Chapter 5
Moving Particles

5.1 Elementary Constituents of Matter

What is matter made of? To find out, we might try breaking any material object into increasingly smaller pieces until we reach a stage when the smallest piece cannot be broken apart. The last step in this imaginary, successive division of matter suggests the existence of unseen *atoms*, a Greek word meaning "indivisible," or something that cannot be divided further.

In the fifth century BC, for example, the ancient Greek philosopher Democritus (ca 470–ca 380 B.C.), and his mentor Leucippus proposed that all matter is composed of combinations of a small number of separate atoms coming together in different ways. They supposed that all substances are composed of four types of elemental atoms: of air, earth, fire, and water. Mud, for example, could be made from earth and water, and fire could turn water into vapor.

Then about two millennia ago, the Roman poet Titus Lucretius Carus (ca 99 BC–ca 55 BC), or Lucretius for short, wrote a wonderful epic poem, in Latin *De Rerum Natura* or in English *On the Nature of the Universe*, which described these indestructible atoms that are so exceedingly small that they are invisible and infinitely vast in number (Lucretius, 55 BC). To Lucretius, atoms were the building blocks of all that exists. This fundamental idea persists to this day. Everything that we see, from a friend to a tree to the Sun and the stars consist of innumerable atoms, all moving randomly about, colliding, gathering together, and breaking apart again. Atoms are immortal, the ingredients of all that existed in the past and the seeds of everything that will exist in the future.

All ordinary matter is composed of elemental *atoms*, and there is a limited number – just 94 naturally occurring atoms, detected directly on the Earth or in astronomical spectra. These atoms are known also as *chemical elements*, because they cannot be decomposed by chemical means. They are very small and exceedingly numerous. A simple drop of water contains about 100,000 billion, billion, or 10^{23} atoms – close to the number of stars in the universe. An additional

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24 atoms have been produced as the result of artificial nuclear reactions within particle accelerators rather than by natural processes.

Atoms combine to form molecules, and there are many more kinds of molecules than there are atoms. The vast numbers of molecules differ only in the kind and relative number of the atoms of which they are constructed. A molecule may be a combination of single chemical atoms, such as the oxygen molecule, O_2, that we breathe, which consist of two oxygen atoms each designated by O. A molecule also may contain different elements, as in water, designated H_2O, which is composed of two atoms of hydrogen, H, and one of oxygen, O. The Earth's transparent atmosphere consists mainly of molecules of oxygen, O_2 (21 %), and nitrogen, N_2 (78 %), with trace amounts of carbon dioxide, CO_2, and water vapor. Methane, CH_4, the natural gas used in a stove, consists of one atom of carbon, C, and four of hydrogen, H. Organic molecules contain more complex combinations of carbon, hydrogen, and other atoms.

But the atom is not indivisible after all. Elemental atoms can be broken into smaller subatomic pieces. The first subatomic particle was found through investigations of electricity. The English physicist Michael Faraday (1791–1867) discovered that the electrical charge carried by different atoms is always an integer multiple of a basic amount, an atomicity of electrical charge – the electron (Faraday 1839, 1844). The concept of such an indivisible quantity of charge was proposed to explain the chemical properties of atoms.

It was in 1894 that the Irish physicist George Johnstone Stoney (1826–1911) coined the word *electron* to describe the fundamental unit of electricity (Stoney 1881, 1894), and electricity is indeed transferred by the flow of electrons. Interactions between electrons hold the atoms in a molecule together in a chemical bond. Similar but much weaker interactions among electrons hold many molecules together.

The English physicist Joseph John Thomson (1856–1940) and his colleagues identified the electron as a particle and determined its charge-to-mass ratio (Thomson 1897a, b). Thomson was studying cathode rays, which carry electrons between electrodes in a tube of gas; he showed that the electrons are deflected when either an external magnetic field or an electric field is applied, which meant that they are electrically charged. Using these curved trajectories, Thomson showed that electrons are very light, roughly $1/1{,}000$th of the mass of the least massive atom, hydrogen. It would take 30 billion, billion, billion, or 3×10^{28}, electrons to make a total mass of just 1 oz, or 28 g.

Thomson received the 1906 Nobel Prize in Physics for his investigations of the conduction of electricity by gases. A few years later, the American physicist Robert A. Millikan (1868–1953) determined the elementary charge of the electron by measuring the electrical force on charged droplets of oil suspended against gravity between two metal electrodes (Millikan 1910, 1913). He was awarded the 1923 Nobel Prize in Physics for this and related work.

The electron, which carries a negative electrical charge and has no known components or substructure, is believed to be a truly elementary particle. The elementary charge of the electron, which is denoted by e, has a value of

$e = 1.602 \times 10^{-19}$ C. The negative electric charge of the electron has a value of $-e$, but no one knows why it has the mass and charge it does. When the electrical current in a house is turned on to light a lamp, about 1 million trillion, or 10^{18}, electrons flow through the wires every second.

In the early 20th century, the New Zealand-born British physicist Ernest Rutherford (1871–1937) and his colleagues showed that radioactivity is produced by the disintegration of atoms, and they discovered that radioactive material emits energetic subatomic particles. When bombarding gold leaf with beams of these particles, they found to their astonishment that about 1 in 20,000 particles bounced right back from where it originated, whereas all of the others passed through the gold. This meant that atoms are largely empty space and that most of the mass of an atom is concentrated in a *nucleus* that is 100,000 times smaller than an atom (Rutherford 1911, 1914). The nucleus of an atom contains less than 10^{-15} of the atomic volume, but it includes almost all the atom's mass.

Within a decade, Rutherford was able to show that the nucleus of different atoms contains various amounts of the nucleus of the simplest atom, hydrogen. He named this nuclear building block a *proton*, from the Greek for "first", since it was the first nuclear particle to be discovered.

A proton is positively charged, with a charge of $+e$, equal in amount to that of an electron but opposite in charge. It is positive. Now, particles with an opposite sign to their electric charge attract one another. So electrons and protons are always attracted to each other. Negatively charged electrons surround positively charged protons in an atom, and the total positive charge of the protons is equal to the total negative charge of the electrons. An atom has no net electrical charge and it is electrically isolated from external space.

Particles with the same electrical charge are driven apart by an electrical repulsion. Rutherford therefore postulated the existence of an uncharged nuclear particle, later called the *neutron*, to help hold protons together in the atomic nucleus and prevent the protons from dispersing as they repelled each other. After an eleven-year search, the English physicist James Chadwick (1891–1974) discovered the neutron (Chadwick 1932a, b).

Protons and neutrons collectively are named *nucleons*, because they are the two constituents of the atomic nucleus. They consist of yet smaller components, known as *quarks*. So, the proton and the neutron are not truly elementary particles. Nevertheless, the nuclear fusion reactions that make stars shine can be understood by assuming that all atomic nuclei are composed of protons and neutrons.

These nucleons are bound together in an atomic nucleus by an exceptionally strong force, the nuclear force or nucleon–nucleon force, which allows them to cling tightly to one another and build the dense, compact atomic nucleus. Although powerful, this attractive force has a short range, operating over very limited distances. The strong force decreases to insignificance at distances greater than about 1 million billionths, or 10^{-15}, of a meter, and closes the nucleus in at an atom's center. The nucleons cannot be pushed any closer together, and this sets the physical size of the nucleus.

Therefore, an atom is largely empty space, like the room in which we are sitting. A tiny, heavy, positively charged nucleus lies at the heart of an atom, surrounded by a cloud of relatively minute, negatively charged electrons that define most of an atom's size and govern its chemical behavior.

The radius of an atom is approximately 10^{-10} m, and it consists of a swarm of electrons orbiting a nucleus, whose radius is about 10^{-15} m for the nucleus of the hydrogen atom and about 10^{-14} m for the nuclei of the heaviest atoms, such as uranium. The fermi unit of length, where 1 fermi = 1 fm = 10^{-15} m, is used in nuclear physics. The diameter of the proton is 1.75 fm, and its radius is 0.875 fm.

A proton and a neutron have about the same mass, which is nearly 2,000 times that of an electron. To be exact, the mass of the proton and the mass of the neutron are respectively 1,836 and 1,839 times the mass of an electron.

The mass of an atomic nucleus is always less than the sum of the masses of its protons and neutrons because they have expended energy to bind themselves together. It is this binding energy that is released in nuclear reactions.

What holds solid objects together? Why doesn't a chair fall apart when the wind blows on it? All durable material objects that surround us consist of atoms and, given the emptiness of an atom, we might wonder why we cannot easily crush them into smaller entities. The answer is that when we push any two pieces of material together the forces of electrical repulsion between the atomic electrons in their adjacent surfaces resist the pressure.

Although the kilogram is a useful unit of mass for describing large objects, the mass of an atom and the mass of its nucleus, which contains nearly all of the atom's mass, are conveniently measured in atomic mass units, abbreviated a.m.u. or u, where

$$\mathrm{u} = 1\,\mathrm{a.m.u.} = 1.660539 \times 10^{-27}\,\mathrm{kg}, \tag{5.1}$$

which is equal to one twelfth of the mass of the carbon-12 atom, denoted ^{12}C. The lightest atom is hydrogen with an atomic weight of 1.007825 u; the heaviest stable atom is lead 208 with an atomic weight of 207.97665 u. Unstable radioactive atoms like uranium 235 are a bit heavier.

Nuclear astrophysicists like to use energy, E, to express mass, m, through the $E = mc^2$ relation, where c is the speed of light, and the convenient unit of nuclear energy is the MeV, where 1 MeV = 1.602176×10^{-13} J, which is also equal to 1,000 keV; note that the conversion factor 1.602176 is the elementary charge of the electron.

Characteristics of these atomic and nuclear particles are given in Table 5.1.

The simplest and lightest atom consists of a single electron circling around a nucleus composed of a single proton without any neutrons. This is an atom of hydrogen. Most of the universe, and the majority of the stars, is composed mainly of hydrogen. The nucleus of helium, the next most abundant atom in the cosmos, contains two neutrons and two protons; so it naturally has two electrons that balance the electrical charge of the protons (Fig. 5.1).

