Chapter 6
Detecting Atoms in Stars

6.1 What is the Sun Made Out Of?

When sunlight is spread out into its different colors or wavelengths, it is cut by several dark gaps. They were first noticed by the English astronomer William Hyde Wollaston (1766–1828) in 1802 (Wollaston 1802), and then investigated in greater detail by German astronomer Joseph von Fraunhofer (1787–1826). Fraunhofer had detected and catalogued more than 300 gaps, assigning Roman letters to the most prominent (Fraunhofer 1817).

By directing the incoming sunlight through a slit and then dispersing it with a prism, Fraunhofer was able to overcome the blurring of colors from different parts of the Sun's disk, discovering numerous dark features in this spectral display. When coarse wavelength resolution is used, adjacent bright emission obscures the dark places, which are no longer found.

The dark gaps of missing colors found in a display of the Sun's radiation intensity as a function of wavelength, or in its spectrum, are now called *lines* because they each look like a line in the spectral display. They are designated further as *absorption lines* because they are produced when atoms in a cool, tenuous gas absorb the radiation of hot, dense underlying material. They are also known as Fraunhofer absorption lines, in recognition of his work. Such lines also can appear in emission when a gas is heated, and they are known as *emission lines*.

Both absorption and emission lines (Fig. 6.1) identify the ingredients of the cosmos. For stars, astronomers mainly use absorption lines to determine the composition of their relatively cool, outer atmospheres. In contrast, emission lines reveal the ingredients of the hotter, rarefied emission nebulae.

The Sun is so bright that its light can be spread out into small wavelength intervals with enough intensity to be detected, thereby displaying numerous dark absorption lines (Fig. 6.2). One instrument used to make and record such a spectrum is called a spectroheliograph, a composite word consisting of *spectro* for "spectrum," *helio* for the "Sun," and *graph* for "record" (Fig. 6.3). It uses the

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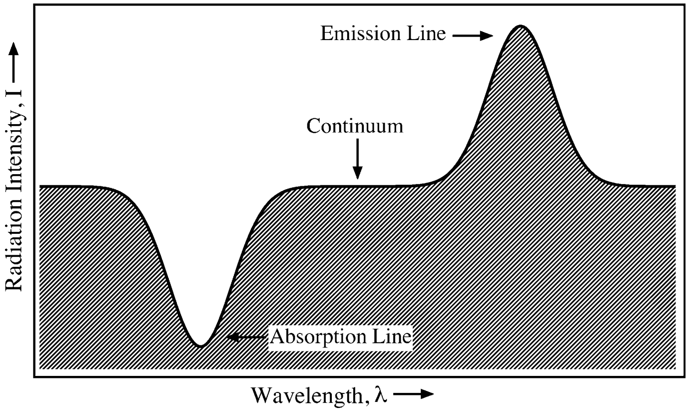

Fig. 6.1 **Absorption and emission lines** The spectrum of a star or other cosmic object displays the intensity of its radiation as a function of wavelength, denoted by the Greek symbol lambda, λ. The object's continuum radiation can be detected at all wavelengths. Atoms can produce absorption at a specific wavelength. This feature is called an *absorption line* because it looks like a line in the spectrum. When atoms are excited at high temperatures, they can radiate an *emission line*. The motion of the absorbing or emitting atoms broadens these lines (see Fig. 6.6). The line wavelength indicates which atom is responsible for the absorption or emission, and the intensity of a stellar line is related to both the number of atoms and the physical conditions in the star's atmosphere. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

grooves of a diffraction grating to reflect sunlight into different locations according to color or wavelength, similar to what a rainbow and compact disk do.

The Sun's absorption lines provided the first clues to the composition of the stars. In the mid-19th century, the German physicist Gustav Kirchhoff (1824–1887) and his chemist colleague Robert Bunsen (1811–1899) showed that every chemical element when burned and vaporized into a gas emits brightly colored lines. And the unique wavelengths of these lines coincide with those of the dark absorption lines in the Sun's spectrum.

When Kirchhoff and Bunsen vaporized an individual element in a flame, the hot vapor produced a distinctive pattern of sharply defined, bright lines. Moreover, when the light produced by a hot radiating object, such as a tungsten lamp, was passed through the cooler vaporized gas, dark lines were produced at exactly the same locations. Kirchhoff generalized this into a law stating that the powers of emission and absorption of a body at any particular wavelength are the same at any given temperature. He also concluded that the visible solar disk was hot and incandescent, producing a continuum spectrum (the sort without lines), which became crossed by the dark Fraunhofer lines when passing through cooler overlying gas.

By comparing the Sun's absorption lines with the emission lines of elements vaporized in the laboratory, Kirchhoff identified in the solar atmosphere several elements known on the Earth. The lines designated by Fraunhofer with the letters H and K were associated with calcium, and iron was assigned the letter E.

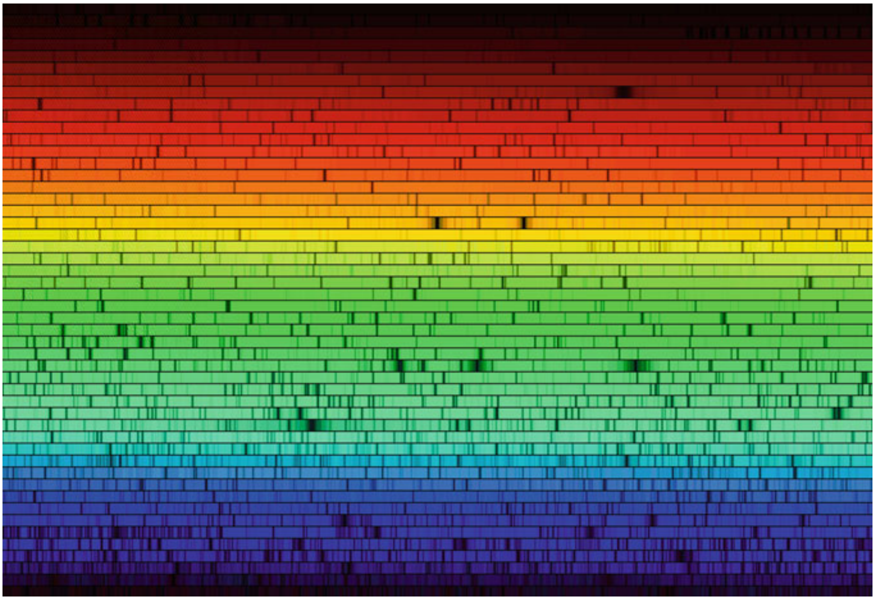

Fig. 6.2 **Solar spectrum** The visible portion of the Sun's radiation has been dispersed into its spectral components, displaying radiation intensity as a function of wavelength. When we pass from long wavelengths to shorter ones (*left–right*, *top—bottom*), the spectrum ranges from *red* thorough *orange*, *yellow*, *green*, *blue* and *violet*. Dark gaps in the spectrum, called Fraunhofer absorption lines, are due to absorption by atoms in the Sun. The wavelengths of these absorption lines can be used to identify the elements in the Sun, and the relative darkness of the lines helps establish the relative abundance of these elements. This high-resolution version of the spectrum of our Sun was created from a digital atlas observed with the Fourier Transform Spectrometer at the McMahon-Pierce Solar Facility at the National Solar Observatory on Kitt Peak, near Tucson Arizona. (Courtesy N. A. Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF.)

The letter b was attributed to magnesium, and the close pair of dark lines in the yellow, specified by the letter D, was attributed to sodium; they produce the distinctive yellow color of sodium vapor streetlights used at the time. This suggested that the Sun – and presumably all stars – are composed of terrestrial elements (Kirchhoff and Bunsen 1860, 1861; Kirchhoff 1861a, b).

The Swedish spectroscopist Anders Jonas Ångström (1814–1874) subsequently identified hydrogen in the solar spectrum; it is associated with Fraunhofer's letters C and F. Ångström published a comprehensive atlas of more than 1,000 absorption lines in the Sun's spectrum, identifying them with hydrogen, sodium, calcium, barium, strontium, magnesium, copper, iron, chromium, nickel, cobalt, zinc, and gold (Ångström 1868). His scale of wavelengths for measuring the spectral lines is still used by some astronomers; it is now named the Ångström, abbreviated Å, with $1 \text{ Å} = 10^{-10} \text{ m} = 0.1 \text{ nm}$.

Unfortunately, the Fraunhofer lines designated A and B are not related to the composition of the Sun. They only appear in spectra gathered beneath the Earth's

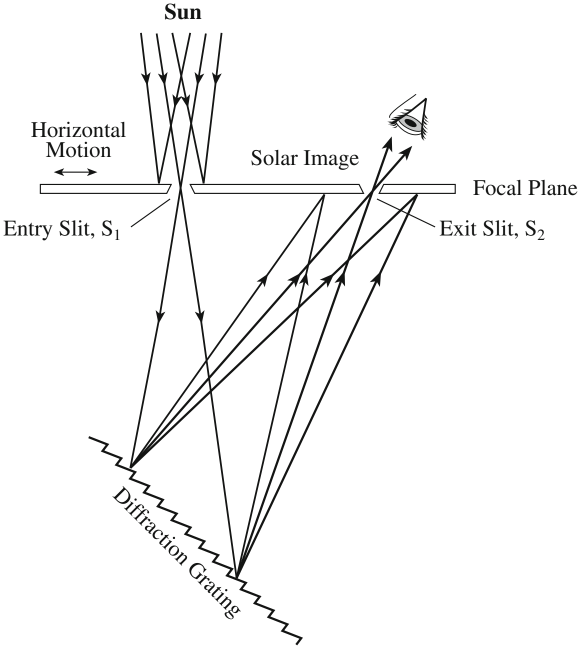

Fig. 6.3 **Spectroheliograph** A small section of the Sun's image at the focal plane of a telescope is selected with a narrow entry slit, S_1, and this light passes to a diffraction grating to produce a spectrum. A second slit, S_2, at the focal plane selects a specific wavelength from the spectrum. If the plate containing the two slits is moved horizontally, then the entrance slit passes adjacent strips of the solar image. The light leaving the moving exit slit then builds up an image of the Sun at a specific wavelength

atmosphere, and are caused by oxygen in the terrestrial atmosphere. The oxygen molecules absorb sunlight at the wavelengths of the A and B Fraunhofer lines, creating the dark lines that are superposed on the Sun's spectrum.

