Chapter 2
Radiation

2.1 Electromagnetic Waves

The physical perception of the universe is governed almost solely by the electromagnetic radiation received from cosmic objects. This radiation carries energy and moves through space in periodic waves at the speed of light, designated by the lower case letter c. The speed of light in empty space is a universal constant, independent of reference in space and time. The radiation is called *electromagnetic* because it propagates by the interplay of oscillating electric and magnetic waves.

Our understanding of electricity and magnetism is founded upon the experimental investigations of the English scientist Michael Faraday (1791–1867), who invented the first rotating electric motor and discovered electromagnetic induction, the principle behind the electric transformer and generator (Faraday 1843). His experiments led Faraday to propose that electromagnetic forces extend into empty space around charged bodies, electrical conductors, and magnets; these invisible forces are now called *electromagnetic fields*.

The Scottish mathematician and theoretical physicist James Clerk Maxwell (1831–1879) was able to express Faraday's results in a precise mathematical form, now known as *Maxwell's equations* (Maxwell 1865). These four partial differential equations depend on variations of the force fields in four dimensions – three for space and one for time.

In regions with no charge or currents, such as a vacuum, Maxwell's equations describe sinusoidal *electromagnetic waves* (Focus 2.1). The waves described by this electromagnetic wave equation have a speed equal to the speed of light, leading Maxwell to comment, "light is an electromagnetic disturbance propagated through the field according to electromagnetic laws." The changing magnetic field creates a changing electric field that, in turn, creates a changing magnetic field. The electric and magnetic field directions are orthogonal to each other and to the direction of travel.

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Focus 2.1 Plane waves of electromagnetic radiation

For the electric field, ***E***, and the magnetic field, ***B***, in free space, Maxwell's equations take the form (Maxwell 1865):

$$\begin{aligned}\nabla \cdot E &= 0\\ \nabla \cdot B &= 0\\ \nabla \times E &= -\frac{\partial B}{\partial t}\\ \nabla \times B &= \mu\varepsilon\frac{\partial E}{\partial t},\end{aligned} \tag{2.1}$$

where $\nabla\cdot$ is the divergence operator, with units of m^{-1}, $\nabla\times$ is the curl operator, with units of s^{-1}, and $\partial/\partial t$ is the partial derivative with respect to time, t. These equations can be written as second-order partial differential equations:

$$\begin{aligned}\left(\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}\right) &= 0\\ \left(\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}\right) &= 0\end{aligned} \tag{2.2}$$

that describes the propagation of electromagnetic waves through a medium or a vacuum. Here c is the speed of light in the medium. In a vacuum:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8\,\mathrm{m\,s^{-1}}, \tag{2.3}$$

where the electric constant, or vacuum permittivity, $\varepsilon_0 = 8.854187817 \times 10^{-12}\ \mathrm{F\,m^{-1}} \approx 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$, and the magnetic constant, or vacuum permeability, $\mu_0 = 1.256632061 \times 10^{-6}\,\mathrm{N\,A^{-2}} \approx 1.257 \times 10^{-6}\ \mathrm{N\,A^{-2}}$. In a medium with refractive index n:

$$c = \frac{1}{n\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\mu\varepsilon}}, \tag{2.4}$$

where ε is the electric permittivity of the medium and μ is the magnetic permeability of the medium.

There are sinusoidal, plane-wave solutions of these equations written as:

$$\begin{aligned}E(r,t) &= E_0\cos(\omega t - k\cdot r)\\ B(r,t) &= B_0\cos(\omega t - k\cdot r),\end{aligned} \tag{2.5}$$

where t is the time variable, and the angular frequency, ω, is related to the wavenumber, k, by the dispersion relation

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}, \tag{2.6}$$

where λ is the wavelength, $\nu = \omega/2\pi = c/\lambda$ is the frequency in s^{-1}, and the constant $\pi \approx 3.154159$.

The energy flux, S, and energy density, U, in the plane wave are

$$S = \frac{c}{8\pi}\sqrt{\frac{\varepsilon}{\mu}}E_0^2 \tag{2.7}$$

which is directed along the direction of wave propagation, and

$$U = \frac{\varepsilon E_0^2}{4\pi}. \tag{2.8}$$

Maxwell realized that c equals the speed of light, which others had previously measured, and concluded that light is a form of electromagnetic radiation.

In common with any wave, electromagnetic radiation has a wavelength, usually denoted by the lowercase Greek letter lambda, λ. The wavelength is the distance between successive crests or successive troughs (Fig. 2.1). Different types of electromagnetic radiation differ in their wavelength, although they propagate at the same speed. Like waves on water, electromagnetic waves have crests and troughs; but, unlike water waves, electromagnetic waves can propagate in vacuous empty space.

In SI units, the wavelength is measured in meters, abbreviated m. Other units of wavelength are the nanometer, or nm for short, where 1 nm $= 10^{-9}$ m, the Ångström, abbreviated Å where 1 Å $= 10^{-10}$ m $= 0.1$ nm, and the micron, denoted μ where 1 μ $= 10^{-6}$ m. Radio astronomers might specify the wavelength in meters or centimeters, abbreviated cm where 1 cm $= 10^{-2}$ m.

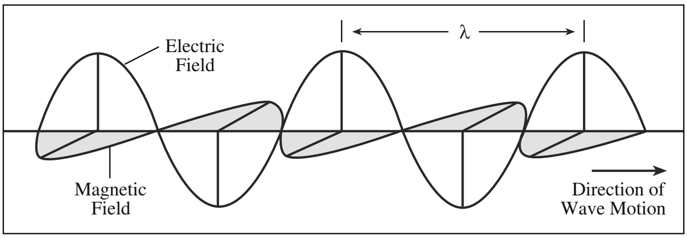

Fig. 2.1 Electromagnetic waves All forms of radiation consist of electrical and magnetic fields that oscillate at right angles to each other and to the direction of travel. They move through empty space at the speed of light. The separation between adjacent wave crests is called the *wavelength* of the radiation and usually is designated by the lowercase Greek letter lambda, λ

Sometimes radiation is described by its frequency, denoted by the lower case Greek letter nu written ν. The frequency indicates how fast the radiation oscillates, or moves up and down. The frequency of a wave is the number of wave crests passing a stationary observer every second, measured in Hertz, abbreviated Hz. One Hertz is equivalent to one cycle per second, or $1\ \text{Hz} = 1\ \text{s}^{-1}$. Radio astronomers use a frequency unit of megahertz, abbreviated MHz, where $1\ \text{MHz} = 10^6\ \text{Hz}$. Radio stations that transmit frequency modulated, or FM, signals, are denoted by their call letters and the frequency of their broadcasts in MHz. The frequency range of FM radio broadcasts is 88–108 MHz. Amplitude modulated, or AM, radio signals are broadcast in several bands of frequency between 0.150 and 30 MHz.

Electromagnetic waves all travel through empty space at the same constant speed – that is, the *speed of light* in a vacuum $c = 299{,}792{,}458\ \text{m s}^{-1}$, or about $2.9979 \times 10^8\ \text{m s}^{-1}$. The product of wavelength, λ, and frequency, ν, is equal to the speed of light, c, or

$$\lambda \times \nu = c. \tag{2.9}$$

So, radiation at shorter wavelengths has a higher frequency and a longer wavelength corresponds to a lower frequency. Any electromagnetic wave, regardless of wavelength or frequency, travels though empty space at the speed of light, and it is the maximum speed possible (Focus 2.2).

Focus 2.2 Light, the fastest thing around

It was once thought that light moves instantaneously through space. But we now know that it travels at a very fast but finite speed. This was first inferred from observations of Jupiter's moon Io in the 17th century. The King of France had directed Giovanni Domenico Cassini (1625–1712), director of the Paris Observatory, to use such observations to improve knowledge of terrestrial longitude and maps of France. Both the Danish astronomer Ole Rømer (1644–1710), who also worked at the observatory, and Cassini noticed a varying time between eclipses of Io by the Jupiter (Rømer 1677). Although the time between Io eclipses was approximately 42 h, it varied by an amount of up to 22 min.

Both astronomers concluded that it was not the orbit of Io around Jupiter that changed, but the time it took Jupiter's light to travel from Io to the Earth, which depended on the Earth's position in its orbit around the Sun. When the Earth was on the side of its orbit that is closest to Jupiter, the observed eclipse period for Io was shortest, and when the Earth was on the opposite side of its annual orbit around the Sun, Io's apparent eclipse period was largest.

Neither astronomer gave a value for the speed of light, which would have been equal to the diameter of the Earth's orbit divided by the time difference between the longest and shortest observed Io period, or a velocity of

$c = 2\ \text{AU}/22\ \text{min}$ and approximately $2.27 \times 10^8\ \text{m s}^{-1}$, where 1 AU = 1.496×10^{11} m is the mean distance between the Earth and the Sun. At Cassini's time this distance was not well known (see following Sect. 2.5).

Jupiter orbits the Sun at a mean distance of 5.203 AU and Jupiter's natural satellite Io orbits Jupiter with a period of 1.769 Earth days. It is eclipsed by the planet with that period. Observations of changes in the eclipse period were interpreted as differences, Δt, in the time, t that light takes to travel from Jupiter to Earth. When Jupiter is furthest from Earth, its distance will be 6.203 AU, since the Earth is 1.00 AU from the Sun, and when Jupiter is closest to the Earth, the giant planet's distance will be 4.203 AU. So the total change in Io's apparent orbital period, from longest to shortest, will be $\Delta t = (6.203–4.203)/c = 2\ \text{AU}/c = 998\ \text{s} = 16.63\ \text{min} = 0.0116$ Earth days, where $c = 2.9979 \times 10^8\ \text{m s}^{-1}$ is the speed of light. This is a relatively small change in the Io's actual orbital period of 1.769 Earth days.

The English astronomer James Bradley (1693–1762) unexpectedly discovered the aberration of starlight about half a century later. It is a change in the observed position of a star that depends on the ratio of the velocity of the Earth and the speed of light. Using then current estimates for the Earth's orbital motion around the Sun, Bradley used his aberration measurements to infer a speed of light of about 294,500 m s^{-1} (Bradley 1728).

More refined laboratory measurements during subsequent centuries indicated that light is always moving at a constant speed with the precise velocity of $c = 299{,}792{,}458\ \text{m s}^{-1}$. Light emitted by any star moves at this speed through empty space for all time. It never stops or slows down, and it never comes to rest. Nothing outruns light; it is the fastest thing around.

Electromagnetic radiation has no way of marking time, and it can persist forever. As long as its rays pass through empty space and encounter no atoms or charged particles like electrons, it will survive unchanged. Radiation emitted from any star or galaxy today might therefore travel for all time in vacuous space, bringing its message forward to the end of the universe. Astronomers on Earth intercept just a small part of this radiation, which is streaming away from both known and unknown objects located throughout the cosmos.

