Chapter 13
Stellar End States

13.1 A Range of Destinies

No material object can exist forever, and stars are no exception. Although their lives may be measured in millions or billions of years, stars do stop shining when all of the available sources of subatomic energy have been exhausted. Their central thermonuclear reactions, which keep the star hot inside, are then turned off. There is no heat and pressure being generated inside such a star, so the internal support has been removed and it begins its ultimate contraction. The demise of such a star results in the simultaneous creation of a new star from its collapsing core, with a final resting state that depends on the star's mass.

As in the beginning of their lives, the central regions of all dying stars are subject to the unsupported, inward pull of gravity from all sides, and the entire stellar mass is compressed into an increasingly smaller radius. It only stops when some outward pressure grows sufficiently large to halt the contraction, which means that there is an enormous range in stellar size and mass density (Table 13.1). The mean mass density of the Sun, for example, is comparable to that of water, whereas the density of a neutron star is similar to that of the nucleus of an atom.

In its earliest stages, a protostar's gravitational contraction is stopped when the star begins to fuse hydrogen nuclei into helium nuclei, generating the internal heat and pressure that halts the formative collapse. A long time later, when the hydrogen runs out, gravity takes over again and compresses the core, heating it up until helium can be consumed in synthesizing carbon. Enough heat is then generated to balance the relentless force of gravity, and the star's outer atmosphere expands to giant or supergiant size.

What happens next depends on the mass of the star. Stars that have a mass comparable to the Sun's mass begin their ultimate collapse when the helium is gone, ending up as burned-out, Earth-sized white dwarf stars. The ultimate destinies of the rare, more massive, and luminous supergiants are explosive. They can leave a city-sized neutron star behind or be crushed into a stellar black hole. Thus,

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Table 13.1 Representative mass, radius, and mean mass density of the stars[a]

Star	Mass ($M_\odot$)	Radius ($R_\odot$)	Mean mass density[b] (kg m^{-3})
Red giant star	1.2	100	0.0014
Sun	1.0	1.0	1,400
White dwarf star	0.6	0.01	0.84×10^9
Neutron star	1.5	0.00001	2.1×10^{18}
Black hole[c]	10.0	0.000004	0.2×10^{18}

[a] The mass is in units of the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, and the radius is in units of the Sun's radius $R_\odot = 6.955 \times 10^8$ m

[b] The mean mass density $= 3\,M/(4\pi R^3)$ for a star of mass M and radius R

[c] A representative radius for the black hole formed by the collapse of a star of mass M is taken as the Schwarzschild, or gravitational, radius $R_g = 2GM/c^2$ for gravitational constant $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2} and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}

there is a range of destinies: Giants turn into tiny white dwarfs; supergiants turn into even smaller neutron stars; and the bigger, heavier supergiants turn into black holes – all compressed into their end states by the never-ending force of gravity. The more massive the star, the smaller it eventually becomes.

13.2 Planetary Nebulae

After discovering Uranus, in 1781, the English astronomer William Herschel (1738–1822) discovered a small glowing object that he designated a *planetary nebula* because of its round shape, which resembled the disks of planets as seen through a small telescope (Herschel 1786). However, planetary nebulae are not made of planets, and planets are not visible in them. The designation *nebula* is from the Latin word for "cloud", and it was used to distinguish the diffuse planetary nebulae, which have resolved disks, from unresolved, point-like stars.

Herschel and other astronomers soon discovered more of these objects, and one of them, named the Cat's Eye Nebula, had a "condensation" in its center, which turned out to be a star (Herschel 1786). It was eventually realized that every planetary nebula has a star at its center, the exposed core of a dying red giant that illuminates the nebula (Fig. 13.1).

Any star with a moderate mass, comparable to that of the Sun, eventually balloons into a red giant star. As the core nuclear reactions cease, the giant sheds its outer layers, which are blown away. All that remains of that part of the star is gas and dust, the planetary nebula. The central regions of a red giant star will collapse into a smaller white dwarf star.

Another English astronomer, William Huggins (1824–1910), used his spectroscope to find a trio of emission lines in the Cat's Eye Nebula (Huggins 1864, 1868). When heated, a low-density gas radiates these emission lines; therefore, their presence indicated that the planetary nebulae contain hot, rarefied gas.

Fig. 13.1 Planetary nebula When a Sun-like star uses up its nuclear fuel, the star's center collapses into an Earth-sized white dwarf star and its outer gas layers are ejected into space. Such a planetary nebula is named after its round shape, which resembles a planet as seen visually in small telescopes, and is not related to planets. The shells of gas in the planetary nebula NGC 6751, shown here, were ejected several thousand years ago. The hot stellar core, exposed by the expulsion of the material surrounding it, has a disk temperature of about 140,000 K. Its intense ultraviolet radiation causes the ejected gas to fluoresce as a planetary nebula. (A *Hubble Space Telescope* image courtesy of the NASA/STScI/AURA/Hubble Heritage Team.)

However, at that time, no one knew how hot the gas was, or much about its elemental constitution.

The wavelength of one of the emission lines detected by Huggins coincided with hydrogen—the Balmer emission line at 486.1 nm, but the chemical identification of the other two emission lines remained a mystery for more than half a century. Although these green nebular lines, at wavelengths of 495.9 and 500.7 nm, were even stronger than hydrogen, they defeated attempts to identify them with elements known on the Earth. These spectral features initially were attributed to a previously unknown element called "nebulium," but the emission lines eventually were shown to be due to known elements that had become ionized by the ultraviolet light of the bright central stars.

When it was realized that the central stars of planetary nebulae are very hot, the English astronomer Arthur Stanley Eddington (1882–1944) proposed that the ultraviolet starlight ionizes surrounding material, and that the electrons liberated by this photoionization will heat the gas to about 10,000 K (Eddington 1926). The American astronomer Donald Menzel (1901–1976) and the Dutch astronomer Herman Zanstra (1894–1972) then showed that the hydrogen emission lines of planetary nebulae are produced when the free electrons liberated by the ultraviolet

Table 13.2 Physical properties of planetary nebulae

N = number density = N_e = electron density = (0.5–20) × 10^9 m^{-3}
T_e = electron temperature ≈ (0.6–1.8) × 10^4 K
R = radius = 0.2–0.8 light-years = 0.07–0.25 pc ≈ (2–7) × 10^{15} m
M = mass = $4\pi R^3 N m_P/3 = 10^{21}$ kg = 5 × 10^{-10} $M_\odot$
V_{exp} = expansion velocity = (1–9) × 10^4 m s^{-1}
τ_{exp} = expansion age ≈ 16,000 years

photoionization recombine with protons to make hydrogen atoms, cascading through the atoms' various allowed electron orbits or energy levels and radiating the Balmer emission line (Menzel 1926; Zanstra 1927, 1928).

In a brilliant piece of detective work, the American astronomer Ira S. Bowen (1898–1973) interpreted the two strong green emission lines as forbidden transitions of doubly ionized oxygen (Bowen 1928). His solution depended on the rarity of atomic collisions in the extremely tenuous planetary nebulae, which allows the occurrence of "forbidden" transitions. They are not actually forbidden but rather so improbable that they seldom take place in a higher-density laboratory situation, where an atom almost always is jostled by collisions into a different state before the forbidden radiation can be emitted.

The observed emission lines indicate gas temperatures of about 10,000 K and electron or ion number densities of about 10 billion per cubic meter, denoted as 10^{10} m^{-3} (Table 13.2). Although this is a big number, such densities are lower than the best vacuum used in a terrestrial laboratory. Kaler (1985) has provided a review of planetary nebulae and their central stars.

The mass density and temperature of planetary nebulae resemble those of the emission nebulae (Sect. 11.1); however, the planetary nebulae are about 10 times smaller and, unlike the emission nebulae, they also are expanding. Both types of nebulae are illuminated by a bright central star, and they both emit similar spectral lines: those of ionized hydrogen and the forbidden emission lines of oxygen and nitrogen ions, designated [O III], [O II], and [N II] (see previous Table 11.2).

As Zanstra realized, the intensity of the hydrogen emission line can be related to the temperature of the exciting star through the theory of the hydrogen atom and the Planck spectrum of thermal radiation. He found that these stars are enormously hot, and modern investigations show that they are the hottest stars known. The luminous central star radiates thousands of times more energy than the Sun and has a temperature of 100,000 K and even over 200,000 K, much higher than any main-sequence star. This places the central star of a planetary nebula right off the scales of the Hertzsprung – Russell diagram, on the far left side (Fig. 13.2). Powerful winds have removed the star's relatively cool, outer layers to reveal its hot interior.

Most of the radiation of such a hot star is at ultraviolet wavelengths, which brighten the surrounding nebula, but the star is relatively dim at the longer visible wavelengths and may even become invisible. However, as a young planetary nebula is blown outward by powerful winds, it slowly grows in size, thins out, and becomes transparent, revealing its source – the exposed core of a dying red giant.

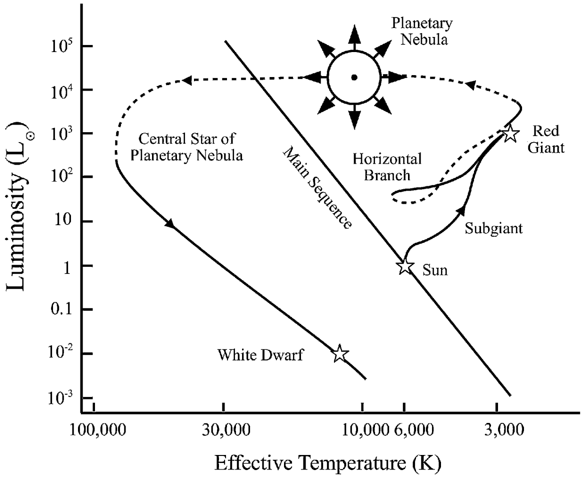

Fig. 13.2 Formation of a planetary nebula and white dwarf star The evolutionary track of a dying Sun-like star in the Hertzsprung-Russell diagram. When the star has exhausted its nuclear hydrogen fuel, which makes the star shine, it expands into a red giant star; after a relatively short time, the giant star ejects its outer layers to form a planetary nebula. The ejected gas exposes a hot stellar core, which collapses to form an Earth-sized white dwarf star that gradually cools into dark invisibility. The luminosity is in units of the Sun's luminosity, denoted $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, and the effective temperature of the stellar disk is in units of degrees kelvin, denoted K. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

When modern telescopes are used to zoom in and resolve the expanding gas and dust, they show that it has not been expelled in a single puff of stellar wind, but instead in multiple gusts that can slam into each other (Fig. 13.3). Fast and slow winds may also play a role in producing the various shapes and forms of planetary nebulae. Balick and Frank (2002) have reviewed the shapes and shaping of planetary nebulae.

The observed expansion speeds of about 10 km s^{-1} and nebular dimensions of about a light-year across indicate the expanding shells of gas were ejected about 16 thousand years before the expansion and size were measured. Their luminescent gas will expand and disperse into interstellar space, cooling into invisibility and becoming indistinguishable from their surroundings in about 20,000 years. This is a relatively brief existence, only about 1 millionth of the stellar lifetime of many billions of years. As a result, planetary nebulae are much less numerous than the stars.

Example: Expansion age of a planetary nebula

The named planetary nebulae have expansion velocities of at least $V_{exp} = 10$ km s^{-1}, and a radius, R, of about 5×10^{15} m or half a light-year. The expansion time $\tau_{\mathrm{exp}} = R/V_{\mathrm{exp}} \approx 5 \times 10^{11}$ s $\approx$ 16,000 years, where 1 year $= 3.1557 \times 10^7$ s.

Fig. 13.3 The Eskimo Nebula About 10,000 years ago, a dying Sun-like star began flinging material into nearby space, producing this planetary nebula that is formally designated as NGC 2392. When first observed more than two centuries ago, it was dubbed the "Eskimo" Nebula because it resembled a face surrounded by a fur parka like those worn by Eskimos. It is located about 5,000 light-years from the Earth. This detailed image, obtained by instruments aboard the *Hubble Space Telescope*, reveals several episodes of ejection from the central star, including an outer ring of objects that are shaped like teardrops pointing outward and elongated, filamentary bubbles, each about 1 light-year in diameter. Dense material enveloping the star's equator has blocked ejected material, and intense winds moving at about 420 km s^{-1} have swept material above and below the equatorial regions. The bright central region contains another wind-blown bubble. (Courtesy of NASA/Andrew Frucher/ERO Team, Slyvia Baggett/STScI/Richard Hook, ST-ECF, and Zolan Levay/STScI.)

Despite their infrequent appearance on cosmic time-scales, thousands of planetary nebulae are known. Some of them are listed in Table 13.3 with the names associated with them, and their number in the New General Catalogue (NGC). Table 13.3 also provides the celestial position, distance, radius, expansion

Table 13.3 Bright named planetary nebulae[a]

Catalog designation	Popular name[c] (Nebula)	RA (2000)		Dec. (2000)		D^b (ly)	R (ly)	V_{exp} (km s^{-1})	$m_V{}^d$	T^d (K)
		h	m	°	′					
NGC 650-1	Little Dumbbell	01	42.4	+51	34.5	2,400	0.85	39	17.5	175,400
NGC 2392	Eskimo	07	29.2	+20	54.7	3,000	0.18	53	10.5	65,000
NGC 3242	Eye of Jupiter	10	24.8	−18	38.5	1,600	0.09	30	12.1	90,000
NGC 3587	Owl	11	14.8	+55	01.0	2,000	0.98	30	16.0	112,000
NGC 3918	Blue Planetary	11	50.3	−57	10.9	3,260	0.16	25	13.2	–
IC 3568	Lemon Slice	12	33.0	+82	34.0	4,500	0.4	–	–	–
MyCn18	Hourglass	13	39.6	−67	22.9	10,100	0.33	10	14.4[e]	–
Menzel 3	Ant	16	17.2	−51	59.2	4,140	0.26	–	17.6[e]	–
M 2-9	Butterfly	17	05.6	+10	08.6	5,542	0.20	31	15.7	–
Hen 3-1357	Stingray	17	16.4	−59	29.6	18,000	0.08	–	15.0	–
NGC 6369	Little Ghost	17	29.3	−23	45.6	2,000	0.15	41	15.9	58,000
NGC 6543	Cat's Eye	17	58.6	+66	38.0	3,000	0.20	20	11.0	50,000
NGC 6720	Ring	18	53.6	+33	01.8	2,300	0.47	30	15.7	150,000
NGC 6751	Dandelion	19	05.9	−05	59.6	≤8,000	≤0.43	40	13.9	76,000
NGC 6826	Blinking Eye	19	44.8	+50	31.5	3,600	0.25	11	10.7	47,000
NGC 6853	Dumbbell	19	59.5	+22	43.3	1,200	1.0	28	14.0	160,000
NGC 7009	Saturn	21	04.2	−11	21.8	2,000	0.38	20	13.0	90,000
NGC 7027		21	07.0	+42	14.2	2,900	0.13	18	16.3	185,000
NGC 7293	Helix	22	29.6	−20	50.2	715	2.0	25	13.4[e]	110,000
NGC 7662	Blue Snowball	23	25.9	+42	32.1	2,500	0.20	30	13.2	110,000

[a] The distance, D, and radius, R, are in units of light-years, abbreviated ly. For conversion use 1 kiloparsec = 1 kpc = 3,260 light-years and 1 light year = 9.46×10^{15} m. The nebula's angular diameter $= 2R/D$ radians, where 1 radian = 2.06265×10^5 s of arc. Data courtesy of James B. Kaler, University of Illinois

[b] The distances of some planetary nebulae are not well known

[c] Other names: Little Dumbell Nebula = M 76, Barbell Nebula or Cork Nebula; Eskimo Nebula = Clownface Nebula; Eye of Jupiter Nebula = Ghost of Jupiter Nebula or Eye Nebula; Owl Nebula = M97; Blue Planetary Nebula = The Southerner Nebula; M 2-9 = Minkowski 2-9 or Butterfly Nebula or Twin Jet Nebula; Ring Nebula = M 57; Dandelion Nebula = Dandelion Puff Ball Nebula; Dumbbell Nebula = M 27; Blue Snowball Nebula = Snowball Nebula

[d] The estimated apparent visual magnitude, m_V, and temperature, T, of the central star

[e] Apparent blue magnitudes

velocity, and apparent visual magnitude and temperature of the central star for the planetary nebulae. The distances are often uncertain, and the radii are between 0.1 and 3 light-years, but it is the popular names that are so fascinating. They describe the resemblance of planetary nebulae to everyday objects, such as the twin weights of a dumbbell, a fur parka enveloping an Eskimo's face, the head of an owl, a cat's eye or simply a ring.

Planetary nebulae are produced by the winds of dying stars. Cassinelli (1979) has provided a review of stellar winds; Kudritzki and Puls (2000) have reviewed winds from hot stars; Willson (2000) has discussed mass loss from cool stars with their impact on the evolution of stars and stellar populations; and Dupree (1986) has discussed mass loss from cool stars.

Stars that produce the planetary nebulae end up as small, dense white dwarf stars. In the very distant future, our Sun will become one, as will the majority of other stars. Their discovery was entirely unexpected.

13.3 Stars the Size of the Earth

13.3.1 The Discovery of White Dwarf Stars

The first white dwarf to be known is a companion of a much brighter star, 40 Eridani, also known as Omicron Eridani from its Greek letter designation. The fainter star is designated 40 Eridani B to distinguish it from the brighter member, A, of the pair. The American astronomer Walter S. Adams (1876–1956) first drew attention to the A0 spectral type of 40 Eridani B, which suggested a disk temperature of about 10,000 K, and noticed that it was surprising that such a hot star should exhibit such a very low luminosity (Adams 1914).

Unfortunately, 40 Eridani B is so far away from its bright companion, with an orbital period of at least 7,300 years, that its mass could not be inferred from its orbital motion. However, this was not the case for Sirius B, the second white dwarf to be discovered. It is also a member of a binary star system, with a luminous companion designated Sirius A, the brightest star in the night sky.

The irregular motion of Sirius A first suggested the presence of its dim companion. As discovered by Friedrich Wilhelm Bessel (1784–1846), the bright star swerves from side to side of a straight-line trajectory, and this swerving motion was attributed to the gravitational attraction of a nearby, unseen companion (Bessel 1844). The American astronomer and telescope maker Alvan Clark (1804–1887) first detected the diminutive star in 1862.

The masses of Sirius A and B have been estimated from the orbital motion of the pair, weighing in at roughly the mass of the Sun; because the companion was about twice as far as Sirius from their common center of mass, the companion had to have about half the mass of the bright star. Modern determinations indicate that the mass of Sirius A is 2.02 solar masses, and that the mass of Sirius B is 0.978 times that of the Sun.

Adams showed that this low-luminosity star has a spectral class A0, like 40 Eridani B (Adams 1915). What Adams did not point out explicitly was that the high surface temperature in combination with the low total luminosity meant that both 40 Eridani B and Sirius B must be very small – only about the size of the Earth. Furthermore, the rather ordinary mass of Sirius B meant that the average mass density of the star must be enormous – about 10^9 kg m^{-3} and about 1 million times the average mass density of the Sun. Later on, it was found that the white dwarf 40 Eridani B has it own companion, designated 40 Eridani C, whose orbital motion allowed a mass determination of about half a solar mass for this white dwarf star, with a mass density comparable to that of Sirius B.