So an atom is composed of a dense and massive nucleus containing protons and neutrons, and surrounded by electrons. As in the example of helium, electrically

Table 5.1 Physical properties of electrons, protons, neutrons, and atoms[a]

Electron
m_e = mass of electron = 5.4858×10^{-4} u = 9.10938×10^{-31} kg = 0.5109989 MeV/c^2
e = elementary charge = 1.6022×10^{-19} C; electron charge = $-e$
r_e = classical electron radius = 2.8179×10^{-15} m
σ_T = Thomson cross section $\frac{8\pi}{3} r_e^2 = 6.65246 \times 10^{-29}$ m^2
Atomic nucleus (protons, neutrons, alpha particles)
m_P = mass of proton = 1.007276466 u = $1.6726218 \times 10^{-27}$ kg = 938.27203 MeV/c^2
e = charge of proton = +1.602×10^{-19} C
m_n = mass of neutron = 1.008664916 u = $1.6749274 \times 10^{-27}$ kg = 939.56536 MeV/c^2
m_α = mass of alpha particle = mass of helium nucleus = 4.001506179 u = $6.6446567 \times 10^{-27}$ kg = 3727.37924 MeV/c^2
Z = total number of protons in nucleus = atomic number
A = total number of protons and neutrons in nucleus = atomic mass number
R = nuclear radius = $r_0 A^{1/3}$ for an atom with mass number A and $r_0 = 1.25 \times 10^{-15}$ m
Atom
a_0 = Bohr radius = 5.2918×10^{-11} m
u = atomic mass unit = $1.66053886 \times 10^{-27}$ kg = 931.494061 MeV/c^2
m_H = mass of hydrogen atom = 1.007825 u = 1.6739326 kg
m_{He} = mass of helium atom = 4.002602 u = 6.646476×10^{-27} kg
Δm_{He} = mass defect of helium atom = $m_{He} - 2m_P - 2m_n - 2m_e$ = 0.030378 u

[a] The mass, m, values are given in atomic mass units u = 1.660539×10^{-27} kg, in kilograms or kg, and as the rest mass energy divided by the square of the speed of light $c = 2.9979 \times 10^8$ m s^{-1}; this energy is given in units of MeV = $1.60217646 \times 10^{-14}$ J

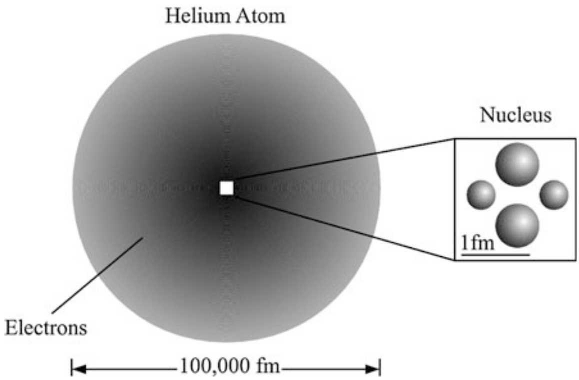

Fig. 5.1 The helium atom An atom of helium contains two electrons that swarm about the atom's nuclear center in a cloud of largely empty space. The *shading* shows that the electrons can be anywhere but are most likely to be found near the center of the atom. The magnified nucleus of the helium atom consists of two protons and two neutrons bound together by a strong nuclear force. The nucleus and each of its four particles are spherically symmetrical. The size of the helium nucleus is about 1 fermi, or 1 fm, which is equivalent to 10^{-15} m. The atom is about 100,000 times bigger than the nucleus, with an atom size of about 10^5 fm, or 10^{-10} m. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

neutral atoms have as many electrons as there are protons and are therefore without net charge. An ionized atom has fewer electrons than protons, and therefore has a positive charge. The atomic number, designated by the symbol Z, is equal to the number of protons in an atom's nucleus. Hydrogen has an atomic number of 1, helium 2, carbon 6, nitrogen 7, oxygen 8, lead 82, and uranium 92. The atomic mass number, designated A, is equal to the total number of protons and neutrons in the nucleus. The radius, R, of the nucleus of an atom of atomic mass number A is $R = r_0 A^{1/3}$, where $r_0 = 1.25 \times 10^{-15}$ m $= 1.25$ fermi $= 1.25$ fm, so a heavier atom has a bigger, and more massive nucleus.

The atoms of a particular element all have the same number of protons in their nucleus, but the number of neutrons can vary, giving rise to different isotopes of the same element. Elements of atomic number 83–94 are composed entirely of radioactive isotopes that are unstable and are known to decay into other elements.

5.2 Heat, Temperature, and Speed

5.2.1 Where Does Heat Come From?

Heat is a form of energy caused by the motion of tiny unseen particles, such as the molecules in a gas, which are in a state of ceaseless motion. These particles move randomly in all directions and do not contribute to the overall motion of the gas in which they reside. The faster they move, the hotter the body.

The energy of motion is called kinetic energy, after the Greek word *kinesis* meaning "motion" – the word cinema has the same root, referring to motion pictures. In a star, individual gas particles are changing direction constantly in an irregular zigzag trajectory, and the heat can be enormous. However, all of the particles in a star move together in the same overall direction, and they are responsible for the bulk kinetic energy of star motion (Fig. 5.2).

All gas molecules are always moving, and the hotter they become the faster they move and the greater their kinetic energy. The lowest possible temperature is absolute zero, or zero on the Kelvin scale denoted K (Kelvin 1848). At absolute zero, molecules cease to move and are completely at rest; they have no kinetic energy. At this temperature, the constituent particles stick together and behave as a frozen solid, resembling ice.

Raise the temperature above absolute zero and molecules move about and collide, turning ice into liquid water. Further increase the temperature and molecules move so fast that they overcome the cohesive forces that bind them together. Gas is formed, capable of nearly unlimited expansion in all directions as when water evaporates.

So, by adding more heat we can transform a solid into a liquid and then a gas, and we can reverse the process by removing heat and lowering the temperature. All our familiar objects exist in one of these three fundamental states – the solid,

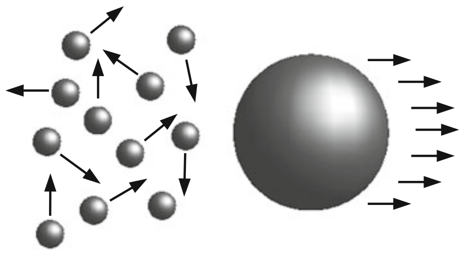

Fig. 5.2 Random and ordered motion Particles within a hot gas (*left*) move here and there in random directions that continually change as the result of collisions between particles. This supports the gas against inward gravitational forces. A planet or star (*right*) moves along a well-defined, ordered trajectory determined by external gravitational forces on it. When a large number of stars has gathered together and is confined within a star cluster, the stars also move in random directions, supporting their combined gravitational pull. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

liquid and gaseous ones. At higher temperatures within cosmic objects, there is a fourth state of matter, known as plasma, in which the atoms are torn into their subatomic ingredients. The range of temperatures found in the universe is illustrated in Table 5.2.

The German physician Jules Robert Mayer (1814–1878) reasoned that heat is a form of energy (Mayer 1842) – generally called "force" in his time and related to the motivating force of fire (Carnot 1824). Mayer found that heat energy can change form, and this had a crucial role in the discovery of the *conservation of energy*. Heat energy can be produced by or transferred into another type of energy, but the energy never disappears. The total energy is conserved (Mayer 1842; Helmholtz 1847).

The English physicist James Prescott Joule (1818–1889) soon provided experimental verification of the conservation of energy for particular cases, in a

Table 5.2 Range of cosmic temperatures

Location	Temperature (K)
Absolute zero	0
Cosmic microwave background radiation	3
Water's freezing (triple) point	273
Water's boiling point	373
Incandescent light bulb	2,500
Visible solar disk	5,780
Center of Sun	1.5×10^7
Atom bomb	3.5×10^8
CERN particle accelerator (proton–proton collision)	10^{13}
Big bang (at 10^{-44} s)	10^{32}

lecture titled “On Matter, Living Force, and Heat” (Joule 1847, 1850). Today, we identify Joule’s “living force” with kinetic energy. The joule unit of energy, abbreviated J, is appropriately named after him.

The German physician and physicist Hermann von Helmholtz (1821–1894) provided the connection between the conservation of energy and the kinetic energy of motion, suggesting that the Sun could gain heat from its gravitational energy by contracting (Helmholtz 1847, 1856, 1908). This mechanism would keep the Sun shining for tens of million of years, but it was eventually realized that the Earth and Sun are 4.6 billion years old, and another source of energy, due to nuclear reactions, was required to conserve energy in the Sun for such long times.

5.2.2 Thermal Velocity

If we bring a hot body into contact with a colder one, the fast-moving particles in the hot body collide at the boundary with the slower-moving particles of the colder body. This transfers to them a part of the kinetic energy. The fast-moving particles gradually slow down and the slow ones accelerate, averaging out the temperature differences until a state of thermal equilibrium is obtained, and then a single temperature characterizes the situation.

The portion of an object’s internal energy that is responsible for its temperature is called thermal energy, and for an equilibrium temperature, denoted by T, the average *thermal energy* given by:

$$\text{Thermal energy} = \frac{3}{2}kT, \tag{5.2}$$

where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. This constant is named after the Austrian physicist Ludwig Boltzmann (1844–1906), whose doctoral thesis pioneered the kinetic theory of gases (Boltzmann 1868).

A particle is said to move at the *thermal velocity* when its kinetic energy is equal to its thermal energy. For a molecule or particle of mass m and velocity V the kinetic energy of motion is:

$$\text{Kinetic energy} = \frac{1}{2}mV^2. \tag{5.3}$$

When this is set equal to the thermal energy, we obtain the thermal velocity, $V_{thermal}$, of a particle of mass, m, at temperature, T:

$$V_{thermal} = \sqrt{\frac{3kT}{m}} = \left[\frac{3kT}{m}\right]^{1/2}. \tag{5.4}$$

The magnitude of the thermal velocity, which is the thermal speed, increases with the temperature of the gas and decreases with increasing particle mass. Hotter particles move faster and more massive ones move slower.

Example: Thermal velocity in our atmosphere and in the Sun's visible disk

At sea level the temperature T is about 288 K, and our air is predominantly composed of nitrogen molecules of mass $m = 2 \times 14 \times \mathrm{u}$, where the atomic mass unit $\mathrm{u} = 1.66054 \times 10^{-27}$ kg. Substituting these numbers into our expression for thermal velocity, $V_{thermal} = (3kT/m)^{1/2}$, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1}, we obtain a thermal speed, the magnitude of the thermal velocity, of about 507 m s^{-1}. For the visible solar disk, the temperature T is 5,780 K and the abundant hydrogen atoms of mass $m_H = 1.67 \times 10^{-27}$ kg, have a thermal speed of about 12,000 m s^{-1}. They move faster because the hydrogen atoms on the Sun's disk are both hotter and less massive than the nitrogen molecules in our air.

Protons and electrons are perpetually steaming away from the Sun in the solar wind. How hot would they have to be to escape from the Sun's gravitational pull? As we saw in Sect. 4.1, the escape speed of the Sun at its visible disk is $V = V_{esc\odot} = (2GM_{\odot}/R_{\odot})^{1/2} = 6.18 \times 10^5\,\mathrm{m\,s^{-1}}$. If that speed was equal to the thermal velocity $V_{thermal} = (3kT/m)^{1/2}$, then the temperature $T = mV^2/(3k)$, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1} and m is the mass of the particle. For a proton of mass $m_P = 1.672 \times 10^{-27}$ kg, the temperature is $T_P = 1.54 \times 10^7$ K, and for the electron of mass $m_e = 9.109 \times 10^{-31}$ kg, the temperature is $T_e = 8.4 \times 10^3$ K. The outer solar atmosphere, the corona, which lies just above the visible solar disk, has a million-degree temperature, so it is hot enough for the electrons to escape from the Sun, and nearly hot enough for the protons to do so. Since the escape velocity falls off with increasing distance from the Sun, this also becomes possible.