2.2 The Electromagnetic Spectrum

Most of us remember the colorful display of a rainbow, which is sunlight bent into separate wavelengths by droplets of water. In the mid-17th century, the English scientist Isaac Newton (1642–1727) showed that sunlight could also be broken into its colors using a prism – a specially cut chunk of glass (Newton 1671, 1704). Furthermore, each color could not be divided into other colors. A crystal

Table 2.1 Approximate wavelengths of colors[a]

Color	Wavelength (nm = 10^{-9} m = 10 Å)
Violet	420
Blue	470
Green	530
Yellow	580
Orange	610
Red	660

[a] Approximate wavelengths good to about 10 nm

chandelier or compact disk also displays the spectrum of visible light, arranging the colors by their different wavelengths.

From short to long waves, the colors in the spectrum of visible light correspond to violet, blue, green, yellow, orange and red (Table 2.1). Their wavelengths might be specified in nanometers, abbreviated nm, where 1 nm = 10^{-9} m or in Ångströms, abbreviated Å, where 1 Å = 0.1 nm = 10^{-10} m. Light from the Sun or an incandescent light bulb often is called white light, because it contains all of the colors, whereas black denotes the absence of color when we see no light.

The *electromagnetic spectrum* describes the types and wavelengths of electromagnetic radiation (Fig. 2.2). From short wavelengths to long ones, this spectrum includes gamma rays, x-rays, ultraviolet radiation, visible light, infrared radiation and radio waves (Table 2.2).

Our eyes detect a narrow range of wavelengths, which include the visible colors. It comprises just one small segment of the much broader electromagnetic spectrum. This band of light is also termed visible radiation, to distinguish it from invisible radiation that cannot be seen with the eye. The radiation we can see is also known as optically visible radiation, since the science of optics is used to describe the lenses and mirrors used to detect the light. The most intense radiation of the Sun and many other stars is emitted at these optically visible wavelengths, and our atmosphere permits it to reach the ground. Other types of radiation, like the invisible x-rays, are absorbed in our atmosphere and do not reach the Earth's surface.

The invisible domains include infrared and radio waves – with wavelengths longer than that of red light – and the ultraviolet (UV) rays, x-rays, and gamma (γ) rays, whose wavelengths are shorter than violet light. They all are *electromagnetic waves* and part of the same family, and they all move in empty space at the speed of light, but we cannot see them.

Gamma rays are the shortest and most energetic electromagnetic waves. Their wavelengths are as small as the nucleus of an atom, or about 10^{-15} m, and their waves are so energetic that they can pass through a thick iron plate.

The *x-ray* region of the electromagnetic spectrum extends from a wavelength of 100 billionth (10^{-11}) of a meter, which is about the size of an atom, to the short-wavelength side of the ultraviolet. The German physicist Wilhelm Röntgen (1845–1923) discovered x-rays, producing them with an electrical discharge in a

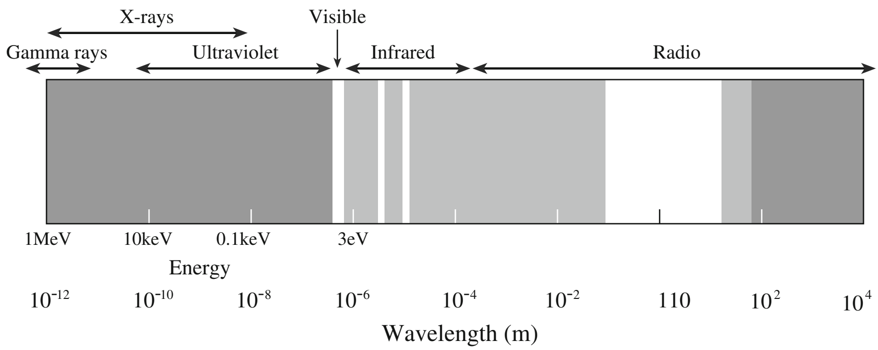

Fig. 2.2 Electromagnetic spectrum Radiation from cosmic objects can be emitted at wavelengths from less than 10^{-12} m to greater than 10^{4} m, where m denotes meters. The visible spectrum that we see with our eyes is a very small portion of the entire range of wavelengths. Lighter shading indicates a greater transparency of the Earth's atmosphere to cosmic radiation. It only penetrates the Earth's atmosphere at visible and radio wavelengths, respectively represented by the narrow and broad white areas. Electromagnetic radiation at short gamma ray, X-ray and ultraviolet wavelengths, represented by the dark areas, is absorbed in our atmosphere. The universe is now observed in these spectral regions from above the atmosphere in Earth-orbiting satellites

Table 2.2 The electromagnetic spectrum

Region	Wavelength range (m)
Radio	10^{-3}–10^{3}
Microwave	10^{-3}–1
Infrared	7×10^{-7}–10^{-3}
Visible	4×10^{-7}–7×10^{-7}
Ultraviolet	10^{-8}–4×10^{-7}
x-ray	10^{-11}–10^{-8}
Gamma ray	Less than 10^{-11}

glass vacuum tube (Röntgen 1896). He used the energetic x-rays to penetrate skin and muscle, detecting human bones and revolutionizing medicine.

The wavelength of *ultraviolet radiation*, abbreviated UV, is just a bit longer, between 10^{-8} and 4×10^{-7} m, with extreme ultraviolet radiation, denoted EUV, lying in the short wavelength part of this range. Most of the ultraviolet radiation from the Sun is absorbed in our air, but prolonged exposure to the amount that reaches the ground can burn your skin.

The infrared part of the electromagnetic spectrum is located at wavelengths between 7×10^{-7} and 10^{-3} m. The German-born English astronomer William Herschel (1738–1822) discovered infrared radiation when he put a beam of sunlight through a prism to spread it into its spectral components. He noticed that an unseen portion of sunlight warmed a thermometer placed beyond the red edge of the visible spectrum (Herschel 1800). The thermometer recorded higher temperatures in the invisible infrared sunlight than in normal visible sunlight. Herschel called them *calorific rays* because of the heat they generated. The term *infrared* did not appear until the late 19th century.

Humans "glow in the dark," emitting infrared radiation, but we cannot see the heat; it is outside our range of vision. Soldiers can locate the enemy at night by using night-vision goggles with infrared sensors that detect their heat, and spy satellites use infrared telescopes to detect heat radiated by rocket exhaust and by large concentrations of troops and vehicles.

Atmospheric molecules such as carbon dioxide and water vapor absorb infrared radiation. So the air that looks so transparent to our eyes is opaque to much of the infrared radiation coming from outer space. Telescopes located above part of the atmosphere, on the tops of mountains in dry climates, can catch some of the incoming infrared radiation before it is completely absorbed. The atmosphere similarly blocks the heat radiation from the Earth's surface, keeping it warmer than it would otherwise be. This warming of the ground is known as the greenhouse effect.

The atmosphere effectively absorbs most of the ultraviolet and infrared radiation from cosmic objects and all of their x-rays and gamma rays, which never reach the ground. To look at the universe at these invisible wavelengths, we must loft telescopes above the atmosphere. This was done first by using balloons and sounding rockets, followed by Earth-orbiting satellites with telescopes that view the cosmos at invisible ultraviolet, infrared, x-ray and gamma ray wavelengths.

Radio waves are between 0.001 and 1,000 m long, too long to enter the eye and not energetic enough to affect vision. The German physicist Heinrich Hertz (1857–1894) discovered *radio waves* by building equipment to both produce and detect the invisible electromagnetic signals (Hertz 1887). The unit of frequency $\nu = c/\lambda$ is now named the *Hertz* in his honor; this unit is abbreviated Hz. *Microwaves* have wavelengths in the short part of the radio-wave region, between 0.001 and 1.0 m.

Radio waves are the only type of invisible radiation that is not absorbed in the Earth's atmosphere. Radio waves even can pass through rain clouds; therefore, the radio universe can be observed on cloudy days and in stormy weather, just as a home or car radio works even when it is raining or snowing. Cosmic radio waves that are longer than about 10 m are nevertheless reflected by an ionized layer in the Earth's atmosphere, called the ionosphere; so these longer radio waves cannot reach the ground and must be observed from space.

2.3 Moving Perspectives

Motion changes our perspective, and observations depend on our relative motion with respect to the object being observed. These moving perspectives are described using inertial frames of reference, which move at a constant velocity, never accelerating or decelerating. The Dutch physicist Hendrik A. Lorentz (1853–1928) derived the coordinate transformation of Maxwell's equations from one inertial system to another, showing that the equations are invariant when subjected to this transformation. The Lorentz transformation utilizes a parameter γ, now known as the Lorentz factor, which is given by (Lorentz 1904):

$$\gamma = \left[1 - \frac{V^2}{c^2}\right]^{-\frac{1}{2}} = \left[1 - \beta^2\right]^{-\frac{1}{2}}, \tag{2.10}$$

where V is the relative velocity of the two inertial frames of reference, $\beta = V/c$ and $c = 2.99792458 \times 10^8$ m s^{-1} is the speed of light.

The German-born physicist Albert Einstein (1879–1955) generalized the Lorentz transformation in the *Principle of Relativity* (Einstein 1905a, b), which states that the laws of nature and the results of experiments performed in an inertial frame are independent of the uniform velocity of the system. Einstein additionally proposed that there exists in nature a limiting, invariant speed, the speed of light, c, now known as a universal constant.

The unvarying speed of light was first demonstrated in the late 19th century by the American physicist Albert A. Michelson (1852–1931), assisted by his friend the chemist Edward W. Morley (1838–1923), when they attempted to precisely measure how the speed of light depends on the Earth's motion through a hypothetical, space-filling medium, the ether, in which light waves were supposed to propagate and vibrate.

As the Earth moves through the stationary ether, a wind would be generated, and the observed speed of light would vary, like the speed of a sailboat going with or against the wind. But Michelson and Morley found that there was no detectable difference in the speed of light measured in the direction of the Earth's motion or at right angles to it (Michelson and Morley 1887). So the experiment meant that there was no light-carrying ether. It also implied that the speed of light is constant, exactly the same in all directions and at all seasons, and independent of the motion of the observer (Focus 2.3).

Focus 2.3 The Michelson-Morley experiment

Many experiments have been carried out to confirm the unvarying speed of light, but the most famous one was conducted in a basement laboratory at the Case School of Applied Science in Cleveland, Ohio in 1887, when the American scientists Albert A. Michelson and Edward W. Morley attempted to use an interferometer to precisely measure how the speed of light depends on the Earth's motion through space.

Scientists of that time firmly believed in an imaginary luminiferous ether, an invisible, frictionless, and unmoving medium that was supposed to permeate all of space. Its presence explained how light waves could travel at high speed through the apparent emptiness of space, providing the medium in which they propagate. Light was supposed to be transmitted in space by the vibrations of the hypothetical, invisible ether.

