Chapter 10
The Sun Amongst the Stars

10.1 Comparisons of the Sun with Other Stars

10.1.1 How Far Away are the Stars?

To determine the distance of a nearby star (other than the Sun), astronomers measure its angular displacement when viewed from opposite sides of the Earth's orbit, or from a separation of twice the AU. The AU is the mean distance between the Earth and the Sun, with a value of about 149.6 million km. This angle is known as the *annual parallax*, from annual for the Earth's yearlong orbit and the Greek word *parallaxis* for the "value of an angle". Once the parallax is combined with the known value of the AU, the star's distance can be established by *triangulation*, the geometry of a triangle.

The measurement involves careful scrutiny of two stars that appear close together in the sky: a bright one relatively nearby and another fainter one much farther away (Fig. 10.1). The annual parallax of the nearer star can then be determined by comparing its position to that of the distant one for a year or more.

By definition, the annual parallax, denoted π_A, is half the apparent angular displacement of a nearby star observed against the more distant stars at intervals of six months from opposite sides of the Earth's orbit. That is:

$$\sin\pi_A = \mathrm{AU}/D \approx \pi_A \tag{10.1}$$

for a star at distance D and 1 AU $= 1.496 \times 10^{11}$ m. As long as the stellar distances are much larger than the AU, which is always the case, the parallax angle π_A is small and $\sin \pi_A \approx \pi_A$. The distance D can therefore be given by:

$$D = \frac{1}{\pi_A}\ \text{parsec}, \tag{10.2}$$

if π_A is given in seconds of arc, denoted by the symbol $''$. The name of this distance unit, the *parsec*, derives from the italicized part of the two words *par*allax and

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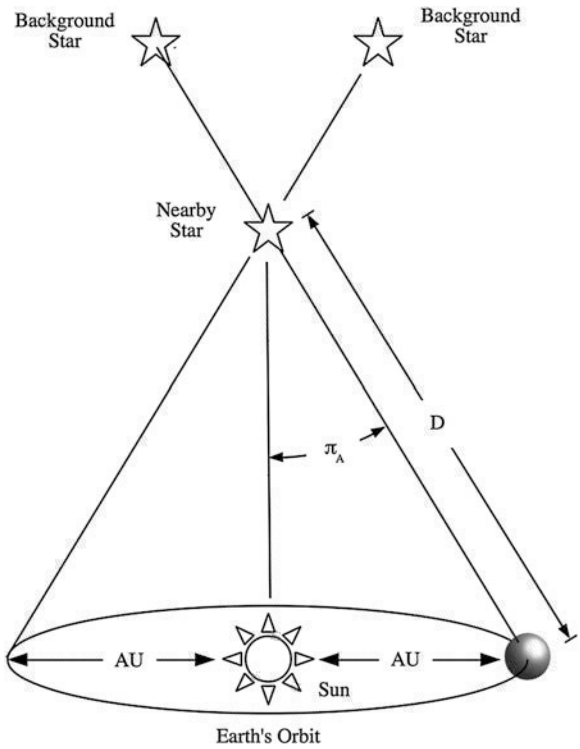

Fig. 10.1 Annual parallax When a distant and nearby star are observed at six-month intervals, from opposite sides of the Earth's orbit around the Sun, astronomers measure the angular displacement between the two stars. It is twice the annual parallax, designated by π_A, which can be used to determine the distance, D, of the nearby star. From trigonometry, sin $\pi_A = AU/D \approx \pi_A$ for small angles, where 1 AU is the mean distance between the Earth and the Sun. The distance D to the star in units of parsecs is given by $1/\pi_A$, if the parallax angle is measured in seconds of arc. This angle is greatly exaggerated in the figure, for all stars have a parallax of less than 1 s of arc or less than 1/3,600th of a degree. The German astronomer Friedrich Wilhelm Bessel (1784–1846) announced the first reliable measurement of the annual parallax of a star in 1838

*sec*onds of arc. The parsec, abbreviated pc, is a convenient unit for measuring stellar distance since neighboring stars often are separated by about 1 parsec.

For conversion purposes, it is useful to know that:

$$1\,\text{parsec} = 1\,\text{pc} = 3.26\,\text{light years} = 3.0857 \times 10^{16}\,\text{m} = 206,265\,\text{AU}. \quad (10.3)$$

The light year is the distance light travels in one year, moving at the speed of light $c = 2.9979 \times 10^8$ m s^{-1} where 1 year = 3.1557×10^7 s.

The fact that light travels through space at a constant speed provides another convenient unit of astronomical distance, by the time it takes light to move through space from the object to the Earth. This is known as the *light-travel time*. Light from the Moon takes 1.5 s to reach the Earth, so we say that the Moon is 1.5 light-seconds away from the Earth. Sunlight takes 499 s or 8.3 min to cover the average distance between the Sun and the Earth. The nearest star (other than the Sun) is at a distance of a little more than 4 light-years.

Many of the brightest stars are hundreds of light-years away, and some stars in our Milky Way are many millions of light-years away. Starlight may reach us from stars that have now extinguished the internal nuclear fires that make them shine. The most distant objects in the universe are billions of light-years away, and the light we now detect from them was generated that long ago. So, radiation provides a method of looking back into time, to decipher the history of the universe – looking back to the time before the Sun and the Earth were formed, some 4.6 billion years ago.

The German astronomer Friedrich Wilhelm Bessel (1784–1846) made the first reliable determination of the annual parallax of a star, reporting a parallax of $0.31''$ for 61 Cygni, with an uncertainty of $\pm 0.02''$ (Bessel 1839). The modern measurement of $0.286''$ indicates that 61 Cygni is at a distance of 3.49 pc or 721,000 AU. Traveling at the speed of light, it takes 11.4 years to cross the distance from 61 Cygni to Earth.

Bessell was closely followed by the Scottish astronomer Thomas Henderson (1798–1844), who obtained a parallax of $1.19'' \pm 0.11''$ for the bright star Alpha Centauri (Henderson 1839). Bessell and Henderson were both observing stars that had exceptionally large motions across their line of sight, suggesting that they would be the nearest stars if all stars move at the same speed. Jackson (1956) and Hirshfeld (2001) describe attempts to obtain the first measurement for the distance of a nearby star (other than the Sun).

There is no known star, other than the Sun, whose annual parallax is greater than $1''$, or whose distance from Earth is less than 1 pc. The star with the largest parallax is Proxima Centauri, with a parallax of $0.769''$, making it the closest star to the Earth other than the Sun. The distance to Proxima is 1.30 pc, and light takes only 4.24 years to reach us from this star. It is about 268,000 times more distant than the Sun, indicating that there are vast, seemingly empty spaces between the stars.

Proxima Centauri is the nearest and dimmest companion of a triple star system that includes Alpha Centauri, the third brightest star in the sky. It takes about a million years for Proxima to orbit Alpha Centauri, and we only think it is associated with it because all three stars move together through space in about the same direction and with about the same speed. The large angular separation of Proxima Centauri from Alpha Centauri is a little more than 2.18°, or four times the angular diameter of the full Moon, which makes it possible to view the much dimmer companion outside the glare of the brighter star.

Because the Earth's atmosphere usually limits the angular resolution of a ground-based telescope to no more than $0.05''$, the annual parallax method can be used only for the very nearest stars, those that are closer than about 65 light-years or 20 pc. However, instruments aboard the ESA *HIPPARCOS* satellite, which orbited the Earth above its atmosphere in the 1990s, pinpointed the position of more than 100,000 stars with an astonishing precision of 0.001″ and obtained accurate measurements for the distances of stars up to 1,000 pc and 3,260 light-years away. This explains the spacecraft's name, which is an acronym for *HIgh Precision PARallax COllecting Satellite*; the name also alludes to the ancient Greek astronomer Hipparchus, who recorded accurate star positions more than 2,000 years ago. A successor to this mission is the ESA *GAIA* mission, short for *Global Astrometric Interferometer for Astrophysics*, currently scheduled for launch in March 2013. This mission is intended to measure 1 billion stellar distances to perhaps 10,000 pc and 32,600 light-years.

10.1.2 How Bright are the Stars?

The apparent brightness of a star is how bright it appears to us when its radiation reaches the Earth. The celestial positions and physical parameters of the ten brightest stars are given in Table 10.1, together with the brightest star, the Sun.

Because a human eye does not register directly the relative amount of radiation entering it, the Greek astronomer Hipparchus (c. 190 BC–c. 120 BC) divided the stars that he could see into six groups to better measure their relative brightness, relative to the eyes. This way of measuring brightness is called the *apparent visual magnitude* and is designated by the lowercase letter m or to be explicit about the visual aspect, by m_V with the subscript V denoting "visual". Hipparchus designated the brightest stars, such as Sirius or Rigel, with the first and most important magnitude, $m = 1$; Polaris and most of the stars in the Big Dipper were designated $m = 2$; and the faintest stars visible to the unaided eye received the sixth magnitude, or $m = 6$. Thus, in the magnitude system, brighter stars have lower magnitudes and fainter stars have higher ones.

About two millennia later, the British astronomer Sir Norman Pogson (1829–1891) noted that the stars of the first magnitude were 100 times as bright as stars of the sixth magnitude and that each magnitude unit is 2.512 times brighter than the next one down, where the number 2.152 is the fifth root of 100, or $100^{1/5}$ (Pogson 1856). The apparent magnitudes m_1 and m_2, of two objects of apparent brightness, or apparent radiation flux f_1 and f_2, are related by:

$$m_1 - m_2 = -2.512 \log\left(\frac{f_1}{f_2}\right) = 2.512 \log\left(\frac{f_2}{f_1}\right), \tag{10.4}$$

where the subscripts denote objects 1 and 2, and log denotes the logarithm to the base ten. An equivalent relation is:

$$\frac{f_1}{f_2} = 2.512^{(m_2 - m_1)} = 10^{0.4(m_2 - m_1)} = 2.512^{-(m_1 - m_2)} = 10^{-0.4(m_1 - m_2)}. \tag{10.5}$$

Table 10.1 The ten brightest stars as seen from Earth[a]

Star name	R.A. (2000)		Dec (2000)		m	Spectral Class[b]	D[c] (light-years)	L ($L_\odot$)	M	Mass ($M_\odot$)	R ($R_\odot$)
	h	m	°	′							
Sun					−26.74	G2 V	0.000016	1.0	+4.83	1.0	1.0
Sirius	06	45.2	−16	43.0	−1.46	A1 V	8.6	25.4	+1.42	2.02	1.71
Canopus	06	24.0	−52	41.8	−0.72	F0 Ib	310	13,600	−5.53	8.5	65.0
Alpha Centauri[d]	14	39.6	−60	50.0	−0.01	G2 V	4.3	1.52	+4.38	1.10	1.23
Arcturus	14	15.7	+19	11.0	−0.04	KI III	36.7	210	−0.29	1.5	25.7
Vega	18	36.9	+38	47.0	+0.03	A0 V	25.0	37	+0.58	2.14	2.5
Capella	05	16.7	+45	42.2	+0.08	G1 III	41	78	+0.20	2.6	9.2
Rigel	05	14.5	−08	12.1	+0.18	B8 Ia	772.5	66,000	−6.7	17.0	78.0
Procyon	07	39.3	+05	13.5	+0.34	F5 IV	11.46	7.73	+2.65	1.42	2.05
Achernar	01	37.7	−57	14.2	+0.50	B3 V	144	3,311	−2.77	6 to 8	10
Betelgeuse	05	55.2	+07	24.4	+0.42v	M2 Ia	643	140,000	−6.05	18 to 19	≈1,180

[a] The stars are listed in order of increasing apparent visual magnitude, m, or decreasing apparent brightness, for the brightest component if it is a binary system. The absolute magnitude is designated as M

[b] The luminosity classes are Ia = Supergiant of high luminosity, Ib = Supergiant of lower luminosity, II = bright giant, III = Normal giant, IV = Sub-giant, V = Main-sequence star, or dwarf star, VI = Subdwarf

[c] The luminosity, L, is in units of the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, the mass is units of the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, and the radius, R, is units of the Sun's radius $R_\odot = 6.955 \times 10^{8}$ m

[d] Alpha Centauri is also known as Rigel Kentaurus

Table 10.2 Apparent visual magnitudes, m_V, of some astronomical objects

Object name	m_V
Sun	−26.74
Full moon	−12.7
Venus[a]	−4.5
Jupiter[a]	−2.5
Sirius	−1.44
Rigel	0.12
Saturn[a]	0.7
Polaris	1.97

[a] At maximum brightness when the planet is in the part of its orbit that brings it closest to the Earth

The apparent magnitude relation takes into account the nonlinear, roughly logarithmic response of the human eye to light intensity, and the apparent radiation flux is a technical name for a star's apparent brightness.

This logarithmic scale caused the very brightest stars to climb to negative apparent magnitudes. Sirius is $m = -1.44$, the planets Venus and Jupiter are a little brighter than Sirius, and the Sun is so close and bright that it is $m = -26.74$ (Table 10.2).

The number of stars increases dramatically with increasing apparent visual magnitude. There are 14 stars brighter than $m = 1$ and about 5,600 stars brighter than $m = 6$, which are all of the stars detectable by the unaided human eye. There are 335,000 stars brighter than $m = 10$, 1.5 million stars brighter than $m = 12$, and 4.8 billion stars brighter than $m = 25$. A backyard telescope can detect stars of apparent magnitude between 10 and 15; the *Hubble Space Telescope* can approach apparent magnitude 30. These stars are 4 billion times fainter than the human eye can see without a telescope, and there are 100 billion of them.

10.1.3 How Luminous are the Stars?

Luminosity is an intrinsic measure of a star, and it is not related to the star's distance from the observer. It is the amount of energy a star radiates per unit time in units of J s^{-1}, which also is the emitted power in watts, where 1 J s^{-1} = 1 watt. A star's luminosity usually is compared to the luminosity of the Sun, designated $L_{\odot}$, with a value of $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}. There are stars that are 1 million times more luminous than the Sun (Table 10.3) and other stars that are 1 million times less luminous than the Sun. The exceptionally luminous beacons are rare, and the most common stars are not even as luminous as the Sun; they are so dim that telescopes are required to see them. The most luminous stars also are amongst the most massive, largest, and hottest stars, and the progressive decrease in stellar luminosity usually corresponds to a decrease in stellar mass and radius (see Table 10.3).

Table 10.3 The range in stellar luminosity[a]

Star name	Luminosity ($L_\odot$)	Mass ($M_\odot$)	Radius ($R_\odot$)	Temperature (K)
R 136a1[b]	8,700,000[c]	265	35.4	53,000
LBV 1806 ~ 20	5,000,000[c]	200	150	36,000
Pistol Star	1,700,000[c]	150	340	20,000
Betelgeuse	200,000[c]	19	1,180	3,500
Rigel	85,000[c]	17	78	11,000
Polaris	2,000	7.5	30	3,200
Aldebaren	425	1.7	44	4,010
Vega	37	2.1	2.5	9,600
Sirius A	25.4	2.0	1.7	9,940
Alpha Centauri A	1.5	1.1	1.2	5,790
Sun	1.0	1.0	1.0	5,780
Sirius B[d]	0.026	0.978	0.0084	25,200
Gliese 229B[e]	0.000006	0.03 to 0.05	0.1	950

[a] The luminosity is in units of the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, the mass is in units of the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, the radius is in units of the Sun's radius $R_\odot = 6.955 \times 10^{8}$ m, and temperature is the effective temperature of the visible stellar disk in degrees kelvin, denoted K

[b] In the Large Magellanic Cloud, a nearby irregular galaxy and satellite of the Milky Way

[c] Bolometric luminosity

[d] Sirius B is an Earth-sized white dwarf star, which has a mass about equal to that of the Sun but has depleted its thermonuclear fuel

[e] Gliese 229B is a sub-stellar brown dwarf object, whose mass is below the lower limit, at about 0.08 solar masses, to sustain hydrogen fusion. This brown dwarf object has a radius about equal to that of Jupiter, which is very close to one tenth the Sun's radius, but a mass of 30–50 times the mass of Jupiter, or about 0.03–0.05 times the mass of the Sun

Astronomers observe an apparent brightness or *apparent radiation flux*, designated by the symbol f_S, which is the amount of radiant energy per unit time per unit area reaching the Earth. This quantity has units of J s^{-1} m^{-2}, and it depends on a star's distance from the Earth.

As stellar radiation travels out into space, it is distributed over an imaginary sphere of surface area $4\pi D_S^2$ at distance D_S from the star. The apparent radiation flux reaching a terrestrial observer from a star of luminosity L_S is:

$$f_S = \frac{L_S}{4\pi D_S^2} \tag{10.6}$$

This is sometimes called the inverse square law of light, since the apparent brightness, or radiation flux, falls off as the inverse square of the distance.

The apparent brightness of stars can be combined with measurements of their distances to determine their luminosity. Some of the stars that appear bright to the eye are relatively nearby and no more luminous than the Sun, but many are distant stars that are hundreds of thousands of times more luminous than the Sun. Thus, the exceptional brightness of the brightest stars, as seen from the Earth, can be due to either the immense power of the radiation they emit or to their relative closeness, compared with other stars.

Example: The Sun's apparent brightness and intrinsic luminosity
Instruments aboard satellites have measured the Sun's apparent brightness, known as the solar constant, $f_{\odot}$. It is the total amount of radiant solar energy per unit area reaching the top of the Earth's atmosphere at the Earth's mean distance from the Sun, and the measurements indicate that $f_{\odot} = 1{,}361$ J s^{-1} m^{-2}. Using a mean distance of 1 AU = 1.496×10^{11} m, we obtain the Sun's luminosity $L_{\odot} = 4\pi f_{\odot}(\mathrm{AU})^2 = 3.828 \times 10^{26}$ J s^{-1}.

By extrapolating the Sun's radiation flux back to its visible disk, we can use that flux, designated $F_{\odot}$, to specify the disk's effective temperature, T_{eff}, by the relation (Sect. 3.4):

$$F_{\odot} = \frac{L_{\odot}}{4\pi R_{\odot}^2} = \sigma T_{eff}^4, \tag{10.7}$$

where the Sun's radius $R_{\odot} = 6.955 \times 10^8$ m and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}. Given these quantities, we obtain $T_{eff} \approx 5{,}780$ K for the visible solar disk.

To compare stellar luminosities, distance has to be removed from the comparison, and a system of absolute magnitudes is used for that. The absolute magnitude, M, is defined as the apparent magnitude the star would have if it was at an arbitrary standard distance of 10 parsecs, 10 pc, or 32.6 light-years. So

$$M = m + 5 - 5\log D, \tag{10.8}$$

where D is the distance in parsecs. The absolute magnitude may also be derived from the apparent magnitude and the annual parallax by the formula

$$M = m + 5 + 5\log \pi_A, \tag{10.9}$$

where π_A is the annual parallax in seconds of arc denoted $''$.

Example: The Sun's absolute visual magnitude
The Sun has an apparent visual magnitude of $m_{v\odot} = -26.74$, and it lies at a mean distance of 1 AU = 1.496×10^{11} m = 4.848×10^{-6} pc, using the conversion of 1 parsec = 1 pc = 3.0857×10^{16} m. So the absolute magnitude of the Sun is given by $M_{v\odot} = -26.74 + 5 - 5 \log(4.848 \times 10^{-6}) = -26.74 + 5 - 5 \log(4.848) + 30 = 4.83$.

The luminosity, L_1 and L_2, of two objects of absolute magnitude, M_1 and M_2, are related by:

$$\frac{L_1}{L_2} = 2.512^{(M_2-M_1)} = 10^{0.4(M_2-M_1)} \tag{10.10}$$

where $2.512 = 100^{1/5} = 10^{0.4}$.

