Chapter 10
Combination of Heat Transfer Resistances

Heat loss from a warm room through a wall to the cold outside involves three heat transfer steps in a series: (i) convection at the inside surface of the wall, (ii) conduction through the wall, and (iii) convection on the outside of the wall. Next, consider a fireplace fire. Here heat reaches the room by radiation from the flames and also by convection of moving air. These processes occur in parallel. There are many processes like these which involve a number of heat transfer steps, sometimes in a series, sometimes in parallel, sometimes in a more involved way.

To find the overall effect of a number of heat transfer steps in a series and in parallel, we draw on the analogy to electrical theory. For *processes in series* the resistances are additive; thus,

$$R_{\text{overall}} = R_1 + R_2 + R_3 \tag{10.1a}$$

or, in terms of conductances,

$$\frac{1}{C_{\text{overall}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{10.1b}$$

For *processes in parallel*, it is the conductances which are additive. Thus,

$$C_{\text{overall}} = C_1 + C_2 + C_3 \tag{10.2a}$$

or, in terms of resistances,

$$\frac{1}{R_{\text{overall}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{10.2b}$$

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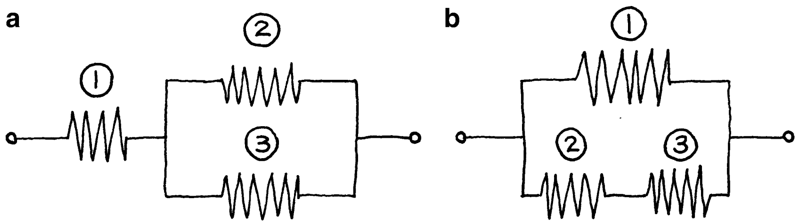

Fig. 10.1 Two series–parallel arrangements of resistances to heat transfer

In the *series–parallel situations* sketched in Fig. 10.1a, we have

$$\frac{1}{C_{\text{overall}}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} \tag{10.3}$$

and for the sketch of Fig. 10.1b,

$$C_{\text{overall}} = C_1 + \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} \tag{10.4}$$

For *processes in series* a glance at equation (10.1a and b) shows that the step with the largest resistance dominates and determines, in most part, the overall resistance. Resistances much smaller than this can be ignored.

For *processes in parallel* matters are quite different, for equation (10.2a and b) shows that the term with largest conductance (hence smallest resistance) dominates and for the most part determines the overall conductance of the process. Conductances much smaller than this can be ignored.

Whenever a particular step in the overall process dominates to the exclusion of all other steps, it is called the *rate controlling step* of the process.

In heat transfer, the individual conductances are represented by the convection coefficient h, the conductivity per unit length $k/\Delta x$, and the radiation coefficient h_r, while the overall conductance for the process is represented by the overall heat transfer coefficient, U. It is this overall transfer coefficient which is of prime importance in heat exchanger design. We now show the form of the overall transfer coefficient U for a few representative situations. The problems at the end of the chapter present other situations.

10.1 Fluid–Fluid Heat Transfer Through a Wall

As shown in Fig. 10.2 the wall material and both liquid films contribute resistance to heat transfer. Thus, we have