If the Earth moved through the stationary ether, a wind would be generated and the observed speed of light would vary, like the speed of an airplane moving with or against the wind. But Michelson and Morley found that there was no detectable difference in the interference pattern produced when a beam of light was sent into the ether wind in the direction of the Earth's motion or directed at right angles to it. Moreover, there was no difference in the measured speed of light when the Earth was traveling toward the Sun and away from it half a year later. That is, Michelson and Morley could measure no difference, Δc, in the speed of light, c, in two perpendicular paths of equal length, in the direction of the Earth's motion or transverse to it, with a precision of $\Delta c/c \leq 0.0001$ (Michelson 1881; Michelson and Morley 1887). Roy J. Kennedy, at the California Institute of Technology, subsequently refined the experiment and improved the measurement precision by a factor of ten (Kennedy 1926; Kennedy and Thorndike 1932).

So the Michelson-Morley experiment meant that there was no light-carrying ether. It also meant that the speed of light is always constant and everywhere the same. In 1907 Michelson was awarded the Nobel Prize in Physics for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid.

The speed of light, c, enters into Albert Einstein's (1879–1955) *Special Theory of Relativity* through the Lorentz factor $\gamma = [1 - (V/c)^2]^{-1/2}$ for an object moving at an observed velocity, V. We normally regard time as absolute and immutable, with nothing disturbing its relentless, steady tick. But for Einstein, time was relative and variable. In rapid travel, the rate at which time flows decreases, so moving clocks run slower by the factor γ. Lengths are diminished at high speed, shrinking in the direction of motion by the amount γ. At very high velocities, mass is also relative, and it increases with the speed by the same infamous γ factor.

In the *Special Relativity*, motions and events are described by coordinates in space (x, y, z) and time, t, within an inertial frame of reference that moves at a constant velocity. The length of an object moving with the reference frame of an observer is called the proper length, and the time read in a clock in that frame is the proper time.

If proper time, t, and time interval, Δt, between two events at one location are measured in system K, then the time interval, $\Delta t'$, between the events as measured in system K' moving with uniform velocity V is:

$$\Delta t' = \Delta t\sqrt{1 - \frac{V^2}{c^2}} = \frac{\Delta t}{\gamma}. \tag{2.11}$$

A moving clock will therefore appear to go slower to an observer in the moving system, which is known as *time dilation*.

Time dilation can prolong the decay time of fast-moving, unstable cosmic ray particles by several orders of magnitude, and noticeably lengthen the lifetime of elementary particles produced in man-made particle accelerators.

Atomic clocks have been flown around the world, first eastward and then westward, and compared with the time recorded by a reference atomic clock on the ground. As predicted by *Special Relativity*, the flying clocks lost time (aged slower) during the eastward trip, in the direction of the Earth's rotation, and gained time (aged faster) during the westward trip (Hafele and Keating 1972).

Lengths are also diminished at high speed, shrinking in the direction of motion. For proper spatial separation or length, Δx, in system K, there is a *Lorentz contraction* or shortening, $\Delta x'$, in the moving K' system given by:

$$\Delta x' = \Delta x \sqrt{1 - \frac{V^2}{c^2}} = \frac{\Delta x}{\gamma}. \qquad (2.12)$$

Thus, both space and time are relative in the *Special Theory of Relativity*.

Mass is also relative, for it increases with the speed. If a particle or object has rest mass, m_0, in a non-moving frame, the mass increases in the moving one to m' given by:

$$m' = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma m_0. \qquad (2.13)$$

The rest-mass energy $E = m_0 c^2$ increases in the moving frame to the energy E' given by:

$$E' = \frac{m_0 c^2}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma m_0 c^2. \qquad (2.14)$$

This expression has been verified in high-energy particle experiments that demonstrate that the energy of a subatomic particle can increase with its speed. The equation also shows that an infinite amount of work would be required to accelerate a particle to the speed of light, with $V = c$, implying that no physical object can move faster than the speed of light in an inertial frame. The mass grows without bound when an object moves as fast as light, and there is nothing that can propel it so fast.

Light is especially difficult to describe using this theory, for any specification of mass, size, or time intervals are undefined when moving at light's speed.

In the *Special Theory of Relativity*, which applies to the non-accelerating and non-gravitational laws of physics, distance is measured by a metric, or line element, ds, that combines space, x, y, z, and time, t. It was first proposed by Einstein's former teacher, Hermann Minkowski (1864–1909) and is given by Minkowski (1908):

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2, \qquad (2.15)$$

where the speed of light, c, is used to give the units of space from time and the all-important negative sign indicates time passing. In spherical coordinates r, θ, ϕ the metric ds is written

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \tag{2.16}$$

In this description, two events don't have a uniquely defined separation in either space or time. Instead, they are separated in space-time. So, the concepts of space and time are interwoven.

Space and time manage to join together in a detectable way when objects move exceptionally fast, approaching the speed of light. In these special circumstances, there is no space without time and no time without space; they are fused together. If the motion is fast enough, it will change the size and shape of things, or slow the passing of time. But these effects only become significant at exceptionally high speeds, close to the speed of light. We never encounter these experiences in normal circumstances, and they are not directly applicable to our everyday lives.

2.4 Thermal (Blackbody) Radiation

An ideal thermal radiator is known as a *blackbody*. By definition, a blackbody absorbs all the radiation that falls upon it and reflects none – hence the term black. A black shirt will similarly absorb most of the visible sunlight falling on it and reflects no colors.

Thermal radiation is emitted by a gas in thermal equilibrium, and arises by virtue of an object's heat, or temperature. A single temperature characterizes *thermal radiation*.

Any hot gas that is in thermal equilibrium, with a temperature above absolute zero, will attempt to radiate its energy away. The emission from such a thermal radiator is found at all wavelengths, or frequencies, but with a varying intensity that depends on the temperature (Fig. 2.3). As the temperature increases, more energy is radiated at all wavelengths. Moreover, the wavelength of maximum radiation shifts toward the shorter wavelengths when the temperature rises.

Since the emission of thermal radiation is present at all wavelengths, astronomers say it emits a continuum spectrum. A display of its radiation intensity as a function of wavelength, known as the spectrum, shows no gaps, breaks or sudden increases or decreases. It is an unbroken continuum ascending to peak intensity and then dropping again as the wavelength increases.

No real object emits a perfect thermal, or blackbody, spectrum, but the Sun shines with roughly such a spectrum. It closely matches the radiation spectrum of a blackbody at a temperature of 5,780 K.

The German physicist Max Planck (1858–1947) derived the formula for the spectrum of a perfect absorber, or blackbody, introducing the idea that it radiates energy in fundamental indivisible units, which he called *quanta*, whose energy is

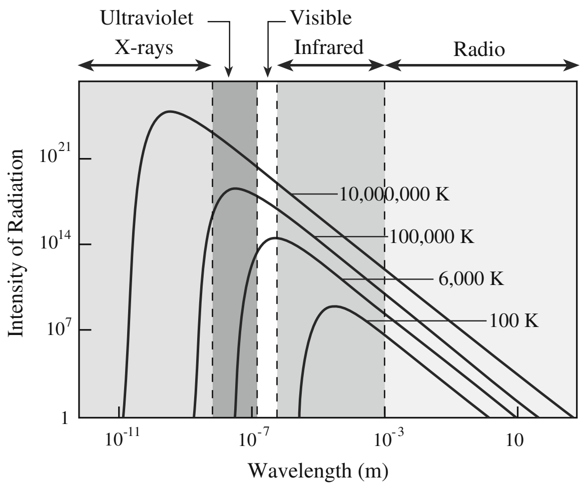

Fig. 2.3 Blackbody radiation The spectral plot of blackbody radiation intensity as a function of wavelength depends on the temperature of the gas emitting the radiation. The German physicist Max Planck (1858–1947) derived the formula that describes the shape and peak of this spectrum in 1900. He proposed that the radiation energy was quantized, which provided a foundation for quantum theory. At higher temperatures the wavelength of peak emission shifts to shorter wavelengths, and the thermal radiation intensity becomes greater at all wavelengths. At a temperature of 6,000 degrees on the kelvin scale, or 6,000 K, the thermal radiation peaks in the visible, or V, band of wavelengths. A hot gas with a temperature of 100,000 K emits most of its thermal radiation at ultraviolet, or UV, wavelengths, whereas the emission peaks in X-rays when the temperature is 1 million to 10 million K

proportional to the frequency of the radiation (Planck 1901, 1910, 1913). The constant of proportionality between the frequency and energy of the radiation is now known as the Planck constant, designated by the lower case letter h. It has a value of $h = 6.626\ 069\ 57 \times 10^{-34}$ J s, or about $h \approx 6.626 \times 10^{-34}$ J s. This marked the beginning of quantum physics, whose history is discussed by Kragh (2002).

Planck found that a blackbody with temperature T emits a *continuum spectrum* of radiation characterized by a brightness distribution, $B_v(T)$, which depends only on the frequency v and temperature T and is given by:

$$B_v(T) = \frac{2hv^3}{c^2} \frac{1}{\exp[hv/(kT) - 1]} \text{ J s}^{-1}\text{ m}^{-2}\text{ Hz}^{-1}\text{ steradian}^{-1} \quad (2.17)$$

where the Planck constant $h \approx 6.626 \times 10^{-34}$ J s, the Boltzmann constant $k \approx 1.381 \times 10^{-23}$ J K^{-1}, and a steradian is the dimensionless SI unit of solid angle, which is related to the area an angle cuts out. The solid angle of a full sphere is 4π and that of a hemisphere is 2π, where $\pi = 3.13159$.

The Planck distribution can also be written per unit wavelength, $B_\lambda(T)$, where the wavelength $\lambda = c/v$ and:

$$B_\lambda(T) = B_\nu\left|\frac{d\nu}{d\lambda}\right| = B_\nu(T)\frac{c}{\lambda^2} = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left[\left(\frac{hc}{\lambda kT}\right)-1\right]}\ \mathrm{J\,s^{-1}\,m^{-2}\,m^{-1}\,steradian^{-1}}. \tag{2.18}$$

The blackbody spectrum is markedly asymmetric. It falls off very rapidly with decreasing wavelength on the short wavelength side of the maximum and decreases gradually with increasing wavelength at long wavelengths. At short wavelengths, or high frequencies, we have the so-called Wien tail of the distribution, derived by the German physicist Wilhelm Wien (1864–1928) near the end of the 19th century. It is given by (Wien 1893):

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{kT}\right)\ \text{for}\ h\nu \gg kT. \tag{2.19}$$

Two English physicists, Lord Rayleigh (John Strutt, 1842–1919) and James Jeans (1877–1946) derived an expression for the brightness of thermal radiation at long wavelengths, or low frequencies. For wavelength λ this *Rayleigh-Jeans law* is (Rayleigh 1900, 1905; Jeans 1905, 1909):

$$B_\lambda(T) = \frac{2ckT}{\lambda^4}\ \text{for}\ hc \ll \lambda kT \tag{2.20}$$

or at frequency ν:

$$B_\nu(T) = \frac{2\nu^2 kT}{c^2}\ \text{for}\ h\nu \ll kT, \tag{2.21}$$

These equations are applicable at radio wavelengths or frequencies for most temperatures.