Still, there are some very strong absorption lines that are due to the Sun, and they extract large amounts of energy from sunlight. They are produced by hydrogen, sodium, magnesium, calcium, and iron (Table 6.1), but iron accounts for more lines than any other element. Because abundant heavy iron accounts for the Earth's high average mass density and because most of the other solar lines corresponded to elements known on the Earth, it was initially thought that the Sun is made of the same material as the Earth; but this turned out to be only partly true. Many of the visible spectral lines are associated with hydrogen, a relatively rare element on Earth (Abundant hydrogen is found bound up in the Earth's water, but hydrogen gas escapes from the Earth and is not found in noticeable quantities in its atmosphere.). Hydrogen is nevertheless the most abundant element in the universe.

The Earth is primarily made of heavy elements that are relatively uncommon in the Sun and the rest of the universe. Hydrogen is about 1 million times more

Table 6.1 Prominent absorption lines and elements detected in sunlight[a]

Wavelength (nm)	Fraunhofer letter	Element symbol and name
393.368	K	Ionized calcium, Ca II
396.849	H	Ionized calcium, Ca II
410.175	h	Hydrogen, H_{δ}, Balmer delta transition
422.674	g	Calcium, Ca I
431.0	G	CH molecule
434.048		Hydrogen, H_{γ}, Balmer gamma transition
438.356	d	Iron, Fe I
486.134	F	Hydrogen, H_{β}, Balmer beta transition
516.733	b_4	Magnesium, Mg I
517.270	b_2	Magnesium, Mg I
518.362	b_1	Magnesium, Mg I
526.955	E	Iron, Fe I
588.997[b]	D_2	Sodium, Na I
589.594	D_1	Sodium, Na I
656.281	C	Hydrogen, $H\alpha$, Balmer alpha transition
686.719	B	Molecular oxygen, O_2, in the Earth's atmosphere
759.370	A	Molecular oxygen, O_2, in the Earth's atmosphere

[a] The photosphere is the visible solar disk. The wavelengths are in nanometer units, where 1 nanometer = 1 nm = 10^{-9} m. Astronomers have often used the Ångström unit of wavelength, where 1 Ångström = 1 Å = 0.1 nm. Joseph von Fraunhofer used letters to designate the spectral lines before they were chemically identified, and the subscripts denote components that were not resolved by Fraunhofer. A Roman numeral I after an element symbol denotes an electrically neutral, or unionized, atom, with no electrons missing, whereas the Roman numeral II denotes a singly ionized atom with one electron missing. The lines A and B are produced by molecular oxygen in the terrestrial atmosphere

[b] Fraunhofer's D line includes the two sodium lines, designated D_1 and D_2, and the helium line at 587.6 nm, designated D_3

abundant than iron in the Sun, but iron is a main constituent of the Earth, which cannot even retain hydrogen gas in its atmosphere for any significant length of time. Asplund et al. (2009) have reviewed observations of the chemical composition of the Sun.

Helium, the second-most abundant element in the Sun, is so rare on the Earth that it was first discovered in the Sun. The French astronomer Pierre Jules César Janssen (1824–1927) observed an unidentified yellow emission line, with a wavelength of 587.49 nm, in the solar spectrum during the solar eclipse on August 18, 1868, which he observed from India (Janssen 1868). The emission originated in the chromosphere, a thin, slightly hotter layer of gas that lies just above the visible solar disk, or photosphere, which became visible when the Moon blocked the bright glare of the photosphere.

On October 20, 1868, the British astronomer Sir Joseph Norman Lockyer (1836–1920) found the same yellow line in the solar spectrum, but he didn't need a solar eclipse (Lockyer 1869, 1887). It was probably not until the following year that Lockyer became convinced that the yellow line he saw could not be identified with any known element on Earth, and named the element "helium" after the

Table 6.2 The twenty most abundant elements in the Sun

Element	Symbol	Atomic number, Z	Abundance[a] (logarithmic)	Discovery on Earth
Hydrogen	H	1	12.00	1766
Helium	He	2	[10.93 ± 0.01]	1895[b]
Carbon	C	6	8.43 ± 0.05	(ancient)
Nitrogen	N	7	7.83 ± 0.05	1772
Oxygen	O	8	8.69 ± 0.05	1774
Neon	Ne	10	[7.93 ± 0.10]	1898
Sodium	Na	11	6.24 ± 0.04	1807
Magnesium	Mg	12	7.60 ± 0.04	1755
Aluminum	Al	13	6.45 ± 0.03	1827
Silicon	Si	14	7.51 ± 0.03	1823
Phosphorus	P	15	5.41 ± 0.03	1669
Sulfur	S	16	7.12 ± 0.03	(ancient)
Chlorine	Cl	17	5.50 ± 0.30	1774
Argon	Ar	18	[6.40 ± 0.13]	1894
Potassium	K	19	5.03 ± 0.09	1807
Calcium	Ca	20	6.34 ± 0.04	1808
Chromium	Cr	24	5.64 ± 0.04	1797
Manganese	Mn	25	5.43 ± 0.04	1774
Iron	Fe	26	7.50 ± 0.04	(ancient)
Nickel	Ni	28	6.22 ± 0.04	1751

[a] Logarithm of the abundance in the solar photosphere, normalized to hydrogen = 12.00, or an abundance of 1.00×10^{12}. Indirect solar estimates are marked with []. The data are from Asplund et al. (2009)

[b] Helium was discovered on the Sun in 1868, but it was not found on Earth until 1895

Greek Sun god, *Helios*, who daily traveled across the sky in a chariot of fire drawn by four swift horses.

Helium was not found in the Earth until 27 years after its discovery in the Sun. Then the Scottish chemist Sir William Ramsay (1852–1916) discovered its spectral signature in a gaseous emission given off by the uranium mineral, cleveite, when it was heated (Ramsay 1895). Ramsay received the Nobel Prize in Chemistry in 1904 for his discovery of noble, or inert, gaseous elements in the air; they include helium, neon, argon, krypton, xenon and radon. These so-called noble gases do not combine with most other chemical elements, which is one reason it took so long to discover helium on Earth.

Today, helium is used on the Earth in a variety of ways, including inflating party balloons and, in its liquid state, to keep sensitive electronic equipment cold. Although plentiful in the Sun, helium is almost nonexistent on the Earth. It is so terrestrially rare that we are in danger of running out of helium during this century. There have been reports that Japanese scientists plan to mine helium from the Moon's surface, where it has been implanted by winds from the Sun.

Altogether, 92.1 % of the atoms of the Sun are hydrogen atoms, 7.8 % are helium atoms, and all of the heavier elements comprise only 0.1 %. In contrast, the main ingredients of the Earth are the heavier elements such as silicon and iron,

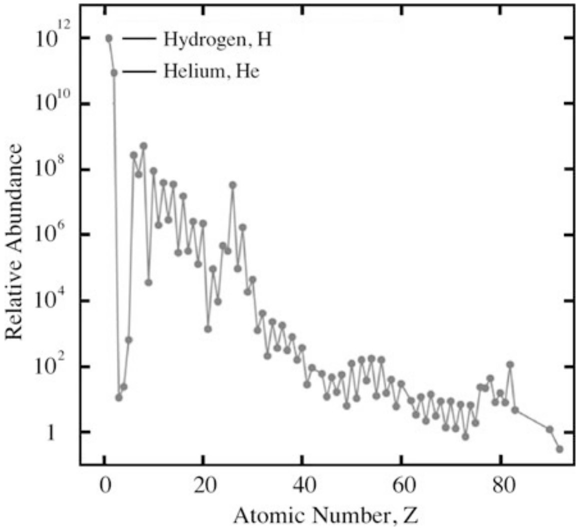

Fig. 6.4 Elemental abundance in solar photosphere The relative abundance of the elements in the Sun's visible disk, the photosphere, plotted as a function of atomic number. The atomic number, denoted by Z, is the number of protons in an atom's nucleus, or roughly half the atomic weight. Heavy elements, with high atomic numbers, are less abundant than light ones, with low atomic numbers; the most abundant element in the Sun is the lightest element, hydrogen. Helium is the second most abundant solar element. The abundance data are plotted in a logarithmic scale normalized to 1 million million, or 1.0 × 10^{12}, for hydrogen. (Adapted from Asplund et al. 2009.)

which explains the Earth's high mass density – about four times that of the Sun, which is only about as dense as water.

By mass, hydrogen accounts for 71.54 % of the Sun; the helium amounts to 27.03 % by mass, because the helium atom is about four times more massive than the hydrogen atom. All of the heavier solar elements amount to only 1.42 % by mass.

The abundance of the 20 most abundant elements in the Sun are listed in Table 6.2, normalized to a hydrogen abundance of 1 million million, or 10^{12}. There is a systematic decrease in the abundance of solar elements with increasing atomic number (Fig. 6.4), but with a noticeable gap of unexpectedly low abundance for the light elements between helium and carbon.

6.2 Quantization of Atomic Systems

Most of the mass of an atom is concentrated in its relatively small nucleus, which is surrounded by electrons (Sect. 5.1). The nucleus has a positive charge due to the protons in it and is about 100,000 times smaller than the atom. The negatively charged electrons keep the atom distended, enlarging its shape. As a result, an atom is mostly empty space.

