Chapter 4
Cosmic Motion

4.1 Motion Opposes Gravity

4.1.1 Everything Moves

All that exists, from atoms to planets and stars to galaxies, is always moving. This motion keeps cosmic objects suspended in space.

Galileo Galilei (1564–1642) imagined an ideal world in which there are no external forces acting on an object, and supposed that such an object will keep on moving at constant speed (Galilei 1632). Isaac Newton (1643–1727) extended the idea in his first law of motion, which states that every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it (Newton 1687). In other words, a moving object continues in motion with the same speed and in the same direction unless an external force is applied to it.

The most significant outside force in the universe is that of gravity, and it is motion that opposes gravitational attraction. Motion and gravity together shape the universe, giving it form and structure. So everything moves, and the way cosmic objects move is governed by the rules of motion and gravitation.

4.1.2 Escape Speed

The energy of motion is known as kinetic energy, and for a mass m moving at speed V, the kinetic energy is $mV^2/2$, so the faster something moves the more kinetic energy it has. If an object moves fast enough, and its kinetic energy becomes large enough, it can overcome the gravitational forces acting upon it and move out of their sphere of influence.

The minimum speed required to counteract and overcome the gravitational force on an object is known as the escape speed, since the object can then escape into surrounding space. The escape speed, denoted V_{esc}, needed for a small body of

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mass, m, to break away from the gravitational pull of a larger mass, M, is obtained by equating the kinetic energy of the small mass to the gravitational potential energy holding it in (Sect. 3.2). That is:

$$\text{Kinetic energy} = \frac{1}{2} m V_{esc}^2 = \frac{GMm}{D} = \text{Gravitational potential energy}, \tag{4.1}$$

or

$$V_{esc} = \left[\frac{2GM}{D}\right]^{\frac{1}{2}} = \sqrt{\frac{2GM}{D}}, \tag{4.2}$$

where D is the distance between the centers of the larger and smaller mass, and the Newtonian constant of gravitation $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

Although the escape speed is often called the escape velocity, the escape speed does not depend on the direction of motion, whereas strictly speaking a velocity includes both the speed and direction. No matter what the direction of travel is, an object moving at the escape speed can break away from another object's gravitational force, provided, of course, that it isn't directed into the surface of the larger mass.

The escape speed is independent of the small mass m, and it is dependent only on the distance D of the small mass and the value of the big mass M. At larger distances the escape speed becomes smaller because the strength of the gravitational force exerted by the big mass is less.

Any object, from an atom to a rocket, must move faster than the escape speed at a planet's surface if it is to move off into surrounding space. The reason why there is no hydrogen in the Earth's atmosphere, for example, is that at large altitudes, up in the ionosphere, the temperature is so high that the light-weight hydrogen atoms move at speeds greater than our planet's escape speed, and evaporate off into space. To obtain the surface escape speed of an object at its radius R, just let $D = R$ in the expression for escape speed.

Example: Escape speed of the Earth, Moon, and Sun

What is the minimum speed needed for a rocket to escape from the gravitational pull of the Earth, V_{escE}, and from the Moon, V_{escM}? We can use the expression $V_{esc} = (2GM/R)^{1/2}$, where the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the M is the mass of the object, R is its radius, and the ½ superscript denotes the square root. For the Earth, the mass $M_E = 5.9736 \times 10^{24}$ kg and the mean radius $R_E = 6.371 \times 10^6$ m, to give $V_{escE} = 1.12 \times 10^4$ m s^{-1}. For the Moon, we have a mass of $M_{moon} = 7.348 \times 10^{22}$ kg and a mean radius of $R_{moon} = 1.737 \times 10^6$ m, to give $V_{escM} = 2.38 \times 10^3$ m s^{-1}. That explains why a lunar lander requires much less rocket propulsion to leave the Moon to rejoin its orbiting command module than either spacecraft needs to leave the Earth. If the command module was orbiting the Moon in synchronous orbit, to remain always above the same point on the Moon, it would have to have an orbital period equal to the Moon's rotation period of 27.3 Earth days = 2.36×10^6 s, and the orbital distance, D, of the command module would be $D = [GM_{moon} P^2/(4\pi^2)]^{1/3} \approx 8.8 \times 10^7$ m, which is a substantial portion of the mean distance between the Earth and the Moon, of 3.844×10^8 m, resulting in poor visibility of the lunar surface from the command module. It has to move much faster around the Moon and closer to it. It is the Sun that dominates the mass of the solar system, with a mass of $M_{\odot} = 1.989 \times 10^{30}$ kg. The escape velocity at the visible disk of the Sun is $V_{esc\odot} = (2GM_{\odot}/R_{\odot})^{1/2} \approx 6.18 \times 10^5$ m s^{-1}, where the Sun's radius $R_{\odot} = 6.955 \times 10^8$ m. The most distant comets reside in a remote reservoir, known as the Oort cloud, located at a distance of $D \approx 100{,}000$ AU = 10^5 AU. The orbital period of such a comet will be $(10^5)^{3/2} \approx 32$ million years, and its orbital speed will be about $V = 2\pi D/P \approx 94$ m s^{-1}, a very slow and leisurely motion by cosmic standards; note 1 AU = 1.496×10^{11} m and 1 year = 3.156×10^7 s. This makes sense, for the orbital speed will fall as the inverse square root of the distance. The Earth orbits the Sun at an average orbital speed of about 30 km s^{-1} = 30,000 m s^{-1}.

Using the mass and radius of the Earth, the Sun, and the Earth's Moon, we obtain respective escape speeds of 11.2, 618 and 2.38 km s^{-1}. If you want to send a rocket off into interplanetary space, it has to move faster than the escape speed at the Earth's surface, about 11.2 km s^{-1} = 1.12×10^4 m s^{-1}. Owing to its larger mass, the escape velocity of the Sun is about 54 times larger than that of the Earth in spite of the Sun's larger radius. At the visible disk of the Sun, we have $V_{esc\odot} = 6.117 \times 10^5$ m s^{-1}. The escape speed from the surface of the Moon is just 2.38×10^3 m s^{-1}, which explains why the relatively small *Lunar Module* spacecraft could land on the Moon and blast off it with relatively low rocket propulsion, returning to its larger, mother spacecraft, the *Lunar Command Module*, that was orbiting the Moon and was launched from the Earth with considerably greater rocket thrust. The Moon's low escape speed also helps explain why it has no atmosphere to speak of.

The mass, radius, and escape speeds of representative planets and stars are given in Table 4.1.

4.2 Orbital Motion

A planet would continue going the way it started, moving along a straight line, if it were not for the Sun's gravitational force that deflects the planet into a curved solar orbit. Therefore, it is the Sun's gravitational attraction that keeps the planets forever moving along their orbital paths. But why doesn't the enormous solar

Table 4.1 Mass, radius and escape speed of some cosmic objects

Object	Mass (kg)	Radius (m)	Escape speed (km s^{-1})
Ceres, largest asteroid	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon	7.348×10^{22}	1.737×10^{6}	2.38
Earth	5.9736×10^{24}	6.371×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.989×10^{30}	6.955×10^{8}	618
Sirius B, white dwarf star	2×10^{30}	1×10^{7}	5,200
Neutron star	2×10^{30}	1×10^{4}	2×10^{5}

gravity pull all of the planets into the Sun? Motion holds the planets in their orbits, opposing the relentless pull of the Sun's gravity and keeping the planets from falling into the Sun.

Each planet is moving in a direction perpendicular to an imaginary line connecting it to the Sun, at exactly the speed required to overcome the Sun's gravitational pull, maintaining an equilibrium between motion and gravitation that keeps the planets in perpetual motion.

For the planetary orbits, or any other orbit of small eccentricity, the length of the orbit is close to a circular one. The mean orbital speed, V_{OP}, of a planet in circular motion about the Sun at a distance D_P, is:

$$V_{OP} = \frac{2\pi D_P}{P_P}, \tag{4.3}$$

where $\pi = 3.14159$, the circumference of a circle with radius D_P is $2\pi D_P$ and P_P is the orbital period. The mean orbital velocity of the Earth around the Sun is, for example, is 29.8 km $s^{-1} = 2.98 \times 10^4$ m s^{-1}, where the mean Earth–Sun distance is 1 AU $= 1.496 \times 10^{11}$ m and the orbital period is 1 year $= 3.1557 \times 10^7$ s.