Absolute magnitudes range from about −12 to fainter than +20, but most stars are between −5 and +15 in absolute magnitude. It can have a negative value, so it is not absolute in a mathematical sense. The brightest star in the night sky, Sirius, has $M = +1.4$, but that is much less luminous than the seventh brightest star Rigel with $M = -8.1$. The apparent and absolute magnitudes of the ten brightest stars were included in Table 10.1. Our apparently brilliant Sun has an absolute magnitude of $M = +4.83$.

Since each step of 15 in absolute magnitude indicates a difference of a million in luminosity, there are stars with an absolute magnitude of −10 that are about a million times more luminous than the Sun and those with an absolute magnitude of 20 that are about a million times less luminous than the Sun.

You can convert absolute magnitude, M, to luminosity L through the relation:

$$\log\left(\frac{L}{L_\odot}\right) = 0.4(4.83 - M), \tag{10.11}$$

or equivalently:

$$L = 10^{0.4\,(4.83-M)} L_\odot \tag{10.12}$$

where the absolute magnitude of the Sun is +4.83 and $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. The absolute magnitude, M, of a star of luminosity L is:

$$M = +4.83 - 2.5\log\left(\frac{L}{L_\odot}\right). \tag{10.13}$$

Example: Distance, luminosity, temperature, and size of the nearest stars

The Sun is the nearest star. It has a mean distance of 1 AU = 1.496×10^{11} m, an absolute luminosity of $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, an effective temperature of $T_{eff} = 5{,}780$ K, and a radius of $R_\odot = 6.955 \times 10^8$ m. The nearest star other than the Sun is Proxima Centauri, located at a distance of 4.243 light-years, where 1 light-year = 9.4605×10^{15} m. The ratio of the distance, D, of Proxima Centauri to the Sun's distance is $D/\mathrm{AU} = 2.68 \times 10^5$, so Proxima Centauri is 268,000 times farther away than the Sun and stars are separated by vast, seemingly empty space. Since 1 parsec = 1 pc = 3.26 light-years, the distance of Proxima Centauri is $D = 1.30$ pc, and its parallax is $\pi_A = 1/D = 0.769$ s of arc = $0.769''$. The spectral type of this star is M5.5, a cool red star with an effective temperature of $T_{eff} = 3{,}042$ K. The apparent visual magnitude of the star is $m_v = 11.05$, so its absolute visual magnitude is $M_v = m_v + 5 - 5\log D = 11.05 + 5 - 5$

log (1.30) $\approx$ 15.5, where D is the distance in parsecs. In the visible range of wavelengths, the absolute magnitude of the Sun is $M_{v\odot} = 4.83$, so in visible light Proxima Centauri has a luminosity given by $\log (L/L_\odot) = 0.4(M_{v\odot} - M_v)$, or $L \approx 10^{-4.27}\, L_\odot = 0.000054\, L_\odot$ in visible light. At a temperature of 3,042 K, the most intense emission, from the Wien displacement law, is at a wavelength of $\lambda = 0.0029/T = 9.53 \times 10^{-7}$ m, or at infrared wavelengths. Most of Proxima Centauri's power is radiated at unseen infrared wavelengths, and the total luminosity over all wavelengths is $L \approx 0.0017\, L_\odot$. We can use the Stefan-Boltzmann law $L = 4\pi\sigma R^2 T_{eff}^4$, where $\pi = 3.14159$ and the Stefan-Boltzmann constant $\sigma = 5.670 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}, to infer the star's radius of $R \approx 10^8$ m $\approx 0.14\, R_\odot$ from the known values of L and T_{eff}. Assuming that the luminosity of a star varies as the fourth power of its mass, the mass of Proxima Centauri would be about a tenth of that of the Sun.

The difference between the apparent magnitude, m, and the absolute magnitude, M, of an object is related to its distance, D, by the distance modulus μ given by:

$$\mu = m - M = 5\log D - 5, \tag{10.14}$$

for D in parsecs, or

$$\mu = m - M = -5(1 + \log \pi_A), \tag{10.15}$$

where π_A is the annual parallax in seconds of arc, denoted by the symbol $''$.

The absorption, or extinction, of light by interstellar dust diminishes the light intensity and increases the apparent magnitude by an amount A. The distance modulus has to be corrected for this to yield the true distance modulus:

$$\mu = m - M - A = 5\log D - 5. \tag{10.16}$$

Example: The absolute magnitude and luminosity of Sirius A and Sirius B

The brightest star in the sky is Sirius, the Dog Star, often denoted Sirius A to distinguish it from its faint companion Sirius B. Sirius A has an apparent magnitude of $m = -1.47$ and a distance of $D = 8.60$ light-years. Divide by 3.26 to get the distance in parsecs, or $D = 2.64$ parsecs, and the absolute magnitude is $M = m + 5 - 5 \log D = -1.47 + 5 - 5 \log (2.64) = 1.42$. The luminosity of Sirius A is $L = 10^{0.4(4.83-M)}\, L_\odot \approx 23\, L_\odot$. The apparent magnitude of Sirius B is $m = 8.30$, which corresponds to an absolute magnitude of $M = 11.19$ and a luminosity of $L = 0.0029\, L_\odot$ for the star's visible light.

There is one more caveat to our magnitude system, for the visual magnitudes that are measured by the eye or a conventional optical telescope only sample the visible wavelengths, and a star can emit intense radiation outside our range of vision, including the ultraviolet or infrared part of the electromagnetic spectrum. Astronomers use the term *bolometric*, from the Greek word meaning "measure of rays" to indicate the total radiation output of a star, and it has both an apparent and absolute bolometric magnitude. Fortunately, stellar temperature, which can be determined by other methods, can be used to determine the amount of radiation that was not detected in the visible range, and apply a bolometric correction to convert visual magnitudes to bolometric ones. And this brings us to the temperature of the stars.

10.1.4 The Temperatures of Stars

An understanding of the physical properties of a star requires knowledge of its temperature as well as its luminosity. The effective temperature of the Sun's visible disk, the photosphere, is inferred from the solar radius and luminosity, with a value of $T_{eff} = 5{,}780$ K. However, we do not have direct knowledge of the radius of most stars. They are too far away and too small in angular size for a telescope to resolve them.

Fortunately, there are two methods to infer a star's temperature even when we do not know its size and luminosity. We can estimate the temperature from the color of the star or infer its temperature from the relative intensities of absorption lines observed in its spectrum. These are the effective temperatures of the visible stellar disks, or the stellar photospheres. The photospheres of the hottest stars that we can see have temperatures of more than 100 times those of the coolest stars, with a range between 2,000 and 50,000 K. Böhm-Vitense (1981) has provided a review of the stellar effective temperature scale.

These are the stars we look at, but our eyes do not see all of the radiation that a star produces. At extreme hot or cold temperatures, a star can become visibly dim, even invisible, because most of its radiation is produced outside the visible part of the radiation spectrum and often is absorbed in the Earth's atmosphere before reaching the ground.

A very hot star, with a temperature of more than 100,000 K, emits most of its light at ultraviolet wavelengths that are absorbed in the atmosphere. These hottest stars are exceptionally luminous and massive.

The coolest star-like objects emit most of their radiation at infrared wavelengths, also absorbed in the Earth's atmosphere and outside our range of vision. There are the substellar, brown dwarf objects, for example, that do not have enough mass to begin nuclear fusion of hydrogen in their core. These stellar disks emit heat associated with their formation or by burning deuterium that was already present in them. The brown dwarf objects are sometimes colder then room temperature, or below 300 K.

10.1.5 The Colors of Stars

There are reddish stars like Betelgeuse and Antares, yellowish stars like the Sun and Capella, and whitish stars like Vega and Sirius. These colors provide a rough indication of the temperature of a star's photosphere. As the temperature rises, the colors change from red – near 3,000 K, to yellow – around 6,000 K, to white – at about 10,000 K.

A star often has a certain color because most of its radiation is emitted at the wavelengths corresponding to that color. The wavelength of maximum starlight intensity varies inversely with temperature. A blue star, for example, is hotter than a red star. The coloring of a star, however, is very subtle, depending on the relative amount of light seen in different colors.

Blue-colored stars, for example, are not just bluer than red stars. For a star of the same radius, a blue star is more luminous than a red one. Exceptionally hot stars emit most of their radiation at invisible ultraviolet wavelengths, and such a star is even more luminous. There is enhanced radiation intensity at adjacent wavelengths, and this ultraviolet spillover produces more blue light than expected for a cooler star. In this case, the temperature of the star is much hotter than that inferred from blue light alone.

Astronomers therefore decided to quantify color by comparing the apparent magnitudes measured in different wavelength bands. They are denoted U, B, and V for ultraviolet, blue, and visual bands, and centered at wavelengths of 350, 450 and 550 nm. The apparent magnitude differences $m_B - m_V = B - V$ or $m_U - m_B = U - B$, are then determined. The apparent magnitude of a star, denoted m, usually refers to its visual apparent magnitude, also written m_V.

The difference between the amount of light received at one color and the amount at another is known as the *color index*, which is usually measured by the difference between blue, designated B, and visual, denoted V, bands with a color index denoted by $B - V$. It provides a reasonable estimate of photosphere temperature by using the ratio of luminosities at two wavelengths, which is better than a temperature estimated from observations at only one wavelength. The temperatures increase from about 3,150 K for $B - V = 2.0$ to 60,000 K at $B - V = -0.4$, reflecting the fact that hotter stars emit more blue light.

However, in addition to ultraviolet spillover into the blue colors, interstellar dust reddens starlight as it travels through space to arrive at the Earth, and the amount of reddening increases with a star's distance. Thus, the observed colors may not reliably reflect the emitted colors. A star's spectral lines provide a more accurate indication of the temperature of a star's photosphere.

10.1.6 The Spectral Sequence

More than a century ago, astronomers noticed that stars of different colors exhibit different spectral lines. Strong absorption lines of hydrogen, for example, dominate the spectra of white stars like Vega and Sirius, whereas some blue stars have noticeable helium absorption lines. Yellow stars like the Sun have strong absorption lines of calcium and heavier elements, called metals, in their spectra.

The different spectral lines that are emitted by stars depend on the physical conditions in the visible disk – the photosphere – and therefore the level of ionization of the emitting atoms (also see Sect. 6.4). Stars that display spectral lines of highly ionized elements must be relatively hot, because high temperatures are required to ionize atoms. These hot stars have relatively weak hydrogen lines because nearly all of the hydrogen is ionized and all of its electrons have been set free from their atomic bonds, no longer emitting or absorbing radiation. In other words, stars that display hydrogen lines have moderate photosphere temperatures. Those exhibiting molecular lines have even cooler temperatures because molecules break apart into their component atoms when the temperature increases.

A system of stellar classification based on spectra was developed in the early twentieth century and is still in use today. Working under the direction of Edward C. Pickering (1846–1914), astronomers at the Harvard College Observatory examined the spectra of hundreds of thousands of stars. The astronomers were mainly women who had studied physics or astronomy at nearby women's colleges, including Wellesley and Ratcliffe. Harvard did not educate women at that time and did not permit women on its faculty.

One of these faithful, stalwart workers was Annie Jump Cannon (1863–1941), who classified the spectra of roughly 400,000 stars in her lifetime (Cannon and Pickering, 1918–1924). She distinguished the stars on the basis of the absorption lines in their spectra and arranged most of them in a smooth and continuous spectral sequence. The hottest stars, with the bluest colors, were designated as spectral type O, followed in order of declining photosphere temperature by spectral types B, A, F, G, K, and M (Table 10.4).

Table 10.4 The spectral classification of stars[a]

Class	Dominant lines	Color	Color index	Effective temperature	Examples
O	He II	Blue	−0.3	28,000–50,000	χ Per, ε Ori
B	He I	Blue–White	−0.2	9,900–28,000	Rigel, Spica
A	H	White	0.0	7,400–9,900	Vega, Sirius
F	Metals; H	Yellow–White	0.3	6,000–7,400	Procyon
G	Ca II; Metals	Yellow	0.7	4,900–6,000	Sun, α Cen A
K	Ca II; Ca I	Orange	1.2	3,500–4,900	Arcturus
M	TiO; Ca I	Orange-Red	1.4	2,000–3,500	Betelgeuse

[a] An H denotes hydrogen, He is helium, Ca is calcium, and TiO is a molecule. The Roman numeral I denotes an electrically neutral, unionized atom, the number II describes an ionized atom missing one electron, and the temperatures are in degrees kelvin, denoted K

Cannon further refined each spectral class by adding numbers from 0 to 9, running from hot to cold; the larger the number, the cooler the star in that class. For example, the hottest F star is designated as F0 and the coolest as F9, followed by G0. In this system, our Sun is classified as G2.

10.1.7 Radius of the Stars

Large stars come in two varieties: the giants and the supergiants. A relatively common type of big star is the red giant star, which is about 100 times bigger than the Sun; the other, exceedingly rare kind, the supergiant, is about 1,000 times larger than the Sun. The benchmark size is the radius of the Sun, denoted $R_{\odot}$, with a value of $R_{\odot} = 6.955 \times 10^8$ m.

The red giants can be found almost anywhere in the night sky, whereas the supergiants are sparsely scattered within the Milky Way. Only one in a million stars is likely to be a supergiant. As the name implies, supergiants are simply extreme examples of the giant stars. They are the rare anomalies that stand out because of their size. They are exceptionally big, massive, luminous and often bright. Well-known examples of both types of large stars are given in Table 10.5.

We can measure the angular size of the largest stars using an interferometer that employs two or more connected mirrors. The radiation waves detected by any two of the mirrors are combined to produce an interference pattern – hence, the name *interferometer*, short for "interference-meter". If the waves of electromagnetic radiation detected by the two mirrors are in phase when combined, their wave

Table 10.5 Some well-known large stars[a]

Star name	Radius ($R_{\odot}$)	Luminosity ($L_{\odot}$)	Mass ($M_{\odot}$)	Temperature (K)
Supergiant stars				
VY Canis Majoris	≈2,000	≈450,000	≈40	≈3,000
VV Cephei A	≈1,900	≈300,000	≈30	≈3,300
Mu Cephei	1,650	60,000	15	3690
Betelgeuse	1,180	140,000	19	3500
Antares	800	65,000	15	3500
Red giant stars				
Mira A	400	9,000	1.2	3,000
R Doradus	370	6,500	≈1.0	2,740
Aldebaren	44.2	425	1.7	4,010
Polaris	30	2,200	7.5	7,200
Arcturus	25.7	210	1.1	4,300
Pollux	8.0	32	1.86	4,865

[a] The radius, R, is units of the Sun's radius $R_{\odot} = 6.955 \times 10^8$ m, the luminosity, L, is in units of the Sun's luminosity $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, the mass, M, is units of the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, and the temperature, T, is the effective temperature of the stellar disk in degrees kelvin, abbreviated K

crests combine and strong light is detected. When they are out of phase, one wave crest matches the trough of the other and they cancel one another. In the earliest applications, the two mirrors were separated gradually to produce a set of light and dark bands, or "fringes"; when the fringes disappeared, the star was resolved. The angular diameter of the source, in units of radians, is the ratio of the wavelength to that mirror separation in which the fringes disappear.

The American physicist Albert A. Michelson (1852–1931) was one of the first to describe the interferometer technique (Michelson 1890), and thirty years later, he teamed up with the American astronomer F.G. Pease (1881–1938) to use an interferometer to measure the size of Betelgeuse. They mounted two moveable mirrors and two fixed mirrors on a 20 foot (6 m) steel beam that was placed across the frame of the 2.5 m (100 inch) Hooker telescope on Mount Wilson. By measuring the mirror separation when the interference fringes disappeared, they concluded that Betelgeuse has an angular diameter of about $\theta = 0.05''$, where the symbol $''$ denotes seconds of arc (Michelson and Pease 1921). By way of comparison, if the Sun were placed at the distance of the next nearest star, Proxima Centauri, it would have an angular diameter of approximately $0.007''$.

Modern visible-light interferometry has been used to measure the angular diameters of about 100 stars, including both supergiant stars and relatively near red giant stars. Current observations of the angular diameter of the supergiant Betelgeuse, at $0.055''$, indicate that it has a radius of 1,180 solar radii, which is equivalent to 5.48 AU – where 1 AU = 1.496×10^{11} m is the mean distance between the Earth and the Sun. The orbital distance of Jupiter from the Sun is 5.2 AU; therefore Betelgeuse and other supergiant stars would fill much of our major planetary system. The *Hubble Space Telescope* (*HST*) was used to obtain an image of Betelgeuse, obtaining the first direct picture of the visible disk of any star other than the Sun. Interferometric measurements in a recent 15 year period suggest that Betelgeuse may be shrinking, even though its visible brightness showed no significant dimming during the same period – a perplexing result (Townes et al. 2009)

Example: Measuring the radius of stars
The radius of the supergiant star Betelgeuse is $R_{\mathrm{B}} = 1180\ R_{\odot} = 8.207 \times 10^{11}$ m, where the solar radius is $R_{\odot} = 6.955 \times 10^{8}$ m. So the radius of Betelgeuse is equivalent to 5.486 AU, where 1 AU = 1.496×10^{11} m is the mean distance between the Earth and the Sun. The star's distance is $D_B = 643$ light-years, with 1 light-year = 9.460×10^{15} m. The angular diameter θ_B of Betelgeuse is therefore $\theta_B = 2R_B/D_B \approx 2.7 \times 10^{-7}$ radians = $0.0556''$, where $''$ denotes seconds of arc and 1 radian = $206{,}265''$. The angular resolution, θ_r, of an interferometer consisting of two mirrors separated by a distance D_I is $\theta_r = \lambda/D_I$ radians for radiation at a wavelength λ. The interference fringes of such an interferometer would disappear when $\theta_r = \theta_B$, or at a mirror spacing of $D_I = \lambda/\theta_B = 2.59$ m for red light from Betelgeuse at a wavelength $\lambda = 700$ nm $= 7 \times 10^{-7}$ m.

The angular diameter of the red giant star R Doradus is $\theta_R = 0.057''$, about the same as Betelgeuse because the red giant is much closer, at a distance of about $D_R = 178$ light-years. After converting this angle into radians, we obtain a radius for Doradus of $R_R = D_R\, \theta_\mathrm{R}/2 \approx 2.3 \times 10^{11}$ m $\approx 330\, R_\odot$, where $\theta_R = 2.76 \times 10^{-7}$ radians and $D_R = 1.68 \times 10^{18}$ m.

The red giant star Arcturus is so close, at a distance of $D_A = 36$ light-years $= 3.40 \times 10^{17}$ m, that its angular size has also been measured, at $\theta_A = 0.021'' = 10^{-7}$ radians. We therefore obtain a radius for Arcturus of $R_A = D_A\theta_A/2 \approx 1.7 \times 10^{10}$ m $\approx 25\, R_\odot \approx 0.1$ AU.

Optical interferometry with the Very Large Telescope Interferometer, abbreviated VLTI, on the Cerro Paranal in Chile, found the angular diameter of Proxima Centauri, the nearest star other than the Sun, to be $\theta_{PC} = 0.0010'' = 5 \times 10^{-9}$ radians, which at its distance of 4.24 light-years $= 4.0 \times 10^{16}$ m corresponds to a radius of about 10^8 m, or one-seventh that of the Sun and 1.4 times the radius of Jupiter $R_J \approx 7 \times 10^7$ m. The star's estimated mass is $0.123\, M_\odot = 2.446 \times 10^{29}$ kg, where $M_\odot$ denotes the mass of the Sun, and 129 times Jupiter's mass $M_J = 1.90 \times 10^{27}$ kg.

