Chapter 4
Molecular Flow

The mean free path (mfp) of molecules increases when the gas pressure is reduced, and at low enough pressure the (mfp) is so large that the molecules begin to bounce from wall to wall of the flow channel rather than collide with each other. When this happens the character of the flow changes. Thus, different flow regimes are encountered depending on the value of the ratio

$$\mathrm{Kn} = \frac{(mfp)}{d} = \frac{\text{mean free path of molecules}}{\text{diameter of flow channel}}$$

where Kn is called the Knudsen number. These flow regimes are as follows:

- *Ordinary laminar flow* ($\mathrm{Kn} \ll 1$): Here Poiseuille's law applies. Flow in this regime is based on the following two assumptions:

  (a) $\tau = (\mu)(du/dy)$ with $\mu = \text{const.}$
  (b) Velocity at the wall is zero.

- *Intermediate or slip flow regime* ($\mathrm{Kn} \cong 1$): Here assumption (b) begins to break down.
- *Molecular flow regime* ($\mathrm{Kn} \gg 1$): Here there are very few collisions between molecules. Most collisions are with the wall. So the concept of viscosity has no meaning and assumption (a) also breaks down.

The velocity profiles in these three regimes are shown in Fig. 4.1.

Now the mean free path of gas molecules varies with pressure and from kinetic theory of gases is found to be about as follows:

$$\begin{array}{ll} \text{at 1 atm}: & (mfp) = 6.8 \times 10^{-8} m \\ \text{at 1 Pa}: & (mfp) = 6.8 \times 10^{-3} m \end{array}$$

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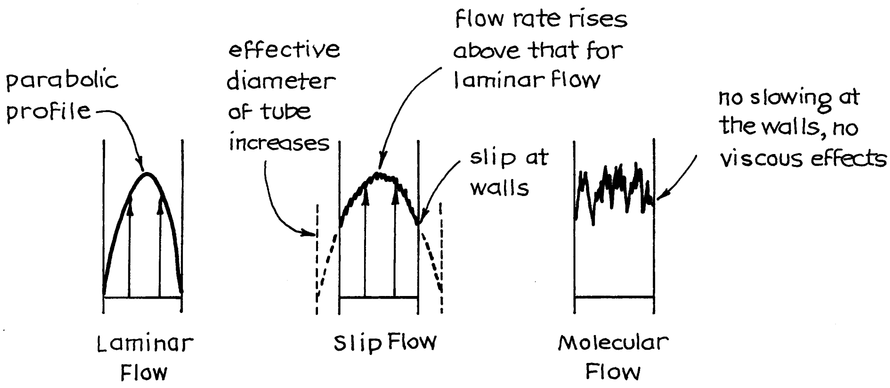


Fig. 4.1 Velocity profiles in various flow regimes

From this we have

$$\left.\begin{array}{ll}\text{Laminar flow}: & \text{when}\, pd > 0.8\,\text{Pa m} \\ \text{Molecular flow}: & \text{when}\, pd < 0.01\,\text{Pa m}\end{array}\right\} \tag{4.1}$$

In finding how fluids flow in high-vacuum systems, we may have to consider all three flow regimes. In just about all cases, our concern reduces to handling the tank–line–pump problem, such as shown in Fig. 4.2.

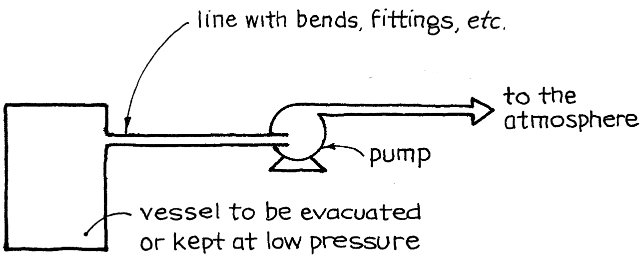


Fig. 4.2 Typical problem for vacuum systems

At one end of the system, flow may be in one regime, at the other end in another. We look at such problems. Also, in evacuating a system the pressure decreases with time, consequently the mass velocity decreases, and the Reynolds number becomes very small. Thus, the transition normally is from laminar to molecular flow, rarely from turbulent to molecular flow.