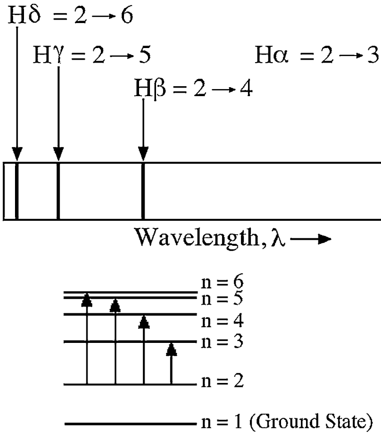

Fig. 6.5 Balmer lines of hydrogen The spectrum of the Sun's optically visible light exhibits four strong absorption lines that are attributed to hydrogen, whose line wavelengths are spaced closer together at shorter wavelengths (*top*). These lines are designated Hα at a *red* wavelength of 656.3 nm, Hβ at a wavelength of 486.1 nm, Hγ at the *blue* 434.1 nm and Hδ at the *violet* 410.2 nm, where 1 nanometer = 1 nm = 10^{-9} m. These spectral features originate when an electron in a hydrogen atom moves from a low to a high electron orbit, the orbital energy of which is a function of the integer n (*bottom*). They were named *Balmer lines* after the Swiss mathematics teacher Johann Balmer (1825–1898), who first derived an equation that describes their wavelengths in terms of integers. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

According to Rutherford's model of the atom, the electrons revolve around the central nucleus, somewhat like the planets that endlessly whirl around the Sun. Unlike a planet, an electron is electrically charged, and a revolving charge emits electromagnetic radiation. That is how radio signals are broadcast, by moving electrons through wires to generate radiation.

This means that something was wrong with Rutherford's model. An electron revolving in an atom-sized orbit radiates light waves, and as a result of this emission, it will lose its kinetic energy of motion. As a result, an electron cannot be perpetually moving around the nucleus. An atomic electron should lose its orbital motion and spiral into the atom's nucleus in less than 1 s. So, the idea of electron orbits did not seem to work.

An unexpected feature of the Sun's hydrogen lines, which are produced by an orbital change of the hydrogen atom's single electron, provided a clue to this enigma. That is, the wavelengths of spectral lines from hydrogen indicate that electrons have to follow certain rules if they want to belong to an atom. Not just any behavior is allowed and only certain orbits are permitted.

Adjacent hydrogen lines in the spectrum of the Sun or any other cosmic object systematically crowd together at shorter wavelengths (Fig. 6.5). The Swiss mathematics teacher Johann Balmer (1825–1898) found a simple equation that describes their regular spacing. The four lines in the Sun's visible hydrogen

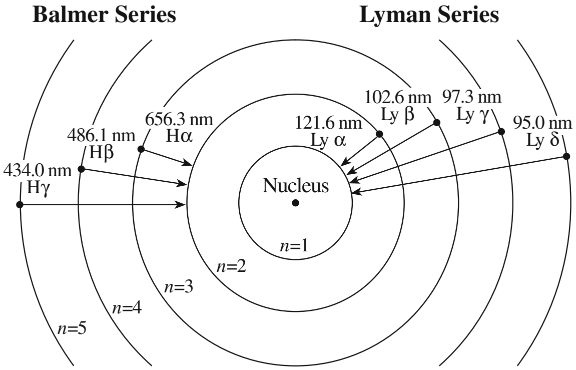

Fig. 6.6 Bohr atom In this model, proposed in 1913 by the Danish physicist Niels Bohr (1885–1962), a hydrogen atom's one electron revolves around the hydrogen nucleus, a single proton, in well-defined orbits described by the integer $n = 1, 2, 3, 4, 5, \ldots$. An electron absorbs or emits radiation when it transitions between these allowed orbits. The electron can jump upward, to orbits with larger n, by absorption of a radiation photon of exactly the right energy, equal to the energy difference between the orbits; the electron can jump down to lower orbits, of smaller n, with the emission of radiation of that same energy and wavelength. Transitions that begin or end on the $n = 2$ orbit define the Balmer series observed at visible wavelengths. They are designated by Hα, Hβ, Hγ,…. The Lyman series, with transitions from the first orbit at $n = 1$, is detected at ultraviolet wavelengths. The orbits are not drawn to scale because the size of their radius increases with the square of the integer n

spectrum have wavelengths of 410, 434, 486, and 656 nm, and they are still known as the Balmer lines. Balmer's equation that specified their wavelengths, λ, is (Balmer 1885):

$$\lambda = K\left(\frac{m^2}{m^2 - n^2}\right) = K\left(\frac{m^2}{m^2 - 2^2}\right), \tag{6.1}$$

where the constant $K = 3.6456 \times 10^{-7}\ m = 364.56$ nm, the integer $n = 2$ and the integers $m = 3, 4, 5, \ldots$. The most intense Balmer line is now known as the *Balmer hydrogen alpha line*, designated Hα, at a red wavelength at 656.3 nm, where 1 nanometer = 1 nm = 10^{-9} m = 10 Å = 10 Ångström.

A few years later, the Swedish physicist Johannes Rydberg (1854–1919) generalized the Balmer equation for all transitions of hydrogen, in the Rydberg formula (Rydberg 1890):

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{6.2}$$

where m and n are integers, the Rydberg constant for hydrogen $R_\infty = 10{,}973{,}731.57\ \mathrm{m}^{-1} = 1.097373157 \times 10^7\ \mathrm{m}^{-1} = 4/K$ where the K is in Balmer's equation where $m = 2$, the $n = 3, 4, 5$ (the m and n have been interchanged from Balmer's formula.) Rydberg's formula was subsequently used to predict spectral lines observed at the invisible infrared, ultraviolet and radio wavelengths.

The Danish physicist Niels Bohr (1885–1962) explained Balmer's equation by a model of the hydrogen atom, now known as the Bohr atom (Bohr 1913). In this model, a single electron in a hydrogen atom revolves about the nuclear proton in specific orbits with definite, quantized values of energy (Fig. 6.6). An electron only emits or absorbs radiation when jumping between those allowed orbits, each jump associated with a specific energy and a single wavelength. If an electron jumps from a low-energy to a high-energy orbit, it absorbs radiation at this wavelength; radiation is emitted at exactly the same wavelength when the electron jumps the opposite way. This unique wavelength is related to the difference between the two orbital energies. Bohr was awarded the 1912 Nobel Prize in Physics for these investigations of the structure of atoms and the radiation emanating from them.

Because only quantized orbits are allowed, spectral lines are produced only at specific wavelengths that characterize or identify an atom. An atom or molecule can absorb or emit a particular type of light only if it resonates to that light's energy. As it turns out, the resonating wavelengths or energies of each atom are unique.

A little more than a decade before Bohr introduced his model, Planck (1901) explained continuum thermal radiation, without lines, by supposing that the emission and absorption of light takes place only in the form of certain discrete portions, or quanta, of energy, now known as *photons* (See Sect. 2.4). He had quantized radiation, giving a photon an energy $h\nu$ at frequency ν. The constant h is the Planck constant, with the value $h = 6.626$ x 10^{-34} J s. The energy of each separate light quantum, or photon, is proportional to the frequency of light, or inversely proportional to its wavelength.

Bohr went one step farther and quantized the energy of motion of the electrons revolving in an atom. He proposed that the mechanical energy of any moving subatomic particle could take on only one of a certain set of discrete values, in an entirely new quantum mechanics. That is, he quantized the electron's angular momentum and energy, using the Planck constant, h. Because the quantum of an electron's orbital energy increases with the decreasing dimensions of the orbit, this suggested that quantum mechanics would become important only for very small, subatomic physical scales.

In the Bohr atom, the single electron of a hydrogen atom is said to orbit the atom's nuclear proton with an angular momentum, $m_e \mathrm{V} r_n$, that is quantized, or (Bohr 1913):

$$m_{\mathrm{e}} \mathrm{V} r_n = hn/(2\pi), \tag{6.3}$$

where the mass of the electron $m_e = 9.109 \times 10^{-31}$ kg, the electron's orbital velocity is V, its orbital radius is r_n for the integer $n = 1, 2, 3 \ldots$, Planck's constant $h = 6.626 \times 10^{-34}$ J s, and $\pi = 3.14159$.

The radius, r_n, of the nth electron orbit is derived by equating the Coulomb force of attraction of the proton, of charge e, on the electron, also of charge e, to the centripetal force, or

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} = \frac{mV^2}{r_n} \tag{6.4}$$

to obtain

$$r_n = a_0 n^2 \tag{6.5}$$

where the radius, a_0, of the first Bohr orbit of hydrogen is:

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m_e e^2} \approx 5.2918 \times 10^{-11}\,\text{m}, \tag{6.6}$$

with an electrical constant $\varepsilon_0 = 8.8542 \times 10^{-12}$ F m^{-1}, the Planck constant $h = 6.6261 \times 10^{-34}$ J s, the constant $\pi = 3.14159$, the electron mass $m_e = 9.1094 \times 10^{-31}$ kg, and the elementary charge $e = 1.6022 \times 10^{-19}$ C.

The electron orbital energy is also quantized, which can be seen by solving for its kinetic energy $m_e V^2/2$. That is, electrons can only occupy orbits with allowed orbital energy, E_n, given by:

$$E_n = \frac{1}{2} m_e V^2 = \frac{m_e e^4}{8\varepsilon_0^2 h^2 n^2} = \frac{hcR_\infty}{n^2} \tag{6.7}$$

where the Rydberg constant, R_∞, is:

$$R_\infty = \frac{m_e e^4}{8\varepsilon_0^2 h^3 c} \approx 10,973,731.568\ \text{m}^{-1} \approx 1.097 \times 10^7 \text{m}^{-1}, \tag{6.8}$$

and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

The constant $hcR_\infty = 13.\ 60569$ eV = 1 Ry is known as the Rydberg unit of energy, and is often used in atomic physics. The electron volt is equivalent to 1 eV = 1.602176×10^{-19} J.