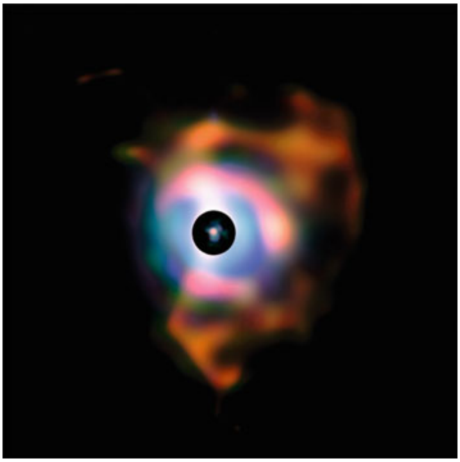

Fig. 10.2 The flames of Betelgeuse The red supergiant star Betelgeuse is slowly shedding its outer atmosphere, producing out-flowing gas that envelops the star and a much bigger nebula of gas and dust that surrounds it. The small circle in the middle of black disk denotes the edge of the supergiant's optically visible disk; it has a diameter of about 5.4 AU, where one AU is the mean distance between the Earth and the Sun. The black disk masks the bright central radiation of the star, in order to detect the infrared radiation of the outer plumes. They stretch to about 400 AU, or 60 million million, or 6×10^{13}, m from the supergiant Betelgeuse. (Courtesy of ESO/VISIR/VLT/Pierre Kervella.)

Supergiant stars are so large that they cannot hold onto their outer atmosphere. They are enveloped by dust and gas blown out by the stars' winds and have no well-defined apparent "edge" (Fig. 10.2).

Example: Mass loss from supergiant stars
The supergiant VY Canis Majoris has a mass, M, of about 40 times that of the Sun, or 40 $M_{\odot}$, but a radius, R, of about 2,000 solar radii, or 2,000 $R_{\odot}$, which is so far from the star's center that the stellar gravity GM/R^2 is but one hundred thousandth, or 10^{-5}, that of the Sun at its radius. The radius of the Sun is $R_{\odot} = 6.955 \times 10^8$ m and the Sun's mass is $M_{\odot} = 1.989 \times 10^{30}$ kg. The escape velocity (Sect. 3.2) required to overcome VY Canis Majoris' gravitational pull is $V_{esc} = (2GM/R)^{1/2} \approx 8.7 \times 10^4$ m s^{-1}, where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The average thermal speed of a hydrogen atom in the visible disk of this star is (Sect. 5.2) is $V_{thermal} = (3kT/m_H)^{1/2} \approx 8.6 \times 10^3$ m s^{-1}, where the Boltzmann constant $k = 1.381 \times 10^{-23}$ J K^{-1}, $m_H = 1.66 \times 10^{-27}$ kg, and the disk temperature of the star is $T = 3{,}000$ K. Since the average thermal speed is just 10 times less than the escape velocity, the atoms in the higher part of the Maxwell speed distribution should have little trouble overcoming the weak gravitational pull in the outer atmosphere of the star and breaking away from it. Images taken from the *Hubble Space Telescope* reveal arcs, filaments, and concentrations of material formed by the massive outflows from this supergiant star, some of them moving close to the star's escape velocity.

The next biggest known supergiant VV Cephei A is surrounded by opaque shells of a highly extended atmosphere, and is not entirely spherical in shape. The supergiant star Betelgeuse is enveloped by gas and dust that extends out to 400 AU from the star (Fig. 10.2).

For most stars, the radius is determined from the luminosity and temperature using the *Stefan-Boltzmann law*, which states that a star's luminosity increases with the square of its radius and the fourth power of its disk temperature. That is, the luminosity L_S of a star is intimately related to the star's radius R_S and effective temperature T_{eff}. If any two of these quantities are known, the third can be found using the Stefan-Boltzmann law

$$L_S = 4\pi\sigma R_S^2 T_{eff}^4 \tag{10.17}$$

where $\pi = 3.14159$ and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J m^{-2} K^{-4} s^{-1} (Sect. 2.4). The Austrian physicist Joseph Stefan (1835–1893) first derived this law (Stefan 1879), on the basis of experimental measurements made by the English physicist John Tyndall (1820–1893), and Stefan's student Ludwig Boltzmann (1844–1906) derived it from theoretical considerations, using thermodynamics (Boltzmann 1872).

This means that

$$R_s = \left[\frac{L_S}{4\pi\sigma T_{eff}^4}\right]^{1/2}. \tag{10.18}$$

In this expression, the effective temperature, T_{eff}, is the temperature of a thermal (blackbody) gas emitting the observed luminosity, which is close to the temperature of the visible stellar disk, known as the photosphere. The radius is that of the photosphere, which is the level at which the stellar gases become opaque at visible wavelengths.

10.1.8 How Massive are the Stars?

The mass of a star usually is expressed in units of the mass of the Sun, $M_\odot = 1.989 \times 10^{30}$ kg, which is determined from Kepler's third law and the orbital period and distance of the Earth, or from the length of the year and the AU (Sect. 3.3). The range in the mass of most stars is relatively small, between about 0.1 and 100 $M_\odot$. The Sun is on the lower side of the stellar mass range, as are most stars. The most massive stars are relatively rare due to their relatively short lifetime.

Although there is not much variation between the masses of the stars, the mass of a star determines a star's luminosity, its effective disk temperature, the length of its life, and its ultimate fate. A small increase in a star's mass, for example, implies a big increase in its luminosity. Stars of lower mass have less weight pressing down on their core, so their core is cooler, the rate of their thermonuclear reactions is slower, and the stars are dimmer. The life span of stars also depends on their mass. The more massive a star is, the shorter its life span. A star of greater mass is more luminous, burns its nuclear fuel at a greater rate, and depletes its available energy in a shorter time.

When the measured masses of stars are combined with observations of the stars' luminosity, we find that stellar luminosity increases rapidly with increasing mass. The reason for this increase is the hotter temperature at the center of a high-mass star when compared to that of a low-mass star. The rate of nuclear reactions is greater at the higher temperature; therefore, the luminosity of the massive star is greater.

The English astronomer Arthur Stanley Eddington (1882–1944) first considered the theoretical aspects of such a mass-luminosity relation (Eddington 1924). For a star whose mass is supported by gas pressure, for example, the internal temperature scales directly with the mass, and the greater the mass, the hotter the central temperature and the greater the stellar luminosity.

Current data show that the luminosity, L_S, of most stars increases in rough proportion to the fourth power of the mass, M_S, with a stellar mass-luminosity relation (Fig. 10.3) given by:

$$L_S = \text{constant} \times M_s^{3.5}. \tag{10.19}$$

and

$$\log\left(\frac{L_S}{L_\odot}\right) = 3.5\log\left(\frac{M_S}{M_\odot}\right), \tag{10.20}$$

or

$$\left(\frac{L_S}{L_\odot}\right) = \left(\frac{M_S}{M_\odot}\right)^{3.5}. \tag{10.21}$$

where the subscript $\odot$ denotes the solar value, with $L_\odot = 3.828 \times 10^{26}$ J s^{-1} and $M_\odot = 1.989 \times 10^{30}$ kg. Notice that although the range of stellar masses is relatively small, usually between about 0.1 and 60 $M_\odot$ in mass, the stellar luminosity varies over about nine orders of magnitude from 10^{-3} to 10^6 $L_\odot$.

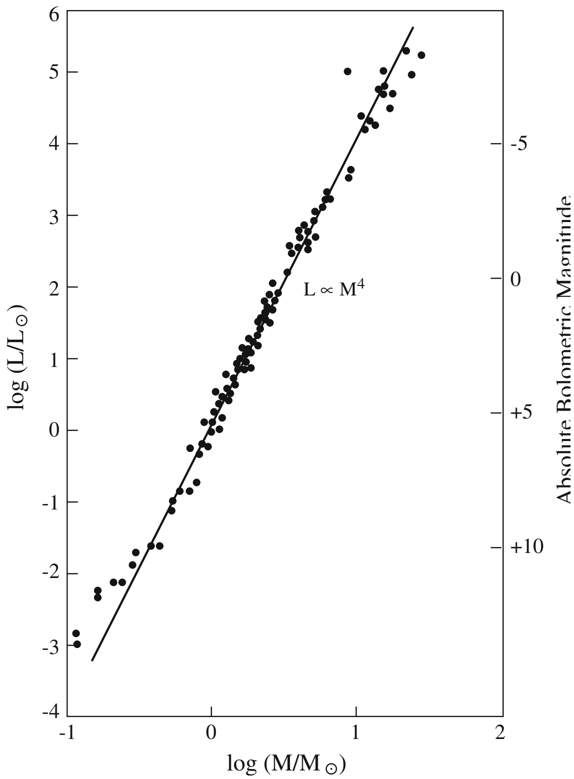

Fig. 10.3 Stellar mass–luminosity relation An empirical mass–luminosity relation for main-sequence stars of absolute luminosity, L, in units of the solar luminosity, $L_\odot$; and mass, M, in units of the Sun's mass, $M_\odot$. The straight line corresponds to a luminosity that is proportional to the fourth power of the mass. The English astronomer Arthur Eddington (1882–1944) proposed a theoretical explanation for this relation in 1924

At about 0.08 $M_{\odot}$ we reach the lower limit for a gaseous body to become a star. Its central regions are too rarefied and too cool to sustain the hydrogen-burning reactions that energize a Sun-like star and make it shine. Some of these nonstellar objects, known as brown dwarfs, can glow for a brief time as the result of heat generated during their formation by gravitational contraction. Although never hot enough for proton fusion, certain brown dwarf stars can shine for a time by burning deuterium that was present in the star at the time of its birth. The low-mass brown dwarfs eventually cool, compressing their near-stellar mass into the size of planets and disappearing from view.

As the mass increases, so does the central temperature. Nevertheless, the temperature inside a star cannot become too high, and therefore its mass cannot exceed an upper bound of 120 $M_{\odot}$. That is, the internal temperature and pressure of a very massive star can become so great that the star will be blown apart from inside (Focus 10.1). One of the most massive known stars is, for example, R136a1, with a mass of about 270 times that of the Sun, and it has been shedding a large fraction of its initial mass through a continuous stellar wind. It is estimated that, at its formation, the star held 320 solar masses and that it has lost 50 solar masses over the past million years.

Focus 10.1 The upper mass limit for a star

At formation, a star cannot keep on getting larger and more massive. There is a limit established when a star gets so big that the outward force of its internal radiation exceeds the inward gravitational force of the entire star.

Although the gas pressure of the hot, moving subatomic particles supports a star like the Sun, radiation pressure becomes important in more massive stars. As realized by the prolific Arthur Stanley Eddington (1882–1944), the outward pressure of radiation can exceed gas pressure in very massive stars (Eddington 1916), and a decade later he showed that this results in a maximum luminosity, now called the Eddington luminosity, at which the radiation blows away the outer atmosphere of a star (Eddington 1926a, b).

The temperatures within stars are high enough to ionize atoms, creating a plasma of free electrons and protons. As the radiation produced within the core of a star works its way out, the free electrons scatter the radiation with a Thomson scattering cross section $\sigma_T = 6.65246 \times 10^{-29}$ m^2. The outward force of the radiation, F_R, on the free electrons at distance r from the center of the star is

$$F_R = \frac{\sigma_T}{c} \frac{L_S}{4\pi r^2} \tag{10.22}$$

where L_S is the radiation luminosity, $\pi \approx 3.14159$, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The inward force of gravitation, F_G, of a star of mass M_S on a proton at distance r is:

$$F_G = \frac{GM_S m_P}{r^2}, \quad (10.23)$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ N m^{2-} kg^{-2} and the proton mass $m_P = 1.6726 \times 10^{-27}$ kg. When the two forces are equal, at the maximum Eddington luminosity L_{Edd}, we have

$$L_S = L_{Edd} = \frac{4\pi G m_P c}{\sigma_T} M_S \approx 6.3\ M_S \approx 1.3 \times 10^{31} \frac{M_S}{M_\odot}\ \mathrm{J\ s^{-1}}, \quad (10.24)$$

or

$$L_S = 3.3 \times 10^4 \frac{M_S}{M_\odot} L_\odot\ \mathrm{J\ s^{-1}}, \quad (10.25)$$

for the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg and the solar luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}.

If the luminosity of a star reaches the Eddington luminosity, a significant proportion of the star's outer layers are ejected into space, and this therefore sets a limit to the mass the star can accumulate at formation. To a first approximation, we can use the mass-luminosity relation to obtain this limiting mass from:

$$L_S = \left(\frac{M_S}{M_\odot}\right)^{3.5} L_\odot \leq L_{Edd} \quad (10.26)$$

or

$$\left(\frac{M_S}{M_\odot}\right)^{2.5} \leq 3.28 \times 10^4 \quad (10.27)$$

to obtain the upper mass limit

$$M_S \leq 64\ M_\odot. \quad (10.28)$$

This is an approximate limit, and a more precise value can be obtained by setting the radiation pressure equal to the gas pressure, and assuming that the gravitational binding energy of the star, GM^2/R, is, from the virial theorem, 3 times the product of the gas pressure and the volume, resulting in an upper mass limit of $M \leq 110\ M_\odot$.

A direct measurement of stellar mass can be obtained from observations of the relative motion of two stars in a binary-star, or double-star, system. Popper (1980) gave us a review of stellar masses.

The members of a double-star system are in mutual orbit around one another, revolving about a common center of mass. If the orbital period and the distance separating the two stars are measured, for example, the sum of their masses, $M_1 + M_2$, can be determined from Kepler's third law (Sect. 3.2):

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} \tag{10.29}$$

for a linear star separation, *a,* and an orbital period, *P*, where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Since orbits are always mutual (each star going about the other), the relative sizes of the orbits provide the ratio of the masses, and the combination of the sum and the ratio gives the individual masses of the stars can be found and compared to the Sun's mass.

Example: Measuring the mass of two stars in a binary system
Suppose the spectral lines of two stars shift back and forth with a period of $P = 2$ years $= 6.312 \times 10^7$ s, that the lines of star *1* shift twice as far as the lines of the other star 2, and Doppler shift observations of spectral lines indicates an orbital speed of $V = 100$ km s^{-1} for star *1* relative to star *2*. The semi-major axis *a* of the system can be determined from the circumference of one orbit $2\pi a = VP$, or $a \approx 10^{12}$ m. Then the sum of the masses can be calculated from Newton's expression of Kepler's third law, $M_1 + M_2 = 4\pi^2 a^3/(GP^2) \approx 1.5 \times 10^{32}$ kg, where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Because the lines of star *1* move twice as far as those of star *2*, star *1* is half as massive as star *2*, with star *1* weighing in at about 0.5×10^{32} kg $= 25\ M_\odot$ and star *2* at about 1.0×10^{32} kg or 50 $M_\odot$, where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg.

Some members of binary-star systems are tens of thousands of AU apart, whereas others touch one another. Moreover, the binary stars come in at least four varieties. There are visual binaries, the two components of which both can be resolved with a telescope and separately observed (Fig. 10.4); however, they are separated so widely that their orbital period about a common center of mass is often greater than a human lifetime.

The periodic motion of just one component of an astrometric binary is observed, whereas its companion is too faint to be seen. An eclipsing binary is a pair of stars whose orbital plane contains the Earth's line of sight, so we periodically observe the stars when they pass in front of or behind one another (Fig. 10.5).

A famous example of an eclipsing binary system is the two brightest stars in the Algol system: they have an orbital period of 2.87 days and a combined mass of about 4.5 solar masses. The stars are located at a distance of 28.5 pc, or 93 light-years, and are separated by only 0.062 AU, where 1 AU $= 1.496 \times 10^{11}$ m is the mean distance between the Earth and the Sun. The two stars are so close to one another that the more massive and bigger component has entered the gravitational sphere of influence of the other, transferring mass to it.

Fig. 10.4 Alpha Centauri Two of the most brilliant stars in the southern sky appear as a single star, named Alpha Centauri, to the unaided eye, but they can be resolved into two stars with the aid of binoculars or a small 5 cm (2 inch) telescope. The yellowish Alpha Centauri A (*lower left*), also known as Rigil Kentaurus, and the blue Alpha Centauri B (*upper right*) are locked together in a gravitational embrace, orbiting each other every 79.91 years. The two components of this binary-star system can approach one another within 11.2 AU and may recede as far as 35.6 AU, where the mean distance between the Earth and the Sun is 1 AU = 1.495×10^{11} m. Both stars have a mass comparable to that of the Sun, denoted $M_{\odot}$, of 1.1 $M_{\odot}$ and 0.90 $M_{\odot}$ for A and B, and a luminosity near that of the Sun, at 1.519 $L_{\odot}$ and 0.500 $L_{\odot}$. They appear bright because they are very nearby, at a distance of just 4.37 light-years. A third and faint companion Proxima Centauri has a luminosity of just 0.0017 $L_{\odot}$ and is located at about 15,000 AU or 2.2° from the two bright stars. At a distance of 4.24 light-years from the Earth, Proxima Centauri is the closest star other than the Sun. (Courtesy of ESO/Yuri Beletsky.)

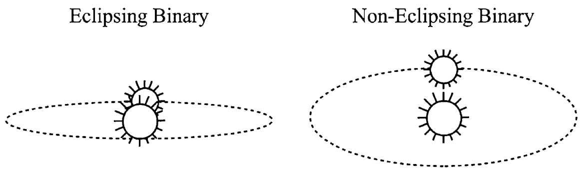

Fig. 10.5 Double stars Two close stars are joined in a gravitational embrace, orbiting each other and forming a binary star system (*left* and *right*). The orbital period and linear star separation can be used to determine the sum of their masses. If the orbital plane of the two companions is sufficiently inclined and within the line of sight, the star system becomes an eclipsing binary (*left*), in which one star is observed to pass behind the other and vice versa; this can provide additional information about the stars. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

Algol also is an example of the spectroscopic binary stars. The separation of this type of star pairs can be inferred from their orbital velocity and period. But this is an approximate determination, since the actual orbital velocities can be greater than the line-of-sight radial velocities measured from the Doppler shift of the spectral lines. The spectral variations of a spectroscopic binary reveal the orbital motion of its unresolved components (Focus 10.2).

Focus 10.2 Determining the stellar mass in a spectroscopic binary system

The analysis of binary stars usually assumes circular motion about a center of mass located between two stars. We have:

$$r_1 M_1 = r_2 M_2 \tag{10.30}$$

with M_1 and M_2 being the masses and r_1 and r_2 their respective distances to the center of mass. Thus, if $a = r_1 + r_2$ is the separation between the masses,

$$r_1 = \frac{M_2}{M_1}(a - r_1) \tag{10.31}$$

or

$$r_1 = \frac{M_2}{M_1 + M_2} a \tag{10.32}$$

and

$$r_2 = \frac{M_1}{M_1 + M_2} a. \tag{10.33}$$

We can find the total mass of the binary system using Newton's version of Kepler's third law. The orbital period, P, is given by:

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3 \tag{10.34}$$

or

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} \tag{10.35}$$

which provides the sum of the two stellar masses.