4.1 Equations for Flow, Conductance, and Pumping Speed

4.1.1 Notation

Molecular flow has its own particular and convenient notation. Let us introduce three essential terms.

1. *Flow rate*. This is measured by

$$
\begin{aligned}
Q &= \left(\frac{\text{m}^3\text{ of gas flowing if the pressure is corrected to unit pressure, or 1 Pa}}{\text{time}}\right) \\
&= p\dot{v} = \dot{n}RT = \frac{\dot{m}RT}{(mw)} = p\frac{\pi d^2}{4}u = \frac{\pi d^2}{4}\frac{GRT}{(mw)} \\
&= \frac{\pi d\,RT\mu}{4\,(mw)}(\text{Re}) \qquad \left[\frac{\text{Pa m}^3}{\text{s}} = \frac{\text{N m}}{\text{s}} = \text{W}\right]
\end{aligned}
\tag{4.2}
$$

2. *Conductance*. In a flow channel such as sketched in Fig. 4.3, the flow rate is proportional to the driving force, Δp. Thus,

$$
Q = -C_{12}\,\Delta p = \underset{\uparrow\ \mathrm{m^3/s}}{C_{12}}\,(p_1 - p_2)
\tag{4.3}
$$

where C_{12} is called the conductance between points 1 and 2 and is inversely proportional to the resistance to flow in that section of flow channel, or

$$
C_{12} \propto \frac{1}{\text{resistance}}
$$

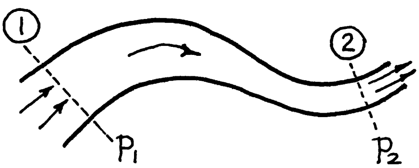


Fig. 4.3 Flow from 1 to 2 due to $p_1 > p_2$

3. *Pumping speed*. The volumetric flow rate of material across a plane normal to flow is called the pumping speed S. Thus, at planes A and B of Fig. 4.4, we have

$$
Q = S_A p_A = \underset{\uparrow\ \mathrm{m^3/s}}{S_B}\, p_B
\tag{4.4}
$$

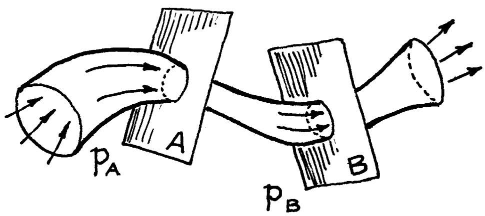


Fig. 4.4 Flow through plane A or B

Note the distinction between pumping speed and conductance. Although they have the same dimensions (m^3/s), they are different measures and should not be confused. C refers to a section of flow system, while S measures what passes across a plane normal to flow. Thus, in Fig. 4.5, C_{12} refers to the section between points 1 and 2 and S_A refers to plane A. The following sections will present equations for conductances, pumping speed, and flow rates for various sorts of equipment: pipes, orifices, pumps, and fittings.

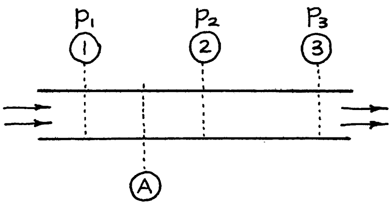


Fig. 4.5 Flow channel showing points 1, 2 and 3 and plane A

4.1.2 Laminar Flow in Pipes

In any differential section of pipe in which there is isothermal laminar flow, the mechanical energy balance of equation (1.7) becomes

$$\underbrace{\cancel{g\,dz}}_{\text{Ignore}} + \underbrace{u\,\cancel{du}}_{\text{Ignore}} + \frac{dp}{\rho} + \underbrace{\cancel{W_s}}_{\substack{=0.\\ \text{no pump}}} + \underbrace{d(\textstyle\sum F)}_{\frac{32u\mu}{d^2\rho}dL\ \text{[from equation (2.5)]}} = 0$$