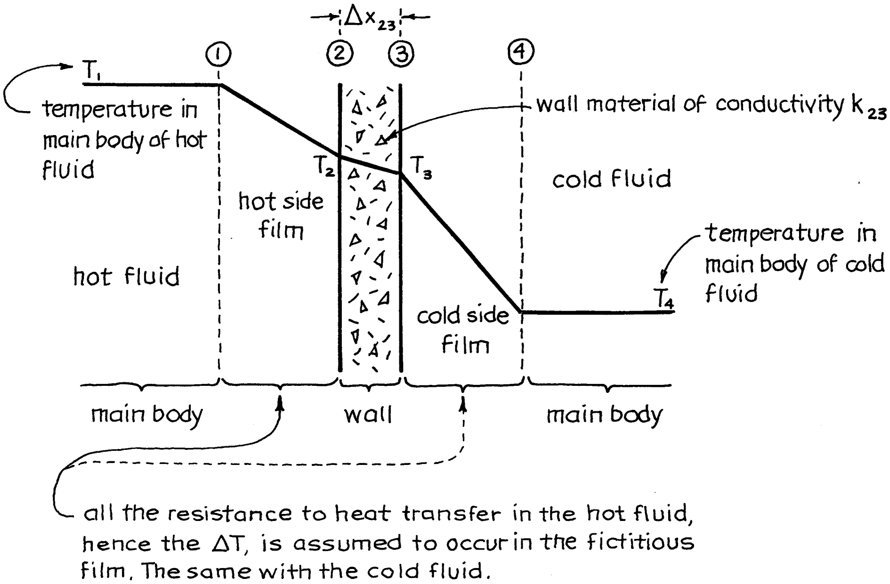

Fig. 10.2 Heat transfer from fluid to fluid that are separated by a wall

$$\dot{q} = h_{12}A(T_1 - T_2)$$
$$\dot{q} = \frac{k_{23}A}{\Delta x_{23}}(T_2 - T_3)$$
$$\dot{q} = h_{34}A(T_3 - T_4)$$

Combining and eliminating the intermediate temperatures T_2 and T_3 gives

$$\dot{q} = -UA\Delta T \quad \text{where} \; \frac{1}{U} = \frac{1}{h_{12}} + \frac{\Delta x_{23}}{k_{23}} + \frac{1}{h_{34}} \tag{10.5}$$

More generally, if there are scale deposits on the surfaces of the separating wall, these deposits represent two more resistances in the series, as shown in Fig. 10.3.

In this case we have

$$\frac{1}{U} = \frac{1}{h_2} + \frac{\Delta x_{23}}{k_{23}} + \frac{\Delta x_{34}}{k_{34}} + \frac{\Delta x_{45}}{k_{45}} + \frac{1}{h_{56}} \quad \left[\frac{m^2 \cdot K}{W}\right] \tag{10.6}$$

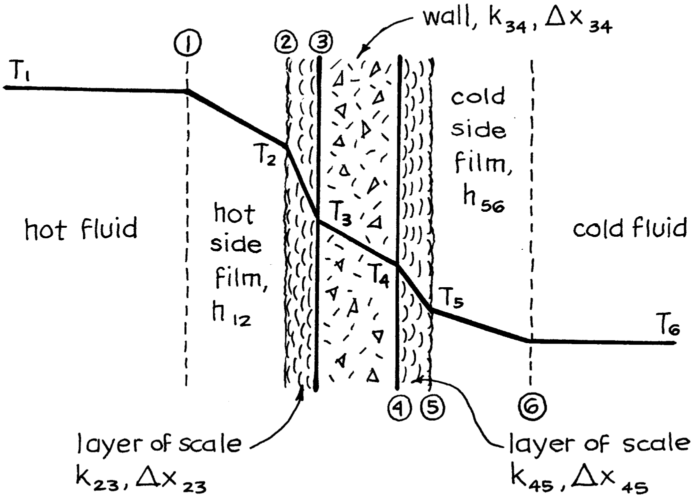

Fig. 10.3 Heat transfer across a flat wall which has scale deposits

Note that equations (10.5) and (10.6) are of the form of equation (10.1a and b) which represents resistances in a series. Thus, if any particular resistance step is very much larger than all the others (low h or low $k/\Delta x$), it will dominate and determine the overall resistance of the process.

10.2 Fluid–Fluid Transfer Through a Cylindrical Pipe Wall

Consider heat transfer from hot fluid at T_1 to cold fluid at T_6 across a pipe with thin scale coatings (thus, $A_2 \cong A_3$ and $A_4 \cong A_5$) as shown in Fig. 10.4.