These stars now are known to be the inner, collapsed leftovers of dying red giant stars, exposed by the planetary nebulae that carried off the outer stellar atmospheres.

13.3.2 Unveiling White Dwarf Stars

Having depleted the hydrogen in their cores, the central regions of solar-mass stars contract to become hot enough to fuse helium into carbon and oxygen; however, there is not enough mass to generate a temperature hot enough to fuse carbon into neon, at about 1 billion K, and an inert carbon–oxygen core is surrounded by an inner helium-burning shell and an outer hydrogen-burning shell.

After shedding most of its outer material to form a planetary nebula, these giant stars leave behind a hot core of carbon and oxygen. Because it cannot generate additional heat by nuclear reactions, the core collapses to form a white dwarf with about the Sun's mass compressed to 1/100th of its former size and about the same radius as the Earth. Such a collapsed star is called a *white dwarf star* because initially it is white in color and it is relatively small for a star.

The concept of a white dwarf star as the exposed carbon–oxygen core of a former giant star initially was difficult to reconcile with the fact that the emitted light of the first white dwarfs contained strong spectral lines of hydrogen and it did not include the oxygen lines found in the surrounding planetary nebula. The French astronomer Evry Schatzman (1920−2010) explained this paradox when he noted that the hydrogen resides in a thin outer atmosphere and that the heavy elements such as carbon and oxygen had to sink out of sight into the dense stellar interior (Schatzman 1945). These elements are drawn by gravity into the unseen interior of the white dwarf and remain hidden by a hydrogen-rich layer that is only about 1/10,000th of the star's mass.

An inert, nonburning star, with no nuclear reactions in its core, which is composed mainly of carbon and oxygen nuclei, represents the final destiny, the end state, of main-sequence stars with a mass of between 0.5 and 8 solar masses, and it accounts for most of the observed white dwarf stars. Main-sequence stars of lower mass, below 0.5 solar masses, bypass the giant stage – never becoming hot enough inside to fuse helium – and they collapse directly into white dwarfs composed of helium. However, these stars have such a low luminosity and temperature that they burn hydrogen slowly and have not yet exhausted their internal supply, even during the 14-billion-year age of the observable universe.

When first exposed, the white dwarf can be very hot because it was previously in the hot interior of a giant star. It may have an initial temperature of 200,000 K due to its hot origin and the collapse that created it, but because there is no thermonuclear fuel, there is nothing left to heat a white dwarf star.

A white dwarf's temperature will drop from an initial high of up to 200,000 K to an observed low that is colder than the Sun, steadily becoming fainter and dimmer. Liebert (1980) has reviewed white dwarf stars, Hansen and Liebert (2003) have provided a review of cool white dwarf stars, and D'Antona and Mazzitelli (1990) have reviewed the cooling of white dwarfs. This cooling and dimming is so slow that the oldest white dwarf has not yet chilled to the point of invisibility. That is, observations indicate that the oldest white dwarf stars, which are remnants of the earliest stars, have not yet had time to cool to the lowest possible luminosity that might be observed. The amount of time that the oldest white dwarf has cooled is estimated to be about 9 billion years, which when combined with its former lifetime as a main−sequence and giant star gives a rough estimate to the age of the observable universe of about 14 billion years (Winget et al. 1987; Wood 1992).

13.3.3 The High Mass Density of White Dwarf Stars

The rather ordinary stellar mass of a white dwarf has been compressed within a star that is comparable to the size of the Earth, which means that it has an enormous mean mass density up to 1 million times that of the Sun or about a billion, or 10^9, kg m^{-3} (Focus 13.1).

Focus 13.1 Radius and mass density of a white dwarf star
The effective disk temperature of Sirius B is $T_{eff} = 25{,}200$ K, while its absolute luminosity, L, is $L = 0.026\ L_{\odot}$, where the Sun's absolute luminosity is $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}. From the Stefan−Boltzmann law,

$$L = 4\pi\sigma R^2 T_{eff}^4, \tag{13.1}$$

where $\pi \approx 3.1416$ and the Stefan−Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}, we obtain an expression for the radius, R:

$$R = \left[\frac{L}{4\pi\sigma T_{eff}^4}\right]^{1/2}, \tag{13.2}$$

which gives a radius $R \approx 5.9 \times 10^6$ m $\approx 0.01\ R_\odot$, for the radius of Sirius B, where the solar radius $R_\odot = 6.955 \times 10^8$ m. By way of comparison, the radius of the Earth is $R_E = 6.378 \times 10^6$ m.

The mass, M, of Sirius B is $M = 0.98\ M_\odot$, where the Sun mass $M_\odot = 1.989 \times 10^{30}$ kg. The mass density, ρ, of Sirius B is therefore $\rho = 3\ M/(4\pi R^3) \approx 2.26 \times 10^9$ kg m^{-3}.

Arthur Stanley Eddington (1882–1944) pointed out that there is nothing inherently absurd about the high mass densities of white dwarf stars (Eddington 1924). Because all of the electrons are stripped away from their atomic nuclei in the hot stellar interiors, the free electrons can be packed closely with the bare nuclei, within the former space of the empty atoms.

Eddington also predicted that the gravitational redshift of Sirius B might be observed once allowance was made for the orbital motion of the double-star system (Eddington 1924). The gravitational redshift is the Doppler shift of a spectral line caused by the loss of energy in overcoming the gravity of the emitting object (Sect. 6.5). Adams apparently confirmed the effect, providing an independent confirmation of the small size of white dwarf stars that had been inferred from their high temperature and low luminosity (Adams 1915). This result was substantiated with greater clarity for Sirius B, 40 Eridani B, and other white dwarf stars using ground-based telescopes (Greenstein et al. 1971; Greenstein and Trimble 1972; Shipman 1972; Wegner 1980; Shipman et al. 1997; Huber et al. 1998), as well as the *Hubble Space Telescope* (Barstow et al. 2005).

The expression for the change in wavelength, $\Delta\lambda$, for a line emitted at wavelength λ_e can be derived from Newton's theory of gravity by assuming that the radiation photons have an energy $h\nu$ at frequency ν, and that this energy can be expressed in terms of an imaginary radiation mass, m, times the square of the speed of light, c, or that $h\nu = mc^2$, where the Planck constant $h = 6.626 \times 10^{-34}$ J s and $c = 2.9979 \times 10^8$ m s^{-1}. When the photon leaves the surface of a mass, M, with radius, R, the photon loses the energy, ΔE, given by:

$$\Delta E = h\Delta\nu = \frac{GMm}{R} = \frac{GMh\nu}{Rc^2}, \tag{13.3}$$

where, $\Delta\nu$ is the change in frequency ν. The gravitational redshift, z_g, caused by this loss of photon energy is given by

$$\begin{aligned} z_g &= \frac{V_r}{c} = \frac{\Delta\nu}{\nu} = \frac{\nu_L - \nu_{observed}}{\nu_{observed}} = \frac{\Delta\lambda}{\lambda_L} = \frac{\lambda_L - \lambda_{observed}}{\lambda_L} = \frac{GM}{Rc^2} \\ &= 2.12 \times 10^{-6}\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right), \end{aligned} \tag{13.4}$$

where V_r is the radial velocity corresponding to the gravitational redshift, ν_L and $\nu_{observed}$ respectively denote the emitted line frequency and observed line frequency, λ_L and $\lambda_{observed}$ respectively denote the emitted line wavelength and observed line wavelength, $\Delta\lambda$ is the change in wavelength caused by overcoming the gravity, and the gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}.

Example: Gravitational redshift and thermal velocity of Sirius B
Precise measurements indicate that the radius of Sirius B is $R = 0.0084$ $R_\odot = 5.84 \times 10^6$ m, for a solar radius $R_\odot = 6.955 \times 10^8$ m, and that its mass is $M = 0.978\ M_\odot$ (Barstow et al. 2005). This indicates a gravitational redshift of $z_g = GM/(Rc^2) \approx 2.5 \times 10^{-4}$, where the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2} and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. This redshift corresponds to a radial velocity $V_r = z_g c \approx 7.5 \times 10^4$ m s^{-1} $\approx$ 75 km s^{-1}, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The measured gravitational redshift of Sirius B is 80.42 ± 4.83 km s^{-1}, which is consistent with the estimate given the uncertainty of the measurement.

The bolometric luminosity of Sirius B is $L = 0.0026\ L_\odot = 9.95 \times 10^{23}$ J s^{-1}, where the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. Measurements indicate an effective temperature of $T_{eff} = 25{,}200$ K for Sirius B. Using the star's luminosity and radius with the Stefan–Boltzmann law $L = 4\pi\sigma R^2\ T_{eff}^4$, where $\pi = 3.1416$ and the Stefan–Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ J m^{-2} K^{-4} s^{-1}, we obtain $T_{eff} \approx 14{,}000$ K. Using $T_{eff} = 25{,}200$ K in the expression for the thermal velocity $V_{thermal} = (3kT/m)^{1/2}$, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1} and a hydrogen atom of mass $m = 1.67 \times 10^{-27}$ kg, we have $V_{thermal} = 2.5 \times 10^4$ m s^{-1} = 25 km s^{-1}, about three times smaller than the gravitational redshift.

For a white dwarf star, R $\approx$ 0.01 R$_\odot$ and $z_g = V_r/c \approx 0.02$ with $V_r \approx 60$ km s^{-1}. The observed gravitational redshifts for Sirius B and 40 Eridani B, in radial velocity units, are 80.42 ± 4.83 km s^{-1} and 23.9 ± 1.3 km s^{-1}, respectively. Observations of these gravitational redshifts confirm the small size of white dwarf stars that had been inferred from their high temperature and low luminosity.

The small size and high mass density of the white dwarf stars have also been substantiated by measurements of their magnetic-field strength. During gravitational collapse, magnetic flux is conserved, and the surface magnetic-field strength increases as the surface area decreases. For a sphere of surface magnetic field strength, B, and radius R, the product BR^2 is a constant, and in solar units:

$$B = B_\odot \left(\frac{R_\odot}{R}\right)^2. \tag{13.5}$$

Table 13.4 Physical properties of white dwarf stars

M_{WD} = mass of white dwarf star $\approx 0.6\ M_{\odot} \approx 1.2 \times 10^{30}$ kg
M_{CWD} = critical upper mass limit for white dwarf star = $1.4\ M_{\odot} \approx 2.3 \times 10^{30}$ kg
R_{WD} = mean radius of white dwarf stars = $0.01\ R_{\odot} \approx 6 \times 10^{6}$ m $\approx R_E$ = radius of Earth
ρ_{WD} = mass density of white dwarf star $\approx 10^{9}$ kg m^{-3}
V_{esc} = escape velocity of white dwarf star = $(2GM_{WD}/R_{WD})^{1/2}$ = 9,000 km s^{-1} = 0.03 c.
L_{WD} = absolute luminosity of white dwarf star $\approx 10^{-3}\ L_{\odot} \approx 3.8 \times 10^{23}$ J s^{-1}
T_{WD} = effective temperature of white dwarf star's visible disk = 4×10^{3} K to 7×10^{4} K
B_{WD} = surface magnetic field strength of white dwarf star = 10^{2} tesla to 10^{4} tesla
z_g = gravitational redshift of white dwarf star = $GM_{WD}/(R_{WD}c^2) \approx 0.02 = V_r/c$ (or radial velocity $V_r = z_g c \approx 60$ km s^{-1}, where c is the speed of light)

The dipolar magnetic field of the solar disk is $B_{\odot} \approx 0.01$ tesla. For a white dwarf of radius R of 1/100th the radius of the Sun, or $R = R_{wd} \approx 0.01\ R_{\odot}$, the surface magnetic field strength will be amplified to $B_{wd} \approx 100$ tesla. Surface magnetic field strengths of 100 to 10,000 tesla were inferred from the circularly polarized light of white dwarf stars, and roughly 10 % of them have magnetic fields in excess of 100 tesla (Kemp et al. 1970). Angel (1978) has reviewed magnetic white dwarfs. Individual papers on magnetism in white dwarfs are referenced in Lang (1999).

The physical properties of white dwarf stars are given in Table 13.4.

13.4 The Degenerate Electron Gas

13.4.1 Nuclei Pull a White Dwarf Together as Electrons Support It

The matter deep inside a white dwarf star is completely ionized and composed of equal numbers of atomic nuclei and electrons. Because most white dwarfs are the crushed remnants of red giant stars, which previously fused helium into carbon, their collapsed cores consist mainly of carbon nuclei and electrons. Stars that are somewhat more massive than the Sun leave behind white dwarfs containing oxygen nuclei. Therefore, white dwarf stars contain various amounts of carbon nuclei or oxygen nuclei, depending on the star, and it is these nuclei that supply the mass and gravity of a white dwarf star.

The nuclei supply the mass and gravity of a white dwarf star, with a mass density, ρ, given by:

$$\rho = \left(\frac{A}{Z}\right) m_p N_e, \tag{13.6}$$

for a white dwarf composed of elements of atomic number Z, the number of protons, and atomic mass number A. The mass number of helium is $A = 4$, that of carbon is $A = 12$ and the mass number of oxygen is $A = 16$, the number of protons and neutrons. Equivalently, $A/Z = N_e/N_p = \mu_e$, the ratio of electron number to proton number, since the number of electrons in an atom is equal to the number of nucleons or protons plus neutrons. For helium, carbon and oxygen $A/Z = 2$ or $Z/A = 0.5$. We also have $A/Z = \rho/(m_p N_e)$. In this expression, m_p is the proton mass, or $m_p = 1.67262 \times 10^{-27}$ kg, and N_e is the electron number density, which is related to the mass density by:

$$N_e = ZN_i = \frac{Z\rho}{Am_P} \approx \frac{0.5\rho}{m_p}, \tag{13.7}$$

where N_i is the ion number density.

Why does the core collapse stop at the white dwarf stage? In other words, what is holding up the star? There are no internal nuclear reactions to provide energy, generate heat, and create pressure to oppose gravity. As the white dwarf radiates away the heat left over from its former life in a red giant star, it eventually might cool down to a temperature of absolute zero, and there would be no motion or thermal energy left to support the white dwarf star.

When the material cooled enough, it could be expected that the electrons would return to their former orbits around the nuclei, making larger atoms, which would force the nuclei apart and make the white dwarf expand in size. However, the stars have no internal energy to push against gravitation and accomplish this feat; therefore, it seemed that a white dwarf star could not become that cold.

This paradox was not resolved until the development of quantum mechanics and the realization that it is the electronic properties of the crushed matter that hold up a white dwarf. The high-speed motions of the densely packed electrons, rather than the nuclei, produce an outward pressure that holds the gravitational forces at bay. These motions are not due to the star's internal temperature or heat; in fact, the internal pressure of a white dwarf star is unaffected by temperature.

The quantum–mechanical description of a very dense, crushed state of matter is statistical, and it is related to the quantum numbers that specify the state of subatomic particles. That is, the properties of the particles, such as energy or location, can take on only specific quantized values and no others. This situation is related to two principles that govern the quantum state of the very small: (1) The *uncertainty principle* that states that at any given time we cannot know exactly both where a particle is and where it is going; and (2) the *exclusion principle* that forbids the existence of two or more particles in exactly the same quantum state.

The German physicist Werner Heisenberg (1901–1976) first stated the uncertainty principle (Heisenberg 1927). It states that the more we know about the location of a subatomic particle, the less we know about its momentum and velocity, and vice versa. In an alternative interpretation, the more we know about the energy of a subatomic particle, the less we know about when it had that energy, and vice versa. Mathematically, the product of the uncertainties, in location and momentum or energy and time, equals the Planck constant *h*.

The Austrian physicist Wolfgang Pauli (1900–1958) proposed the exclusion principle (Pauli 1927), which states that two identical subatomic particles cannot occupy the same quantum state at the same time. It applies to electrons, protons, and neutrons, and it dictates how electrons behave in an atom, occupying their various energy levels. Each electron in an atom has its own space, which prevents the electrons from either joining together in the same location or falling into their atomic nucleus. The exclusion principle ensures the very existence of atoms.

It also means that the free electrons in a white dwarf star, which are not attached to atoms, cannot be in precisely the same place at the same time. They instead resist being squeezed into one another's territory, darting away at high-speeds just to keep their own space. This provides the pressure of the crushed state of matter, which is caused by the electrons' resistance to crowding.

The Italian physicist Enrico Fermi (1901–1954) first worked out the statistical description of a large number of identical subatomic particles based on the exclusion principle (Fermi 1926, 1928). He specified the conditions in which all of the particles have the least possible energy without violating Pauli's exclusion principle, which specifies that these particles cannot all occupy the very lowest energy, called the *ground state*, at the same time. In Fermi's solution, all of the states up to a certain limiting energy can be occupied, and all of those above that energy are not occupied.

Under such conditions the collection of subatomic particles, or gas, is said to be *degenerate*. In mathematics, a degenerate case is a limiting one in which a class of objects changes its nature so as to belong to another, usually simpler, class; for example, a point is a degenerate circle.

As shown by Ralph H. Fowler (1889–1944), who was Eddington's colleague at Cambridge University, it is the degenerate pressure of the electron gas that supports a white dwarf star. Fowler applied Fermi's statistical description to such a dense star, showing that its electrons are completely degenerate. They produce an outward push, known as *degeneracy pressure*, to keep their own space and support the star (Fowler 1926). This kind of pressure is proportional to the $5/3$ power of mass density, and accordingly increases rapidly with it.

Not only is the electron degeneracy pressure strong enough to withstand the crushing gravity of a white dwarf star; it is also independent of the temperature of the electrons and involves no nuclear reactions. Because the pressure does not depend on temperature, it will persist even if the star cools to absolute zero, without the electrons ever rejoining the nuclei. As Fowler showed, the individual electrons in the crushed matter, even at a temperature of absolute zero still would have a kinetic energy comparable to the thermal energy of particles in an expanded gas with a temperature as large as 10 million K.

Because the equation of state of the degenerate electron gas is unaffected by temperature, any heating by hypothetical nuclear reactions will increase the temperature and rates of those reactions. The temperature would continue increasing until the star exploded, so we conclude that white dwarf stars do not shine by nuclear reactions. Their light must come from the slow leakage of the heat contained in the nondegenerate nuclei. Eventually, the white dwarf star will fade into a gigantic black molecule, a frozen star.