Integrating and combining with equation (4.3) gives, between points 1 and 2,

$$\left.\begin{aligned}
Q_{\mathrm{lam}} &= C_{\mathrm{lam}}\,(p_1 - p_2) \qquad \left[\frac{\mathrm{Pa\,m^3}}{\mathrm{s}}\right] \\
\text{where} \quad \bar{p} &= \frac{p_1 + p_2}{2} \\
C_{\mathrm{lam}} &= \frac{\pi d^4 \bar{p}}{128\,\mu L} \;\underset{20\,^\circ\mathrm{C}}{\overset{\text{air}}{=\!=}}\; 1{,}364\,\frac{d^4 \bar{p}}{L} \\
&\phantom{=\frac{\pi d^4 \bar{p}}{128\,\mu L}}\;\underset{20\,^\circ\mathrm{C}}{\overset{\mathrm{H_2O\ vapor}}{=\!=}}\; 2{,}584\,\frac{d^4 \bar{p}}{L} \qquad \left[\frac{\mathrm{m^3}}{\mathrm{s}}\right]
\end{aligned}\right\} \tag{4.5}$$

Equation (4.5) represents laminar flow in the "language" of molecular flow. Note that it looks different from the corresponding equation of Chap. 2.

4.1.3 *Molecular Flow in Pipes*

In this regime we assume no collision between molecules; they simply float from wall to wall of the pipe. But how do molecules leave the wall? Do they bounce off the wall (elastic collision) as shown in Fig. 4.6a, or do they hesitate for a long enough time on the surface to forget the direction they originally came from (diffuse reflection) as shown in Fig. 4.6b?

Let f = fraction of molecules diffusely reflected. For these Knudsen showed that

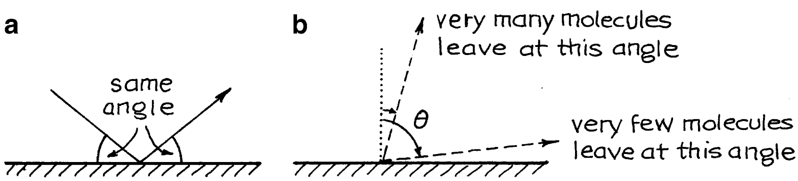


Fig. 4.6 Two types of collisions of molecules with the pipe wall: (**a**) elastic collisions (**b**) diffuse reflections

the number leaving at any particular angle is given by

$$n = k \cos\,\theta$$

Then $1 - f$ = fraction reflected or bouncing off the wall.

Very little information is available on the value of f, but roughly

$$f \cong 0.77 \text{ for copper and glass tubing}$$
$$f \cong 0.90 \text{ for iron pipe}$$

Also, f values are suspected to vary with flow regime; for example, see Fig. 4.7.

Because of the uncertainty in f value and because it is close to unity, we will assume throughout that $f = 1$. Then, on applying the kinetic theory of gases with this assumption, it can be shown that

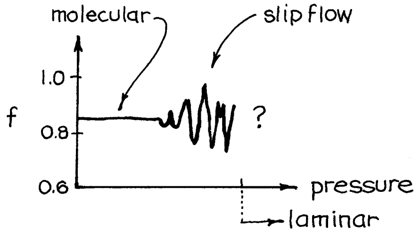


Fig. 4.7 Fraction of molecules diffusely rejected

$$Q_{\text{mol}} = C_{\text{mol}}(p_1 - p_2) \qquad \left[\frac{\text{Pa m}^3}{\text{s}}\right]$$

where

$$\left.\begin{aligned} C_{\text{mol}} = \frac{d^3}{L}\left[\frac{\pi RT}{18(mw)}\right]^{1/2} &\underset{20\,^\circ\text{C}}{\overset{\text{air}}{=\!=}} \left[\frac{\pi(8.314)293}{18(0.0289)}\right]^{\frac{1}{2}}\frac{d^3}{L} = 121.3\frac{d^3}{L} \\ &\underset{20\,^\circ\text{C}}{\overset{H_2O\text{ vapor}}{=\!=\!=}} 153.7\frac{d^3}{L} \qquad \left[\frac{\text{m}^3}{\text{s}}\right] \end{aligned}\right\} \tag{4.6}$$