The *Rayleigh–Jeans law* agrees with experimental results at large wavelengths, with $\lambda \gg hc/(kT)$, or, equivalently, at low frequencies $\nu \ll kT/h$, but strongly disagrees at the short ultraviolet wavelengths (or high frequencies). This inconsistency between observations and the predictions of classical physics is commonly known as the *ultraviolet catastrophe*; Planck (1901) explained the inconsistency when he introduced radiation quanta.

The blackbody, or thermal, spectrum has a maximum intensity at a wavelength, $\lambda_{\max}$, which can be found by taking the derivative of the Planck distribution and setting the equation to zero, or from $dB_\lambda(T)/d\lambda = 0$, giving:

$$\lambda_{\max} = \frac{b}{T} = \frac{0.00289777}{T}\ \text{meters} \approx \frac{0.0029}{T}\ \text{meters}, \tag{2.22}$$

where b is the Wien displacement constant and T is the temperature on the kelvin scale, abbreviated K. This expression is called the *Wien displacement law*, after the German physicist Wilhelm Wien (1864–1928) who formulated the relationship based on a thermodynamic argument (Wien 1893). It is known as a displacement law because the wavelength peak $\lambda_{\max}$ is displaced when the temperature, T, is changed. The expression indicates that colder objects radiate most of their energy at longer wavelengths, and that hotter objects are most luminous at shorter wavelengths. In other words, as the temperature of a gas increases, most of its thermal radiation is emitted at shorter and shorter wavelengths.

Example: The most intense thermal radiation at different temperatures

The Sun radiates its most intense radiation in the visible colors. At an orange wavelength of $\lambda = 500$ nm, the effective temperature of the Sun's photosphere is $T = 0.0029/(5.00 \times 10^{-7}) \approx 5{,}800$ K. The average body temperature of a human is about $T = 310$ K. From the Wien displacement law, the wavelength of maximum thermal radiation at this temperature is $\lambda_{\max} = 0.0029/310 \approx 9.35 \times 10^{-6}$ m, corresponding to infrared wavelengths. This heat radiation can be detected by rattlesnakes and by night-vision goggles. The primary mirror of the *Spitzer Space Telescope* has a diameter of $D_T = 0.85$ m, and its angular resolution θ at this infrared wavelength is $\theta = \lambda/D_T \approx 1.1 \times 10^{-5}$ rad $\approx$ 2.27″, where 1 rad $= 2.06265 \times 10^5$ ″. Suppose this telescope was pointed down at the ground to act as a spy satellite from a geosynchronous orbit where the orbital period is equal to the Earth's rotation period of 24 h. The semi-major axis of such an orbit is equal to $a = 42{,}164$ km, so the altitude H above the ground in $H = a - R_E = 3.579 \times 10^7$ m, where the radius of the Earth is $R_E = 6{,}371$ km (see Sect. 4.1). The smallest feature this telescope could resolve on the ground would have a linear size of $L = H \times \theta \approx 394$ m, bigger than a human but comparable to a convoy of vehicles. An x-ray telescope operating at a wavelength of $\lambda = 1.24 \times 10^{-9}$ m would detect the thermal radiation of a gas at a temperature of $T = 0.0029/\lambda \approx 2.3 \times 10^6$ K, or about 2 million K. In contrast, the cosmic microwave background radiation has a temperature of $T = 2.725$ K, and the wavelength at which its emission is most intense is $\lambda_{\max} = 0.0029/2.725 \approx 0.001$ m or 1 mm.

The Wien displacement law helps explain why stars have different colors. Since red wavelengths, at about 660 nm, are longer than blue wavelengths, at around 470 nm, you would expect that the visible disk of a red star would be cooler than the disk of a blue star. The Wien displacement law yields effective disk temperatures of about 4,400 K for the red star and roughly 6,200 K for the blue star.

However, the radiation from exceptionally hot stars, which peaks at short, invisible ultraviolet wavelengths, also enhances the radiation intensity at adjacent blue wavelengths. A star that is most intense at an unseen ultraviolet wavelength of 30 nm might have a disk temperature as great as 100,000 K, and such a star will also emit more radiation in blue visible light than a cooler star (Fig. 2.4). Careful spectral calibration of stellar colors indicates that blue stars, of spectral class O, can indeed have disk temperatures as high as 280,000 K, while the red stars of spectral class M can be about 100 time cooler, at 2,800 K (Sect. 10.10).

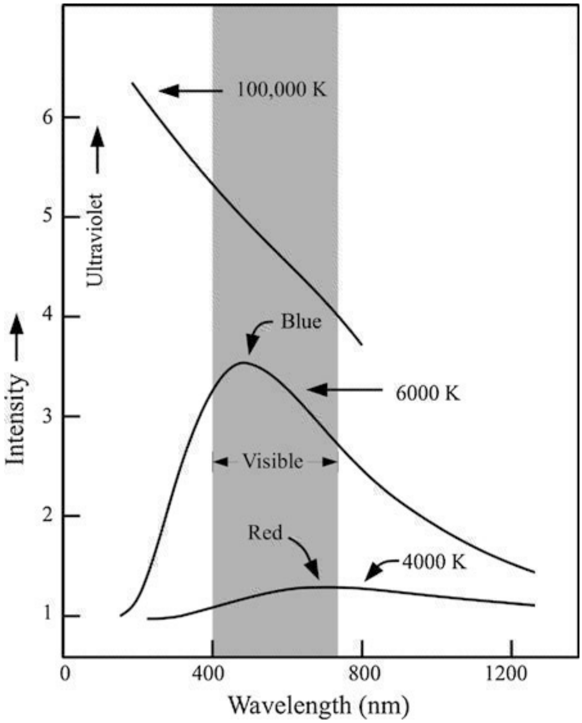

Fig. 2.4 Ultraviolet overflow The continuum spectrum of a star's thermal radiation changes with the effective temperature of the stellar disk, and this results in different star colors within the visible range of wavelengths, from 400 to 700 nm (*middle*). They range from blue stars, at relatively short visible wavelengths to red stars, at the longer wavelengths detected by our eyes. The thermal radiation of a star with an effective disk temperature of about 4,000 degrees on the kelvin scale, denoted K, peaks at the red wavelengths, and a hotter star with a temperature of about 6,000 K emits its most intense emission at blue wavelengths. A much hotter star at 100,000 K will be most intense at invisible ultraviolet wavelengths (*left*), but because the total energy emitted by a star increases dramatically with temperature, the very hot star will also appear bright at blue wavelengths

In terms of frequency, Wien's displacement law for the maximum frequency, $\nu_{\max}$, is determined from $dB_\nu(T)/d\nu = 0$ and is given by: $\nu_{\max} \approx 2.8\,kT/h \approx 5.8 \times 10^{10}\,T\,\text{Hz}$.

Because the spectrum of blackbody radiation per unit frequency interval, $B_\nu(T)$, differs from the Planck distribution per unit wavelength, $B_\lambda(T)$, the $\nu_{\max}$ does not equal $c/\lambda_{\max}$.

The energy density, $u_\nu(T)$, of blackbody radiation, per unit frequency interval, is

$$u_\nu(T) = \frac{4\pi}{c} B_\nu(T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\exp\left[\left(\frac{h\nu}{kT}\right) - 1\right]}. \tag{2.23}$$

Table 2.3 Radiation constants

a = Radiation density constant = $8\pi^5 k^4/(15c^3h^3) = 4\sigma/c = 7.5657 \times 10^{-16}$ J K^{-4} m^{-3}	
σ = Stefan-Boltzmann constant = $2\pi^5 k^4/(15c^2h^3) = ac/4 = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}	
c_1 = First radiation constant = $2\pi hc^2 = 3.741771 \times 10^{-16}$ J s^{-1} m^2	
c_2 = Second radiation constant = hc/k = 0.0143877 m K	
$b = \lambda_{\max} T$ = Wien displacement law constant = 0.002897768 m K ≈ 0.002898 m K	

The energy density has SI units of J m^{-3} Hz^{-1}. The radiation is isotropic, or the same in all directions, and the solid angle of a full sphere is 4π sr, where the constant $\pi \approx 3.14159$. When this expression is integrated over all frequencies, we obtain the total energy density, u, of a blackbody:

$$u = \int_0^\infty u_\nu(T) d\nu = aT^4, \tag{2.24}$$

where the radiation constant a is given by

$$a = \frac{8\pi^5 k^4}{15c^3h^3} \approx 7.57 \times 10^{-16}\ \mathrm{J\,K^{-4}\,m^{-3}} \tag{2.25}$$

The radiant flux, $f_\nu(T)$, of energy flowing out of the blackbody over π sr, or over the hemisphere facing an observer, is

$$f_\nu(T) = \pi B_\nu(T)\ \mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}, \tag{2.26}$$

in units of energy per unit time per unit area per unit frequency interval. The radiant flux is what is observed from astronomical objects. When integrating the flux over all frequencies one obtains the total radiant output per unit area, f, given by:

$$f = \int_0^\infty f_\nu(T) d\nu = \int_0^\infty \pi B_\nu(T)\, d\nu = \frac{ac}{4} T^4 = \sigma T^4, \tag{2.27}$$

where the Stefan-Boltzmann constant, σ, is given by: $\sigma = ac/4 = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}.

This and other radiation constants are given in Table 2.3.

We can add up, or integrate, the contributions to the blackbody spectrum at every wavelength to obtain the total luminosity of a thermal radiator. This results in the Stefan-Boltzmann law in which the luminosity increases with the square of the radius and the fourth power of the effective temperature. Luminosity is intrinsic to a star, establishing its power and energy output per unit time.

The Stefan-Boltzmann law states that the total power, or intrinsic luminosity L, at the visible disk of a star or other astronomical object with radius, R, and effective temperature, T_{eff}, is:

$$L = 4\pi\sigma R^2 T_{\mathrm{eff}}^4, \tag{2.28}$$

where $\pi = 3.1416$ and the Stefan-Boltzmann constant $\sigma = 2\pi^5 k^4/(15c^2h^3) = 5.6704 \times 10^{-8}\,\mathrm{J\,s^{-1}\,m^{-2}\,K^4}$. The effective temperature, T_{eff}, is the disk temperature that the object would have if it were a perfect blackbody radiating at luminosity L.

