Chapter 8
What Makes the Sun Shine?

8.1 Can Gravitational Contraction Supply the Sun's Luminosity?

When we measure the total amount of sunlight that illuminates and warms our globe, and extrapolate back to the Sun, we find that it is emitting an enormous power of 385.4 million, million, million, million, or 3.828×10^{26}, watts, where one watt = 1 J s^{-1}. This brilliance is far too great to be perpetually sustained, and we therefore wonder what heats the Sun, and how long that heat will last.

In the mid-nineteenth century, the German physicist Hermann von Helmholtz (1821–1894) proposed that the Sun's luminous energy is due to its gravitational contraction (Helmholtz 1856, 1908). If the Sun were gradually shrinking, the compressed matter would become hotter and the solar gases would be heated to incandescence; in more scientific terms, the Sun's gravitational energy would be converted slowly into the kinetic energy of motion and heat up the Sun, so that it would continue to radiate. This follows from the principle of conservation of energy, which Helmholtz was one of the first to propose (Helmholtz 1847). It states that energy can be neither created nor destroyed; it can only change form.

The Irish physicist William Thomson (1824–1907), later Lord Kelvin, subsequently showed that the Sun could have illuminated the Earth at its present rate for about 100 million years by slowly contracting (Kelvin 1862, 1899; Burchfield 1975). We can follow his reasoning by noting that the gravitational potential energy, Ω, of a star of mass M_S and radius R_S is given by (Sect. 3.2):

$$\Omega = -\frac{3GM_S^2}{5R_S}, \tag{8.1}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}. The change, $\Delta\Omega$, in gravitational potential energy created by a decrease in radius, ΔR_S, is:

$$\Delta\Omega = \frac{3GM_S^2}{5R_s^2}\Delta R_S \approx 3.27 \times 10^{32}\left(\frac{M_S}{M_\odot}\right)^2\left(\frac{R_\odot}{R_s}\right)^2 \Delta R_s \text{ J}, \tag{8.2}$$

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where the radius decrease is in meters, and we have normalized the mass and radius in terms of the Sun's mass $M_{\odot} = 1.989 \times 10^{30}$ kg and the Sun's radius $R_{\odot} = 6.955 \times 10^{8}$ m.

If the energy change provides an absolute luminosity, L_S, in a time interval, Δt, then $L_S = \Delta\Omega/\Delta t$, and the rate of change in radius is:

$$\frac{\Delta R_S}{\Delta t} = \frac{5L_S R_S^2}{3GM_S^2} \approx 1.17 \times 10^{-6}\left(\frac{L_S}{L_{\odot}}\right)\left(\frac{M_{\odot}}{M_S}\right)^2\left(\frac{R_S}{R_{\odot}}\right)^2 \text{ m s}^{-1}, \tag{8.3}$$

where the Sun's absolute luminosity is $L_{\odot} = 3.828 \times 10^{26}$ J s^{-1}. Since one year is 3.156×10^{7} s, this shows that a contraction of only 36.9 m per year will power the Sun at the present rate. That is a very small change considering the much larger radius of the Sun.

The problem with this mechanism is the long duration of the Sun and other stars. If the source of a star's present luminosity were gravitational potential energy, then the current radius would shrink to zero in the Kelvin–Helmholtz time, denoted by the symbol τ_{K-H}, given by Kelvin (1863):

$$\tau_{K-H} = \frac{R_S}{(\Delta R_S/\Delta t)} = \frac{\Omega}{L_S} = \frac{3GM_S^2}{5R_S L_S} = 5.95 \times 10^{14}\left(\frac{M_S}{M_{\odot}}\right)^2\left(\frac{R_{\odot}}{R_S}\right)\left(\frac{L_{\odot}}{L_{-S}}\right) \text{ s.} \tag{8.4}$$

Using 1 year = 3.156×10^{7} s, the Kelvin–Helmholtz time for the Sun is about 1.89×10^{7} years. A contraction of 36.9 m per year will power the Sun at its present rate by converting gravitational potential energy into heat. However, if the Sun continues to shine this way, it will shrink down to practically nothing and vanish from sight in 18.9 million years. The astonishing thing, which was not realized at the time Lord Kelvin wrote his articles, was the Sun's durability. It has lasted much longer than he envisioned.

The problem is much worse for a giant star that has both a larger radius and a greater luminosity. As shown by the British astronomer Arthur Stanley Eddington (1882–1944), gravitational contraction can only keep a giant star shining for no more than 100,000 years, and he therefore proposed that energy that is locked up inside the atom was a likely alternative candidate for making the stars shine (Eddington 1920).

Kelvin had assumed the Sun would be about as old as the Earth, and calculated the age of the Earth under the assumption that it began in an initially molten state and cooled from the outside in. Using the equation of heat conduction with the known conductivity of rock, he calculated that it would take about 100 million years to reach the then observed temperature gradient between the hot lower levels of mines and the cold surface of the Earth (Kelvin 1863, 1899).

The discovery of radioactivity provided an entirely new perspective on our planet's internal heat and age. Radioactive elements could heat the planet from inside, emitting energetic particles that produced a rise in internal temperature (Rutherford 1905), so the Earth's hot interior is not a result of cooling from an earlier, hotter state. Radioactivity also clocked the Earth's age, by establishing the relative amounts of radioactive parent elements and stable, non-radioactive daughters. When this ratio is combined with the known rates of radioactive decay, they indicated that the Earth is at least 2–3 billion years old (Boltwood 1907; Rutherford 1929). Modern measurements using this method establish an age of about 4.6 billion years for the Earth, and presumably for the Sun.

Moreover, in looking back at the Earth's history, we find that the Sun has been shining steadily and relentlessly for eons, with a brilliance that could not be substantially less than it is now. The radioactive clocks in rock fossils indicate, for example, that the Sun was hot enough to sustain primitive creatures on the Earth 3.4 billion years ago (Tice and Lowe 2004).

Even in the early 20th century, no one had any clue as to why the Sun, or any other star, could shine so brightly for billions of years. That understanding had to await the discovery of subatomic particles, and the realization that the Sun is composed mainly of hydrogen. Of equal importance was the fact that the center of the Sun is much hotter than an ordinary fire, enabling it to consume atomic nuclei.

8.2 How Hot is the Center of the Sun?

The most abundant atom in the Sun is hydrogen, with a single proton at its nuclear center and one electron outside of the nucleus. It is so hot within most of the Sun, except its cool outer atmosphere, that all of the protons and electrons have been liberated from their atomic bonds and move about unattached to one another.

Protons are 1,836 times more massive than electrons; therefore they dominate the gravitational effects inside the star. The temperature, $T_{C\odot}$, at the center of the Sun can be estimated by assuming that each proton down there is hot enough and moving fast enough to counteract the gravitational compression it experiences from the rest of the star. That is we can equate the thermal energy of the proton to the gravitational energy, expressed by the relation:

$$\frac{3}{2}kT_{C\odot} = \frac{Gm_P M_\odot}{R_\odot}, \tag{8.5}$$

where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, the Newtonian gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}, the mass of the proton is $m_P = 1.6726 \times 10^{-27}$ kg, the mass of the Sun $M_\odot = 1.989 \times 10^{30}$ kg and the radius of the Sun $R_\odot = 6.955 \times 10^8$ m.

Solving for the central temperature of the Sun we obtain:

$$T_{C\odot} = \frac{2Gm_P M_\odot}{3kR_\odot} \approx 1.5 \times 10^7 \text{ K}. \tag{8.6}$$

So deep down inside the Sun, within its dense central core, the protons have a temperature of 15 million K. This and other physical parameters of the Sun are given in Table 8.1.

Example: Gas pressure and mass density at the center of the Sun
The gas pressure $P_{C\odot}$ at the center of the Sun, will be the force per unit area due to the gravitational force per unit area of the material above it. To a rough approximation:

$$P_{C\odot} \approx \frac{M_\odot}{R_\odot^2}\frac{GM_\odot}{R_\odot^2} \approx \frac{GM_\odot^2}{R_\odot^4} \approx 10^{15}\text{ pascal}, \tag{8.7}$$

where the gravitational constant $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}, the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, and the Sun's radius $R_\odot = 6.955 \times 10^8$ m. A more exact calculation gives a value about ten times as large or $P_{C\odot} \approx 10^{16}$ pascal.

From the ideal gas law (Sect. 5.4):

$$P_{C\odot} = NkT_{C\odot} = \frac{\rho_{C\odot}kT_{C\odot}}{m_P}, \tag{8.8}$$

where N is the number density of protons, the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, the $\rho_{C\odot}$ is the central mass density of the Sun, and the mass of the proton $m_P = 1.6726 \times 10^{-27}$ kg. Solving for the central mass density we obtain:

$$\rho_{C\odot} = \frac{P_{C\odot}m_P}{kT_{C\odot}} \approx 10^5\text{ kg m}^{-3}, \tag{8.9}$$

using a central temperature of $T_{C\odot} \approx 1.5 \times 10^7$ K.

Such extreme central conditions were recognized more than a century ago, when Jonathan Homer Lane (1819–1880), an American astronomer and inventor working at the U.S. Patent Office, assumed that gas pressure supports the weight of the Sun (Lane 1870; Ritter 1898). Although no one knew about nuclear protons at the time, Lane's basic reasoning still applies. The hot protons move about with high speeds, frequently colliding with one another and creating the gas pressure that holds up the Sun. For the Sun, a central temperature of 15 million degrees K establishes equilibrium between the outward pressure of the moving protons and the inward gravitational pull at the Sun's center.

In any layer within the Sun, the weight of the overlying gas must be equal to the outward-pushing pressure; otherwise the Sun would expand or contract, which is not observed. At greater distances from the center, there is less overlying material to support and the compression, pressure, and temperatures are less, so the solar material becomes progressively thinner and cooler.

As one might suspect, a more massive star produces greater compression at its center, and a higher central temperature is required to hold it up. The central temperature, T_{CS}, of a star of mass, M_S. and radius, R_S, is given by:

Table 8.1 Physical properties of the Sun

$M_\odot$ = mass of Sun = 1.989×10^{30} kg
$R_\odot$ = radius of Sun = 6.955×10^{8} m
$\rho_\odot$ = mean mass density of Sun = $3M_\odot/(4\pi R_\odot^3) \approx 1{,}409$ kg m^{-3}
$\rho_{c\odot}$ = central mass density of Sun = 151,300 kg m^{-3}
$T_{C\odot}$ = central temperature of Sun = $2Gm_pM_\odot/(3kR_\odot) \approx 1.5 \times 10^{7}$ K
$V_{esc\odot}$ = escape velocity from photosphere of Sun = $(2GM_\odot/R_\odot)^{1/2} \approx 6.177 \times 10^{5}$ m s^{-1}
$D_\odot$ = 1 AU = mean Earth–Sun distance = 1.4959787×10^{11} m $\approx 1.496 \times 10^{11}$ m
$\theta_\odot = R_\odot/D_\odot$ = angular radius of Sun = 959.63″ where 1″ = 1 s of arc
(At the Sun 1 s of arc = 1″ = 7.253×10^{5} m)
$P_{r\odot}$ = sidereal rotation period of visible solar disk at the equator = 25.67 days
$V_\odot$ = rotation velocity of visible solar disk at the equator = 1,971 m s^{-1}
$f_\odot$ = solar constant = 1,361 J s^{-1}m^{-2}
$L_\odot$ = absolute luminosity of Sun = $4\pi f_\odot D_\odot^2 \approx 3.828 \times 10^{26}$ J s^{-1}
$T_\odot$ = effective temperature of visible solar disk = $[L_\odot/(4\pi\sigma R_\odot^2)]^{1/4} \approx 5{,}780$ K
$m_{v\odot}$ = apparent visual magnitude of the Sun = −26.74 mag
$m_{bol,\odot}$ = apparent bolometric magnitude of the Sun = −26.83 mag
$M_{v\odot}$ = absolute visual magnitude of the Sun = +4.83 mag
$M_{bol\odot}$ = absolute bolometric magnitude of Sun = +4.74 mag
$B_\odot$ = magnetic field strength at visible solar disk = 100–1,000 G = 0.01–0.1 T
X = mass fraction of hydrogen = 0.7154
Y = mass fraction of helium = 0.2703
Z = mass fraction of all other atoms = 0.0142
Age = 4.6×10^{9} year

$$T_{CS} = T_{C\odot}\left(\frac{M_S R_\odot}{M_\odot R_S}\right). \tag{8.10}$$

It is the moving particles inside any star, including the Sun, which holds up the star. This motion, pushing, and pressure of the particles prevent a star from collapsing under its enormous weight. What keeps the particles down there hot, to sustain their rapid motion? It is nuclear-fusion reactions in the compact, dense core of a star that energizes the particles there, sustaining their heat and making them move rapidly. Once the nuclear reactions begin, the subatomic energy that is liberated keeps the nuclei sufficiently hot to ensure continuation of the reactions.