For the Earth, we have an escape velocity of $V = V_{escE} = (2GM_E/R_E)^{1/2} = 1.12 \times 10^4$ m s^{-1}. Hydrogen atoms do not remain in the Earth's atmosphere, and since their mass is approximately the same as that of their nuclear proton, the temperature required for this escape is $T = mV^2/(3k) \approx 5 \times 10^3$ K. The mean surface temperature of the Earth is about 281 K, not hot enough for hydrogen to escape immediately from the surface. However, the temperature of the Earth's atmosphere increases at higher levels to more than 10^4 K, in its ionosphere, permitting hydrogen to leak off the planet by thermal evaporation, which has been detected from satellite observations as a geocorona.

The equilibrium temperature is sometimes called the *kinetic temperature*, denoted by T_K, and it is given by:

$$\text{Kinetic temperature} = T_K = \frac{mV^2}{3k}. \tag{5.5}$$

5.2.3 Collisions

The numerous particles in a gas are always colliding with each other, and another German scientist, Rudolf Clausius (1822–1888), introduced the concept of the mean free path between collisions. He asked how far, on average, can a molecule move before it comes under the sphere of action of another molecule. When the number of particles per unit volume increases, for example, the mean distance between collisions will decrease.

The average distance covered by a moving particle between successive collisions is called the *mean free path*, denoted l. It is given by (Clausius 1858):

$$l = \frac{1}{(\sigma N)}, \tag{5.6}$$

where N is the number density of gas particles and the cross sectional area $\sigma = \pi r_0^2$ for collision radius r_0. For an atom we might take $r_0 = a_0 = 5.298 \times 10^{-11}$ m, the Bohr radius to obtain $l \approx 10^{20} N^{-1}$ m. A molecule would be somewhat larger, depending on the number of atoms it contains.

For a gas at temperature T and pressure P,

$$l = \frac{kT}{\pi r_0^2 P}, \tag{5.7}$$

where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. In our atmosphere at sea level, with a temperature $T = 288$ K and a pressure $P = 10^5$ Pa, the mean free path between collisions of nitrogen molecules with radius $r_0 \approx 2 \times 10^{-10}$ m, is about $l \approx 3 \times 10^{-7}$ m, and each air molecule is subjected to many billions of collisions every second.

The approximate mean time, τ, between collisions is given by the ratio of l divided by the thermal velocity, or:

$$\tau \approx l\left(\frac{m}{3kT}\right)^{1/2}, \tag{5.8}$$

where l is the mean free path, m is the particle mass, and k is the Boltzmann constant.

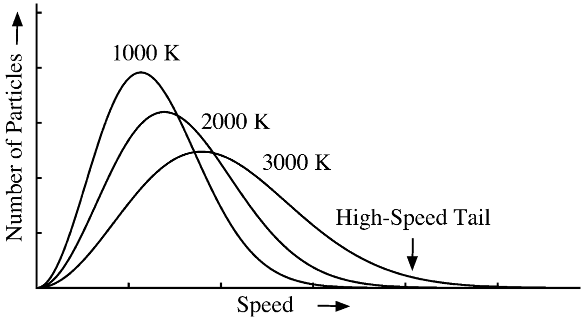

Fig. 5.3 Maxwell distribution of particle speeds The speeds of particles with the same mass and three different temperatures. The peak of this distribution shifts to higher speeds at higher temperatures. There is a small fraction of particles having high speed, residing in the high-speed tail of the distribution, and this fraction increases with temperature. The fraction of particles with low speed becomes smaller at higher temperatures but does not vanish. The peak also shifts to higher speeds at lower mass when the temperature is unchanged. The Scottish scientist James Clerk Maxwell (1831–1879) derived this distribution in 1873. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

5.2.4 The Distribution of Speeds

Also in the mid-nineteenth century, the Scottish physicist James Clerk Maxwell (1831–1879) introduced a statistical approach to the kinetic theory of gases, which recognizes that every gas particle has a different speed and that each collision between particles changes the speeds of those particles. He proposed that the numerous collisions between the large numbers of molecules in a gas produce a statistical distribution of speeds, in which all of the speeds might occur with a different and known probability. This is called the *Maxwell speed distribution.*

In thermal equilibrium, the average value of the kinetic energy of particles in a gas might be distributed equally among all of the particles, but this equality is only statistically true. Most particles move with the same average speed, but not all of them. Some are faster than average and others slower. Gas particles can gain or lose speed by collisions with one another, so they do not all move at the same average speed. In any given instant, the speed and kinetic energy of most of the particles are close to the average value, but there is always a small percentage that moves faster or slower than the average.

The Maxwell speed distribution, illustrated in Fig. 5.3, gives the fraction of gas molecules, or other particles, moving at a particular speed (the magnitude of the velocity vector) at any given temperature and mass (Maxwell 1860). The most probable speed is close to the average in thermal equilibrium; therefore, the most likely speed for any given type of particle increases with its temperature. However, there is a range of speeds, both higher and lower than the average value, and this range also increases with the temperature. In other words, the Maxwell distribution function becomes broader and its peak shifts to higher speeds when the temperature rises.

Although this distribution function appears to be symmetrical, it has a high-speed tail. Particles in the high-speed tail have greater kinetic energy than other particles in the distribution, and they have an important role in the nuclear-fusion reactions that make the Sun and other stars shine.

The Maxwell speed distribution applies to all types of particles in thermal equilibrium, as long as there are many of them. In addition to atoms or molecules, it can be used to describe the speeds of numerous subatomic particles inside the Sun, which have been freed from their atomic bonds at very high temperatures. The distribution also describes the speeds of millions of stars that are collected together in star clusters, in which the stellar motions are analogous to those of gas particles.

The number of particles, $N(V)dV$, with speeds between V and $V + dV$ is given by:

$$N(V)dV = N_{tot} f(V)dV \tag{5.9}$$

where N_{tot} is the total number of particles of a given mass in the system. The Maxwell speed probability density distribution, $f(V)$, is given by (Maxwell 1860):

$$f(V) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} V^2 \exp\left[\frac{-mV^2}{2kT}\right], \tag{5.10}$$

or equivalently

$$f(V) = \left[\frac{2}{\pi}\left(\frac{m}{kT}\right)^3\right]^{1/2} V^2 \exp\left[\frac{-mV^2}{2kT}\right], \tag{5.11}$$

where T is the temperature, m is the mass of the molecule, or particle, under consideration, and the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. This function has three parts, a constant term, a V^2 term, and an exponential term.

The most probable speed, V_P, is obtained by setting the differential $df(V)/dV$ equal to zero and solving for V, which yields

$$V_P = \sqrt{\frac{2kT}{m}}. \tag{5.12}$$

The *most probable speed* is the speed associated with the highpoint in the Maxwell distribution, and it is the speed most likely to be possessed by any particle of mass m at temperature T. For the diatomic nitrogen molecule at sea level temperature of 288 K, we have a most probable speed of $V_P = 414$ m s^{-1}, since the mass $m = 2 \times 14 \times$ u for atomic mass unit u $= 1.66054 \times 10^{-27}$ kg.

Only a small fraction of the particles have the most probable speed, since there is also a range of speeds, both higher and lower than the most probable one, and this range also increases with the temperature. In other words, the Maxwell distribution function becomes broader and its peak shifts to higher speeds when the temperature rises.

A measure of the range of possible speeds, or the spread of the Maxwell distribution, is the full width to half maximum, ΔV, of the exponential term, which is a Gaussian function of the form $\exp[-V^2/(2\sigma^2)]$ of standard deviation σ. Such a function has $\Delta V = 2.355\ \sigma$, and for the Maxwell distribution we have:

$$\Delta V \approx 2.355(kT/m)^{1/2} \approx 1.66 V_p. \tag{5.13}$$

This is only approximate since there is an enhancement at high velocities owing to the V^2 term in the Maxwell distribution, but the main point is that both the most probable velocity and the spread in the Maxwell distribution increase with temperature.

The most probable speed is very close to the average speed in thermal equilibrium, since $V_P = (2/3)^{1/2}\ V_{thermal} = 0.816\ V_{thermal}$. The mean speed $<V>$ is the mathematical average, denoted by $< >$, of the speed distribution. It is given by:

$$<V> = \int_0^\infty Vf(V)dV \tag{5.14}$$

$$<V> = \left(\frac{8kT}{\pi m}\right)^{1/2} = \frac{2}{\sqrt{\pi}} V_P. \tag{5.15}$$

The root mean square speed, V_{rms}, is the square root of the average squared speed

$$V_{rms} = \left(\int_0^\infty V^2 f(V)dV\right)^{\frac{1}{2}} \tag{5.16}$$

or

$$V_{rms} = \left(\frac{3kT}{m}\right)^{1/2} = <V^2>^{1/2} = \left(\frac{3}{2}\right)^{1/2} V_P. \tag{5.17}$$

The Maxwell speed distribution can be expressed as an energy distribution for the kinetic energy $E = mV^2/2$. The number $N(E)$ of gas particles with a kinetic energy between E and $E + dE$ is given by:

$$N(E)dE = \frac{2N_{tot}}{\pi^{\frac{1}{2}}(kT)^{\frac{3}{2}}} E^{\frac{1}{2}} \exp\left(-\frac{E}{kT}\right) dE = N_{tot} f(E)dE. \tag{5.18}$$

The total internal energy, U, of a total of N_{tot} particles in thermal equilibrium is given by (Maxwell 1860; Boltzmann 1868):

$$U = \int_0^\infty EN(E)dE = \frac{3}{2} N_{tot} kT. \tag{5.19}$$

Table 5.3 Atmospheres of Venus, Mars, and Earth[a]

	Venus	Mars	Earth
Constituent			
Carbon dioxide, CO_2	96	95	0.038
Nitrogen, N_2	3.5	2.7	78
Argon, Ar	0.007	1.6	0.93
Water vapor, H_2O	0.010	0.03 (variable)	1 (variable)
Oxygen, O_2	0.003	0.13	21
Surface pressure (bar)	92	0.007–0.010	1.0 (at sea level)
Surface temperature (K)	735	183–268	184–330
		Mean 210	Mean 287.2

[a] Percentage composition of atmospheric constituent

Table 5.4 Atmospheres of the giant planets and the Sun[a]

Constituent	Sun	Jupiter	Saturn	Uranus	Neptune
Hydrogen, H_2	84	86.4	97	83	79
Helium, He (atom)	16	13.6	3	15	18
Water, H_2O	0.15				
Methane, CH_4	0.07	0.21	0.2	2	3
Ammonia, NH_3	0.02	0.07	0.03		

[a] The percentage abundance by number of molecules for the Sun, cooled to planetary temperatures so that the elements combine to form the compounds listed, and for the outer atmospheres of the giant planets below the clouds. Blanks indicate unobserved compounds. (Courtesy of Andrew P. Ingersoll.)