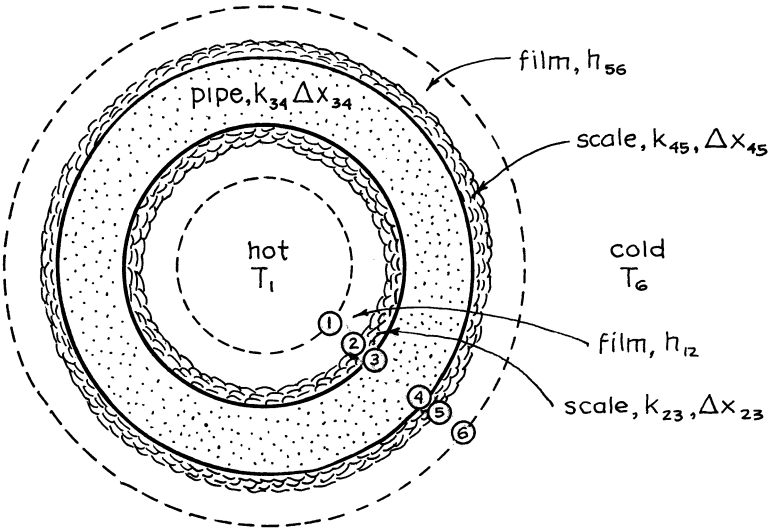

Fig. 10.4 Heat transfer across a scale-coated pipe wall

Noting that all the resistances occur in a series, we can show that

$$q = -UA\Delta T_{\text{overall}} = U_3A_3(T_1 - T_6) = U_4A_4(T_1 - T_6) \quad [\text{W}]$$

with

$$\frac{1}{U_3A_3} = \frac{1}{U_4A_4} = \frac{1}{h_{12}A_3} + \underbrace{\frac{\Delta x_{23}}{k_{23}A_3}}_{\text{Scale}} + \underbrace{\frac{\Delta x_{34}}{k_{34}A_{34,lm}}}_{\text{Wall}} + \underbrace{\frac{\Delta x_{45}}{k_{45}A_4}}_{\text{Scale}} + \frac{1}{h_{56}A_4} \quad \left[\frac{\text{K}}{\text{W}}\right] \tag{10.7}$$

where

$$A_{34,lm} = \frac{A_4 - A_3}{\ln\frac{A_4}{A_3}} \cong \frac{A_4 + A_3}{2} \quad \text{if} \frac{A_4}{A_3} < 2$$

Note that the value of U obtained depends on the area basis chosen, whether it is the inside area or the outside area of the pipe.

10.3 Conduction Across a Wall Followed by Convection and Radiation

Heat transfer by conduction or convection is proportional to the temperature difference of the two locations, while radiation transfer is proportional to the difference of T^4. Thus, the latter is very much more temperature sensitive and should dominate at high temperatures, but be negligible at low temperatures.

To be able to assess the relative contributions of the various mechanisms and to be able to combine them, we need to express them in the same measure. We can cast the radiation transfer in terms of a heat transfer coefficient if we put

$$q = h_r A_1 \; T_1 - T_2 \; = \sigma A_1 \underline{\mathscr{F}_{12} \; T_1^4 - T_2^4}$$

Or the appropriate expression from Chapter 9

Thus, the radiation heat transfer coefficient is

$$h_r = \frac{\sigma \mathscr{F}_{12}\left(T_1^4 - T_2^4\right)}{T_1 - T_2} \tag{10.8}$$

Now consider the conduction of heat through a slab followed by convection to a fluid at T_3 and radiation to a facing surface at T_3, as sketched in Fig. 10.5. This is a series–parallel situation as shown in equation (10.3), or