When the electrons in a white dwarf star are moving at non-relativistic speeds, considerably less than the speed of light, the degenerate electron pressure, P_e, is given by:

$$P_e = \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{5m_e} N_e^{5/3}. \tag{13.8}$$

where the Planck constant $h = 6.626 \times 10^{-34}$ J s, the electron mass $m_e = 9.109 \times 10^{-31}$ kg, and N_e is the electron density. Notice that the degenerate electron pressure does not depend on the temperature. An equivalent expression for the equation of state of non-relativistic degenerate electron pressure is (Fowler 1926).

$$P_e \approx \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}, \tag{13.9}$$

for a fully ionized gas composed of elements of atomic number Z and atomic mass number A. The proton mass $m_P = 1.67262 \times 10^{-27}$ kg. For a white dwarf star, which is composed of helium, carbon and oxygen with $Z/A = 0.5$, and:

$$P_e \approx 3.074 \times 10^6 \rho^{5/3} \text{ Pa for } \rho \ll 10^{10} \text{ kg m}^{-3} \tag{13.10}$$

Example: Gas pressure, degenerate electron pressure, and magnetic pressure in a white dwarf

A white dwarf star has a mass density of $\rho = 10^9$ kg m^{-3} and an initial temperature of $T = 10^7$ K. The gas pressure, P_G, is given by:

$$P_G = N_i kT = \frac{\rho}{A m_p} kT, \tag{13.11}$$

where N_i is the ion number density, the mass number $A = 4$ for helium, $A = 12$ for carbon and $A = 16$ for oxygen, the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, and the proton mass is $m_p = 1.67262 \times 10^{-27}$ kg. Using these numbers for A = 4, the gas pressure is $P_G \approx 2 \times 10^{19}$ Pa. The non-degenerate electron gas pressure is $P_e = 3.074 \times 10^6\ \rho^{5/3} \approx 3 \times 10^{21}$ Pa. So even on formation, before a white dwarf star has cooled, the degenerate electron pressure is about one hundred times greater than the gas pressure.

Since the white dwarf has no nuclear fusion reactions to supply heat, it gradually cools. The effective temperature of Sirius B, for example, is now 2.5×10^4 K, so its gas pressure is about 5×10^{16} Pa.

Since magnetic flux is conserved in gravitational collapse, the magnetic field strength B is amplified by the inverse square of the radius, to as much as $B \approx 10^4$ tesla for a white dwarf star which is 100 times smaller than the Sun. The magnetic pressure, $P_B = B^2/(2\mu_0) \approx 4 \times 10^{13}$ Pa, is much smaller than the degenerate electron gas pressure, where the permeability of free space is $\mu_0 = 1.2566 \times 10^{-6}$ N A^{-2}.

By 1929, the Estonian astrophysicist Wilhelm Anderson (1880–1940) demonstrated that the electrons in highly compressed, degenerate matter begin to attain velocities on the order of the speed of light, and that in this case the variation of the electron mass with velocity must be taken into account by using the equations of *Special Relativity*. For a relativistic degenerate electron gas the equation of state is (Anderson 1929; Stoner 1930):

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \rho^{4/3}, \tag{13.12}$$

which differs from the non-relativistic case in that the electron mass does not appear and the pressure varies as the $4/3$ power of the mass density, ρ, rather than the $5/3$ power for a non-relativistic degenerate electron gas. Using $Z/A = 0.5$ and evaluating $h = 6.626 \times 10^{-34}$ J s^{-1}, $c = 2.9979 \times 10^8$ m s^{-1}, and $m_p = 1.67262 \times 10^{-27}$ kg, we obtain

$$P_e \approx 0.49 \times 10^{10} \rho^{4/3} \text{ Pa} \quad \text{for } \rho \gg 10^{10} \text{ kg m}^{-3}, \tag{13.13}$$

for the relativistic, degenerate electron gas. The non-relativistic and relativistic degenerate electron pressures become equal at a mass density of $\rho \approx 4 \times 10^9$ kg m^{-3}.

13.4.2 Radius and Mass of a White Dwarf

A higher-mass white dwarf will be squeezed into a smaller space by its gravity, so the star's radius decreases with increasing mass. For a large enough mass, we might imagine that the star's radius would become very small, perhaps even shrinking to almost zero; however, this is preposterous and there must be a limit to the mass.

An approximate expression for the radius, R_{WD}, of a non-relativistic white dwarf star of mass, M, is:

$$R_{WD} \approx \frac{h^2}{20 m_e m_p^{5/3} G} \left(\frac{Z}{A}\right)^{5/3} M^{-1/3} \approx 4.82 \times 10^{16} M^{-1/3} \text{ m} \tag{13.14}$$

where h is the Planck constant, m_e is the electron mass, m_p is the proton mass, $Z/A = 0.5$ and the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. This means that

$$R_{WD} \approx 3.8 \times 10^6 (M_\odot/M)^{1/3}\, \text{m}, \tag{13.15}$$

where the solar mass $M_\odot = 1.989 \times 10^{30}$ kg; see Provencal (1998) for observational tests of the mass-radius relation of white dwarf stars.

As the white dwarf shrinks in size, its mass and mass density become higher as does the degeneracy pressure of its electron gas. Under extreme compression, however, the average speed of the electrons increases and eventually approaches the speed of light. This means that there is an upper limit to the mass that can be supported by their pressure. This makes common sense, since the electrons will move at greater speeds with increasing stellar mass and density, although they cannot move faster than the speed of light.

The limiting mass for a white dwarf star is determined under high-speed, relativistic conditions, when the electrons approach the speed of light. As both the German-Estonian astrophysicist Wilhelm Anderson (1880–1940) and the English physicist Edmund C. Stoner (1899–1968) demonstrated, a star can contract only until the gravitational potential energy becomes insufficient to balance the increase in the kinetic energy of the electrons, which occurs for stellar masses of about 1 solar mass (Anderson 1929; Stoner 1929, 1930).

The two forms of energy are roughly equal at:

$$P_e \left(\frac{4\pi R_{WD}^3}{3} \right) \approx \frac{G M_c^2}{R_{WD}}, \tag{13.16}$$

or at a critical mass M_c given by

$$M_c \approx \left[\frac{4\pi P_e R_{WD}^4}{3G} \right]^{1/2}, \tag{13.17}$$

where P_e is the relativistic electron pressure, R_{WD} is the radius of the white dwarf star, and the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Assuming a mass density of $\rho = 10^9$ kg and $R_{WD} = 6 \times 10^6$ m, we obtain $M_c \approx 10^{30}$ kg $\approx M_\odot = 1.989 \times 10^{30}$ kg.

Thus, for stellar masses larger than about 1 solar mass, there can be no equilibrium white dwarf configurations. More massive stars collapse under their own weight to form a neutron star or a black hole at the endpoints of stellar evolution.

During his voyage from India to Cambridge University for his graduate studies, the Indian astrophysicist Subrahmanyan Chandrasekhar (1910–1995) derived the detailed equation of state of a degenerate electron gas in the extreme relativistic limit (Chandrasekhar 1931). The exact solution for the critical mass is given by

$$M_c = 0.21 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p \,. \tag{13.18}$$

or for a white dwarf star with $Z/A = 0.5$,

$$M_C \approx 1.46\, M_{\odot}, \tag{13.19}$$

where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg.

Although both Anderson and Stoner previously called attention to the existence of this upper mass limit, it is known now as the *Chandrasekhar limit*, because he was the first to derive the detailed equilibrium conditions in which degenerate electron gases support a dense star's gravity.

In 1983, the Nobel Prize in Physics was awarded equally to Chandrasekhar, for his theoretical studies of the physical processes of importance to the structure and evolution of the stars, and to William A. "Willy" Fowler (1911–1995) for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe.

13.5 Exploding Stars

13.5.1 Guest Stars, the Novae

For at least 2,000 years, astronomers, hunters, mariners, and others familiar with the brightest stars must have been amazed by a *nova*, or "new star", that would appear suddenly at a place in the sky where no star previously had been seen. For a few days, the nova might be among the brightest stars in the dark night sky. But then the star would begin to fade away, and in about a month it would disappear back into invisibility, without a trace. The Chinese called them "guest stars" or "visiting stars" because they were not permanent members of the celestial sphere, instead appearing suddenly and then departing abruptly, like uninvited guests.

Every 20 years or so, a nova is luminous enough and close enough to be conspicuous without the aid of a telescope, attracting the attention of both astronomers and the superstitious. Like good wine, they are specified by the year of their occurrence, and their location is specified by the constellation in which they appear. Nova Aquilae 1918 was the brightest of the twentieth century, at apparent visual magnitude −1.1; Nova Herculis 1934 has historical importance; and Nova Cygni 1992 was the brightest nova in recent history.

Something of substance had to be at a nova's location before it appeared, to supply the energy of its outburst. By the mid-twentieth century, it was realized that this "mysterious something" was an inconspicuous star that previously had been recorded during systematic surveys of the dark night sky, using large telescopes and photographic exposures to record the dim light of faint stars.

Table 13.5 Physical properties of some novae[a]

Star Name	Year	m_{max}	L_{max} ($L_{\odot}$)	D (ly)	P_{orb} (hours)	M_1 ($M_{\odot}$)	M_2 ($M_{\odot}$)	V_{exp} (km s^{-1})
Classical novae								
GK Persei	1901	+0.2	$10^{5.3}$	1,500	47.92	0.9	0.25	1,200
V603 Aquilae	1918	−1.4	$10^{5.6}$	800	3.31	0.66	0.2	265
DQ Herculis	1934	+1.4	$10^{3.9}$	316	4.65	0.62	0.44	315
V1974 Cygni	1992	+4.4	$10^{4.9}$	10,430	19.53	0.83	–	–
Dwarf nova								
SS Cygni	b	8.3	≈10	≈541	6.603	0.60	0.40	–
Recurrent nova								
RS Ophiuchi	c	4.5	0.1	≥2,000	455.7 d	1.4	–	–

[a] Maximum visual magnitude, m_{max}, and maximum luminosity L_{max} in units of the Sun's luminosity $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, distance in light-years, abbreviated ly, orbital period, P_{orb}, white dwarf mass, M_1, and companion mass, M_2 in units of the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and expansion velocity V_{exp}

[b] The dwarf nova SS Cygni undergoes frequent and regular outbursts every 7–8 weeks, with an apparent visual magnitude of $m_{min} = 12.2$ at minimum and $m_{max} = 8.3$ at maximum. More than 800 outbursts have been observed since its discovery in 1896

[c] The recurrent nova RS Ophiuchi erupted in 1898, 1907, 1933, 1945, 1958, 1967, 1985 and 2006. It is a binary system with a red giant star in a 455.7-day orbit around a white dwarf star of mass near the Chandrasekhar limit

So, the bright, short-lived novae are neither new nor temporary but instead existing stars that suddenly increase in brightness by as much as 100,000 times, returning to their original states after several months or a few years.

Telescopic observations also have enabled the detection of faint dwarf novae. They brighten repeatedly, on a time-scale from days to decades, although by a smaller amount and with lower luminosity than the classical novae whose explosive outbursts are visible without the aid of a telescope. More than 900 outbursts of the dwarf nova SS Cygni, for example, have been observed since its discovery in 1896. It varies from apparent visual magnitude of 12.2 at minimum to 8.3 at maximum, every 7–8 weeks.

The properties of some of the classical novae, visible by the unaided eye, are listed in Table 13.5 with the dwarf nova SS Cygni.

13.5.2 What Makes a Nova Happen?

A major new understanding of novae occurred in the 1950s and 1960s when a few American astronomers began to examine the total light and spectra of ex-novae, long after the intense light of the nova outburst had faded to a relatively weak level. It then was discovered that a nova is not one star but rather two stars very close together. Twenty years after the 1934 eruption of Nova Herculis, for example, Merle F. Walker (1926–) found that this nova is an eclipsing binary system with a remarkably brief orbital period of only 4.6 h (Walker 1954). The shortness of the period indicated that the two stars are very close together, practically touching one another. Nearly a decade later, Walker was able to show that Nova T Aurigae 1891 also is an eclipsing binary system with a short period, of 4.8 h (Walker 1963).

Alfred H. Joy (1882–1973) had examined the absorption and emission lines of the dwarf nova SS Cygni, identifying it as a binary-star system with a short orbital period of 6.6 h (Joy 1956). The emission lines originated in a blue-white dwarf star, whereas the absorption lines came from a red main-sequence star, the size of which was estimated to be roughly half the distance between the two stars. They were so close to one another that mass could spill from the red star into the blue star, and the nova process might be related to this mass flow.

In the meantime, Robert P. Kraft (1927–) demonstrated that membership in a short-period binary system is a necessary condition for a star to become a nova of either the classical or dwarf type (Kraft 1964). One of the stellar pair was usually a blue-white dwarf star; the other red component was usually a cool main-sequence star of spectral type G, K, or M or one of the same spectral type that is aging and expanding into a red giant. The short orbital period indicated that the two stars are so close that hydrogen flows from the red companion onto the white dwarf, reviving the “dead” star and giving it a brief new life in a cataclysmic nuclear explosion. The term *cataclysmic variable star* is used now to designate such close binary star systems, in which one of the components – conventionally called the *primary star* – is a white dwarf that accretes matter from its secondary companion. The category includes classical, dwarf, and recurrent novae.

Example: Dwarf nova SS Cygni

SS Cygni is a double star system with an orbital period of $P = 0.275$ days $= 6.603$ h $= 23{,}770$ s. It consists of a white dwarf of mass $M_1 = 0.60\ M_\odot$ and a main sequence star of mass $M_2 = 0.40\ M_\odot$, where the Sun’s mass $M_\odot = 1.989 \times 10^{30}$ kg. The linear separation, a, of the two stars can be inferred from Kepler’s third law:

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} \tag{13.20}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}. Using $M_1 + M_2 = M_\odot$ and solving for the linear separation we obtain $a \approx 1.24 \times 10^9$ m $\approx 1.78\ R_\odot$, where the Sun’s radius $R_\odot = 6.955 \times 10^8$ m. The white dwarf star has a radius of about $0.01\ R_\odot$ and a low-mass main-sequence star of spectral class K5 might have a radius of about $0.8\ R_\odot$, so the two stars are practically touching each other. It therefore is not surprising that mass flows from the main-sequence star onto the white dwarf star, resulting in outbursts from SS Cygni every 7–8 weeks.

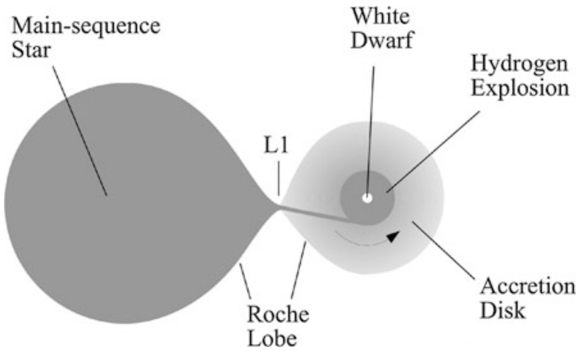

Fig. 13.4 **Nova** A classical nova is a thermonuclear explosion that occurs on the surface of a white dwarf star that is in a close orbit with a main-sequence star. The strong gravitational attraction of the white dwarf pulls its nearby companion into an elongated shape, the outer edge of which is designated the *Roche lobe*. Some of the hydrogen in the outer atmosphere of the main-sequence star spills over at the inner Lagrangian point, denoted L1, where the gravitational pull of the two stars is equal. This hydrogen spirals into a rotating accretion disk and down to the white dwarf, igniting an explosion, like a colossal hydrogen bomb. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

The immense gravity of the white dwarf star distorts the shape of its nearby companion, stretching it into an elongated configuration (Fig. 13.4). This is a tidal effect in which the side of the companion star that is nearest the white dwarf is pulled toward it, and the companion's center is pulled away from the side that is farthest from the white dwarf. Material that is pulled close enough to the white dwarf can pass outside the gravitational control of the red companion and go into orbit around the white dwarf, eventually spiraling into it.

The region in which the companion star retains gravitational control of its substance is known as its *Roche lobe*, named after the French astronomer Édouard A. Roche (1820–1883) who described it more than a century ago (Roche 1849/1850/1851). When the outer hydrogen atmosphere of the close companion star overflows its Roche lobe, it does not fall directly into the white dwarf star. In the absence of an intense magnetic field, the gas is accreted into an orbiting disk that resembles the proto-planetary disks that circle protostars (Sect. 12.3); the gas streams onto the magnetic poles when there is a strong magnetic field directing the flow.

In either case, the hydrogen is pulled slowly into the white dwarf, which compresses and heats the gas to very high temperatures. The accumulating hydrogen is a potential bomb that remains harmless until detonated.

A thin layer of hydrogen slowly builds up on the white dwarf as its companion keeps feeding matter into it. The pressure and temperature rise until hydrogen fusion suddenly is ignited, at about 10 million K, or 10^7 K. A runaway thermonuclear explosion then occurs, and a white dwarf that normally would cool and fade away if left alone suddenly shines as brightly as 100,000 Suns (Starrfield et al. 1974, 1985). The explosion subsides after a few weeks, but the overflow of the companion continues. Gallagher and Starrfield (1978) have reviewed the theory and observation of classical novae.

The envelope of the white dwarf star is thrown off during the nova explosion, at high speeds of up to several thousand kilometers per second. However, despite the violence, the amount of material ejected is only about 0.000005 of a solar mass. The white dwarf therefore can retain its stability, and potentially generate additional novae as its companion continues to feed matter into it. An example of such a recurrent nova is RS Ophiuchi, which has exploded into a bright nova state at least six times between 1898 and 2006.

Eventually, the hydrogen may build up until it pushes the white dwarf above its limiting mass; the entire star then explodes, not just the thin outer atmosphere. This is a supernova that suddenly and unpredictably brightens with the light of 1 billion Suns.

13.5.3 A Rare and Violent End, the Supernovae

On rare occasions, an entire star is annihilated and suddenly becomes so bright that it can be seen easily in daylight rather than just at night like the novae. The Chinese emperor's astronomers in the Sung dynasty recorded one on July 4, 1054, near the constellation now known as Taurus, the Bull. The Chinese chronicles indicate that the new star initially was brighter than everything in the night sky except the full Moon; could be seen during the daytime for three weeks after its first appearance; and remained visible in the night sky for 22 months, without the aid of telescopes, which had not yet been invented. The "new" star of 1054 was definitely far brighter and longer lasting than any other guest star.

More than four centuries passed before other exceptionally brilliant guest stars were noticed, and this time they shook the very foundations of European thought. As Aristotle taught, heavenly bodies were supposed to be eternal, pure, changeless, incorruptible, and perfect, unlike anything on the Earth. Yet, in a span of just 32 years, two new daytime stars could be seen by almost anyone in the Earth's Northern Hemisphere who happened to look up. Each star remained fixed in the heavens for about a year, and then disappeared from view.

Both events were also discovered at a time before telescopes were invented. The Danish astronomer Tycho Brahe (1546–1601) witnessed the first visitor in 1572, which initially was brighter than the planet Venus. Perhaps because of the excitement caused by his discovery, Brahe built an observatory in which detailed measurements were made of stars and the planets. Johannes Kepler (1571–1630), who used Brahe's observations of planets to determine the laws of their motion, spied another bright new star as it lit up the heavens in 1604.