4.1.4 *Intermediate or Slip Flow Regime*

If we simply add the laminar and molecular contributions to the total flow as the pressure shifts from one flow regime to the other, we find the behavior shown in Figs. 4.8 and 4.9. Actually, the observed flow in the slip flow regime is somewhat lower (at most 20 %) than the sum of the individual contributions. Since the more exact treatment of this situation would lead to complications, we will assume simply that

$$Q_{\substack{\text{total in}\\ \text{slip flow}}} = Q_{\text{mol}} + Q_{\text{lam}} \qquad \left[\frac{\text{Pa m}^3}{\text{s}} = \frac{J}{\text{s}} = W\right] \tag{4.7}$$

More precise equations for this flow regime are found in Dushman (1949).

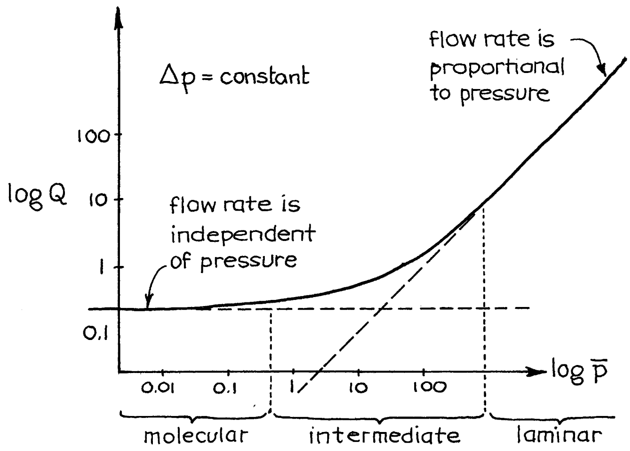


Fig. 4.8 Flow rate of a gas in a pipe for a fixed Δp between the two ends

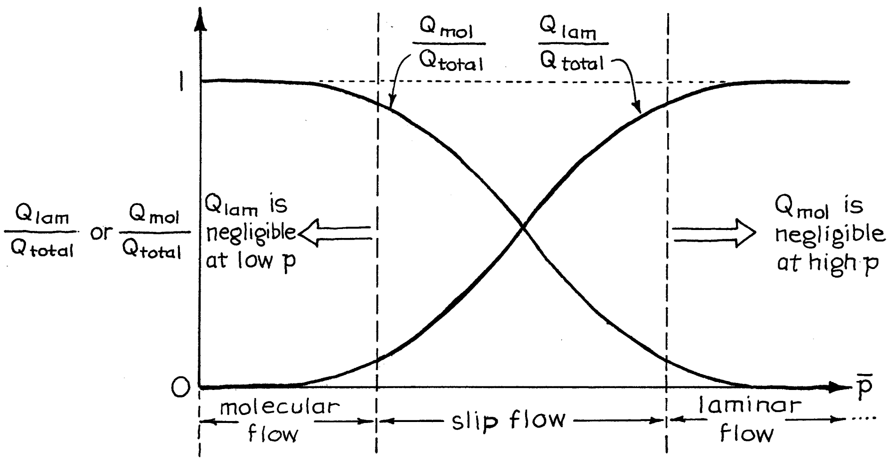


Fig. 4.9 Relative contribution of laminar and molecular mechanisms to the flow of gases in pipes

