one parsec = 1 pc = 3.0857×10^{16} m, 1 Mpc = 10^6 pc, and the Newtonian gravitational constant $G = 6.693 \times 10^{-11}$ N m^2 kg^{-2}.

We also have the relation:

$$\frac{kc^2}{R^2(t_0)} = H_0^2(2q_0 - 1) \text{ for } \Lambda = 0. \tag{14.67}$$

For model universes with zero cosmological constant, or $\Lambda = 0$, we have three possibilities with three different values of the space curvature constant k, describing a closed, Euclidean or open universe (Fig. 14.19).

If $k = 1$ then $q_0 > 0.5$, $\rho_0 > \rho_C$ and $\Omega_0 > 1$ for elliptical closed space and an oscillating universe in which $R(t)$ reaches a maximum in the future. The universe would then eventually turn back upon itself and reform the dense fireball of its youth.

If $k = 0$ then $q_0 = 0.5$, $\rho_0 = \rho_C$ and $\Omega_0 = 1$ for a flat, Euclidean space without curvature and an ever-expanding Einstein-De Sitter universe (Einstein and De Sitter 1932), in which $R(t)$ continues to forever increase with time, t. The universe is then poised between open and closed, right on the dividing line.

If $k = -1$ then $0.0 < q_0 < 0.5$, $\rho_0 < \rho_C$, and $\Omega_0 < 1$ for a hyperbolic open space and an ever-expanding Milne universe (Milne 1935) in which $R(t)$ also continues to forever increase with time, t. The inward pull of gravity is too weak to ever quell the outward expansion of the universe.

The cosmological constant has been revived with the discovery of mysterious dark energy that is now accelerating the expansion of the universe, and the appropriate adjustments to our equations are given in Sect. 15.6.2.