In spectroscopic binaries, the stars are usually not resolved, and their separations cannot be measured, but oscillations in the line-of-sight velocities are inferred from Doppler shifts of spectral lines. Because the perpendicular to the orbital plane is inclined to the line of sight by an angle i, the Doppler velocity amplitudes will be related to the true orbital velocity amplitudes by:

$$|V_{1obs}| = |V_1| \sin i, \tag{10.36}$$

and

$$|V_{2obs}| = |V_2| \sin i, \tag{10.37}$$

where the symbol $|x|$ denotes the absolute value of x. Since

$$|V_1| = \frac{2\pi r_1}{P}, \tag{10.38}$$

and

$$|V_2| = \frac{2\pi r_2}{P}, \tag{10.39}$$

then

$$\frac{|V_{1obs}|}{|V_{2obs}|} = \frac{r_1}{r_2} = \frac{M_2}{M_1}. \tag{10.40}$$

Replacing a with $r_1 + r_2 = P(|V_{1obs}| + |V_{2obs}|)/(2\pi \sin i)$ in Kepler's third law and using these expressions for r_1 and r_2 we can obtain:

$$(M_1 + M_2) \sin^3 i = \frac{P(|V_{1obs}| + |V_{2obs}|)^3}{2\pi G}. \tag{10.41}$$

If the spectrum of only one star, designated by the subscript *1*, is detected, due to the faintness of the second one, we can use:

$$(M_1 + M_2) \sin^3 i = \frac{P|V_{1obs}|^3 \left(1 + \frac{M_1}{M_2}\right)^3}{2\pi G} \tag{10.42}$$

or

$$\frac{M_2^3}{(M_1 + M_2)^2} \sin^3 i = \frac{P|V_{1obs}|^3}{2\pi G}. \tag{10.43}$$

When M_2 is much less than M_1, which would account for the faint luminosity of the second star, we obtain:

$$M_2 \sin i \approx \left(\frac{P}{2\pi G}\right)^{\frac{1}{3}} |V_{1obs}| M_1^{\frac{2}{3}}. \tag{10.44}$$

As might be expected, bigger stars are more massive, and there are fewer stars of high mass than those with low mass. The distribution of stars relative to mass is known as the *initial mass function*, with the term *initial* meaning the mass with which the stars were formed before their subsequent evolution. The Austrian-born American astronomer Edwin E. Salpeter (1924–2008) derived the initial mass function for stars more massive than the Sun (Salpeter 1955), and found that the number of stars with masses in the range M to $M + dM$ is proportional to $M^{-2.35}$. The number falls off roughly as the inverse square of the mass and indicates that the star-formation process results in many more stars of low mass than high mass. When compared to the number of stars with a mass equal to that of the Sun, denoted $M_{\odot}$, there are roughly 100 times more stars with 1/10th of that mass, at 0.1 $M_{\odot}$, and about 1/100th fewer stars with a mass of 10 solar masses, or 10 $M_{\odot}$. Stars of higher mass are also bigger, whereas those of low mass are relatively small; the small stars outnumber the large stars.

10.2 Main-Sequence and Giant Stars

10.2.1 The Hertzsprung–Russell Diagram

Once the luminosity of stars was obtained from their brightness and measurements of their distance, astronomers were able to show that most stars exhibit a systematic decrease in luminosity as one progresses through the spectral sequence O, B, A, F, G, K, M. This progression is exactly what we would expect because the spectral sequence also denotes a scale of decreasing stellar temperatures, and the luminosity of a radiating body depends strongly on temperature.

The luminosity drop is illustrated in the famous *Hertzsprung–Russell diagram* (*H-R*) diagram, of luminosity or absolute magnitude plotted against the spectral class or effective temperature (Fig. 10.6). The diagram's name derives from the Danish astronomer Ejnar Hertzsprung (1873–1967), who plotted such diagrams for the Pleiades and Hyades star clusters, and the American astronomer Henry Norris Russell (1877–1957), who published an early version of this diagram for both noncluster and cluster stars (Hertzsprung 1911; Russell 1914).

Most stars, including the Sun, lie on the main sequence that extends diagonally from the upper left to the lower right, or from the high-luminosity, high-temperature blue stars to the low-luminosity, low-temperature red stars. The stars on the main sequence are the most common type in the Milky Way, constituting about 90 % of its stars.

The *Stefan-Boltzmann law* describes the general characteristics of the H-R diagram. It is given by:

$$L_S = 4\pi\sigma R_S^2 T_{eff}^4, \tag{10.45}$$

where L_S is the luminosity of the star, $\pi = 3.14159$, the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}, the radius of the star is R_S, and T_{eff} is the effective temperature of the visible stellar disk. This expression indicates that for a fixed radius, the luminosity of a star increases with the fourth power of the

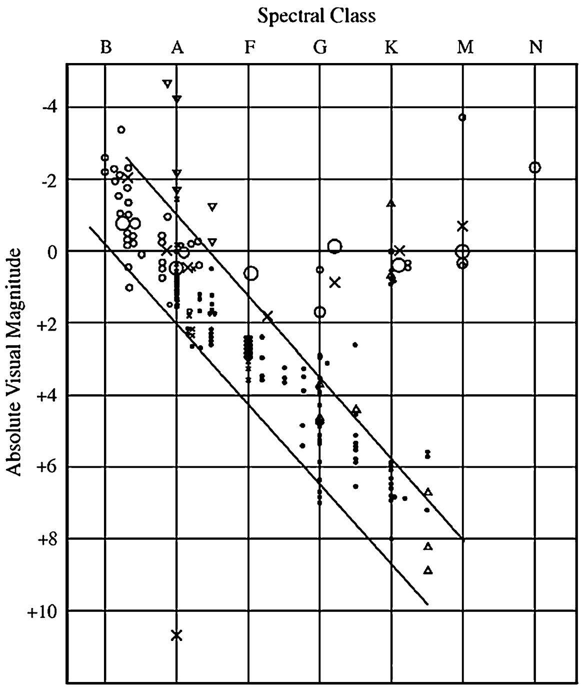

Fig. 10.6 Original Hertzsprung–Russell diagram The absolute luminosity, in magnitude units (*vertical axis*) plotted in 1914 by Henry Norris Russell (1877–1957) as a function of spectral class (*top horizontal axis*) for four moving star clusters: the Hyades (*black dots*), the Ursa Major group (*small crosses*), the large group in Scorpius (*small open circles*), and the 61 Cygni group (*triangles*). The *large circles* and *crosses* represent points calculated from the mean parallaxes and magnitudes of other groups of stars. The two *diagonal lines* mark the boundaries of Ejnar Hertzsprung's (1873–1967) observations of the Pleiades and Hyades open star clusters in 1911; this now is known as the main sequence along which most stars, including the Sun, are located. The giant stars are located at the *upper right*. In his publication, Russell included a similar diagram for individual bright stars, the distances of which had been established from stellar parallax measurements. It closely resembled the diagram shown here with an exceptional point in the lower left-hand corner, which is included here with an "x" mark. This star is the faint companion of a double-star system Omicron^2 Eridani, or 40 Eridani, now known to be a white dwarf star. [Adapted from Russell (1914).]

effective temperature; therefore, colder stars are less luminous. That is exactly what happens along the main sequence, for although the radius varies by a relatively small amount along the main sequence, the luminosity variation is due mainly to a change in temperature.

The observations also showed a different, unanticipated effect that gives the H-R diagram a peculiar shape. Some of the stars retain a high luminosity at decreasing temperature, in a band that extends to the upper right of the H-R diagram (see Fig. 10.6). This could be explained if the luminous cool stars were larger in radius than the less luminous ones, with the increase in size offsetting the

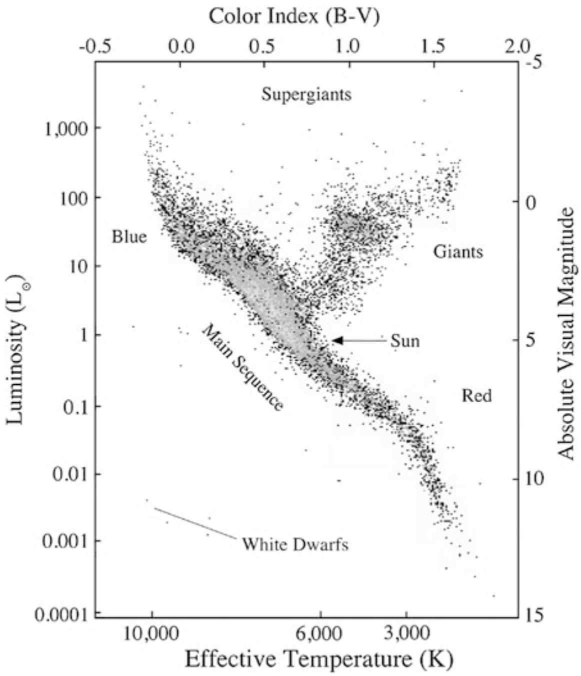

Fig. 10.7 Hertzsprung–Russell diagram for nearby stars A plot of the luminosity (*left vertical axis*) in units of the Sun's absolute luminosity, denoted $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, against the effective temperature of the star's disk in degrees kelvin, designated K (*bottom horizontal axis*) for 22,000 stars in the catalogue of the *HIPPARCOS* satellite. This plot is known as the Hertzsprung-Russell (H-R) diagram. The absolute visual magnitude (*right vertical axis*) and color index, B-V (*top horizontal axis*) are also designated. Most stars, including our Sun, lie along the main sequence, which extends from the high-temperature blue-white stars at the top left to the low-temperature red stars at the bottom right. The Sun is a main-sequence star with an absolute visual magnitude $M_V = 4.8$ and color index $B\text{-}V = 0.68$. The radiation from all main-sequence stars is sustained by hydrogen-burning reactions in their core. Stars of about the Sun's mass evolve into helium-burning red giant stars, located in the upper-right side of the diagram. Very rare bright giant stars and extremely scarce and luminous supergiants are found above the giant stars and along the top of the diagram. Faint and initially hot white dwarf stars are located in the lower left side. Due to their low luminosity, these endpoints of stellar evolution are relatively difficult to observe. (Data courtesy of the ESA/*HIPPARCOS* mission. From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

drop in temperature. The Stefan-Boltzmann law indicates that for a fixed temperature, the luminosity of a star increases with the square of the radius. If a star is 25,000 times more luminous than the Sun, with the same temperature, it follows from the Stephan-Boltzmann law that it will be 158 times bigger than the Sun.

Russell realized that he had found another type of star, which he named *giants* for their large size (Russell 1913). These stars have a radius as large as the mean distance between the Earth and the Sun. Russell also used the name *dwarfs* for the more numerous main-sequence stars, because they are smaller than the giants, but the designation is confusing. There is no observable difference between the size and luminosity of the hottest dwarf and most giant stars, and the white dwarf stars are not even on the main sequence. In this book, therefore, we retain the designation *giant stars*, but use the term *main-sequence stars* for the other stars.

There are relatively few giant stars when compared to the number of stars on the main sequence. This is because stars spend the majority of their lifetime on the main sequence, and the giant stars belong to a subsequent and shorter-lived part of a star's evolution.

Nearly a century of increasingly accurate and extensive observations confirmed the initial characteristics of the H-R diagram (Fig. 10.7). To assist physical interpretations, the luminosities are displayed along the left vertical axis, and the color index, or equivalently, the effective temperature of the stellar disk is on the bottom horizontal axis.

Chiosi et al. (1992) have reviewed developments in our understanding of the H-R diagram. Reid (1999) reviewed the H-R diagram and the galactic distance scale after the *HIPPARCOS* mission and Lebreton (2000) has reviewed the mission's implications for stellar structure and evolution.

10.2.2 The Luminosity Class

There was an unresolved uncertainty in the H-R diagram, which created a dilemma for specifying the physical characteristics of a star. A star could be small or large as well as hot or cold. A red cool star, for example, might be either much more luminous than the Sun or much fainter. Once the spectral type establishes the temperature, the star could be on either the luminous giant or the dimmer main-sequence part of the H-R diagram. To resolve this ambiguity, astronomers found a way of classifying stars by their luminosity in addition to their spectral type.

Pioneering investigations by the American astronomer Walter S. Adams (1876–1956) and the German astronomer Arnold Kohlschütter (1883–1969) found that the relative intensities of certain neighboring spectral lines could be used to

Table 10.6 The Morgan–Keenan (M–K) luminosity classes

Ia	Bright supergiants
Ib	Supergiants
II	Bright giants
III	Giants
IV	Subgiants
V	Main sequence stars (or dwarfs)
VI (or SD)	Subdwarfs
D (or VII)	White dwarfs

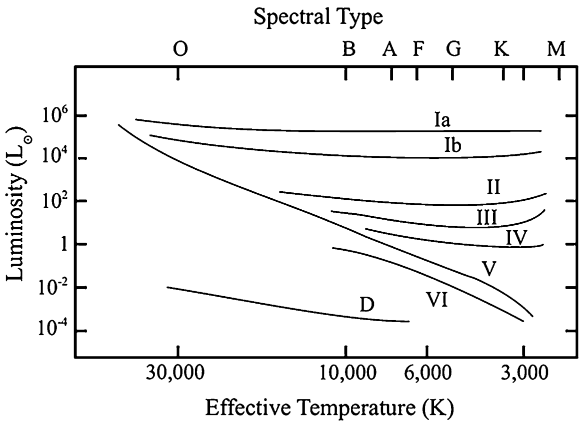

Fig. 10.8 Spectral type and luminosity class in the H–R diagram When both a star's spectral type (*top horizontal axis*) and luminosity class (*Roman numerals*) are known, the star's luminosity (*left vertical axis*) in units of the Sun's luminosity, denoted $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, and effective disk temperature (*bottom horizontal axis*) can be obtained. The spectral types are shown at the coolest temperature for each type. A Roman numeral V designates the main-sequence stars: subgiants by IV, giants by III, bright giants by II, and supergiants by Ia and Ib. VI or SD denotes the subdwarfs, and D or VII designates the white dwarf stars. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

determine the luminosities of both main sequence and giant stars (Adams and Kohlschütter 1914).

In the mid-twentieth century, William W. Morgan (1906–1994) and Philip C. Keenan (1908–2000), of the Yerkes Observatory in Chicago, introduced the M–K system (Morgan et al. 1943), in which the most luminous and largest stars have the lowest numbers, given in Roman numerals (Table 10.6). In the M–K system, the Roman numeral III designates the giant stars, and V denotes the main-sequence stars; Class IV of subgiants is located between them. The most luminous, Class I stars are the supergiants, shown near the rarely occupied, upper edge of the H-R diagram. Both the spectral type and the M–K luminosity class can be specified in the H-R diagram (Fig. 10.8).

Because the spectral type O, B, F, G, K, or M depends solely on the physical properties of a star's outer atmosphere – the photosphere – it is not sufficient to determine the star's internal properties and evolutionary status. To solve this problem, both the spectral type and luminosity class are provided in a two-dimensional scheme; for example, the Sun is designated as G2 V.

If we know a star's luminosity class, we can find its luminosity, or absolute magnitude, and a star's distance can be inferred from the apparent magnitude and luminosity. This is known as the *spectroscopic distance*, or *spectroscopic parallax*.

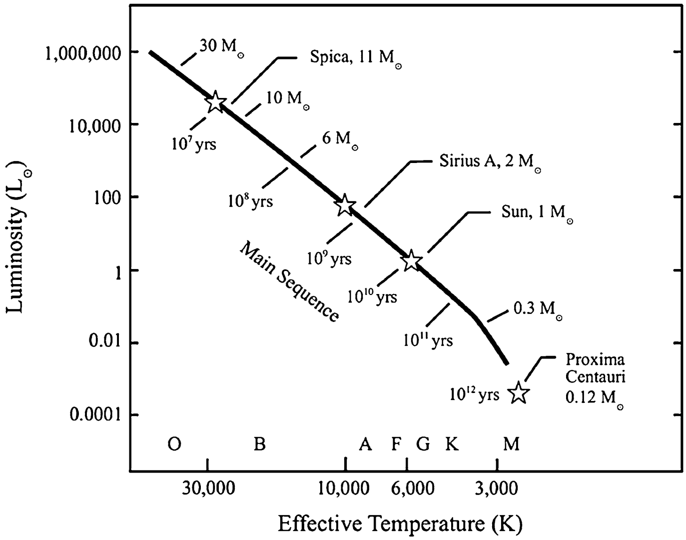

Fig. 10.9 Stellar mass and lifetime on the main sequence The relation between a star's luminosity (*left vertical axis*), in units of the Sun's luminosity, denoted $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, and the star's effective disk temperature (*bottom horizontal axis*) in degrees kelvin, designated K, for the main-sequence stars in the Hertzsprung-Russell diagram. The stellar masses given along the main-sequence curve are in units of the Sun's mass denoted $M_\odot = 1.989 \times 10^{30}$ kg. Stars of higher mass are hotter and more luminous. All of these stars shine by hydrogen burning with a lifetime that also is denoted along the main-sequence curve. More massive stars burn their hydrogen fuel at a faster rate and have a shorter lifetime. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

10.2.3 Life on the Main Sequence

Today, the H-R diagram remains a primary tool for tracing the path of stellar evolution, but the routes are more complex than initially supposed. Russell, for example, thought that most stars began life as hot, blue-white stars and ended their life as cool red ones, moving from upper left to lower right along the main sequence, which is what might happen if the stars cool with time. But once scientists understood the ways that nuclear fusion makes a star shine, the early speculations proved to be wrong. The main sequence is not a singular evolutionary pathway, as once thought; it is simply a portrait of the sky at one moment, depicting different stars of varying mass. The giant stars represent later rather than earlier stages in a star's life cycle. As it turns out, a star begins its bright shining life on the main sequence.

Like our Sun, other stars on the main sequence generate energy by converting hydrogen into helium; as long as it shines in this way a star's position on the main sequence does not change substantially. It simply slowly becomes more luminous

Table 10.7 The main-sequence stars[a]

Spectral type	Effective temperature (K)	Mass ($M_{\odot}$)	Luminosity ($L_{\odot}$)	Radius ($R_{\odot}$)	Lifetime (years)
O5	44,500	60	7.9×10^5	12	3.7×10^6
B0	30,000	17.5	5.2×10^4	7.4	1.1×10^7
B5	15,400	5.9	8.3×10^2	3.9	6.5×10^7
A0	9,520	2.9	5.4×10	2.4	2.9×10^8
F0	7,200	1.6	6.5	1.5	1.5×10^9
G0	6,030	1.05	1.5	1.1	5.1×10^9
K0	5,250	0.79	0.42	0.85	1.4×10^{10}
M0	3,850	0.51	0.077	0.60	4.8×10^{10}
M5	3,240	0.21	0.011	0.27	1.4×10^{11}

[a] The mass, M_S, is in units of the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg, the absolute luminosity, L_S, is in units of the Sun's absolute luminosity, $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}, and the radius, R_S, is in units of the Sun's radius, $R_{\odot} = 6.955 \times 10^8$ m. The lifetimes are the amount of time required to exhaust the nuclear hydrogen fuel that supplies the energy of stars on the main sequence

and moves slightly to the upper right in the H-R diagram. Moreover, the different positions along the main sequence are closely related to only one property of the stars – their mass – and not to their different evolutionary status. The mass sets the central temperature and nuclear fusion rate at which the outward pressure and the inward gravitational force remain in balance.

The stellar masses decrease downward from upper left to lower right on the main sequence (Fig. 10.9). The high-mass stars are more luminous than the low-mass stars because the central temperatures of the former are higher, to support the greater mass, and their nuclear-reaction rates are faster, producing radiation of much greater luminosity. The hot, luminous O stars can have masses as high as 150 times that of the Sun, whereas the cool, dim, main-sequence M stars might have as little as 0.08 solar masses.

All of these main sequence stars shine by converting hydrogen into helium. The effective disk temperature, mass, luminosity, radius, and lifetime of main-sequence stars of different spectral types are listed in Table 10.7.

Because a star begins shining with a limited supply of hydrogen, it can remain on the main sequence for only a limited lifetime – that is, the time it takes to deplete all of the hydrogen fuel in its hot core. Although more massive stars certainly contain more hydrogen in their larger core, they are much hotter inside and fuse this hydrogen into helium at a faster rate, resulting in a shorter life. As indicated in Table 10.7, main-sequence lifetimes range from a few million to 100 billion years from spectral type O5 to M5.