4.1.5 Orifice, Contraction, or Entrance Effect in the Molecular Flow Regime

As shown in Fig. 4.10, we have two situations here: an orifice or obstruction in a length of pipe (case A) and a smaller pipe leading from a larger pipe or a tank (case B). For both cases the kinetic theory of gases gives

$$\left.\begin{aligned}
&Q_{\text{or, mol}} = C_{\text{or,mol}}(p_1 - p_2) \quad \left[\frac{\text{Pa}\,\text{m}^3}{\text{s}}\right] \\
&\text{where} \\
&C_{\text{or, mol}} = d^2\left(\frac{D^2}{D^2 - d^2}\right)\left[\frac{\pi RT}{32(mw)}\right]^{1/2} \\
&\underset{20\,^\circ\text{C}}{\overset{\text{air}}{=\!=}} 91d^2\left(\frac{D^2}{D^2 - d^2}\right) \\
&\text{water} \\
&\underset{20\,^\circ\text{C}}{\overset{\text{vapor}}{=\!=}} 115d^2\left(\frac{D^2}{D^2 - d^2}\right) \quad \left[\frac{\text{m}^3}{\text{s}}\right]
\end{aligned}\right\} \tag{4.8}$$

1. *Equivalent length of pipe*. Comparing conductances with flow in a tube, or equation (4.8) versus equation (4.6), shows that the length of pipe which has the same resistance as the orifice or contraction is:

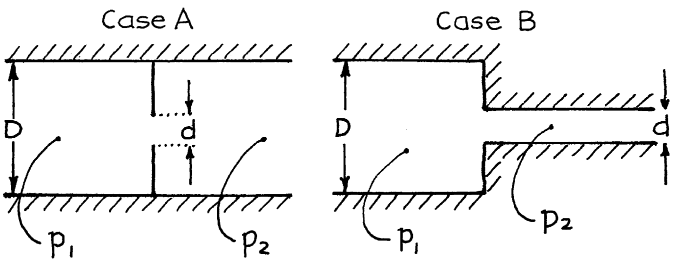


Fig. 4.10 Reduction in the area available for flow. A = orifice inside a pipe, B = reduction in pipe diameter

- For case A, in terms of a pipe of diameter D,

$$\frac{L_{\text{eq}}}{d} = \frac{4}{3}\left(\frac{D^2}{d^2} - 1\right) \tag{4.9}$$

This value of L_{eq} can be quite large. Thus, if $d = 0.1D$, then the resistance of the orifice is equivalent to the resistance of a pipe which is 132 pipe diameters long.

- For case B, in terms of the smaller following pipe of diameter d,

$$\frac{L_{\mathrm{eq}}}{d} = \frac{4}{3}\left(1 - \frac{d^2}{D^2}\right) \tag{4.10}$$

This expression shows that the resistance contributed by the contraction is equivalent to a length of about one diameter of small pipe. This is often negligible compared to the other resistances in the vacuum system.

4.1.6 Contraction in the Laminar Flow Regime

Consider laminar flow of gases, not molecular flow, at not too high velocities (not near critical flow) through a contraction going from D to d. From the values of Table 2.2, we can show that

$$\left.\begin{aligned} Q_{\mathrm{or,\ lam}} &= C_{\mathrm{or,\ lam}}(p_1 - p_2) \\ \text{where} \quad C_{\mathrm{or,\ lam}} &= \frac{\pi \bar{p}}{\rho u}\left(\frac{d^2 D^2}{D^2 - 0.8d^2}\right) \end{aligned}\right\} \tag{4.11}$$

The equivalent length of this contraction, in terms of the leaving pipe d, is then found to be

$$\frac{L_{\mathrm{eq}}}{d} = \frac{\mathrm{Re}}{160}\left(1.25 - \frac{d^2}{D^2}\right) \tag{4.12}$$

Here the equivalent length can be as much as 18 diameters of small pipe.

4.1.7 Critical Flow Through a Contraction

When the pressure ratio across a contraction is ≥ 2, the contraction behaves as a critical flow orifice. For this situation equation (3.27) can be written as

$$Q = \frac{\pi}{4} d^2 p_{\mathrm{upstream}} \left[\frac{kRT}{(mw)}\left(\frac{2}{1+k}\right)^{(k+1)/(k-1)}\right]^{1/2} \tag{4.13}$$

4.1.8 Small Leak in a Vacuum System

Suppose we have a tiny leak in a vacuum system. One may look at this in one of a number of ways, for example, as a narrow channel or as a pinch point. These two extremes are shown in Fig. 4.11. Let us estimate the leak rate for these two extremes, remembering that $p_{\text{sytem}} \ll p_{\text{surroundings}}$.