The permitted orbital energy increases with decreasing integer, or quantum number, n, which corresponds to decreasing radius or size of the electron orbit. The closer an electron is to the nucleus, the greater is its allowed energy. Both the energy and radius of an electron's orbit vary as the inverse square of the quantum number.

The nth energy level of the hydrogen atom has energy $E_n = 13.6\ \text{eV}/n^2 = hcR_\infty/n^2$, where 13.6 eV is the ionization potential of hydrogen and 1 eV = 1.602×10^{-19} J. The temperature required to ionize hydrogen, removing its sole electron from its ground state is about 1.6×10^5 K, obtained from setting the temperature equal to 13.6 eV divided by the Boltzmann constant $k = 1.38 \times 10^{-23}$ J K^{-1}.

The energy difference between two energy levels specified by integers n and m is:

$$E_n - E_m = \left(\frac{1}{n^2} - \frac{1}{m^2}\right) \times 13.5609 \text{ eV}. \tag{6.9}$$

Radiation can be emitted or absorbed by a hydrogen atom when the electron changes between allowed orbits, with the frequency ν_{mn} of the $m - n$ transition given by

$$\nu_{mn} = \frac{1}{h}|E_m - E_n| = cR_\infty\left|\frac{1}{n^2} - \frac{1}{m^2}\right| \approx 2cR_\infty\frac{(m-n)}{n^3}, \tag{6.10}$$

where $||$ denotes the absolute value, and the wavelength λ_{mn} of the $m - n$ transition can be inferred from the frequency by $\lambda_{mn} = c/\nu_{mn}$ where $c = 2.9979 \times 10^8$ m s^{-1} is the speed of light.

The wavelength, λ_{mn}, of the radiation emitted or absorbed in a radiative transition between the two levels will be:

$$\lambda_{mn} = \frac{hc}{E_m - E_n} = \left(\frac{1}{n^2} - \frac{1}{m^2}\right)^{-1} \times 911.5 \text{ Å}, \tag{6.11}$$

where 1 Å $= 10^{-10}$ m $= 0.1$ nm is one Ångström.

The most intense spectral line for any given value of n is the transition with $m - n = 1$, known as the alpha, or α, transition, and the next most intense line is for the beta, or β, transition with $m - n = 2$, and so on for increasing $m - n$. The difference in orbital energies for the α transition is smaller than any other transition, and therefore the easiest to accomplish; more atoms will undergo this transition than other atoms, and the line is therefore the most intense. At higher $m - n$, the difference in orbital energies is greater and the transition is more energetic; but there are fewer atoms undergoing the transition and the intensity of the line is lower.

For any n, the frequency of the α transition is given by:

$$\nu_{mn} = \frac{2cR_\infty}{n^3}. \tag{6.12}$$

Example: Recombination lines at radio wavelengths

Electron transitions at high quantum numbers n have been detected from hydrogen atoms surrounding very hot stars. For the $m - n = 1$, or the α transition, radiation from the $n = 109$ transition has been observed. The frequency of this transition is $\nu_{mn} = 2cR_\infty/n^3 = 5.079 \times 10^9$ Hz $=$ 5,079 MHz, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1} and the Rydberg constant $R_\infty = 1.097 \times 10^7$ m^{-1}. The radius of the electron orbit is $r_n = a_0 n^2 \approx 6.29 \times 10^{-7}$ m, where the Bohr radius $a_0 = 5.2918 \times 10^{-11}$ m, so the orbit radius is much larger than the average radius of an atom, usually about 10^{-10} m.

The α transition detected at visible wavelengths is called the Balmer α transition, at a red wavelength of 6562.8 Å while the ultraviolet α transition is known as Lyman α at 1215.67 Å, where 1 Å = 10^{-10} m.

The Lyman series, for $n = 1$, includes

Designation	m	λ_{mn} (Å)
$Ly\alpha$	2	1,216
$Ly\beta$	3	1,026
$Ly\gamma$	4	973
Lyman limit	∞	911.5

And the Balmer series, for $n = 2$, includes:

Designation	m	λ_{mn} (Å)
$H\alpha$	3	6,563
$H\beta$	4	4,861
$H\gamma$	5	4,341
Balmer limit	∞	3,646

The important hydrogen transitions between low n orbits have been given the last names of the persons who first observed them. They are called the Lyman transitions for $n = 1$ (Lyman 1906), the Balmer transitions for $n = 2$ (Balmer 1885), the Paschen transitions for $n = 3$ (Paschen 1908), the Brackett transitions for $n = 4$ (Brackett 1922), and the Pfund transitions for $n = 5$ (Pfund 1924).

The wavelengths of the relevant m to n transitions are given in Table 6.3 for m between 1 and 10, as well as for $m = \infty$, which is the limiting wavelength at which the lines merge together (Fig. 6.7). For wavelengths shorter than this limit,

Table 6.3 Wavelengths of the m to n transitions of hydrogen for $n = 1$ to $n = 5$ and $m = 2$ to $m = 10$[a]

Series (m)	Lyman ($n = 1$)	Balmer ($n = 2$)	Paschen ($n = 3$)	Brackett ($n = 4$)	Pfund ($n = 5$)
2	121.567				
3	102.572	656.280			
4	97.2537	486.132	1875.10		
5	94.9743	434.046	1281.81	4051.20	
6	93.7803	410.173	1093.81	2625.20 7	7457.8
7	93.0748	397.007	1004.94	2165.50	4652.5
8	92.6226	388.905	954.598	1944.56	3739.5
9	92.3150	383.538	922.902	1817.41	3296.1
10	92.0963	379.790	901.491	1736.21	3038.4
∞	91.15	364.6	820.36	1458.4	2278.8

[a] The wavelengths are given in nanometers where 1 nm = 10^{-9} m

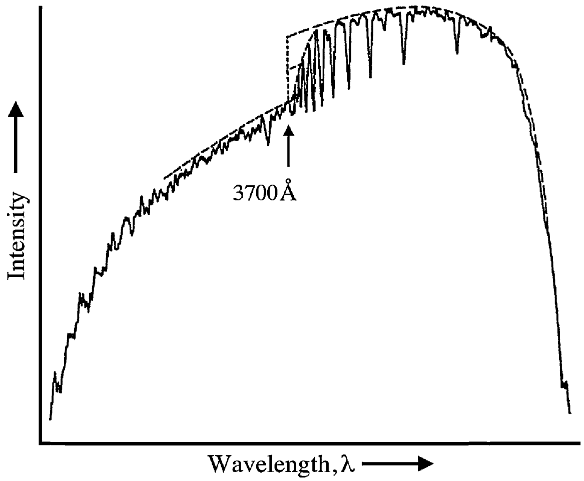

Fig. 6.7 Balmer limit As shown in this spectrum of a star, the lines of the Balmer series of hydrogen crowd together and merge at a limiting wavelength, the Balmer limit at 364.6 nm or approximately 3,700 Ångström. At wavelengths less than this limit, the radiation of hydrogen is emitted at a continuous range of wavelengths, known as the Balmer continuum, resulting from transitions between states with principal quantum number $n = 2$ and states in which the single electron is freed from the atom

the radiation is emitted at a continuous range of wavelengths, or as a continuum, resulting from transitions in which the electron leaves the atom.

For an atom of atomic number Z, which is the charge of the nucleus in units of the proton charge, the electron orbital energy, E_n, is given by

$$E_n = \frac{m_e e^4 Z^2}{8\varepsilon_0^2 h^2 n^2} = \frac{hcR_A}{n^2} \tag{6.13}$$

where n is an integer and the atomic Rydberg constant, R_A, is given by:

$$R_A = R_\infty \left(1 + \frac{m_e}{M_A}\right)^{-1} \approx R_\infty \left(1 - \frac{m_e}{M_A}\right) \tag{6.14}$$

where the electron mass $m_e = 5.4858 \times 10^{-4}$ u, the atomic mass M_A is often given in units of the atomic mass unit u = 1.660539×10^{-27} kg. The electron mass in atomic mass units is $m_e = 0.000548579867$ u. For hydrogen $Z = 1$ and for helium $Z = 2$. The frequency for the transition from an upper level m to a lower level n is given by

$$\nu_{mn} = cR_A Z^2 \left(\frac{1}{n^2} - \frac{1}{m^2}\right) \approx 2cR_A Z^2 \frac{(m-n)}{n^3} \tag{6.15}$$

Table 6.4 Atomic number Z, atomic mass M_A, and atomic Rydberg constant R_A for the most abundant atoms in the cosmos[a]

Atom	Atomic number, Z	Atomic mass, M_A (u)	Rydberg constant, R_A (10^7 m^{-1})
Hydrogen, H^1	1	1.007 825	1.096776
Helium, He^4	2	4.002 603	1.097223
Carbon, C^{12}	6	12.000 000	1.097323
Nitrogen, N^{14}	7	14.003 074	1.097330
Oxygen, O^{16}	8	15.994 914	1.097335
Neon, Ne^{20}	10	19.992 440	1.097343

[a] The atomic mass is given in atomic mass units u = 1.660539×10^{-27} kg

The atomic number and Rydberg constant for abundant atoms are given in Table 6.4.

6.3 Some Atoms are Excited Out of Their Lowest-Energy Ground State

Because atoms reside together in great numbers, we must use a statistical approach to determine their average properties. Their level of excitation depends on the temperature and the density, which influence how often the particles collide and become excited. The higher the temperature, the faster particles move, and the more frequent their collisions. When the particles are packed together in greater numbers per unit volume, with a greater density, collision frequency also increases.