A more massive star is hotter at its center than a less massive star, and it ought to be more luminous. It turns out that a star's nuclear energy supply is proportional to the mass, as indicated by Einstein's famous expression $E = Mc^2$, where E is the energy, M the mass, and c is the speed of light. The rate at which energy is being radiated away, the luminosity, L_S, also increases with the mass, but as the fourth power of the mass, M_S. So, the length of time, τ, that a star shines, is another dramatic function of the mass; these times were also given in Table 10.7 for main-sequence stars.

How do we determine the length of time that a main-sequence star can continue to shine by converting protons into helium nuclei? These nuclear-fusion reactions are limited to the hot, dense stellar core. Outside of the core, where the overlying weight and compression are less, the gas is cooler and thinner so nuclear fusion cannot exist. For instance, the energy-generating core of the Sun extends about one quarter of the distance from the center to the visible solar disk. When all of the hydrogen within the core has been converted into helium, a star has exhausted its nuclear fuel supply and can no longer reside on the main sequence.

More than a half-century ago, the Brazilian astrophysicist Mario Schönberg (1914–1990) and the Indian-American astrophysicist Subrahmanyan Chandrasekhar (1910–1995) considered stellar models in which hydrogen is burned inside a star's core, or in a thin shell between the burned-out core and the overlying material (Schönberg and Chandrasekhar 1942). They found that it was impossible to construct models in which more than 12 % of the mass of the star is included in the exhausted core. This meant that the lifetime of a star on the main sequence is limited to the time it takes to convert 12 % of its hydrogen into helium.

The energy, ΔE, released in the conversion of four protons into one helium nucleus by hydrogen burning (Sect. 8.3) is $\Delta E = \Delta mc^2 = 0.007\ (4m_P c^2)$, and the rest-mass-energy conversion process is just 0.007 or 0.7 % efficient. In this expression, Δm is the mass difference between the mass of four protons and the mass of the helium nucleus, m_P denotes the mass of a proton, and c is the speed of light. The mass difference is due to the binding energy liberated during nuclear fusion to make a star shine.

If we convert 12 %, or 0.12, of the Sun's mass into energy in this way, then the energy released is $E = 0.12 \times 0.007\ M_\odot c^2$ where the mass of the Sun $M_\odot = 1.989 \times 10^{30}$ kg and the speed of light $c = 2.9979 \times 10^8\ \text{m s}^{-1}$. The Sun's main-sequence lifetime, τ_{ms}, required to convert 12 % of its mass into helium is therefore:

$$\tau_{ms} = \left(\frac{E}{L_\odot}\right) = 0.12(0.007)\left(\frac{M_\odot c^2}{L_\odot}\right) \approx 3.92 \times 10^{17}\ \text{s} \approx 1.24 \times 10^{10}\ \text{years}, \tag{10.46}$$

where the luminosity of the Sun is $L_\odot = 3.828 \times 10^{26}\ \text{J s}^{-1}$, and 1 year $= 3.156 \times 10^7$ s. Assuming that the luminosity of main-sequence stars increases with the 3.5 power of the mass, the main-sequence lifetime for a star of mass, M_S, is:

$$\tau_{ms} = 3.90 \times 10^{17}\left(\frac{M_\odot}{M_S}\right)^{2.5}\ \text{s}. \tag{10.47}$$

An exceptionally massive star, of say 100 times the mass of the Sun, will survive on the main sequence for only about 40 thousand years; in contrast, a star of moderate mass, say 10 times the mass of the Sun, will survive for 40 million years. That is why the more massive stars are so rare and hard to find.

Stars of intermediate mass, such as the Sun, will shine for 10–20 billion years. The Sun formed about 4.6 billion years ago; in another 7.8 billion years, it is expected to end its life on the main sequence.

So, the position of a star on the main sequence depends on its mass – the most massive stars being the most luminous and the more massive a star, the shorter it lives and the sooner it evolves off of the main sequence. Ninety percent of all main-sequence stars have a mass below 0.8 solar mass, and they have not yet had time to perish. They have been on the main sequence ever since they were born, thereby providing us with no information about stellar evolution. In contrast, some of the more massive stars, which were born long ago, have had enough time to burn up their available hydrogen fuel and advance to the next stage of stellar life. Thus, to understand *stellar evolution* we must examine the upper part of the main sequence in the *H-R diagram*, which applies to the more massive, shorter-lived stars.

10.2.4 The Red Giants and Supergiants

After the low-mass, main-sequence stars, the most common type of star is the red giant found in the upper right side of the H-R diagram (Fig. 10.10). These low-temperature stars are not exceptionally massive. They have an intermediate mass of roughly 1–10 times that of the Sun and are in a late state of stellar evolution from somewhat hotter main-sequence stars. Although cooler than the Sun, the red giants are about 100 times more luminous due to their much larger size, about 50 times the radius of the Sun. Prominent, bright-red giants include Aldebaran and Arcturus.

Because they are so luminous, we can see red giant stars that are relatively distant without using a telescope. However, they also are much less common than main-sequence stars because relatively few stars have entered this later stage of life. The red giants last only a few million years, which is a brief existence compared to the billions of years that stars of roughly solar mass spend on the main sequence.

The giant stars are enormously distended stars with a low mean mass density and a high luminosity. If we assume that the inner temperatures of giant stars are high enough to generate a gas pressure sufficient to balance gravitation, then their luminosity would greatly exceed that which is actually observed. This enigma was resolved almost a century ago when the great English astronomer Arthur Stanley Eddington (1882–1944) showed that radiation pressure must stand with gravitation and gas pressure as the third major factor in maintaining the equilibrium of a star

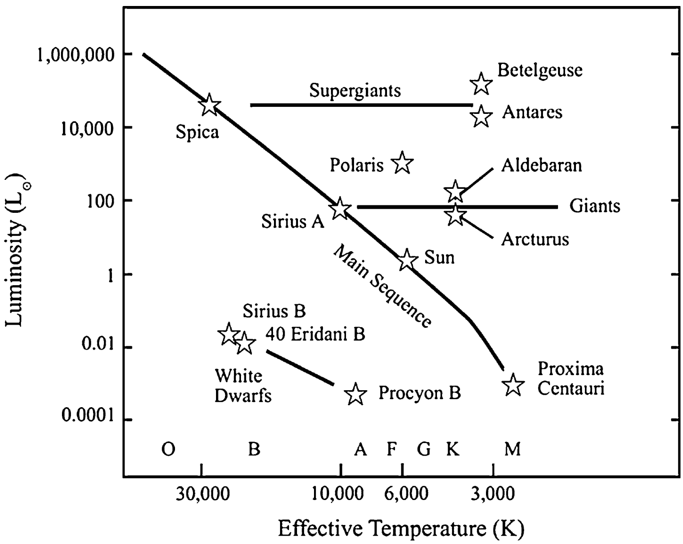

Fig. 10.10 Giants, supergiants, and white dwarfs The majority of stars occupy the main sequence in the H-R diagram. Stars with a mass comparable to that of the Sun will evolve into helium-burning giant stars, illustrated by Aldebaran and Arcturus in this diagram (*middle right*). These red giants are somewhat cooler than the Sun but about 100 times more luminous. The luminosity (*left vertical axis*) is in units of the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. More massive stars, which have a shorter lifetime on the main sequence, evolve into supergiant stars that are between 10 thousand and 1 million times as luminous as the Sun. Antares and Betelgeuse (*top right*) illustrate the supergiant stars on this diagram. After depleting all of their helium fuel, which is after the core hydrogen is exhausted, the less-luminous giant stars evolve into white dwarf stars of very low luminosity and initially hot disk temperatures. They are illustrated in the bottom left of this diagram by Sirius B, 40 Eridani B, and Procyon B. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

(Eddington 1917). The radiation pressure is the pressure exerted by electromagnetic radiation.

Although the outward pressure caused by the motion of gas particles, or the gas pressure, indeed does support the Sun and most other stars against the inward force of their immense gravity, it is insufficient for the much larger giant stars. They are also supported by radiation pressure, which increases with the fourth power of the temperature. In contrast, gas pressure is simply proportional to the temperature; so, if we sufficiently increase the central temperature, radiation pressure will become much larger than gas pressure.

Eddington also showed how some of the giant stars that are in radiative equilibrium could pulsate, with outer envelopes that move in and out, becoming alternately ionized and neutral during the course of pulsation (Sect. 14.1, Focus 14.1, Eddington 1918, 1919).

The supergiants are very massive, evolving from main-sequence stars of 10 to 100 solar masses, and exceptionally large, with radii of hundreds of times that of the Sun. They are also 10–100 times more luminous than the red giants. Antares and Betelgeuse are supergiant stars.

The supergiants are so exceedingly rare that we can only see them as a sparse sprinkling across the top edge of the H-R diagram. They are even less common than the O stars; rarely seen in any given part of the night sky, but so intrinsically luminous that we can see a few without a telescope.

Example: Properties of a large star

Suppose a star is ten thousand times more luminous than the Sun, with a stellar luminosity of $L_S = 10^4\ L_\odot$, where the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, and a spectral type and effective temperature the same as that of the Sun, whose effective temperature is $T_{eff} \approx 5780$ K. We can use the Stefan-Boltzmann law $L_S = 4\pi\sigma\, R_S^2 T_{eff}^4$, where $\pi \approx 3.14159$ and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J m^{-2} K^{-4} s^{-1}, to infer the star's radius $R_S \approx 6.9 \times 10^{10}$ m $\approx 100\ R_\odot$, where the Sun's radius $R_\odot = 6.955 \times 10^8$ m. The mass-luminosity relation indicates that the luminosity scales roughly as the 3.5 power of the mass, so the mass of the star is about $M_S = (L_S/L_\odot)^{1/3.5}\ M_\odot \approx 10\, M_\odot$ where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg. The main-sequence lifetime of the star is about $\tau_{ms} \approx 10^{10}\ (M_\odot/M_S)^{2.5}$ years $\approx$ 14 million years.

Instruments aboard *HIPPARCOS* have obtained a parallax of $\pi_A = 3.78 \times 10^{-3}\ ''$ for the blue supergiant star Rigel, to give a distance of $D = 1/\pi_A \approx 264$ pc $= 8.15 \times 10^{18}$ m, where 1 pc $= 3.0857 \times 10^{16}$ m. Interferometer measurements indicate that Rigel has an angular diameter of $\theta = 2.75 \times 10^{-3}\ '' = 1.33 \times 10^{-8}$ radian, where 1 radian $= 2.06264 \times 10^5\ ''$, so its radius is $R = \theta D/2 \approx 5 \times 10^{10}$ m $= 78\ R_\odot$, where the Sun's radius $R_\odot = 6.955 \times 10^8$ m. Rigel's absolute bolometric magnitude $M = -7.84$, so its luminosity $L = 10^{0.4(4.83-M)} L_\odot \approx 10^5 L_\odot$, where the Sun's absolute magnitude is 4.83 and the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. We can determine the star's effective temperature, T_{eff}, from the Stefan-Boltzmann law $L = 4\pi\sigma\, R^2\ T_{eff}^4$, where $\pi = 3.14159$ and the Stefan-Boltzmann constant $\sigma = 5.6704 \times 10^{-8}$ J m^{-2} K^{-4} s^{-1}, obtaining $T_{eff} \approx 10^4$ K.

Both the giant and the supergiant stars are so large that their atmospheres are slowly blowing away with strong winds that carry their outer atmospheres into surrounding space. For example, the giant star Mira, "The Wonderful," is pulsating and also losing mass at about one millionth of a solar mass per year. That is more than 1 million times the mass loss rate of the Sun's wind.

Because giants and supergiants are former main-sequence stars that have exhausted their core supply of nuclear hydrogen, they must be undergoing other forms of nuclear fusion. We now consider these various nuclear burning reactions.

10.3 Nuclear Reactions Inside Stars

Clayton (1984) and Rolfs and Rodney (2005) have provided textbooks that include stellar nuclear reactions and nucleosynthesis.

10.3.1 The Internal Constitution of Stars

Although we cannot see the inside of a star, its internal structure can be explained by a few simple concepts, one of which is a star's equilibrium. Like the Sun, almost every star we see is neither collapsing nor expanding, and it remains the same size throughout most of its long life. At every point inside such a star, the inward pull of its gravity is balanced precisely by the outward push of its internal pressure.

As with the Sun, all of the other main-sequence stars are composed mainly of the lightest element, hydrogen, but it is too hot for whole hydrogen atoms to exist within them. These atoms are fragmented into their subatomic constituents by frequent collisions. The material in these stellar interiors therefore is in the plasma state, composed almost entirely of hydrogen nuclei, protons, and free electrons no longer attached to atoms. Compressed to high density, protons still occupy the vast empty spaces of former atoms, so plasma behaves like a perfect gas with a pressure that increases with the temperature.

The central temperature of any main-sequence star can be estimated by assuming that a proton at the center is hot enough and moving fast enough to counteract the gravitational compression on the proton from the rest of the star. For the Sun, this balance is achieved at a central temperature of 15.6 million K (Sect. 8.2). A more massive star produces greater compression at its center, so a higher central temperature is required to hold it up.

The temperature, T_C, at the center of a star like the Sun can be estimated by assuming that each proton down there is hot enough and moving fast enough that the thermal energy $3kT_C/2$ counteracts the gravitational compression it experiences from all the rest of the star. That is:

$$\frac{3}{2}kT_C = \frac{Gm_PM_S}{R_S}, \tag{10.48}$$

where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the mass of the proton is $m_P = 1.6726 \times 10^{-27}$ kg, and M_S and R_S respectively denote the mass and radius of the star.

Solving for the central temperature we obtain:

$$T_C = \frac{2Gm_P M_S}{3kR_S} = 1.54 \times 10^7 \left(\frac{M_S}{M_\odot}\right)\left(\frac{R_\odot}{R_S}\right) \text{K}. \tag{10.49}$$

In the numerical approximation, the mass and radius of the star are given in solar units, denoted by the subscript $\odot$, where $M_\odot = 1.989 \times 10^{30}$ kg and $R_\odot = 6.955 \times 10^8$ m. Thus, the temperature at the center of the Sun is about 15 million K.

The temperature at the center of a main-sequence star is proportional to its mass, so a star that is 10 times more massive than the Sun is 10 times hotter in its center. At a large enough mass, the star becomes so hot that it is blown apart; this explains why there are no known stars with a mass greater than about 120 times the mass of the Sun.

We have assumed that the outward gas pressure of the moving protons supports the inward pull of gravity, a condition known as hydrostatic equilibrium. The ideal gas law (Sect. 5.4) gives this gas pressure, P_g:

$$P_g = NkT, \tag{10.50}$$

which is equivalent to:

$$P_g = \frac{\rho kT}{\bar{m}} = \frac{\rho kT}{\mu m_H} \tag{10.51}$$

for a gas of mass density ρ and mean mass per particle given by:

$$\bar{m} = \rho/N = \mu m_H \tag{10.52}$$

for mean molecular weight μ. The mass of the hydrogen atom $m_H = 1.00794$ u $\approx 1.67 \times 10^{-27}$ kg, which is roughly equal to the atomic mass unit u $= 1.6605 \times 10^{-27}$ kg and good enough for order of magnitude estimates. For a fully ionized hydrogen gas, the mass per particle is $m_P/2$ or half the proton mass.

The gas pressure is the outward pressure caused by the motion of gas particles and increases with their temperature. Gas pressure does indeed support the Sun and most other stars against the inward force of their immense gravity, but this does not apply for much larger, and relatively rare, giant stars.

Eddington (1917) showed that in addition to the kinetic gas pressure, P_g, the radiation photons in a giant star exert an additional radiation pressure, P_r, given by:

$$P_r = \frac{aT^4}{3} \tag{10.53}$$

where the radiation constant $a = 7.5657 \times 10^{-16}$ J m^{-3} K^{-4}. For radiative equilibrium at the center of star of radius, R_S, and mass, M_S:

$$\frac{4}{3}\pi R_S^3 P_r = \frac{GM_S^2}{R_S}, \tag{10.54}$$

where the gravitation constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

The total pressure, P, inside a star is given by the sum of the gas pressure and the radiation pressure. The radiation pressure is much less than the gas pressure at the center of the Sun, but it can compete with gas pressure in supporting giant stars. The complete equation of state for the pressure, P, is then given by:

$$P = P_g + P_r = \frac{\rho kT}{\bar{m}} + \frac{1}{3}aT^4. \tag{10.55}$$

Example: Supporting a star by gas pressure or radiation pressure
At the center of a star of mass density ρ_c and temperature T_c, the gas pressure is $P_g = NkT \approx \rho_c kT/m_p$, where the proton number density $N \approx \rho_c/m_p$, the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1} and the proton mass $m_p = 1.6726 \times 10^{-27}$ kg. The radiation pressure, P_r, is given by $P_r = aT_c^4/3$, where the radiation constant $a = 7.5657 \times 10^{-16}$ J m^{-3} K^{-4}.

At the center of the Sun $\rho_c \approx 1.5 \times 10^5$ kg m^{-3} and $T_c \approx 1.5 \times 10^7$ K, so the gas pressure $P_g \approx 2 \times 10^{16}$ Pa and the radiation pressure $P_r \approx 1.3 \times 10^{13}$ Pa, which is 1 thousand times less than the gas pressure. In other words, the gas pressure is about 1 thousand times greater than the radiation pressure at the center of the Sun.

Suppose a main-sequence star is one hundred times as massive as the Sun, with a mass $M_S = 100\ M_\odot$, where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg. The mass-luminosity relation indicates that the luminosity scales roughly as the 3.5 power of the mass, so the star's luminosity would be $L_S \approx 10^7\ L_\odot$, an exceptionally luminous star, where the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. If the effective temperature was about the same as that of the Sun, and because the luminosity varies as the square of the radius, the star's radius will be $R_S \approx 10^{3.75}\ R_\odot$, a large star, where the Sun's radius $R_\odot = 6.955 \times 10^8$ m. Assuming that the star is entirely composed of protons, then the central temperature, T_{CS}, of the star, which scales as M_S/R_S, will be $10^{-1.75}$ that of the Sun. The gas pressure, P_{gS}, varies as $M_S T_{CS}/R_S^3$ or as M_S^2/R_S^4, so it will be 10^{-11} that of the Sun. The central radiation pressure, P_{rS}, varies as T_C^4, or as $(M_S/R_S)^4$, which will be 10^{-7} that of the Sun, and the ratio of radiation pressure to gas pressure in the star center will be ten thousand times greater than that of the Sun. For this more massive and luminous star the central radiation pressure is estimated to be 10 times the gas pressure at the star's center.

Since the radiation pressure increases with the fourth power of the temperature, and the temperature has to increase with the mass, the radiation pressure can overcome the gravity of an exceptionally massive star. That is, a giant star cannot remain in equilibrium if the central temperature and mass become too high, and this occurs for masses greater than about 120 solar masses (see Focus 10.1, Sect. 10.1).

But where does a star's heat come from? The energy released by nuclear fusion in the stellar core heats the gas and generates its pressure. That is, nuclear reactions that transform a light element into a heavier one liberate subatomic energy that sustains the high temperatures within a star. This energy also makes its way out of the star to provide its luminosity and keep it shining.

Thus, two other fundamental concepts in understanding a star's interior are: (1) the way energy is generated by nuclear reactions near its center, and (2) the methods in which the radiation produced by these reactions works its way out to the observed stellar disk, its photosphere. The energy generation depends on the nuclear fuel, as well as the mass density and temperature in a star's core. The radiation-energy transfer depends on a star's internal opacity to radiation, which prevents some of the radiation from escaping.