5.3 Molecules in Planetary Atmospheres

The molecular ingredients of an atmosphere can be determined by observing the unique spectral signatures of different molecules. The atmospheres of the eight major planets in our solar system are mainly composed of molecules of the cosmically abundant atoms – hydrogen, H, carbon, C, oxygen, O, and nitrogen, N, but in various percentages given in Tables 5.3 and 5.4. The main ingredient of the atmospheres of Venus and Mars is carbon dioxide, abbreviated CO_2. Our breathable air contains 21 % oxygen molecules, denoted O_2, and 78 % nitrogen molecules, abbreviated N_2. The atmospheres of the giant planets are mainly composed of molecular hydrogen, denoted H_2.

As suggested by George Johnstone Stoney (1826–1911), the ability of a planet or satellite to retain an atmosphere depends on both the temperature of that atmosphere and the gravitational pull of the planet or satellite (Stoney 1898, 1900). An atmosphere is held near a planet by its gravity, but since gas has a natural tendency to expand into space, only planets with a sufficiently strong gravitational pull can retain an atmosphere of a given composition. The ability of a planet to retain an atmosphere also depends on its temperature, determined by both the planet's distance from the Sun and the atmospheric greenhouse effect. Lighter, hotter molecules will move faster than heavier, colder ones, and the fast ones will be more likely to escape the planet's gravitational grasp.

If the gas is hot, the molecules move about with a greater speed and are more likely to escape the gravitational pull of the planet. This is one of the reasons that Mercury, the closest planet to the Sun and therefore the hottest, has no atmosphere. The other reason is that Mercury has a relatively small mass, as far as planets go, and thus has a comparatively low gravitational pull. On the other hand, a planet with a larger mass is more likely to retain an atmosphere, which helps explain why massive Jupiter retains the lightest element, hydrogen. Jupiter is also relatively far away from the Sun's heat, so molecules in Jupiter's atmosphere move at a relatively slow speed.

As shown by John S. Lewis (1941–), the composition of the planets and their atmospheres is intimately connected with their distance from the Sun (Lewis 1974, 2004).

An atom, ion, or molecule moves about because it is hot. Its kinetic temperature, T, is used to define its thermal velocity, $V_{thermal}$, given by equating the thermal energy to the kinetic energy of motion, or

$$V_{thermal} = \sqrt{\frac{3kT}{m}} = \left[\frac{3kT}{m}\right]^{1/2}. \tag{5.20}$$

where the Boltzmann constant $k = 1.3806 \times 10^{-23}$ J K^{-1}, and the particle's mass is denoted by m. We see right away that at a given temperature, lighter particles move at faster speeds. Colder particles of a given mass travel at slower speeds. Anything will cease to move when it reaches absolute zero on the kelvin scale of temperature.

The thermal velocity can be compared to the planet's escape velocity, V_{esc}, given by

$$V_{esc} = \sqrt{\frac{2GM}{D}} = \left[\frac{2GM}{D}\right]^{1/2}, \tag{5.21}$$

where M is the planet's mass, the universal gravitational constant is $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and D is the distance between the center of the planet and the gas particle. The escape velocities at the surfaces or cloud tops of the planets range between 4 and 60 km s^{-1}.

A planet tends to retain molecules that are moving at velocities less than the planet's escape velocity, and a molecule's velocity increases with temperature. At a given temperature, a molecule's velocity increases with decreasing molecular mass, so lighter molecules move at faster speeds and are more likely to escape a given planet or satellite than heavier ones. The high-speed tail in the Maxwell–Boltzmann distribution means that at any instant, a tiny fraction of the molecules are moving fast enough to escape even when the average thermal velocity is less than the escape velocity.

When the thermal velocity exceeds the escape velocity for a given type of molecule, all of those molecules will promptly flow out into space, and if this happens for every type of molecule, an airless body is left behind – like Mercury, the Earth's Moon, and every natural planetary satellite in the solar system except Titan.

For the Earth, Mars and Venus, the thermal velocity of all molecules is smaller than the escape velocity, but the lightest, fastest molecules can still slowly leak out or evaporate from the top of the atmosphere where collisions no longer dominate the velocity distribution. At lower altitudes, collisions confine the particles, but above a certain altitude known as the exobase, the atmosphere is so tenuous that gas particles hardly ever collide. Nothing stops an atom or molecule with sufficient velocity from flying away from the exobase into space.

The method of molecular escape from the exobase is known as *Jeans escape*, after the British scientist James Jeans (1877–1946) who introduced it (Jeans 1916). It is also known as thermal evaporation since it is analogous to the slow evaporation of water from the ocean. On average, the molecules in ocean water do not have enough thermal energy to escape from the liquid, but some of them acquire enough as the result of collisions near the ocean's surface.

At and below the exobase in the atmosphere, collisions between particles drive the speed distribution into a Maxwellian one, while above the exobase collisions are essentially absent and particles that have velocities greater than the escape velocity may leave the planet. The upward moving atoms in the high-speed tail of the Maxwell distribution can leave or exit the planet, hence the name exobase.

The lightest element, hydrogen, is the one that most easily overcomes the gravity of a terrestrial planet, but first it must reach the exobase. On Earth, the exobase is located about 500 km above the surface, and calculations indicate that about a billion, billion, billion, or 10^{27}, hydrogen atoms are still being lost from the Earth's exobase every second. This value is confirmed by satellite ultraviolet observations of hydrogen escaping from the Earth's upper atmosphere. So hydrogen gas is very rare in the Earth's atmosphere, present at about 1 part per million by volume. Nevertheless, it is still the third most abundant element in the Earth's surface, in the form of chemical compounds such as hydrocarbons and water.

Notice that lighter particles are lost by thermal evaporation at a much faster rate than heavier ones. Even over the Earth's lifetime of 4.6 billion years, the total mass of all the hydrogen atoms lost by thermal evaporation is 2×10^{17} kg, and the amount lost by heavier molecules would be much less. By way of comparison the total mass of the Earth's atmosphere is about 5×10^{18} kg. Lammer (2008) has reviewed the atmospheric escape and evolution of the terrestrial planets and satellites.

5.4 Gas Pressure

5.4.1 What Keeps Our Atmosphere Up?

Why doesn't the sky fall down, as Chicken Little once said was happening? After all, the Earth's atmosphere is pulled down by the planet's relentless gravity. The answer is that the atmosphere is warmed by the Sun, so its molecules are in continuous motion and collide with one another, producing a gas pressure that prevents them from falling to the ground. So, the atmosphere holds itself up.

The British scientist Robert Boyle (1627–1691) likened air to a "heap of little bodies, lying one upon another," which acted like springs that resist compression. He discovered that the total pressure exerted by all the little springs is inversely proportional to the volume of space in which they are confined (Boyle 1660). Other things being equal, the product of pressure and volume is conserved, so when a gas is compressed into a smaller volume the pressure rises.

When you compress a gas, the molecules move about more rapidly and collide more often. They are resisting being crowded together and pushed into a confining place. If you remove the confinement, the gas will expand out into surrounding space.

Radiation, wind, or a magnetic field also can produce a pressure, known as radiation pressure, wind pressure or magnetic pressure. For example, radiation pressure of sunlight (Debye 1909) can be used to propel a spacecraft by using a solar sail, and both solar radiation pressure and solar wind pressure push different types of comet tails away from the Sun. The solar radiation pressure can also cause a dust grain to slowly spiral into the Sun; a drag due to the component of radiation pressure tangential to the grain's motion. This drag is known as the *Poynting–Robertson effect* (Poynting 1904; Robertson 1937).

The SI unit of pressure is the pascal, abbreviated Pa, which is a force per unit area with $1 \text{ Pa} = 1 \text{ N m}^{-2}$, where the newton, abbreviated N, is the unit of force. Atmospheric pressure on the Earth and other planets often is measured in the bar unit of pressure, where $1 \text{ bar} = 100{,}000 \text{ Pa} = 10^5 \text{ Pa}$. The Earth's standard atmospheric pressure at sea level is defined as 1.01325 bar.

The radiation pressure of sunlight on a perfectly reflecting surface at the Earth's distance from the Sun is about 10^{-5} Pa. The pressure of the Sun's winds just outside our planet is a startling 10^{18} Pa, but it drops to about 10^{-13} Pa when the winds spread out to their boundary with interstellar space. The pressure of intense magnetic fields just above the visible solar disk is about 30 Pa.

Some representative pressures, displaying their enormous range in the universe, are given in Table 5.5.

Table 5.5 Range of cosmic pressures

Location	Pressure[a] (Pa)
Interstellar space	10^{-13}
Sunlight radiation pressure at Earth orbit	10^{-5}
Beneath foot of a tarantula	1
Visible solar disk[b]	10
Atmospheric pressure on Mars	10^{3}
Earth's atmosphere at sea level[c]	10^{5}
Inside a champagne bottle	5×10^{5}
Surface pressure of atmosphere on Venus	9×10^{6}
Inside a fully charged scuba tank	10^{7}
Center of the Earth	4×10^{11}
Center of Jupiter	7×10^{12}
Center of the Sun	2×10^{16}

[a] Unless otherwise stated in the location, the pressure is the gas pressure
[b] The gas pressure at the visible solar disk, known as the photosphere, is about the same as the vacuum pressure inside an incandescent light bulb
[c] The standard atmosphere of Earth has a sea-level pressure of 1.01325 bar = 101,325 Pa

5.4.2 The Ideal Gas Law

Under most conditions, an atmosphere behaves like an ideal gas, in which the randomly–moving molecules have no volume, like a point, and do not interact with each other, except by collisions. The ideal gas law for the gas pressure, P_g, is given by:

$$P_g V = N_{tot} k T, \tag{5.22}$$

where V is the volume, N_{tot} is the total number of molecules, the Boltzmann constant $k = 1.3806 \times 10^{-23}$ J K^{-1}, and T is the temperature. Such a relation between pressure, volume and temperature is known as an equation of state.

Using the particle number density $N = N_{tot}/V$, the ideal gas law can also be written

$$P_g = N k \mathrm{T}. \tag{5.23}$$

Example: **Gas pressure and number of molecules at sea level in the Earth's atmosphere**
The standard atmospheric pressure on Earth at sea level is equal to 1.01325×10^{5} Pa. The gas temperature is 288 K, so the number density of atmosphere molecules at sea level is $N = N_{tot}/V = P/kT = 2.5 \times 10^{25}$ m^{-3}.