The exceptionally brilliant guest stars of 1054, 1572, and 1604 were much brighter and longer lasting than the conventional novae known at the time or

Table 13.6 Historical supernovae visible with the unaided eye[a]

Explosion Date	m_{max}	M_{max}	L_{max} ($L_{\odot}$)	Visible (months)	Type	Remnant name	D (ly)	θ ($'$)	R (ly)
SN 185	−8.0	−20.2	$10^{10.0}$	8	Ia	RCW 86	9,100	45	56
SN 386	+1.5	–	–	3	–	–	–	–	–
SN 393	−1.0	−11.0	$10^{6.3}$	8	II/Ib	*b*	3,000	70	30
SN 1006	−7.5	−19.2	$10^{9.6}$	21	Ia	PKS 1451−41	7,200	31	32
SN 1054	−6.0	−17.5	$10^{8.9}$	22	II	Crab nebula	6,500	7	6.6
SN 1181	−1.0	–	–	6		*c*	>8,000	–	–
SN 1572[d]	−4.0	−16.4	$10^{8.5}$	16	Ia	Tycho	11,500	8.3	14
SN 1604[e]	−2.5	−16.4	$10^{8.5}$	12	Ia	Kepler	20,000	3.2	9

[a] Maximum apparent visual magnitude, m_{max}, maximum absolute magnitude, M_{max}, maximum luminosity, L_{max}, in units of the Sun's luminosity $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, length of visibility to the unaided eye, supernova type, supernova remnant name, distance D in light-years, abbreviated ly, angular diameter θ of supernova remnant in minutes of arc, denoted $'$, and remnant radius, R, in light-years

[b] Supernova remnant RX J1713.7-3946

[c] Radio source 3C 38. The radio and x-ray pulsar J0205+6449 may not be associated with SN 1181

[d] Supernova explosion also known as Tycho's star, supernova remnant 3C 10

[e] Supernova explosion also known as Kepler's star, supernova remnant 3C 358

subsequently. They therefore have been dubbed *supernovae*, a term coined by the Swiss astronomer Fritz Zwicky (1898–1974).

Now that we know the distances, a supernova also refers to a stellar outburst the maximum luminosity of which exceeds by factors of several billions the luminosity of our Sun (Table 13.6). This is millions of times the peak luminosity of a classical nova. Moreover, unlike novae, the new breed of exploding stars has nothing conventional to return to after the explosion. They require so much energy that the mass of an entire star is annihilated.

In 1934, Walter Baade (1893–1960) and Fritz Zwicky communicated to the United States National Academy of Sciences a remarkable pair of papers on supernovae (Baade and Zwicky 1934a, b). In one paper, they showed that the enormous energy emitted in the supernova process corresponds to the total conversion of an appreciable fraction of a star's mass into energy. In the second paper, they predicted that a supernova explosion will accelerate charged particles to very high energies and that supernovae that occur only once in a millennium can account for the energetic cosmic-ray particles that now rain down on the Earth's atmosphere from all directions in outer space. In this more speculative paper, the two Caltech astronomers also said that the collapsing core of the explosion might become a neutron star, of very small radius and extremely high mass density. It took a half-century for astronomers to realize that Baade and Zwicky were correct on all counts.

The initial evidence for supernovae was extraordinarily sparse. In the Milky Way, they are seen at intervals of roughly 100 years, which is about 3,000 times less common than dwarf novae; so it might take centuries before the next supernova could be observed in the Milky Way. Fortunately, it was found that supernovae occur more frequently in the many spiral nebulae outside our Milky Way.

In the early decades of the twentieth century, astronomers discovered numerous faint novae in spiral nebulae, which suggested that the spirals were very distant if these "new" stars were like the classical novae seen in the Milky Way. At that distance, an exceptionally bright nova, observed in the nearest spiral nebula Andromeda in 1885, would have the luminosity of a supernova.

When the enormous distances to the spiral nebulae were confirmed, it was realized that they are not nebulae at all but instead galaxies that each contain about 100 billion stars (Sect. 14.2). Moreover, at maximum, a supernova briefly will outshine 1 billion stars in the same galaxy. Light from hundreds of supernovae in distant galaxies might be on its way to us now. Their light could take many thousands or even billions of years to travel to the Earth.

Because there are many of these extragalactic spirals, now called galaxies, Zwicky realized that a systematic photographic survey quickly would catch at least one star in the act of supernova explosion – and he was right. He detected the first one in 1937, with a camera attached to a modest telescope placed on the roof of a building at Caltech (Zwicky 1937). This supernova occurred in the spiral galaxy NGC 4157, now known to be about 55 million light-years away, so the actual explosion occurred 55 million years before Zwicky saw it, the time it took for its light to travel the vast distance separating the galaxy from us.

By observing the spectra and fading light of supernovae in distant galaxies, astronomers subsequently found that there are two methods for stars to come to such a violent end. The German-American astronomer Rudolph Minkowski (1895–1976), for example, divided the supernovae into two categories, denoted Type I and Type II, distinguished by the absence or presence of hydrogen in their spectra (Minkowski 1941). Type I was subsequently divided into three categories. Type Ia exhibits a strong absorption feature of singly ionized silicon at a wavelength of 615 nm in their spectra near peak light; Types Ib and Ic do not display this spectral feature. Modern identifying characteristics of supernovae of different types are given in Table 13.7, including the important Type Ia and Type II. Filippenko (1997) has reviewed the optical spectra of supernovae.

The peak light output from a Type I supernova typically is one or two orders of magnitude more luminous than that of the fainter Type II supernovae, which fade more slowly. We now know that the decay of radioactive elements produced during high-temperature Type II explosions heats the expanding gas and produces the optically visible light.

Walter Baade used Tycho's observations of the decaying light from the brilliant 1572 supernova to demonstrate that it is consistent with the superluminous emission of a supernova of Type I (Baade 1945; Van Den Bergh 1993); Kepler's meticulous observations of the 1604 event indicated a Type II light variation. Both

Table 13.7 Characteristics of supernova types[a]

Characteristic	Type Ia	Type Ib	Type II
Optical spectrum	No hydrogen Si II at 615.0 nm	No hydrogen He I at 587.6 nm	Hydrogen present at 656.3 nm
Maximum luminosity	$10^{9.8}\ L_{\odot}$	$\approx 10^{9.1}\ L_{\odot}$	$10^{9.1}\ L_{\odot}$
Ejection velocity	$\geq 10^4$ km s^{-1}	$\geq 10^4$ km s^{-1}	$\leq 10^4$ km s^{-1}
Ejected mass	$\approx 1\ M_{\odot}$	$\approx 1\ M_{\odot}$	$\approx 5\ M_{\odot}$
Progenitor star	White dwarf	Wolf-Rayet	Supergiant
Progenitor star mass	$1\ M_{\odot}$	4–$7\ M_{\odot}$	$\geq 8\ M_{\odot}$

[a] Maximum luminosity in units of the Sun's luminosity $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, and mass values in units of the Sun's mass $M_{\odot} = 3.854 \times 10^{30}$ kg

of these supernovae were observed before telescopes were invented and spectra were obtained from cosmic objects.

Although the two types of supernovae release comparable amounts of total energy during their explosion, there is a radical difference in the mass and kinetic energy of their ejected material. The expanding shells of Type Ia contain roughly 1 solar mass of material, whereas those of Type II events are about five times more massive. The large mass difference suggests that the progenitor stars of Type Ia supernovae are less massive than those of Type II. There is comparatively little discussion of the reasons for Type Ib and Ic supernovae in the scientific literature. Like supernovae of Type II, they are probably massive stars that have run out of nuclear fuel at their centers. Type Ib supernovae may be related to the core collapse of massive Wolf-Rayet stars that have lost hydrogen by strong winds.

13.5.4 Why do Supernova Explosions Occur?

Both types of supernovae involve the explosive conversion of a star's entire mass into energy but by different physical mechanisms. A Type Ia stellar explosion is due to external causes. It involves a white dwarf star pushed into nuclear explosion by too much mass overflow from a nearby companion. The other, Type II, supernova is an internal event, which occurs during the gravitational collapse of the iron core of a massive star that has depleted all of its energy. The nuclear supernovae of Type Ia and gravity-powered ones of Type II are believed to occur with about equal likelihood in the Milky Way, at the rate of 1 every 50–100 years.

Weiler and Sramek (1988) and Trimble (1982, 1983) have reviewed our knowledge of supernovae and supernova remnants. Woosley and Weaver (1986) have reviewed the physics of supernova explosions. Bethe (1990) has reviewed our understanding of supernova mechanisms.

Hillebrandt and Niemeyer (2000) review Type Ia supernova explosion models. Smartt (2009) has reviewed the progenitor stars of Type II supernovae. Heger et al.

(2003) describe how massive single stars end their lives. Langer (2012) has reviewed the pre-supernova evolution of both massive single and massive binary stars, and Paczynski (1971) has reviewed evolutionary processes in close binary systems. The next two subsections of the text provide a general description of the two types of supernova explosions.

13.5.5 When a Nearby Star Detonates Its Companion

Like the novae, there is one type of supernova that gets assistance from the outside, being pushed over the edge into explosion, shattering an entire star. Such a supernova – now known as Type Ia and characterized by the absence of emission from hydrogen – occurs in a close binary-star system, with a white dwarf star – the shrunken dense remnant of a former low-mass star – circling a main-sequence star.

The English astrophysicist Fred Hoyle (1915–2001) and his American colleague William A. "Willy" Fowler (1911–1995) introduced the detailed mechanisms for this type of supernova (Hoyle and Fowler 1960; Fowler and Hoyle 1964). When the nearby companion star expands as a result of its normal evolution, hydrogen from its outer atmosphere spills onto the white dwarf. The overflow, for example, might happen when the ordinary visible companion runs out of core hydrogen fuel and swells into a red giant star. As the hydrogen overflow continues, a steady increase in the mass of the white dwarf will compress and heat the star. As the increasing mass approaches the upper mass limit for a white dwarf star at 1.46 solar masses, the rise in internal temperature ignites a nuclear explosion.

Because the white dwarf is supported against gravity by temperature-independent, degenerate electron pressure, adding heat to the star's interior increases the temperature but not its pressure; therefore, the white dwarf does not expand and cool in response. Instead, the increased temperature initiates the fusion of carbon nuclei in a runaway nuclear explosion that obliterates the star in a few seconds and releases about 10^{49} J in energy.

In other words, the added mass detonates a carbon bomb, triggering explosive nuclear reactions that quickly spread throughout the star and completely shatter it. The entire star explodes into a Type Ia supernova that shines with the light of billions of Suns, and there is nothing left.

Because every one of these explosions is triggered at the same mass limit, under similar conditions, and also because the star is completely destroyed, Type Ia supernovae are expected to produce about the same maximum light output every time they occur. The typical absolute visual magnitude is -19.3, or about 5 billion times the luminosity of the Sun, with little variation. Astronomers use this bright, uniform luminosity as a "standard candle" to measure the distances to their host galaxies located far beyond the Milky Way in the remote parts of the observable universe, thereby determining the pace of its expansion. They have shown that a repulsive force, called dark energy, is making the universe expand at an accelerating rate (Sect. 15.6).

13.5.6 Stars that Blow Themselves Up

There is more than one way to explode a star, and some of the supernovae are gravity-powered, catastrophic outbursts from very old massive stars. This method of shattering a star applies to an isolated star with the right mass – between about 8 and 20 times the Sun's mass – that blows itself apart. This Type II supernova, which exhibits hydrogen in its spectra, follows the creation of an iron core within an evolving, massive supergiant star. Smartt (2009) has reviewed the progenitors of such core-collapse supernovae.

The material in the core of such a massive supergiant star is not degenerate, in the mathematical sense used for white dwarf stars; nuclear reactions proceed in the advanced burning stages at ever-increasing central temperatures until an iron core is produced and all of the available nuclear fuel has been exhausted. Deprived of these resources, the iron core collapses under its own weight into a neutron star or black hole in less than 1 s, and an explosion blows away the rest of the in-falling matter.

When the iron nuclei in the core of such a star are pushed together, no energy is released. The iron does not burn, regardless of how hot the star's core becomes. So, there is no longer any energy being generated to sustain the star's structure. Then, a massive star, having burned brightly for perhaps 10 million years, can no longer support its own crushing weight and the iron core collapses.

The central iron core can be crushed into a ball no bigger than New York City in less than 1 s, accruing energy from its in-fall. Electrons are squeezed inside the iron nuclei, combining with their protons to make neutrons. The material is compacted to nuclear density, and the center collapses to form a neutron star. If the collapsing core is more massive than about 3 solar masses, however, the collapse proceeds to the formation of a black hole.

Having lost the supporting core, the surrounding material first plunges in toward the center. When reaching mass densities approaching that of an atomic nucleus, the collapsing core bounces back and a powerful shock wave pushes out against the rest of the star. With the help of a dense shower of neutrinos produced in the collapsing core, the star's outer layers are torn apart and expelled into deep space at supersonic speeds. The doomed star suddenly increases in brightness 100 million fold, becoming a Type II supernova that briefly outshines up to 1 billion of its neighboring stars combined (Fig. 13.5).

This type of supernova hurls into surrounding space all of the elements synthesized inside the star and residing in shells surrounding the iron core before its collapse. During the high-temperature explosions, the supernova also produces vast amounts of other heavy elements, including radioactive elements such as uranium. Newly formed radioactive nickel, for example, eventually decays into

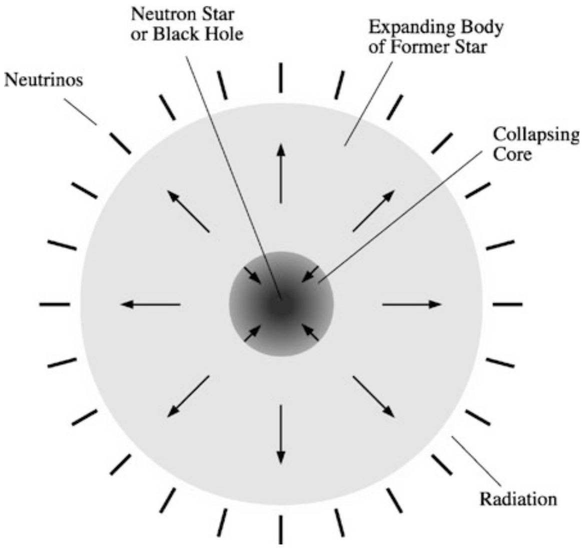

Fig. 13.5 Type II supernova In this type of supernova explosion, an isolated star blows up and its shattered remains are propelled into surrounding space. Radio and x-ray radiation from the expanding supernova remnant can be observed for thousands of years after the explosion. The core of the star is compressed by gravitational contraction into a neutron star or a black hole. Neutrinos emitted from the collapsing core remove most of the supernova energy and assist shock waves in pushing the stellar remains into an expanding remnant. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

iron, producing most of the iron now found in the universe. Such a supernova occurred within a nearby satellite galaxy of the Milky Way in 1987, which led to new insights about how these explosions occur.

13.5.7 Light of a Billion Suns, SN 1987A

For more than three and a half centuries, nobody was fortunate enough to see a supernova with the unaided eye. Then, late in the evening of February 24, 1987, astronomers discovered one (Fig. 13.6), which was designated SN 1987A (SN is for supernova; 1987A denotes the first one discovered that year). The star exploded 168,000 years before that night in the Large Magellanic Cloud, one of the two satellite galaxies of the Milky Way visible from the Earth's Southern Hemisphere. It generated an intense burst of light that peaked in visual brightness at the third magnitude.

Astronomers at the Las Campanas Observatory, a barren mountaintop near La Serena, Chile, were the first to notice the new star when Ian Shelton (1957–)

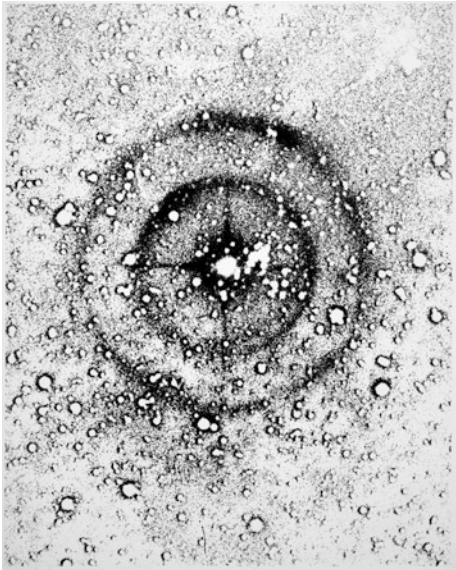

Fig. 13.6 Light echoes from SN 1987A Two complete rings of light surround the exploded star SN 1987A in this negative image taken with the 3.9 m (153.5 inch) Anglo-Australian Telescope on 15 July 1988. The initial flash of light from the supernova explosion has been reflected off clouds of interstellar dust and observed 14 months after the explosion was brightest, somewhat like an echo of sound. These light echoes arise in two thin sheets of microscopic dust grains located about 470 light-years (*inner ring*) and 1,300 light-years (*outer ring*) in front of the supernova. The rings have been made more prominent by photographically subtracting an image taken 3 years before the supernova exploded, canceling much that existed previously. Stars, however, are still visible as faint haloes. (Courtesy of David Malin and the Anglo-Australian Observatory.)

photographed it at 3 o'clock in the morning using a small 0.25 m (10 inch) telescope placed in an unheated shed. The discovery was relayed to the International Astronomical Union's clearinghouse for such events, which relayed the news to astronomers throughout the world. By the next evening, nearly all major radio and optical telescopes south of the Equator were observing the supernova. One month later, it was featured on the cover of *Time* magazine with just one word: "BANG!" Astronomers were still watching its expanding debris years after the exploding star hurled it into space.

McCray (1993), Arnett et al. (1989), and Trimble (1988) have provided us with reviews of Supernova 1987A.

SN 1987A was a gravity-powered, iron-catastrophe Type II supernova, generated during the core collapse of a former blue supergiant star that began life with a mass of about 20 times that of the Sun. The physical properties of this progenitor star and its subsequent supernova explosion are listed in Table 13.8.

Table 13.8 Supernova SN 1987A[a]

Progenitor star
Name: Sanduleak −69°202
Spectral Type: B3 Ia (blue supergiant)
Location: Large Magellanic cloud
Distance: 168,000 light-years
Radius: 3×10^{10} m $\approx 50\ R_{\odot}$
Effective temperature = 16,000 K
Luminosity = 4.6×10^{31} J s^{-1} $\approx 10^{5}\ L_{\odot}$
Mass $\approx 16\ M_{\odot}$
Neutrino burst
Number of neutrinos detected: 19 anti-neutrinos
Energy of each neutrino: $\approx$ 20 MeV = 3.2×10^{-12} J
Neutrino flux at Earth: 5×10^{14} m^{-2}
Number of neutrinos emitted: $\approx 10^{58}$ neutrinos
Energy released in neutrinos: $\approx 10^{48}$ J
Duration of neutrino burst: $\approx$ 10 s
Neutrino luminosity: $\approx 10^{48}$ J s^{-1} $\approx 10^{22}\ L_{\odot}$
Visible explosion
Peak visible luminosity: 3.8×10^{33} J s^{-1} $\approx 10^{7}\ L_{\odot}$
Velocity of ejected material: $\approx 10^{7}$ m s^{-1}
Mass of ejected material $\approx 8 \times 10^{30}$ kg $\approx 4\ M_{\odot}$
Kinetic energy of ejected material $\approx 4 \times 10^{44}$ J

[a] The symbols $R_{\odot}$, $L_{\odot}$ and $M_{\odot}$ respectively denote the radius, luminosity and mass of the Sun

One of the more interesting observations of SN 1987A was made from beneath the Earth's surface, when massive subterranean instruments detected a few neutrinos emitted during the explosion. Although solar neutrinos had been observed coming from nuclear reactions that power the Sun, all other stars are so far away and the number of their neutrinos striking the Earth is so low that they had never been detected coming from any other cosmic object. It had nevertheless been proposed that a supernova might generate a great number of neutrinos (Focus 13.2).