8.3 Nuclear Fusion Reactions in the Sun's Core

8.3.1 Mass Lost is Energy Gained

The only known method for keeping the Sun shining with its present luminosity for billions of years involves nuclear fusion reactions under the intense pressures and exceptionally high temperatures at great depths within the Sun. They are termed "nuclear" because it is the interaction of atomic nuclei that powers the

Sun. In nuclear-fusion reactions, two or more atomic nuclei fuse together to produce a heavier nucleus, releasing energy, subatomic particles, and radiation. For the Sun, it is protons, the nuclei of abundant hydrogen atoms that fuse together to make the nuclei of helium atoms, the next most abundant element in the Sun.

Energy can be derived only from energy, and the source of energy in nuclear fusion is mass loss. The basic idea was provided by Albert Einstein (1879–1955) in his *Special Theory of Relativity*, which included the famous formula $E = mc^2$ for the equivalence of mass, m, and energy, E (Einstein 1905a, b, 1906, 1907). Because the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, or about 300 million meters per second, is a very large number, only a small amount of mass is needed to produce a huge amount of energy.

Important evidence for the then unknown source of the Sun's energy was being obtained at the Cavendish Laboratory at Cambridge University in England about a decade after Einstein's seminal work. Here Ernest Rutherford (1871–1937) showed that the massive nuclei of all atoms are composed of hydrogen nuclei, which he named protons. At about the same time, the chemist Francis Aston (1877–1945), also working at the Cavendish, invented the mass spectrograph and used it to show that the mass of the helium nucleus is slightly less massive, by a mere 0.7 %, than the sum of the masses of the four hydrogen nuclei, or protons, that enter into it (Aston 1919, 1920).

While Rutherford and Aston were discovering the inner secrets of the atoms, the astronomer Arthur Stanley Eddington (1882–1944), director of the nearby Cambridge Observatory, was examining the internal workings of the Sun and other stars, and reasoned that the stars are the crucibles in which the heavier elements are made from lighter ones.

Realizing that such stellar alchemy would release energy, Eddington (1920) proposed that hydrogen is transformed into helium inside stars, with the resultant mass difference released as energy to power the Sun. The mass that is lost goes into energizing the Sun and other stars. The mass difference, Δm, is converted into energy, ΔE, to power the Sun, all in accordance with Einstein's equation $\Delta E = \Delta mc^2$. Eddington rightly concluded that this could supply the Sun's current luminous output for an estimated 15 billion years.

During the ensuing decade it was realized that the lightest known element, hydrogen, is the most abundant element in the Sun, so hydrogen nuclei, or protons, must play the dominant role in nuclear reactions within our daytime star. Physicists were nevertheless convinced that protons could not react with each other inside the Sun. Even in a high-speed, head-on collision at the enormous central temperature of the Sun, two protons did not have sufficient energy to overcome their mutual electrical repulsion and merge together. The thermal velocity of the average proton at a temperature of 15 million K was far too slow, and the central temperature had to be at least a thousand times hotter to raise the average kinetic energy of the protons above the electrical barrier (Focus 8.1).

Focus 8.1 The temperatures necessary for thermonuclear reactions
Protons are positively charged, and like charges repel one another with an electrostatic force given by Coulomb's law (Coulomb 1785), discovered by the French physicist Charles Augustin de Coulomb (1736–1806). This means that there is an electrified barrier that prevents protons from becoming too close to each other.

The electrical force, F, on a charge q_1 due to the presence of another charge q_2 separated from it by a distance, D, is given by Coulomb's law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{D^2} \approx 8.9875 \times 10^9 \frac{q_1 q_2}{D^2}, \tag{8.11}$$

where $\pi \approx 3.14159$ and the electric constant $\varepsilon_0 = 8.8542 \times 10^{-12}$ F m^{-1}. A positive force implies an electrical repulsion and a negative force implies an electrical attraction.

The repelling force is proportional to the square of the electrical charge of the protons, and it is inversely proportional to their separation. Therefore, the force of repulsion between two protons becomes increasingly larger as they are brought closer together. Stated another way, the protons do not move fast enough, with enough kinetic energy, to overcome the electrical barrier and merge together.

The potential energy U_{21} at charge 1 by charge 2 is given by:

$$U_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{D}, \tag{8.12}$$

where for two protons $q_1 = q_2 = e$, the elementary charge. To determine the velocity that is just fast enough for one proton to move into another, we equate the kinetic energy of motion to the electrical potential energy, or:

$$\frac{1}{2} m_P V^2 = \frac{e^2}{4\pi\varepsilon_0 D}, \tag{8.13}$$

and solve for the velocity, V,

$$V = \left[\frac{2e^2}{4\pi\varepsilon_0 m_P D}\right]^{1/2} = 1.66 \times 10^7 \text{ m s}^{-1}, \tag{8.14}$$

where the charge of a proton is $e = 1.6022 \times 10^{-19}$ C, the mass of the proton is $m_P = 1.6726 \times 10^{-27}$ kg, and we have assumed that the separation of two protons is comparable to the size of an atomic nucleus, or $D = 10^{-15}$ m, when they touch and merge together.

The mean speed of a proton inside the Sun is determined by equating the kinetic energy of the moving proton to the thermal energy that keeps it hot (Sect. 5.2), or

$$\frac{1}{2} m_p V_{th}^2 = \frac{3}{2} kT , \tag{8.15}$$

where T is the temperature and the Boltzmann constant $k = 1.38066 \times 10^{-23}$ J K^{-1}. Solving for the thermal velocity, V_{th}:

$$V_{th} = \left[\frac{3kT}{m_P}\right]^{1/2} \approx 157\ T^{1/2}\ \text{m s}^{-1}. \tag{8.16}$$

For the center of the Sun, where the temperature $T = 15$ million K, the speed is only $V_{th} \approx 6.1 \times 10^5$ m s^{-1}, more than twenty times lower than that required to overcome the electrical repulsion between two protons. A proton would have to be at a temperature of $T \approx 10^{10}$, or 10 billion, K for the nuclear fusion to occur at the speed of 1.6×10^7 m s^{-1}.

Even at the enormous central temperature of the Sun, two protons do not seem to have enough energy to overcome their electrical repulsion and move into each other. As subsequently described in the text, when the quantum–mechanical penetration probability of a nucleus is combined with the Maxwellian distribution of particle speeds, a few protons at the higher speeds can fuse together in the center of the Sun.

As it turned out Eddington was correct, for the Russian physicist George Gamow (1904–1968) already had provided an explanation for this paradox. While at the University of Göttingen, in what is now Western Germany, Gamow showed how a subatomic particle could escape from the nucleus of a radioactive atom. He used the quantum theory of the very small, in which a subatomic particle can act like a spread-out wave with no precisely defined position, to determine the penetrability of the barrier surrounding a nucleus during radioactive decay (Gamow 1928; Gurney and Condon 1928). This tunnel effect works in the opposite way when nuclear particles merge rather than separate, and it explains why nuclei can fuse together inside a star.

With Gamow's encouragement, two young students, the English astronomer Robert d'Escourt Atkinson (1898–1982) and the Austrian physicist Fritz Houtermans (1903–1966), applied and extended his quantum-tunneling theory to the process of nuclear fusion in stars (Atkinson and Houtermans 1929). Using Gamow's penetration probability with the Maxwellian distribution of particle speeds, they showed that the fusion of light nuclei could create stellar energy in accordance with Einstein's formula connecting mass loss to energy gained.

It was immediately clear that the most effective nuclear interactions were those involving light nuclei with low charge and that only a few particles in the high-speed tail of the Maxwellian speed distribution would be able to penetrate nuclei. For this reason, nuclear reactions proceed slowly in the Sun and other stars. Atkinson and Houtermans also demonstrated that the rate of nuclear reactions substantially increases with the increasing central temperature of stars.

Atkinson then addressed this problem in far greater detail, arguing that the observed relative abundances of the elements might be explained by the synthesis of heavy nuclei from lighter ones within stars (Atkinson 1931). By this time the great stellar abundance of hydrogen had been established, and Atkinson subsequently demonstrated that the most likely nuclear reaction within stars is the collision of two protons to form a deuteron and a positron (Atkinson 1936).

The probability of penetration depends on the kinetic energy, or speed, and the electrical charges of the colliding particles. They have a greater impact when moving at faster speeds, but the electrical repulsion increases with the charge. For this reason, the lightest nuclei, the protons, are more likely to fuse together than the heavier ones, which have greater nuclear charge and mass. That is, the lightest elements carry the smallest charge, with less electrical repulsion between them, and they also move faster than heavier nuclei at a given temperature.

Even with this enhanced penetration probability, the average kinetic energy of two colliding protons is not enough for fusion to occur at the center of the Sun. However, the particles in a hot gas do not all move at the same average velocity. There is a relatively small number, in the high-speed tail of the Maxwellian speed distribution, which moves at much faster speeds, permitting fusion once the penetration probability also is considered.

The number of high-speed protons decreases exponentially with increasing speed and kinetic energy, whereas the tunneling probability increases exponentially with the energy (Fig. 8.1). In the overlap region, where the exponential decline meets the exponential rise, there are protons that can participate in the nuclear fusion reactions that make the Sun shine. Thus, protons do sometimes get close enough to move into each other and fuse together, even though their average energy is well below that required to overcome their electrical repulsion.

If two nuclei, designated by 1 and 2, undergo fusion, the nuclear reaction is written

$$1 + 2 \rightarrow 3 + 4 + Q, \tag{8.17}$$

where Q is the amount of energy released during this reaction, often given in units of MeV with 1 MeV = 1.602×10^{-13} J. The reaction rate, r_{12}, is given by

$$r_{12} = N_1 N_2 < \sigma V > \ \mathrm{m^3 s^{-1}}, \tag{8.18}$$

where N_1 and N_2 are the number densities of the two nuclei and $<\sigma V>$ denotes an averaged product of the interaction cross-section σ and the relative velocity V of the two nuclei. It takes into account both the tunneling cross section and the Maxwellian speed distribution, which depends on the temperature.

As an example, the fusion of two protons at the center of the Sun, where the temperature is 15.6 million K, has $<\sigma V> = 1.19 \times 10^{-49}$ m^3 s^{-1}, which is determined from both the theory of thermonuclear reactions and laboratory measurements of the proton–proton reaction.

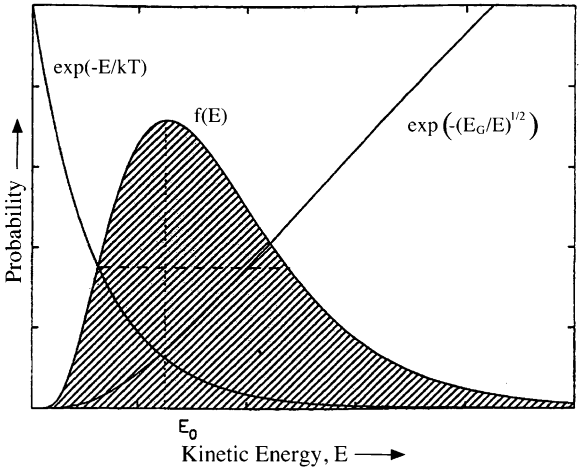

Fig. 8.1 Nuclear tunneling in the Sun's core The high-speed tail (*left*) of the Maxwellian distribution of nuclear particle speeds plotted as a function of kinetic energy, E, for protons near the center of the Sun. The P(E) function (*right*) describes the quantum mechanical probability of two protons overcoming the electrical repulsion between them; it depends on the Gamow energy E_G and the relative energy E of the two colliding protons. The shaded area (*center*) illustrates the function f(E), which is the product of the speed and penetration function; it determines the nuclear reaction rate in the core of the Sun

In stellar model calculations one uses the mass density, ρ, and the mass fraction X_i of the nuclei i, which are related to the number density N_i by:

$$N_i = \rho N_A \frac{X_i}{A_i} \tag{8.19}$$

where the Avogadro constant $N_A = 6.022141 \times 10^{23}$ mol^{-1}, and A_i is the atomic mass of nuclear species i in atomic mass units.

The mean lifetime τ_1 of nucleus 1 until destruction by fusion with nucleus 2 is:

$$\tau_1 = \frac{1}{N_2 < \sigma V >}. \tag{8.20}$$

where N_2 is the number density of nucleus 2 and $<\sigma V>$ is the average product of the interaction cross section σ and the relative velocity V of the two nuclei.

At the center of the Sun the mass density is $\rho = 1.513 \times 10^5$ kg m^{-3}, and since the mass is established by the protons with a mass $m_P = 1.6726 \times 10^{-27}$ kg, we have $N_1 = N_2 = \rho/m_P \approx 10^{32}$ m^{-3}. The mean lifetime for destruction of a proton by fusion with another proton at the center of the Sun is therefore $\tau \approx 10^{17}$ s $\approx$ 3 billion years, and since the density decreases with distance from the very center of the Sun there is plenty of time to keep the Sun shining by this reaction.