When there are two objects orbiting a common center of mass, and one of them has a very small mass when compared to the mass of the other one, as is the case of planets orbiting the massive Sun, the orbital speed depends only on the dominant mass and the distance of the orbiting object from it. For the planets, the orbital speed depends only on the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg and the distance of the planet, or:

$$V_{OP} = \left[\frac{GM_\odot}{D_P}\right]^{1/2} = \frac{V_{esc\odot}}{\sqrt{2}}. \tag{4.4}$$

which follows from Kepler's third law assuming a circular orbit or one of small eccentricity. It tells us that the more distant planets move at a slower speed. The orbital speed is independent of the planet's mass, which is why the planetary realm, known as the solar system, is dominated by the Sun.

This equation also indicates that the escape speed is $\sqrt{2}$ times larger than the orbital speed of a body. The $\sqrt{2}$ factor is a very small number, just 1.414, so the orbital speed is very close to the escape speed at the relevant distance. Of course, a planet couldn't be moving just as fast as the escape speed, or any faster than that, for it would then escape from the solar system, moving off into interstellar space; and if the planet moved any slower than its orbital speed, it would be pulled into the Sun and consumed by it.

We can square both sides of the previous two equations and collect terms to obtain

$$P_P^2 = \frac{4\pi^2}{GM_\odot} D_P^3 \tag{4.5}$$

which is the Newtonian expression for Kepler's third law (Sect. 3.3).

Example: How fast does the Moon move around the Earth?

The Moon orbits our planet at a mean distance from the Earth of $D_M = 3.844 \times 10^8$ m with an orbital period P_M of 27.32 days, where 1 day = 86,400 s. This is the Moon's sidereal orbital period, from fixed star to fixed star. For a circular orbit, the Moon's mean orbital speed about the Earth would be $V_{OM} = 2\pi D_M / P_M = 1.02 \times 10^3$ m s^{-1}. We can compare this orbital speed to the escape speed, V_{escE}, from the Earth's gravity at the Moon's mean distance, $V_{escE} = (2GM_E/D_M)^{1/2} = 1.44 \times 10^3$ m s^{-1}, where the mass of the Earth is $M_E = 5.9736 \times 10^{24}$ kg and the Newtonian gravitational constant is $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The mean orbital speed of the Moon V_{OM} is just equal to $V_{escE}/\sqrt{2}$ at the Moon's distance, which shows that the Moon is bound to the Earth by its gravitational pull, diminished by the distance to the Moon, and that the Moon is perpetually falling toward the Earth while moving around it.

Calculations of the speed of an orbiting object also apply to communications, military, and weather satellites, which might be launched into geosynchronous orbits with an orbital period equal to the Earth's rotation period.

Example: Geosynchronous orbits

In a geosynchronous orbit, a satellite's orbital period equals the Earth's rotation period, so the satellite stays in the same location above the planet's surface. The distance, D_{GS}, of this kind of satellite above the center of the Earth can be obtained from a rearrangement of Kepler's third law:

$$D_{GS} = \left[\frac{GM_E P_r^2}{4\pi^2}\right]^{1/3}, \tag{4.6}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the mass of the Earth $M_E = 5.9736 \times 10^{24}$ kg, the rotation period of the Earth is $P_r = 24$ h $= 86{,}400$ s, and the constant $\pi = 3.14159$. Substitution into this formula gives $D_{GS} \approx 4.22 \times 10^7$ m. The more exact value of $D_{GS} = 4.2164 \times 10^7$ m is obtained using the Earth's sidereal rotation period of $P_r = 86{,}164$ s. For a geosynchronous satellite orbiting the Earth's equator, the altitude H of the satellite above sea level will be $H = D_{GS} - a_e = 3.5786 \times 10^7$ m, where the equatorial radius of the Earth is $a_e = 6.3781 \times 10^6$ m.

When two orbiting objects have comparable mass, as is the case for some binary stars, then the mean orbital velocity, V_{O1} of an object of mass M_1 orbiting another mass M_2 at a distance a is given by:

$$V_{O1} = \left[\frac{GM_2^2}{(M_1 + M_2)a}\right]^{1/2}. \tag{4.7}$$

Here a is the separation of the two objects. If r_1 and r_2 denote their respective distances from a common center of mass, and we assume circular orbits, then $r_1 M_1 = r_2 M_2$ with $a = r_1 + r_2 = r_1\,(M_1 + M_2)/M_2$.

Astronomers record the spectral lines of a star, and look for periodic variations in the observed line-of-sight velocity, $V_{OBS1} = V_{O1} \sin i$, where i is the inclination angle between the perpendicular to the orbital plane and the line of sight. The observed period of variations in the detected radial velocity along the line of sight is the orbital period, P, given by Kepler's third law:

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}. \tag{4.8}$$

We can use these equations to obtain an expression for the mass, M_2:

$$M_2^3 \sin^3 i = \frac{PV_{OBS1}^3}{2\pi G}(M_1 + M_2)^2. \tag{4.9}$$

When the mass of object 1 greatly exceed the mass of object 2, as is the case when looking for previously unseen exoplanets orbiting a nearby star, or when $\mathrm{M}_1 \gg \mathrm{M}_2$,

$$M_2 \sin\ i \approx \left(\frac{P}{2\pi G}\right)^{1/3} V_{OBS1}\, M_1^{2/3}, \tag{4.10}$$

and the mass of the star, M_1, can be inferred from other observations (Sect. 10.1).

4.3 The Moving Stars

4.3.1 Are the Stars Moving?

Each night the stars rise, move slowly across the dark sky, and then disappear from view; but this slow apparent movement of the stars is not due to the motions of the stars themselves. It is caused by the rotating Earth, which spins beneath the celestial sphere. Despite eons of stellar observations in antiquity, there was no evidence that any of these stars were moving.

Yet, if the stars were motionless, their mutual gravitation eventually would pull them together into a single mass. Without motion, there would be nothing to keep the stars apart, and they could not be suspended in space. So there is no star that is completely at rest, and the stars must be moving ever so slightly from their apparent places in the night sky.

Moreover, the speeds of the moving stars are not modest. Observations indicate that the stars are moving at speeds of about 10 km s^{-1} relative to their stellar neighbors. The Sun, for example, is currently traveling at a speed of about 20 km s^{-1}, or 20,000 m s^{-1}, relative to other nearby stars. This is about 1,000 times faster than a car moves on a highway.

Stars also move together at larger speeds in directed motions. Both the Sun and nearby stars, for example, are whirling about the remote center of the Milky Way at a speed of 220 km s^{-1}. If these stars traveled at faster speeds, they would move off into space, even out of the Milky Way; if they were moving at slower speeds, they would be pulled by gravitation into the center of the Milky Way. But because they are so far away, the stars seem to be moving slowly through space, only gradually changing their apparent separation and grouping.

4.3.2 Components of Stellar Velocity

Stars seem to be moving here, there and everywhere, so it is not easy to figure out where they are going. However, a star's motion manifests in two ways, depending on the method used to observe it, and these two components of velocity can be combined to give the direction of motion (Fig. 4.1). The "sideways" velocity component is directed perpendicular or transverse to the line of sight, with a speed designated by $V_{\perp}$, where the subscript $\perp$ denotes perpendicular. The other component, a radial velocity with a speed denoted by V_r, is the velocity moving toward or away from us along the line of sight to a star. When these two velocity components are known, we can determine the speed and direction of a star in three dimensions. The Pythagorean theorem gives the magnitude of the star's space velocity, its true speed in space, V_S, given by $V_S^2 = V_r^2 + V_{\perp}^2$.