Focus 13.2 Neutrinos generated during a supernova

As first realized by George Gamow (1904–1968) and his Brazilian colleague Mario Schönberg (1914–1990), the temperature of a collapsing stellar core can become high enough to create both neutrinos and antineutrinos, which can easily escape and carry away prodigious amounts of energy (Gamow and Schönberg 1941). This would lead to further loss of supporting pressure, the implosion of the core with a rapid rise in temperature, and an explosion as a supernova. They named the nuclear transformation associated with neutrino energy loss the Urca process, after its similarity to the gambling operations at the Casino de Urca near Rio de Janeiro. There also, no matter how you played the game, you always seemed to lose.

Today we know that supernova neutrinos may be emitted during nuclear processes other than the Urca process, but the basic idea was correct. Vast numbers of neutrinos are released as the crushed iron core of a dying star is broken into its subatomic components. A neutrino is produced each time a proton and an electron combine, and pairs of neutrinos and antineutrinos are created when the core temperature reaches 100 billion K. All of these neutrinos are without electrical charge, have almost no mass, and move unimpeded at nearly the speed of light through nearly any amount of matter, even the entire Earth. But if enough cosmic neutrinos were directed at the Earth, massive, subterranean instruments might detect a small number of them.

For several seconds, the relatively nearby supernova explosion SN 1987A generated such enormous amounts of the elusive neutrinos that a small number were detected. They were recorded in two underground neutrino detectors 3 hours before the first visible sighting of SN 1987A.

Only 19 neutrinos were detected flashing through the underground darkness. Yet, even this small number signaled the presence of an awesome energy, vastly exceeding the amount contained in the radiation and expanding debris of the supernova (also see Table 13.8). They indicated that 10 billion trillion trillion trillion, or 10^{58}, neutrinos were produced by the exploding star. As the iron core imploded, the neutrinos carried away energy of about 5×10^{46} J, or roughly 99 % of the explosion energy, and emitted a total neutrino luminosity comparable to that of the optically visible luminous output of more than ten thousand billion billion stars like the Sun.

Example: Energy of a supernova

During the supernova explosion of SN 1987A, about a solar mass, or $1.0\, M_\odot = 1.989 \times 10^{30}$ kg was ejected with a velocity V of about 10^7 m s^{-1}. The kinetic energy of the ejected mass is equal to $0.5\, M_\odot V^2 \approx 10^{44}$ J. The binding energy released in forming a central neutron star of radius R_{NS} is equal to $E_G = GM_\odot^2/R_{NS} \approx 1.9 \times 10^{46}$ J, where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and $R_{NS} = 14$ km. The energy, E, released in completely destroying a solar-mass star is $E = M_\odot c^2 \approx 1.8 \times 10^{47}$ J, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The energy of each detected neutrino was 20 MeV $= 3.2 \times 10^{-12}$ J, and the neutrino flux at Earth was 5×10^{14} m^{-2}. The number of neutrinos emitted is equal to the product of this flux times the area at the Earth's distance, or $4\pi D^2$, where the distance $D = 168{,}000$ light-years $= 1.59 \times 10^{21}$ m and 1 light-year $= 9.46 \times 10^{15}$ m, so the total number of neutrinos emitted is $5 \times 10^{14} \times 4\pi D^2 = 1.58 \times 10^{58}$. The total energy emitted by the neutrinos, $E_{neutrino}$, is about $1.58 \times 10^{58} \times$ energy per neutrino $\approx 5 \times 10^{46}$ J.

The neutrinos detected from SN 1987A (Bionta et al. 1987; Hirata et al. 1987) solved one of the thornier problems in understanding such a supernova explosion. It was known that a stellar collapse generates tremendous amounts of energy, but there was difficulty explaining how that energy was transferred from the collapsing core into the outer layers of the star in sufficient amounts to produce an explosion. The core might rebound, sending shock waves propagating into the surrounding material, but computer simulations indicated that the shock waves could not blow away the rest of the star. They always became stalled when encountering the in-falling matter from the outer layers.

Unlike the stalled shock waves, the flood of escaping neutrinos carries tremendous amounts of energy far away from the stellar core, a very small fraction of which gets caught in the in-falling outer layers of the collapsing star, heating up the gas to a temperature of more than 10 billion K. This produces a buoyant, convecting bubble of energy that reverses the in-fall and powers the explosion.

Three hours after the initial collapse and generation of neutrinos in SN 1987A, its heated bubble expanded, driving shock waves before it, and burst through the surrounding material, breaking the star apart and hurling its pieces into space, which produced the dazzling light of the supernova. This explains why the neutrinos were detected 3 hours before any light was seen.

13.5.8 Will the Sun Explode?

There is no explosion forecast for the Sun's future. It is going out alone, passing into its final resting state unaccompanied by a close companion. Although it will end up as a dense, high-gravity white dwarf star, even the nearest star still will remain far beyond the dead Sun's gravitational embrace, never orbiting it. Moreover, our Sun is nowhere near massive enough to ever explode by itself. So, no nova or supernova is expected when the Sun runs out of nuclear fuel. It will go quietly into the oblivion of permanent night.

13.6 Expanding Stellar Remnants

In their explosive death, stars that go supernova blast their outer layers into surrounding space, expelling much or all of the stellar material at supersonic speeds of up to 30,000 km s^{-1}, or $1/10$th of the speed of light. A strong shock wave forms ahead of the ejected material, colliding with the surrounding interstellar gas and heating it up to temperatures of tens of millions of K. The high-temperature material emits intense x-rays that have been observed with instruments aboard spacecraft located above the Earth's obscuring atmosphere, thereby recording the debris of cataclysmic stellar explosions that occurred even thousands of years ago, before recorded history.

Like the explosions that cast this material out, there are two types of supernova remnants. They can be the remains of a white dwarf star sent into explosion by a nearby companion in a Type Ia supernova or the explosive debris of a single massive star that has expired in a Type II supernova. Weiler and Sramek (1988) and Trimble (1982, 1983) have reviewed our knowledge of both supernovae and supernova remnants.

The *Chandra X-ray Observatory*, for example, has imaged the *Tycho supernova remnant*, named for Tycho Brahe (1546–1601) who reported observing the original explosion in 1572. A Type Ia supernova formed this remnant, when a white dwarf was sent into annihilation by overflow from a nearby companion star. An arc of x-ray emission in the supernova remnant was attributed to material blown off the companion star, which otherwise survived the destruction of its neighbor and now is moving within the remnant more quickly than its neighbors as the result of the explosion. The properties of the arc and remaining star indicate that the former white dwarf star and its companion once orbited one another in a five-day period at a separation of less than 1/10th of the mean distance between the Earth and the Sun, or 0.1 AU (Lu et al. 2011).

Supernova remnants often emit intense radio radiation. The majority of these radio supernova remnants appear as bright rings, or shells, in projection against the sky. The intense radio radiation cannot be produced by a hot gas, like x-rays, but instead is emitted by electrons accelerated to high speeds by the supernova explosion and spiraling in a magnetic field.

A beautiful example of an expanding shell-like supernova remnant is the *Cassiopeia A supernova remnant*, abbreviated Cas A. It is located roughly 10,000 light-years away in the direction of the constellation Cassiopeia and is the brightest radio source in the sky. The expanding shell of Cas A is also a strong source of x-rays, emitted by a 50-million-K gas (Fig. 13.7).

Despite its radio and x-ray brilliance, the remnant is faint at optically visible wavelengths, which nevertheless indicate that it is rich in oxygen and now expanding at a speed of about 5,000 km s^{-1}. The Type II supernova explosion would have been observed as a daytime star around 1680, but there are no historical records of the event. Thick clouds of interstellar dust, or material ejected from the massive star's outer layers, may have absorbed the light and rendered the explosion optically invisible.

The most spectacular example of a Type II supernova remnant is the *Crab Nebula supernova remnant* (Fig. 13.8, Baade 1957), also one of the brightest radio sources in the sky. The Crab Nebula, also designated M 1, NGC 1952, or Taurus A, is the remnant of a supernova explosion that was observed in 1054. It is about 6,500 light-years away, has a diameter of 11 light-years, and expands at a speed of about 1,500 km s^{-1}. The physical properties of this fascinating object are given in Table 13.9.

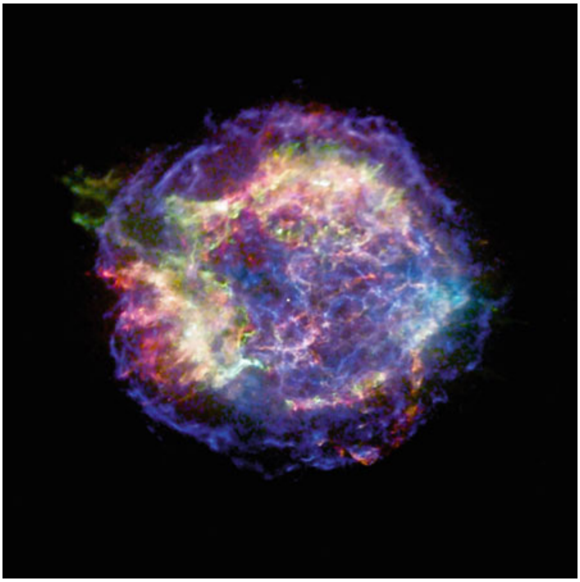

Fig. 13.7 X-ray image of Cassiopeia A supernova remnant The expanding supernova remnant Cassiopeia A has a temperature of about 50 million K and therefore is a luminous x-ray source, seen in this image from the *Chandra x-ray Observatory*. Still visible in x-rays, the tiny point-like source near the center of Cas A is a neutron star, the collapsed core of the star that exploded about 330 years ago, as observed from the Earth. (Courtesy of NASA/CXC/MIT/University of Mass. Amherst/M. S. Stage, et al.)

The English amateur astronomer John Bevis (1695–1771) discovered the nebula in 1731. The French astronomer Charles Messier (1730–1817) independently found it a few decades later (Messier 1781), listing it as the first entry in his famous catalogue. The Irish astronomer William Parsons (1800–1867), the third Earl of Rosse, gave it its present name, the Crab Nebula, after sketching it to resemble a crab (Rosse 1850).

The Crab is the first nebula to be associated with expanding material – by the American astronomer John C. Duncan (1882–1962); and the first to be recognized as the remnant of a stellar explosion – by the American astronomer Edwin Hubble (1889–1953), who identified the Crab Nebula with the guest star recorded by Chinese astronomers in 1054 A.D. in the same region of the sky (Duncan 1921; Hubble 1928). The Crab supernova remnant is also the first cosmic radio source (Bolton et al. 1949) and the first cosmic x-ray source (Bowyer et al. 1964) to be discovered outside the solar system; it was identified as one of the brightest, persistent emitters of gamma rays (Haymes et al. 1968). Hester (2008) has provided a review of the Crab Nebula, as an astrophysical chimera. Reynolds (2008) reviewed supernova remnants at high energy.

Photographs taken by Walter Baade using the Mt. Wilson 2.5 m (100 inch) telescope in the early 1940s indicated that the visible nebula consists of two

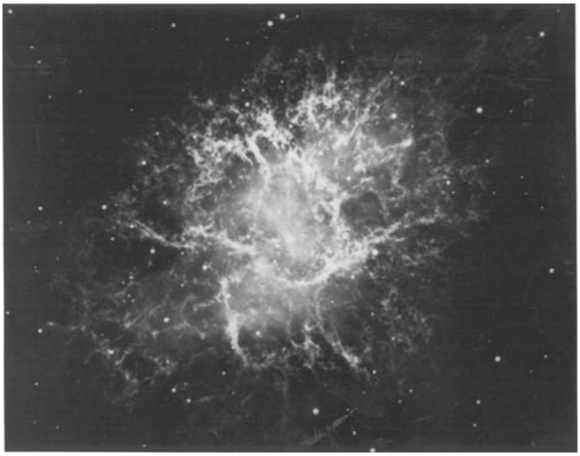

Fig. 13.8 The Crab Nebula supernova remnant The optically visible light of the Crab Nebula, designated as M 1 and NGC 1952, consists of two distinct parts: (1) A system of expanding filaments forms an outer envelope in which emission lines occur at well-defined wavelengths, and (2) an inner amorphous region that emits continuum radiation at all wavelengths. Walter Baade (1893–1960) took this photograph of the expanding filaments in the wavelength range of 640 nm to 670 nm using the 5.0 meter (200 inch) telescope on Mount Palomar, California in 1955. A Type II supernova explosion observed nearly 1,000 years ago, in 1054, ejected the filaments. The continuum glow that is concentrated in the inner parts of the nebula is the nonthermal radiation of high-speed electrons spiraling in magnetic fields (see Fig. 13.9). This continuum emission is powered by a spinning neutron star, the southwesternmost (*bottom right*) of the two central stars. The neutron star is the crushed, ultradense core of the exploded star. It also is a radio pulsar that acts like a lighthouse spinning 30 times a second. (Courtesy of Hale Observatories.)

distinct parts that emit radiation differently (Baade 1942). A tangled, oval-shaped network of red and green filaments, seen in the light of bright emission lines from ionized atoms, encases the inner blue and milk-white continuum radiation. The filamentary remnants of the explosion contain about 4 solar masses of material, consisting mostly of ionized helium and hydrogen along with lesser amounts of carbon, oxygen, nitrogen, iron, neon, and sulfur. The progenitor star that was annihilated to make the explosion had an estimated mass of between 9 and 11 solar masses, some of that mass was sent into invisibility within surrounding space and the rest remained within a central neutron star.

In addition to an expanding shell, the radio nebula has a filled center that coincides with the inner visible light. These types of supernova remnants are named *plerions*, from the Greek word *pleres* for "full" or "filled."

The inner radiation contains practically all of the energy emitted by the Crab Nebula, and it is 1,000 times more intense at radio wavelengths than at optically visible wavelengths. It is impossible to reconcile the observed radio emission with the optical emission through the thermal radiation of a hot gas at any plausible temperature. As pointed out by the Russian astronomer Iosif Shklovskii (1916–1985), both the radio and optical emission of the Crab Nebula come from

Table 13.9 Physical properties of the Crab Nebula supernova remnant

Date of explosion = 1054 AD
M_S = progenitor star mass $\approx 9\ M_{\odot}$
M_{ejec} = ejected mass = 2–3 $M_{\odot} \approx 5 \times 10^{30}$ kg
M_N = neutron star mass = 1.4 $M_{\odot}$
D = distance $\approx$ 6,500 light-years $\approx$ 2.0 kpc $\approx 6.2 \times 10^{19}$ m
θ = angular extent = $4.5' \times 7.0'$
R = radius = $\theta D/2$ = 4.1 light-years × 6.1 light-years = 3.9×10^{16} m × 6.1×10^{16} m
V = velocity of ejected material $\approx$ 1,450 km $s^{-1} \approx 1.45 \times 10^6$ m s^{-1}
L = total luminosity = $10^{31.14}$ J s^{-1}
S = radio flux density = 1,040 Jy at 1 GHz
L_X = x-ray luminosity = $10^{30.38}$ J s^{-1}
N_e = electron density $\approx 4 \times 10^7$ m^{-3}
B = magnetic field strength $\approx 3 \times 10^{-8}$ tesla
M_{ns} = neutron star mass $\approx$ 1.4 $M_{\odot} \approx 2.8 \times 10^{30}$ kg
E_b = binding energy released in forming neutron star $\approx GM_{\odot}^2/R_{ns} \approx 10^{46}$ J
P = radio pulsar period = 0.033326 s
dP/dt = radio pulsar period time derivative = 421.288×10^{-15} s s^{-1}
$T = P/(2dP/dt)$ = approximate age of pulsar $\approx 3.95 \times 10^{10}$ s $\approx$ 1,250 year

synchrotron radiation emitted by high-energy electrons spiraling around magnetic fields at nearly the speed of light (Shklovskii 1953).

The Crab Nebula was the first supernova remnant known to emit nonthermal synchrotron radiation, although the radio emission of the Milky Way had been attributed to the synchrotron radiation of cosmic ray electrons spiraling about the interstellar magnetic field (Sect. 11.3). Electrons of extremely high energy emit optically visible light, whereas electrons of slightly lower energy radiate at radio wavelengths. Because the more energetic electrons lose their energy faster and also radiate at shorter wavelengths, the synchrotron-radiation mechanism provides a natural explanation for the nonthermal spectrum of the Crab's radiation, which is more intense at longer wavelengths. At every wavelength of observation, from x-rays to radio waves, the bulk of radiation from the Crab Nebula is accounted for by the synchrotron-radiation mechanism. It explains the nonthermal spectrum of the optical and radio emission, as well as the polarization of the optical continuum (Fig. 13.9, Baade 1957). The polarized radiation has a preferred orientation or direction due to the high-speed electrons moving in a large-scale, well-ordered magnetic field.