The ideal gas law, also known as the perfect gas law, is a simplified equation of state that is a good approximation to many gases under many different conditions.

It describes the pressure within the Earth's atmosphere, and it is also used to specify the gas pressure exerted by subatomic particles in the hot interiors of stars. The ideal gas law was first stated by the French engineer Émile Clapeyron (1799–1864), and derived from kinetic theory by the German physicists August Krönig (1822–1879) and Rudolf Clausius (1822–1888) (Clapeyron 1834, 1856; Clausius 1850, 1857, 1870).

An alternative expression for the ideal gas law is:

$$P_g = \frac{\rho kT}{\bar{m}} = \frac{\rho kT}{\mu m_H} \tag{5.24}$$

for a gas of mass density ρ and average mass per particle given by:

$$\bar{m} = \rho/N = \mu m_H \tag{5.25}$$

for mean molecular weight μ or the mean particle mass in units of the mass of the hydrogen atom of $m_H = 1.00794$ u $\approx 1.67 \times 10^{-27}$ kg, which is roughly equal to the atomic mass unit u $= 1.6605 \times 10^{-27}$ kg and good enough for order of magnitude estimates.

For a diatomic molecule composed of atoms of atomic mass number A, the mean molecular weight is $\mu = 2A$, which is 28 for molecular nitrogen N_2, where the mass number of the nitrogen atom ^{14}N is 14.

Stellar interiors consist of ionized gas. For a fully ionized hydrogen gas, there will be an equal number of protons and electrons, so the mean mass will be:

$$\bar{m} = \frac{m_e + m_P}{2} = \frac{m_H}{2}, \tag{5.26}$$

where the mass of the electron, m_e, is negligible when compared to the mass of the proton, m_P, which is equal to that of the hydrogen atom $m_H = 1.67 \times 10^{-27}$ kg. For a fully ionized helium gas $\bar{m} = 4m_H/3$.

When the ionized gas is composed of three different kinds of particles, each with its own mass m_i and number density N_i denoted by subscript i, the mean particle mass is

$$\bar{m} = \frac{N_1 m_1 + N_2 m_2 + N_3 m_3}{N_1 + N_2 + N_3} = \frac{\rho}{N}. \tag{5.27}$$

For an ionized gas containing hydrogen, H, helium, He, and an element of mass number A,

$$\frac{\bar{m}}{m_H} = \frac{\rho}{N m_H} = \frac{2}{1 + 3X + 0.5Y} \tag{5.28}$$

where X, Y, and Z represent the concentration by mass of hydrogen, helium and heavier elements and $X + Y + Z = 1$. Their number densities are given by:

$$N_H = \frac{X\rho}{m_H}, N_{He} = \frac{Y\rho}{4m_H}, N_A = \frac{Z_A \rho}{A m_H}, \tag{5.29}$$

where the atomic number $Z_A \approx A/2$ is the mass abundance of an element of atomic mass number A. As an example, the abundances observed in the disk of the Sun are $X = 0.71$, $Y = 0.27$ and $Z = 0.02$; in the solar core nuclear fusion reactions have converted about half the hydrogen into helium and $X = 0.34$, $Y = 0.64$ and $Z = 0.02$.

Any hot gas exerts gas pressure, and the gas pressure will vary with distance from whatever is heating the gas. The Earth's atmosphere, for example, is heated from below, at the warm ground, and from above, by the Sun's radiation. Unlike our atmosphere, the Sun is heated from inside, at the center of its hot dense core.

5.4.3 The Earth's Sun-Layered Atmosphere

Our thin atmosphere is pulled close to the Earth by its gravity and suspended above the ground by molecular motion. And because air molecules are mainly far apart, our atmosphere is mostly empty space, and it always can be squeezed into a smaller volume. The atmosphere near the ground is compacted to its greatest density and pressure by the weight of the overlying air. Yet, even at the bottom of the atmosphere the density is only about 1/1000th of that of liquid water; an entire liter of this air weighs only 1 g.

At greater heights there is less air pushing down from above, so the compression is less and the pressure and density of air gradually fall off into the vacuum of space. There is a simple formula that expresses the drop in the atmosphere pressure at increasing distance, or radius from a planet center. The gas pressure, $P_g(r)$, at radius, r, is given by the *barometric equation*, also known as the *barometric law*:

$$P_g(r) = P_g(R) \exp\left[-\frac{(r-R)}{H}\right] \tag{5.30}$$

where $P_g(R)$ is the surface pressure at radius R, the height above the ground is $r - R$, and H is the *atmosphere scale height* given by:

$$H = \frac{kT}{\bar{m}g} \tag{5.31}$$

and

$$H \approx \frac{kTR^2}{\bar{m}GM} \tag{5.32}$$

where k is the Boltzmann constant, $\bar{m} \approx \mu m_H$ is the mean molecular mass for mean molecular weight μ, where $m_H = 1.67 \times 10^{-27}$ kg is the mass of the hydrogen atom, and the g is the local acceleration of gravity given by $g = GM(r)/r^2 \approx GM/R^2$ for a planet of mass M, at least as long as $r - R$ is much less than R.

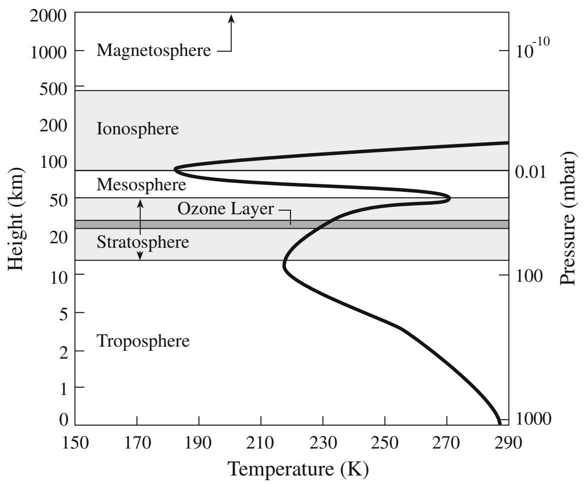

Fig. 5.4 Earth's layered atmosphere The atmospheric pressure (*right scale*) decreases with altitude (*left scale*). This is because fewer particles are able to overcome the Earth's gravitational pull and reach higher altitudes. The temperature (*bottom scale*) also decreases steadily with height in the ground-hugging troposphere, but the temperature increases in two higher regions that are heated by the Sun: the *stratosphere*, with its critical *ozone layer*, and the *ionosphere*. The stratosphere is heated mainly by ultraviolet radiation from the Sun, and the ionosphere is created and modulated by the Sun's x-rays and extreme ultraviolet radiation, which breaks apart atmospheric molecules, and strips electrons from atoms to produce ions. The process of ionization by the Sun's invisible rays releases heat to warm the ionosphere, so the temperature rises with altitude. In the ionosphere, at about 100–500 km above the ground, temperatures skyrocket to higher than anywhere else in the atmosphere. At higher altitudes, the atmosphere thins out into the exosphere, or the "exit to the outside sphere." The temperature is so hot out there and the particles move so fast that some atoms and molecules slowly evaporate

The temperature of the atmosphere also decreases at increasing height near the ground; however, it is not a simple fall-off when we keep going upward. It falls and rises in two full cycles as we move off into space (Fig. 5.4). That's because the Earth's atmosphere is heated from both the warm ground below and from above by ultraviolet and x-ray radiation from the Sun.

The temperature decreases steadily with height in the lowest regions of our atmosphere, since it is heated from the warmer ground below. When warm currents move up from the Earth's surface, they expand in the lower pressure and become cooler. The average air temperature drops below the freezing point of water of 273 K at only a kilometer or two above the Earth's surface, and bottoms out at a height of about 12 km above sea level. This is the greatest height achieved by the air currents. All our weather occurs below this altitude, controlled by visible sunlight.

Global atmospheric circulation, driven by differential solar heating of the equatorial and polar surfaces, creates complex, wheeling patterns of weather in this region, leading to the designation troposphere, from the Greek *tropo* for "turning."

The average extent of the ground-hugging troposphere varies with latitude, from about 16 km above the warm equator to roughly 8 km over the cold poles.

The vertical extent of the troposphere was detected near the end of the nineteenth century when Leon Philippe Teisserenc de Bort (1855–1913), a French meteorologist, launched hundreds of unmanned balloons that carried thermometers and barometers to altitudes as great as 15 km (de Bort 1902). At this height, the temperature no longer decreased with altitude, and seemed to remain nearly constant. If the temperature was unchanging, the ingredients of the atmosphere above the troposphere might settle down into layers, or strata, depending on their weight, so de Bort named this region the stratosphere.

Contrary to everyone's expectations, the temperature increases at greater heights within the stratosphere, rising to nearly ground-level temperatures at about 50 km above the Earth's surface; but we still use the name stratosphere to designate the layer of the atmosphere that lies immediately above the troposphere. The Sun's invisible ultraviolet radiation is largely absorbed in the stratosphere, where the radiation warms the gas and helps make ozone.

Above the stratosphere we come to the mesosphere, from the Greek *meso* for "intermediate." The temperature declines rapidly with increasing height in the mesosphere, from temperatures of about 265 K at 50 km altitude to far below freezing at about 85 km, where the temperature reaches the lowest levels in the entire terrestrial atmosphere.

The mesosphere has been known as the "ignorosphere" because it is too high to be reached by airplanes and too low to be studied by most spacecraft. The air at this height is too thin to support research balloons or aircraft, but thick enough for atmospheric friction, or air drag, to cause satellites to decay quickly from orbit. Sounding rockets pass through this region too rapidly to permit detailed study.

Instruments aboard rockets, as well as radio signals from the ground, have been used to examine the higher levels of the Earth's atmosphere, where the temperatures rise to above those on the ground owing to extreme ultraviolet and x-ray radiation from the Sun. This radiation is energetic enough to break the atmospheric molecules apart, and to strip electrons off their component atoms, producing ions. They reside in the ionosphere (Focus 5.1), where the temperature can rise to about 1200 K.

Focus 5.1 The Earth's ionosphere

On December 12, 1901, the Italian electrical engineer Guglielmo Marconi (1874–1937) startled the world by transmitting a radio signal in Morse code across the Atlantic, from England to Newfoundland. Marconi became an international hero and established the American Marconi Company, which later evolved into the Radio Corporation of America, abbreviated RCA. In 1909 Marconi and the German inventor Karl F. Braun (1850–1918) received the Nobel Prize in Physics for "their contribution to the development of wireless telegraphy."

Because radio waves travel in straight lines, and cannot pass through the solid Earth, no one expected that Marconi could send a radio signal halfway around the world. Radio waves get around the Earth's curvature by reflection from an electrically charged layer, now called the ionosphere, extending into space from roughly 70 to 500 km above the Earth's surface. The atoms in the ionosphere are highly ionized, and many of their electrons have therefore been set free from atomic bonds. As independently shown by Arthur E. Kennelly (1861–1939), then at the Harvard School of Engineering, and Oliver Heaviside (1850–1925) in England, these electrons give the ionosphere a high electrical conductivity, which enables them to turn the radio signals back toward the ground, reflecting them like a metal mirror and not allowing the radio signals to pass through (Kennelly 1902).