A star's motion across the line of sight produces an angular change in position, called proper motion, which depends on both the star's distance and the

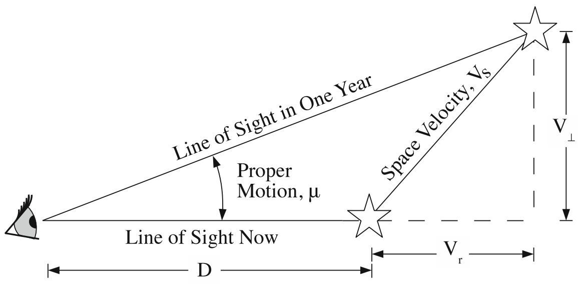

Fig. 4.1 **A star moves** The space velocity, V_S of a star relative to an observer can be resolved into two mutually perpendicular components: (1) the radial velocity, V_r, directed along the line of sight; and (2) the tangential velocity, $V_\perp$, which is perpendicular or transverse to the line of sight. From the Pythagorean theorem $V_S^2 = V_r^2 + V_\perp^2$. Over a given interval of time, shown here as one year, the star will move through a proper motion angle μ, which depends on $V_\perp$ and the star's distance, D, from the observer. In this figure, the proper motion $\mu = V_\perp / D$ is exaggerated greatly by more than 10,000 for even the closest star. At a distance of only 6 light-years, Barnard's star has the largest known proper motion of 10.3 s of arc per year

perpendicular speed $V_\perp$. The radial velocity is observed through the Doppler effect, which measures how the star's spectral lines appear to shorten or lengthen in wavelength depending on the relative velocity of the star and the observer, and whether the motion is toward or away from the observer. When a star is moving directly away, then there is no perpendicular component to its motion, and if the star is moving directly across the line of sight, then the radial component of the star's motion is reduced to zero.

It is difficult to judge a star's speed if it is headed straight toward or away from us, just as it is difficult to determine how fast a distant car is moving on a highway. However, if a star crosses at right angles to our line of sight, we could see a change in its position. To detect that change, astronomers needed to look at the nearest stars where the angular change in position is greatest.

Given enough time, the displacement of a nearby star's celestial position can be detected. The English astronomer Edmond Halley (1656–1742) first noticed the change when he compared the positions of extremely bright stars, such as Sirius and Arcturus, with those measured by the Greek astronomer Hipparchus around 150 BC and recorded in Ptolemy's reproduction of Hipparchus' catalogue. Halley's comparison indicated that at least three stars had changed position and moved (Halley 1717).

So it took more than 1,800 years before anyone noticed that a star could move. Nowadays, with vastly improved technology and observations from spacecraft, the motions of many tens of thousands of stars are known with great accuracy.

4.3.3 Proper Motion

The stellar motion that Halley detected is an angular change in a star's position over time, due to its velocity transverse or perpendicular to the line of sight. The angular rate of change is known as *proper motion*, which is intrinsic to the star and belongs to it, in contrast to any improper motion that might be caused by the Earth's movement in space.

Proper motion is not a velocity; it is the angular rate at which a star moves across the sky over years or centuries, and it does not by itself determine the speed of motion. To convert a star's proper motion into a velocity or speed, we must know the star's distance, and in Halley's time no one knew the distance of any star other than the Sun.

For a star located at distance D, the proper motion μ is:

$$\mu = V_{\perp}/D \, \text{rad s}^{-1}, \tag{4.11}$$

where 1 rad = $2.06265 \times 10^{5}\,''$ and $''$ denotes seconds of arc. Proper motion is designated by the Greek letter mu, or the symbol μ. The speed perpendicular to the line of sight, $V_{\perp}$, is known as the *transverse velocity*. If $V_{\perp}$ is given in units of km s^{-1} and D is in units of parsecs, we have:

$$\text{Annual Proper Motion} = \mu = 0.211 V_{\perp}/D \,''\, \text{yr}^{-1}, \tag{4.12}$$

and

$$V_{\perp} = 4.74 \, \mu D \, \text{km s}^{-1}, \tag{4.13}$$

where 1 yr = 3.156×10^{7} s, and the μ in this case is called the annual proper motion. One parsec is abbreviated 1 pc, and 1 pc = 3.08568×10^{16} m is the typical separation between adjacent stars. The coefficient 0.211 comes from $2.06265 \times 3.156/3.08568$ in the various conversion factors, and $4.74 = 1.0/0.211$.

4.3.4 Radial Velocity

The other component of a star's velocity, the *radial velocity* directed along the line of sight, can be measured using the Doppler shift of a spectral feature in the star's radiation. Such a feature, called a spectral line, has a definite, well-known wavelength(Sect. 6.1).

Just as a source of sound can vary in pitch or wavelength, depending on its motion, the wavelength of electromagnetic radiation shifts when the emitting source moves with respect to the observer. Such a shift is named after the Austrian scientist, mathematician, and schoolteacher Christiaan Doppler (1803–1853) who discovered it more than one and a half centuries ago (Doppler 1842; Andrade 1959). If the motion is toward the observer, the shift is to shorter wavelengths; when the motion is

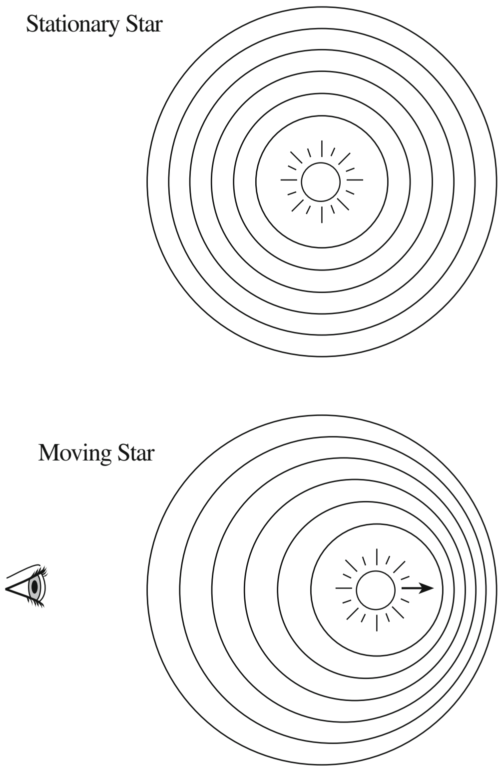

Fig. 4.2 Doppler effect A stationary source of radiation (*top*) emits regularly spaced light waves that get stretched out or scrunched up if the source moves (*bottom*). Here a star moving away (*bottom right*) from the observer (*bottom left*) is shown. The stretching of light waves that occurs when the source moves away from an observer along the line of sight is called a *redshift*, because red light waves are relatively long visible light waves; the compression of light waves that occurs when the source moves along the line of sight toward an observer is called a *blueshift*, because blue light waves are relatively short. The wavelength change, from the stationary to moving condition, is called the *Doppler shift*, and its size provides a measurement of radial velocity, or the speed of the component of the source's motion along the line of sight. The Doppler effect is named after the Austrian physicist Christiaan Doppler (1803–1853), who first considered it in 1842

away, the wavelength becomes longer (Fig. 4.2). We notice the effect for sound waves when listening to the changing pitch of a passing ambulance siren. The tone of the siren is higher as the ambulance approaches and lower when it moves away.

If the spectral line is emitted at a specific wavelength, $\lambda_{emitted}$, by a source at rest, the wavelength, $\lambda_{observed}$, observed from a moving source is given by the relation:

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}} = \frac{V_r}{c} \quad \text{for } V_r \ll c, \tag{4.14}$$

where the quantity z is known as the redshift, V_r is the speed of the source's radial motion along the line of sight away from the observer; since the speed and direction are known, V_r denotes the radial velocity. The speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The parameter z is called the redshift since the Doppler shift is toward the longer, redder wavelengths in the visible part of the electromagnetic spectrum. When the motion is toward the observer, V_r is negative and there is a blue shift to shorter, bluer wavelength. The greater the speed along the line of sight in either direction, the bigger the wavelength shifts.

The notation $V_r \ll c$ in our equation for the redshift, z, means that the formula applies for radial velocities, V_r, much less than the speed of light, c. This is the case for the motions of stars in our Milky Way. Nevertheless, remote collections of billions of stars, the galaxies, are moving at radial velocities that increase with their distance, and the exceptionally remote ones can have radial velocities that approach the speed of light, or $V_r \approx c$. In this case our equation has to be modified (see Sect. 14.3).

4.3.5 Observed Proper Motions of Stars

The star with the largest proper motion races across the sky at about 10.4 s of arc, denoted as 10.4″, each year. This is *Barnard's star*, named after the American astronomer Edward E. Barnard (1857–1923) who discovered it (Barnard 1916). In our lifetime this star will move by roughly half the angular diameter of the Moon; however, because it is a dim, faint star a telescope is required to see it. Barnard's star is $1/27$th of the brightness of the faintest star that can be seen with the unaided eye. It is a relatively nearby star, located at a distance of just 5.98 light-years, or 1.834 pc, and its large proper motion is attributed to both a high transverse speed and the closeness of the star.