The unit of energy is the joule, and the unit of luminosity is joule per second, abbreviated J $\mathrm{s^{-1}}$. Power is often expressed in units of watts, where 1 watt = 1 W = 1 J $\mathrm{s^{-1}}$.

The Stefan-Boltzmann law indicates that at a given effective temperature, bigger stars have a greater luminosity than smaller stars, and at the same size, hotter stars are intrinsically more luminous than cooler stars. The Austrian physicist Joseph Stefan (1835–1893) obtained this law using experimental measurements made by the English physicist John Tyndall (1820–1893), and Stefan's student Ludwig Boltzmann (1844–1906) derived it from theoretical considerations, using thermodynamics (Stefan 1879; Boltzmann 1872).

The intensity of radiation striking a unit area decreases as the radiation spreads out into an increasing volume. The area of an imaginary sphere located at a distance, D, from the Sun or any other star is given by $4\pi D^2$, so the intensity per unit area, designated by l, is given by $l = L/(4\pi D^2)$, which falls off as the inverse square of the distance. You can notice this effect when watching the increased brightness of a car's headlight when the car approaches you and its distance decreases, or when watching the car's taillights dim as it moves away to greater distance.

The radiant flux, f, of a blackbody, thermal radiator of radius R and absolute luminosity, L, and temperature T, observed at a distance, D, is

$$f = \frac{L}{4\pi D^2} = \frac{\sigma R^2 T_{\mathrm{eff}}^4}{D^2}. \tag{2.29}$$

2.5 How Far Away is the Sun, and How Bright, Big and Hot is it?

2.5.1 *Distance of the Sun*

How far away is the Sun? The mean distance separating the Earth and the Sun is known as the astronomical unit, abbreviated AU, and it provides the crucial unit of planetary distance. Yet, for a very long time no one knew exactly how big it was. We now know that it is about 149.6 million km.

By the end of the 17th century, astronomers and other scientists had a good understanding of how the planets move around the Sun, but they could produce a scale model of the solar system that only provided relative distances of the planets from the Sun. The true distances and speeds of motion of the planets remained unknown.

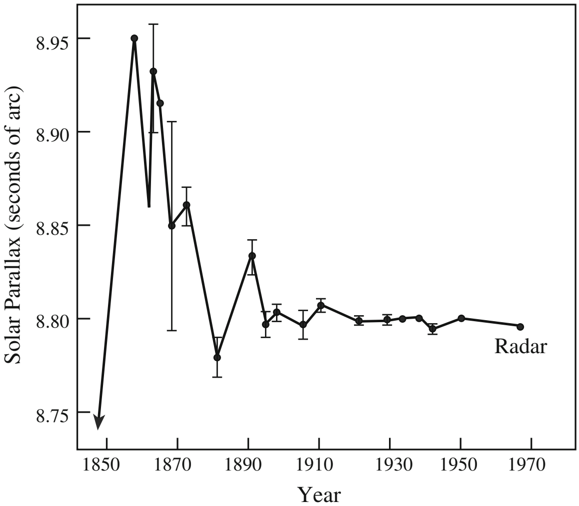

Fig. 2.5 Distance to the Sun Values of the solar parallax obtained from measurements of the parallaxes of Venus, Mars, and the asteroid Eros between 1850 and 1970. The solar parallax, designated by $\pi_{\odot}$, is half the angular displacement of the Sun viewed from opposite sides of the Earth. The error bars denote the probable errors in the determination; the points for 1941, 1950 and 1965 all have errors smaller than the plotted points. In the 1960s, the newly developed radar (i.e., radio detection and ranging) technology enabled determination of the Sun's distance with an accuracy of about 1,000 m. The radar value of the solar parallax is 8.79405 s of arc

It is no wonder then that obtaining a precise value for the Sun−Earth distance played an important role in the astronomy of the 18th and 19th century. The quest for accurately measuring that distance involved hundreds of trips to remote countries, tens of thousands of observations and photographs, and the lifetime work of several astronomers. They first determined the separations of the Earth and a nearby planet, such as Venus or Mars, and then used this planetary distance to infer the separation of the Earth and the Sun.

The distance of a nearby planet can be estimated by measuring the angular separation in the apparent direction of the planet when observed simultaneously from two widely separated locations on the Earth. This angle is known as the *parallax*, from the Greek *parallaxis*, meaning the "value of an angle." If both the parallax and the separation between the two observers are known, then the distance of a planet can be determined by triangulation. This is based on the geometric fact that if we know the length of one side of a triangle and the angles of the two corners, then all of the other dimensions can be calculated.

Since angular measurements were involved, the astronomical unit was naturally specified by an angle called the solar parallax, which is defined as half the angular separation of the Sun as viewed from opposite sides of the Earth. More than a century of estimates for the solar parallax are shown in Fig. 2.5 and discussed in Focus 2.4 – also see Hirshfeld (2001) and Van Helden (1985).

Focus 2.4 The solar parallax and the Sun's distance

The distance separating the Earth and the Sun, known as the astronomical unit or AU for short, is determined by first estimating the distance between the Earth and a nearby planet. This planetary distance then can be used to specify the AU. The distance of Venus from the Sun, for example, is equal to one half of the distance between the Earth and Venus when it is closest and farthest away, on the other side of the Sun. When the Venus−Sun distance is known, we can infer the distance of any other planet from the Sun using Kepler's third law (see Sects. 3.1, 3.2), which relates the orbital periods and orbital distances of the planets.

For more than a century, the distances of Venus and Mars were determined by triangulation from different points on the Earth. It involved measurements of the parallax, or angular difference in the apparent direction of the planet, as observed from widely separated locations. The solar parallax, designated by the symbol $\pi_{\odot}$ was then inferred. It is defined mathematically by $\sin \pi_{\odot} = R_E/AU$, where the equatorial radius of the Earth is $R_E = 6.378 \times 10^8$ m. The ratio of R_E and the AU provided an angle in radian units, and one radian is equivalent to $2.06265 \times 10^5\ ''$ where the symbol $''$ denotes a second of arc or an arc second.

In 1672, Giovanni Domenico Cassini (1625–1712), an Italian astronomer and the first director of the Paris Observatory, obtained an early triangulation of Mars, combining his observations from Paris with those taken by his colleague Jean Richer (1630–1696) from Cayenne, French Guiana. The planet was then in opposition, at its closest approach to the Earth. From the two sets of observations of Mars, made from opposite sides of the Earth and about 7,200 km apart, it was possible to estimate the distance to Mars and to infer an approximate value of 9.5″ for the solar parallax (Van Helden 1985).

Astronomers in the 18th and 19th century attempted to improve the measurement accuracy of the Sun's distance during the rare occasions when Venus crossed the face of the Sun in 1761 and 1769, with an estimate for the solar parallax of $8.57'' \pm 0.04''$ and in 1874 and 1882 with a wide range of results between 8.76″ and 8.88″ from world-wide observations. The method also involved comparison of observations from widely separated locations to determine the distance by triangulation.

In 1877, David Gill (1843–1914), an unemployed Scottish astronomer with no university degree, traveled to the small island of Ascension near the equator where he could use the Earth's rotation to view the near approach of Mars from different directions, obtaining a solar parallax of $8.78'' \pm 0.01''$.

Subsequent determinations of the distance to the nearby asteroid named 433 Eros, during its closest approaches to the Earth in 1900–1901 and 1930–1931, resulted in respective estimates for the solar parallax of $8.807'' \pm 0.0027''$ and $8.790'' \pm 0.001''$.

Significant improvements in the precision of planetary distances came in the late 1960s by bouncing pulsed radio waves off of Venus and timing the echo. The round-trip travel time – about 276 s when Venus is closest to the Earth – was measured using atomic clocks, and a precise distance to Venus then was obtained by multiplying half of the round-trip time by the speed of light.

The distance of Venus from the Sun is equal to one half of the difference between the Earth and Venus when it is closest and farthest away, on the other side of the Sun. The resulting radar value for the solar parallax was 8.79405″. The corresponding value of the astronomical unit, inferred from the radar determination of the distance of Venus, is 149,597,870 km, with an accuracy of about 1 km, or for the accuracy required in most astronomical calculations 1 AU $= 1.496 \times 10^{11}$ m (Ash et al. 1967; Muhleman 1969).

Nowadays the accuracy of the mean Earth−Sun distance is fixed by the exact value for the speed of light. The Earth–Sun light travel time, τ_{AU} – or the time for light to travel across 1 AU – is given as a primary astronomical constant and has the value $\tau_{AU} = 499.0047863852$ s, with a derived value for the mean Earth-Sun distance of $1\text{AU} = c\tau_{AU} = 1.495978707 \times 10^{11}$ m, where the speed of light $c = 299792458$ m s^{-1}. The derived value of the solar parallax is $\pi_{\odot} = 8.7941433''$ where ″ denotes second of arc.

The time for light to travel from the Sun to the Earth is used now as a primary astronomical constant. It is approximately 499 s, which corresponds to an AU of about 149.6 million km or 1.496×10^{11} m, and approximately 10,000 times the diameter of the Earth. Once scientist's determined the Sun's distance, they could determine the Earth's mean orbital velocity, by assuming – to a first approximation – a circular orbit and dividing the Earth's orbital circumference by its orbital period of $P_E =$ one year $= 3.1557 \times 10^7$ s. The Earth's velocity is $2\pi\ \text{AU}/P_E =$ 29,800 m s^{-1}, which is equivalent to approximately 107,000 km per hour, much faster than the fastest airplane or car.

By way of comparison, the light travel time from the next nearest star other than the Sun to the Earth is 4.24 years, or approximately 134 million s. This star is called Proxima Centauri and it is located at a distance of 4.01×10^{16} m and 268,000 AU.

Therefore, the Sun is about a quarter million times closer to the Earth than the next nearest star. Because of this closeness, the Sun is approximately 100 billion times brighter than any other star. This brilliance and proximity permit detailed investigations that are not possible for any other star. As a result, studies of the Sun provide the foundation and benchmark for an understanding of other stars.

Although our unaided eyes can see about six thousand stars in the night sky, and telescopes reveal hundreds of billions of them in the Milky Way, our own daytime star, the Sun, is a special star. It is the source of all our power. Its radiation energizes our planet, warms the ground and sea, lights our days, strengthens our bodies, and sustains life on Earth.

The life-sustaining Sun also links us to the other stars, and to understand how the Sun or any other star operates, we must examine their radiation, which spreads out and carries energy in all directions.

2.5.2 How Big is the Sun?

Any incandescent body shines because it is hot. The wire filament in an incandescent light bulb is, for example, heated to a white-hot temperature of about 3,000 K to produce its luminous glow. As it turns out, the visible solar disk is just about twice that hot, and it owes its much greater luminosity to its vastly larger size.