Despite the successes of the synchrotron-radiation theory, explaining the origin of the energetic electrons that gave rise to the radiation remained a fundamental difficulty. The electrons radiating at optically visible wavelengths will dissipate their energy by synchrotron radiation in about 180 years, and the more energetic electrons that produce the short-wavelength x-rays should lose their energy and disappear in less than a year. Because the supernova radiation was emitted more than 900 years ago, the high-speed electrons producing the synchrotron radiation

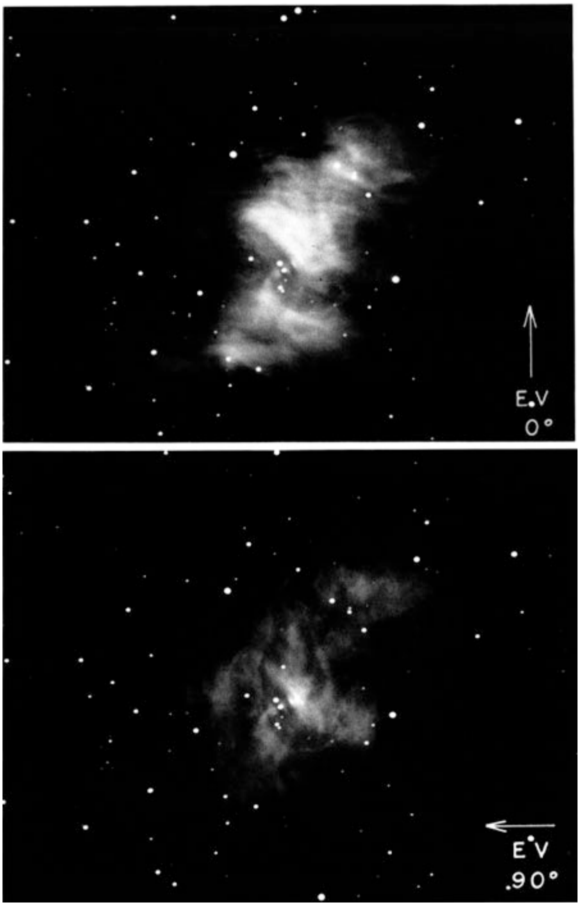

Fig. 13.9 Polarized light of the Crab Nebula The inner continuum emission of the Crab Nebula supernova remnant should be polarized, or emitted in a preferred direction, if it is caused by nonthermal synchrotron radiation. This was confirmed in detail by these visible-light photographs taken in the wavelength range of 540–640 nm through a polarized filter by Walter Baade (1893–1960) in 1955 using the 5.0 meter (200 inch) telescope on Mount Palomar, California. The arrows indicate the direction of the electric vector of the light recorded. The south westernmost of the two central stars is the remnant neutron star and a radio pulsar. This inner amorphous region of the Crab Nebula supernova remnant is a powerful source of radio radiation, while also emitting optically visible light and x-rays. This radiation is most intense at the longer, radio wavelengths, a characteristic of non-thermal radiation. It has been attributed to the synchrotron radiation of relativistic electrons whirling at nearly the speed of light around magnetic fields. Because the more energetic electrons lose their energy faster and also radiate at shorter wavelengths, the synchrotron radiation mechanism provides a natural explanation for the nonthermal emission of the Crab's inner regions and for the fact that its radio emission is a thousand times more intense than its visible light. (Courtesy of Hale Observatories.)

cannot be survivors of the original explosion. Instead, some unknown source must be continuously replenishing these energetic electrons.

Example: Expansion age, pulsar age, and x-ray synchrotron lifetime for the Crab Nebula

The material ejected from the explosion that gave rise to the Crab Nebula supernova remnant is now expanding at a velocity of $V = 1.45 \times 10^6$ m s^{-1}. The largest angular extent of the supernova remnant is $\theta = 7.0' = 420'' \approx 2.035 \times 10^{-3}$ radians, where 1 radian $= 2.063 \times 10^5\,''$. At the Crab Nebula's distance of $D =$ 6,500 light-years $= 6.15 \times 10^{19}$ m, where 1 light-year $= 9.46 \times 10^{15}$ m, this angular extent corresponds to a radius of $R = \theta D/2 \approx 6.26 \times 10^{16}$ m. If the ejected material has been moving at a constant velocity across this radius, the expansion age is $T = R/V \approx 4.3 \times 10^{10}$ s $\approx$ 1,367 years, where 1 year $= 3.1557 \times 10^7$ s.

The supernova explosion observed by the Chinese in this region of the sky occurred in 1054 AD, or about 955 years ago, which is slightly younger and might indicate that the debris initially expanded at a faster rate than that observed today.

The Crab pulsar, described in the next section, has a period of $P = 0.033326$ s, and that period is increasing at the rate of $dP/dt = 421.288 \times 10^{-15}$ s s^{-1}. The approximate pulsar age is $T = P/(2dP/dt) = 3.95 \times 10^{10}$ s = 1,250 years. This upper limit to the age is consistent with the historical age of about 950 years, for the periods lengthen with time and the initial pulsar period might have been shorter. Setting $P = 0.025$ s for that initial period would give the correct age.

The Crab Nebula emits synchrotron radiation at radio, visible light, and x-ray wavelengths. It is emitted by energetic electrons spiraling in a magnetic field of strength $B \approx 3 \times 10^{-8}$ tesla. At a soft x-ray photon energy of $E = 10$ keV $= 1.602 \times 10^{-15}$ J, the frequency of the radiation is $\nu_s = E/h = 2.42 \times 10^{18}$ Hz, where the Planck constant $h = 6.626 \times 10^{-34}$ J s. The wavelength of the radiation is $\lambda_s = c/\nu_s \approx 1.23 \times 10^{-10}$ m, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The Lorentz factor γ of high-speed electrons giving rise to synchrotron radiation at this frequency or wavelength is given by $\gamma = ([\nu_s/(2.8 \times 10^{10}\,B)]^{1/2} \approx 5.34 \times 10^7$, and the synchrotron lifetime of the electron radiating this x-ray radiation is $\tau_s = 8.187/(B^2\gamma) \approx 1.7 \times 10^8$ s $\approx$ 5.4 years. More energetic x-rays correspond to a higher γ and a shorter lifetime for electrons emitting this radiation; synchrotron radiation emitted at gamma-ray wavelengths has even briefer electron lifetimes. The electrons emitting synchrotron x-rays and gamma rays from the Crab Nebula could not have been accelerated in the supernova explosion that occurred nearly 1,000 years ago.

An early clue to the energy source of the Crab Nebula was provided when the American astronomer Carl Lampland (1873–1951) observed moving wisps and knots that originated at the center of the nebula (Lampland 1921). Walter Baade subsequently showed that these features move at up to a tenth of the speed of light, suggesting that the central star was injecting high-speed particles into the nebula. It was eventually realized that a neutron star was born in the crushed center of the explosion.

The Italian astronomer Franco Pacini (1939–) pointed out that gravitational energy released during the collapse of a normal star into a neutron star will be converted into rotational energy (Pacini 1967). It follows from the conservation of angular momentum that the neutron star will be spinning rapidly, with a rotation period of less than a second, and it will also have an intense magnetic field, owing to the conservation of magnetic flux in collapse. Provided that the magnetic axis and rotation axis are not aligned, a rotating dipole magnetic field will convert the rotational energy into electromagnetic energy, thereby providing the luminosity of the Crab Nebula for its entire lifetime.

In the same year as Pacini's prescient publication, the first radio pulsar was discovered. The following year, pulsars were attributed to rapidly rotating neutron stars, and a pulsar was found at the center of the Crab Nebula, spinning 30 times a second. The Crab was the first object whose luminous output was related to a central pulsar.

The nebula is powered by the pulsar wind that is composed of charged particles accelerated to nearly the speed of light by the rapidly rotating, intense magnetic field of the spinning pulsar. Short-lived, flaring bursts of radiation from the Crab Nebula at gamma ray wavelengths may be due to sudden restructuring of the pulsar magnetic field, accelerating electrons to energies more than 100 times greater than can be attained by any particle accelerator on Earth.

Pulsar winds are found inside the shells of other filled supernova remnants, but Cassiopeia A is an exception. Although the x-ray emission of a central neutron star has been located within this supernova remnant, using instruments aboard the *Chandra X-ray Observatory* (Lu et al. 2011), the neutron star is relatively quiet and does not emit any detectable pulsar wind activity. One possible explanation is that the magnetic fields are so extremely strong that they have stifled pulsar wind activity rather than enhanced it. We now turn to early speculations about the possible existence of neutron stars and the subsequent discovery of radio and x-ray pulsars.

13.7 Neutron Stars and Pulsars

13.7.1 Neutron Stars

Walter Baade (1893–1960) and Fritz Zwicky (1898–1974) proposed the possibility that neutron stars might exist just two years after James Chadwick's (1891–1974) discovery of the neutron (Chadwick 1932a, b). They speculated that a supernova

Table 13.10 Physical properties of neutron stars

M_{NS} = mass of neutron star $\approx 2\ M_{\odot} = 3.978 \times 10^{30}$ kg
M_{CNS} = critical upper mass limit for neutron star $\approx 3\ M_{\odot} = 5.967 \times 10^{30}$ kg
R_{NS} = radius of neutron star $\approx$ 12 km $= 1.2 \times 10^{4}$ m
ρ_{NS} = mass density of neutron star $\approx 5 \times 10^{17}$ kg m^{-3}
V_{esc} = escape velocity of neutron star $= (2GM_{NS}/R_{NS})^{1/2}$ = 210,000 km s^{-1} = 0.70 c
E_{NS} = binding energy released to form a neutron star $= GM_{NS}^2/R_{NS} \approx 8.8 \times 10^{46}$ J
P = rotation period = 0.001–10 s
B = magnetic field strength = 10^{8} tesla

explosion is driven by the gravitational energy released when a massive star runs out of fuel and its core collapses, but the explosion may not completely destroy the stellar core. A dense cinder could remain at the center or, in their prescient words: "With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a *neutron star*, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density." (Baade and Zwicky 1934b).

We now know that if a collapsing star is more massive than 1.4 solar masses, the inward force of its gravitation will overcome the outward degenerate electron pressure that halts the collapse at the white-dwarf stage. The young Russian physicist Lev Landau (1908–1968) speculated that dead stars with a mass above this white-dwarf limit might collapse until neutron degeneracy pressure would halt the crush of gravity (Landau 1938). Baym and Pethic (1979) provide an early review of the physics of neutron stars.

The electrons are pushed into direct contact with the atomic nuclei, and thus packed together at nuclear densities. The enormous pressure of the rapid collapse would create neutrons when the electrons merged with the nuclear protons, forming a nuclear gas with a mass density surpassing that of the Sun, and water, by a factor of a million, billion, at 5×10^{17} kg m^{-3} (Table 13.10). At these densities, a normal star like the Sun would have collapsed to a radius of only 12 km. Such a neutron star has a powerful gravity, with an escape velocity of about 0.7 or 70 % of the speed of light.

Within a year of Landau's speculations, George Gamow (1904–1968) pointed out that the high central mass densities of neutron stars can be expected only in those stages of stellar evolution subsequent to the exhaustion of the thermonuclear fuel in normal stars, and that only the very massive stars would have evolved rapidly enough to reach this stage during the lifetime of the universe (Gamow 1939).

At about the same time, the American physicist J. Robert Oppenheimer (1904–1967) and a Canadian graduate student George M. Volkoff (1914–2000) found that neutron stars have a limiting mass of their own (Oppenheimer and Volkoff 1939). They showed that at these large mass densities both the degenerate neutron pressure and the effects of gravitation on space–time must be considered. The equation of state of the nuclear material was obtained in the first approximation by identifying it with a degenerate, relativistic gas of neutrons fulfilling Fermi statistics. The macroscopic structure of the star, its mass, radius, and density distributions, were determined from Einstein' *General Theory* of gravitation. Calculating equilibrium configurations along these lines, Oppenheimer and Volkoff found that a stable neutron star could exist only in a finite range of masses.

Just as degenerate electron pressure supports a white dwarf star, it is degenerate neutron pressure that supports a neutron star. This means that the radius, R_{NS}, of a neutron star of mass, M_{NS}, can be estimated by:

$$R_{NS} = \frac{m_e}{m_n} R_{WD} \approx 10^7 \left(\frac{m_e}{m_n}\right) \left(\frac{Z}{A}\right)^{5/3} \left(\frac{M_{NS}}{M_\odot}\right)^{-1/3} \text{ m} \tag{13.21}$$

or

$$R_{NS} \approx 10^4 (M_\odot / M_{NS})^{1/3} \text{ m}, \tag{13.22}$$

where the electron mass $m_e = 9.1094 \times 10^{-31}$ kg, the neutron mass $m_n = 1.6749 \times 10^{-27}$ kg, R_{WD} denotes the radius of a white dwarf star, $Z/A = 1.0$ for a neutron star, and the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg. So, for a neutron star of mass $M_{NS} = 1.4\ M_\odot$ and radius $R_{NS} = 10$ km, the star's mass density, $\rho_{NS} = 3\ M_{NS}/(4\pi R_{NS}^3) \approx 7 \times 10^{17}$ kg m^{-3}, which is comparable to the mass density of the nucleus of an atom.

Although the radius of a neutron star shrinks with increasing mass, you can't increase the mass of a neutron star without limit any more than you can for a white dwarf star, and the mass of a neutron star is now believed to be between 1.4 and 3.0 solar masses.

Theoretical considerations of a dense neutron gas suggest that it could be superfluid, with no resistance to flow, and superconducting, without electrical resistance. *Chandra* x-ray observations of the neutron star at the center of the Cassiopeia A supernova remnant indicate a rapid decline in the temperature of the ultra-dense neutron core, of about 4 % over a 10 year period, suggesting that it is made of superfluid and superconducting material (Page et al. 2011; Shternin et al. 2011).

To sum up, when an isolated massive star can no longer support its own crushing weight, the center collapses, and obtains energy from its in-fall. If the collapsing stellar core weighs between 1.4 and 3.0 solar masses, it is compacted to nuclear density, and forms a neutron star. When the core weighs more than 3 solar masses, it collapses into a black hole.

Although important in hindsight, these early considerations about the possibility of neutron stars did not evoke much interest at the time. They would have remained a speculative curiosity if it were not for the serendipitous discovery of radio pulsars.

13.7.2 Radio Pulsars from Isolated Neutron Stars

Before discussing the discovery of pulsars, we provide references to modern reviews for further reading. Lyne and Graham-Smith (2012) have discussed many aspects of pulsar astronomy, and Taylor and Stinebring (1986) provided an earlier review of our understanding of pulsars. Gaensler and Slane (2006) have reviewed the evolution and structure of pulsar wind nebulae; and Phinney and Kulkarni (1994) have reviewed binary and millisecond pulsars.

Pulsars were discovered accidentally during a survey of the *scintillations*, or "twinkling", caused when radio radiation from cosmic sources passes through the Sun's winds. When the radio waves are viewed through the wind-driven material, they blink on and off, varying on time-scales of a few tenths of a second – in much the same way that stars twinkle when seen through the Earth's varying atmosphere. Repeated observations of several scintillating radio sources at different angles in relation to the Sun provide information about the properties of the solar wind and the angular structure of the radio sources. The fluctuations are greatest for the smaller emitters, just as stars twinkle more than the Moon or planets, which have larger angular extents.

To study these effects, Antony Hewish (1924–) and his colleagues at Cambridge University built a large array of 2048 dipole antennas, spread over four and a half acres and operated at a long radio wavelength of 3.7 m, since the scintillating fluctuations were known to be more prominent at longer wavelengths. The combined signals from all the antennas were connected to a radio receiver and chart recorder with a time constant of 0.1 s, the time-scale of the scintillations. When examining the charts in July 1967, graduate student Jocelyn Bell (1943–) found a strong fluctuating signal in the middle of the night, when the array was pointed away from the Sun and the effects of the solar wind should have been small.

Further investigations led to the astonishing detection of periodic radio pulses, with an exceedingly precise repetition period of 1.3372795 s. The first radio pulsar had been detected (Hewish et al. 1968). No one had foreseen its existence, and no other known astronomical object kept time so accurately.

By the time the discovery was ready for publication, in 1968, evidence of other radio pulsars was found in the existing chart recordings; within three weeks, a second paper announced the discovery of three additional radio pulsars (Pilkington et al. 1968). This triggered searches for other previously unknown pulsars with large radio telescopes using rapid time sampling rather than the long integration times formerly used. In less than a year, the list of pulsars was expanded to more than two-dozen, and a pulsar was detected at the position of the very star thought to be the neutron star remnant of the Crab Nebula supernova explosion.

We now know that the term *pulsar* is misleading for the compact stars do not pulsate – they rotate – but the name has stuck. It designates repeating pulses of radio emission rather than a pulsating star.

The physical properties of radio pulsars are given in Table 13.11.

Table 13.11 Physical properties of radio pulsars

P_{RP} = period of radio pulsar = 0.001–4 s
dP_{RP}/dt = rate of increase of radio pulsar period = 10^{-15}–10^{-12} s s^{-1}
$T = P_{RP}/(2dP_{RP}/dt)$ = characteristic radio pulsar age = 10^3–10^{10} years (where 1 year = 3.156 × 10^7 s)
B_{RP} = magnetic field strength of radio pulsar = 10^9–10^{13} G = 10^5–10^9 tesla
L_{RP} = radio luminosity = 10^{18}–10^{24} J s^{-1}
M_{RP} = mass of binary pulsar PSR 1913 + 16 = 1.44 $M_{\odot} \approx 2.85 \times 10^{30}$ kg

The pulsars probably could have been discovered many years earlier, when other large radio antennae were constructed, but radio astronomers were used to adding up signals over long time intervals to detect faint cosmic radio signals. The long time resolutions precluded detection of the pulsars. They are relatively faint radio sources when averaged over their period because there is no emission between the brief radio pulses. If time resolutions comparable to the pulsar burst durations of milliseconds had been used, the intense radio bursts would have been detected easily. It is because Hewish specifically designed a new type of radio telescope for a study of the rapidly changing solar wind effects that the radio pulsars were discovered accidentally.

The extreme regularity of the periodic radio bursts suggested that they are controlled by the rotation of a massive body, and the short duration of the pulsar bursts suggested that their radiation originates in a body that cannot be much larger than the Earth. That is, the size should be smaller than the product of the speed of light and the burst duration; otherwise, it may be violating nature's upper speed limit, the speed of light. A rotating white dwarf was initially suspected, but it could not spin faster than about once per second. Rotation with a shorter period would tear the white dwarf apart, since its outer atmosphere would be rotating faster than the star's escape velocity.

The Austrian-born American astronomer Thomas Gold (1920–2004) proposed that radio pulses are produced by a rapidly rotating neutron star with an intense magnetic field (Gold 1968). He assumed that a pulsar would emit radio radiation in a beam, like a lighthouse, oriented along the magnetic axis (Fig. 13.10). An observer sees a pulse of radio radiation each time the rotating beam flicks across the Earth. Because the neutron star's beam could be oriented at any angle, the beams of many pulsars would miss the Earth and would remain forever unseen.

Gold suggested definitive observational tests of his ideas. He noticed that a spinning neutron star gradually loses its rotational energy and slows down, successfully predicting that this would cause a slow lengthening of the radio pulsar periods with time. He also predicted that radio pulsars with much shorter periods would be found, as they were.

If a slowly rotating star collapses down to a small size, the rate of rotation increases. It's a result of the conservation of angular momentum, which means that the period of rotation is proportional to the square of the radius. When an ordinary visible star runs out of nuclear fuel and is compressed to the size of a neutron star,

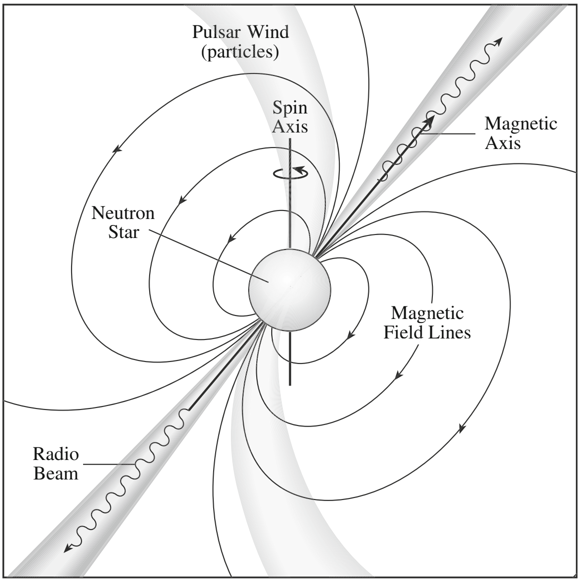

Fig. 13.10 Radio pulsar A spinning neutron star has a powerful magnetic field the axis of which intersects the north and south magnetic poles. The rotating fields generate strong electric currents and accelerate electrons, which emit an intense, narrow beam of radio radiation from each magnetic polar region. Because the magnetic-field axis can be inclined to the neutron star's rotation axis, these beams can wheel around the sky as the neutron star rotates. If one beam sweeps across the Earth, a bright pulse of radio emission, called a *pulsar*, is observed once every rotation of the neutron star

its rotation period would speed up from days to milliseconds. Since the time between successive radio pulses is the same as the rotation period of the neutron star, it has to be initially spinning at a very fast rate.