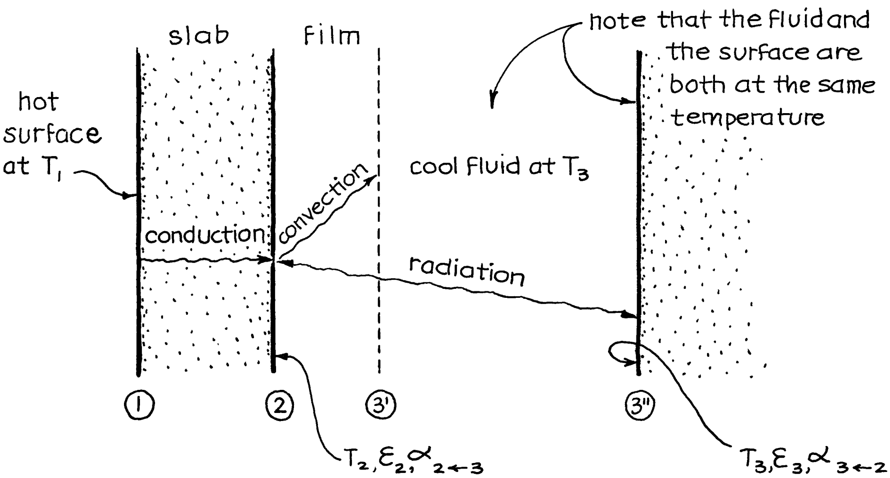

Fig. 10.5 Conduction through a wall followed by convection and radiation in parallel

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_{12}} + \frac{1}{C_{23,\text{conv}} + C_{23,\text{rad}}}$$

In heat transfer language this combination of conductances gives

$$\left.\begin{aligned} &\dot{q}_{13} = -UA\Delta T = UA(T_1 - T_3) \quad [\text{W}] \\ &\text{where} \\ &\frac{1}{U} = \frac{\Delta x_{12}}{k_{12}} + \frac{1}{h_{23,\text{conv}} + h_{23,\text{rad}}} \quad \left[\frac{\text{m}^2 \cdot \text{K}}{\text{W}}\right] \\ &\text{and where, from equation (9.67) for parallel facing source and sink,} \\ &h_{23,\text{rad}} = \frac{\sigma\left[\alpha_{3\leftarrow 2}\varepsilon_2 T_2^4 - \alpha_{2\leftarrow 3}\varepsilon_3 T_3^4\right]}{(\alpha_{2\leftarrow 3} + \alpha_{3\leftarrow 2} - \alpha_{3\leftarrow 2}\alpha_{2\leftarrow 3})(T_2 - T_3)} \end{aligned}\right\} \quad (10.9)$$

10.4 Convection and Radiation to Two Different Temperature Sinks

Now consider a more complex case where heat is lost from surface 1 by convection to fluid at T_2, but also by radiation through the transparent fluid to a parallel surface at T_3, as shown in Fig. 10.6.

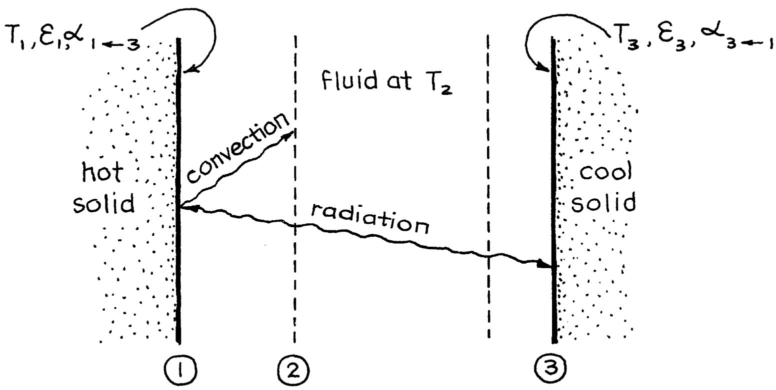

Fig. 10.6 Heat transfer to two different temperature sinks

The heat lost by surface 1 is given by

$$\begin{aligned}\dot{q}_1 &= \dot{q}_{12,\mathrm{conv}} + \dot{q}_{13,\mathrm{rad}} \\ &= h_{12,\mathrm{conv}} A(T_1 - T_2) + h_{13,\mathrm{rad}} A(T_1 - T_3)\end{aligned}$$

where, from equation (9.67),

$$h_{13,\mathrm{rad}} = \frac{\sigma A\left[\alpha_{3\leftarrow 1}\varepsilon_1 T_1^4 - \alpha_{1\leftarrow 3}\varepsilon_3 T_3^4\right]}{(\alpha_{1\leftarrow 3} + \alpha_{3\leftarrow 1} - \alpha_{1\leftarrow 3}\alpha_{3\leftarrow 1})(T_1 - T_3)}$$

In terms of $T_1 - T_2$, we get

$$\dot{q}_{1\rightarrow} = U_{12} A(T_1 - T_2) \quad \text{where } U_{12} = h_c + h_r\left(\frac{T_1 - T_3}{T_1 - T_2}\right) \tag{10.10}$$

or, in terms of $T_1 - T_3$,

$$\dot{q}_{1\leftarrow} = U_{13} A(T_1 - T_3) \quad \text{where } U_{13} = h_c\left(\frac{T_1 - T_2}{T_1 - T_3}\right) + h_r \tag{10.11}$$

Note that the temperature ratio appears in an h term whenever that h term is based on one ΔT, while the U and other h term are based on another ΔT.