The number of atoms in the lowest possible energy state, called the ground state, is greater than the number in any other state of energy, essentially because its easier to stay in the lowest energy state. Because it requires less energy, an atom acts as if it prefers the ground state. It is simply more difficult to enter or remain in an excited state of energy. Outside agencies are required to sustain an excited state of energy.

The allowed energy levels of the electron orbits within an atom can be compared to the rungs of a ladder. Electrons can climb the ladder's energy rungs when an atom either collides with other atoms or absorbs radiation. Once an electron is up on a rung, it can jump downward, releasing the energy it attained to get there. Since it is easier to stay on the ground and never climb the ladder, most atoms are usually in the ground state. Because each type of atom has a different type of ladder, with energy rungs located at different places, every element exhibits unique absorption or emission lines.

It takes more energy to excite the higher states, and the number of atoms that exist in a given energy level varies inversely and exponentially as a function of the energy. When collisions are the dominant process that influences the energy-level population, the ratio of the population of two energy levels of a given atom depends on the temperature. At higher temperatures, more atoms are pumped up to the more energetic states.

Under conditions of local thermodynamic equilibrium, the ratio of the number of atoms at two different energies depends on their energy difference divided by the temperature. At a higher temperature, there are more atoms with higher energy; but there always are more atoms with the lowest possible energy, the ground state. The number ratio is known as the *Boltzmann distribution*, named after Ludwig Boltzmann (1894–1906) who derived it (Boltzmann 1872).

Individual atoms in a collection of atoms are always moving about, undergoing collisions, becoming excited and radiating that excitation away. The total energy emitted depends on the number of excited atoms, which varies with the temperature, the energy of the transition, and the transition probabilities, which are tabulated in spectroscopic data bases found on the Internet.

Under conditions of local thermodynamic equilibrium at a temperature T, the number densities, N_n, and N_m, of atoms in levels n and m with energies E_n and E_m are related by the Boltzmann distribution (Boltzmann 1872)

$$\frac{N_n}{N_m} = \frac{g_n}{g_m} \exp\left(-\frac{E_n - E_m}{kT}\right), \tag{6.16}$$

where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, the g_n is the degeneracy of level n. The degeneracy of a level is the number of quantum states with the energy of that level. The quantity E_n is the energy of level n relative to a fundamental, unexcited ground state $n = 1$.

For the hydrogen atom, we have $g_n = 2n^2$ and

$$E_n = 13.6\left[1 - \frac{1}{n^2}\right] \text{eV}, \tag{6.17}$$

where n is the quantum number of the atomic energy level under consideration and 1 eV $= 1.602 \times 10^{-19}$ J.

Example: Exciting hydrogen atoms

Find the temperature at which the number density of hydrogen atoms in the fundamental ground state is equal to that of the second excited state for $n = 3$. From the Boltzmann equation, we set $N_1/N_3 = (g_1/g_3) \exp\left[-(E_1 - E_3)/(kT)\right] = 1$, and using $g_1 = 2$, $E_1 = 0$ for reference, $g_3 = 18$, $E_3 = 12.09$ eV $= 1.939 \times 10^{-18}$ J and $k = 1.38 \times 10^{-23}$ J K^{-1}, we obtain $(1.4036 \times 10^5)/T = \ln(9) = 2.197$, or $T \approx 63{,}900$ K. (The ln denotes the natural logarithm.)

If N_0 is the total number of atoms or ions per unit volume in the ground state with $n = 1$, the total number of excited atoms, N_E, with energy, E_n, above the ground state will be:

$$\frac{N_E}{N_0} = \frac{g_n}{U} \exp\left(-\frac{E_n}{kT}\right) \approx \exp\left(-\frac{E_n}{kT}\right), \tag{6.18}$$

where the partition function for the atom or ion under consideration is given by

$$U = \sum_{1}^{\infty} g_n \exp\left(-\frac{E_n}{kT}\right), \tag{6.19}$$

and g_n is the degeneracy of level n. The fraction of atoms or ions in a given energy level is equal to the portion of the partition function related to this level.

When the electron stays within the atom or ion, going from one bound state to another, then the transition is known as a bound–bound transition. If the electron breaks free of the atom or ion, then it is called a bound-free transition. The probability per unit time, P_{mn}, that an atom will undergo a bound–bound transition from a high state of energy, E_m, to a lower state of energy, E_n, is (Einstein 1917a):

$$P_{mn} = A_{mn} + B_{mn} U_v, \tag{6.20}$$

where A_{mn} is the Einstein coefficient for spontaneous transition between the two states (spontaneous emission), and B_{mn} is the Einstein stimulated coefficient for a transition induced by radiation of energy density U_v in the frequency range v to $v + dv$. The probability per unit time for a radiation-induced absorption is

$$P_{nm} = B_{nm} U_v \tag{6.21}$$

where B_{nm} is the Einstein coefficient for photo-absorption. The energy density of black body radiation at temperature, T, in the frequency range v to $v + dv$ is (Planck 1901)

$$U_v = \frac{8\pi h v^3}{c^3} \left[\exp\left(\frac{hv}{kT}\right) - 1\right]^{-1} = \frac{4\pi}{c} B_v(T), \tag{6.22}$$

where $B_v(T)$ is the brightness of the radiator at frequency v and the temperature is T. Because the number of downward transitions must equal the number of upward transitions, and because in thermodynamic equilibrium each state has a population determined by the Boltzmann distribution, it follows that

$$A_{mn} = \frac{8\pi h v^3}{c^3} B_{mn} \tag{6.23}$$

and

$$g_m B_{mn} = g_n B_{nm}, \tag{6.24}$$

where g_m is the statistical weight of the m level.

The A_{mn} is the probability per second that an atom with an electron in level m will spontaneously emit a photon of energy $h v_{mn} = E_m - E_n$ so the energy, E, emitted per unit volume by the spontaneous bound–bound transition is:

$$E = N_m h\nu_{mn} A_{mn}, \tag{6.25}$$

where N_m is the volume density of atoms in level m and $h\nu_{mn} = E_m - E_n$.

6.4 Ionization and Element Abundance in the Sun and Other Stars

Because a greater number of atoms will absorb more light, the relative darkness of the absorption lines in the Sun's spectrum should establish the relative abundance of the elements there. That is, darker, stronger absorption lines generally indicate high absorption and therefore larger amounts of the absorbing element. However, the strength of an element's absorption lines depends only to some extent on the element's abundance.

For example, atoms exist in altered physical states at the high temperatures that prevail within stars. This can result in a change in the wavelength and intensity of the spectral lines observed in stellar atmospheres. In the latter part of the 19th century, Joseph Norman Lockyer (1836–1920) had already shown experimentally that elements exhibit different spectra under varying conditions of temperature and pressure in the terrestrial laboratory, and that, in particular, the arc and higher temperature spark spectra of the same element differed (Lockyer 1887). The Sun is obviously hotter than typical laboratory temperatures, or it would not shine so brightly, and different temperatures and pressures prevail at various locations within the solar atmosphere. Astronomers eventually realized that an element displays different spectral lines depending on the physical conditions of the solar region in which it is located.

Moreover, some stars show conspicuous lines other than the dominant lines in the Sun's spectrum, suggesting that different stars have different compositions. Scientists began to think that this was instead related to the stellar temperature.

Example: Intensity of the red hydrogen alpha transition in the solar atmosphere

The intensity of a spectral line emitted by atoms or ions in an excited state s will depend on the number density, N_s, of atoms or ions occupying the excited state, the energy E_s of that state, and the temperature T. Under conditions of local thermodynamic equilibrium, the Boltzmann distribution indicates that (Boltzmann 1872):

$$\frac{N_s}{N_{tot}} = \frac{g_s}{U}\exp\left(\frac{-E_s}{kT}\right), \tag{6.26}$$

where g_s is the degeneracy, or the statistical weight, of level s, and the partition function for free particles of mass m is $U = (2\pi mkT)^{3/2}V/h^3$ where V is the volume occupied by the gas, the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1} and the Planck constant $h = 6.626 \times 10^{-34}$ J s.

For the red hydrogen alpha transition, the wavelength $\lambda = 656.28$ nm $= 6.65628 \times 10^{-7}$ m, and the energy $E_s = hc/\lambda = 3.027 \times 10^{-19}$ J, where the speed of light $c = 2.9989 \times 10^8$ m s^{-1}. The number of atoms undergoing this transition in the solar chromosphere and the photosphere depends on the temperature, $T_P = 5{,}780$ K for the photosphere and $T_C = 2 \times 10^4$ K for the chromosphere. The relative number of atoms, N_{sC}/N_{sP}, in the two layers of the solar atmosphere is given by:

$$\frac{N_{sC}}{N_{sP}} = \left(\frac{T_P}{T_C}\right)^{3/2} \frac{\exp\left(\frac{-E_S}{kT_C}\right)}{\exp\left(\frac{-E_S}{kT_P}\right)} \approx 2.3. \tag{6.27}$$

So there are more atoms excited into this state in the chromosphere than the underlying photosphere, because at higher temperatures the exponential function is closer to one and at lower temperatures it is a smaller number.

When the theory of ionization in stellar atmospheres was developed, it became clear that the presence or absence of specific spectral lines did not necessarily indicate the chemical composition of a star's atmosphere. In 1920, Meghnad Saha (1893–1956), a young lecturer at the University of Calcutta, demonstrated that the spectral lines of different elements are excited under different conditions of temperature and pressure. This set the stage for showing that many stars have similar compositions (Saha 1920).

In his analysis, Saha demonstrated the analogy between the dissociation of molecules and the ionization of atoms. He replaced the mass of the atom with the mass of the electron in the expression for the degree of dissociation of a molecule, thereby obtaining his now-famous ionization equation. This formula, known as the Saha equation, relates the degree of ionization of an atom to temperature and pressure, and therefore indicates that the relative intensities of a star's different spectral lines are caused, in part, by differences in the pressure and temperature of the stellar atmosphere.