A neutron star is also a powerful magnet, and the magnetism is attached to the material within the star. When the star collapses it carries its magnetism with it, packing it into a smaller volume and amplifying its strength by factors of billions. The magnetic field strength increases in inverse proportion to the surface area, which is itself proportional to the square of the radius. So a typical surface magnetic field on a visible-light star, with a strength of 0.01 tesla, would be strengthened by a factor of a ten thousand million, or to 10^8 tesla, if collapsing to a neutron star.

Example: Period and magnetic field of a rotating neutron star or white dwarf star

We can infer the rotation period of a neutron star using the conservation of angular momentum. For a sphere of mass, M, radius, R, rotation period, P, and rotation velocity $V = 2\pi R/P$, the conservation of angular momentum in gravitational collapse requires that $MVR = 2\pi MR^2/P =$ constant, or that the rotation period varies as R^2 if the mass is not changed. For the Sun, $M = M_{\odot} = 1.989 \times 10^{30}$ kg, $R = R_{\odot} = 6.955 \times 10^8$ m, $P = 25.67$ days at the solar equator, where one day = 86,400 s, and $V = 1{,}971$ m s^{-1} at the solar equator. If a star of this mass and rotation period collapsed to form a neutron star of radius $R_{NS} = 10$ km, the rotation period would be $P_{NS} \approx 4.6 \times 10^{-4}$ s.

Conservation of magnetic flux in gravitational collapse provides an estimate for the magnetic field of a neutron star. For a sphere with a surface magnetic field strength, B, and radius, R, the conservation of magnetic flux in gravitational collapse requires that the magnetic flux $BR^2 =$ constant, or that the magnetic field strength B varies as R^{-2}. For the Sun, $B = B_{\odot} = 10^{-2}$ tesla and $R = R_{\odot} = 6.955 \times 10^8$ m. If a star of this magnetic field strength and radius collapsed to a neutron star of radius $R_{NS} = 10$ km it would have a surface magnetic field strength of $B_{NS} = 4.8 \times 10^7$ tesla.

The same formulae apply for a white dwarf star, which would collapse to a radius of about 0.01 solar radii, to give a rotation period of $P_{WD} = 221$ s and a magnetic field strength of 100 tesla. Although such strong magnetic fields have been detected for white dwarfs, many white dwarfs rotate with periods of days or even years rather than seconds to minutes. The slow rotation of white dwarf stars may be attributed to the powerful winds that created their surrounding planetary nebulae, removing rotational energy and slowing the white dwarf.

The association of radio pulsars with neutron stars in supernova remnants became accepted when Australian radio astronomers found a pulsar with an extremely short period – 89 ms – in the center of the extended radio source Vela X, believed to mark the debris of a supernova explosion. Soon afterward the Crab Nebula was found to contain a radio pulsar with an even shorter period, 33 ms (Reifenstein et al. 1969) and the neutron star theory received further impressive support when optically visible pulsed light was observed (Nather et al. 1969) from the very star that Walter Baade and Rudolph Minkowski had identified in 1942 as the central stellar remnant of the Crab Nebula supernova explosion (Baade 1942; Minkowski 1942). Astronomers have used the powerful *Chandra X-ray Observatory* to trace out the jets, rings, winds, and shimmering shock waves generated by the highly magnetized, rapidly spinning pulsar.

As Franco Pacini previously demonstrated, before the discovery of radio pulsars, a rapidly spinning and highly magnetized neutron star is a powerful source of electromagnetic radiation (Focus 13.13), which could keep the Crab Nebula supernova remnant shining for thousands of years, and ever since the observation of the supernova explosion (Pacini 1967). Moreover, the rotational periods of radio pulsars are increasing, just as Gold predicted. If P denotes the radio pulsar period and $dP/dt = \dot{P}$ designates the rate of increase of that period with time, then the age, τ, of the radio pulsar can be estimated from:

$$\tau = \frac{P\dot{P}}{2}. \tag{13.23}$$

The ages determined from the observed slow-down rate and periods of radio pulsars range from young pulsars with ages of 1,000–10,000 years to old pulsars with ages up to 100 million years. A supernova remnant will expand and dissipate into interstellar space, becoming undetectable in less than 100 thousand years. Most observed radio pulsars have therefore outlived any observable supernova remnant they might be associated with, and most pulsars are not found in one. Only a few rare, young pulsars, such as the Crab Nebula pulsar are found in a supernova remnant.

Focus 13.3 Luminosity, rotational energy, and magnetic field strength of a radio pulsar

A neutron star with a dipolar magnetic field will behave as a rotating magnetic dipole, with radiation luminosity L_{NS} given by:

$$L_{NS} = \frac{\mu_0 m_\perp^2 \omega^4}{6\pi c^3}, \tag{13.24}$$

where the magnetic constant $\mu_0 = 1.2566 \times 10^{-6}$ N A^{-2}, the symbol $m_\perp$ denotes the component of the magnetic dipole moment perpendicular to the rotation axis, the angular rotation velocity $\omega = 2\pi/P$ for a rotation period P, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. For a uniformly magnetized neutron star of radius R_{NS} and surface magnetic field of strength B_{NS}, the magnetic dipole moment is:

$$m_\perp = \frac{4\pi}{\mu_0} B_{NS} R_{NS}^3 \sin\theta, \tag{13.25}$$

where θ is the angle between the rotation axis and the magnetic axis. We can therefore express the neutron star luminosity by:

$$L_{NS} = \frac{8\pi\left(B_{NS} R_{NS}^3 \sin\theta\right)^2}{3\mu_0 c^3}\left(\frac{2\pi}{P}\right)^4. \tag{13.26}$$

The magnetic dipole radiation extracts rotational energy, E_{rot}, from the neutron star. The rotational energy is related to the moment of inertia $I = 2MR^2/5$ for a uniform rotating sphere of mass M and radius R, and given by:

$$E_{rot} = \frac{1}{2} I\omega^2 = \frac{2\pi^2 I}{P^2} = \frac{4\pi^2}{5}\frac{MR^2}{P^2}. \tag{13.27}$$

For a neutron star $M = M_{NS} \approx 1.4\, M_{\odot}$, where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and the radius $R = R_{NS} = 10$ km $= 10^4$ m.

The rate of change, dE_{rot}/dt of the rotational energy with increasing time, t, is related to the increase $dP/dt = \dot{P}$ of the rotation period P, and given by the expression:

$$\frac{dE_{rot}}{dt} = -I\omega\frac{d\omega}{dt} = -\frac{4\pi^2 I\dot{P}}{P^3} = -\frac{8\pi^2}{5}MR^2\frac{\dot{P}}{P^3}. \tag{13.28}$$

The pulsar period and its rate of change can be combined with this equation to give the power extracted from the rotation of a neutron star, with $M = M_{NS} \approx 1.4\, M_{\odot}$, where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and the radius $R = R_{NS} = 10$ km $= 10^4$ m.

These measurements can also be used to provide a lower limit to the magnetic field strength B at the surface of the pulsar. We do not know the inclination angle θ between the rotation axis and the magnetic axis, but we do know that θ is less than 90° and that sin θ is less than or equal to 1, or that $\sin\theta \leq 1$. Setting the loss in rotational energy dE_{rot}/dt equal to the neutron star luminosity L_{NS} and combining terms from the previous equations we obtain:

$$B \geq \left(\frac{3\mu_0 c^3 M}{80\pi^3 R^4}\right)^{1/2} \left(P\dot{P}\right)^{1/2}. \tag{13.29}$$

For a neutron star with $M = M_{NS} \approx 1.4\, M_{\odot}$, where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and a radius $R = R_{NS} = 10$ km $= 10^4$ m, this equation becomes:

$$B \geq 3 \times 10^{15} (P\dot{P})^{1/2} \text{ tesla}. \tag{13.30}$$

As demonstrated in the next example, the loss of rotational energy inferred from the period increase of the pulsar is exactly what is needed to keep the Crab Nebula shining at the present rate for about 1,000 years, ever since the observation of the supernova explosion that was associated with the pulsar's birth. So it is the central pulsar that makes the nebula glow. Moreover, it was soon shown that radio pulsars could efficiently accelerate particles to nearly the speed of light, accounting for both the synchrotron radiation of the Crab Nebula and the beamed radio waves that are observed as pulsars.

Example: Energy loss and magnetic field strength of the Crab Nebula pulsar

The pulsar at the center of the Crab Nebula has a period $P = 0.033326$ s and a period increase of $dP/dt = \dot{P} = 4.213 \times 10^{-13}$ s s^{-1}. Assuming that this pulsar has a mass equal to the mass of a neutron with a mass $M = M_{NS} \approx 1.4\, M_{\odot}$, where the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and a radius $R = R_{NS} = 10$ km $= 10^4$ m, the rate of change of the rotational energy is $dE_{rot}/dt = (8\pi^2/5)MR^2\dot{P}/P^3 \approx 5 \times 10^{31}$ J s^{-1}. This rate at which the Crab Nebula pulsar loses its rotational energy is comparable to the energy requirements of the surrounding supernova remnant, including its non-thermal synchrotron radiation and its expansion.

We can also provide a lower limit to the pulsar's surface magnetic field from $B \geq [3\mu_0 c^3 M/(80\pi^3 R^4)]^{1/2} (P\dot{P})^{1/2} \approx 3 \times 10^{15} (P\dot{P})^{1/2}$ tesla $\approx 4 \times 10^8$ tesla. This magnetic field strength is comparable to that expected from the gravitational collapse of the Sun to a neutron star.

When the observed slow-down rate of most radio pulsars is combined with their periods, typical ages of between 1 million years to 100 million years are obtained. A supernova remnant will expand and dissipate into the vastness of interstellar space, becoming unrecognizable in less than 100 thousand years, removing all signs of the pulsar's birth. Pulsars therefore outlive their supernova remnants and most pulsars are not found in one. Only a few rare, young pulsars, such as the Crab Nebula Pulsar, are rotating fast enough to efficiently accelerate particles to very high energies.

Although most radio pulsars are alone in space without a nearby companion, some binary pulsars have been discovered, the most famous being PSR 1913 + 16 with a period of 0.05898 s; the PSR designates pulsar and 1913 + 16 specifies its position in the sky. Russell A. Hulse (1950–) and Joseph H. Taylor, Jr. (1941–) found it as a result of a deliberate, high-sensitivity, computerized search for new radio pulsars at the Arecibo Observatory in Puerto Rico (Hulse and Taylor 1975). The discovery of a radio pulsar that is a member of a double-star system indirectly suggested the emission of gravitational waves, which had never been seen before (Focus 3.6, Sect. 3.5).

A second binary, millisecond radio pulsar, designated PSR J1614–2230, has now been found, with a period of just 3.15 ms. It is attributed to a neutron star in orbit around a white dwarf star with an orbital period of 8.7 days, and has been used to test aspects of general relativity theory other than gravitational waves. These investigations have shown that the pulsar is the most massive neutron star known so far, with a mass of 1.97 solar masses; the mass of the white dwarf companion is 0.50 solar masses (Demorest et al. 2010).

Kramer and Stairs (2008) have summarized knowledge of the double pulsar; Hughes (2009) has discussed gravitational waves from merging compact binaries; and Joss and Rappaport (1984) have reviewed neutron stars in interacting binary systems.

Individual stars are bound together so tightly that a supernova that leads to the formation of a neutron star in a binary-star system may not disrupt its companion; the two stars can remain together, as evidenced by the binary pulsars. This is important for understanding pulsars that have been detected at x-ray wavelengths. Unlike most radio pulsars, they are members of close binary-star systems rather than single, isolated neutron stars.

13.7.3 X-ray Pulsars from Neutron Stars in Binary Star Systems

Because x-rays are absorbed in our atmosphere, cosmic x-ray sources must be observed with instruments launched above the obscuring air, in rockets or satellites. By the mid-twentieth century brief, 5-minute rocket flights had shown that the Sun radiates detectable x-rays, and it was thought that lunar material also might emit them when illuminated by solar x-rays.

Riccardo Giacconi's (1931–) group at the American Science and Engineering Company (AS&E) concluded that x-rays emitted by conventional stellar objects other than the Sun would be too faint to be detected with existing instruments. They designed the sensitive equipment needed to detect the Moon's x-rays and to search for other unknown sources of x-ray radiation. They unexpectedly found the first known discrete x-ray source outside of the solar system, which led to the discovery of a new class of cosmic objects and new physical processes (Giacconi et al. 1962).

This pioneering rocket flight set the stage for a host of rocket and satellite observations of discrete x-ray sources, including x-ray stars that are 1,000 times brighter in x-rays than the Sun at all wavelengths and that are 1,000 times more luminous in x-rays than in visible light. The x-rays signaled the presence of 1-million-degree gas spiraling from a close companion star into a neutron star or black hole.

Giacconi was awarded the 2002 Nobel Prize in Physics for these pioneering contributions, which led to the discovery of cosmic x-ray sources. He shared the prize with Raymond Davis, Jr. (1914–2006), and Masatoshi Koshiba (1926–), who were recognized for their detection of cosmic neutrinos.

One of the brightest sources in the newly discovered x-ray sky, designated Centaurus X-3, pulses in x-rays every 4.84 s. The rapid pulsating x-ray variations were discovered shortly after the launch of the first dedicated x-ray satellite, on December 12, 1970 from the offshore San Marco platform, near the Coast of Kenya (Giacconi et al. 1971). Because this date coincided with the seventh

Table 13.12 Physical properties of binary x-ray pulsars

P_{XP} = period of x-ray pulsar = 0.7–800 s
dP_{XP}/dt = rate of decrease of x-ray pulsar period = $-(10^{-5}$–$10^{-2})$ P_{XP} year^{-1}
P_0 = orbital period = 5–10 days
L_{XP} = x-ray luminosity = (0.1–10) × 10^{30} J s^{-1}
M_{XP} = mass of x-ray pulsar = 1.05–1.87 $M_{\odot}$ ≈ (2.1–3.7) × 10^{30} kg

anniversary of the independence of Kenya, the satellite was given the name *Uhuru*, the Swahili word for freedom.

After analyzing a year of observations of Centaurus X-3, the *Uhuru* scientists found a regular pattern of intensity changes of the x-ray pulses, which increased and decreased in strength with a much longer period of 2.087 days and systematic changes in the timing of pulses in the same period. These effects were attributed to a companion star, which was orbiting the x-ray source and regularly eclipsing its emission.

During the next year, accurate measurements showed that the average pulsation period of Centaurus X-3 was getting shorter, which meant that its rotation was speeding up, not slowing down like radio pulsars. This indicated that the rotational energy of the x-ray pulsar was increasing, rather than decreasing, with time. The physical properties of such binary x-ray pulsars are given in Table 13.12.

The gain in rotational energy is attributed to matter drawn in from a nearby optically visible companion star (Fig. 13.11). Because matter is being pulled toward the surface of an x-ray-emitting neutron star, instead of being expelled from it, the neutron star is knocked up to a faster rotation. The material spirals in at the same direction as the neutron star's rotation. When it lands, it gives the neutron star a sideways kick, increasing its rotational energy, speeding it up, and causing the rotation period to become shorter as time goes on. Because most radio pulsars are not members of binary-star systems, they expel material, lose rotational energy, slow down, and have periods that lengthen.

So a neutron star can be seen in x-rays when it is in very close orbit with a normal star that shines in visible light detected with an optical telescope. This stellar companion does not radiate detectable x-rays, but gas flowing from it fuels the x-ray emitting neutron star.

And there are two ways of looking at the stellar duo. The visible picture portrays only the normal star, and the x-ray image just reveals its compact neighbor, the neutron star. A complete understanding of the double-star system, one visible and the other unseen, can only be understood when the two perspectives are combined.

Because the two stars are rapidly orbiting around one another, the gas from the ordinary star does not fall directly onto the neutron star but instead shoots past the neutron star, swinging around it to form a whirling disk of hot gas, known as an *accretion disk*. It spirals around and down onto the central neutron star. The inner portions of the swirling accretion disk revolve more rapidly than the outer

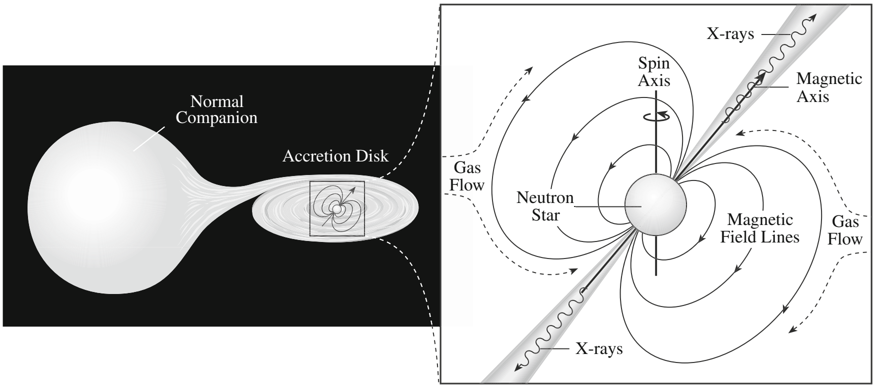

Fig. 13.11 X-ray pulsar The outer atmosphere of an ordinary star, detected in optically visible light, spills onto its companion, an invisible neutron star. The flow of gas is diverted by the powerful magnetic fields of the neutron star, which channel the in-falling material onto the magnetic polar regions. The impact of the gas on the star creates a pair of x-ray hot spots aligned along the magnetic axis at each magnetic cap. Because the magnetic-field axis can be inclined to the neutron star's rotation axis, the x-ray radiation from the hot spots can sweep across the sky once per rotation, which is observed as periodic x-rays if one of the hot spots intersects the observer's line of sight

portions, just as the closer planets orbit the Sun at faster speeds than more remote planets.

The rapidly spinning inner parts of the disk constantly rub against the slower-moving outer parts. This viscous friction heats up the accretion disk and causes the material in it to spiral inward. The closer the material moves toward the central neutron star, the hotter the in-falling gas becomes, eventually reaching temperatures of millions of K and emitting luminous x-rays.

The intense magnetic field of a rotating neutron star acts as a funnel to guide in-falling matter onto a neutron star's magnetic north and south poles, creating an x-ray pulsar. As the accreting material heats up and falls onto the polar surfaces of the neutron star, it emits two beams of x-rays that flash in and out of view as the neutron star rotates and one or two of the beams sweep past the Earth.