The solar radius, denoted $R_{\odot}$, can be determined from observations of the Sun's angular size and distance, and these measurements indicate that the solar radius $R_{\odot} = 6.955 \times 10^8$ m, which is 109 times the radius of the Earth (see Sect. 1.4).

2.5.3 The Unit of Energy

The joule is the unit of energy in the International System of units, abbreviated by SI from the French Système International d'unitès. The SI unit of energy is named after the English physicist James Prescott Joule (1818–1889), who described the relationship of heat to mechanical work (Joule 1847), leading to the theory of conservation of energy. When an SI unit is spelled out in English, it begins with a lower case letter, like joule, but it is abbreviated with a capital version of the first letter, such as J.

A joule is the work required to produce one watt of power for one second, so a power of 1 J s^{-1} is equivalent to one watt.

One joule is twice the kinetic energy of a mass of one kilogram, abbreviated 1 kg, moving at a speed of one meter per second, or 1 m s^{-1}. This amount of energy is a very small number as far as the mass and speed of cosmic objects are concerned. The Sun, for example, has a mass of about 2,000 billion billion billion kg, or 2×10^{30} kg, and moves though space at a speed of about 220,000 m s^{-1}.

Even an ordinary table lamp with a 100 watt light bulb uses just 100 J s^{-1}, whereas the Sun liberates a lot more power, some 382.8 million billion billion J s^{-1} written 3.828×10^{26} J s^{-1}.

2.5.4 The Sun's Luminosity

The Sun emits radiation in all directions, and as the solar radiation spreads out into space, it is dispersed into an ever-increasing volume. The distant Earth therefore collects only a small fraction of the total energy radiated by the Sun. The solar constant specifies the amount of the Sun's radiation that arrives at our planet. It is denoted by the symbol $f_\odot$, and is precisely defined as the total amount of radiant solar energy per unit time per unit area reaching the top of the Earth's atmosphere at the Earth's mean distance from the Sun. (Any physical parameter of the Sun is denoted by a subscript $\odot$, a circle with a dot at the center.)

Artificial satellites have been used to accurately measure the Sun's total irradiance, or radiant flux, just outside the Earth's atmosphere, establishing the value of the solar constant (Kopp et al. 2005):

$$f_\odot = 1{,}361\,\mathrm{J\,s^{-1}\,m^{-2}}. \tag{2.30}$$

We can use the solar constant and Earth-Sun distance to determine the total amount of energy radiated by the Sun every second. At the Earth's mean distance of 1 AU from the Sun, the solar radiation per unit area is diminished by $4\pi\,(\mathrm{AU})^2$, the surface area of a sphere at this distance. We therefore infer the Sun's luminosity, denoted $L_\odot$, by multiplying the solar constant with this area to obtain:

$$L_\odot = 4\pi f_\odot (\mathrm{AU})^2 = 3.828 \times 10^{26}\,\mathrm{J\,s^{-1}}. \tag{2.31}$$

where 1 AU = 1.496×10^{11} m.

2.5.5 Taking the Sun's Temperature

The Sun, like any incandescent body, shines because it is hot. How hot? Once we know the radius and luminosity of the Sun, we can determine the temperature of the Sun's visible disk. Using the Stefan-Boltzmann law, the effective temperature, $T_{\mathrm{eff}\odot}$, of the visible solar disk is given by:

$$T_{\mathrm{eff}\odot} = \left[\frac{L_\odot}{4\pi\sigma R_\odot^2}\right]^{1/4} \approx 5{,}780\,\mathrm{K}, \tag{2.32}$$

where the Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ J m K^{-1} s^{-1}, and the Sun's radius is $R_\odot = 6.955 \times 10^8$ m. At this temperature, all elements in the Sun are present in gaseous form. The Sun is only about twice as hot as the wire filament in an incandescent light bulb, so its much greater luminosity is due to its vastly larger size.

Astronomers use the kelvin temperature scale that starts from absolute zero, the temperature at which atoms and molecules cease to move. The unit for this scale is written kelvin, without a capital K, or just denoted by a capital K. Water freezes at

273 K and boils at 373 K, and to convert to degrees Celsius, abbreviated by C, just subtract 273, or C = K − 273. The conversion to degrees Fahrenheit, denoted by F, is more complicated, with F = (9 K/5) − 459.4.

2.5.6 How Hot are the Planets?

Solar radiation warms a planet's surface, and as we would expect, the heat is greatest for objects that are closest to the Sun. That is because the intensity of sunlight falls off as the inverse square of distance from the Sun.

We can make an initial estimate for the temperature of a planet by assuming that the surface of a terrestrial planet or the cloud tops of a giant planet are not noticeably warmed by heat rising from the planet's interior and that there is no atmosphere above them. The planet is then heated solely by the Sun's radiation, and we can calculate the planet's effective temperature, T_{ep}, from the relation $T_{ep} = 279\ (\mathrm{AU}/D_p)^{1/2}$ K, where D_p is the planet's distance from the Sun and the mean distance between the Earth and the Sun is 1 AU = 1.496×10^{11} m.

To derive this expression, notice that the radiant energy per unit time, L_P that a planet of radius R_P receives from the Sun is:

$$L_P = \pi R_P^2 f = \frac{R_p^2 L_\odot}{4D_p^2} = \pi R_P^2 \frac{\sigma R_\odot^2 T_{e\odot}^4}{D_P^2}\ \mathrm{J\,s^{-1}}, \qquad (2.33)$$

where R_P is the radius of the planet, f is the total amount of radiant solar energy perunit time per unit area reaching the top of the planet's atmosphere, D_P is the planet's distance from the Sun, the Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ J m^{-2} K^{-4} s^{-1}, the solar radius $R_\odot = 6.955 \times 10^8$ m, and the effective temperature of the visible solar disk is $T_{e\odot} = 5{,}780$ K.

The Stefan-Boltzmann law can also be applied to a planet, giving its radiant luminosity, L_P, or the amount of radiation energy lost per unit time, as a function of its radius and effective temperature, T_{ep}.

$$L_P = 4\pi\sigma R_P^2 T_{ep}^4\ \mathrm{J\,s^{-1}}. \qquad (2.34)$$

Assuming thermal equilibrium between energy lost and received, so $L_P = \pi R_P^2 f$, and combining equations we obtain:

$$T_{ep}^4 = \frac{L_p}{4\pi\sigma R_p^2} = \frac{L_\odot}{16\pi D_P^2 \sigma} = \frac{R_\odot^2 T_{e\odot}^4}{4\,D_P^2} \approx 1.35 \times 10^{32}\left(\frac{1}{D_P^2}\right) \approx 6.03 \times 10^9 \left(\frac{\mathrm{AU}}{D_P}\right)^2, \qquad (2.35)$$

or

$$T_{ep} \approx 279 \left(\frac{AU}{D_P} \right)^{1/2} K, \quad (2.36)$$

where 1 AU = 1.496×10^{11} m is the mean distance of the Earth from the Sun.

Notice that the effective temperature is independent of the planet's radius, and that the effective temperature for planets around other stars depends upon the star's disk temperature and the square root of the star's radius, as well as the planet's distance from the star, or on the star's absolute luminosity and the planet's distance from the star. This is of interest in determining the habitable zone, in which the planet surface temperature might permit liquid water, at temperatures between 273 and 373 K; it is located closer to a star that is less luminous.

This expression assumes that all of the sunlight falling on the planet is absorbed, but some of it is always reflected. The extent to which a planet or satellite reflects light from the Sun is specified by its albedo, A, the percentage of reflected light. The visual albedo measures the fraction of incoming visible sunlight that is reflected directly into space, on a scale of 0.0–1.0. Rocky bodies like the planet Mercury or the Earth's Moon absorb a lot of incident sunlight, while clouds or icy surfaces reflect it. Thus, the Moon and Mercury have a visual albedo of 0.12, while cloud-covered Venus has an albedo of 0.65, helping to make it the brightest planet we detect with our eyes.

Taking the albedo, A, into account, we have:

$$T_{ep} = 279(1 - A)^{1/4} \left(\frac{AU}{D_P} \right)^{1/2} K. \quad (2.37)$$

There are two kinds of albedo, the Bond albedo (Bond 1863), which measures the total proportion of electromagnetic energy reflected, and the visual geometric

Table 2.4 Distances, visual albedos, effective temperatures, and mean temperatures of the planets[a]

Planet	Average distance, D_P (AU)	Visual geometric albedo, A	Effective temperature, T_{eff} (K)	Mean Temperature[b] (K)
Mercury	0.387	0.106	436	440
Venus	0.723	0.65	252	730
Earth	1.000	0.367	249	281
Mars	1.524	0.150	217	210
Jupiter	5.203	0.52	102	165
Saturn	9.537	0.47	77	134
Uranus	19.19	0.51	53	76
Neptune	30.07	0.41	45	73

[a] Distance and mean temperature from the Jet Propulsion Laboratory. Effective temperatures are calculated from the visual geometric albedos, which are from http://ssd.jpl.nasa.gov

[b] The mean surface temperatures for the terrestrial planets and the mean cloud-top temperatures for the giant planets

albedo that refers only to electromagnetic radiation in the visible spectrum. The geometric albedo of an astronomical body is the ratio of its actual brightness to that of an idealized flat, fully and isotropically reflecting disk with the same cross-sectional area. The Bond albedos for Mercury, Venus, Earth and Mars are 0.119, 0.75, 0.29, and 0.16, respectively, while their visual geometric albedos are 0.106, 0.65, 0.367, and 0.150. When our formula is applied to the Earth we obtain T_{ep} (Earth) $\approx$ 256 K using the Bond albedo and T_{ep} (Earth) $\approx$ 249 K using the visual geometric albedo. The Bond albedo for the Earth's Moon is 0.123, so its effective temperature would be higher, at about 270 K.

The effective temperatures of the planets are compared to their mean observed surface or cloud-top temperatures in Table 2.4. The surface of Venus is much hotter than expected, and the surface of the Earth is somewhat hotter, both a consequence of the greenhouse effect (Focus 2.5). The giant planets are also hotter, due to the heat left over from their formation or to helium raining down inside them.

Focus 2.5 Global warming by the greenhouse effect

The surface temperature of a terrestrial planet can increase when its atmosphere traps heat near the surface, warming it to a higher temperature than would be achieved by the Sun's radiation in the absence of an atmosphere. Incoming sunlight is partly reflected by clouds, but the rest passes through the atmosphere to warm the planet's surface. Much of the surface heat is re-radiated in the form of long infrared waves that are absorbed by atmospheric molecules such as carbon dioxide or water vapor. Some of the trapped heat is re-radiated downward to warm the planet's surface and the air immediately above it. The atmosphere thus acts as a one-way filter, allowing the warmth of sunlight in, and holding it close to the planet's surface and elevating the temperature there.