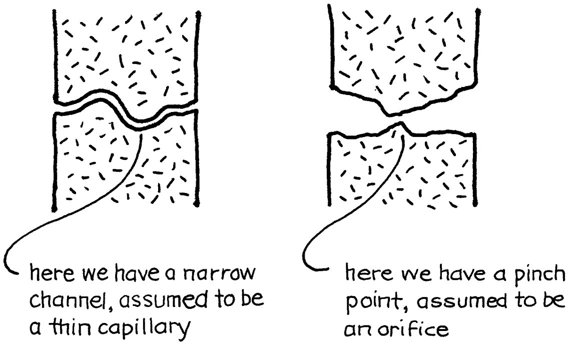


Fig. 4.11 Two ways of looking at a leak in a vacuum system

1. *Assume a capillary.* If the diameter of the capillary is small compared to the mean free path of molecules at 1 atm, then we have molecular flow of the leaking gas, and from equation (4.6),

$$Q_{\text{leak}} = Q_{\text{mol}} = \frac{d^3 p_{\text{upstream}}}{L} \left[\frac{\pi RT}{18(mw)} \right]^{1/2} \tag{4.14}$$

   However, if the diameter of the capillary is large compared to the mean free path of the molecules at 1 atm, then we have laminar flow of the leaking gas most of the way through the capillary (see Problem 4.3), in which case equation (4.5) applies. This gives

$$Q_{\text{leak}} = Q_{\text{lam}} = \frac{\pi d^4 \left(p_{\text{upstream}}^2 - p_{\text{downstream}}^2 \right)}{256 \mu L} \tag{4.15}$$

2. *Assume an orifice.* Since the pressure ratio across the orifice is many times greater than 2, we should use the critical orifice expressions for compressible flow. Thus, from equation (3.27) we have

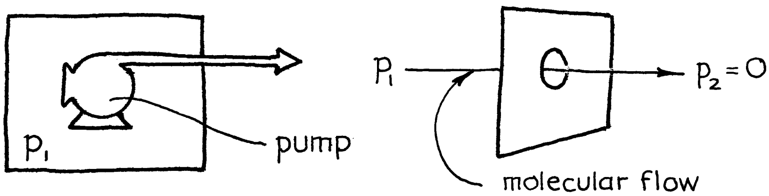
$$
\begin{aligned}
Q_{\text{leak}} = Q_{\text{crit}} &= \frac{\dot{m}RT}{(mw)} = \frac{G^{*}ART}{(mw)} \\
&= \frac{\pi}{4} d^2 p_{\text{upstream}} \left[\frac{kRT}{(mw)} \left(\frac{2}{1+k} \right)^{(k+1)/(k-1)} \right]^{1/2}
\end{aligned} \tag{4.16}
$$

Which equation you use, (4.14), (4.15), or (4.16), depends on what you know of the leak and how you view it. However, if you know nothing of the nature of the leak, assume the critical orifice. Chances are that this extreme more closely represents the leak.

4.1.9 Elbows and Valves

In the molecular flow regime and $\mathrm{Re} < 100$, the resistance of elbows and valves which have no flow restrictions is negligible. So just take the mean flow length of the fitting, bend, open valve, etc. However, if the pipe fitting or valve has a restriction, find the smallest cross section and apply equation (4.9).

4.1.10 Pumps



We define pumping speed S_p as follows

$$
\begin{aligned}
S_{p,\,\text{at}\,p_1} &= \left\{ \begin{matrix}\text{volume of gas removed,} \\ \text{measured at } p_1\end{matrix} \middle/ \text{time} \right\} \\
&= \left\{ \begin{matrix}\text{volume of gas entering the throat} \\ \text{of the pump, measured at the} \\ \text{entrance of the pump}\end{matrix} \middle/ \text{time} \right\} \\
&= \frac{Q}{p_1} \left[\frac{\mathrm{m}^3}{\mathrm{s}} \right]
\end{aligned} \tag{4.17}
$$