The fraction x of atoms that are ionized in a gas at a certain temperature T and pressure P is given by:

$$\frac{x^2}{1-x} = \frac{(2\pi m_e)^{3/2}}{h^3} \frac{(kT)^{5/2}}{P} \exp\left(-\frac{\chi}{kT}\right) \tag{6.28}$$

where χ is the ionization potential, or the amount of energy to be supplied to an atom to ionize it, h is the Planck constant and m_e is the mass of the electron. The ionization potentials of some cosmically abundant atoms are given in Table 6.5.

Table 6.5 Ionization potentials χ for different stages of ionization of the most abundant atoms in the cosmos

Z		Stage of Ionization							
		I	II	III	IV	V	VI	VII	VIII
1	H	13.598							
2	He	24.587	54.416						
6	C	11.260	24.383	47.887	64.492	392.077	489.981		
7	N	14.534	29.601	47.448	77.472	97.888	552.057	667.029	
8	O	13.618	35.116	54.934	77.412	113.896	138.116	739.315	871.4

Under conditions of local thermodynamic equilibrium, the number density, N_r, of atoms in the rth stage of ionization is related to that of the $(r+1)$ state, N_{r+1}, by the Saha equation (Saha 1920, 1921):

$$\frac{N_{r+1}}{N_r}N_e = \frac{U_{r+1}}{U_r}\frac{2(2\pi m_e kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{-\chi_r}{kT}\right), \tag{6.29}$$

where N_e is the free electron density, U_r is the partition function of the rth stage, the χ_r is the energy required to remove an electron from the ground state of the r-times ionized atom, T is the temperature, m_e is the electron mass, and h and k are respectively the Planck and Boltzmann constants. The electron density N_e can be expressed in terms of the electron pressure $P_e = N_e kT$.

The population density, N_n, of the nth quantum level is given by:

$$N_n = N_e N_i \frac{h^3}{(2\pi m_e kT)^{\frac{3}{2}}}\frac{g_n}{2}\exp\left(\frac{\chi_r - \chi_n}{kT}\right), \tag{6.30}$$

where N_e and N_i are, respectively, the free electron and ion densities, g_n is the statistical weight of the nth level, and χ_n is the excitation energy of the nth level above ground level. For hydrogen like atoms,

$$\chi_n = I_H Z^2\left(1 - \frac{1}{n^2}\right) \tag{6.31}$$

where $I_H = 13.6$ eV $= 2.179 \times 10^{-18}$ J is the ionization potential of the hydrogen atom, and

$$g_n = 2n^2. \tag{6.32}$$

Saha used his ionization equation in a physical theory for stellar spectra, specifying temperatures of stars of different spectral type. His result showed that differences in stellar spectra are caused by differences in excitation rather than in chemical composition. The English astrophysicists Ralph A. Fowler (1899–1944) and Edward Milne (1896–1950) then showed that the number of atoms or ions responsible for the production of a spectral line can be estimated from the line intensity once the temperature and pressure of the stellar atmosphere are known (Fowler and Milne 1924). This paved the way for the work of the American astronomer Cecilia H. Payne (1900–1979), who showed that stars with different spectra have essentially the same composition, and it eventually led to the realization that the lightest element, hydrogen, is by far the most abundant element in most stars (Focus 6.1). This discovery also had a fundamental role in understanding how the Sun shines – by fusion reactions of the nucleus of the hydrogen atom, the proton.

Focus 6.1 Hydrogen, the most abundant element in the Sun and most stars

In a brilliant doctoral dissertation written in 1925, the American astronomer Cecilia H. Payne (1900–1979) showed that the atmospheres of virtually every luminous, middle-aged star have the same ingredients (Payne 1925). Her calculations also indicated that hydrogen is by far the most abundant element in the Sun and most other stars. However, she could not believe that the composition of stars differed so enormously from that of the Earth, where gaseous hydrogen is rarely found, so she mistrusted her understanding of the hydrogen atom. Prominent astronomers of the time also did not think that hydrogen was the main ingredient of the Sun and other stars.

Subsequent detailed investigations of the Sun's absorption-line intensities, by the German astronomer Albrecht Unsöld (1905–1995), the British astronomer William H. McCrea (1904–1999) and the American astronomer Henry Norris Russell (1877–1957) showed that the Sun is composed mainly of the lightest element, hydrogen, accounting for 92.1 % of the number of atoms in the Sun (Unsöld 1928; McCrea 1929; Russell 1929). Hydrogen is a million times more abundant than any other element in solar atmosphere, and the number density of hydrogen atoms at the base of the chromosphere is about 10^{18} atoms m^{-3}.

The Danish astronomer Bengt Strömgren (1908–1987) next calculated the hydrogen content in the interior of stars, assuming that they are chemically homogeneous, and showed that their observed luminosities require that the entire star, not just its atmosphere, be composed predominantly of hydrogen (Strömgren 1932).

We now know that very old stars have very few elements other than hydrogen and helium; these stars probably have existed since our Galaxy formed. Middle-aged stars, like the Sun, contain noticeable, but still small, amounts of heavier elements.

Hydrogen is the most abundant element in the stellar universe, so there was nothing wrong with Miss Payne's calculations. The Earth just does not have sufficient gravity to retain hydrogen in its atmosphere for any length of time. Any hydrogen gas that our young planet might have once had must have evaporated away while the Earth was forming and has long since escaped, or become locked into water or surface rocks.

In addition to specifying the compositions of stars, detailed observations of absorption or emission lines yield information about the temperature, density, motion, and magnetism of the Sun and other stars, as well as rarefied nebulae, interstellar matter, and a various cosmic objects.

6.5 Wavelengths and Shapes of Spectral Lines

6.5.1 Radial Motion Produces a Wavelength Shift

Just as a source of sound can vary in pitch or wavelength depending on its motion, the wavelength of electromagnetic radiation shifts when the emitting source moves with respect to the observer (See Sect. 4.2). Such a wavelength change is named the *Doppler effect* after the Austrian scientist, mathematician and schoolteacher, Christiaan Doppler (1803–1853) who first explained how it works (Doppler 1842).

The Doppler effect shows that the wavelength of a spectral line that an astronomer observes can differ from the emitted line wavelength. The size of the wavelength change depends on the relative speed of the radiating source along the observer's line of sight, known as the radial velocity. The greater the radial velocity, the larger the Doppler shift. In 1868, the English astronomer William Huggins (1824–1910) was the first to use this method to determine the velocity of a star moving away from the Earth (Huggins 1868).

For a source radial velocity directed away from the observer along the line of sight, the shift is toward longer, redder wavelengths in the visible part of the electromagnetic spectrum, and therefore also is known as a *redshift*. When the motion is toward the observer, there is a *blueshift* to shorter, bluer wavelengths. For a source radial velocity, V_r, away from the observer along the line of sight, the Doppler redshift formula is applicable:

$$z = \frac{\Delta\lambda}{\lambda_L} = \frac{\lambda_{observed} - \lambda_L}{\lambda_L} = \frac{\nu_L - \nu_{observed}}{\nu_{observed}} \approx \frac{V_r}{c} \text{ for } V_r \ll c, \tag{6.33}$$

or

$$\frac{\lambda_{obs}}{\lambda_L} = 1 + z \tag{6.34}$$

and

$$\lambda_{obs} \approx \lambda_L\left(1 + \frac{V_r}{c}\right) \text{ for } V_r < < c, \tag{6.35}$$

where the observed line wavelength is, $\lambda_{observed}$, the emitted line wavelength is λ_L, the z is known as the redshift since visible spectral lines are Doppler shifted to longer, redder wavelengths, ν denotes frequency and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

For large radial velocities approaching c, the relativistic Doppler effect yields:

$$1 + z = \left[\frac{c + V_r}{c - V_r}\right]^{\frac{1}{2}} \text{ for } V_r \approx c \tag{6.36}$$

and

$$\frac{V_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \text{ for } V_r \approx c. \tag{6.37}$$

Example: Redshifting Lyman α into the visible

The Lyman α transition from hydrogen atoms in a star that is moving at relatively low speeds with respect to the Earth will be emitted at the rest wavelength of $\lambda_L = 121.567$ nm, in the ultraviolet part of the electromagnetic spectrum. If a galaxy is moving at a high enough speed, it might redshift the emission into a red wavelength of $\lambda_{obs} = 600$ nm, with a redshift $z = (\lambda_{obs} - \lambda_L)/\lambda_L = 3.936$. For redshifts greater than one, the velocities approach that of light, and the radial velocity $V_r = 2.76 \times 10^8$ m s^{-1} using the high-speed equation $\frac{V_r}{c} = \frac{(z+1)^2-1}{(z+1)^2+1}$ for the radial velocity V_r with the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

6.5.2 Gravitational Redshift

For massive, collapsed stars, there also is a detectable gravitational redshift caused by the loss of radiation photon energy in overcoming the immense gravitational pull of a star. This is a small effect for stars like the Sun, whose gravitational redshift is about 2×10^{-6}, but it increases for collapsed stars.