10.5 Determination of Gas Temperature

As a final example of the interaction of these different modes of heat transfer, consider the determination of the temperature of a hot gas flowing in a pipe. As shown in the sketch of Fig. 10.7, the thermocouple is protected by a shield, a common practice with corrosive gases.

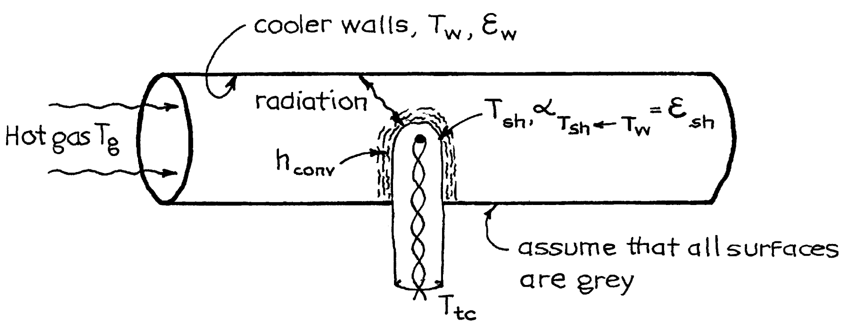

Fig. 10.7 Finding the temperature of a gas with a shielded thermocouple

First of all, no heat is lost by the thermocouple or the gas within the shield.

$$T_{tc} = T_{sh}$$

Now make a heat balance for the shield. Heat enters by convection from the hot gas; however, heat leaves by net radiation to the cooler walls. Thus, at steady state,

$$\underset{\text{to shield}}{\dot{q}_{\text{convection}}} + \underset{\text{between shield and walls}}{\dot{q}_{\text{radiation interchange}}} = 0$$

or

$$h_{\text{conv}}(T_g - T_{sh}) = \sigma\varepsilon_{sh}(T_{sh}^4 - T_w^4)$$

For gas flow outside and normal to a cylinder (for h_{conv})

For a completely enclosed greybody; use equation (9.65) (for $\sigma\varepsilon_{sh}$)

and noting that $T_{sh} = T_{tc}$

$$T_g = T_{tc} + \frac{\sigma\varepsilon_{sh}}{h_{\text{conv}}}(T_{tc}^4 - T_w^4) \tag{10.12}$$

($\sigma\varepsilon_{sh}$ in $\frac{W}{m^2K^4}$; h_{conv} in $\frac{W}{m^2K}$)

The temperature of the shield (hence of thermocouple) will be somewhere between the gas and wall temperature. High h_{conv} and low ε_{sh} will result in the thermocouple measuring the gas temperature, while low h_{conv} and high ε_{sh} will result in the thermocouple measuring the wall temperature.

This type of radiation correction is very important at high temperatures.

10.6 Extensions

For other more complicated situations, either:

- Write out all the individual heat interchange terms and eliminate all intermediate temperatures. This procedure is illustrated in section A, above.
- Develop the electrical analog, find the overall conductance or overall resistance, and then replace with heat flow terms. This procedure is illustrated in section C, above.

Usually the latter procedure is simpler, but with more than one source and one sink, one must be careful in using this approach, as shown in section D, above.

Problems on Combining Resistances

10.1 *From battleships to skating rinks.* My neighbor can't resist auctions, and just last week he bought the World War II battleship *USS Iowa* for $277.00. He has great plans for this war relic and wants to implement them as soon as possible. One of his schemes is to build a neighborhood skating rink using ½-inch-thick steel plates instead of pipes to carry the refrigerant. Thus, the floor of the rink would consist of a double layer of steel plates a short distance apart with refrigerant flowing in between and ice above. However, it is important that no water form on the surface of the ice. With this restriction in mind, how thick should the ice layer be? The refrigerant is at – 18 °C. The air in the rink is at 25 °C.