The maximum theoretical pumping speed can be looked upon as the flow rate into an orifice which has no back pressure or with equation (4.16)

$$S_{p,\max} = \frac{Q_{or}}{p_1} \frac{\text{air}}{20\,°\text{C}} 91d^2 \tag{4.18}$$

The speed factor of a pump is defined as follows:

$$\text{Speed factor} = \left(\frac{\text{speed of actual pump}}{\text{speed of a perfect vacuum pump}} \right) \tag{4.19}$$

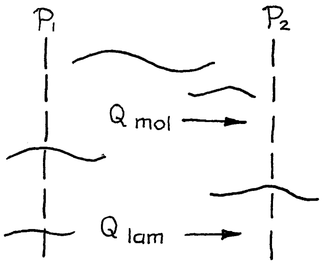
At pressures between 10^{-4} and 1 Pa, the speed factor is equal to 0.4 ~ 0.6 for an oil diffusion pump and is equal to 0.1 ~ 0.2 for a mercury vapor pump. The maximum practical speed factor of vacuum pumps $\cong$ 0.4.

4.2 Calculation Method for Piping Systems

Suppose we have the piping system shown in Fig. 4.12. Its flow resistances, included in this figure, consist of a rather complex series–parallel combination. Let us see how to evaluate the overall resistance to flow for systems of this kind.

1. For *resistances in parallel*, an example being molecular and laminar flow, we write

$$Q_{mol} = C_{mol}(p_1 - p_2)$$
$$Q_{lam} = C_{lam}(p_1 - p_2)$$



Adding these flow contributions gives

$$(Q_{mol} + Q_{lam}) = (C_{mol} + C_{lam})(p_1 - p_2)$$

or

$$Q_{tot} = C_{tot}(p_1 - p_2) \tag{4.20}$$

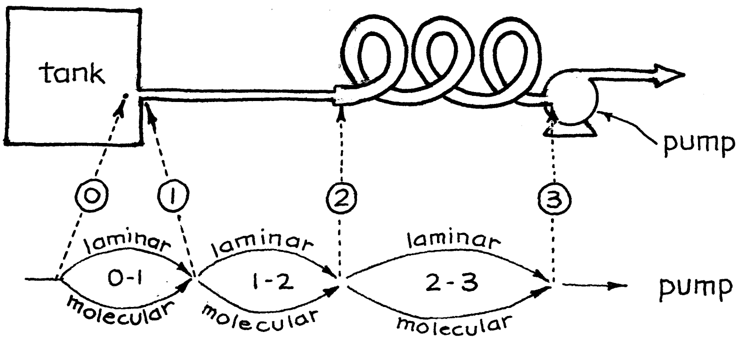


Fig. 4.12 A piping system and its corresponding series–parallel resistances

2. For *resistances in series*, noting that the flow rate is the same for each section, we can write

$$-Q = C_{12}\,\Delta\,p_{12} = C_{23}\,\Delta\,p_{23}$$

or

$$Q = C_{12}\,(p_1 - p_2) = C_{23}\,(p_2 - p_3)$$

Eliminating the intermediate pressure p_2, we find

$$Q = \frac{1}{(1/C_{12}) + (1/C_{23})}(p_1 - p_3) = C_{\text{tot}}\,(p_1 - p_3) \tag{4.21}$$

Extending this procedure to any number of regions in series is direct, and generalization to any arrangement of series and parallel resistance is not too difficult. For example, for the tank–line–pump system of Fig. 4.13, a frequently met situation, we have

$$Q = C_{01}(p_0 - p_1) = C_{12}(p_1 - p_2) = S_2 p_2$$

Combining these expressions and eliminating intermediate partial pressures p_1 and p_2 gives

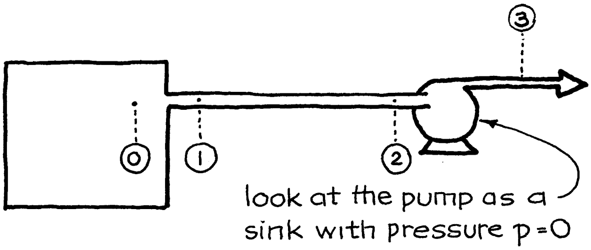


Fig. 4.13 Tank-line-pump series system

Fictitious total conductance between 0 and 3 where $p_3 = 0$.
It is in fact the pumping speed at the tank, or S_0.