When a photon of energy $h\nu = mc^2$ leaves the surface of a massive body of mass M and radius R, it loses an energy ΔE given by

$$\Delta E = h\Delta\nu = \frac{GMm}{R} = \frac{GMh\nu}{Rc^2} \tag{6.38}$$

where the radiation moves at the speed of light c and is imagined to have an effective mass m. The gravitational redshift is given by

$$z_g = \frac{\Delta\nu}{\nu} = \frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}. \tag{6.39}$$

The gravitational redshift for the Sun is $z_g \approx 2 \times 10^{-6}$, which is a value that would be produced by the Doppler effect at a radial velocity of only 0.64 km s^{-1}, and so is very difficult to measure. Nevertheless, such a measurement has been reported (Blamont and Roddier 1961; Forbes 1970; Snider 1972). The effect increases for collapsed stars, and it has been measured for white dwarf stars with a mass about equal to that of the Sun and a size about equal to that of the Earth (Adams 1925; Popper 1954; Greenstein et al. 1971; Greenstein and Trimble 1972; Shipman 1972, Sect. 13.3)

The most accurate tests of the gravitational redshift are made in the Earth's gravity, which is much weaker than that of the Sun or white dwarf stars. The greater terrestrial accuracy in the measurement of this effect is possible because the wavelengths of nuclear spectral lines are more precisely known than those of the spectral lines of any celestial object. The discovery of narrow gamma ray spectral lines of nuclei (Mössbauer 1958) led to the first detection of the terrestrial gravitational redshift. Using the Mössbauer effect, Pound and Rebka (1959, 1960) measured a value of $z_g = (2.57 \pm 0.26) \times 10^{-15}$ as opposed to the predicted value of $z_g = 2.46 \times 10^{-15}$. The agreement between theory and experiment was subsequently improved to about one percent (Pound and Snider 1964, 1965)

Example: Measuring the gravitational redshift on the Earth and stars

Due to the Earth's relatively low gravity, the gravitational redshift measurement requires radiation at a very precise gamma-ray wavelength, generated by a nuclear resonance Mössbauer effect. In going from the ground to a height H, the wavelength change due to the gravitational redshift will be $\Delta\lambda/\lambda = GM_E H/(R_E^2 c^2) \approx 10^{-16} H$, where $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}, the mass of the Earth $M_E = 5.974 \times 10^{24}$ kg, the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, and the radius of the Earth $R_E = 6.371 \times 10^6$ m. Pound and Rebka (1959, 1960) measured the change over a distance of 22.5 m, requiring a measurement precision of 2×10^{-15}.

For the Sun, the gravitational redshift is $\Delta\lambda/\lambda = GM_\odot/(R_\odot c^2) \approx 2 \times 10^{-6}$, where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg and the Sun's radius $R_\odot = 6.955 \times 10^8$ m. By way of comparison, the thermal broadening $\Delta\lambda/\lambda_L = V_{thermal}/c = (3kT/m)^{1/2}/c = 4 \times 10^{-5}$ for a spectral line of hydrogen, of mass $m = 1.67 \times 10^{-27}$ kg, emitted from the visible disk of the Sun at temperature $T = 5780$ K, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1} (also see next example). So the thermal broadening effect is about 16 times greater than the gravitational redshift effect, making the latter very difficult to detect.

A white dwarf star has about the same mass as the Sun, but a radius that is comparable to that of the Earth and 100 times smaller than the Sun, at about 6.37×10^6 m, so the gravitational redshift is about 100 times larger and easier to detect (Sect. 13.3).

A more exact, relativistic expression for the gravitational redshift, which is applicable to a very massive, collapsed object is (Einstein 1911, 1916):

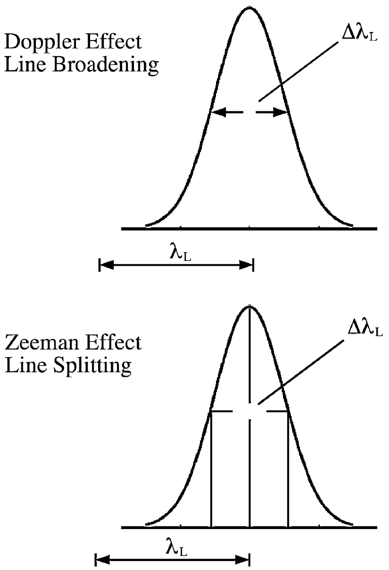

Fig. 6.8 Effects that broaden a spectral line The motion of absorbing or emitting atoms can broaden a line to the short-wavelength and long-wavelength side of the resting, or nonmoving, wavelength, here denoted by λ_{L} (*top*). The Doppler effect describes the broadening (see Fig. 4.2). When the motion is due to the heat or temperature of the radiating atoms, the effect produces thermal broadening; when the average temperature of a collection of atoms increases, the thermal broadening becomes wider. The Doppler effect broadening can also be caused by the rotation or expansion of the source. An intense magnetic field can split a single line at wavelength λ_{L} into two components by the Zeeman effect (*bottom*). The wavelength difference $\Delta\lambda_{\mathrm{L}}$ between the split lines is proportional to the strength of the magnetic field. (From “The Life and Death of Stars” by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

$$z_g = \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} - 1. \tag{6.40}$$

Vessot et al. (1980) have compared the time of a hydrogen maser clock in a rocket with a similar clock on the ground, confirming the relativistic gravitational redshift within the uncertainties of the measurement.

6.5.3 Thermal Motion Broadens Spectral Lines

Any observed spectral line is the superposition of the lines emitted by many individual atoms in different physical conditions. Rather than appearing at a single wavelength, the observed line therefore is broadened over a range of wavelengths (Fig. 6.8). Van Vleck and Haber (1977) have reviewed absorption and emission lines and line breadths.

The amount of wavelength broadening increases with the temperature of the source or, to be exact, it varies as the thermal velocity of the moving atoms and the square root of the temperature.

For atoms in thermodynamic equilibrium at temperature T, a line emitted at wavelength λ_L will be Doppler broadened by an amount $\Delta\lambda_D$ given by:

$$\Delta\lambda_D = \lambda_L\left(\frac{V_{thermal}}{c}\right), \tag{6.41}$$

where $c = 2.9979 \times 10^8$ m s^{-1}, and the thermal velocity $V_{thermal}$ of an atom of mass m at temperature T is given by:

$$V_{thermal} = \sqrt{\frac{3kT}{m}} = \left[\frac{3kT}{m}\right]^{1/2}, \tag{6.42}$$

where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1}.

Example: Thermal broadening of atomic hydrogen lines in the photosphere

The temperature of the visible solar disk, the photosphere, is $T = 5{,}780$ K, so the thermal velocity, $V_{thermal}$ of a hydrogen atom of mass $m_H = 1.007825$ u $= 1.6735 \times 10^{-27}$ kg, will be $V_{thermal} = (3kT/m_H)^{1/2} = 1.20 \times 10^4$ m s^{-1}, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1}. The wavelength broadening $\Delta\lambda$ in wavelength λ is given by $\Delta\lambda/\lambda = V_{thermal}/c = 4.0 \times 10^{-5}$, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

6.5.4 Rotation or Expansion of the Radiating Source can Broaden Spectral Lines

If a source is rotating, the Doppler effect of the object's side rotating toward the observer produces a blueshift to shorter wavelengths; the other side, which is rotating away, shifts a line to longer wavelengths. The combined effect produces a line broadening that increases with the rotation velocity and that depends on the projected linear equatorial velocity or the observed rotational velocity of the line.

If a source is rotating with an equatorial rotation velocity V_{rot} it will produce a line broadening by an amount $\Delta\lambda_{rot}$ given by:

$$\Delta\lambda_{rot} = \lambda_L\left(\frac{V_{rot}\sin i}{c}\right), \tag{6.43}$$

where i is the inclination of the source's equator to the celestial equator. The quantity of V_{rot} sin i is the projected linear equatorial velocity or the observed rotational velocity of the line.

A similar broadening applies to an expanding source, which exhibits a line broadening that increases with the expansion velocity, $V_{\exp}$, producing

$$\Delta\lambda_{\exp} = \lambda_L\left(\frac{V_{\exp}}{c}\right). \tag{6.44}$$

6.5.5 Curve of Growth

The Belgian astronomer Marcel Minnaert (1893–1970) and his Dutch colleague Gerard F.W. Mulders (1908–1993) introduced the equivalent width, W_λ, or the area of an absorption line profile, as a measure of line intensity (Minnaert and Mulders 1930). By plotting the equivalent widths against the number of absorbers (in a logarithmic form), they showed for the first time the empirical curve of growth, which permits the comparisons of line intensities and widths with theoretical expectations. It is mainly of concern for advanced studies of stellar atmospheres.

The two classical model atmospheres are known as the Schuster-Schwarzschild atmosphere (Schuster 1905; Schwarzschild 1906) and the Milne-Eddington atmosphere (Milne 1921, 1930; Eddington 1917, 1926, b). In local thermodynamic equilibrium they both lead to the approximate relation for the equivalent width, W_λ, of a line of intensity I_λ at wavelength, λ. If we define

$$W_\lambda = \int \frac{I_C - I_\lambda}{I_C}\,d\lambda, \tag{6.45}$$

where the subscripts C and λ denote the continuum adjacent to the line and the line wavelength, we have:

$$\frac{W_\lambda}{\lambda} = \frac{e^2}{4\varepsilon_0 m_e c^2} N f \lambda, \tag{6.46}$$

where the electron charge is e, the electric constant is ε_0, the electron mass is m_e, the speed of light is c, the N is the column density of the atoms producing the spectral line and the oscillator strength f is related to the absorption cross section σ by

$$\sigma = \frac{e^2}{4\varepsilon_0 m_e c} f. \tag{6.47}$$

Each spectral line is characterized by the oscillator strength *f*, and the larger the *f* the stronger the spectral line that is seen. We can determine *N* once the oscillator strength is known and the equivalent width measured.

Observations of the intensities and widths of spectral lines can be compared with theoretical expectations in order to determine the excitation temperature, the turbulent velocity, the electron and gas pressures, the surface gravity, and the abundance of the elements in a stellar atmosphere. Nowadays there are complex theoretical models for the atmospheres, and relevant data is on the Internet.