Barnard's star moves across the line of sight at a speed of $V_\perp = 90.4$ km s^{-1}. When combined with its radial velocity of $V_r = -110.6$ km s^{-1}, with the negative sign indicating that the star is approaching us, a space velocity of $V_S = 142.7$ km s^{-1} relative to the Sun is obtained, from $V_S^2 = V_r^2 + V_\perp^2$. At its radial velocity, Barnard's star will move one light-year closer to us in about 2,100 years, using 1 light-year $= 9.461 \times 10^{15}$ m and 1 year $= 3.156 \times 10^7$ s to convert between units.

Example: How fast does Barnard's star move?

Barnard's star has an annual proper motion of $\mu = 10.4''$ year^{-1}. Its distance, D, inferred from its parallax (see Sect. 10.1) is $D = 5.98$ light-years $= 1.834$ pc, where 1 pc $= 3.26$ light-years. The star's transverse velocity, perpendicular to the line of sight, is $V_\perp = 4.74\ \mu D = 90.4$ km s^{-1}. The star's redshift is $z = -3.689 \times 10^{-4} = -0.0003689$, so its radial velocity is $V_r = z \times c = -110.6$ km s^{-1}, where the speed of light $c = 2.9979 \times 10^5$ km s^{-1}. The space velocity, V_S, of Barnard's star is inferred from $V_S^2 = V_\perp^2 + V_r^2$, or $V_S = 142.8$ km s^{-1}.

The closest star, Proxima Centauri, is just 4.24 light-years away, and at a distance of 5.98 light-years Barnard's star is nearly that close. However, Proxima Centauri is also moving closer, with a radial velocity of –21.7 km s^{-1}, so it will keep its status as the closest star for a very long time to come. The proper motion of Proxima Centauri is 3.85″ year^{-1}.

The star with the second largest proper motion, at 8.7″ yr^{-1}, is Kapteyn's star, named for the Dutch astronomer Jacobus C. Kapteyn (1851–1922), who first catalogued it (Kapteyn 1898). It has a distance of 12.8 light-years or 3.92 pc, so it is moving across the line of sight at a speed of $V_\perp = 162$ km s^{-1}. Kapteyn's star has a radial velocity of $V_r = 245.5$ km s^{-1}, giving it a true space velocity relative to the Sun of $V_S = 293.6$ km s^{-1}. This intriguing star moves around the center of the Milky Way in the opposite direction to the other nearby stars. It may have originated outside the Milky Way disk and is now hurtling through it.

Most proper motions are exceedingly small and usually measured in seconds of arc per century, or milliarcseconds per year, which means the same thing. Due to atmospheric blurring the angular resolution of the best telescope at the best location on the Earth is only about 0.2″, and we would have to wait more than 20 years to measure a proper motion of this size. However, the effect is cumulative; therefore successive generations of astronomers can measure proper motion. After 20 centuries, the proper motion of many stars might be 20″, which explains why Halley was able to detect the effect using ancient observations.

It is much easier to measure proper motion from space, outside the Earth's atmosphere. Instruments aboard the *HIPPARCOS* satellite have pinpointed the positions and established the proper motions of more than 100 thousand stars with an astonishing precision of 0.001″. The stellar distances are inferred from parallax measurements, and that explains the spacecraft's name, an acronym for *High Precision PARallax Collecting Satellite*. The perpendicular velocities can be determined from the proper motions and distances.

Astronomers specify the proper motion μ_α in right ascension α and the proper motion μ_δ in declination δ. The magnitude of the total proper motion, μ, is given by the vector addition of its components $\mu^2 = \mu_\delta^2 + \mu_\alpha^2 \cos^2 \delta$, where the *cos* δ factor accounts for the projection of μ_α on the celestial sphere. The components of proper motion and the radial velocities of stars with exceptionally high proper motion are listed in Table 4.2, where the proper motions are in units of milliarcseconds per year, or 10^{-3} ″ year^{-1}, and abbreviated mas year^{-1}, and the + or − sign of the radial velocity indicates motion away or toward the observer, respectively.

Table 4.2 Stars with the highest proper motion[a]

Star	$\mu_\alpha \cos\delta$ (mas year^{-1})	μ_δ (mas year^{-1})	Parallax (mas)	Radial velocity (km s^{-1})
Barnard's star	−798.71	1,0337.77	549.30	−110.6
Kapteyn's star	6,500.34	−5,723.17	255.12	+245.5
Groombridge 1830	4,003.69	5,814.64	109.22	−98.0
Lacaille 9352	6,766.3	1,327.99	303.89	+9.7
Gliese 1 (GJ 1)	5,633.95	−2,336.69	229.32	+23.6

[a] The designation mas is short for milliarcseconds or $0.001 = 10^{-3}$ s of arc

4.3.6 Motions in Star Clusters

Gravitation can constrain the paths of stars that are congregated within star clusters (Table 4.3). As many as 1 million stars, for example, are crowded together in a typical globular star cluster. The cluster is tightly bound by gravity, which gives it a distended spherical shape and relatively high stellar density toward the center (Figs. 4.3, 4.4). The name of this category of star cluster is derived from the Latin *globules*, for "a small sphere". Another type of stellar grouping, known as an open star cluster, includes up to a few thousand stars that were formed at the same time, but are only bound loosely to one another by mutual gravitational attractions (Fig. 4.5). Unlike globular star clusters, which can be held together by its stars' mutual gravitational pull for tens of billions of years, an open star cluster will disperse within a few million years.

The stars in a globular cluster are moving around like a swarm of bees, or like hot, subatomic particles inside a star. The stellar motions oppose the combined gravitational attraction of all of the stars, preventing them from gathering together and collapsing to the center of the star cluster.

In a short elegant discussion, the great British astronomer Arthur Stanley Eddington (1882–1944) demonstrated that the internal kinetic energy of a star cluster is half its gravitational potential energy (Eddington 1916). He also pointed out that this result could have been obtained at once from what is known as the virial theorem, a formula whose previous use had been almost entirely restricted to gases. In Eddington's application, stars replace the atoms and molecules of a gas.

The *virial theorem* describes the stability of a finite, self-gravitating collection of particles, either atoms or stars, which is bound by gravitational forces. It states that the total kinetic energy averaged over time is just equal to half the total

Table 4.3 Physical properties of star clusters

Open star cluster	Globular star cluster
N_S = total number of stars = 100 to 1,000	N_S = total number of stars = 10^4 to 10^6
R_C = radius = 1 to 10 pc ≈ (3 to 31) × 10^{16} m	R_C = radius = 10 to 100 pc ≈ (3 to 31) × 10^{17} m
Age = 10^7 to 10^9 year	Age = (10 to 14) × 10^9 year = 10 to 14 Gyear

Fig. 4.3 Globular star cluster NGC 6934 Several hundred thousand stars swarm around the center of the globular star cluster NGC 6934, which lies at a distance of about 50,000 light-years from the Earth. These ancient stars are estimated to be about 10 billion years old. This sharp image, obtained from the *Hubble Space Telescope*, is about 3.5 min of arc and 50 light-years across. (Courtesy of NASA/ESA.)

gravitational potential energy. For N_S stars of individual mass M_S the star cluster has a total mass of $M_C = N_S M_S$, and it will be gravitationally bound together in a stable configuration if:

$$\frac{1}{2} M_S < V_S >^2 = \frac{G M_C M_S}{2 R_C}, \tag{4.15}$$

and

$$< V_S > = \left[\frac{G N_S M_S}{R_C}\right]^{1/2} = \frac{V_{esc}}{\sqrt{2}} \tag{4.16}$$