After arrival on the main sequence, which is designated the *zero age*, the internal structure of a star can be determined by only four equations, which describe the equilibrium, energy transport, conservation of mass, and conservation of energy within the star. The crucial equations, given in Focus 10.3, can be solved without any knowledge of the properties of the star before arrival on the main sequence. Kippenhahn et al. (2012) provide a good textbook of stellar structure and evolution.

Focus 10.3 The equations of stellar structure

To obtain information on the interior constitution of the stars, astrophysicists have to integrate basic equations. Pioneering work in this field can be found in the books of Eddington (1926a, b) and Chandrasekhar (1939). The four differential equations that determine a stars initial position on the main sequence of the Hertzsprung–Russell diagram and its subsequent evolutionary history are:

The equation of hydrostatic equilibrium. This equation states that the inward force of gravity caused by the mass, $M(r)$, within a distance, r, from the stellar center is just balanced by the outward gas pressure, $P(r)$, at radius, r, so that:

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM(r)}{r^2}, \tag{10.56}$$

or equivalently

$$\frac{dP(r)}{dM(r)} = -\frac{GM(r)}{4\pi r^4}, \tag{10.57}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the mass density is denoted by $\rho(r)$, and the gas pressure is given by the ideal gas law (Sect. 5.4).

The equation of mass continuity or *the equation of mass conservation*. This equation specifies the mass, $M(r)$, contained within radius, r, in terms of the mass density, $\rho(r)$, by:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \tag{10.58}$$

or equivalently

$$\frac{dr}{dM(r)} = \frac{1}{4\pi r^2 \rho(r)}. \tag{10.59}$$

This equation is subject to the boundary conditions of zero mass at zero radius, or $M(r) = 0$ at $r = 0$, and a mass that is now equal to the total mass of the star, M_S, at the visible stellar radius, R_S, or $M(R_S) = M_S$. For the Sun, $M_S = M_\odot = 1.989 \times 10^{30}$ kg and the radius $R_S = R_\odot = 6.955 \times 10^8$ m.

The equation of energy conservation. This equation states that the energy generated per unit mass per unit time in the star's core, denoted by $\varepsilon(r)$, supplies the energy flux, $L(r)$, carried across radius, r, or that:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \varepsilon(r), \tag{10.60}$$

or equivalently

$$\frac{dL(r)}{dM(r)} = \varepsilon(r). \tag{10.61}$$

The energy generation, $\varepsilon(r)$, is a function of the initial composition, mass, density, and temperature. This equation has the boundary condition provided by the current luminosity, L_S, for a star of total mass, M_S, and radius, R_S. For the Sun, we have $L_S = L_\odot = 3.828 \times 10^{26}$ J s^{-1}.

The equation for radiative energy transfer. This equation relates the temperature, $T(r)$, at radius, r, to the amount of energy being transferred by radiation to that distance. It is related to the opacity to radiation, $\kappa(r)$, which measures the resistance of the material to energy transport by radiation. The equation is:

$$\frac{dT(r)}{dr} = -\frac{3\kappa(r)\rho(r)L(r)}{16\pi r^2 ac[T(r)]^3}, \tag{10.62}$$

or equivalently

$$\frac{dT(r)}{dM(r)} = -\frac{3\kappa(r)L(r)}{64\pi^2 acr^4[T(r)]^3}, \tag{10.63}$$

where the radiation density constant $a = 7.5657 \times 10^{-16}$ J m^{-3} K^{-4}, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The total luminosity of a star, L_S, with a radius, R_S, is given by the Stefan-Boltzmann law $L_S = 4\pi\sigma\, R_S^2\, T_{eff}^4$ where the Stefan-Boltzmann constant $\sigma = ac/4 = 5.6704 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4} and T_{eff} is the effective temperature of the visible stellar disk.

The chemical composition of a zero-age main-sequence star is assumed to be homogenous, and is often specified by $X = \rho_H/\rho$ the fraction by mass of material in the form of hydrogen, $Y = \rho_{He}/\rho$ the fraction by mass of material in the form of helium, and $Z = 1 - X - Y = \rho_{metals}/\rho$ the fraction by mass of material heavier than helium. The mass density for fully ionized plasma of total number density, N, is:

$$\rho = \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)^{-1} Nm_H, \tag{10.64}$$

where the mass of the hydrogen atom is $m_H = 1.6726 \times 10^{-27}$ kg.

The approximate abundances observed in the disk of the Sun are $X = 0.71$, $Y = 0.27$, and $Z = 0.02$, and $\bar{m} = 0.61\, m_H$; in the solar core nuclear fusion reactions have converted about half the hydrogen into helium and $X = 0.34$, $Y = 0.64$ and $Z = 0.02$, and $\bar{m} = 0.85\, m_H$.

At any given time, stars of the same composition have radii, luminosities, effective temperatures, and mean densities determined solely by the star's mass. The German astronomer Heinrich Vogt (1875–1936) demonstrated this concept in 1926, and the American astronomer Henry Norris Russell (1877–1957) derived it independently the following year in his textbook; therefore, it is known as the Vogt–Russell theorem (Vogt 1926; Russell et al. 1927). It implies that a star of a given mass, age, and chemical composition occupies a unique position, related to the star's evolutionary history, on the H-R diagram. The mass, age, and composition are all we need to know to understand the life history of a star.

A star will continue shining with a luminosity and temperature determined by its mass, remaining stable and fundamentally unchanged for millions to billions of years. The only caveat to this understanding of stellar life is that the core of a star is the only place hot enough for nuclear reactions to occur. The composition of the core slowly changes as the result of these reactions; eventually, there is no more nuclear energy in the stellar core so it loses its equilibrium.

It appears to be simple, but the theory is complex – with detailed applications that are found in advanced texts. Moreover, our understanding of the internal constitution of stars includes explanations of how their energy is generated, by thermonuclear reactions in the stellar cores, beginning with the fusion of hydrogen into helium in main-sequence stars.

10.3.2 Two Ways to Burn Hydrogen in Main-Sequence Stars

All main-sequence stars generate energy by the thermonuclear fusion of hydrogen nuclei, the protons, into helium nuclei. Because the hydrogen is "burned up" or consumed to fuel the nuclear fires, we call this process *hydrogen burning*, although it is a chain of nuclear-fusion reactions rather than the combustion of an ordinary fire.

There are two methods of burning, or fusing, hydrogen into the heavier element helium; the dominant mechanism, which produces the most power, depends on the mass of a star. The main source of energy for main-sequence stars with a mass less than 1.5 times the Sun's mass is the *proton–proton chain* of nuclear reactions, abbreviated as the *p–p chain*. A different sequence of nuclear reactions converts protons into helium nuclei inside main-sequence stars more massive than 1.5 times the mass of the Sun. This is known as the *carbon–nitrogen–oxygen* (*CNO*) *cycle*. As the name suggests, this is a cyclic set of nuclear reactions.

The thermonuclear process responsible for the energy production in any star is not limited to a single nuclear transformation, but rather consists of a sequence of linked transformations that together form a nuclear chain reaction. The p–p chain is linear, with only one direction, whereas the CNO cycle occurs in a closed circular chain. For both types of hydrogen burning, four protons combine to make one helium nucleus, thereby releasing energy.

In the lower right-hand, low-mass side of the main sequence, where the great majority of stars are located, nuclear energy is generated as a result of the proton–proton chain that makes the Sun shine. In 1939, the German-born American physicist Hans A. Bethe (1906–2005) first delineated this complete nuclear transformation of four protons into one helium nucleus (Bethe 1939).

Because the proton–proton chain was discussed in detail in Sect. 8.3, we now limit the discussion to the general result. Four protons, each designated by ^{1}H or p, combine to form a helium nucleus, denoted by ^{4}He, releasing powerful gamma ray radiation, designated γ, positrons with the symbol e^+, electron neutrinos, denoted by the symbol ν_e, and an energy of about 4×10^{-12} J per reaction chain. We can use the notation of nuclear reactions to describe the p–p chain, with nuclei on the left side of a reaction designated by an arrow, →, fusing to make the nuclei and other particles or radiation on the right side of the arrow. The proton–proton chain is described using this notation in Focus 10.4, with the net result for each reaction chain:

$$4\,^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 6\gamma + 2\nu_e. \tag{10.65}$$

Focus 10.4 The proton–proton chain

The proton–proton chain of reactions begins with the merger of two protons, each designated by ^{1}H, in the reaction:

$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{D} + e^+ + \nu_e. \tag{10.66}$$

It is followed by

$$^{2}D + {}^{1}H \rightarrow {}^{3}He + \gamma, \quad (10.67)$$

with a last step

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H. \quad (10.68)$$

In the process, additional gamma rays are released when the positrons combine with electrons, denoted e^{-}, during pair annihilation, denoted by $e^{+} + e^{-} \rightarrow \gamma + \gamma$, and the net result of the p–p chain is:

$$4\,{}^{1}H \rightarrow {}^{4}He + 6\gamma + 2\nu_e. \quad (10.69)$$

At about the same time that Bethe delineated the proton–proton chain, the German physicist Carl Friedrich von Weizsäcker (1912–2007) examined the thermonuclear reactions that might occur in stars, publishing his results in two papers that appeared in the *Physikalische Zeitschrift* (Weizsäcker 1937, 1938). In the first paper, he reasoned that the merger of two protons must have started hydrogen burning in the Sun. In the second paper, Weizsäcker proposed that elements that are heavier than hydrogen already were created before the formation of stars, as we know them now. He no longer was limited to reactions that began with the lightest element, hydrogen, and this led him to the important discovery of the cyclic CNO chain of reactions in which carbon acts as a catalyst for the synthesis of helium from hydrogen (Focus 10.5). The overall result for each CNO reaction chain is as follows:

$$^{12}C + 4\,{}^{1}H \rightarrow {}^{12}C + {}^{4}He + 5\gamma + 2\nu_e, \quad (10.70)$$

where the carbon nucleus, denoted ^{12}C, is forever being regenerated and acts like a catalyst. That is, the CNO cycle is a circular reaction chain in which carbon is destroyed and then re-created. Therefore it is available for the sequence of reactions to occur repeatedly. Like the non-circular, linear proton–proton chain, each CNO reaction chain releases about 4×10^{-12} J of energy.

Focus 10.5 The CNO cycle

The cyclic CNO chain of reactions starts when a carbon nucleus, ^{12}C, fuses with a proton, ^{1}H, to produce a nucleus of nitrogen, ^{13}N, and gamma ray radiation, γ. The nitrogen decays to form a nucleus of heavier carbon, ^{13}C; a positron, e^{+}; and an electron neutrino, ν_e. These beginning nuclear-fusion reactions are as follows:

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma, \quad (10.71)$$

and

$$^{13}\text{N} \rightarrow {}^{13}\text{C} + e^{+} + \nu_e. \tag{10.72}$$

The cycle then continues when the heavy carbon combines with a proton to form heavier nitrogen, ^{14}N, which then fuses with another proton to form oxygen, ^{15}O. The oxygen decays to make ^{15}N, which then combines with a proton to make the original carbon, ^{12}C, together with a helium nucleus, ^{4}He. The nuclear-fusion reactions are as follows:

$$^{13}\text{C} + {}^{1}\text{H} \rightarrow {}^{14}\text{N} + \gamma \tag{10.73}$$

$$^{14}\text{N} + {}^{1}\text{H} \rightarrow {}^{15}\text{O} + \gamma \tag{10.74}$$

$$^{15}\text{O} \rightarrow {}^{15}\text{N} + e^{+} + \nu_e \tag{10.75}$$

and

$$^{15}\text{N} + {}^{1}\text{H} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}. \tag{10.76}$$

The positrons, e^{+}, annihilate with the electrons, e^{-}, to produce energetic gamma radiation by the following reaction:

$$e^{+} + e^{-} \rightarrow 2\gamma. \tag{10.77}$$

This occurs for both positrons generated during the each CNO reaction chain.

Like the proton–proton chain, the net result of the CNO cycle is that four protons are fused together to form one helium nucleus, gamma rays and electron neutrinos. By summing the left and right sides of all the participating reactions, we obtain the following:

$$^{12}\text{C} + 4{}^{1}\text{H} \rightarrow {}^{12}\text{C} + {}^{4}\text{He} + 5\gamma + 2\nu_e. \tag{10.78}$$

The circular CNO reaction chain is induced by high temperatures in the cores of massive main-sequence stars and becomes self-sustaining by the catalytic action of carbon. This cycle also could begin at the intermediate stages with nitrogen or oxygen, so the entire cycle is called the carbon–nitrogen–oxygen, or CNO, cycle.

It was Bethe who realized that the proton–proton reaction, which explains the luminous output of the Sun, fell short of the much greater luminosity of the hotter and more massive stars. So, he systematically examined a great number of nuclear reactions that would not operate within stars and eliminated them. He independently found that the CNO cycle would generate about the same energy as the proton–proton process for each nuclear reaction chain. Bethe also showed that the greater rate and temperature dependence of the CNO cycle could account for the

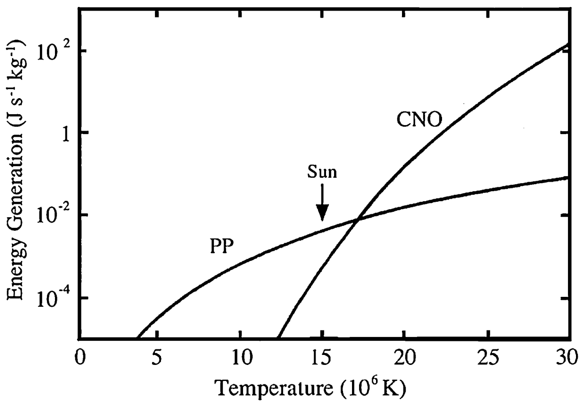

Fig. 10.11 Energy generation by two hydrogen-burning processes The energy output (*left vertical axis*), in units of power per kilogram, or J s^{-1} kg^{-1}, as a function of core temperature (*bottom horizontal axis*) in millions, or 10^6, degrees kelvin, designated K. The proton–proton chain, denoted PP, dominates the hydrogen-burning energy production for the Sun and less massive stars that have lower core temperatures. At the center of the Sun, where the temperature is 15.6×10^6 K, the PP chain is the dominant nuclear-reaction chain for converting hydrogen nuclei into helium nuclei, with an energy output of 0.016 J s^{-1} kg^{-1}, or 51 million MeV g^{-1} s^{-1} in the units used by nuclear astrophysicists. In more massive main-sequence stars, the central temperature is higher and the CNO cycle of hydrogen burning is the most efficient process. Main-sequence stars of mass less than 1.5 solar masses shine by the PP chain of nuclear reactions, whereas the main-sequence stars with mass greater that 1.5 solar masses burn hydrogen by the CNO set of nuclear reactions. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

high luminosity of the massive stars. Subsequently, this conclusion was placed on a firm basis when William A. "Willy" Fowler (1911–1995) and his colleagues at the California Institute of Technology made laboratory measurements of the nuclear reaction cross sections for every reaction in the chain; the results were published in a series of papers spanning several decades.

Given today's knowledge, it is the mass and therefore the central temperature of a main-sequence star that determine which hydrogen burning reaction supplies most of its power. The relevant formula for the energy production rate, ε_{pp}, of the proton–proton chain is given by

$$\varepsilon_{pp} = 0.24\rho X^2 \left[\frac{10^6}{T}\right]^{2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right] \text{ J s}^{-1} \text{ kg}^{-1}, \qquad (10.79)$$

where ρ is the mass density and X is the mass fraction of hydrogen. The energy production rate for the CNO cycle is:

$$\varepsilon_{CNO} = 8.7 \times 10^{20} \rho X_{CNO} X \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-152.3\left(\frac{10^6}{T}\right)^{1/3}\right] \text{ J s}^{-1} \text{ kg}^{-1}, \quad (10.80)$$

where X_{CNO} is the sum of the mass fractions for carbon, nitrogen, and oxygen. The variations of the two energy-producing reactions as a function of the core temperature T is illustrated in Fig. 10.11 for a typical stellar composition.

Both the proton–proton chain and the CNO cycle operate within main-sequence stars, and release about the same amount of energy during each reaction chain; however, the total amount of energy generation differs depending on the mass and central temperature of a star (Fig. 10.11). The CNO cycle contributes little energy in low-mass stars with low central temperatures, and the proton–proton chain produces almost all of the energy radiated by main-sequence stars with a mass less than or equal to the Sun's mass (Bahcall et al. 2003). However, the CNO cycle is faster and generates more energy in massive stars that have high central temperatures. Stars with a mass of about 2 solar masses or above generate almost their entire energy output by the CNO cycle.

In the Sun, with a central temperature of 15.6 million K, only 1.5 % of its energy is generated by the CNO cycle. That is why the proton–proton chain explains the Sun so well. However, with increasingly more mass and an increasingly hotter stellar core, the CNO cycle becomes the dominant energy source for a main-sequence star. For a star with a mass of 1.5 times the mass of the Sun, where the central temperature reaches 18 million K, each method of burning hydrogen produces the same amount of total energy and half the luminosity of a star.

The role of radiation and convection in transporting energy out of the stellar core also depends on the mass of a main-sequence star (Fig. 10.12). When the stellar mass is comparable to that of the Sun, the energy-generating core is surrounded by a radiative zone and topped by a convective zone (see Sect. 8.5). Main-sequence stars with a mass of more than 2 solar masses have a convective core. In these stars, the rate of energy generation by the CNO cycle is sensitive to temperature, so the fusion is highly concentrated in the core. Consequently, there is a high-temperature gradient in the core region, which results in a central convection zone. The outer regions of such a massive star transport energy by radiation with little or no convection.

All stars that begin their relatively long and placid life on the main sequence are composed mainly of hydrogen. These stars are initially uniform balls of plasma with the same composition throughout. As time passes, the central stellar core is changed slowly from hydrogen to helium, so the inside of a main-sequence star eventually becomes different from the outside. Eventually, the core is all used up, exhausting its supply of hydrogen by converting it into helium. The star has to leave the main sequence and become hot enough inside to burn helium – the ash of its former hydrogen-burning fires.

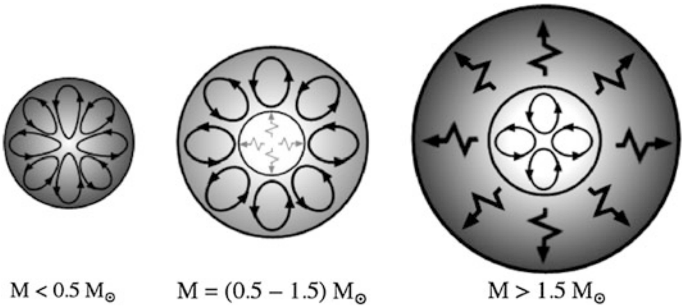

Fig. 10.12 Convection inside stars of different mass Most stars have convective zones in which energy is transported by the wheeling motion of convection, denoted here by closed curves with arrows for stars of different mass, designated by M, and compared to the Sun's mass denoted $M_{\odot}$. The symbol $<$ means less than and the symbol $>$ denotes greater than. Low-mass stars, with less than half a solar mass, are fully convective from core to visible disk and therefore of uniform composition. Their low temperatures result in a high opacity to radiation. In intermediate-mass stars, such as the Sun, radiation transport dominates convection in the hot central regions, which are enveloped by a cooler convective region. The visible disks of these stars do not include the nuclear-fusion products from their core but rather retain the same composition as the interstellar medium from which these stars were formed. High-mass stars, with more than 1.5 times the mass of the Sun, have a large radiative zone that is not enveloped by a convective zone. The temperature-sensitive hydrogen-burning reactions of the CNO cycle cause the development of a convective core in these stars. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

10.3.3 Helium Burning in Giant Stars

The fact that giant stars are connected to the main sequence of the H-R diagram suggested that the giants are the next stage of stellar evolution. However, because giant stars have larger luminosities at lower disk temperatures than main-sequence stars, they seemed to shine by a different and unknown process. The enigma was resolved partially when the Estonian astronomer Ernst Öpik (1893–1985), working at the Armagh Observatory in Northern Ireland, argued that the inside of a giant star can become very hot and dense at the same time that its outer parts become cool and rarefied (Öpik 1938).