The idea that our atmospheric blanket might warm the Earth was suggested by the French mathematician Jean-Baptiste Fourier (1768–1830) and developed by the Irish scientist John Tyndall (1820–1893). Fourier wondered how the Sun's heat could be retained to keep the Earth hot, concluding that sunlight passes through the atmosphere, which also prevents the escape of heat from the planet's surface (Fourier 1824, 1827).

Tyndall built an instrument to measure the heat-trapping properties of various gases, examining the transmission of infrared radiation through them. He found that the main constituents of our atmosphere – oxygen and nitrogen – were transparent to both visible and infrared radiation. Oxygen molecules, denoted O_2, account for 21 % of our atmosphere, while nitrogen molecules, designated N_2, accounts for 78 %. These diatomic, or two-atom, molecules are incapable of absorbing any noticeable amounts of infrared heat radiation.

Tyndall also found that water vapor, designated H_2O, and carbon dioxide, denoted CO_2, absorb significant heat even though they are minor ingredients of the Earth's atmosphere (Tyndall 1861, 1863). As Tyndall realized, these gases are transparent to sunlight, which warms the ground, but partially opaque to the infrared rays, which are trapped near the surface and warm our globe. Water vapor and carbon dioxide molecules consist of three atoms and are more flexible and free to move in more ways than diatomic molecules, so they absorb the heat radiation.

Global warming by heat-trapping gases in the air is now known as the *greenhouse effect*, but this is a misnomer. The air inside a garden greenhouse is heated because it is enclosed, preventing the circulation of air currents that would carry away heat and cool the interior. Nevertheless, the term is now so common that we continue to use it to designate the process by which an atmosphere traps heat near a planet's surface.

As Tyndall pointed out, our environment would be much colder at nighttime in the absence of the greenhouse effect, and the Earth might otherwise be covered with frost. The warming is crucial to life on Earth. If the Earth had no atmosphere, it would be directly heated by the Sun's light to temperatures below the freezing point of water. Fortunately, the extra heat from the greenhouse effect keeps the oceans, lakes and streams from turning into ice.

Nevertheless, humans have increased *global warming* by burning coal, oil, and gas and releasing carbon dioxide into the atmosphere. This extra warming has been rising ever since the industrial revolution. The effect was suggested by the Swedish scientist Svante Arrhenius (1859–1927), realized as an environmental threat by the American scientists Roger Revelle (1909–1991) and Hans E. Suess (1909–1920), and documented by Charles D. Keeling's (1928–2005) measurements of the atmospheric carbon dioxide (Arrhenius 1896; Revelle and Suess 1957; Keeling 1960, 1978, 1997). Weart (2008) describes the discovery of global warming.

The Nobel Peace Prize was awarded in 2007 jointly to the Intergovernmental Panel on Climate Change and to Albert Arnold (Al) Gore Jr. (1948–) for their efforts to build up and disseminate greater knowledge about man-made climate change, and to lay the foundations for the measures that are needed to counteract such change. The film entitled *An Inconvenient Truth* (2006) documents Gore's campaign to make the issue of global warming, by human emissions of heat-trapping gases, a recognized problem.

2.6 The Energy of Light

When radiation moves in space from one place to another, it will behave like trains of waves. But when radiation is absorbed or emitted by atoms, it behaves not as a wave but as a package of energy, or like a particle, a *photon*. A photon is a discrete quantity of energy associated with electromagnetic radiation. Thus, light has a wave-particle duality; it can act light a wave and a particle depending on the situation (De Broglie 1923).

Photons have no electric charge and travel at the speed of light. They are created whenever a material object emits electromagnetic radiation, and they are consumed when matter absorbs radiation. And each atom, ion, or molecule can only absorb and radiate at a very specific set of photon energies. (An ion is an atom that has lost one or more electrons.)

The ability of radiation to interact with matter is determined by the energy of its photons.

Photon energy depends on the wavelength or frequency of the radiation. Waves with shorter wavelengths, or higher frequencies, correspond to photons with higher energy. That is, the energy, E, transported by a particular photon is directly proportional to the radiation frequency, v, and inversely proportional to the radiation wavelength, λ. The photon energy, E, is given by:

$$E = hv = \frac{hc}{\lambda}, \tag{2.38}$$

where h is the Planck constant with the value $h = 6.626 \times 10^{-34}$ J s, the frequency is given in Hz or s^{-1}, and the wavelength is in m.

The idea that light acts like a particle, the photon, when interacting with matter originated when Albert Einstein (1879–1955) explained the *photoelectric effect*, in which some metals release a current of electrons when light shines on them. Measurements of this effect indicated that the kinetic energy of the individual escaping electrons increases with the frequency of the incoming light wave. Einstein explained the observations by supposing that individual electrons are not hit by a continuous stream of light energy, but by an individual photon of light with an energy hv (Einstein 1905a, b).

Einstein was awarded the 1921 Nobel Prize in Physics for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect. The American scientist Robert A. Millikan (1868–1953) subsequently endorsed the photon interpretation, despite his initial reservations, and used the effect to measure the value of Planck's constant h (Millikan 1916); he received the 1924 Nobel Prize in Physics for his work on the elementary charge of electrons and on the photoelectric effect.

The amount of energy transported by a single photon is quite small. For yellow light, the wavelength $\lambda = 580$ nm, so the frequency $v = 5.17 \times 10^{14}$ Hz, and the photon energy, E, is only 3.42×10^{-19} J. A hundred-watt light bulb radiates a power of 100 J s^{-1}, so it sends out an incredible 2.9 million million million, or 2.9×10^{18}, photons every second.

Radio waves have even smaller photon energy, when compared with the photons of visible light. The low energies of the radio photons cannot easily excite the atoms of our atmosphere, so radio photons easily pass through the air. Visible radiation can also slip through the Earth's atmosphere with little trouble. Its photons are too energetic to resonate with molecular vibrations and they are too feeble to excite atoms.

Ultraviolet photons are sufficiently energetic to tear off electrons from atoms and many molecules in the Earth's atmosphere, particularly in the ozone layer. That's a good thing, since most of these ultraviolet photons cannot reach the ground. If they did they would cause lots of damage to our skin and eyes.

Astronomers often describe energetic, short-wavelength radiation, such as x-rays or gamma rays, in terms of their energy rather than their wavelength or frequency. At the atomic level, the natural unit of energy is the electron volt, or eV. One electron volt is the energy an electron gains when it passes across the terminals of a 1-volt battery. A photon of visible light has an energy of about two electron volts, or 2 eV. Much higher energies are associated with nuclear processes; they are often specified in units of millions of electron volts, denoted MeV. A somewhat lower unit of energy is 1,000 electron volts, called kilo-electron volts and abbreviated keV; it is often used to describe x-ray radiation. For conversion between energy units, 1 eV = 1.602×10^{-19} J and 1 keV = 1.602×10^{-16} J.

The x-ray region lies between 1 and 100 keV of energy. There are soft x-rays with relatively low energy and modest penetrating power, with energies of 1–10 keV. The hard x-rays have higher energy and greater penetrating power, at 10–100 keV. Gamma rays are even more energetic than x-rays, exceeding 100 keV in energy.

2.7 Radiation Scattering and Transfer

2.7.1 Why is the Sky Blue and the Sunsets Red?

Our atmosphere is a colorless gas, as you can see in looking at the air in your room, but the sky is usually blue and sunsets are red. The incident sunlight contains all colors, but molecules in our atmosphere scatter blue light from the Sun more than they scatter red sunlight. John Tyndall (1820–1893) discovered the effect when passing light through a clear fluid holding small particles in suspension (Tyndall 1861), and Lord Rayleigh (1842–1919) derived the relevant equations for atmosphere molecules a decade later. When the Sun is overhead, the light that reaches us is mostly scattered sunlight, and this causes the sky to appear blue. When the Sun sets, its rays pass through a maximum amount of atmosphere, and most of the blue light is scattered out before it reaches us. The setting Sun is therefore reddened, and atmospheric dust also contributes to its apparent red color.

2.7.2 Rayleigh Scattering

The scattering of radiation by a particle depends on the size, a, of the particle and the wavelength, λ, of the radiation. When the particle is much smaller in size than the wavelength, or $a \ll \lambda$, then the effect is known as *Rayleigh scattering*, named after Lord Rayleigh (1842–1919). It applies to gas molecules that scatter visible sunlight, explaining why the sky is blue and why a sunset red.

The intensity of Rayleigh scattered radiation, I, by a spherical particle of radius a at wavelength λ for an incident wave of intensity I_0 is (Rayleigh 1871, 1899):

$$I \approx \frac{8\pi^4 a^6}{D^2\lambda^4}\left[\frac{n^2-1}{n^2+2}\right]^2 I_0\left(1+\cos^2\theta\right), \tag{2.39}$$

where D is the distance from the sphere to the observation point, the scattering angle θ is the angle between the direction of propagation of the incident wave and the direction of observation, and n is the relative index of refraction $n = [\varepsilon_2\mu_2/(\varepsilon_1\mu_1)]^{1/2}$ between the sphere, denoted by subscript 2, and the surrounding medium, labeled with subscript 1, the ε denotes the dielectric constant and μ is the magnetic permeability.

The amount of Rayleigh scattering from a single particle can also be expressed as a scattering cross section, σ_S given by (Rayleigh 1871):

$$\sigma_S = \frac{128\pi^5 a^6}{3\lambda^4}\left[\frac{n^2-1}{n^2+2}\right]^2. \tag{2.40}$$

The major molecular constituent in our atmosphere, nitrogen, has $\sigma_S = 5.1 \times 10^{-31}$ m^2 in green light at a wavelength of $\lambda = 530$ nm.

The strong wavelength dependence of the Rayleigh scattering, which varies as λ^{-4}, means that the shorter blue wavelengths are scattered much more than the longer, red wavelengths. Since the molecules in our atmosphere are much smaller than the wavelengths of colored light, the blue component of sunlight is more strongly scattered down to our eyes than the other colors, creating our bright blue sky. At sunset the Sun's rays pass through a maximum amount of atmosphere; most of the blue sunlight is then scattered out of our viewing direction, and the setting Sun is colored red. Dust in the air also helps redden the sunset.

Dust particles are larger than molecules, and comparable in size to the wavelength of visible light. The equations that describe the scattering are then more complicated; it is known as Mie scattering after the German physicist Gustav Mie (1869–1957) who first published its mathematical equations. They are used to describe the scattering of starlight by interstellar dust, which reddens the light of distant stars and has a relatively weak dependence on wavelength, varying as λ^{-1}.

2.7.3 Thomson and Compton Scattering

Electromagnetic radiation can be scattered by a free electron, which is unattached to an atom. The electric field of the incident wave accelerates the electron, which moves in the direction of the oscillating electric field and emits radiation at the same wavelength, or frequency, as the incident wave. The scattering is described by the *Thomson scattering* cross section, which is independent of the wavelength, or frequency, of the incident radiation.