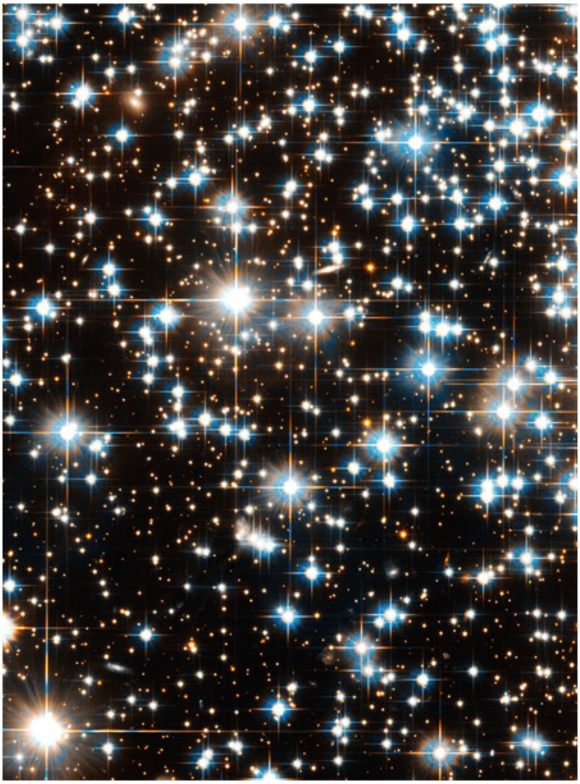

Fig. 4.4 Faint stars in a globular cluster This five-day exposure from an instrument aboard the *Hubble Space Telescope* includes the faintest detectable stars in the globular star cluster NGC 6397, which is located about 8,500 light-years away from the Earth. Some of these objects are white dwarf stars, the collapsed, burned-out relics of former stars like the Sun. White dwarfs cool down at a predictable rate, which can be used to measure the age of this globular cluster, estimated to be about 12 billion years. The crossed lines radiating from the bright stars are diffraction spikes caused by the struts that support the telescope mirror. (Courtesy of NASA/ESA/Harvey Richer, University of British Columbia.)

where V_{esc} denotes the escape velocity of the cluster, R_C is the radius of the star cluster, V_S is a star's velocity and the brackets $< >$ denote a time average with a time-averaged stellar speed of $<V_S>$, and the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

Example: How fast do stars move in a bound star cluster?

The number of stars, N_S, in a globular star cluster can be about a million, or $N_S = 10^6$, each with a mass, M_S, about equal to that of the Sun $M_S = M_\odot = 1.989 \times 10^{30}$ kg. They are apparently bound together in a sphere with a radius of $R_C = 10$ pc $= 3.086 \times 10^{17}$ m. According to the virial theorem, the kinetic energy of the stars, moving at an average star velocity $<V_S>$, must balance just half the gravitational pull of all the stars on any one star, or that $M_S <V_S>^2/2 = GN_S M_S^2/(2R_C)$, where the Newtonian gravitational constant $G = 6.674 \times 10^{11}$ m^3 kg^{-1} s^{-2}. Substituting the numbers into this equation we obtain $<V_S> = 2.07 \times 10^4$ m s^{-1} $=$ 20.7 km s^{-1}.

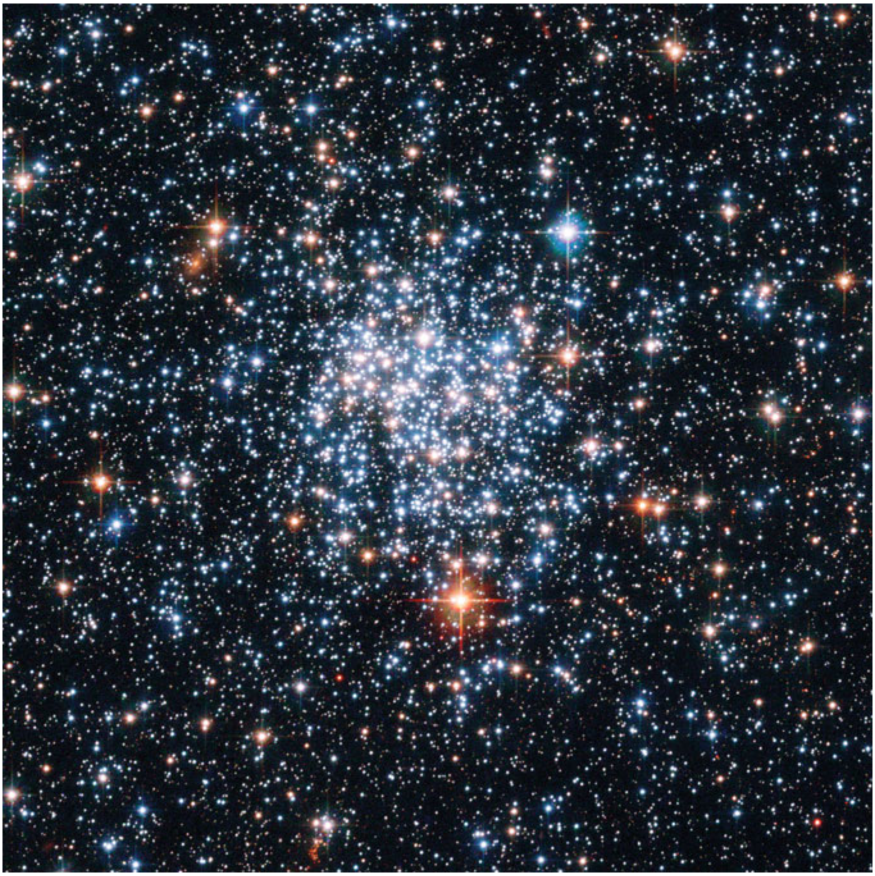

Fig. 4.5 Open star cluster NGC 265 A brilliant cluster of bright blue stars is located in the Small Magellanic Cloud, about 200,000 light-years away and about 65 light-years across. This *Hubble Space Telescope* image subtends an angle of about 70 s of arc. (Courtesy of NASA/ESA/E.Olszewski University of Arizona.)

If the stars move on average at a slower speed than $<V_S>$, they will be pulled gravitationally into each other and the cluster will collapse. If the stars move at an average speed that is faster than $<V_S>$, they eventually will disperse because the cluster cannot hold together. This is what is happening to open star clusters, and to star associations that are bound even more loosely. In fact, some stars are moving out of certain stellar associations at unexpectedly rapid speeds.

4.3.7 Runaway Stars

Some stars race through space with an abnormally high velocity relative to the surrounding interstellar medium. These high-speed stars are known as *runaway*

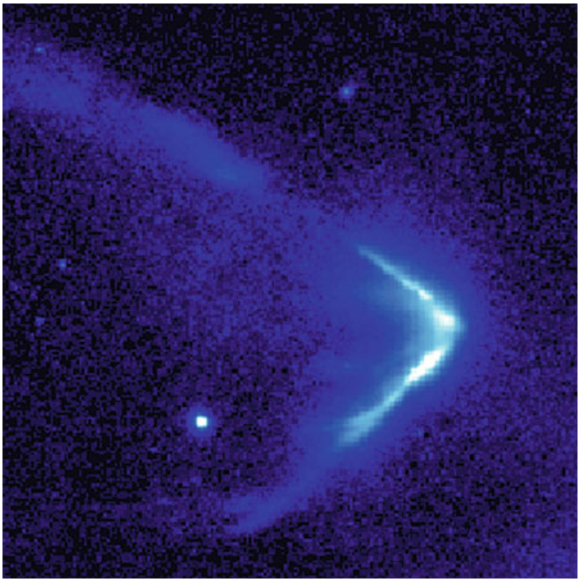

Fig. 4.6 Runaway star A high-speed star slams into dense interstellar gas, creating a bow shock wave that may be a million kilometers wide. The star is thought to be relatively young, only millions of years old. Moving at a speed of about 100 km s^{-1}, it has journeyed 160 light-years since its birth, most likely in a loosely bound stellar association. (Courtesy of NASA/ESA/R. Sahai/JPL.)

stars, because they are moving away from their place of origin. They are former members of very loose star clusters, known as stellar associations, containing bright, and relatively hot, massive and young stars, that are designated as O and B stars.

As first noticed by the Armenian astronomer and statesman Viktor Ambartsumian (1908–1996), these O and B stars are expanding away from one another and from a common origin at speeds of about 10 km s^{-1} (Ambartsumian 1949). The associations are now dispersing and disintegrating, but still moving together in a roughly spherical shape due to their relatively young age. They are not expected to stay together for longer than a few tens of millions of years.