The power generated per unit mass, ε_{12}, by this reaction is given by:

$$\varepsilon_{12} = \frac{r_{12}Q}{\rho} \text{ J s}^{-1} \text{ kg}^{-1}. \tag{8.21}$$

The total energy released by the proton–proton chain of reactions that make the Sun shine is $Q \approx 26$ MeV $\approx 4 \times 10^{-12}$ J, and $\varepsilon_{12} \approx 10^{-3}$ J s^{-1} kg^{-1}. If the core of the Sun has a mass of 0.2 $M_\odot \approx 0.4 \times 10^{30}$ kg, the product of $\varepsilon_{12} \times 0.2\, M_\odot \approx 4 \times 10^{26}$ J s^{-1}, which is the present luminosity of the Sun, $L_\odot$.

Example: How frequent are proton collisions at the center of the Sun? The number density, N_P, of protons at the center of the Sun is $N_P = \rho_{C\odot}/m_P \approx 10^{32}$ m^{-3}, where the central density $\rho_{C\odot} = 1.5 \times 10^5$ kg m^{-3} and the proton mass $m_P = 1.6726 \times 10^{-27}$ kg. That is 100 million trillion trillion protons per cubic meter at the center of the Sun. The mean free path l between collisions is $l = 1/(N_P \pi R^2)$ (Sect. 5.2), the reciprocal of the product of the proton's area, πR^2 and the number density of protons. With a proton number density of 10^{32} m^{-3} a proton with a radius of 10^{-15} m will move about 0.003 m before striking another proton. The thermal velocity, $V_{th} = (3kT/m_P)^{1/2} \approx 6.1 \times 10^5$ m s^{-1} for a proton at a temperature of $T_{C\odot} = 15$ million degrees, where the Boltzmann constant $k = 1.38065 \times 10^{-23}$ J K^{-1}, so the time between collisions is $l/V_{th} \approx 5 \times 10^{-9}$ s and there are about 200 million collisions occur every second.

Despite the exceptionally large number of collisions, a fusion reaction between two colliding protons does not happen very often. The protons nearly always bounce off one another without triggering a nuclear reaction during a collision. Even with the help of tunneling, the average proton must make about 10 trillion trillion, or 10^{25}, collisions before nuclear fusion can happen. It only occurs when the collision is almost exactly head on, and between exceptionally fast protons. This explains why the Sun does not expend all of its nuclear energy at once, like an immense hydrogen bomb.

8.3.2 Understanding Thermonuclear Reactions

What is of primary interest in fueling stars is the rate at which the nuclear reactions occur and the power they generate. But before considering these details, it is useful to know the units that nuclear astrophysicists commonly use when considering thermonuclear reactions. The include:

$$\begin{aligned} \text{Size} &= 1\,\text{Fermi} = 1\ \text{fm} = 10^{-15}\ \text{m} \\ \text{Cross section} &= 1\,\text{barn} = 10^{-28}\ \text{m}^2 = 100\ \text{fm}^2 \\ \text{Energy} &= 1\,\text{MeV} = 1.60217646\text{x}\ 10^{-13}\ \text{J} = 1000\ \text{keV} \\ \text{Mass} &= 1\,\text{atomic mass unit} = \text{u} = 1.66053886 \times 10^{-27}\ \text{kg} \\ \text{Rest mass energy} &= \text{Mass} \times c^2 \end{aligned} \tag{8.22}$$

$$\begin{aligned} \text{Proton} &= m_P = 938.2720\,\text{MeV} \\ \text{Neutron} &= m_n = 939.5654\,\text{MeV} \\ \text{Helium nucleus} &= 3727.379\,\text{MeV} = \text{Alpha particle} \\ \text{Electron} &= m_e = 0.511\,\text{MeV} \\ \text{Atomic mass unit} &= \text{u} = 931.494\,\text{MeV} \\ \text{Boltzmann constant} &= k = 1.38065 \times 10^{-23}\ \text{J K}^{-1} \\ &= 8.61733 \times 10^{-11}\ \text{MeV K}^{-1} \end{aligned} \tag{8.23}$$

The mass of an atom's nucleus is, for example, always less than the sum of the individual masses of its constituent protons and neutron, or nucleons. Energy is removed to bind the subatomic nucleons together and form a nucleus, and this energy has mass. This mass is removed from the total mass of the original particles, and it is missing in the resulting nucleus. The missing mass is known as the nuclear *mass defect*, and represents the energy released when the nucleus formed. The mass defect, ΔM, for a nucleus containing A nucleons, Z protons, and A–Z neutrons is:

$$\Delta M = [Zm_p + (A - Z)\,m_n - m_{nuc}], \tag{8.24}$$

where A is the mass number of the nucleus, Z is the atomic number, m_P is the mass of the proton, m_n is the mass of the neutron, and m_{nuc} is the mass of the nucleus.

The *binding energy*, E_B, used to assemble the nucleus from its constituent nucleons is

$$E_B = \Delta M\,c^2. \tag{8.25}$$

The binding energy measures how tightly bound a nucleus is. The binding energy is the energy required to separate the nucleus into its constituent nucleons, and it also is a measure of the energy released during nuclear fusion of light nuclei into heavier ones. The binding energy released during nuclear fusion is responsible for energy production in the interior of stars.

The *binding energy per nucleon*, f, is given by $f = E_B/A$ and illustrated in Fig. 8.2. It shows that nuclei near iron have the largest nuclear binding, while lighter and heavier nuclei are less tightly bound. The graph indicates that energy can be released by combining lighter nuclei into heavier ones, known as *nuclear fusion*, provided they are less massive than iron. For example, it shows that about 7 MeV $\times$ 4 = 28 MeV in energy would be released during the fusion of four

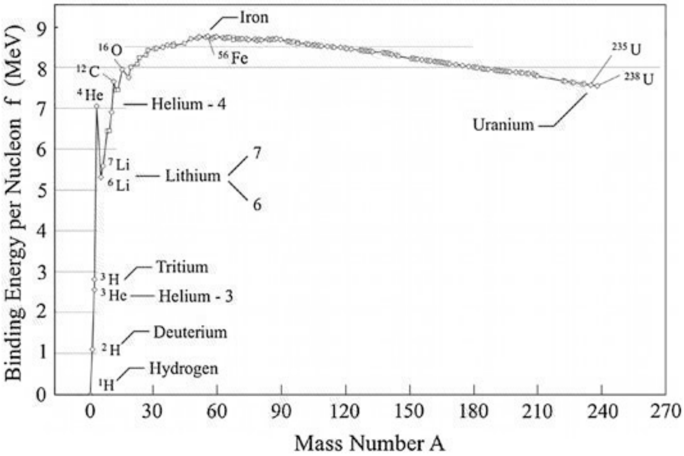

Fig. 8.2 Binding energy The binding energy per nucleon, denoted *f*, is shown as a function of atomic mass number, *A*. The highest nuclear binding energy is found near iron, denoted ^{56}Fe. Energy can be released by the nuclear fusion of lighter nuclei into heavier ones, provided they are less massive than iron. For example, the Sun shines by fusing four protons, denoted ^{1}H, into one helium nucleus, designated ^{4}He. About 7 MeV is released for each nucleon, and since four nucleons are involved the total energy release in the synthesis of one helium nucleus is about 28 MeV. Further energy is released in giant stars by the nuclear fusion of helium into carbon, denoted ^{12}C, but note that energy would not be released by fusing helium into lithium, denoted ^{6}Li or ^{7}Li, due to its lower binding energy per nucleon

protons, denoted ^{1}H, into one helium nucleus, designated ^{4}He; the exact value of the energy release is Q = 26.73 MeV.

The amount of energy released during nuclear reactions, or the Q value, results from the mass difference between the initial and final nuclei. The assumption that nuclear mass values are equal to the measured mass values usually makes little difference, for the binding energy and mass defect contribution of electrons are negligible compared to those of the nucleons. (As discussed subsequently, there is a small difference owing to the production of positrons.)

The helium is synthesized by fusing four protons together, so the fraction of mass converted during each one of these transmutations is $Q/(4m_P c^2) = 0.007$, since Q = 26.73 MeV and the energy equivalent of the mass of the proton is $m_P c^2 = 938$ MeV. Under the assumption that the Sun continues to emit its current luminosity $L_\odot$ by this process, it could last a time of $0.007\, M_\odot c^2/L_\odot = 3.2 \times 10^{18}$ s $\approx 10^{11}$ years, but since the nuclear reactions are limited to the core of the Sun, the nuclear lifetime is closer to 10^{10} years.

Example: Binding energy of a deuteron

A deuteron, denoted ^2H or ^2D, is the nucleus of the deuterium atom, and consists of one proton and one neutron. The mass of the deuterium atom, which is the mass of the deuteron for the accuracy needed, is $m_D = 2.013553$ u, and the mass of the proton and neutron are, respectively, $m_P = 1.007276$ u and $m_n = 1.008665$ u, where the atomic mass unit u $= 1.66053886 \times 10^{-27}$ kg. The mass defect $\Delta M = m_P + m_n - m_D = 0.002388$ u $= 3.9654 \times 10^{-30}$ kg, and the binding energy $E_B = \Delta M c^2 = 3.56 \times 10^{-13}$ J $= 2.224$ MeV, where the speed of light $c = 2.9979 \times 10^8$ m s^{-1} and 1 J $= 6.24150974 \times 10^{12}$ MeV.

The binding energy and binding energy per nucleon for several nuclei are given in Table 8.2.

As suggested by Table 8.2 and Fig. 8.2, the binding energy per nucleon $f = E_B/A$ for cosmically abundant elements exhibits a nearly smooth increase with increasing A up until iron, with a steady decrease beyond that. This means that binding energy is released in the fusion of very light nuclei into somewhat heavier nuclei, and that iron is the most tightly bound, abundant nucleus. The reason the trend reverses after iron is the growing positive charge of the nuclei. It means that you cannot gain energy by synthesizing elements heavier than iron inside stars. Energy can only be released from these very heavy nuclei by nuclear fission into intermediate-mass nuclei.

The calculation of thermonuclear reaction rates is enormously complex, for it depends upon the specific reaction, whether or not there is a resonance in the reaction cross section, and on accelerator measurements of the reaction, carried out for decades by William A. "Willy" Fowler (1911–1995) and his colleagues; Fowler was awarded the 1983 Nobel Prize in Physics for his theoretical and experimental studies of nuclear reactions of importance in the formation of the

Table 8.2 Binding energy, E_B, and binding energy per nucleon, $f = E_B/A$, for some nuclei of atomic mass number A[a]

Nucleus	Symbol	A	E_B (MeV)	$f = E_B/A$ (MeV)
Proton	^{1}H	1	0.0000136	0.0000136
Deuteron	^{2}H or ^{2}D	2	2.22452	1.11226
Tralphium	^{3}He	3	7.7181	2.5727
Helium	^{4}He	4	28.3007	7.0752
Carbon	^{12}C	12	92.1617	7.6801
Oxygen	^{16}O	16	127.619	7.9762
Iron	^{56}Fe	56	492.258	8.7903
Uranium	^{238}U	238	1801.7688	7.5704

[a] The number preceding the letter symbol of the nucleus is the atomic mass number A, the number of nucleons

chemical elements in the universe. The fascinating details of the calculations can be found in Clayton (1968, 1984) and Rolfs and Rodney (1988), with summaries of the various reactions in Lang (1999). The details for non-resonant reactions are discussed in Focus 8.2.

Focus 8.2 Non-resonant thermonuclear reaction rates

All the nuclei in a star are positively charged, and they must tunnel through each other's Coulomb barrier of electrical repulsion for fusion to occur, even at the high temperatures within stars. Quantum mechanical considerations show that in the absence of resonances the cross section for the tunneling, $\sigma(E)$, can be written (Gamow 1928; Gurney and Condon 1928):

$$\sigma(E) = \frac{S(E)}{E} \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right], \tag{8.26}$$

where the kinetic energy, E, of two nuclei of masses M_1 and M_2 in their center-of-mass system is:

$$E = \frac{\mu V^2}{2} = \frac{M_1 M_2}{M_1 + M_2}\left(\frac{V^2}{2}\right), \tag{8.27}$$

for a reduced mass $\mu = M_1 M_2/(M_1 + M_2)$ and relative velocity V of the two nuclei. The Gamow energy E_G is given by:

$$E_G = (\pi\alpha Z_1 Z_2)^2 2\mu c^2 = \left[0.98948 Z_1 Z_2 A^{1/2}\right]^2 \text{MeV}. \tag{8.28}$$

The fine structure constant $\alpha = e^2/(2\varepsilon_0 hc) = 7.29735 \times 10^{-3} = [137.0356]^{-1}$, for elementary charge e, electric constant ε_0, Planck constant h and speed of light c, the charges of the two nuclei, in units of the proton charge, are Z_1 and Z_2, and the reduced nuclear mass number $A = A_1 A_2/(A_1 + A_2)$, where the mass number A_1 denotes the number of nucleons, or the number of protons plus the number of neutrons, in nucleus 1. For the fusion reaction of two protons, $Z_1 = Z_2 = 1$, the $\mu = m_P/2$ for proton mass m_P, the $A = 0.5$, the proton rest energy $m_P c^2 = 938.2723$ MeV, and the Gamow energy is $E_G = (\pi/137)^2 \times 938.2723 = 0.494$ MeV ≈ 0.5 MeV $=$ 500 keV.