The rapid expansion of radio broadcasting in the 1920s, as well as the concurrent development of pulsed radio signals, helped specify the structure and daily variation of the ionosphere. Edward Appleton (1892–1965) and his students measured the height of the reflecting layer by determining the elapsed time between transmitting a radio pulse and receiving its echo from the ionosphere; like all electromagnetic radiation, the radio waves travel at the speed of light. They showed that there are at least three such reflecting layers, now labeled D, E and F, at respective altitudes of 70, 100 and 200–300 km (Appleton 1932; Appleton and Barnett 1925).

In 1947, Sir Appleton was awarded the Nobel Prize in Physics for his investigations of the physics of the upper atmosphere, especially for his discovery of the so-called Appleton layer.

The mystery of exactly what produces and controls the ionosphere was not solved until after World War II, when captured German V-2 rockets were brought to the United States. These and subsequent rockets, built by American engineers, were used by the Naval Research Laboratory to loft detectors above the atmosphere, showing that the Sun emits very energetic radiation at invisible x-ray and extreme ultraviolet wavelengths (Byram et al. 1956). When this radiation reaches the upper atmosphere it breaks the nitrogen and oxygen molecules into their constituent atoms and ionizes them, producing free electrons and atomic ions. The ionosphere above your head therefore develops as the Sun rises and decays as the Sun sets; it lingers on during the night but is not energized then.

The process of ionization by the Sun's invisible rays releases heat to warm the ionosphere, so the temperature increases with altitude in it. Within the ionosphere, the temperatures rise to higher values than anywhere else in the entire atmosphere. Indeed, some scientists prefer to call this region the thermosphere, or "hot" sphere. The thermosphere overlaps with the E and F regions of the ionosphere, beginning at about 90 km and extending upward to about 500 km.

At higher altitudes, the atmosphere thins out into the *exosphere*, or the "exit and outside sphere." In the exosphere the gas density is so low that an atom can completely orbit the Earth without colliding with another atom. The temperature is so hot out there, and the atoms move so fast, that some atoms can escape the Earth's gravitational pull and travel into outer space. It is therefore the thermosphere at the top of the ionosphere that caps our Sun-layered atmosphere and provides the Earth's threshold into space.

5.4.4 Pressure, Temperature, and Density Inside the Sun

Our Sun is a giant sphere of extremely hot gas, rarefied on the outside and compacted on the inside. Unlike the Earth, it has no solid surface. The high compressibility of the solar gas brings about a rapid increase in density as we go from its visible disk to its center, and as the result of this crowding, the gas particles collide more frequently with higher speeds than elsewhere in the Sun. The compacted gas particles also push more vigorously outward, producing strong gas pressure that keeps the Sun from collapsing.

Within the Sun's dense, central core, the density has increased to 151,300 kg m^{-3}, which greatly exceeds that of any solid or liquid bodies in our everyday environment. The central density is more than ten times greater than the density of solid lead, at 11,340 kg m^{-3}, but still behaving like a gas.

The center of the Sun is just slightly more than 100 times denser than the Sun taken as a whole, at a mean value of 1409 kg m^{-3}. That's the number you get when dividing the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg by its volume, $4\pi R_{\odot}^{3}/3$, for a solar radius of $R_{\odot} = 6.955 \times 10^{8}$ m. The mean density of the Sun is near that of water, at 1,000 kg m^{-3}, but it is an average density and the Sun is much too hot to be solid or liquid anywhere inside.

The central temperature can be estimated by assuming that a proton at the center of the Sun is hot enough and moving fast enough to counteract the gravitational compression it experiences from the rest of the star. When you do the arithmetic, this balanced condition occurs at a central temperature of 15.6 million K (Sect. 8.2). That is how hot the center of the Sun has to be to avoid collapsing under its own weight, and something has to keep it that hot. The heat is supplied by nuclear reactions at the core of the Sun (Chap. 8).

At this temperature and the central density, the central pressure needed to resist the weight of the overlying gas is 2×10^{16} Pa, or 200,000 million times the pressure of our atmosphere at sea level.

Example: **Gas pressure at the center of the Sun**
The mass density at the center of the Sun is $\rho = 1.51 \times 10^{5}$ kg m^{-3}, and the mass is provided by protons of mass $m_P = 1.67 \times 10^{-27}$ kg. So the number density of protons at the center of the Sun is $N = \rho/m_P = 0.90 \times 10^{32}$ m^{-3}. The central solar temperature is $T_C = 1.56 \times 10^{7}$ K. So the gas pressure at the center of the Sun is $P = NkT = 1.9 \times 10^{16}$ Pa, using the

Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. This is about 200 billion, or 2×10^{11}, times as great as the atmospheric pressure at sea level on Earth, at about 10^5 Pa. That's because there is both a much higher temperature and a much higher particle number density in the core of the Sun.

In contrast to both the central and mean densities, the outer layers of the Sun are quite rarefied. This is because there is less overlying material to support at greater distances from the center, so there is a drop in pressure, density and temperature. The compression is less, so the gas gets thinner and cooler (Fig. 5.5). Halfway from the center of the Sun to the visible disk, the density is the same as that of water, and about nine tenths of the distance from the center to the Sun's apparent edge, we find material as tenuous as the transparent air that we breathe on Earth.

At the visible solar disk, the rarefied gas is about one thousand times less dense than our atmosphere at sea level. Out there, in the more rarefied outer parts of the Sun, the temperature has fallen to 5,780 K. Examination of this outer, cooler solar atmosphere tells us about the elemental constituents of the Sun. But in most of the Sun, from its core to its upper atmosphere, there are no atoms; at these hot temperatures the atoms have been torn by collisions into their subatomic ingredients, mainly protons and electrons, to make an ionized gas known as plasma.

5.5 Plasma

5.5.1 Ionized Gas

Plasma has been called the *fourth state of matter*, to distinguish it from the solid, liquid and gas states (Fig. 5.6). The *plasma* is a hot, completely ionized gas, consisting of ions and electrons that have been pulled free of atoms. The high temperatures result in all the atoms losing their normal complement of electrons to leave positively charged ions behind, and it is too hot for the free electrons and ions to join together and form permanent atoms. Because the total negative electrical charge of the free electrons is equal to the total positive charge of the ions, plasma is electrically neutral over a sufficiently large volume.

The ionosphere is plasma. The interiors of most stars are plasma consisting mainly of electrons and protons, but this plasma still behaves like a gas that is described by thermal equilibrium and the Maxwellian speed distribution. Most of the matter in the universe is in the plasma state.

Ions are atoms that have lost one or more electrons, and the ionization energy is the amount of energy needed to remove electrons from an atom. This energy is also known as the ionization potential, and in atomic physics, the ionization

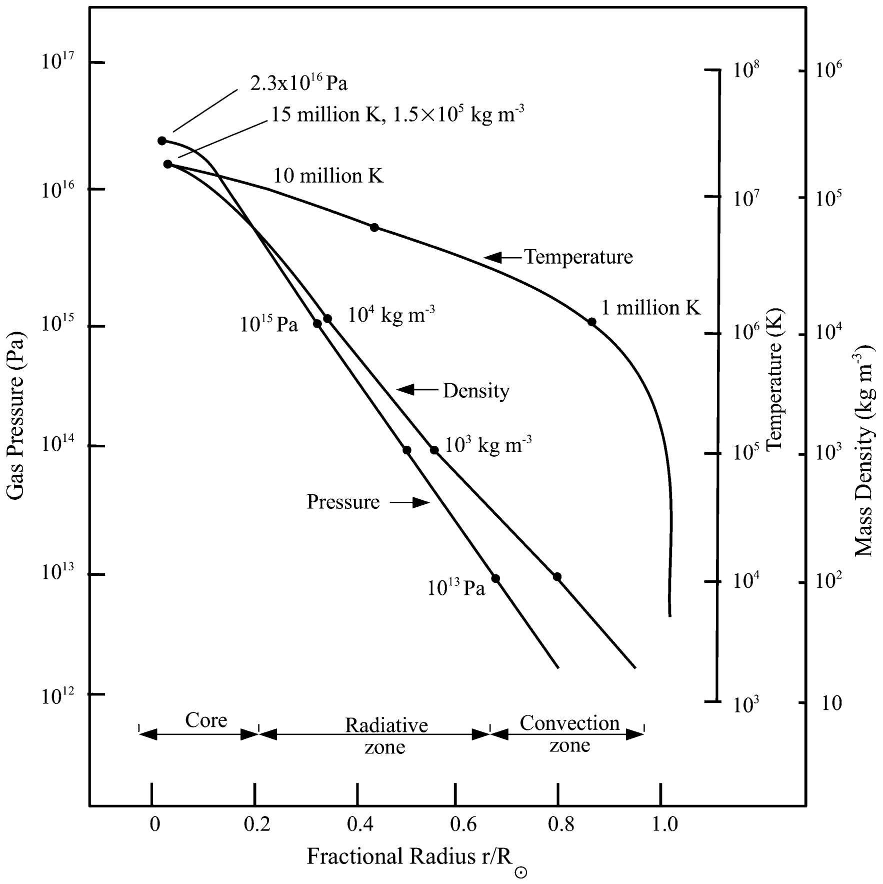

Fig. 5.5 Internal compression of the Sun The variation of pressure, temperature, and mass density with fractional radial distance from the Sun's center (*left*) to its visible disk (*right*). At the center of the Sun, the temperature is 15.6 million K and the mass density is 151,300 kg m^{-3}; the central pressure is 2.33×10^{16} Pa, or 233 billion times that of the Earth's atmosphere at sea level (one bar is equivalent to 100,000 Pa). Nuclear reactions occur only in the central core to about 25 % of the Sun's radius. The energy produced in the core is transported by radiation to 71 % of the star's radius, where the temperature has dropped to about 2 million K and the density has fallen to about 200 kg m^{-3}. The energy then is transported by convection out to the Sun's visible disk, known as the photosphere, where the temperature is 5,780 K, and the pressure and density have dropped off the scales of the graph. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

energy, or potential, is measured using the unit of electron volt, abbreviated eV. By definition, the electron volt is the amount of kinetic energy gained by a single unbound electron when it is accelerated through an electric potential difference of 1 V. It is the energy an electron gains when it passes across the terminals of a 1 V battery.