Runaway stars are moving with faster speeds than other stars in the OB associations but with proper motions that often point away from the stellar association to which they once belonged. These runaways are most likely escaped members of former binary star systems that once belonged in the association, until one of the two stars exploded. As described by the Dutch astronomer Adriaan Blaauw (1914–2010), runaway stars are very massive stars whose high space velocities are comparable to the orbital velocities expected for massive binary-star systems (Blaauw and Moran 1954; Blaauw 1961, 1964). Because massive stars burn their thermonuclear fuel faster, and have a shorter lifetime than normal stars, one member of such a binary system will quickly exhaust its thermonuclear reserves and explode as a supernova, thereby releasing the other member as a high-velocity star. The evolution and explosive fate of such massive stars is considered in Sect. 13.5.

The *Hubble Space Telescope* has captured striking images of runaway stars plowing through regions of dense interstellar gas and creating brilliant bow-shock structures and trailing tails of glowing gas (Fig. 4.6). These features are formed when the stars' powerful stellar winds slam into the surrounding gas. The shocks indicate that the runaway stars are traveling at speeds between 50 and 100 km s^{-1} relative to the dense gas through which they are moving. This is five or ten times faster than the expansion speeds of the stellar associations or the average speeds of stellar motions with respect to nearby stars or the local interstellar medium.

4.4 Cosmic Rotation

In addition to moving through space, an astronomical object also rotates or spins about its axis. This rotation often can be traced back to an origin from a more distended object of slower spin, but sometimes it is related to a glancing collision in the past.

The period of rotation is the time it takes to complete one revolution, or the time for the planet or star to spin into the same orientation in space. For the planets and the Sun, this intrinsic rotation period is known as the sidereal rotation period, from fixed star to fixed star; it has been corrected for any observational effects such as the Earth's orbital motion around the Sun.

4.4.1 Unexpected Planetary Rotation

For solid rocky planets, the rotation period is everywhere the same on the planet's surface. The Earth, for example, rotates once very 24 h or 86,400 s at all latitudes, or at every angular distance north or south of the equator. As a result, all points of the globe take the same amount of time to complete one rotation and a day lasts 24 h everywhere on the planet. If the rotation period differed at different latitudes, the solid planet would break apart.

You might think that it's easy to determine the rotation period of a planet. All you need to do is watch how long it takes for a prominent surface feature to spin around behind the planet and reappear. But this was not the case for Mercury, which is so close to the Sun that most people have never even seen the planet, let alone resolved anything on its surface. And the situation was even worse for cloud-covered Venus, whose surface can never be seen.

Astronomers once supposed that solar tides in the body of Mercury would cause the planet to rotate on its axis once every 87.97 Earth days, in step with its orbital period. Just as the Earth's Moon always presents the same face to the Earth, it was thought that one side of Mercury was always turned toward the Sun. To test this idea, the Italian astronomer Giovanni Schiaparelli (1835–1910) monitored Mercury's surface markings seen though his 0.46-m (18-inch) telescope, and he concluded that the same side of the planet did, indeed, always face the Sun

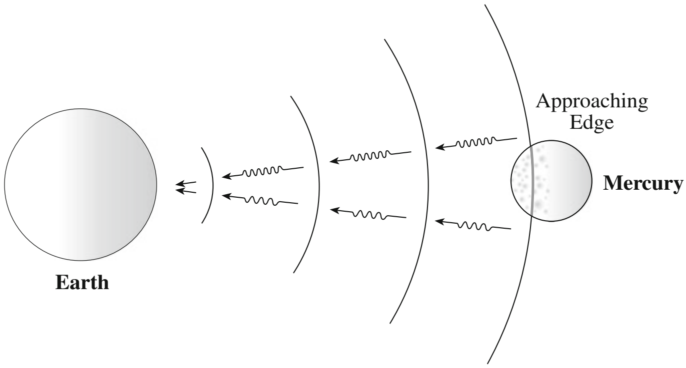

Fig. 4.7 Radar probes of Mercury A radio signal spreads out as a spherical wave, and Mercury intercepts a small fraction of them. As the wave sweeps by the planet, it is reflected in spherical wavelets whose wavelengths are Doppler-shifted by the rotational motion of Mercury's surface. The waves from the receding side are red-shifted toward longer wavelengths and those from the approaching side are blue-shifted to shorter wavelengths. The total amount of wavelength change, from red to blue, reveals the speed of rotation, and the rotation period can be obtained by dividing the planet's circumference by this speed

(Schiaparelli 1889; Defrancesco 1988). For three-quarters of a century, telescopic observers agreed with his conclusion. All of these astronomers were dead wrong!

In 1967, Mercury's true rotational period was determined with radio signals that rebounded from the planet (Dyce et al. 1967). The world's largest radio telescope, located in Arecibo, Puerto Rico, was used to transmit 2 million watts of pulsed radio power at the planet, and to receive the faint echo. This technique is known as radio detection and ranging, abbreviated radar, and it is also used to locate and guide airplanes near airports.

Each pulse was finely tuned, within a narrow range of wavelengths around 0.497 m, and emitted for only about a millisecond. Upon hitting the planet, its rotation de-tuned the pulse, slightly spreading it over a wider range of wavelengths (Fig. 4.7). One side of the globe was rotating away from the Earth, while the other side was rotating toward our planet. These motions produced slight changes in the wavelength of the echo, which arrived back at Arecibo shortly before the next radar pulse was sent. The rotational velocity and period were calculated from the broadened, wavelength-shape of the return echo, using the well-known expression for the Doppler effect.

A rotating object will produce a blueshift on the side spinning toward an observer, and a redshift on the opposite side. Their combined effect will broaden a narrow spectral line or a finely tuned radio pulse at wavelength λ by an amount $\Delta\lambda$ given by the expression:

$$\frac{\Delta\lambda}{\lambda} = \frac{V_{rot}}{c}. \tag{4.17}$$

The period, P, of rotation for an object of radius R is $P = 2\pi R/V_{rot}$, and V_{rot} is the rotation velocity.

The radar result for Mercury came as an unexpected surprise. Its rotation period was 58.646 Earth days, or exactly two-thirds of the 87.969-day period that had been accepted so long. Thus, with respect to the stellar background, Mercury spins on its axis three times during two full revolutions about the Sun, which follows from $3 \times 58.646 = 2 \times 87.969$, and it is technically known as spin–orbit coupling. In comparison, the Earth's Moon has a 1:1 spin–orbit resonance in which its rotation period is equal to its orbital period.

The Italian scientist Giuseppe Colombo (1930–1984) provided an explanation for this result in terms of the Sun's varying tidal forces as Mercury revolves about its elongated orbit (Colombo and Shapiro 1966; Goldreich and Peale 1966). The solar gravity pulls hardest on Mercury when the planet is closest to the Sun, at perihelion, and least at the opposite side of its eccentric orbit, at aphelion. This extra gravitational pull of the Sun at perihelion gives an abrupt twist to Mercury's non-spherical body, speeding up the rotation rate and forcing it into synchronism at perihelion with the 3:2 resonance. If Mercury's orbit around the Sun were much closer to a circular shape, like the nearly round orbit of the Moon around the Earth, then the Sun's tidal forces would have slowed Mercury's rotation into synchronism with its orbital motion, in a 1:1 resonance with a rotation period equal to its orbital period.

No human eye has ever gazed on the surface of Venus, which is forever hidden by a thick overcast of impenetrable clouds, but radio waves can penetrate this obscuring veil and touch the landscape hidden beneath. By bouncing pulses of radio radiation off the surface of Venus, the radar astronomers also discovered in 1967 that this planet spins in the backward direction, opposite to that of its orbital motion. That is, unlike the other terrestrial planets, Venus does not rotate in the direction in which it orbits the Sun.

The radar observations also showed that Venus spins with a period longer than any other planet, at 243.018 Earth days. This rotation period is even longer than the planet's 224.7 Earth-day period of revolution around the Sun, so the day on Venus is longer than its year. Tides raised by the Sun in the planet's thick atmosphere may explain why Venus turns very slowly and in the wrong way, but it might have alternatively been knocked into a backwards rotation by a collision with a planet-sized object in the early epochs of the solar system, when such collisions were more common.

Unlike the rocky terrestrial planets, the gaseous giant planets do not rotate at a uniform rate, and this results in a non-spherical shape. The outward force of rotation opposes the inward gravitational force, and this reduces the pull of gravity in the direction of spin. As a result, the giant planets rotate faster in their equatorial middle, where there is a perceptible bulge, and slower at the flattened poles. So they have an oblate shape that is elongated along the equator (Table 4.4). The same thing even happens to the Earth, but by a relatively small amount since it is solid instead of gaseous inside.