For most nuclear reactions in stars, the strength factor $S(E)$ is between 10 MeV-barns and 1 keV-barns, but it usually cannot be calculated from theoretical considerations. Far from a nuclear resonance, the factor $S(E)$ is a slowly varying function of E and may conveniently be expressed as a power series expansion:

$$S(E) = S_0\left[1 + \frac{S'(0)}{S_0} E + \frac{S''(0)}{2S_0} E^2\right]. \tag{8.29}$$

References to the experimental measurements of the constants in this expression are provided in Lang (1999).

For a number density N_1 and N_2 of the reacting nuclei, the reaction rate r_{12} is given by:

$$r_{12} = N_1 N_2 < \sigma V > \quad \mathrm{m}^{-3}\mathrm{s}^{-1}, \tag{8.30}$$

where the Gamow tunneling cross section is combined with the Maxwellian speed distribution at temperature, T, to obtain:

$$\langle \sigma V \rangle = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty \exp\left(\frac{-E}{kT}\right) \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right]. \tag{8.31}$$

The integration is over the function:

$$f(E) = \exp\left(\frac{-E}{kT}\right) \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right], \tag{8.32}$$

where the first term is due to the fall in the number of particles with increasing energy, found in the Maxwellian distribution, and the second term expresses the exponential rise in tunneling probability with increasing energy. The overlap of these two terms, previously shown in Fig. 8.1, defines the region of energy in which the nuclear reactions occur.

Most of the reactions take place at the effective thermal energy, E_0, or Gamow peak, given by Fowler and Hoyle (1964):

$$E_0 = \left(\frac{kT}{2}\right)^{2/3} E_G^{1/3} = 0.1220\left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3}\,\mathrm{MeV} \tag{8.33}$$

where T_9 is the temperature in billions of K, or $T_9 = T/10^9$. For light nuclei and temperatures of some tens of millions of degrees, the most effective energy E_0 is usually 10 to 30 keV. This energy is greater than $kT = 86\ T_9$ keV, reflecting the fact that the particles in the high-speed tail of the Maxwellian distribution contribute to the reactions. For the proton–proton reaction at the center of the Sun, $Z_1 = Z_2 = 1$, the $A = 0.5$, and $T_9 = 0.0156$ for a central temperature of 15.6 million K, with $kT = 1.3$ keV, so the $E_0 \approx 0.006$ MeV = 6 keV for this solar reaction.

Nuclei with energies close to E_0 and spread over the energy range ΔE_0 contribute mainly to the total rate of the thermonuclear reactions, where ΔE_0 is given by full width to half maximum of the function $f(E)$, or

$$\Delta E_0 = 4\left(\frac{E_0 kT}{3}\right)^{1/2} \approx 0.65 E_G^{1/6} (kT)^{5/6} \approx 0.237\left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6}\,\mathrm{MeV}. \tag{8.34}$$

The spread ΔE_0 is usually between 4 and 10 keV, just a bit smaller than E_0. For the proton–proton fusion reaction, the two are about equal.

The reaction rate is therefore approximated by (Burbidge et al. 1957; Fowler et al. 1975; Clayton 1968, 1984; Rolfs and Rodney 1988)

$$r_{12} = N_1 N_2 \left(\frac{2}{\mu}\right)^{1/2} \frac{S_0 \Delta E_0}{(kT)^{3/2}} \exp\left[-\frac{3E_0}{kT}\right], \tag{8.35}$$

or equivalently

$$r_{12} \approx 0.65 N_1 N_2 \left(\frac{2}{\mu}\right)^{1/2} \frac{S_0 E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]. \tag{8.36}$$

For the fusion of two protons, $S_0 = 3.89 \times 10^{-25}$ MeV barns $= 3.89 \times 10^{-53}$ MeV m^2 (Kamionkowski and Bahcall 1994) and for the Sun center where $T = 15.6$ million K, this reaction has a rate of $r_{12} \approx 10^{-49}\ N_1\ N_2\ \mathrm{m}^3\ \mathrm{s}^{-1} \approx 10^{15}\ \mathrm{m}^3\ \mathrm{s}^{-1}$, where $N_1 = N_2 \approx 10^{32}\ \mathrm{m}^{-3}$.

The energy generation, ε_{12}, or the power generated per unit mass, for a reaction that generates an energy Q is

$$\varepsilon_{12} = \frac{r_{12} Q}{\rho} \quad \mathrm{J\,s^{-1}\,kg^{-1}}, \tag{8.37}$$

where ρ is the mass density at the reaction site. In the core of the Sun, the complete chain of thermonuclear reactions releases a $Q \approx 26$ MeV $\approx 4.2 \times 10^{-12}$ J and $\varepsilon_{12} \approx 10^{-3}\ \mathrm{J\ s^{-1}\ kg^{-1}}$.

8.3.3 Hydrogen Burning

The detailed nuclear reactions inside stars could not be understood until the 1930s when several subatomic particles were known, including the neutron, discovered in 1932, the positron, detected in cosmic ray showers in 1932, and the neutrino, hypothesized in 1933.

The German physicist Carl Friedrich von Weizsäcker (1912–2007) proposed that the solution to the solar-energy problem lay in the fusion of protons, which Atkinson previously suggested (Weizsäcker 1937; Atkinson 1936). Then Gamow, who had immigrated to the United States, suggested to one of his graduate students, Charles Critchfield (1910–1994), that he calculate the details of the reaction. The results were sent to the German-born American physicist Hans A. Bethe (1906–2005) at Cornell University, who found them to be correct, and the two published a joint paper titled "The Formation of Deuterons by Proton Combination" (Bethe and Critchfield 1938).

In April 1939, Gamow, who was teaching physics at George Washington University in Washington, DC, organized a conference to bring astronomers and physicists together to discuss the problem of stellar energy generation, under the sponsorship of the Department of Terrestrial Magnetism of the Carnegie Institution. At this conference, the astronomers told the physicists what they knew about the internal constitution of stars, which was quite a bit.

By then, it was known that the lightest element, hydrogen, is by far the most abundant element in the outer atmosphere of the Sun (Unsold 1928; McCrea 1929; Russell 1929), as well as its interior (Strömgren 1931, 1932). At Gamow's conference, the Danish astronomer Bengt Strömgren (1908–1987) additionally reported that because the Sun was predominantly hydrogen, it would have a central temperature of about 15 million K rather than 40 million K, as estimated by Eddington under the assumption that the Sun had approximately the same chemical composition as the Earth, with a preponderance of heavy elements rather than hydrogen. The lower temperature meant that the calculations of Bethe and Critchfield correctly predicted the Sun's luminosity. Bethe, who attended the conference, was so stimulated by the meeting that within six months he had published a paper titled "Energy Production in Stars," which explains how the Sun fuses hydrogen into helium, releasing energy to heat the Sun's core and generate the radiation that makes it shine (Bethe 1939, 1967).

At about the same time, Weizsäcker showed how other nuclear reactions could fuel stars using carbon as a catalyst in the synthesis of helium from hydrogen, but he did not investigate the rate of energy production or its temperature dependence, which Bethe subsequently did (Bethe 1939; Weizsäcker 1938). He eventually received the Nobel Prize in Physics, in 1967, for his contributions to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars.

The sequence of nuclear reactions that make the Sun shine is called the proton–proton chain because it begins with the fusion of two protons. The complete chain of nuclear reactions also is known as the hydrogen-burning reaction – for it is hydrogen nuclei, protons, which are being consumed to make helium. However, it is not combustion in the ordinary chemical sense; in the proton–proton chain (Fig. 8.3), four protons are fused together to form a helium nucleus that contains two protons and two neutrons.

Nuclear reactions often are written in shorthand notation using letters to denote the nuclei and other subatomic particles. An arrow → specifies the reaction. Nuclei on the left side of the arrow react to form the products given on the right side. The amount of energy released during the reaction can also be given on the far right side of the arrow, and is often specified in units of MeV where $1\ \text{MeV} = 1.692 \times 10^{-13}$ J. A letter and a preceding superscript designate a nucleus. For historical reasons, the nuclei of the hydrogen isotopes ^{1}H, ^{2}H, and ^{3}H, also are named protons, deuterons and tritons, and the nucleus of ^{4}He is called an alpha particle. A Greek letter γ denotes energetic gamma-ray radiation. A positron

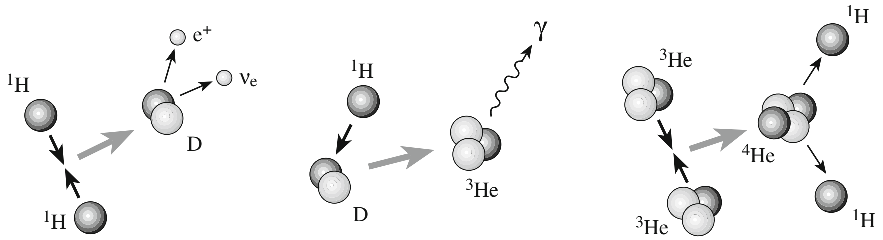

Fig. 8.3 Proton-proton chain Hydrogen nuclei, or protons, are fused together to form helium nuclei within the solar core, providing the Sun's energy. In 1939, the German-born American physicist Hans Bethe (1906–2005) described the detailed sequence of nuclear-fusion reactions, called the proton–proton chain. It begins when two protons, here designated by the letter ^{1}H, combine to form the nucleus of a deuterium atom, the deuteron that is denoted by D, together with the emission of a positron, denoted by e$^+$, and an electron neutrino, designated by ν_e. Another proton collides with the deuteron to make a nuclear isotope of helium, denoted by ^{3}He, and then a nucleus of helium, designated by ^{4}He, is formed by the fusion of two ^{3}He nuclei, returning two protons to the gas. Overall, this chain fuses four protons together to make one helium nucleus. Even in the hot, dense core of the Sun, only rare, fast-moving particles can take advantage of the tunnel effect and fuse in this way

is denoted by e^+ and an electron is designated by e^-. The symbol ν_e denotes an electron neutrino.

The superscript that precedes a nucleus letter is the mass number, A, which is the sum of the neutrons and the protons and the total number of nucleons in the nucleus. Different isotopes of an element have the same number of protons and the same letter symbol but a different number of neutrons and a different superscript. For instance, a rare isotope of helium, designated ^3He, has two protons and one neutron in its nucleus, whereas the nucleus of the common form of helium, ^4He, has two protons and two neutrons.

In the first step of the proton–proton chain, two protons, each designated by either ^1H or p, meet head on and merge into each other, tunneling through the electrical barrier separating them. The two protons combine to make a deuteron, ^2D, the nucleus of a heavy form of hydrogen known as deuterium.

Because a deuteron consists of one proton and one neutron, one of the protons must be neutralized. It is turned into a neutron, n, with the ejection of a positron, e^+, to carry away the proton's charge, and an electron neutrino, ν_e, to balance the energy in the reaction. This is the positive beta decay reaction, denoted by:

$$p \rightarrow n + e^+ + \nu_e, \tag{8.38}$$

which applies to only one of the two protons making the deuteron. The initiating proton–proton reaction that involves both protons therefore is written:

$$p + p \rightarrow {}^2D + e^+ + \nu_e, \tag{8.39}$$

which releases 0.425 MeV in energy.

Each proton inside the Sun is involved in a collision with other protons millions of times every second, but only exceptionally hot ones can tunnel through their electrical repulsion and fuse together. Only one collision in every ten trillion trillion initiates the proton–proton chain.

The electron neutrinos produced in the first step of the proton–proton chain escape from the Sun without reacting with matter, carrying energy away. However, the positron or positive electron – the anti-matter particle of the electron – is consumed immediately. Anti-matter and matter cannot coexist. As soon as any anti-matter is produced, it is immediately wiped out of existence by colliding with an electron, and the reason why we live in a material world is simply because there is more matter than anti-matter.

The positron, e^+, created in the first step of the proton–proton chain almost instantly encounters a free electron, e^-, and both become pure energy. The two subatomic particles collide and annihilate one another in a flash of radiation at gamma ray wavelengths, denoted γ. This energy-producing pair-annihilation reaction can be written symbolically as:

$$e^- + e^+ \rightarrow 2\gamma, \tag{8.40}$$

where each gamma-ray photon has an energy of 0.511 MeV, corresponding to the rest mass energy of an electron, for a total reaction energy release of 1.022 MeV.

The next step follows with little delay. In less than 1 s, the deuteron collides with another proton to form a nucleus of light helium, ^3He, and releases yet another gamma-ray photon, with about 5.49 MeV in energy. In symbolic terms, the second step of the proton–proton chain is written:

$$^2D + p \rightarrow {}^3He + \gamma. \tag{8.41}$$

This reaction occurs so easily that deuterium cannot be synthesized inside stars; it is consumed quickly to make heavier elements.