Thomson scattering is very important deep within the Sun, where the temperatures are high enough to ionize the atoms, producing numerous free electrons that scatter radiation produced by nuclear fusion reactions in the solar core and determine how that radiation works it way out to the visible disk of the Sun (see Sect. 8.5). It also establishes the upper limit to the luminosity of a star, known as the Eddington luminosity, which is related to the largest mass a star may have (Sect. 10.1, Focus 10.2).

The English physicist Joseph John Thomson (1856–1940) first provided the expression for the total scattered power, *P,* or the energy scattered per unit time in all directions (Thomson 1903, 1906), which is given by:

$$P = \sigma_T c U, \tag{2.41}$$

where U is the energy density of the incident radiation, c is the speed of light, and the Thomson scattering cross section, σ_T, is given by:

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 = 6.6525 \times 10^{-29}\,\mathrm{m}^2, \tag{2.42}$$

where the classical electron radius $r_e = 2.818 \times 10^{-15}$ m, the electron charge $e = 1.602 \times 10^{-19}$ C, the electric constant in vacuum $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}, the rest mass of the electron $m_e = 9.1094 \times 10^{-31}$ kg, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. Notice that the free electron acts as if it had the classical electron radius when interacting with radiation.

The Thomson scattered radiation is polarized along the direction of the electron's motion, or along the direction of the oscillating electric field of the incident radiation. The power scattered per unit solid angle, $dP/d\Omega$, therefore depends on the angle θ between the direction of the electron's motion and the direction of the observer, or:

$$\frac{dP}{d\Omega} = \frac{3}{8\pi}\sigma_T U \sin^2\theta. \tag{2.43}$$

Shortly after the big bang origin of the expanding universe, it was so hot that the universe was completely opaque to electromagnetic radiation as the result of Thomson scattering. The cosmic microwave background radiation, dating back to shortly after the big bang, is thought to be linearly polarized as a result of Thomson scattering (see Sect. 15.2).

The Thomson scattering cross section is applicable whenever the incident photon energy is much less than the rest mass energy of the electron, for radiation frequency $\nu \ll m_e c^2/h \approx 10^{20}$ Hz. When the photon energy of the incident electromagnetic radiation is comparable to, or larger than, the rest mass energy of the free electron, or for frequencies $\nu \geq 10^{20}$ Hz, the incident radiation transfers energy to the electron, and the scattered photon has less energy, or a lower frequency and longer wavelength, than the incident one. The effect is named *Compton scattering* or the *Compton effect*, after the American physicist Arthur H. Compton (1892–1962) who first observed and explained it, receiving the 1927 Nobel Prize in Physics for his discovery. The change in wavelength, $\Delta\lambda$, caused by Compton scattering from an electron that is at rest, or not moving, is given by (Compton 1923a, b):

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_C(1 - \cos\theta), \tag{2.44}$$

where λ_1 is the wavelength of the incident radiation, λ_2 is the wavelength of the Compton scattered radiation, h is the Planck constant, m_e is the rest mass of the electron, c is the speed of light, and θ is the scattering angle, or the angle by which the incident radiation is deflected. The quantity $\lambda_C = h/(m_e c) = 2.426 \times 10^{-12}$ m is known as the *electron Compton wavelength*. The Compton wavelength for any other subatomic particle is given by the same expression with m_e replaced by the mass of the particle.

In the *inverse Compton effect*, the electrons are not at rest, and may be moving at high speeds. These high-energy electrons scatter low energy photons, and the photons now gain energy in the Compton interaction and the electrons lose energy. When the electron's speed is large, approaching that of light, the scattered frequency, ν_2, for incident radiation of frequency ν_1 is given by:

$$\nu_2 \approx \gamma^2 \nu_1 \text{ for } \gamma h \nu_1 \ll m_e c^2, \tag{2.45}$$

the scattering cross section is $\sigma_S = \gamma^2 \sigma_T$ for Thomson scattering cross section σ_T, and the total energy radiated per unit time, P, by an electron passing through radiation of energy density U is given by

$$P \approx \gamma^2 \sigma_T c U, \tag{2.46}$$

where the energy of the electron is $\gamma m_e c^2$ and γ is the Lorentz factor $\gamma = [1 - (V/c)^2]^{-1/2}$ for an electron moving at velocity V.

The scattered radiation from high-energy electrons with Lorentz factors $\gamma = 1{,}000$ has a frequency that is a million times that of the incident radiation. Thus radio radiation becomes ultraviolet radiation, infrared radiation becomes x-rays and optical radiation becomes gamma rays.

When the electron velocity is high and $\gamma h \nu_1 \gg m_e c^2$, all of the electron energy is transferred into the scattered radiation regardless of the incident photon frequency and $\nu_2 = \gamma m_e c^2/h$ for $\gamma h \nu_1 \gg m_e c^2$.

Inverse Compton scattering can quench the synchrotron radiation of cosmic radio sources, which is emitted by high-speed electrons, and the effect can be important in the x-ray radiation of relativistic electrons being accreted by a black hole. Longair (2011) has provided applications of scattering formulae to astronomical objects in high-energy situations.

2.7.4 Radiation Transfer

Once radiation is emitted from an astronomical object, it must pass through intervening space before it reaches the observer. The radiation can be absorbed when passing through a layer or cloud of matter, and the same material can also emit radiation. The material's effect on the radiation is therefore characterized by an absorption coefficient per unit length α_ν at frequency ν and emission coefficient ε_ν at frequency ν. For matter in thermodynamic equilibrium at temperature T,

$$\varepsilon_\nu = \alpha_\nu B_\nu(T), \tag{2.47}$$

a result derived by Gustav Kirchhoff (1824–1887) and hence known as *Kirchhoff's law* (Kirchhoff 1860). Here $B_\nu(T)$ is the Planck distribution for thermal, or black-body, radiation discussed in the previous Sect. 2.4, and the emission coefficient ε_ν is the power per unit volume per unit frequency interval per unit solid angle.

The optical depth, denoted by τ_ν, characterizes the absorption capability of the intervening matter at radiation frequency ν. It is given by:

$$\tau_\nu = \int_0^L \alpha_\nu dx \tag{2.48}$$

for a cloud or layer of thickness L in the x direction and an absorption coefficient per unit length α_ν. For radiation of incident intensity $I_\nu(0)$, the intensity of radiation, $I_\nu(L)$, on leaving the cloud will be:

$$I_\nu(L) = I_\nu(0)\exp(-\tau_\nu) + \frac{\varepsilon_\nu}{\alpha_\nu}[1 - \exp(-\tau_\nu)], \tag{2.49}$$

and the intensity $I_\nu(\text{cloud})$ of the thermal radiation emitted by the cloud is given by:

$$I_\nu(\text{cloud}) = \int_0^L \varepsilon_\nu \exp(-\alpha_\nu x)dx \approx B_\nu(T)[1 - \exp(-\tau_\nu)], \tag{2.50}$$

with $B_\nu(T) = \varepsilon_\nu/\alpha_\nu$. If the cloud or layer has negligible radiation at frequency ν, its emission coefficient is effectively zero and:

$$I_\nu(L) = I_\nu(0)\exp(-\tau_\nu). \tag{2.51}$$

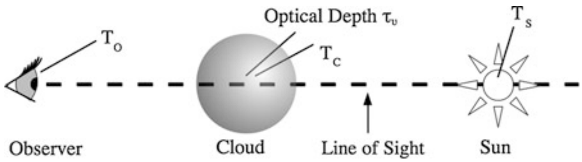

Fig. 2.6 Looking through a cloud When an interstellar cloud happens to lie along the line of sight to a star, the observed temperature, denoted T_O, can differ from the star's temperature, abbreviated T_S. That is because the cloud, with temperature T_C, will shine like any hot gas, emitting its own radiation, and the cloud can also absorb and scatter the star's radiation that is passing through it. A thick, dense cloud can absorb all the incident star's radiation, so you don't even see the star; just the cloud is detected. In contrast, a thin, rarefied cloud can be transparent; therefore, you look right through it, detecting the star as if the cloud wasn't even there. The cloud is characterized by its optical depth, denoted by the symbol τ_ν, which depends on the substance in the cloud, the thickness of the cloud along the line of sight, and the observation frequency, designated by ν. An optically thin cloud is transparent to the radiation at this frequency and the cloud optical depth $\tau_\nu \ll 1$. An optically thick cloud, with $\tau_\nu \gg 1$ is opaque, and at this frequency we cannot observe anything behind the cloud

The term optical depth implies that we are talking about radiation at the visible wavelengths we detect with our eyes. If the optical depth $\tau_\nu \gg 1$, along a ray path through a cloud or layer, then that cloud or layer is known as optically thick. On the other hand, a transparent cloud or layer is known as optically thin if $\tau_\nu \ll 1$. It follows that an optically thick object extinguishes the light of a source behind it, whereas an optically thin object absorbs negligible amounts of light passing through it. More generally, the terms optically thick and optically thin roughly mean opaque and transparent at the wavelength or frequency of electromagnetic radiation we are considering.

When the emission coefficient is not zero, the brightness, $B_{C\nu}(T)$, of the thermal emission from the intervening cloud or layer is given by:

$$\begin{aligned} B_{C\nu}(T) &= B_\nu(T)[1 - \exp(-\tau_\nu)] \\ &= B_\nu(T) \text{ if } \tau_\nu \gg 1 \text{ (optically thick)} \\ &= \tau_\nu B_\nu(T) \text{ if } \tau_\nu \ll 1 \text{ (optically thin)}, \end{aligned} \tag{2.52}$$

where $B_{C\nu}(T)$ denotes the cloud brightness at frequency ν and temperature T.

The observed brightness and temperature of a source that lies behind a cloud will depend on the temperature of the source, the temperature of the cloud, and the optical depth of the cloud (Fig. 2.6). When the cloud is transparent at the observation wavelength or frequency, which corresponds to the completely optically thin situation, the source temperature is observed. If the cloud is opaque, or optically thick, the cloud's temperature is observed.

If a source of brightness, $B_S(T)$, at frequency, ν, and temperature, T_S, is irradiating a cloud or layer of temperature, T_C, the total observed brightness $B_{O\nu}(T_{\text{TOT}})$ is given by

$$B_{O\nu}(T_{\mathrm{TOT}}) = B_{S\nu}(T_S)\exp(-\tau_\nu) + B_{C\nu}(T_C), \tag{2.53}$$

where

$$T_{\mathrm{TOT}} = T_S \exp(-\tau_\nu) + T_C[1 - \exp(-\tau_\nu)]. \tag{2.54}$$

For the optically thin case $\tau_\nu \ll 1$ the total observed temperature would be given by:

$$T_{\mathrm{TOT}} = T_S(1 - \tau_\nu) + \tau_\nu T_C. \tag{2.55}$$