The apparent outward force that draws a rotating body away from the center of gravitational acceleration is known as *centrifugal force*, from the *Latin centrum* meaning "center" and *fugere*, meaning "to flee". It tends to push the equatorial

Table 4.4 Oblateness of the giant planets and the Earth[a]

Planet	Equatorial radius, R_e (km)	Polar radius, R_p (km)	Oblateness $(R_e–R_p)/R_e$
Earth	6,378.140	6,356.755	0.003353
Jupiter	71,492	66,854	0.0649
Saturn	60,268	54,364	0.0980
Uranus	25,559	24,973	0.0229
Neptune	24,766	24,342	0.0171

[a] The radii are given in units of kilometers, abbreviated km. The radii of the giant planets are those at the level where the atmospheric pressure is equal to one bar, the pressure of air at sea level on Earth

regions out. The ratio of the centrifugal acceleration at the equator to the gravitational acceleration at the equator is

$$m = \frac{\omega^2 R^3}{GM} = \frac{4\pi^2 R^3}{P^2 GM} \tag{4.18}$$

for a planet rotating at an angular velocity $\omega = 2\pi/P$, rotation period P, radius R and mass M, where the universal constant of gravitation $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

The rapid rotation of a planet or star might push the equatorial regions out so far that it rips the object apart, and this provides an upper limit to the possible rotation speed and a lower limit to the rotation period (Focus 4.1). If the rotation is too fast, and the equatorial push is too much, there is nothing left to rotate.

Focus 4.1 How fast can a planet or star rotate?

The rotation of a planet or star forces its equatorial regions out, and if the speed of rotation is too fast the object will fall apart. This will happen if the equatorial rotation velocity of the star, V_{rot}, exceeds the escape velocity, V_{esc}, which for an object with mass, M, and radius, R, occurs when:

$$V_{rot} = \frac{2\pi R}{P} \geq V_{esc} = \left(\frac{2GM}{R}\right)^{1/2}, \tag{4.19}$$

where the symbol $\geq$ denotes greater than or equal, the constant $\pi \approx 3.14159$ and the Newtonian gravitation constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1}s^{-2}. Collecting terms in this equation, we see that the break up happens for rotation periods P of

$$P \leq \left(\frac{2\pi^2 R^3}{GM}\right)^{1/2}. \tag{4.20}$$

where the symbol $\leq$ denotes less than or equal. We would obtain the same condition, within a factor of the square root of two, or $\sqrt{2}$, if we set the ratio of the centrifugal acceleration at the equator equal to the gravitational acceleration at the equator. For a planet or star of uniform mass density $\rho = M/(4\pi R^3/3)$, the mass per unit volume, the upper limit to the period of rotation is:

$$P \leq \left(\frac{8\pi}{3G\rho}\right)^{1/2} = \sqrt{\frac{8\pi}{3G\rho}}. \qquad (4.21)$$

For a rocky planet with a mass density of 3,000 kg m^{-3}, like that of the Earth's crust, the fastest possible rotation period is P ≈ 6,470 s ≈ 1.8 h. Most asteroids, for example, do not rotate faster than once every 2.2 h.

Stars with a mean mass density like that of the Sun, at 1,410 kg m^{-3}, would be expected to rotate with periods longer than about 2.6 h, but stars do not have a uniform mass density. Moreover, collapsed stars, like neutron stars and pulsars have very high mass densities approaching nuclear densities of 5×10^{17} kg m^{-3}, which is what happens when you press a solar mass into a star about 10^4 m in radius. Our equation then shows that the fastest pulsar probably has a period of about 0.5 ms, or 0.5×10^{-3} s, and thus rotates about 2,000 times a second. Such stars also have exceptionally high escape velocities owing to their compact size.

The instability of uniformly rotating spherical masses was first described by the Scottish mathematician Colin Maclaurin (1698–1746), and the general result ever since then (Maclaurin 1742) is that a rotating sphere becomes unstable when the angular rotational velocity $\omega = 2\pi/P$ rises above $(G\rho)^{1/2}$ (Tassoul 1978). The detailed theory for rotating fluid masses, as well as gaseous ones, has a long, rich history that can be found in Todhunter (1962) and Chandrasekhar (1969).

Despite its great size, Jupiter rotates so fast that day and night each last about 5 h and its full day is less than one-half Earth day. The precise rotation period of 9.9249 h is found by tracking radio bursts that are linked to the planet's spinning magnetic field, which emerges from deep within the planet. Saturn rotates with a day of only 10.6562 h, which is also inferred from the observed periodic modulation in Saturn's radio emission, generated in its spinning magnetic fields. The visible clouds at different latitudes on both giant planets rotate at different speeds and even in different directions. The rotation periods of the some planets and stars, including the Sun, are given in Table 4.5.

4.4.2 The Sun's Differential Rotation

Observations of sunspots have long indicated that the visible solar disk rotates differently at different latitudes, with a faster rate at the equator than at higher

Table 4.5 Rotation periods and rotation velocities of some planets and stars

Object	Rotation period	Radius[a] (m)	Rotation velocity[a] ($m\ s^{-1}$)
Earth	0.99727 Earth days[b]	6.378×10^6	4.651×10^2
Earth's Moon	27.322 Earth days[c]	1.738×10^6	4.627
Mercury	58.6462 Earth days	2.440×10^6	3.026
Venus	−243.018 Earth days	6.052×10^6	1.81
Jupiter	9.9249 h	7.149×10^7	1.26×10^4
Saturn	10.6562 h	6.027×10^7	9.87×10^3
Sun (equator)	25.67 Earth days	6.955×10^8	1.97×10^3
Vega	12.5 h	1.933×10^9	2.7×10^5
White dwarf star[d]	186.5 s	6.378×10^6	2.1×10^5
Crab pulsar	0.033 s	10^4	1.9×10^6

[a] The equatorial radius is given when the object has a known oblate shape, and in this situation the equatorial rotation velocity is provided

[b] One Earth day is defined as the time for our planet to revolve once with respect to the Sun, and such a solar day is 24 h or 86,400 s long. The Earth's rotation period with respect to stars, or sidereal time, runs about 4 min slower than the solar day. The sidereal day lasts 23 h 56 min 04 s or 8.6164×10^4 s

[c] The sidereal rotation period of the Earth's Moon, from fixed star to fixed star, is 27.322 Earth days. The time from new Moon to new Moon, known as the synodic month, is 29.53 Earth days

[d] The radius of a white dwarf star is assumed to be equal to that of the Earth and its rotation period inferred from the rotation period of the Sun and conservation of angular momentum, so the period scales as the inverse square of the radius

Table 4.6 Differential rotation of the Sun[a]

Solar latitude (degrees)	Rotation period (days)	Rotation speed ($km\ h^{-1}$)	Rotation speed ($m\ s^{-1}$)	Angular velocity (nHz)
0 (Equator)	25.67	7,097	1,970	451
15	25.88	6,807	1,891	447
30	26.64	5,922	1,645	434
45	28.26	4,544	1,262	410
60	30.76	2,961	823	376
75	33.40	1,416	393	347

[a] Data from the MDI instrument aboard the *SOHO* spacecraft

latitudes (Carrington 1863; Newton and Nunn 1951; Snodgrass 1983). This is known as differential rotation, since the rate of rotation differs at different latitudes. It indicates that the Sun is not solid, for it would be torn apart by differential rotation if it was solid; instead, most of the Sun is a gaseous plasma. Gilman (1974) has provided a review of the rotation of the Sun.

The rotation speed of the visible solar disk, the photosphere, can be inferred from the Doppler effect of an absorption line originating there, and the results confirm the differential rotation suggested by sunspot observations (Table 4.6). The Sun spins about its axis with a period of about 25 days at the equator, which corresponds to a rotation speed of about 2 $km\ s^{-1}$ or roughly 7,200 km per hour. But since the Earth is orbiting the Sun in the same direction that the Sun rotates, the solar rotation period observed from the Earth is about 27 days. The shorter rotation period is the star's true rotation period, and is technically known as the sidereal rotation period, from fixed star to fixed star; the longer period required for a fixed feature to rotate back to the same position as viewed from Earth, is called the synodic rotation period.