In the final part of the proton–proton chain, two such light helium nuclei meet and fuse together to form a nucleus of normal heavy helium, ^4He, and return two protons to the solar gas. This step takes about 1 million years on average and is written:

$$^3He + {}^3He \rightarrow {}^4He + 2p, \tag{8.42}$$

generating another 12.86 MeV of energy. This normal helium nucleus contains two protons and two neutrons. So, two of the protons that contributed to the formation of helium were converted into neutrons by the positive beta-decay reaction. The total energy released in the proton–proton chain is the sum of that released by all of the contributing reactions,

$$Q = 2 \times 1.022 + 2 \times 0.425 + 2 \times 5.49 + 12.86 = 26.73 \text{ MeV}. \tag{8.43}$$

This last reaction in the proton–proton chain happens 86 % of the time. Less frequent terminations involve the interaction of light helium with heavy helium to form beryllium (Focus 8.3).

Focus 8.3 Secondary nuclear fusion reactions in the Sun
The proton–proton chain will end by making beryllium about 14 % of the time. A nucleus of light helium, ^3He, will fuse with a nucleus of heavier helium, ^4He, to form a nucleus of light beryllium, ^7Be, according to the nuclear fusion reaction:

$$^3He + {}^4He \rightarrow {}^7Be + \gamma, \tag{8.44}$$

where γ denotes energetic gamma-ray radiation.

Most of the time, the light beryllium will combine with an electron, e^-, to make a nucleus of lithium, ^7Li, which then joins a proton, ^1H, to make two nuclei of heavy helium, ^4He. The reactions are (Parker et al. 1964):

$$^7Be + e^- \rightarrow {}^7Li + v_e, \tag{8.45}$$

and

$$^7Li + {}^1H \rightarrow {}^4He + {}^4He, \tag{8.46}$$

where v_e denotes an electron neutrino.

About 0.02 % of the time, the light beryllium combines with a proton to make boron, ^8B. The boron is a radioactive nucleus that decays in just one second into beryllium, ^8Be, together with the emission of a positron, e^+, and an electron neutrino. The heavy beryllium then decays to make two nuclei of heavy helium, completing the conversion of protons into helium. These secondary nuclear fusion reactions are:

$$^7Be + {}^1H \rightarrow {}^8B + \gamma \tag{8.47}$$

and

$$^8B \rightarrow {}^8Be + e^+ + v_e \tag{8.48}$$

where the positron annihilates with an electron to create a gamma ray γ, and

$$^8B \rightarrow {}^4He + {}^4He \tag{8.49}$$

A total of six protons are required to produce the two ^3He nuclei that go into this last reaction, but two protons are returned to the solar interior to be reused later. Because two protons and a helium nucleus are produced, the net result of the proton–proton chain is:

$$4p \rightarrow {}^4He + \text{gamma ray radiation} + \text{neutrinos}, \tag{8.50}$$

which releases a net energy of 26.73 MeV or 4.28×10^{-12} J each time it occurs, using 1 MeV = 1.602×10^{-13} J. That corresponds to the energy $\Delta E = \Delta mc^2$, where Δm is the mass loss in converting four protons into one helium nucleus and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The energy released in each reaction is very small, but there are a lot of them, about 10^{38} every second.

Deducting the 2 × 0.511 MeV = 1.022 MeV from the annihilation of the two pre-existing electrons by interaction with protons, we have an energy of 25.71 MeV, which corresponds to the rest mass difference between four protons and a ^{4}He nucleus. That is:

$$25.71 \text{ Mev } = (4m_P - m_{He})c^2 = 0.007\left(4m_P c^2\right), \tag{8.51}$$

where m_P denotes the proton mass, m_{He} designates the mass of a helium nucleus, the speed of light $c = 2.9979 \times 10^8$ m s^{-1}, the $m_P c^2 = 938.2720$ MeV and $m_{He}c^2 = 3727.379$ MeV. Thus, the rest-mass-to-energy conversion of the proton–proton chain is 0.007 or 0.7 %.

This energy leaves the Sun as radiation, and the part of this radiation that constitutes visible light is what makes the Sun shine. The subatomic energy that is liberated also keeps the core of the Sun hot, assuring continuation of the nuclear reactions. The time, τ, to radiate away just 10 % of the energy available from this source is:

$$\tau = \frac{0.1(0.007)M_\odot c^2}{L_\odot} \approx 3.26 \times 10^{17}\text{s} \approx 10^{10} \text{ years}, \tag{8.52}$$

where the Sun's mass $M_\odot = 1.989 \times 10^{30}$ kg, the Sun's luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, and 1 year = 3.1557×10^7 s.

Example: The Sun is losing mass

The Sun shines by the energy released, ΔE, from the mass loss, Δm, every time four protons are converted into a helium nucleus. That is, the helium nucleus is slightly less massive, by a mere 0.007, or 0.7 %, than the four protons that combine to make it, so there is an energy, ΔE, released given by:

$$\Delta E = \Delta m\, c^2 = (4m_p - m_{He})c^2 = 0.007\left(4m_p\right)c^2 \approx 4.2 \times 10^{-12} \text{ J} \tag{8.53}$$

where the mass of the proton is $m_p = 1.6726 \times 10^{-27}$ kg, the mass of the helium nucleus is $m_{He} = 6.644 \times 10^{-27}$ kg, and $c = 2.9979 \times 10^8$ m s^{-1} is the speed of light.

The number of reactions, N, that must occur every second to make the Sun shine with its present luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}, is:

$$N = \frac{L_\odot}{\Delta E} \approx 10^{38} \text{ reactions per second}, \tag{8.54}$$

and the mass loss, ΔM, from the Sun in just one second is

$$\Delta M = \frac{\Delta m L_\odot}{\Delta E} = \frac{L_\odot}{c^2} \approx 4.26 \times 10^9 \text{ kg s}^{-1}. \quad (8.55)$$

Since 1 ton is equal to 1,000 kg, about 4 million tons of matter disappears from the Sun every second, vanishing to provide the Sun's energy. Provided the Sun has been shining at the same rate ever since it formed 4.6 billion years ago, it has lost about 6.2×10^{26} kg, using 1 year = 3.1557 $\times 10^7$ s. However, this mass loss is trivial compared to the Sun's total mass of $M_\odot = 1.989 \times 10^{30}$ kg. The Sun has lost only 0.0003 of its original mass in all that time.

The time, t, required to consume the entire solar mass this way would be:

$$t = \frac{M_\odot}{\Delta M} = 0.47 \times 10^{21} \text{ s} \approx 10^{13} \text{ years}, \quad (8.56)$$

but since the nuclear reactions are confined to the hot, central core, the hydrogen is depleted in a lifetime of about one-tenth this value.

8.3.4 Why Doesn't the Sun Blow Up?

Unlike a nuclear bomb on the Earth, the temperature-sensitive reactions inside the Sun act like a thermostat, releasing energy in a steady, controlled manner at exactly the rate needed to keep the Sun in equilibrium between the inward pull of gravity and the outward pressure of the moving subatomic particles. If the rate of the thermonuclear reactions in the central regions of the Sun rises as the result of a temperature increase, nuclei move faster and create more pressure. The entire body of the Sun then would expand and thereby bring down its central temperature. If the rate of core nuclear reactions were to drop, gravity would pull the Sun inwards, and the resulting increase in central temperature would bring the rate of energy production back into equilibrium. So the pendulum continues to swing between gravity and fusion, with no winner. That is how the Sun harnesses its nuclear energy, which it has been doing for 4.6 billion years.

8.4 The Mystery of Solar Neutrinos

8.4.1 The Elusive Neutrino

Neutrinos, or "little neutral ones," are tiny, invisible packets of energy with no electrical charge and almost no mass, traveling at nearly the speed of light (see Sect. 7.4). They are produced in great profusion by thermonuclear reactions in the

Sun's core, removing substantial amounts of energy that is never seen again. Every second, trillions upon trillions of the solar neutrinos pass right through the Earth without even noticing that it is there (Focus 8.4). At night, the solar neutrinos travel through the Earth before passing through the walls of our houses and even through our bodies, without us ever noticing them.

Focus 8.4 Trillions upon trillions of neutrinos

The aggregate number of solar neutrinos is staggering. Every time the proton–proton chain creates one helium nucleus, it releases an energy, $\Delta E = \Delta mc^2 = 0.007\ (4m_p)\ c^2 = 4.2 \times 10^{-12}$ J, where Δm is the mass difference between the helium nucleus and the four protons that went into making it. The mass of the proton is $m_p = 1.6726 \times 10^{-27}$ kg, and the speed of light $c = 2.9979 \times 10^8$ m s^{-1}. The energy released by the Sun's ongoing nuclear fusion reactions works its way out of the Sun to provide its present luminosity $L_\odot = 3.828 \times 10^{26}$ J s^{-1}. The total number of helium-producing proton–proton chains required to fuel the Sun's energy every second is $L_\odot / \Delta E \approx 10^{38}$, and since two neutrinos are emitted every time one helium nucleus is made, we conclude that the Sun emits 2×10^{38} neutrinos every second. If the Sun has been shining at the same rate for the past 4.6 billion years, it has emitted an astonishing 3×10^{55} neutrinos.

The number of neutrinos passing through the Earth each second is less than those emitted by the Sun, diminished by the ratio of the Earth's cross sectional area to the area of a sphere with a radius equal to the mean distance from the Sun to Earth. Thus, the number of neutrinos passing through Earth per second is:

$$\left(\frac{2L_\odot}{\Delta E}\right)\left(\frac{\pi R_E^2}{4\pi (D_\odot)^2}\right) \approx 10^{29}, \tag{8.57}$$

where the radius of the Earth is $R_E = 6.378 \times 10^6$ m and the mean distance between the Earth and the Sun is $D_\odot = 1$ AU $= 1.496 \times 10^{11}$ m.

So, there are 0.1 million trillion trillion neutrinos passing through the Earth every second. To obtain the number of neutrinos passing through every square meter of the side of the Earth facing the Sun, just divide by the Earth's area πR_E^2 to get about 7×10^{14}, or 700 thousand billion, neutrinos per square meter.

The flux of solar neutrinos expected at the Earth is calculated using super-computers that culminate in the Standard Solar Model that describes the Sun's luminous output, size, and mass at its present age. Such calculations have been developed and refined by John N. Bahcall (1934–2005) of the Institute for Advanced Study at Princeton (Bahcall 1964, 1978; Bahcall and

Pinsonneault 2004), and other astrophysicists throughout the world, such as Sylvaine Turck-Chièze (1951–) at Saclay, France (Turck-Chièze et al. 1988).

The computer models always include three basic assumptions:

(1) Energy is generated by hydrogen-burning reactions in the central core of the Sun, and there is no mixing of material between the core and overlying regions. The nuclear reaction rates depend on the density, temperature, and composition of the core, as well as coefficients extrapolated from laboratory experiments.
(2) The outward thermal pressure, due to the energy-producing reactions, just balances the inward pressure due to gravity, thereby keeping the Sun from either collapsing or blowing up.
(3) Energy is transported from the deep interior to the visible solar disk via radiation and convection (see the next Sect. 8.5). The great bulk of energy is carried by radiative transport with an opacity determined from atomic physics calculations.

One begins with a newly formed Sun having a uniform composition, and an element abundance that is observed in the visible solar disk today. The model then imitates the evolution of the Sun to its present age of 4.6 billion years by slowly converting hydrogen into helium within the model core. The central nuclear reactions supply both the radiated luminosity and the local heat or pressure, while also creating neutrinos and producing composition changes in the core. The Sun's current luminosity, size, and neutrino flux are obtained after 4.6 billion years when about 37 % of the hydrogen in the core has now been transformed into helium. Once the Standard Solar Model has specified the neutrino flux the predictions are extended to specific experiments that detect neutrinos of different energies.

8.4.2 Solar Neutrino Detectors Buried Deep Underground

Unlike a conventional optical telescope, which is placed as high as possible to minimize distortion by the Earth's obscuring atmosphere, a solar neutrino detector is buried beneath a mountain or deep within the Earth's rocks inside mines. This shields the instrument from deceptive signals caused by cosmic rays. There, beneath tons of rock that only a neutrino can penetrate, detectors unambiguously measure neutrinos from the Sun. If neutrino detectors were placed on the Earth's surface, they would detect high-energy particles and radiation produced by cosmic rays interacting with the Earth's atmosphere.

Thus, solar-neutrino astronomy involves massive subterranean detectors that look right through the Earth and observe the Sun at night or day. The first such neutrino telescope, constructed in 1967 by Raymond Davis, Jr. (1914–2006), was a 615-ton tank located 1.5 km underground in the Homestake Gold Mine near Lead, South Dakota. The huge cylindrical tank was filled with 378,000 liters of cleaning fluid, technically called perchloroethylene or "perc" in the dry-cleaning trade;

each molecule of the stain remover consists of two carbon atoms and four chlorine atoms (Davis 1964; Davis et al. 1968).

Most solar neutrinos passed through the tank unimpeded. Occasionally, however, a neutrino scored a direct hit with the nucleus of a chlorine atom, turning one of its neutrons into a proton, emitting an electron to conserve charge, and transforming the chlorine atom into an atom of radioactive argon. The new argon atom rebounded from the encounter with sufficient energy to break free of the perc molecule and enter the surrounding liquid. Because argon is chemically inert, it can be culled from the liquid by bubbling helium gas through the tank. The number of argon atoms recovered in this way measured the incident flux of solar neutrinos.