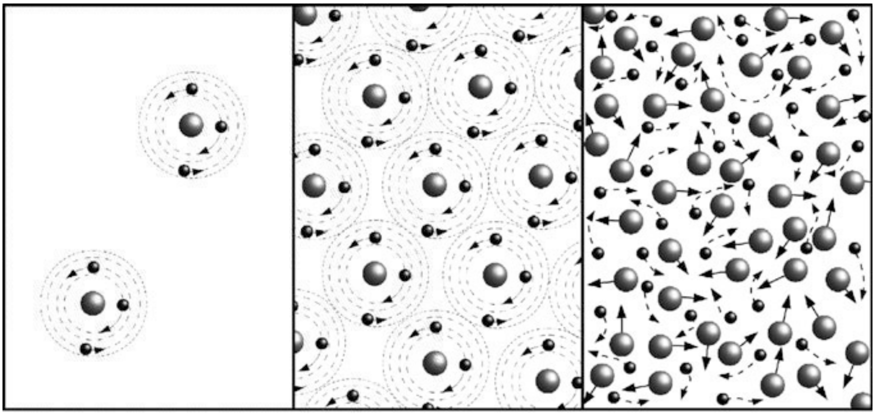

Fig. 5.6 States of matter The locations of atoms, *large dashed circles*; their component electrons, *small filled circles*; and central nucleus, *large filled circles*, for the gaseous (*left*), liquid or solid (*center*), and plasma (*right*) states of matter. In the gaseous state, the atoms are widely separated and free to move about. The atoms are practically touching one another in the solid and liquid states. At sufficiently high temperature and pressure, the atoms cease to exist and the plasma state is created. The atoms are torn into their constituents by frequent collisions at high temperatures. Plasma consists of bare nuclei and unattached electrons moving about in random directions within the former empty space of atoms. In the plasma state, matter regains the compressibility of the gaseous state and plasma behaves like a gas. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

The eV unit of energy is also used to describe x-rays, which were first produced by connecting a high-voltage power supply across the ends of an evacuated glass tube. The x-rays are in the 1–100 keV range of energy, where 1 keV = 1,000 eV.

The conversion from the eV unit to the joule unit of energy is

$$1\,\text{eV} = 1.602176 \times 10^{-19}\,\text{J} \approx 1.602 \times 10^{-19}\,\text{J}, \tag{5.33}$$

which is numerically equal to the elementary charge, in coulombs, of the electron.

The amount of energy required to remove the least tightly bound electron from a neutral atom is called the first ionization potential, denoted by the Roman numeral I. The additional energy needed to remove the next least tightly bound electron is the second ionization potential. More generally, the nth ionization potential, or the nth ionization energy, is the energy required to strip off the nth electron after the first $n - 1$ electrons have been removed.

For the one and only electron of the hydrogen atom, the first ionization potential is 13.5984 eV. The first, second and third ionization potentials of atomic oxygen are 13.6181, 35.117 and 54.934 eV. The temperature, *T,* required to ionize hydrogen, ripping off its sole electron and leaving a proton behind, is obtained by dividing its first ionization potential by the thermal energy *kT*, for the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, giving a temperature $T \approx 1.58 \times 10^5$ K.

At this temperature, or any higher one, hydrogen atoms will be completely torn apart, into their subatomic ingredients, the electrons and protons, which happens inside most stars.

5.5.2 Plasma Oscillations and the Plasma Frequency

If all the electrons in plasma were displaced by a small amount with respect to the ions, the force of electrical attraction between the electrons and ions would pull the electrons back, but when pushed the displacement can continue. The back and forth motion is a natural oscillating one, with electrons moving at the *plasma frequency*, designated ν_P, given by (Tonks and Langmuir 1929):

$$\nu_P = \left[\frac{e^2 N_e}{4\pi^2 \varepsilon_0 m_e}\right]^{1/2} = 8.98\, N_e^{1/2}\ \mathrm{Hz}, \tag{5.34}$$

where the electron density N_e is in units of m^{-3}, the electron charge $e = 1.602 \times 10^{-19}$ C, the electron mass $m_e = 9.1094 \times 10^{-31}$ kg, the permittivity of free space is $\varepsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{F\ m}^{-1}$, and $\pi \approx 3.14159$.

The ionosphere reflects radio waves at the plasma frequency ν_P or at the plasma wavelength, denoted λ_P. Since the radio radiation travels at the speed of light, c, we have $\lambda_P \times \nu_P = c = 2.9979 \times 10^8\ \mathrm{m\ s}^{-1}$.

Example: **The temperature, origin, and plasma frequency of the Earth's ionosphere**

The F layer of the ionosphere, located about 200 km above our heads, contains oxygen atoms that are missing two electrons and a number density of free electrons of $N_e = 10^{12}\ \mathrm{m}^{-3}$. The temperature, T, required to create these ions can be estimated by equating the thermal energy $3kT/2$ to the third ionization potential for oxygen atoms of 54.934 eV, using the Boltzmann constant $k = 1.38065 \times 10^{-23}\ \mathrm{J\ K}^{-1}$ and $1\ \mathrm{eV} = 1.6022 \times 10^{-19}$ J. The result is $T = 4.229 \times 10^5$ K. If the photon energy $h\nu$ of the incident solar radiation is equal to this thermal energy, where the Planck constant $h = 6.626 \times 10^{-34}$ J s, then the wavelength of the radiation is $\lambda = c/\nu = 2hc/(3kT) \approx 2.3 \times 10^{-8}$ m, or an x-ray wavelength, so x-rays from the Sun can produce the ionization. The plasma wavelength λ_P corresponding to the plasma frequency ν_P in this layer of the ionosphere is $\lambda_P = c/\nu_P \approx 33$ m, where the speed of light $c = 2.9979 \times 10^8\ \mathrm{m\ s}^{-1}$, and $\nu_P = 8.98\ N_e^{1/2} = 8.98 \times 10^6$ Hz. Radio transmissions from the Earth at this long wavelength are reflected back down to the ground, and cannot get through the ionosphere. They can nevertheless be reflected at an angle, enabling long-distance radio communications.

We can infer the height and electron density of a given layer in the ionosphere by sending radio signals up into the atmosphere at successively longer wavelengths or shorter frequencies. The ionosphere will not mirror radio waves, and send a signal back, unless their wavelength is longer than the plasma wavelength or their frequency is less than the plasma frequency. This provides a measure of the electron density in the ionosphere, the shorter the reflection wavelength or the higher the reflection frequency, the larger the electron density. Radio waves with wavelengths that are less than the plasma wavelength can pass right through the ionosphere, because, roughly speaking, they are short enough to pass among the electrons. These shorter wavelengths are used in communications with satellites or other spacecraft in outer space beyond the ionosphere. The longer wavelengths are used in radio communications around the Earth through reflection off the ionosphere.

Example: ***Sputnik***
The F layer in the ionosphere has an electron density of about $N_e \approx 10^{12}\ \mathrm{m}^{-3}$; therefore, it has a plasma frequency of $\nu_p \approx 8.98 \times 10^6$ Hz ≈ 8.98 MHz with a corresponding wavelength of $\lambda_P = c/\nu_\mathrm{P} \approx 33$ m. Radio or microwave signals with wavelength shorter than this value can see through the ionosphere and communicate with satellites in and above it. At longer wavelengths, radio signals sent up into the ionosphere are reflected back down to the ground and this is how the ionosphere's electron densities are measured. The orbit of the first artificial satellite, *Sputnik*, which was launched on October 4, 1957, had a semi-major axis of 6,955 km, placing it about 584 km above the mean radius of the Earth; the mean radius is 6,371 km. *Sputnik* therefore orbited within the outer ionosphere; the atmospheric friction led to rapid orbital decay and the demise of the satellite in 3 months. Amateur radio operators monitored the beep of its radio signals throughout the world. At a signal frequencies of 20.0 and 40.0 MHz, or wavelengths of about 15 and 7.5 m, the radio signals from *Sputnik* were just short enough in wavelength to pass through the ionosphere.

5.5.3 Atoms are Torn Apart into Plasma Within the Sun

Whole atoms are only found in the outer visible layers of the Sun, where the temperature is a relatively cool 5,780 K. Raise the temperature by just a factor of three, to about 17,000 K, which happens just beneath the solar disk we see with our eyes, and the Sun's hydrogen atoms are stripped bare, losing their identity.

The hot atoms move rapidly here and there, colliding with each other at high speeds, and the violent force of these collisions is enough to fragment the atoms into their subatomic constituents. And since the Sun is mostly hydrogen, its interior consists mainly of protons, the nuclei of hydrogen atoms, and free electrons that have been torn off the atoms by innumerable collisions and set free to move throughout the Sun.

What is left is plasma, a seething mass of electrically charged particles, the electrons and protons. The electrical charge of the protons balances and cancels that of the electrons, which have been removed from atoms to also release the protons, so the plasma has no net electric charge. The Sun is just one huge mass of incandescent plasma, compressed on the inside and more tenuous further out.

Plasma can be packed more tightly than complete atoms. This is because the electrons in an atom are located at relatively remote distances from the atomic nuclei, so atoms are largely empty space and once the electrons are removed the protons can be compressed together more than atoms can.

5.6 Sound Waves and Magnetic Waves

5.6.1 Sound Waves

Sound is transmitted in waves that are produced by perturbations in an otherwise undisturbed gas or liquid. These waves can be described as a propagating change in the mass density. For a fluid medium, which can be either a gas or a liquid, we assume an initial equilibrium in which the fluid is at rest, with initial velocity $V_0 = 0$ and a constant mass density ρ_0 and pressure P_0, where the subscript 0 denotes the initial undisturbed condition. We then assume a perturbation ρ_1 in the mass density ρ that becomes $\rho = \rho_0 + \rho_1$; the perturbation velocity is denoted as V_1, and the perturbation pressure designated P_1 with a subscript 1 for the perturbed condition. The equations of hydrodynamics have a plane wave solution for the perturbed density in the x direction given by:

$$\rho_1 \propto \exp\left[i\left(\frac{2\pi x}{\lambda} - \omega t\right)\right] \tag{5.35}$$

where the frequency, ω, is related to the wavelength, λ, by:

$$\omega^2 = \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{\partial P}{\partial \rho}\right). \tag{5.36}$$

The pressure, P_1, and velocity, V_1, also satisfy the wave equation. The French mathematician and astronomer Pierre-Simon Laplace (1749–1827) used the ideal gas law to describe the pressure, by $P = NkT$ and $P/\rho = kT/\bar{m}$ to obtain the speed of sound, c_S (Laplace 1816):

$$c_S = \left(\frac{\omega\lambda}{2\pi}\right) = \left(\frac{\partial P}{\partial \rho}\right)^{\frac{1}{2}} = \left(\frac{\gamma P_o}{\rho_o}\right)^{\frac{1}{2}} = \left(\frac{\gamma k T_o}{\bar{m}}\right)^{\frac{1}{2}}, \tag{5.37}$$

where γ is the adiabatic index. For a monatomic gas, the γ is $5/3 = 1.667$ and for a diatomic gas $\gamma = 7/5 = 1.400$. The Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. The mean molecular mass is $\bar{m} = \mu \times u$ where the mean molecular weight is μ and the atomic mass unit u = 1.66054×10^{-27} kg. Isaac Newton (1642–1727) considered the speed of sound in an isothermal calculation in his *Principia* (*Book II*, *Proposition* 49), instead of an adiabatic one, obtaining essentially the same result with a $\gamma = 1.000$ for an isothermal perturbation.