6.5.6 Magnetic Fields Split Spectral Lines

When an atom is placed in a magnetic field, it acts like a tiny compass, and it adjusts the energy levels of its electrons. If the atomic compass is aligned in the direction of the magnetic field, the electron's energy increases. If it is aligned in the opposite direction, the energy decreases. Because each energy change coincides with an alteration in the wavelength or frequency of the radiation emitted by that electron, a spectral line emitted at a single wavelength by a randomly oriented collection of atoms becomes a group of three lines of slightly different wavelengths in the presence of a magnetic field (Fig. 6.9). The size of an atom's internal

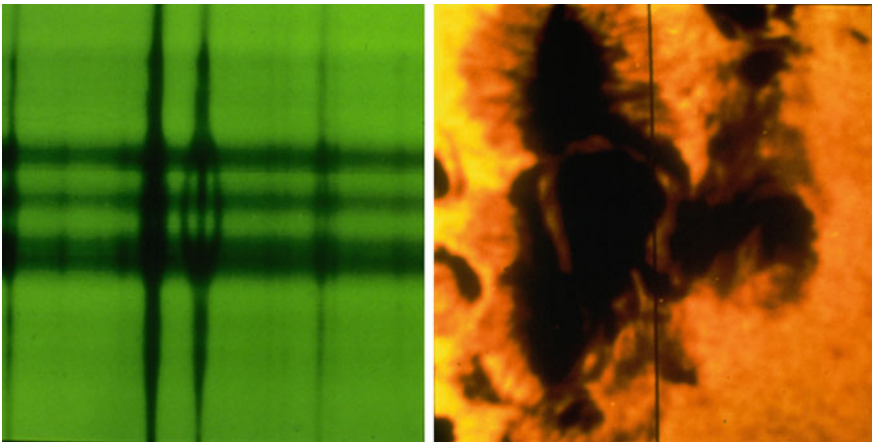

Fig. 6.9 The Zeeman effect The magnetic field in a sunspot can be measured using the Zeeman effect. In a sunspot (*right*) the spectral lines that are normally at a single wavelength become split into two or three components (*left*), depending on the orientation of the field with respect to the line of sight. The vertical line crossing the sunspot denotes the alignment of the observing instrument slit. The separation of the outermost components is proportional to the strength of the magnetic field, in this sunspot about 0.4 tesla, or 4,000 gauss. The components also have a circular polarization, which indicates the direction of the longitudinal magnetic field. (Courtesy of NOAO.)

adjustments, and the extent of its spectral division, increases with the strength of the magnetic field.

This magnetic transformation was named the *Zeeman effect*, after Pieter Zeeman (1865–1943), who first noticed it in a laboratory (Zeeman 1896, 1897a, b). His Dutch colleague Hendrik A. Lorentz (1853–1928) predicted the effect (Lorentz 1898). The pair received the 1902 Nobel Prize in Physics for their work.

We can gain some insight to the Zeeman effect by considering the motion of a free electron in the presence of a magnetic field. A charged particle cannot move straight across a magnetic field, but instead gyrates around it. If the particle approaches the magnetic field in a perpendicular direction, a magnetic force pulls it into a circular motion about the magnetic field line. Because the particle can move freely in the direction of the magnetic field, it spirals around it with a helical trajectory.

The size of the circular motion, called the radius of gyration and designated R_g, depends on the velocity, $V_\perp$, of the particle in the perpendicular direction, the magnetic field strength, B, and the mass, m, and charge, Ze, of the particle. That gyration radius is described by the equation (Larmor 1897):

$$R_g = \left(\frac{mV_\perp}{ZeB}\right), \tag{6.48}$$

provided that the velocity is not close to the speed of light, c. An electron will circle about the magnetic field with a radius, R_g, and with a period, P, given by:

$$P = \frac{2\pi R_g}{V_\perp} = \frac{2\pi m_e}{eB}. \tag{6.49}$$

At the velocity $V_\perp$, the electron goes once around the circumference $2\pi R$ in the period P. The rotational frequency, ν_g, of this circular motion, in a plane perpendicular to the magnetic field, is

$$\nu_g = \frac{1}{P} = \frac{eB}{2\pi m_e} \approx 2.80 \times 10^{10} B\,\mathrm{Hz}, \tag{6.50}$$

where the elementary charge of the electron is $e = 1.6022 \times 10^{-19}$ C, the mass of the electron is $m_e = 9.1094 \times 10^{-31}$ kg, and B is the magnetic field strength in tesla.

When an atom is placed in a magnetic field, a very similar thing happens to its electrons and the spectral lines they emit. A line that radiates at a wavelength λ_L without a magnetic field becomes split into two or three components depending on the orientation of the magnetic field. This has been detected in sunspots (Fig. 6.8).

We can represent the orbital motion of the electron using the Bohr model of the atom for the electron in the ground state orbit with quantum number $n = 1$. The orbital angular momentum in a circular orbit will be $m_e VR = nh/(2\pi) = h/(2\pi)$ for orbital velocity V, orbital radius R, and h denotes the Planck constant with $h = 6.6261 \times 10^{-34}$ J s. When the atom is placed in a magnetic field of strength B, the magnetic moment, designated μ, is the Bohr magneton, denoted by μ_B, given by:

$$\mu_B = \frac{e}{2m_e}(mVR) = \frac{eh}{2\pi m_e} = 9.274 \times 10^{-24}\ \mathrm{J\ T^{-1}}, \tag{6.51}$$

where J denotes the joule units of energy and T designates the tesla unit of magnetic field strength.

When the atom is placed in the magnetic field, a single spectral line is split into three closely spaced spectral lines. The adjacent energy levels differ in energy, ΔE_B, corresponding to a photon energy difference $h(\nu_1 - \nu_2)$ related by:

$$\Delta E_B = \mu_B B = \frac{eh}{4\pi m_e} B = h(\nu_1 - \nu_2). \tag{6.52}$$

The two outer components of the three Zeeman-split spectral lines will be separated by an energy difference $2\Delta E_B$, and a total frequency separation $\Delta\nu$ given by:

$$\Delta\nu = \frac{2\mu_B}{h} B \approx 2.80 \times 10^{10} B\ \mathrm{Hz}. \tag{6.53}$$

For the three component split, the shift, $\Delta\lambda$, in wavelength of the two outer components is given by:

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu \approx 93.3\lambda^2 B\ \mathrm{m}, \tag{6.54}$$

where the shift in frequency is $\Delta\nu$, and the speed of light is $c = 2.9979 \times 10^8\ \mathrm{m\ s^{-1}}$, both $\Delta\lambda$ and λ are in meters, and B is in tesla. The separation is thus proportional to the magnetic field strength B.

Example: Zeeman effect in sunspots and interstellar space

The magnetic field strength in a sunspot measured by Hale (1908a, b) was $B = 0.3$ tesla. For the red Hα transition at a wavelength of $\lambda = 656.3\ \mathrm{nm} = 6.56 \times 10^{-7}$ m, the Zeeman effect will give a line splitting of $\Delta\lambda = 93.3\ \lambda^2\ B \approx 1.20 \times 10^{-11}$ m or $\Delta\lambda/\lambda \approx 1.80 \times 10^{-5}$. The Doppler broadening of this line due to the thermal motion of the hydrogen atoms at a temperature of 5,780 K is $\Delta\lambda/\lambda = V_{thermal}/c = 4.0 \times 10^{-5}$ (see previous example), only twice the expected.

The interstellar spaces are very cold, about 100 K or less, with less thermal broadening, but an accuracy of $\Delta\lambda/\lambda = 7 \times 10^{-10}$ is required to make the measurement at a wavelength of the $\lambda = 21$ cm transition of interstellar hydrogen.

The American solar astronomer George Ellery Hale (1868–1938) first made measurements of this Zeeman splitting in sunspots, showing that they have magnetic field strengths of about 0.3 tesla (Hale 1908a, b). This is about 10,000 times the strength of the terrestrial magnetic field that orients our compasses. The split lines

Table 6.6 Cosmic magnetic fields

Object	Magnetic field strength, B (tesla)[a]
Earth (equator to pole)	3×10^{-5}–6×10^{-5}
Solar wind (at Earth orbit)	2.5×10^{-9}
Sunspot	0.3
Sun (global)	3×10^{-4}
Interstellar space	10^{-10}
White dwarf star	10^{2}
Pulsar	10^{8}

[a] 1 tesla = 1 T = 10^4 gauss = 10^4 G

are circularly polarized, and the direction of polarization indicates the direction of the magnetic field, pointing into or out of the Sun (Hale et al. 1919).

As suggested by Bolton and Wild (1957), the magnetic field strength in interstellar space, with about $B = 10^{-10}$ tesla, can be observed at radio wavelengths (Verschuur 1968, 1971; Troland and Heiles 1977).

Ledoux and Rensen (1966) have provided an early summary of magnetic stars; Donati and Landstreet (2009) reviewed magnetic fields in nondegenerate stars. Angel (1978) provided a review of magnetic white dwarf stars. Crutcher (2012) discusses magnetic fields in molecular clouds; Beck et al. (1996) review galactic magnetism; Sofue et al. (1986) reviewed the global structure of magnetic fields in galaxies, and Carilli and Taylor (2002) reviewed observations of magnetic fields in clusters of galaxies.

The magnetic field strengths, B, of various cosmic objects are listed in Table 6.6.

All of this refers only to the outer atmosphere of the Sun and other stars. As it turns out, atoms do not exist in most of the Sun, except in the cool visible layer in which dark absorption lines are formed. It is too hot everywhere else for whole atoms to survive. Innumerable collisions fragment the abundant hydrogen atoms into their constituent pieces. Their protons and electrons have been set free from their atomic bonds, wandering throughout the solar material unattached to each other. Some of these protons merge together near the Sun's center, but an understanding of these nuclear fusion reactions had to await investigations of how nuclear particles occasionally escape the tight confines of radioactive elements.