Every few months, Davis and his colleagues flushed the tank with helium, extracting about 15 argon atoms from a tank the size of an Olympic swimming pool. That was a remarkable achievement considering that the tank contained more than 1 million trillion trillion, or 10^{30}, chlorine atoms. The scientists persisted for nearly thirty years, capturing signs of only 2,000 neutrinos in all that time. However, the consequences were enormous. The measurements implied not only that nuclear-fusion reactions indeed were providing the Sun's energy, making it shine, but there also was a small unexpected problem with the result that led to a new understanding of the physics of neutrinos.

The neutrino reaction rate with atoms in neutrino detectors is so slow that a special unit was invented to specify the experiment-specific flux. The Solar Neutrino Unit, abbreviated SNU and pronounced "snew," is equal to one neutrino interaction per second for every trillion trillion trillion, or 10^{36}, atoms. Even then, the predictions were only a few SNU per month for even the largest, most-massive, detectors first constructed.

The Homestake detector always yielded results in conflict with the most accurate theoretical calculations. The final experiment value was 2.55 ± 0.25 SNU, where the $\pm$ value denotes an uncertainty of one standard deviation (Cleveland et al. 1998). (A standard deviation is a statistical measurement of the uncertainty of a measurement; a definite detection must be above three standard deviations and preferably above five.) In contrast, theoretical results using the Standard Solar Model predicted that the Homestake detector should have observed a flux of 8.5 ± 1.8 SNU. So the tank full of cleaning fluid captured almost one third of the expected number of neutrinos (Fig. 8.4).

The discrepancy between the observed and calculated values is known as the Solar Neutrino Problem. Its significance was confirmed in 2002, when Davis received the Nobel Prize in Physics; sharing it with Japanese scientist Masatoshi Koshiba (1926–), whose group used another giant, underground detector, named Kamiokande, to detect both solar and supernova neutrinos.

In 1987, the Kamiokande neutrino detector began to monitor solar neutrinos, confirming the neutrino deficit observed by Davis. This second experiment, located in a mine at Kamioka, Japan, consisted of a 4,500-ton, or 4.5 million-liter, tank of pure water. Nearly 1,000 light detectors were placed in the tank walls to measure signals emitted by electrons knocked free from water molecules by passing neutrinos.

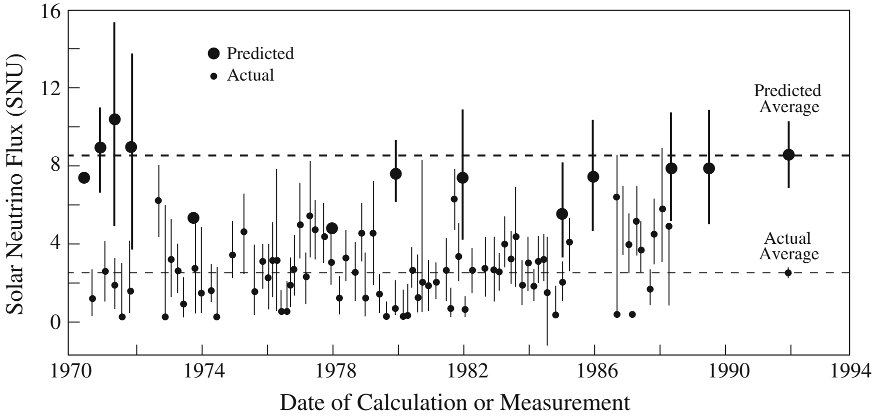

Fig. 8.4 Solar neutrino fluxes Calculated and measured solar neutrino fluxes have disagreed for several decades. The fluxes are measured in solar neutrino units (SNU) which are defined as 1 neutrino interaction per trillion trillion trillion, or 10^{36}, atoms per second. Measurements from the chlorine neutrino detector (*small dots*) give an average solar neutrino flux of 2.6 ± 0.2 SNU (*lower broken line*), well below theoretical calculations (*large dots*) that predict a flux of 8.5 ± 1.8 SNU (*upper broken line*) for the Standard Solar Model. Other experiments also have observed a deficit of solar neutrinos, suggesting that either some process prevents neutrinos from being detected or there is an incomplete understanding of the nuclear processes that make the Sun shine

When an energetic solar neutrino collides with an electron in the water, the neutrino knocks the electron out of its atomic orbit, pushes it forward in the direction of the incident neutrino, and accelerates the electron to nearly the speed of light. In water, the electron moves faster than the light it radiates, and as a result the electron produces a cone-shaped pulse of light about its path. The faint blue glow is technically known as Cherenkov radiation, named after the Soviet physicist Pavel A. Cherenkov (1904–1958) who discovered the effect (Cherenkov 1937).

The axis of the light cone gives the electron's direction, which is the direction from which the neutrino arrived. Because the observed electrons were preferentially scattered along the direction of an imaginary line joining the Earth to the Sun, the Kamiokande water experiment also confirmed that the neutrinos indeed are produced by nuclear reactions in the Sun's core. After 1000 days of observation, Yoji Totsuka (1942–2008), speaking for the Kamiokande collaboration led by Masatoshi Koshiba (1926–), could therefore report that neutrinos really are coming for the Sun, where nuclear fusion reactions are taking place, while also confirming the neutrino deficit observed by Davis (Totsuka 1991).

The deficit of solar neutrinos was subsequently confirmed in the 1990s by two massive underground detectors using gallium, a rare and expensive metal used in the red lights of hand calculators and other pieces of electronic equipment. When combined with the chlorine and the Kamiokande results, all four experiments seemed to have confirmed that solar neutrinos are missing, and that the Solar Neutrino Problem is real.

8.4.3 Solving the Solar Neutrino Problem

After almost 40 years of meticulous measurements and calculations, the neutrino count still came up short! Massive underground detectors always observed fewer neutrinos than theory states they should detect, and many scientists thought that the discrepancy was due to an incomplete understanding of neutrinos (Bahcall and Bethe 1990).

As it turned out, neutrinos are transforming into a different form during their journey from the center of the Sun, escaping detection by changing character. We haven't mentioned it yet, but scientists have learned that there are three separate types, or flavors, of neutrinos, each named after the fundamental, subatomic particle with which it is most likely to interact (Focus 8.5). All of the neutrinos generated inside the Sun are electron neutrinos, designated ν_e; this is the type that interacts with electrons, denoted e^-. The other two types, the muon neutrino, ν_μ, and the tau neutrino, ν_τ, interact with muons, μ, and tau particles, τ, respectively.

Focus 8.5 Leptons

A lepton is an elementary, subatomic particle, whose name comes from the Greek word *lepton* meaning "fine, small, thin, subatomic or slender." Altogether there are six types of leptons, which are divided into two classes: the three charged leptons and the three neutral, or uncharged leptons, known as neutrinos.

The electron, denoted by e, is the best-known lepton and the first to be discovered. Unlike the stable electron, the other two charged leptons, the muon denoted by μ and the tau particle designated τ, are unstable subatomic particles. They are unfamiliar to most of us because they die shortly after birth. The muon decays into an electron, a muon neutrino and an electron anti-neutrino in just 2 millionths, or 2×10^{-6}, of a second, and the tau particle disappears just three-tenths of a million-millionth, or 3×10^{-13}, of a second after it is made.

Every charged lepton has a corresponding antiparticle of opposite charge but equal mass. The antiparticle of the electron is thus known as the positron, or positive electron. The lepton of negative charge is denoted with a – superscript, as e^-, μ^-, and τ^-, and the corresponding antiparticle with the positive charge is designated by a + subscript, with e^+, μ^+, and τ^+.

The three electrically neutral leptons, or neutrinos, have a small but non-zero mass, and rarely interact with anything. Only the weak subatomic force affects them, and this weak interaction enables them to travel great distances through matter without being affected by it. The three types, or flavors, of neutrinos are the electron neutrino, designated ν_e, the muon neutrino, denoted by ν_μ, and the tau neutrino, denoted ν_τ. Each type also has a corresponding antiparticle, called an antineutrino and denoted by a bar above the symbol, or by $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$.

Neutrinos apparently have an identity crisis! Each type of neutrino is not completely distinct, and the different types can be transformed into one another. In the language of quantum mechanics, neutrinos do not occupy a well-defined state; they instead consist of a combination or mixture of states. As neutrinos move through space, the states come in and out of phase with one another, so the neutrinos change form with time.

The effect is called *neutrino oscillation* because the probability of metamorphosis between neutrino types has a sinusoidal, in and out, oscillating dependence on path length. The change in identity is not one way, for a neutrino of one type can change into another kind of neutrino and back again as it moves along. The three possible types of neutrinos are called electron neutrinos, muon neutrinos, and tau neutrinos, each named for the type of particle it interacts with.

In 1967, the Italian atomic physicist Bruno Pontecorvo (1913–1993) proposed that one type of neutrino might transform, or oscillate, into another type in the vacuum of space, and two years later, Pontecorvo and Vladimir Gribov (1930–1997), proposed that the Solar Neutrino Problem could be explained if solar neutrinos switch from electron neutrinos to another type as they travel in the near vacuum of space from the Sun to Earth, thereby escaping detection (Gribov and Pontecorvo 1969). Almost a decade later, the American physicist Lincoln Wolfenstein (1923–) showed that the neutrinos could oscillate, or change type, more vigorously by interacting with matter, rather than in a vacuum (Wolfenstein 1978), and the Russian physicists, Stanislav P. Mikheyev (1940–) and Alexei Y. Smirnov (1951–) subsequently explained how the matter oscillations might explain the Solar Neutrino Problem (Mikheyev and Smirnov 1985).

The theory, named the MSW effect after the first letters of the last names of the scientists who developed it, proposed that the electron neutrinos generated in the solar core could change type on their way out of the Sun, and therefore remain invisible to the first solar neutrino detectors.

Such a transformation was suggested first by observations of nonsolar neutrinos using the Super-Kamiokande detector, which replaced the older, nearby Kamiokande instrument in 1996. Super-Kamiokande can observe both solar electron neutrinos and atmospheric muon neutrinos. The former are created by nuclear fusion at the center of the Sun, whereas the latter are created when fast-moving cosmic rays enter the Earth's atmosphere from outer space. Solar electron neutrinos are distinguished by their relatively low energy, near the 5 MeV lower threshold of the detector. A high energy of 1,000 MeV is typical of an atmospheric muon neutrino. Neutrinos of higher energy produce a tighter cone of light, so a solar electron neutrino makes a fuzzy, blurred and ragged light pattern, while an atmospheric muon neutrino produces a neat, sharp-edged ring of light.

After monitoring light patterns for more than 500 days, the Super-Kamiokande scientists reported that there were roughly twice as many muon neutrinos coming from the atmosphere directly over the Super-Kamiokande detector than those coming from the other side of the Earth (Fukada et al. 1998a, b). The muon neutrinos are produced in the atmosphere above every place on our planet, but some of them apparently disappeared while traveling through the Earth to arrive at the detector from below.

Subsequent experiments using neutrinos generated by particle accelerators on the Earth confirmed the effect (Eguci et al. 2003). They suggest that although all the neutrinos produced by nuclear reactions in the Sun are electron neutrinos, they do not stay that way. Nevertheless, the terrestrial neutrinos did not come from the Sun and are not directly related to nuclear fusion reactions there. So the solution to the Solar Neutrino Problem was not known definitely until 2001, when a new underground solar neutrino detector in Canada, the Sudbury Neutrino Observatory, demonstrated that solar neutrinos are changing type when traveling to the Earth.

The Sudbury Neutrino Observatory, abbreviated SNO and pronounced "snow", is located 2 km underground in a working nickel mine near Sudbury Ontario. It is a water detector, but unlike Kamiokande or Super-Kamiokande, the SNO detector contains heavy water.

Heavy water is chemically similar to ordinary water, and it does not appear or taste any different. In fact, heavy water exists naturally as a constituent of ordinary tap or lake water in a ratio of about 1 part in 7,000; expensive chemical and physical processes can separate it.

The hydrogen in heavy water has a nucleus, called a deuteron, which consists of a proton and a neutron. For ordinary water, the hydrogen is about half as light, with a nucleus that contains only a proton and no neutron. It is the heavier deuteron that makes SNO sensitive to not just one type of neutrino but instead to all three known varieties.

One thousand tons, or 1 million liters, of heavy water, was placed in a central spherical cistern with transparent acrylic walls. A geodesic array of about 10,000 photo-multiplier tubes surrounds the vessel to detect the flash of light given off by heavy water when it is hit by a neutrino. Both the light sensors and the central tank are enveloped by a 7,800-ton jacket of ordinary water (Fig. 8.5), to shield the heavy water from emissions of the underground rocks. As with other neutrino detectors, the overlying rock blocks energetic particles generated by cosmic rays.

The Sudbury Neutrino Observatory can be operated in two modes: one sensitive only to electron neutrinos and the other equally sensitive to all three types of neutrinos. Observations with both modes have confirmed that the Solar Neutrino Problem is caused by changes in the neutrinos as they travel from the solar core. When this is taken into account, the total number of electron neutrinos produced in the Sun is as predicted (McDonald 2005). Haxton et al. (2013) have provided a recent review of solar neutrinos.