The hydrogen-burning process of a main-sequence star is confined to the central stellar core, which is surrounded by an inert, nonburning envelope in which no nuclear reactions take place. When the core hydrogen is expended, the core is forced to contract, for it can no longer support itself under the crush of gravity. The central temperature will rise to about 100 million, or 10^8, K once gravitational forces compress the core to a smaller volume and increase the mass density 1,000 fold, to about 10 million kg m^{-3}. The rapid increase in core temperature causes the surrounding hydrogen envelope to expand, producing a vast, cool envelope of low mass density (Fig. 10.13). These spectacular changes in both the inside and

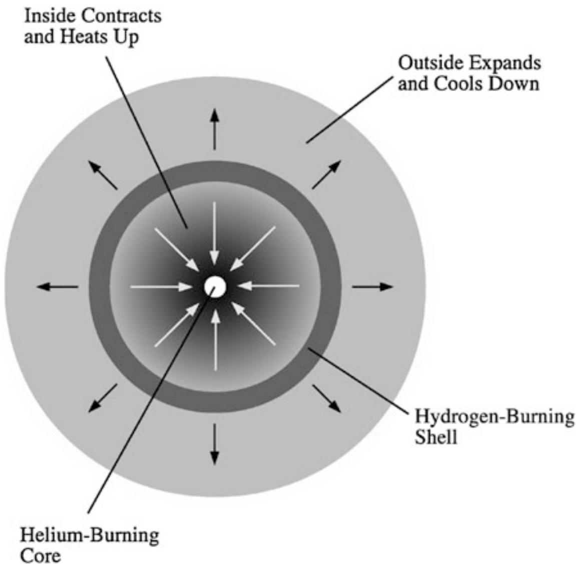

Fig. 10.13 Formation of a giant star When a main-sequence star consumes the hydrogen in its core, the inside of the star contracts and heats up, causing the outside to expand and cool down. Hydrogen burning resumes in a shell that envelops the collapsing core. The center of the star eventually heats up to about 100 million K, which is hot enough to burn helium and stop the core collapse. A giant star then has been created with a luminosity of about 100 times that of the Sun and a radius of approximately 50 times the radius of the Sun. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

outside of a dying main-sequence star account for the observed characteristics of red giant stars.

Öpik realized that the large increase in central temperatures and densities of giant stars open up a new source of energy not available to the main-sequence stars, and he proposed that the helium ash produced by hydrogen burning would serve as the nuclear fuel for giant stars.

The main difficulty with this scenario was that there is no stable nucleus of atomic weight 5, and this gap seemed to provide an impenetrable barrier for the synthesis of heavier elements from helium of weight 4 using protons of weight 1. A proton could not be attached to a helium nucleus to make the next heavier substance.

It took more than a decade to resolve the difficulty, which was explained almost simultaneously and independently by Öpik and the American astronomer Edwin E. Salpeter (1924–2008), at Cornell University (Öpik 1951; Salpeter 1952). When the core of a star reaches a sufficiently high temperature, of about 100 million, or 10^8, K, helium nuclei can be converted to carbon nuclei by triple collisions of helium nuclei, thus circumventing the mass 5 difficulties.

This release of energy by fusing helium into carbon within a star is known as *helium burning*. It also is called the *triple alpha process* because the helium nucleus is an alpha particle and a triple collision is required to make a nucleus of the carbon atom.

The net result of the helium burning, triple alpha chain reaction is

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma, \tag{10.81}$$

where ^{4}He is a helium nucleus or alpha particle, ^{12}C is a carbon nucleus, the γ denotes gamma radiation. About 7.275 MeV or 1.165×10^{-12} J of energy is released each time it occurs. The triple alpha process happens so frequently at the high central temperatures of giant stars that it can power their intense luminosity.

The difficulty in pushing together even two helium nuclei is exacerbated by their electrical charge. Each helium nucleus contains two protons; therefore, the electrical repulsion between two helium nuclei is four times that between two protons.

Example: Hot enough to burn helium

To overcome the electrical, or Coulomb, repulsion between two helium ions, of charge $2e = 2 \times 1.6022 \times 10^{-19}$ C each, the kinetic energy $m_{He}V^2/2$ of a colliding helium ion, of mass $m_{He} = 6.6445 \times 10^{-27}$ kg and velocity V must be equal to the electrical potential energy $4e^2/(4\pi\varepsilon_0 R_{He})$ when the two ions touch, where $\pi = 3.14159$, the permittivity of free space is $\varepsilon_0 = 10^{-9}/(36\pi) = 8.854 \times 10^{-12}$ F m^{-1}, and the radius of the helium ion is $R_{He} \approx 10^{-15}$ m. Solving for the velocity $V \approx 10^7$ m s^{-1}, and setting it equal to the thermal velocity $V = V_{thermal} = (3kT/m_{He})^{1/2}$, with the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, we obtain a temperature of $T \approx 1.6 \times 10^{10}$ K to overcome the electrical repulsion.

This is an impossibly high temperature, about 4 times hotter than that required for the direct fusion of two protons, but quantum–mechanical tunneling lowers the central star temperature needed to about five billion, or $T_C \approx 5 \times 10^9$ K. The increase in temperature is created by core collapse to a radius R_c. By equating the thermal energy $3kT_c/2$ to the gravitational energy $Gm_{He}M_c/R_c$, where the gravitational constant $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and assuming a core mass about equal to that of the Sun with $M_\odot = 1.989 \times 10^{30}$ kg, we can infer a core radius of $R_c \approx 10^7$ m $\approx 0.014\ R_\odot$ for the Sun's radius $R_\odot = 6.955 \times 10^8$ m and a mass density of $\rho = 3\ M_\odot/(4\pi R_c^3) \approx 0.5 \times 10^9$ kg m^{-3}.

It turns out that certain nuclear resonances also enhance the reaction rate of the triple helium burning process, and modern calculations indicate that a core temperature of 100 million, or 10^8 K might suffice to initiate the helium fusion, but we have the basic idea. The central core must collapse to increase the temperature and enable helium fusion.

The quantum–mechanical tunneling effect will help, as it does in permitting two protons to fuse together in the Sun (see Sect. 8.3). The rise in the central temperature of a giant star is needed to increase the number of helium nuclei moving fast enough to penetrate the larger electrical barrier with the aid of tunneling, which incidentally also explains why helium burning doesn't occur in low-temperature, main-sequence stars. Only the helium nuclei in the high-velocity tail of the Maxwellian speed distribution can merge together in a giant star, which means that most of the helium nuclei are not moving fast enough to merge and that the helium-burning reactions occur relatively slowly.

Under most circumstances, helium burning still would be exceedingly unlikely because it involves the nearly simultaneous collision of not two but rather three helium nuclei. Such a triple collision is favored by two exceptionally large collision cross sections, termed *resonance reactions* in the parlance of nuclear physics. The bigger the cross section for a collision, the more likely it will occur.

In 1952, Salpeter, who was unaware of Öpik's work the previous year, presented some of these more detailed explanations for the formation of carbon in the triple alpha, helium burning process. Two helium nuclei can combine to form a beryllium 8 nucleus, but because the beryllium is unstable only a tiny fraction remains at any instant. As Salpeter noticed, a beryllium nucleus can occasionally combine with a third helium nucleus to form carbon 12. Nevertheless because beryllium is so extremely rare, it must have a large cross section for helium capture if any substantial amount of carbon is to be produced.

Two years later, Fred Hoyle (1915–2001) showed that the triple alpha process occurs at a rapid enough rate to provide the luminosity of a red giant star if the carbon goes through an excited state and the giant core temperature reaches about 10^8 K at a mass density of about 10^7 kg per cubic meter (Hoyle 1954). William Fowler and his colleagues subsequently showed that the required excited state of carbon does in fact exist. The formation of carbon from helium is thus enhanced enormously by two facts: the existence of beryllium 8 (itself a kind of resonance) and the existence of the excited state of carbon.

The details are somewhat complicated but suffice it to say that the mass 5 barrier can be overcome; Fynbo et al. (2004) provide relatively recent estimates for the rates of the stellar triple-alpha process. In retrospect, it seems almost miraculous that nature has conspired in this way to make helium burning and giant stars exist. Our understanding of the evolution of main-sequence stars into these larger, more luminous counterparts was stimulated by investigations of star clusters.

10.4 Using Star Clusters to Watch How Stars Evolve

Entire stars have had a beginning, followed by a long period of growth and inevitable decay, eventually turning into something else, and we can use the H-R diagrams of star clusters to map out the stages of stellar transfiguration. As time elapses, the more massive stars evolve into the next phase of stellar life and the

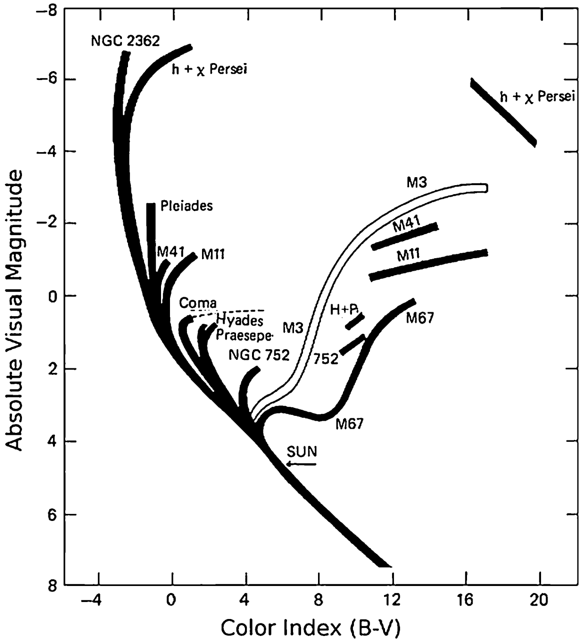

Fig. 10.14 Open star clusters in the H–R diagram Open star clusters are relatively young, and most of their stars have not yet left the main sequence in the H-R diagram. The youngest clusters, such as the Pleiades, retain all but the topmost part of the main sequence. The turnoff point of the Pleiades star cluster from the main sequence indicates an age of roughly 100 million years. The Hyades star cluster, which turns off about halfway down the main sequence, is about 600 million years old. The lowest open cluster in this diagram, M 67, is an estimated 5 billion years old, with a main sequence that stops just above the Sun. One globular star cluster, M 3, is shown for comparison. [Adapted from Allan Sandage (1957).]

main sequence disappears from the top down. Very massive stars at the upper left of the main sequence become supergiants; those with intermediate masses comparable to the Sun become red giants.

Benacquista (2013) gives a fine introduction to the evolution of both single and binary stars. Gallart et al. (2005) have provided a review of stellar evolution models and color magnitude diagrams. Iben and Renzini (1983) have reviewed the asymptotic giant branch (AGB) evolution and beyond, and Winckel (2003) has reviewed the post AGB stars. Chiosi and Maeder (1986) have reviewed the evolution of massive stars with mass loss; Gilles and Baraffe (2000) have reviewed the theory of low-mass stars and substellar objects; and Maeder and Meynet (2000) have discussed the evolution of rotating stars.

Stars within a star cluster are all of the same approximate age, within a few million years, dating back to the formation of the cluster. They also began with the same initial composition of material and exhibit a full range of stellar mass. Because the stars in a given cluster are all at the same distance from the Earth, we

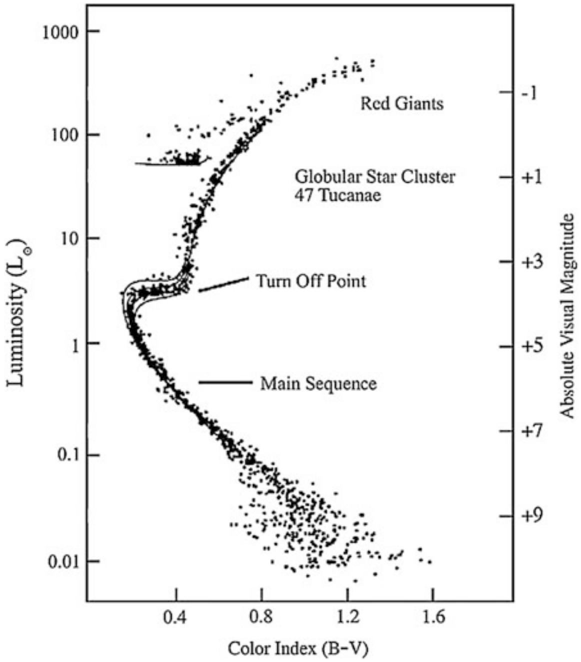

Fig. 10.15 How old is a globular star cluster? A plot of the luminosity (*left vertical axis*), in units of the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, against the color index, B-V (*bottom horizontal axis*), for the stars in the southern globular star cluster 47 Tucanae, also designated NGC 104. The absolute visual magnitudes, M_V, of the stars also are shown (*right vertical axis*). Although low-mass, relatively faint stars are still on the main sequence (*diagonal line from middle left to bottom right*), the massive, bright stars in the cluster have left the main sequence and are evolving into giant stars (*top right*). Theoretical tracks, called *isochrones*, show the evolutionary distributions at different ages of 10 billion, 12 billion, 14 billion and 16 billion years from top to bottom and left to right. The best fit to the observed data corresponds to an age between 12 billion and 14 billion years for this star cluster. (Courtesy of James E. Hesser.)

can obtain direct observations of their relative luminosity without knowing the distance.

A cluster H-R diagram can be used as a clock, dating the age of the cluster and the stars in it by the place of their turnoff from the main sequence to become supergiants or giants. Stars with a luminosity and temperature greater than the turnoff value all have evolved away from the hydrogen-burning state of stellar life, and the age of the cluster is equal to the main-sequence lifetime of stars at this turnoff point. The lower its luminosity or temperature, the older is the star cluster.

The main-sequence turnoffs of the loosely bound open star clusters (Fig. 10.14) indicate that they are approximately 100 million years old (Mermilliod 1981; Sandage 1957). Such relatively young clusters are identified by the membership of O and B stars, which would leave the main sequence in a relatively short time.

The main sequence of the H-R diagram for globular star clusters does not contain these hot, luminous stars (Fig. 10.15), and these star clusters have ages between 10 billion and 14 billion years – or about two to three times the age of the Sun. This indicates that the masses of the stars at the main-sequence turnoff point of globular clusters are less than the Sun. Soderblom (2010) provided a review of the age of stars, whereas Van den Berg et al. (1996) reviewed the age of the galactic globular cluster system.

Due to their great age and numerous stars, the H-R diagrams of these dense stellar concentrations help us watch how stars evolve to the later stages of stellar life. Such investigations involve theoretical calculations of precisely how long a main-sequence star's central fuel supply can last and models of what happens when its fuel is used up. Martin Schwarzschild (1912–1997), the son of German astronomer Karl Schwarzschild (1873–1916), was one of the first to examine this phase of stellar evolution. After emigrating to the United States, Martin Schwarzschild used theoretical models and primitive computers, developed by his Princeton colleague John von Neumann (1903–1957), to chart the evolutionary trajectory of a star and compare it to the various kinks, bends, and gaps of missing stars on the H-R diagrams of globular star clusters. His theoretical evolutionary models were facilitated by the fact that all of the stars in globular star clusters have the same initial chemical composition.

Martin teamed with Allan Sandage (1926–2010), then a graduate student at the California Institute of Technology, whose H-R diagrams included faint stars that connected the main sequence to the red-giant branch (Fig. 10.16). When Sandage's data were compared to the model results, it improved our understanding of evolution away from the main sequence and this provided a sound observational basis for stellar aging (Sandage and Schwarzschild 1952; Sandage 1957; Schwarzschild 1958).

When the hydrogen-burning fires are quenched in a star of roughly solar mass, the shrinking stellar core heats up and causes the star as a whole to swell into a bloated red giant. The gravitational energy released by the collapsing, nonburning core is then spread over a much larger area, resulting in a lower disk temperature and a shift of the visible starlight into the red part of the spectrum. This accounts for the red-giant branch in the H-R diagram of globular star clusters.

The rising heat from the collapsing core ignites hydrogen burning in an internal shell that envelops a core of inactive helium. As realized by Schwarzschild and the English astronomer Fred Hoyle (1915–2001), the giant core is compressed into a degenerate state and eventually heats up to a temperature of about 10^8 K, when core helium burning begins (Hoyle and Schwarzschild 1955). Once the star fuses helium into carbon within its core, it enters the horizontal branch of the cluster H-R diagram and is technically no longer considered a red giant. As then demonstrated by Schwarzschild and the Richard Härm (1909–1996), helium begins to

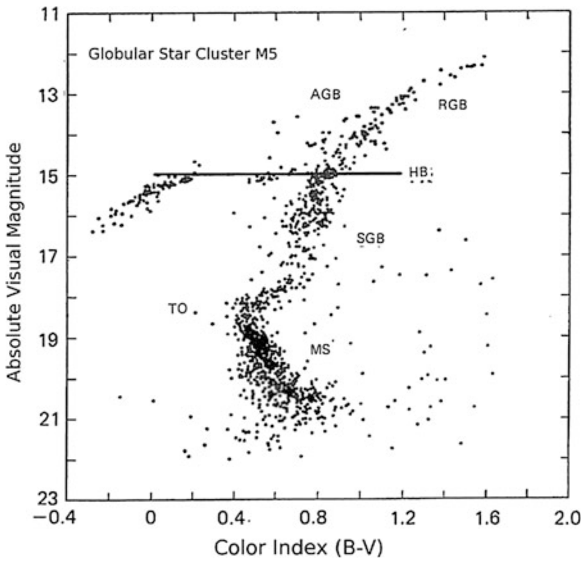

Fig. 10.16 Globular star cluster in the H–R diagram The Hertzsprung-Russell diagram for the globular star cluster M 5, where the absolute visual magnitude (*left vertical axis*) is plotted as a function of color index (*bottom horizontal axis*). It is very different from the H-R diagrams for open star clusters shown in Fig. 10.14. The high-mass stars in this globular star cluster have left the main sequence (*MS*) at a relatively low turnoff point, denoted TO, indicating a greater age than the open star clusters. This diagram illustrates the evolutionary tracks of these stars into the red-giant branch, designated RGB (*top right*), as well as other evolutionary stages such as the subgiant branch, denoted SGB; the asymptotic giant branch, designated AGB; and the horizontal branch, denoted HB, that extends to the left. The gap of missing stars in the horizontal branch for the globular star cluster M 5 shows the instability strip of pulsating stars, known as RR Lyrae stars. [Adapted from Arp (1962).]

burn abruptly in the core, releasing a flash of intense energy that rejuvenates a star's luminous output and initiates core helium fusion into carbon, which can last about 10^8 years (Schwarzschild and Härm 1962). Härm was born and educated in Estonia, but moved to the United States during World War II (1939–1945), spending his entire subsequent career at Princeton University.

When the core helium is exhausted, the carbon and oxygen core collapses, helium is burned in a surrounding shell, and the star briefly rises to giant status a second time, along the asymptotic branch of the H-R diagram. A short period of instability then begins, where the star can pulsate, but it is approaching its end.

An even briefer stellar life, with more violent winds, is expected for the more massive main-sequence stars that become supergiants. They help explain where most of the heavier elements originate.