The speed of sound in an ideal gas is proportional to the square root of the temperature, but it is nearly independent of pressure or mass density for a given gas. At a constant temperature, the ideal gas pressure has no effect on the speed of sound because the pressure and the density, which is also proportional to pressure, have equal but opposite effects on the speed of sound, and the two contributions cancel out exactly.

Our atmosphere consists mainly of diatomic molecules N_2 at 78 % and oxygen O_2 at 21 %. For diatomic molecules $\gamma = 1.400$, and the mean molecular weight $\mu = 2A$ for a diatomic molecule composed of atoms of atomic mass number A. Nitrogen and oxygen have A = 14 and A = 16, respectively; therefore, the mean molecular weight of our atmosphere is $2 \times 14 \times 0.78 + 2 \times 16 \times 0.21 = 28.56$. Substituting these numbers with the other known constants into the equation we find that sound moves though the air at a speed of about 20 $T^{1/2}$ m s^{-1}, or to be precise:

$$c_{air} = 20.0457[T]^{1/2}\,\mathrm{m\,s^{-1}}, \tag{5.38}$$

where T is temperature on the kelvin scale, and $T = 273.15 + T_c$ if you are using a temperature T_C in °C. At sea level, the temperature is 288 K or 15 °C and the sound speed is about 340 m s^{-1}. The speed of sound decreases with altitude, due to lower temperatures found there, but even at the cruising altitudes of most aircraft the temperature is less than about 216 K corresponding to a speed of sound of less than 294 km s^{-1}.

The speed of motion divided by the speed of sound is called the Mach number, and anything that moves at a speed greater than Mach 1 is said to be traveling at supersonic speed. Most modern fighter aircraft are supersonic. Such aircraft have broken the sound barrier and can produce a sonic boom.

Example: **Sound waves in the Earth's atmosphere**
Assuming that the Earth's atmosphere is mainly composed of diatomic nitrogen molecules, with an adiabatic index of $\gamma = 7/5$ and a molecular mass of 2×14 u, where the atomic mass unit u = 1.660539×10^{-27} kg, the sound speed at ground level, where the temperature is $T = 288$ K, is about $c_s = [\gamma kT/(28\mathrm{u})]^{1/2} \approx 346$ m s^{-1}, where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}. The first shout from a drowning man, located 1 km out in the ocean, would be heard at the beach in just 2.9 s.

We detect sound by small changes in the sound pressure against our vibrating eardrum, above and below the normal atmospheric pressure. The threshold of hearing for most individuals is a sound pressure of 2×10^{-5} Pa. Sound pressure is inversely proportional to distance from the source of the sound, unlike sound intensity that falls off with the inverse square of the distance.

At sea level, sound waves move about 4.3 times faster in water than in air. The larger mass density of water slows the sound waves in water relative to air, and this nearly makes up for the compressibility differences of the two media. Submarines use pulses of sound waves to detect the direction and distance of ships or the depth of the ocean floor. The acronym SONAR is used for such SOund Navigation And Ranging.

Sound waves are generated by turbulence (Lighthill 1952, 1954; Proudman 1952), and it is such turbulent motions that give rise to the roar of a jet airplane engine. The convective rise and fall of gas in the outer layers of the Sun also create sound waves (Biermann 1948; Schwarzschild 1948; Schatzman 1949) that are produced by the turbulent motion (Goldreich and Kumar 1990). Most of the solar sound waves are trapped inside the Sun, and they are used to investigate its internal properties (Sect. 8.5).

Within the Sun we can assume, to a first approximation, that it consists of a fully ionized monatomic gas with an adiabatic index of $\gamma = 5/3 = 1.667$ and the mass is dominated by the protons, with a mass $m_P = 1.67 \times 10^{-27}$ kg. Similar assumptions apply to the hot expanding solar atmosphere, resulting in the supersonic solar wind.

Example: **Sound speed in the solar wind**
A perpetual wind of protons and electrons is blowing out from the Sun in all directions through interplanetary space. At its origin near the Sun, the temperature is 10^6 K, and the highly conducting wind stays nearly that hot all the way to the Earth and beyond. Using this temperature with $\gamma = 1.667$, a proton mass of $m_P = 1.6726 \times 10^{-27}$ kg, and the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, we find that the sound speed in the solar wind is $c_S \approx (\gamma kT/m_P)^{1/2} \approx 10^5$ m s^{-1} or 100 km s^{-1}. The solar wind has a fast component moving at a speed of about 750 km s^{-1} and a slow one moving at about half that speed, so the solar wind is always supersonic, with a speed exceeding the speed of sound.

5.6.2 Magnetic Waves

In addition to ponderous material particles, like atoms, molecules, electrons, and ions, there are also magnetic fields that permeate the universe. The trajectories of charged particles, the electrons and the ions, are guided by these magnetic fields, which act as a wall to them. The charges can spiral around the magnetic fields but cannot cross them.

The theory that deals with the interaction of a hot gas, or plasma, and a magnetic field is called magnetohydrodynamics, or MHD for short. As the ponderous name suggests, the equations are a combination of those of electromagnetism and fluid mechanics. The Swedish electrical engineer Hannes Alfvén (1908–1995) pioneered the study of MHD, receiving the 1970 Nobel Prize in Physics for this work.

Alfvén proposed the possible existence of oscillations produced by magnetic tensions, which are now known as *Alfvén waves*. These waves propagate in the direction of the magnetic field with the Alfvén velocity V_A given by (Alfvén 1942a, b):

$$V_A = \frac{B}{\sqrt{\mu_0 \rho}}, \tag{5.39}$$

where B is the magnetic field strength in tesla, ρ is the mass density in units of kg m^{-3}, and the magnetic permeability $\mu_0 = 1.2566 \times 10^{-6}$ N A^{-2}.

Neglecting the contribution of electrons to the mass density and assuming that there is a single ion species we obtain:

$$V_A = \frac{B}{\sqrt{\mu_0 N_i m_i}}, \tag{5.40}$$

where N_i is the number density of the ions in m^{-3} with mass m_i in kg, for protons $m_i = 1.6726 \times 10^{-27}$ kg. The ion mass density provides the inertia for the oscillations and the magnetic field provides the restoring force. The ion motion and the magnetic field perturbations are in the same direction, both transverse to the direction of propagation.

The kinetic energy density of the ions is given by:

$$\frac{1}{2} m_i V_A^2 N_i = \frac{B^2}{2\mu_0}, \tag{5.41}$$

which is equal to the magnetic field energy density $B^2/(2\mu_0)$.

Cosmic magnetic fields are always being jostled, twisted and stirred around, and tension acts to resist the motions and pull the disturbed magnetism back. This generates Alfvén waves that propagate along magnetic fields, somewhat like a vibrating string. These waves do not form shocks, and once generated they can propagate large distances, directing their energy along open magnetic fields. Alfvén suggested that these waves could contribute to the heating of the outer solar atmosphere (Alfvén 1947).

Instruments aboard *Mariner 5* detected magnetic fluctuations attributed to large-amplitude Alfvén waves in the interplanetary medium during the spacecraft's voyage to Venus in 1967 (Belcher et al. 1969). The pressure of these waves may push the solar wind to a higher speed than it would otherwise have (Cranmer

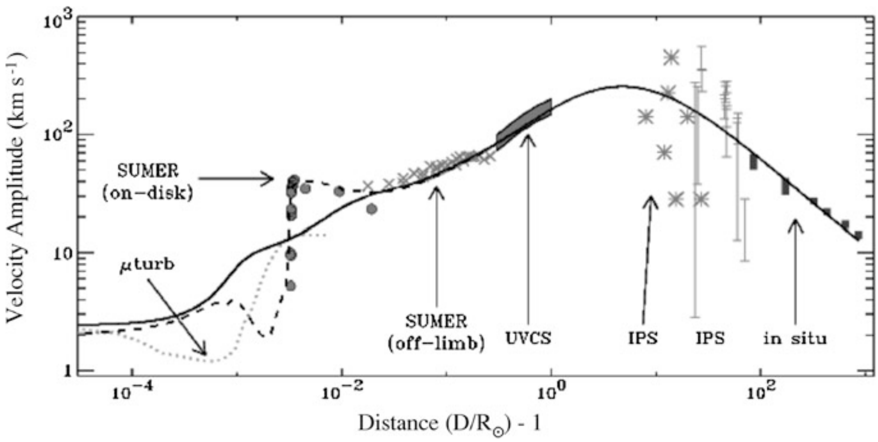

Fig. 5.7 Alfvén waves The velocity amplitudes, or speeds, of Alfvén waves, expressed as transverse velocities of the oscillating magnetic field lines, versus distance, D, above the visible disk of the Sun or photosphere, given in units of the solar radius $R_{\odot}$. The *solid curve* fits the data observed from spacecraft whose instruments are specified by their acronym. The left-most two sets of data (*dotted* and *dashed curves*) represent radial motions and may not correspond directly to the transversely oscillating Alfvén waves (adapted from a figure provided by Steven R. Cranmer)

and van Ballegooijen 2005). The magnetic waves have been measured for more than 40 years throughout the plane of our solar system, both near to and far from the Sun (Fig. 5.7). Magnetometers aboard the *Ulysses* spacecraft have detected the effects of Alfvén waves above the Sun's polar regions, and observations from the *Hinode* spacecraft have found their signatures near the visible solar disk, with perhaps enough energy to power the Sun's winds (De Pontieu et al. 2007).

Example: **Alfvén waves in the interplanetary medium**
Spacecraft measure an interplanetary magnetic field strength just outside the Earth of $B = 2.5 \times 10^{-9}$ T. The observed proton density in the solar wind at the Earth's orbit is $N_P = 5 \times 10^6$ m^{-3} for protons of mass $m_P = 1.67 \times 10^{-27}$ kg. The associated Alfvén velocity is $V_A = B/(\mu_0 N_P m_P)^{1/2} = 2.44 \times 10^4$ m s^{-1} = 24 km s^{-1}, with $\mu_0 = 1.2566 \times 10^{-6}$ N A^{-2}. The magnetic fields are tied to the Sun at one end, and stretch out into interplanetary space at the other, with a strength that is inversely proportional to the distance. The proton density is proportional to the inverse square of the distance, but the Alfvén velocity goes as the inverse square root of this density. So the two distance variations cancel, and we expect comparable Alfvén velocities throughout interplanetary space, which have in fact been observed from 10 to 100 km s^{-1} (Fig. 5.7).