The English amateur astronomer Richard Christopher Carrington (1826–1875) determined the solar rotation rate from low latitude sunspots in the 1850s, and defined a fixed solar coordinate system that rotates once every 25.38 days (Carrington 1863), which would correspond to the sidereal rotation period near the solar equator. To compare locations on the Sun over a period of time, the mean observed, or synodic, solar rotation period has been arbitrarily taken to be 27.2753 days. Each rotation of the Sun is then given a unique number called the Carrington rotation number, with rotation 1 beginning on 9 November 1853.

The synodic equatorial rotation period, as observed from Earth, is 26.75 ± 0.05 days, with a differential synodic rotation given by (Sheeley et al. 1992):

$$\omega(\theta) = 13.46 - 2.7\cos^2\theta + 1.2\cos^4\theta - 3.2\cos^6(\theta), \tag{4.22}$$

where ω is the angular velocity in degrees per day, and θ is the co-latitude measured from the poles instead of the equator. An angular velocity of 13.46 degrees per day corresponds to 451 nHz.

Helioseismologists have more recently used 5 min oscillations of the photosphere, produced by internal sound waves, to investigate the internal structure of the Sun (see Sect. 8.5), and the rotation rate inside the Sun has been measured by a change in the periods of the sound waves. Waves propagating in the direction of rotation are carried along by the moving gas, and move faster than they would in a non-rotating Sun. A bird or a jet airplane similarly moves faster when traveling with the wind and takes a shorter time to complete a trip. The resonating sound-wave crests moving with the rotation therefore appear, to a fixed observer, to have shorter periods. Waves propagating against the rotation are slowed down, with longer periods. These opposite effects make the observed solar oscillation periods divide, and such rotational splitting depends on both the depth and the latitude of the sound waves moving within the Sun.

The solar oscillations have a period of about 5 min, so the rotational splitting is roughly 5 min divided by 25 days, or about one part in seven thousand since 1 day = 1,440 min. The photosphere oscillations have to be measured ten or a hundred times more accurately than this to determine subtle variations in the Sun's rotation, or as accurately as one part in a million.

The solar oscillation data indicate that differential rotation, in which the equator spins faster than the poles, is preserved throughout the outer third of the Sun, known as the convective zone (Fig. 4.8). Within this zone, there is little variation of rotation with depth, and the inside of the Sun does not rotate any faster than the outside at the same latitude. At greater depths, the interior rotation no longer mimics that of sunspots, and differential rotation disappears. The internal accord

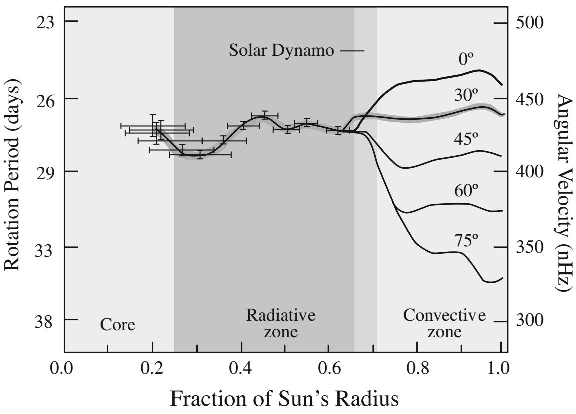

Fig. 4.8 Internal rotation of the Sun The rotation rate inside the Sun, determined by helioseismology using instruments aboard the *SOHO* spacecraft. The outer parts of the Sun exhibit differential rotation, with material at high solar latitudes rotating more slowly than material at equatorial latitudes. This differential rotation persists to the bottom of the convective zone at 28.7 percent of the way down to the center of the Sun. The rotation period in days is given at the left axis, and the corresponding angular velocity scale is on the right axis in units of nanoHertz (nHz), where 1 nHz $= 10^9$ Hz, or 1 billionth, of a cycle per second. A rotation rate of 320 nHz corresponds to a period of about 36 days (*solar poles*) and a rate of 460 nHz to a period of about 25 days (*solar equator*). The rotation in the outer parts of the Sun is given at latitudes of 0 (*solar equator*), 30, 45, 60, and 75 degrees. Just below the convective zone, the rotational speed changes markedly, and shearing motions along this interface may be the dynamo source of the Sun's magnetism. There is uniform rotation in the radiative zone, from the base of the convective zone at 0.713 to about 0.25 solar radii. The sound waves do not reach the central part of the energy-generating core. (Courtesy of Alexander G. Kosovichev/convective zone/Sebastien Couvidat, Rafael Garcia, and Sylvaine Turck-Chièze/radiative zone. *SOHO* is a project of international cooperation between ESA and NASA.)

breaks apart just below the base of the convective zone, where the rotation speed becomes uniform from pole to pole. Lower down, within the radiative zone, the rotation rate remains independent of latitude, acting as if the Sun were a solid body. Although gaseous, the radiative interior of the Sun rotates at a nearly uniform rate intermediate between the equatorial and high-latitude rates in the overlying solar material. Lang (2009) provides detailed references to pioneering spacecraft studies of the internal rotation of the Sun; also see the review by Thompson et al. (2003).

Thus, the Sun's internal rotation velocity changes sharply at about one third of the way to its center, where the outer parts of the radiative interior, which rotate at one speed, meet the overlying convective zone, which spins faster in its equatorial middle. The transition between these two different regimes takes place in a narrow region of strong rotational shear that most likely plays an important role in the generation of the large-scale solar magnetic field. The differential rotation of other stars may also play a significant role in the generation of their magnetic fields.

4.4.3 Stellar Rotation and Age

The rotation of stars other than the Sun is inferred from the Doppler broadening of their spectral lines. However, astronomers did not realize at first that some stars rotate fast enough for such measurements to be meaningful (Abney 1877). The Doppler effect of atoms moving in the hot stellar atmosphere once was thought to be substantially greater than that of stellar rotation. The Sun, for example, has an equatorial rotational speed of about 2 km s^{-1}, but the thermal speed of hydrogen atoms in the Sun's visible disk, at a temperature of 5,780 K is about 12 km s^{-1}. The Doppler broadening of the hot, moving atoms would therefore be greater than that attributed to rotation, and therefore make the rotational motion very difficult to detect. The only reason that solar rotation could be measured using spectral lines, rather than sunspots, was that the Sun is resolved and one can map out the speeds at various places along the visible disk, but that is not possible for almost all other stars that remain unresolved with even the best telescope.

The turning point came in a seminal paper by two Russian astronomers, Grigory Ambramovich Shajn (1892–1956) and Otto Struve (1896–1963), who showed that relatively young stars rotate faster than older ones and therefore exhibit exceptionally broad spectral lines. They concluded that some stars have equatorial rotational velocities ranging up to 100 km s^{-1} (Shajn and Struve 1929; Struve 1930).

The observed component of radial velocity, V_r, depends on the inclination, *i,* of the star's pole to the line of sight, with $V_r = V_e \sin i$, where V_e is the rotational velocity at the equator. The inclination is often unknown, so the measurements give a minimum value for the star's rotation velocity, sometimes referred to as the projected rotational velocity.

Stars with rapid rotation are massive, hot and young (Slettebak 1949, 1954, 1955). They include the bright stars Achernar, Alpha Arae, Pleione, and Vega (Peterson et al. 2006), with respective equatorial rotational velocities of up to 300, 470, 329, and 274 km s^{-1}. These stars are rotating so rapidly that their equators bulge outward, giving them a flattened shape. Achernar, for example, is thought to have an equatorial diameter that is about 50 % greater than the distance between its poles. The stars with rapid rotation exhibit intense x-ray emission, due to hot coronae and presumably intense magnetic fields (Pallavicini et al. 1981).

Less massive, cooler and older stars like the Sun rotate with much slower speeds of 10 km s^{-1} or less, and do not exhibit a pronounced equatorial bulge. These stars most likely were formed with fast rotations like the more massive stars, but have slowed down as they aged. Stellar magnetic fields coupled to the surrounding interstellar material act as magnetic brakes over long time intervals. The Sun, for example, probably originated 4.6 billion years ago with a rotation velocity of about 100 km s^{-1}; its magnetism helps to explain why it is now rotating at about 2 km s^{-1} (Sects. 12.1 and 12.2).