10.5 Where did the Chemical Elements Come From?

10.5.1 Advanced Nuclear Burning Stages in Massive Supergiant Stars

A supergiant star has a much greater mass, interior compression, central temperature, and luminosity than a giant star, which is why we call them "super." Supergiants pass through the same early stages of stellar life as giants but at a faster rate. Unlike their counterparts of lesser mass, massive stars with a mass above 10 solar masses quickly consume the hydrogen in their cores and transform into colossal supergiants that burn the next available nuclear fuel: helium. A star with a mass of 25 times that of the Sun, for example, will complete hydrogen burning by the CNO cycle in about 7 million years and helium burning in a mere 660,000 years. In contrast, the Sun will take roughly 10 billion years to complete hydrogen burning by the proton–proton chain and another 100 million years to consume the helium within its core.

When the core helium is consumed, the evolutionary paths of high-mass and moderate-mass stars diverge. The cores of supergiant stars are so massive that they can contract and heat up enough to burn carbon, thereby stopping the core collapse and shining with renewed vigor. Carbon nuclei have an electrical charge equivalent to six protons; therefore, a formidable electrical repulsion separates them, and collisions at great speed are required for its penetration. This can happen when the temperature rises to about a billion K, or 10^9 K.

In contrast, the giant stars are not sufficiently massive to burn anything heavier than helium because they never get hot enough inside. In technical terms, degeneracy pressure halts the contraction of the inert, nonburning carbon core in a giant star before it can become hot enough for carbon fusion. These stars expel their outer layers into surrounding space and collapse inside to an Earth-sized white dwarf star.

A supergiant is so massive that it can enter progressively more advanced nuclear-burning stages (Salpeter 1957; Weaver et al. 1978; Heger et al. 2003). The core helium is converted into carbon, and some of the newly formed carbon nuclei can fuse with helium nuclei to make oxygen. When the helium is gone, the core contracts until it becomes hot enough to burn carbon into neon, which temporarily re-stabilizes the core. Each time the core depletes the elements that it is fusing, or "burning," it shrinks and heats up until it becomes hot enough for fusion reactions of the nuclear ash, continuing up the chain of successively heavier abundant elements. At the same time, nuclear fusion of the earlier fuel continues in overlapping shells at lower temperatures. Layer upon layer of nuclear burning shells are created deep down inside a supergiant.

The aging of a supergiant star accelerates rapidly, consuming its internal fuel sources at ever-increasing central temperatures and rates (Table 10.8). Due to the higher temperatures needed for these nuclear reactions to occur, they also proceed at a much more rapid rate than hydrogen burning, and the thermonuclear lifetime

Table 10.8 Nuclear fusion processes in a supergiant star of 25 solar masses[a]

Core fusion process	Central temperature (K)	Central density (kg m^{-3})	Duration (years)
Hydrogen burning (H → He)	3.7×10^7	3.8×10^3	7,300,000
Helium burning (He → C and O)	1.8×10^8	6.2×10^5	660,000
Carbon burning (C → Ne)	7.2×10^8	6.4×10^8	165
Neon burning (Ne → Mg and Si)	1.4×10^9	3.7×10^9	1.2
Oxygen burning (O → Si)	1.8×10^9	1.3×10^{10}	0.5
Silicon burning (Si → Fe)	3.4×10^9	1.1×10^{11}	0.004

[a] Adapted from Weaver et al. (1978)

of the supergiant stars therefore is much shorter than those of even the giant stars. This is one reason that so few supergiant stars are observed.

In the terminal stages of a supergiant's life, its inner core is converting silicon into iron, and onion-like overlying layers are burning lighter elements, such as magnesium, oxygen, neon, carbon, and helium. However, when the iron nuclei in the core of such a star are pushed together, no energy is released. The iron does not burn, regardless of how hot the star's core becomes. The nuclear fires are extinguished; and the star has reached the end of its life. It can never again shine by any slow nuclear-fusion process, and there is no energy left to support the core. The star has become bankrupt, having completely spent all of its internal resources. There is nothing left to do but collapse.

The inert core collapses in less than 1 s, bounces and then explodes as a supernova with the light of 1 billion Suns. The shattered star and all of the elements made inside it are then dispersed into surrounding space. This material provides the seeds for future planets and stars, which explains where most of the heavy elements that are now found in the Earth and the Sun came from.

10.5.2 Origin of the Material World

What accounts for the origin of the chemical elements that make up our everyday world? The English astronomer Arthur Stanley Eddington (1882–1944) was one of the first to propose that the light elements are compounded into more complex elements within stars (Eddington 1920). Then, within a decade, the great stellar abundance of hydrogen had been established, and Robert d'Escourt Atkinson (1898–1982) showed how element synthesis in stars, starting with the proton–proton reaction, might account for both stellar energy and the origin of the elements (Atkinson 1931). In his view, the observed relative abundance of the elements could be explained by the creation of less abundant, heavy nuclei from more abundant, lighter nuclei, particularly hydrogen and helium. The formation of heavy nuclei from the nuclear reactions of lighter nuclei is termed *nucleosynthesis*.

Baron Carl Friedrich von Weizsäcker (1912–2007) proposed two mechanisms for nucleosynthesis: either within stars or in the primeval “fireball” explosion, out of which the expanding universe arose (Weizsäcker 1937, 1938). Today, we know that our material world indeed was synthesized in both places. Most of the heavy elements that now are found in the universe were created within former stars by various nuclear reactions at different times and under different physical conditions. However, all of the hydrogen and most of the helium now observed in the universe was synthesized in the big-bang “fireball” explosion. Alpher and Herman (1950), Trimble (1975), Penzias (1979), and Fowler (1984) have reviewed the early history of ideas concerning element formation in stars and the big bang. Our understanding of the details of these processes is intimately related to the observed element abundance in the Sun.

10.5.3 The Observed Abundance of the Elements

An important key to understanding how stars synthesize the elements is obtained from their relative abundances, initially studied by chemists rather than astronomers. The American chemist William D. Harkins (1873–1951), for example, found an important clue to the mystery of the origin of the elements when he noticed that elements of low atomic weight are more abundant than those of high atomic weight and that, on average, the elements with even atomic numbers are about 10 times more abundant than those with odd atomic numbers of about the same number. These features led Harkins to conjecture that the relative abundances of the elements depend on nuclear rather than chemical properties and that heavy elements must have been synthesized from lighter ones (Harkins 1917, 1931).

Decades later, astronomers showed that stars are the crucibles in which all but the lightest elements are formed by nuclear reactions that convert light elements into heavier ones, and that the systematic decline in the abundance of heavier elements can be attributed to the relative scarcity of stars that have evolved to the stage that creates them.

Two other American chemists, Hans E. Suess (1909–1993) and Harold Clayton Urey (1893–1981), provided a detailed discussion of the elemental and isotopic abundances of the Sun and similar stars, calling attention to the many fluctuations that appear in the general trend of an exponential decline of abundance with increasing atomic weight (Suess and Urey 1956). This discussion served as a major stimulus for modern ideas concerning stellar nucleosynthesis (Fig. 10.17).

Asplund et al. (2009) have reviewed observations of the chemical composition of the Sun; Wilson and Rood (1994) provided an early summary of abundances in the interstellar medium; and Savage and Sembach (1996) reviewed interstellar abundances from absorption-line observations with the *Hubble Space Telescope*.

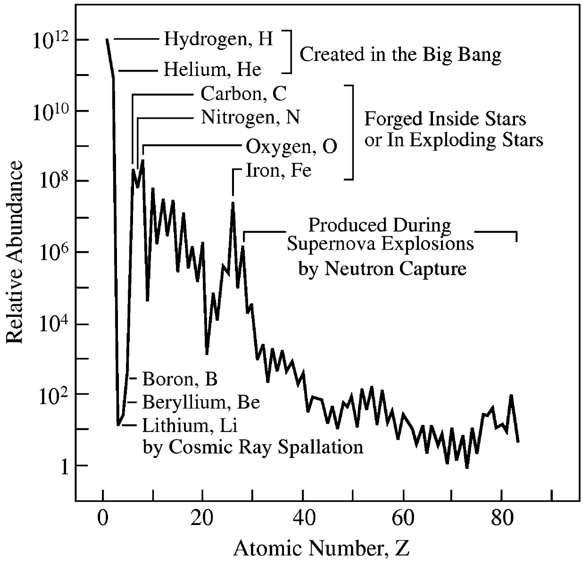

Fig. 10.17 Abundance and origin of the elements in the Sun The relative abundance of elements in the solar photosphere, plotted as a function of their atomic number Z, which is the number of protons in an atom's nucleus and roughly half the atomic weight. The abundance is plotted on a logarithmic scale and normalized to a value of 1 million million, or 1.0×10^{12}, for hydrogen. Hydrogen, the lightest and most abundant element in the Sun, was formed about 14 billion years ago in the immediate aftermath of the big bang that led to the expanding universe. Most of the helium now in the Sun was also created then. All of the elements heavier than helium were synthesized in the interiors of massive stars that were then wafted or blasted into interstellar space where the Sun subsequently originated. Carbon, nitrogen, oxygen, and iron were created over long time intervals during successive nuclear burning stages in former massive stars as well as during their explosive death. Elements heavier than iron were produced by neutron-capture reactions during the supernova explosions of stars that lived and died before the Sun was born. The light elements boron, beryllium and most lithium are believed to originate from heavier cosmic-ray particles that were stripped of some of their ingredients by collisions, in a process called *spallation*. The exponential decline of abundance with increasing atomic number and weight can be explained by the rarity of stars that evolved to later stages of life. (Data courtesy of Nicolas Grevesse.)

10.5.4 Synthesis of the Elements Inside Stars

The English astronomer Fred Hoyle (1915–2001) introduced the grand concept of nucleosynthesis in stars in the mid twentieth century (Hoyle 1946, 1954). He showed that both theoretical and experimental considerations were required, and placed the concept of stellar nuclear reactions within the framework of stellar structure and evolution, using the then-known nuclear data.

By 1957 the detailed abundance data of Suess and Urey had served as an inspiration for a comprehensive review of the major element-producing reactions in stars by Geoffrey R. Burbidge (1925–2010), E. Margaret Burbidge (1919–), Fred Hoyle and William A. "Willy" Fowler (1911–1995). Their seminal publication, entitled "Synthesis of the Elements in Stars" was published in the *Reviews of Modern Physics* (Burbidge et al. 1957); it is known as the B^2FH paper, after the surnames of the four authors. It provided the fundamental framework on which most subsequent studies of stellar nucleosynthesis are based (Fig. 10.17).

As a star ages, it experiences successive core-contraction and stable-core burning stages, in which heavier elements are synthesized from lighter ones, and the nuclear ash of one stage becomes the fuel of the next one. These core nuclear-burning reactions proceed at a progressively hotter, denser, and faster pace, such as successive hydrogen, helium, carbon, neon, oxygen, and silicon burning. Elements with an even number of protons in their nuclei, such as carbon and oxygen, are more abundant because they are formed by nuclear reactions with helium, which contains two protons rather than one in its nucleus.

As realized by B^2FH, the temperature is not the same everywhere inside a star; therefore, its nuclear evolution is most advanced in the central regions and least or not at all advanced near its outer regions. Thus, the composition of an aging star is not uniform throughout. At the end of its life, a massive star would be layered with successively thinner shells of helium, carbon, oxygen, and silicon surrounding an inert, nonburning iron core that explodes as a supernova, casting out the heavy elements from the overlying layers of the star.

Although the general flow of the observed relative abundance of the more abundant elements can be explained by successive static, or nonexplosive, burning stages within stars, elements heavier than iron – as well as many of the detailed ups and downs of the abundance curve for lighter elements – were attributed to fast nuclear reactions during the explosive, supernova death of massive stars. The American hydrogen-bomb tests in the 1950s had indicated that heavy elements are created by rapid neutron bombardment during the explosions, and Hoyle and Fowler knew that similar processes occur in supernovae (Hoyle and Fowler 1960).

B^2FH used slow, s, neutron capture; rapid, r, neutron capture; and proton, p, capture to explain some of the details of the abundance curve. Some of the nuclear-burning reactions produced free neutrons, residing outside any atomic nucleus. These neutrons can slowly fuse with abundant heavy nuclei to produce the relatively rare ones, encountering no electrical obstacle because the neutron is electrically uncharged. Unhindered by electrical repulsion, the neutrons permit the extension of stellar nucleosynthesis from iron all the way to uranium. A free neutron also can decay into a proton, and the rapid capture of both neutrons and protons during supernova explosion helps to forge the heavier elements.

Busso et al. (1999) reviewed nucleosynthesis in asymptotic giant branch (AGB) stars. Arnett (1973, 1995) reviewed explosive nucleosynthesis in stars, whereas Meyer (1994) gave us a review of the r-, s- and p- processes in nucleosynthesis.

10.5.5 Big-Bang Nucleosynthesis

The synthesis of elements inside stars is an incomplete scenario for it does not explain the origin of any of the hydrogen and most of the helium in the observable universe. Moreover, because deuterium is destroyed rapidly inside stars, there also must be another explanation for its cosmic existence. As proposed by the eclectic George Gamow (1904–1968) and his colleagues, these elements must have been produced during the exceptionally hot and dense first moments of the big-bang explosion that gave rise to the expansion of the universe. In fact, it once was thought that all of the elements found today might have been created back then.

Working with his young colleague Ralph A. Alpher (1921–2007), Gamow proposed that all of the elements were produced in a chain of nuclear reactions during the earliest stages of the expanding universe. They supposed that the original substance of the material universe, the cosmic "ylem," consisted solely of neutrons at high temperature. Some of these neutrons decayed into protons, and successive captures of neutrons by protons led to the formation of the elements.

This novel idea was published in 1948, in a paper titled "The Origin of the Chemical Elements," with Hans A. Bethe (1906–2005) added as an author, even though he contributed nothing to the research or the writing. This was done to make a pun on the first letters of the Greek alphabet – alpha, beta, and gamma or α, β, and γ (Alpher et al. 1948).

Two years after the α–β–γ paper, the Japanese astrophysicist Chushiro Hayashi (1920–2010) showed that in the first moments of the expansion, the temperature was hot enough to create particles such as neutrinos and positrons, the anti-matter particles of electrons (Hayashi 1950). The mutual interaction of all of the subatomic particles present in the first moments of the big bang establishes the relative number of neutrons and protons, which in turn determine the amount of helium produced.

With this correction, modern computations by Robert V. Wagoner (1938–), David Schramm (1945–1997) and their colleagues conclusively demonstrated that all of the hydrogen and most of the helium found in the cosmos today were synthesized in the immediate aftermath of the big bang (Wagoner et al. 1967; Peebles et al. 1991).

These processes are known as *big-bang nucleosynthesis*, and they tell us how much matter is now in the universe in both visible and invisible forms. Boesgaard and Steiman (1985) have reviewed the theory and observations of big-bang nucleosynthesis.

So Gamow was partly right. The lightest and most abundant element, hydrogen – and therefore the majority of atoms that we see today – indeed were formed even before the stars existed, in the immediate aftermath of the big bang that produced the expanding universe. All of the hydrogen that is now found in stars, interstellar space, and the rest the universe was created then, about 14 billion years ago, and so was most of the helium, the second most abundant element.

Why weren't all of the heavier elements produced in the early stages of the big bang? By the time that the expanding universe became sufficiently cool to allow nucleosynthesis to occur, at a temperature of about 1 billion K, it had expanded into a greater volume and become less dense. The rate of helium burning by the triple alpha process is proportional to the square of the density at a given temperature, and at 1 billion degrees the density of the expanding universe is too low by many orders of magnitude for appreciable operation of the triple alpha process.

In simpler terms, the expanding universe was not and is not in equilibrium, so it rapidly cooled down and thinned out to low density, making the simultaneous collision of three helium nuclei – alpha particles – nearly impossible. The dense cores of giant stars, however, are in equilibrium for the long intervals of time needed to accrue noticeable amounts of carbon by the triple alpha process. Only the very light elements, such as deuterium and helium could be produced by two-body reactions during the big bang, whereas carbon had to be synthesized by three-body reactions in the interior of stars.

This completes our account of the origin of the elements inside stars and during the big bang, with one oversight: the under-abundant, light elements with atomic weights between hydrogen and helium. These light elements – boron, beryllium, and most of the lithium – probably were produced by *spallation reactions* in which energetic charged particles, known as cosmic rays, strip off the components of heavy nuclei to form light nuclei (Reeves et al. 1973; Reeves 1994). Wallerstein and Conti (1969) reviewed lithium and beryllium in stars. For references to research papers on the observations and explanations of the abundance of light elements see Lang (1999).

10.5.6 The First and Second Generation of Stars

Because big-bang nucleosynthesis produced no elements heavier than helium, the earliest stars had to be composed of the lightest abundant elements: hydrogen and helium. This first generation of stars is known as *Population I stars*. Then, as a result of ongoing stellar alchemy, the most massive first-generation stars forged heavier elements in their cores, scattering them into space by winds or explosions as they died. These heavy elements, known as *metals* to astronomers, then were recycled and incorporated into second generation stars. Because some of their material came from previous stars, these *Population II stars* are polluted somewhat by the heavier elements, the metals.

Baade (1952) proposed the existence of two stellar Populations, finding that pulsating variable stars of the two types have different period-luminosity relations. Ivezic et al. (2012) have provided a recent review of galactic stellar Populations. The first stars to be formed in the universe have been designated *Population III*.

Observations of stellar spectra confirm this scenario. Very old Population I stars, which formed when the universe was young, have less than 0.1 % of their mass in elements heavier than hydrogen or helium. We see these survivors of the earliest times in the oldest globular star clusters. Astronomers have not yet found completely pure stars with absolutely no metals, but they are confident that the second-generation Population II stars contain a greater proportion of heavy elements, at 2–3 % of their mass.

Audouze and Tinsley (1976) provided an early discussion of the chemical evolution of galaxies; Rana (1991) subsequently reviewed the chemical evolution of our Galaxy; and McWilliam (1997) reviewed abundance ratios and galactic chemical evolution.

Beers and Christlieb (2005) reviewed the discovery and analysis of very metal-poor stars in the Galaxy, whereas Sandage (1986) reviewed the population concept and related topics such as the collapse of our Galaxy. Gratton et al. (2004) reviewed abundance variations within globular star clusters; Wheeler et al. (1989) have reviewed the abundance ratios as a function of metallicity.

10.5.7 Cosmic Implications of the Origin of the Elements

During the billions of years before the Sun was born, massive stars reworked the chemical elements, fusing lighter elements into heavier ones within their nuclear furnaces. Carbon, oxygen, nitrogen, silicon, iron, and most of the other heavy elements were created this way. The enriched stellar material then was cast out into interstellar space by the short-lived massive stars, gently blowing out in their stellar winds or explosively ejected within supernova remnants.

The Sun and its retinue of planets condensed from this material about 4.6 billion years ago. They are composed partly of heavy elements that were synthesized in stars that lived and died before the Sun and planets were born. The Earth and everything on it spawned from this recycled material.

Moreover, the first stars could not have had rocky planets like the Earth because there initially was nothing but hydrogen and helium. The only possible planets would have been icy balls of frozen gas. Without carbon, life as we know it could not evolve on these planets.

Perhaps the most fascinating aspect of stellar alchemy is its implications for life on the Earth. Most of the chemical elements in our bodies, from the calcium in our teeth to the iron that makes our blood red, were created billions of years ago in the hot interiors of long-vanished stars. Therefore, we are all made of "star stuff". If the universe were not very, very old, there would not have been enough time to forge the necessary elements of life in the ancient stars. The lightest element, hydrogen, needed for the water in our bodies, was synthesized when the observable universe was very young, in the first instants of the big bang. Therefore, we are the offspring of both the star and the big bang – true children of the cosmos.