8.5 How the Energy Gets Out

All of the Sun's nuclear energy is created deep down inside its high-temperature core, and no energy is created in the cooler regions outside of it. The energy-generating core extends to about one quarter of the distance from the center of the

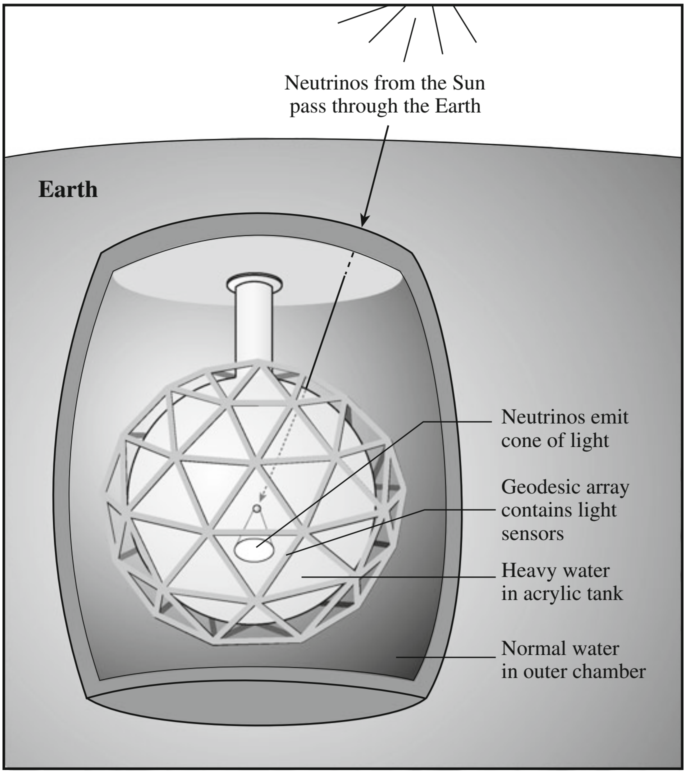

Fig. 8.5 How Sudbury works Neutrinos from the Sun travel through more than 2 km of rock, entering the acrylic tank of the Sudbury Neutrino Observatory, which contains 1,000 tons (1 million liters) of heavy water. When one of these neutrinos interacts with a water molecule, it produces a flash of light that is detected by a geodesic array of photo-multiplier tubes. Some 7,800 tons (7.8 million liters) of ordinary water surrounding the acrylic tank blocks radiation from the rock, and the overlying rock blocks energetic particles generated by cosmic rays in our atmosphere. The heavy water is sensitive to all three types of neutrinos

Sun to the visible solar disk, accounting for only 1.6 % of the Sun's volume. However, about half of the Sun's mass is packed into its dense core.

Because we cannot see inside the Sun, astronomers combine basic theoretical equations, such as those for equilibrium and energy generation or transport, with observed boundary conditions, such as the Sun's mass and luminous output, to create models of the Sun's internal structure. These models consist of two nested spherical shells that surround the hot, dense core (Fig. 8.6).

The innermost shell, called the radiative zone, extends from the core to 71.3 % of the Sun's radius. As the name implies, energy moves through this region by radiation. The outermost layer is known as the convective zone, where energy is transported in a churning, wheeling motion called convection.

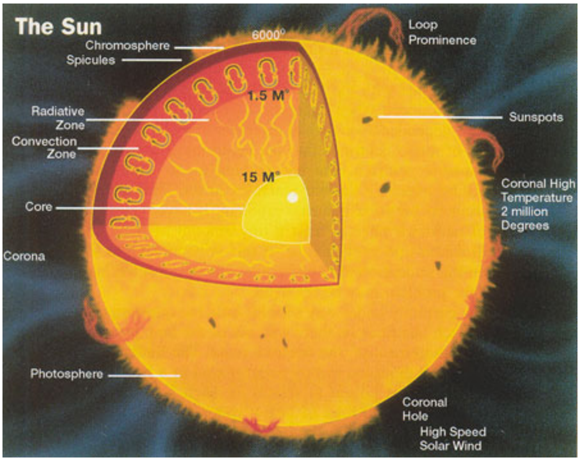

Fig. 8.6 Anatomy of the Sun The Sun is an incandescent ball of ionized gas powered by the fusion of hydrogen in its core. As shown in this interior cross-section, energy produced by nuclear fusion is transported outward, first by countless absorptions and emissions within the radiative zone and then by convection. The visible disk of the Sun, called the *photosphere*, contains dark sunspots, which are Earth-sized regions of intense magnetic fields. A transparent atmosphere envelops the photosphere, including the low-lying chromosphere with its jet-like spicules and the 1 million-degree corona that contains holes with open magnetic fields, the source of the high-speed solar wind. Loops of closed magnetic fields constrain and suspend the hot million-degree gas within coronal loops and cooler material in prominences

Radiation does not move quickly through the solar interior. A single gamma ray produced by nuclear fusion in the core of the Sun cannot move even a fraction of a millimeter before encountering a subatomic particle, where the radiation is scattered or absorbed and reemitted with less energy. This radiation quickly interacts with another particle in the radiative zone and is eventually reradiated at yet lower energy. The process continues again countless times as the radiation moves outward on a haphazard, zigzag path, steadily losing photon energy at each encounter.

Example: Scattering of radiation inside the Sun

Free electrons, which are not attached to atoms, scatter radiation with a Thomson scattering cross section, σ_T, given by Thomson (1903) (Sect. 2.7):

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{8\pi}{3}\left[\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right]^2 = 6.6525 \times 10^{-29}\ \text{m}^2, \tag{8.58}$$

where $r_e = 2.8179 \times 10^{-15}$ m is the classical electron radius, $e = 1.6022 \times 10^{-19}$ C is the fundamental unit of charge, $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1} is the electric constant, and $c = 2.9989 \times 10^{8}$ m s^{-1} is the speed of light.

A radiation photon moving into plasma with an electron number density of N_e will travel a mean free path, l, before encountering an electron, where (Clausius 1858) (Sect. 5.2)

$$l = \frac{1}{N_e \sigma_T}. \tag{8.59}$$

Hydrogen is by far the most abundant element in the Sun, and throughout the solar core and radiation zone it is completely ionized into free protons and free electrons. In the solar core, the mass density $\rho \approx 1.5 \times 10^5$ kg m^{-3}, and since it is the protons that contribute the vast majority of this mass the proton number density is $N_P = \rho / m_P \approx 10^{32}$ m^{-3}, where the proton mass $m_P = 1.6726 \times 10^{-27}$ kg. (The proton is 1836 times more massive than the electron, so we can ignore the electrons' contribution to the mass.) That is, there are 100 million trillion trillion protons per cubic meter at the center of the Sun, so it is not surprising that they don't move very far before colliding with one another. Since each former hydrogen atom contained one proton and one electron $N_P = N_e$. Within the core, a photon will only travel a length $l = 1/(N_e \sigma_T) \approx 1.5 \times 10^{-4}$ m before encountering an electron. Further out, the mass density is lower, but even at the mean solar mass density of 1,409 kg m^{-3}, we still obtain average values of $N_e \approx 8.4 \times 10^{29}$ m^{-3} and $l = 0.018$ m. If the radiation was always headed straight out of the Sun, it would make about 2×10^{10}, or 20 billion, collisions in working its way across the radiation zone, whose thickness is about half the solar radius, or about 3.5×10^8 m.

Each photon will, however, be scattered in a random direction, which will generally be different from the outward direction. The decrease in temperature with increasing distance from the Sun center still assures that more radiation moves outward than inward, just as heat normally flows from a hotter region to a colder one. The radiation therefore follows the path of least resistance, heading for regions of lower density and temperature.

The total time for the radiation to diffuse through the radiation zone is a random walk problem, with a lengthening step, or path length, at larger distances from the solar center. The photon spends most of its time close to the core where the mean free path is shortest, and the average step length required to reach the inner edge of the convective zone is $l = 9.0 \times 10^{-4}$ m. The diffusion time for the radiation to move from the bottom to the top of the radiative zone is about 170,000 years (Mitalas and Sills 1992).

As a result of this continued ricocheting and innumerable collisions in the radiative zone, it takes about 170,000 years, on average, for radiation to work its way out from the Sun's core to the bottom of the convective zone (Mitalas and Sills 1992), where the temperature has become cool enough for heavy nuclei to capture electrons and form atoms that absorb radiation. These atoms block the outward flow of radiation like dirt on a window, and the radiation heats the bottom of the convective zone.

This material becomes hotter than it otherwise would be, and it must find a way to release the pent-up energy. In response to heating from below, gases in the bottom layer of the convective zone expand, thereby becoming less dense than the gas in the overlying layers. Due to its low density, the heated material rises to the visible solar disk in about 10 days and then cools by radiation. The cooled gas then sinks because it is denser than the hotter gas, only to be reheated and rise again (Focus 8.6). Such convective motions can occur whenever a layer of fluid is heated from below (Jeffreys 1926), as in a kettle of boiling water or a simmering pot of oatmeal, with hot rising bubbles and cooler sinking material.

Focus 8.6 Convection

Convection is a method of transferring heat from hotter to cooler regions within a gas or liquid, and when it occurs in a star, energy is transported within the stellar interior by a wheeling gas motion. When a region of high density is displaced upward into a region of lower density and pressure, convection will take place if the displaced volume expands and becomes less dense than its surroundings. It will then continue to be buoyed up like a balloon or bubbles in a boiling pot of water.

For an adiabatic expansion in which no heat is exchanged with the new surroundings, convection will occur if the structural temperature gradient of the star is greater than the adiabatic gradient, or when (Schwarzschild 1906):

$$\left(\frac{dT(r)}{dr}\right) > \frac{\gamma - 1}{\gamma}\frac{T(r)}{P(r)}\frac{dP}{dr}, \tag{8.60}$$

where the adiabatic index $\gamma = 5/3$ for ionized hydrogen, $T(r)$ is the gas temperature at radius r, the dT/dr is the gas temperature gradient in the radial, r, direction, and $P(r)$ is the gas pressure at radius r given by the ideal gas law, in which $P = \text{constant} \times \rho T$, for a mass density ρ.

The structural temperature gradient in a star is given by Eddington (1917)

$$\frac{dT(r)}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2}, \tag{8.61}$$

where the radiation constant $a = 7.5657 \times 10^{-16}$ J m^{-3} K^{-4}, the Rosseland mean opacity is κ, and $L(r)$ is the radiation luminosity at radius r.

If the temperature in a star falls fast enough with increasing radius, convection sets in. This happens in the outer, cooler layers of stars like the Sun, where the opacity from heavy elements becomes high enough to produce a steep temperature gradient.

In stars that are more massive than the Sun, the nuclear fusion of hydrogen into helium occurs by the CNO cycle rather than the proton–proton chain (see Sect. 10.3), and the CNO process is a very sensitive function of temperature. As a result, the centers of these stars are very hot, but the temperature falls off rapidly with distance from the center and convection occurs in the stellar core.

Chandrasekhar (1961) has written a comprehensive book that includes convective instability of a layer of gas heated from below. The conditions for the instability can be expressed in terms of a Rayleigh number (Rayleigh 1916). Convective energy transport in stars is described by the mixing-length theory of convection (Böhm-Vitense 1953, 1958), Schwarzschild (1958), and standard textbooks of stellar astrophysics). Galloway and Weiss (1981), Wilson (1966, 1978), and Spiegel (1971) have also discussed convection in stars.

The convective zone is capped by the photosphere, the place where the gaseous material changes from being completely opaque to being transparent to radiation. In the photosphere, a process of absorption and reemission of radiation carries the Sun's energy out. Rupert Wildt (1905–1976) explained the detailed observations of sunlight by showing that both hydrogen atoms and negative hydrogen ions absorb radiation in the photosphere (Wildt 1939). Collisions between unionized, or neutral, hydrogen atoms and free electrons lead to the formation of the negative hydrogen ions. Despite their low concentration, they provide the absorption and extra opacity needed to account for the sunlight that escapes from the photosphere.

As first noticed by William Herschel (1738–1822), the photosphere contains a fine granular pattern (Herschel 1801). These closely packed granulation cells now can be examined using high-resolution images taken from ground-based telescopes under conditions of excellent observation (Fig. 8.7) or from spacecraft located outside of the Earth's obscuring atmosphere. The images reveal a host of granules with bright centers surrounded by dark lanes, exhibiting a non-stationary, overturning motion caused by the underlying convection.

The bright center of each granule, or convection cell, is the highest point of a rising column of hot gas. The dark edges of each granule are the cooled gas, which sinks because it is denser than the hotter gas. Each individual granule lasts only about 15 min before it is replaced by another one, never reappearing in precisely the same location.

The mean angular distance between the bright centers of adjacent granules is about 2.0 s of arc, corresponding to about 1,500 km at the Sun. That seems very large, but an individual granule is about the smallest thing you can see on the Sun when peering through our turbulent atmosphere.