But there is a limit to the x-ray emission produced by the accreting material, owing to the radiation pressure it develops. The maximum luminosity is now called the Eddington limit, after Arthur Stanley Eddington (1882–1944) who showed that greater luminosity would blow away any surrounding matter – long before the discovery of any x-ray star (Eddington 1926). The outward force of radiation pressure becomes equal to the inward gravitational force on the accreting material when the radiation luminosity is at the Eddington luminosity. A neutron star that is accreting material at close to the Eddington limit will heat up to a temperature of about 20 million K and emit intense x-ray radiation (Focus 13.4).

Focus 13.4 Accretion luminosity and the Eddington limit
The luminous output resulting from mass falling, or accreting, onto a compact object will depend upon the rate of mass transfer, denoted by $\dot{M} = dM/dt$, as well as the mass, M, and radius, R, of the compact object. From the conservation of energy, half the release in gravitational potential energy will be equal to the gain in kinetic, or thermal, energy of the hot accreting gas, and the remaining half will be converted into heat. The luminous output, L_{acc}, of the thermal radiation from the accreting material will be:

$$L_{acc} = \frac{G\dot{M}M}{2R}. \tag{13.31}$$

The temperature, T, of the radiating gas can be obtained from the Stefan–Boltzmann law, which results in

$$T = \left[\frac{L_{acc}}{4\pi\sigma R^2}\right]^{1/4} = \left[\frac{G\dot{M}M}{8\pi\sigma R^3}\right]^{1/4}, \tag{13.32}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-1}.

When the compact receiving object is a white dwarf star, the mass is $M \approx M_{\odot} = 1.989 \times 10^{30}$ kg, the radius is $R_{WD} \approx 6 \times 10^6$ m, and the mass accretion rate $\dot{M} = 10^{-9}\ M_{\odot}$ yr^{-1}, where 1 year = 3.1557×10^7 s. The accretion luminosity is $L_{acc} \approx 7 \times 10^{26}$ J s^{-1} $\approx 2\ L_{\odot}$, and the temperature $T \approx 3 \times 10^5$ K. For a neutron star with the same rate of mass transfer, the luminosity will be about a thousand times that of the Sun and the temperature will be about a million K, but higher rates of mass accretion are possible.

Arthur Stanley Eddington (1882–1944) showed that there is a maximum luminosity, now called the Eddington luminosity, L_{Edd}, for any source of radiation before it blows away the surrounding matter. In effect, the outward force of radiation pressure from the compact accreting star pushes against the inward gravitational force on the accreting material. The two forces become equal when the radiation luminosity is (Eddington 1926a, b Sect. 10.1, Focus 10.1):

$$L_{Edd} = \frac{4\pi G m_p c M}{\sigma_T} \approx 6.3M \text{ J s}^{-1}, \tag{13.33}$$

or

$$L_{Edd} \approx 1.25 \times 10^{31} \frac{M}{M_{\odot}} \text{ J s}^{-1}, \tag{13.34}$$

where M is the mass of the compact accreting star, the Thomson scattering cross section for the electron is $\sigma_T = 6.65246 \times 10^{-29}$ m^2, the mass of the proton is $m_p = 1.6726 \times 10^{-27}$ kg, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}.

The numerical approximation can also be written:

$$L_{Edd} \approx 3.3 \times 10^4 \frac{M}{M_\odot} L_\odot \text{ J s}^{-1}. \quad (13.35)$$

where the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1},

The Eddington limit sets an upper limit to the accretion luminosity of a compact object, since for L_{acc} greater than L_{Edd}, the radiation pressure will inhibit further accretion. The maximum accretion rate, $\dot{M}_{Edd}$, is obtained by equating the accretion luminosity to the Eddington limit, or

$$\dot{M}_{Edd} = \frac{8\pi m_p c R}{\sigma_T}, \quad (13.36)$$

or

$$\dot{M}_{Edd} \approx 2 \times 10^{11} R \text{ kg s}^{-1} \approx 3 \times 10^{-12} R \, M_\odot \text{ yr}^{-1}, \quad (13.37)$$

for a compact object of radius R. At the Eddington limit, a white dwarf star with $R = 6 \times 10^6$ m, the accretion rate would be about 2×10^{-5} solar masses per year. It can be substantially less than this amount.

Sometimes a compact, invisible object with a visible companion can be force-fed with more matter than it can consume, and it hurls the in-falling matter out in two oppositely directed jets. This can happen when the invisible star is a black hole, which we discuss next.

Example: Accretion luminosity and temperature from mass transfer to a neutron star

The maximum mass transfer rate $\dot{M}_{Edd}$ onto a neutron star of radius $R_{NS} = 10$ km $= 10^4$ m will be $\dot{M}_{Edd} = 8\pi m_P c R_{NS}/\sigma_T \approx 2 \times 10^{15}$ kg s^{-1}, where the proton mass $m_P = 1.6726 \times 10^{-27}$ kg, the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, and the Thomson scattering cross section $\sigma_T = 6.65246 \times 10^{-29}$ m^2. The accretion luminosity onto a solar-mass neutron star of mass $M = 1.4\, M_\odot = 2.785 \times 10^{30}$ kg will be $L_{acc} = GM_\odot \dot{M}_{Edd}/2R_{NS} \approx 1.25 \times 10^{31}$ J s^{-1}, where the Newtonian constant of gravitation $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}. The temperature, T, of the gas can be estimated from the Stephan-Boltzmann law $L_{acc} = 4\pi\sigma R_{NS}^2 T^4$, or $T = \left[L_{acc}/\left(4\pi\sigma R_{NS}^2\right)\right]^{1/4} \approx 2 \times 10^7$ K, where the Stefan–Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}. From the Wien displacement law (Sect. 2.4), a thermal gas at this temperature will emit most of its radiation at a wavelength $\lambda_{max} = 0.0029/T = 1.4 \times 10^{-10}$ m, or at x-ray wavelengths.

13.8 Stellar Black Holes

13.8.1 Imagining Black Holes

John Michell (1724–1793), a British clergyman and natural philosopher, suggested more than two centuries ago that certain stars could remain forever invisible. He reasoned that a star might be so massive and its gravitational pull so powerful that light could not escape it. He wrote: "All light from such a body would be made to return to it by its own proper gravity" (Michell 1784).

The French astronomer and mathematician Pierre Simon de Laplace (1749–1827) popularized the idea, including it in his *Exposition du système du monde*. He subsequently showed that light could never move fast enough to escape the immense gravitational attraction of some compact stars (Laplace 1796). Their matter might be so concentrated, and the pull of gravity so great, that light could not emerge from them, making these stars forever dark and invisible.

When it was realized that light travels at a definite speed of 2.9979×10^8 m s^{-1}, or roughly 300,000 km s^{-1}, a *black hole* could be defined as any object whose escape velocity, the velocity required for escape from an object's gravitational pull, exceeds the velocity of light. All stars that we can observe have escape velocities smaller than the speed of light, which is why we can observe their light. The Sun, for example, has an escape velocity of 600 km s^{-1}. Compressing a star into a smaller size raises its escape velocity. When a dead stellar core of approximately the Sun's mass collapses into a white dwarf star, its escape velocity increases to about 9,000 km s^{-1}, or 3 % of the speed of light. At a neutron star's radius, the escape velocity becomes about 210,000 km s^{-1}, or 70 percent of the speed of light.

If a massive star has consumed all of its available nuclear fuel and its core mass exceeds the upper limit to a neutron star's mass, at about 3 solar masses, the stellar core's gravity will overcome the degenerate neutron pressure of even nuclear matter. At this point, there is no known force that can halt the collapse. The smaller the collapsing core becomes, the larger is its escape velocity, until it exceeds the speed of light and a stellar black hole is formed. It is black, or rather invisible, because no light can leave it, and it is a hole because nothing that falls into it can escape.

In other words, if a star has used up its thermonuclear fuel and its core is sufficiently massive, there is nothing left to hold back the inexorable force of gravity. The core continues to collapse forever, vanishing from the directly observable universe.

13.8.2 Observing Stellar Black Holes

Since a black hole is invisible, and it does not absorb, emit, or reflect radiation, how do we know it is there? We detect a black hole by its gravitational effect on the motion of a visible star.

With remarkable foresight, the Reverend John Michell also speculated, in 1784, that the unseen star might betray its presence by its gravitational effects on a nearby, luminous star in orbit around it (Michell 1784). In modern extensions of this idea, a black hole may be detected if it is in a tight, close orbit with a visible star whose outer atmosphere spills over into the dominant gravitational influence of the black hole. This material swirls around and down into the black hole, orbiting faster and faster as it gets closer – as a result of the ever-increasing gravitational forces. The rapidly moving particles collide as they are compressed to fit into the hole, heating the material to temperatures of millions of K. At these temperatures, the gas emits almost all of its radiation at x-ray wavelengths. It is analogous to the accretion disks of binary x-ray pulsars, except that the invisible companion is a black hole rather than a neutron star.

Remillard and McClintock (2006) have reviewed the x-ray properties of black-hole binaries, whereas Eardley and Press (1975) provided an earlier review of astrophysical processes near black holes. Mirabel and Rodriguez (1999) review black-hole sources of relativistic jets in our Galaxy.

So, the way to find a stellar black hole is to look for two stars that are in close orbit, one a normal visible star and the other unseen except for its x-rays. The mass, velocity, and orbital period of the visible star can be used to determine the mass of its orbital partner, which emits no visible light. If that mass is noticeably greater that the upper mass limit for a neutron star, set at about 3 solar masses, the unseen star is thought to be a stellar black hole. Any normal star with this mass would be very bright and easily seen through a telescope, but a black hole is dark, emitting no detectable visible light.

The archetype of a stellar black hole is Cygnus X-1, located in the constellation Cygnus and one of the first x-ray sources to be discovered. Rapid, irregular x-ray bursts from this object were detected from the *Uhuru* satellite in 1970. The x-rays flickered on and off as rapidly as a few milliseconds, and because nothing travels faster than the speed of light the emitter had to be less than 300 km across. This meant that it was smaller than a white dwarf star, and had to be either a neutron star or a black hole.

Cygnus X-1 is accompanied by a bright, blue supergiant star of spectral class O, located at a distance of about 6,000 light-years from the Earth. Observations of the continuously and periodically shifting spectral lines of this bright visible star – by the English astronomers B. Louise Webster (1941–1990) and Paul Murdin (1942–), and independently confirmed by the Canadian astronomer Charles Thomas Bolton (1943–) – indicated that it is revolving every 5.60 days about an invisible companion of more than eight times the mass of the Sun for the unseen companion (Webster and Murdin 1972; Bolton 1972, 1975). It emits no light and its mass is greater than that of the upper mass limit for a neutron star, of about 3.0 solar masses; therefore, by elimination, it must be a stellar black hole.

Recent measurements of the Cygnus X-1 binary star system indicate that the visible supergiant star has a mass of 19.2 solar masses and the unseen companion has a mass of 14.8 solar masses, which confirms that its mass is well above the upper mass limit for a neutron star. In addition, the two stars are separated by only 0.2 AU, or 20 percent of the distance from the Earth to the Sun and about half the separation of Mercury from the Sun (Orosz et al. 2011; Reid et al. 2011). Like other supergiant stars of its spectral type, the visible star is thought to be shedding mass in a stellar wind at a rate of about 2.3 solar masses every 1 million years. Due to the proximity of the invisible companion, a significant portion of this wind is being drawn into the black hole to form its x-ray-emitting accretion disk.

We now know of many stellar black holes identified in this way. The orbital properties of visible companions of cosmic x-ray sources indicate masses beyond the neutron-star limit. Like Cygnus X-1, many of these black holes can also exhibit highly luminous, rapid, and irregular x-ray outbursts, showing that they are very small on a cosmic scale. The transient x-ray flickering, sometimes brightening a million-fold in milliseconds, is most likely emitted as in-falling material takes the final plunge and vanishes into the black hole.

13.8.3 Describing Black Holes

The outer edge of a black hole can be defined as the radius at which the escape velocity, required to escape from its gravitational pull, is equal to the speed of light, or when the kinetic energy of an object moving at this speed is equal to the gravitational potential energy of the mass holding it in. This radius is 3,000 m for a stellar black hole with the mass of the Sun. It is known as the *Schwarzschild radius* in recognition of the German astronomer Karl Schwarzschild (1873–1916), who first recognized its mathematical significance, and also sometimes called the *gravitational radius*. While serving as an artillery lieutenant on the Russian front during World War I (1914–1918), Schwarzschild derived the solution to Einstein's *General Theory of Relativity* for a spherical, non-rotating black hole. His publication, titled "On the Field of Gravity of a Point Mass in the Einsteinian Theory" can be used to specify the space time intervals outside a black hole (Schwarzschild 1916).

The radius of a black hole can be defined as the radius, R, at which the escape velocity, V_{esc}, of a particle of mass, m, from a larger mass, M, and radius, R, is equal to the speed of light, c, or when $V_{esc} = (2GM/R)^{1/2} = c$, for a gravitational constant G. That's obtained from classical physics by equating the kinetic energy of a moving object to the gravitational potential energy of the mass holding it in, with $V_{esc} = c$. That is the kinetic energy $mV_{esc}^2/2 = GMm/R$, the gravitational potential energy. Solving for the radius, we obtain the Schwarzschild radius R_g given by

$$R_g = \frac{2GM}{c^2} \approx 2.95 \times 10^3 \left(\frac{M}{M_\odot}\right) \text{ m}, \tag{13.38}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, and the solar mass $M_\odot = 1.989 \times 10^{30}$ kg.

Example: Schwarzschild radius of the Earth and Sun
The Schwarzschild radius, also known as the gravitational radius, of the Earth is $R_{SE} = 2GM_E/c^2 \approx 8.87 \times 10^{-3}$ m, where the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}, the mass of the Earth $M_E = 5.974 \times 10^{24}$ kg, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The physical radius of the Earth is $R_E = 6.378 \times 10^6$ m, which is almost a billion times larger than the planet's gravitational radius. For the Sun, the gravitational radius $R_{S\odot} \approx 2.95 \times 10^3$ m ≈ 3 km, for a solar mass $M_\odot = 1.989 \times 10^{30}$ kg. The physical radius of the Sun $R_\odot = 6.955 \times 10^8$ m is more than a hundred thousand times larger than its gravitational radius. At the time that Schwarzschild derived the metric involving this term, there was no known physical object whose linear size was smaller than its gravitational, or Schwarzschild, radius.

In the *Special Theory of Relativity*, space and time were combined to define a metric, ds, or space time interval, given by (Minkowski 1908; Hargreaves 1908)

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \tag{13.39}$$

for time interval dt and space coordinates x, y, z. The speed of light, c, has been added to give dt the units of distance. This metric applies to Euclidean space without gravity and no curvature of space.

In the *General Theory of Relativity*, for which gravity manifests itself in the curvature of space time, the metric outside a non-rotating mass in a vacuum is the *Schwarzschild metric* given by (Schwarzschild 1916):

$$ds^2 = \left[1 - \frac{2GM}{c^2 r}\right] c^2 dt^2 - \frac{dr^2}{\left[1 - \frac{2GM}{c^2 r}\right]} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2. \tag{13.40}$$

Here r, θ, ϕ are spherical coordinates whose origin is at the center of the massive object, and M is the mass. The trajectories of free-falling particle and radiation are described by a null geodesic, for which $ds = 0$.

A proper time τ is defined by the proper time interval $d\tau$ given by:

$$d\tau = \frac{ds}{c}. \tag{13.41}$$

For a clock at rest, the space coordinates are not changing, so $dr = d\theta = d\phi = 0$, and the proper time interval is:

$$d\tau = \left[1 - \frac{2GM}{c^2 R}\right]^{1/2} dt. \tag{13.42}$$

The proper time interval approaches zero times dt when the radius R approaches the Schwarzschild radius $R_S = 2GM/c^2$. A clock placed on a collapsing star will appear, to a distant observer, to tick more slowly as the star's radius approaches the Schwarzschild radius. This is known as gravitational *time dilation*. The clock will appear to stop, at this critical radius, when time will seem to go on forever and the star has become "frozen".

The decline in the observed radiation due to gravitational time dilation is expressed as a decrease in the number, N_{ph}, of photons observed, with (Oppenheimer and Snyder 1939):

$$N_{ph} \propto \exp\left(\frac{-t}{2R_S}\right), \tag{13.43}$$

or with a characteristic observed free-fall time, τ_{ff}, given by:

$$\tau_{ff} = \frac{2R_S}{c} = \frac{4GM}{c^2} \approx 2.94 \times 10^3 \left(\frac{M}{M_\odot}\right) \text{ s}, \tag{13.44}$$

where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg.

Example: A star's core collapse time to become a black hole

If a sufficiently massive star consumes all the available thermonuclear fuel, and its core is more massive than about 3 solar masses, it will collapse to a black hole in a free-fall time given by $\tau_{ff} = 4GM/c^2$, with the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. For a 5 solar mass star with $M = 5\ M_\odot$, where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, the free-fall time is $\tau_{\mathrm{ff}} \approx 10^4$ s.

During this time, the number of observed photons will have decreased by exp (−1,000), or e^{-1000}, a very small number, so the star will have effectively disappeared, becoming invisible and black.

The Schwarzschild metric has a one over zero term, a singularity, which blows up at the Schwarzschild, or gravitational, radius $R_g = 2GM/c^2$, which is the radius at which the escape velocity becomes equal to the speed of light. In 1963 the New Zealand mathematician Roy Kerr (1934–) described the space–time metric outside a rotating black hole, and it reduces to Schwarzschild's metric when there is no rotation. The gravitational field outside black holes are uniquely determined by their mass and angular momentum and described by Kerr metrics.

The singularity can be moved down to the center of the black hole by a clever change in geometry, but it never disappears. It is a location where matter is compressed into an undefined state of infinite mass density. That is, in 1960 two mathematicians, the American Martin Kruskal (1925–2006) and the Hungarian-Australian George Szekeres (1911–2005), defined a line element that removes the singularity at the Schwarzschild radius through a coordinate transformation, but that didn't mean that the singularity disappeared – you can't form a black hole within the constraints of *General Relativity* without having one; the English astrophysicist Roger Penrose (1931–) demonstrated that (Penrose 1965). The singularity was just moved down to the center of the black hole in the Kruskal-Szekeres transformation.

Nevertheless, the defining notion of a black hole, in terms of an escape velocity that exceeds the speed of light, and the method of inferring its presence in an x-ray emitting binary star system with a close visible companion, do not depend on Einstein's theory and can be determined from classical Newtonian physics without any singularity.

The Schwarzschild radius, which can be located outside the singular center, has not lost its significance. It marks the event horizon – literally, a horizon in the geometry of space–time beyond which no event can be seen, just as the Earth's horizon is the boundary for our vision. Nearby space then is said to curl into a black hole, carrying light and matter and any other form of energy with it. They are so intensely wrapped around a black hole that it becomes a cocoon disconnected from the outside and cut off forever from the rest of the universe.

Black holes are mysterious objects. They cannot be observed directly, because any radiation they might emit cannot escape. A black hole's presence can be inferred only from indirect, circumstantial evidence, using measurements in the accessible parts of the universe – the visible stars – to make inferences about the dark places that cannot be observed.