(a) Do your first calculation ignoring heat transfer by radiation and the resistance of the steel pipe.
(b) Then include the resistance of the steel plate.

Do you feel that the resistance of the steel plate can be reasonably ignored?

10.2 *Pottery kilns.* Business is so good at *Pottery West* that the master potter plans to construct a new and larger kiln about 2 m high in which to bake his artistic creations. Estimate:

(a) The outside temperature of a vertical wall of this kiln
(b) The heat loss through this wall

Data: The inside temperature of the kiln wall will be 1,150 °C; room temperature is 20 °C. The kiln wall will be 20 cm thick, made of high-temperature firebrick ($k = 0.1$ W/m · K, $\varepsilon = 0.8$).

Note: The findings of the previous problem suggest that you should not ignore the radiation from wall to room.

10.3 *Temperature of a space voyager.* Estimate the temperature of a spherical space probe as it passes Mars on its way to the outer planets.
Data: Effective color temperature of the Sun = 6,150 K

Radius of the Sun = 695,000 km
Distance from the Sun to Earth = 148,000,000 km
Distance from the Sun to Mars = 228,000,000 km
The skin of the voyager is # 301 stainless steel

For related information, see *Science* **127** 811 (1958) and **128** 208 (1958).

10.4 *Earth's temperature.* What would be the mean temperature of Earth if it were a gray body? See previous problem for data.

10.5 *The insulation of hot-air ducts*. Energy Savers, Inc., was upset to discover that the hot-air ducts under our building are completely uninsulated—just naked shiny 300-mm tin-plated pipes ($\varepsilon = 0.05$). What a waste of energy.

They urge that we immediately insulate because each minute of delay costs us money. We could opt for their preformed pop-on foam insulation. However, for a real first-class job, they strongly recommend their patented double protection formula—a 1.6-mm layer of especially thick insulating cardboard ($k = 0.15$ W/m K), glued snugly to the pipe, and then a coating of long-lasting, nonbiodegradable, insect-repellent, low-emissivity aluminum paint ($\varepsilon = 0.55$). Though more labor intensive and more costly, we are assured that this is the best that modern technology can offer.

I suppose they are right. However, before I sign a contract, I'd like to know whether the energy saving would really be substantial. So would you please determine what fraction of the original energy loss is saved with this insulation. For these calculations, take the temperature of the pipe walls to be 75 °C and the surrounding crawl space to be at 5 °C.

A thermocouple, protected by a stainless steel shield, is inserted in an air preheater duct. At the air velocity flowing in the duct, it is estimated that $h_{\text{conv}} = 100$ W/m^2 K. Find the temperature of the hot air:

10.6 If the thermocouple reads $T_{tc} = 400$ K and if the temperature of the steel walls is $T_w = 300$ K.

10.7 If $T_{tc} = 1{,}000$ K and if $T_w = 900$ K. Note how sharply the error in T_{tc} reading increases (because of the intrusion of radiation) as the temperature level rises.

10.8 *Heat transfer to the walls of gas/solid fluidized beds*. Observation shows that part of the time (fraction δ) the hot wall of a bubbling fluidized bed is bathed by the rising gas bubbles, the rest of the time by the gas/solid emulsion. In addition, the layer of emulsion right at the wall surface has somewhat different properties (larger voidage) than the rest of the emulsion.

Let us develop a bed/wall heat transfer model based on these observations, as follows. When bubbles bathe the surface, heat flows by convection h_1 and by radiation h_2 from the hot wall directly into the cold bed. However, when the emulsion bathes the surface, things are a bit more complicated because then heat flows by convection h_3 and radiation h_4 to the first layer of the emulsion. This first layer then passes the heat into the main body of the emulsion by unsteady-state heating with mean heat transfer coefficient h_5.

With this model, develop an expression to represent the overall heat transfer coefficient h in terms of δ and the five individual coefficients.