There are at least a million granules on the visible solar disk at any moment. They are constantly evolving and changing, producing a honeycomb pattern of rising and falling gas that is in constant turmoil, bubbling away and completely changing on time-scales of minutes.

The granules are superimposed on a larger cellular pattern, called the supergranulation, studied at the California Institute of Technology by Robert B.

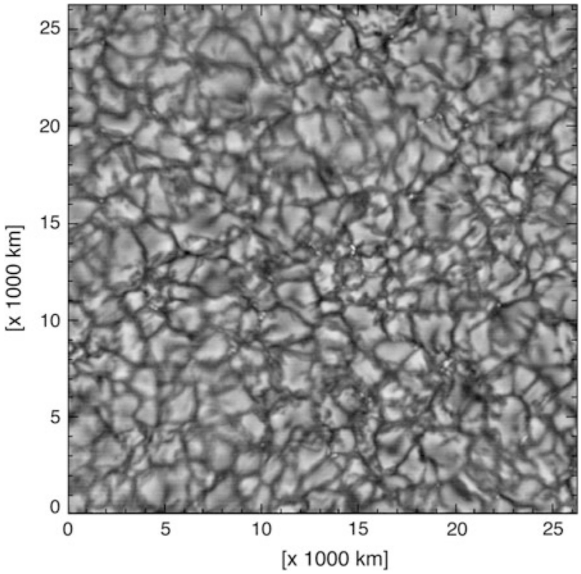

Fig. 8.7 The solar granulation Underlying convection shapes the photosphere, producing tiny, varying regions called *granules*. They are places where hot and therefore bright material reaches the visible solar disk. The largest granules are approximately 1,400 km across. They are not circular but rather angular in shape. This honeycomb pattern of rising (*bright*) and falling (*dark*) gas is in constant turmoil, completely changing on time-scales of minutes and never exactly repeating itself. This image was taken with exceptional angular resolution of 0.2 s of arc, or 150 km, at the Sun using the National Solar Observatory's Vacuum Tower Telescope at the Sacramento Peak Observatory. (Courtesy of Thomas R. Rimmele/AURA/NOAO/NSF.)

Leighton (1919–1997) and his collaborators in the early 1960s. They subtracted a long-wavelength image of the Sun from a short-wavelength one, revealing a supergranulation pattern of horizontal flow, each supergranulae is an estimated 16,000 km across, or almost three times as large as the Earth, with a lifetime of roughly 24 h. Because the motion is predominantly horizontal, the supergranules were not detected when looking directly at the center of the solar disk, but further out toward the sides of the round solar disk, where the horizontal motion is partially directed along the line of sight. Leighton (1963) has provided a review of the solar granulation.

Roughly 3,000 supergranules are seen on the visible solar disk at any moment. And like the ordinary granulation, the changing pattern of supergranulation is caused by convection. But unlike the granules, whose gases move up and down, the material in each supergranule cell rises in the center, and exhibits a sideways motion as it moves away from the center with a typical velocity of about 0.4 km s^{-1}. Only after this prolonged horizontal motion does the material eventually sink down again at the cell boundary. The supergranular flow carries the magnetic field across the photosphere, sweeping the magnetism to the edges of the supergranulation cells where it collects, strengthens and forms a network of concentrated magnetic field.

When studying the supergranulation in the 1960s, Leighton and his co-workers unexpectedly discovered vertical up and down motions in the subtracted difference between long-wavelength and short-wavelength solar images. They exhibited a periodic oscillation with a period of about five minutes (Leighton 1961; Leighton et al. 1962; Noyes and Leighton 1963). These oscillations have subsequently been used to investigate the unseen depths of the Sun (Focus 8.7).

Focus 8.7 Helioseismology

Vigorous turbulent motion in the convective zone produces sound waves (Goldreich and Kumar 1990), which drive five-minute oscillations in the overlying photosphere (Ulrich 1970; Leibacher and Stein 1971; Deubner 1975). Each five-minute period is the time it takes for the localized motion to change from moving outward to moving inward and back outward again. Such five-minute oscillations are imperceptible to the unaided eye, for the photosphere moves a mere hundred-thousandth (0.00001) times the solar radius, but they can be detected using the Doppler effect of a single absorption line formed in the photosphere. Deubner and Gough (1984) provided a review of helioseismology at that stage of its development. More recent accomplishments of helioseismology are included with references to the relevant research papers in Lang (2009).

The information obtained from oscillations produced by sound waves that traveled to various levels within the Sun can be combined to create a picture of the Sun's large-scale internal structure (Fig. 8.8). The technique is known as helioseismology, a hybrid name combining the Greek words *Helios* for the "Sun" and s*eismos* for "earthquake" or "tremor."

Observations from space, where night never falls, provide the best data for helioseismology. The *SOlar and Heliospheric Observatory*, abbreviated *SOHO*, has provided them. Instruments aboard this spacecraft have observed the solar oscillations 24 h a day, every day for more than ten years. By considering a sequence of waves with longer and longer wavelengths, that penetrate deeper and deeper, the radial profile of the sound speed has been determined and used to establish the lower boundary of the convective zone, at a radius of 71.3 % of the radius of the Sun.

Rotation imparts a clear signature to the oscillation periods, lengthening them in one direction and shortening them in the other. These opposite effects make the oscillation periods divide, and such rotational splitting depends on both depth and latitude within the Sun. The helioseismological observations indicate that differential rotation, in which the equator spins faster than the poles, is preserved throughout the convective zone, but it disappears in the radiative zone that rotates at one speed (Sect. 4.3).

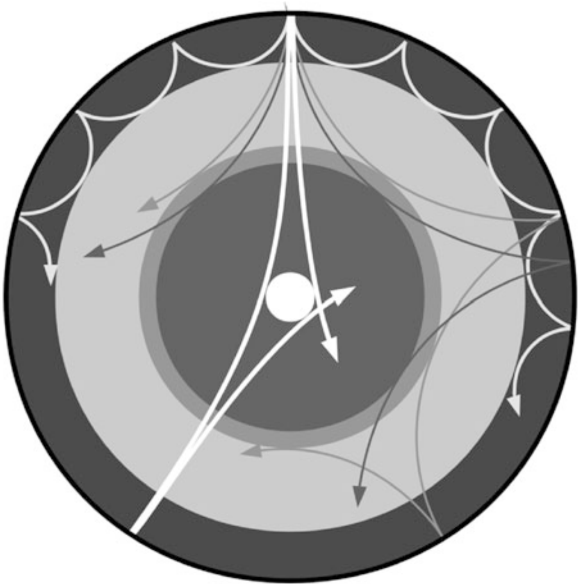

Fig. 8.8 Sound paths in the Sun The trajectories of sound waves are shown in a cross section of the solar interior. The rays are bent inside the Sun, like light within the lens of an eye. They circle the solar interior in spherical shells called *resonant cavities*. Each shell is bounded at the top by a large, rapid density drop near the photosphere and bounded at the bottom at an inner turning point where the bending rays undergo total internal refraction due to the increase in sound speed with depth inside the Sun. (From "The Life and Death of Stars" by Kenneth R. Lang, published by Cambridge University Press, 2013. Reprinted with permission.)

8.6 The Faint-Young-Sun Paradox

The Sun has grown slowly in luminous intensity since it formed; with a steady, inexorable brightening that is a consequence of the increasing amount of helium accumulating in the Sun's core. As the hydrogen in the Sun's center slowly depletes, and is steadily replaced by heavier helium, the core must continue producing enough pressure to prevent the Sun from collapsing. The only way to maintain the pressure and keep supporting the weight of a heavier material is to increase the central temperature. As a result of the slow rise in temperature, the rate of nuclear fusion gradually increases and so does the Sun's luminosity. The Sun is, for example, now 30 % more luminous than it was 4.6 billion years ago.

The Sun's luminosity increases as time goes on, so of course the Sun was significantly dimmer in the remote past. Therefore, the Earth should have been noticeably colder then. However, this does not agree with geological evidence. Assuming an unchanging terrestrial atmosphere, with the same composition and reflecting properties as today, the lower solar luminosity in the past would have caused the Earth's global surface temperature to be below the freezing point of water during the planet's first 2.6 billion years. The oceans would have been frozen solid, there would have been no liquid water, and the entire planet would have been locked into a global ice age.

Yet, sedimentary rocks, which must have been deposited in liquid water, date back to a time when the Earth was less than 800 million years old. There is fossil evidence in those rocks of living things at about that time. Thus, for billions of years, the Earth's surface temperature was not very different from today; conditions have remained hospitable for life on the Earth throughout most of the planet's history.

There are several possible explanations for the discrepancy between the Earth's warm climatic record and an initially dimmer Sun, which is known as the faint-young-Sun paradox. It can be resolved if the Earth's primitive atmosphere contained about a thousand times more carbon dioxide than it does now (Sagan and Chyba 1997). Greater amounts of carbon dioxide would enable the early atmosphere to trap more solar heat near the Earth's surface, warming it by the greenhouse effect, which would prevent the oceans from freezing. Another possibility is that the Sun was more magnetically active in its youth, expelling strong winds, energetic particles, and radiation that might have kept the Earth warm (Schilling 2001; Sackmann and Boothroyd 2003; Minton and Malhotra 2007).

What about the future? In only 1 billion years the Sun will have brightened by another 10 %. Calculations suggest that the Earth's oceans could evaporate then at a rapid rate, resulting in a hot, dry, uninhabitable Earth. In about 3 billion years from now, the Sun will then be hot enough to boil the Earth's oceans away, leaving the planet a burned-out cinder, a dead and sterile place.

8.7 The Sun's Destiny

The Sun cannot shine forever, because eventually it will deplete the hydrogen fuel in its core. Although it has converted only a trivial part of its original mass into energy, the Sun has processed a substantial 37 % of its core hydrogen into helium in the past 4.6 billion years. There will be no hydrogen left in the solar core about 7 billion years from now. When that hydrogen is exhausted, the central part of the Sun will undergo a slow collapse, and the gradually increasing core temperature will cause the outer layers of the Sun to expand into a red giant star, with a dramatic increase in size and a powerful rise in luminosity (Sackmann et al. 1993). Eventually the Sun will become 170 times larger and 2,300 times more luminous than it is now (Fig. 8.9). This will result in a substantial rise in temperatures throughout the solar system, becoming hot enough to melt the Earth's surface.

Meanwhile, the core of the Sun will continue to contract until the central temperature is hot enough to ignite helium – which is at about 100 million K. However, this conversion of helium into carbon will not last long compared to the Sun's 12 billion years of hydrogen burning. In about 35 million years, the core helium will have been used up and there will be no heat left to hold up the Sun. In

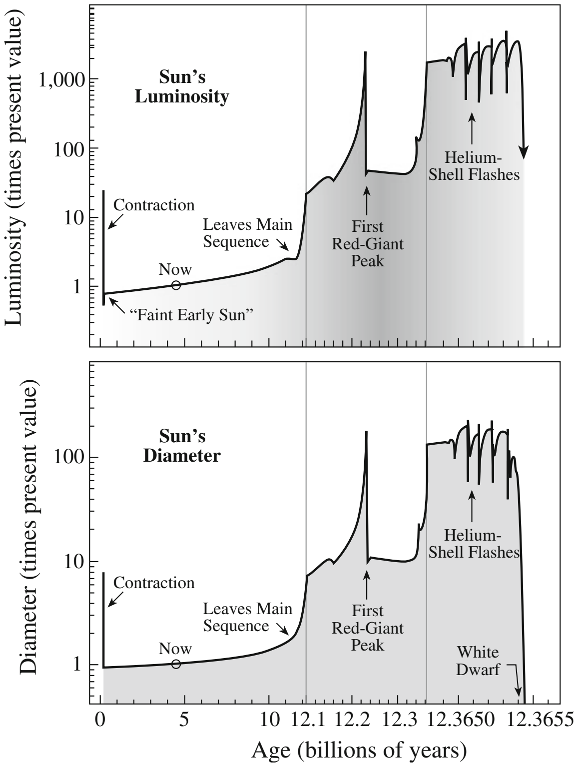

Fig. 8.9 The Sun's fate In about 7 billion years, the Sun will become much brighter (*top*) and larger (*bottom*). The time-scale is expanded near the end of the Sun's life to show relatively rapid changes. (Courtesy of I-Juliana Sackmann and Arnold I. Boothroyd.)

a last spurt of activity, the Sun will shed the outer layers of gas to produce an expanding "planetary" nebula around the star, and the core will collapse into a white dwarf star (see Sects. 13.1, 13.2).

By this time, the intense winds will have stripped the Sun down to about half of its original mass, and gravitational collapse will squeeze the remaining part to about the size of the present-day Earth. Nuclear reactions then will be a thing of the past, and there will be nothing left to warm the Sun or planets. The former Sun will gradually cool down and fade away, plunging all of the planets into a deep freeze.

Such events are in the very distant future, of course; but even now, the Sun threatens the Earth with its perpetually expanding atmosphere that envelops our planet and with explosive outbursts that can send energetic particles, intense radiation, and huge magnetic bubbles toward the Earth.