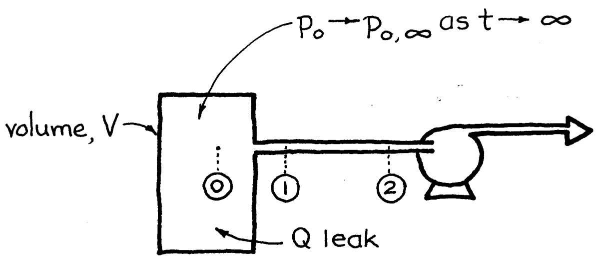
$$
\left.
\begin{aligned}
Q &= C_{\text{tot}}\ \ p_0 - p_3 \ = S_0 p_0 \\
&= \frac{1}{\dfrac{1}{C_{\text{or}}} + \dfrac{1}{C_{\text{line}}} + \dfrac{1}{S_p}}\, p_0 \\
&= \frac{1}{\dfrac{1}{C_{01,\text{lam}} + C_{01,\text{mol}}} + \dfrac{1}{C_{12,\text{lam}} + C_{12,\text{mol}}} + \dfrac{1}{S_2}}\, p_0 \\
&\underset{20\,^{\circ}\mathrm{C}}{\overset{\text{air}}{=\!=}} \frac{1}{\dfrac{1}{\underbrace{\dfrac{\pi p d^2}{\rho u}}_{\text{Eq.(4.11)}} + \underbrace{91 d^2}_{\text{Eq.(4.8)}}} + \dfrac{1}{\underbrace{1364 d^4 \bar{p}}_{\text{Eq.(4.5)}} + \underbrace{121.3 d^3}_{\text{Eq.(4.6)}}} + \dfrac{1}{S_p}}\, p_0
\end{aligned}
\right\}
\qquad (4.22)
$$

These two terms can usually be ignored

$\bar{p} = \dfrac{p_1 + p_2}{2}$, and for anything but the shortest pipes take $\rho_1 \cong \rho_0$

4.3 Pumping Down a Vacuum System

Consider the changing conditions where gas is being pumped from the system, gas leaks into the system, and the pressure within the system



decreases with time. At any instant we can express the net flow rate of gas out of the system as

$$\left.\begin{aligned} Q = -\frac{d}{dt}(p_0 V) = -V\frac{dp_0}{dt} &= \begin{pmatrix}\text{removal rate}\\ \text{from tank}\end{pmatrix} - \begin{pmatrix}\text{leak rate}\\ \text{into tank}\end{pmatrix} \\ &= S_2 p_2 - Q_{\text{leak}} \\ &= C_{\text{tot}} p_0 - Q_{\text{leak}} \end{aligned}\right\} \tag{4.23}$$

At $t = \infty$ the system reaches steady state, $Q = 0$, so the above expression reduces to

$$Q_{\text{leak}} = C_{\text{tot}} p_{0,\infty} \tag{4.24}$$

Combining equations (4.23) and (4.24) at any time gives

$$\left.\begin{aligned} -\frac{dp_0}{dt} &= \frac{C_{\text{tot}}}{V}\Big(p_0 - \underset{Q_{\text{leak}}/C_0}{p_{0,\infty}}\Big) \\ \frac{1}{C_{\text{tot}}} &= \frac{1}{C_{\text{or}}} + \frac{1}{C_{\text{line}}} + \frac{1}{S_p} \end{aligned}\right\} \tag{4.25}$$

In general C_{tot} varies with pressure.

Now if C_{tot} is constant and independent of p (this means that C_{line} and S_p do not change as the pressure is lowered), integration gives

$$\ln\left(\frac{p_{0,\text{start}} - p_{0,\infty}}{p_{0,\text{finish}} - p_{0,\infty}}\right) = \frac{C_{\text{tot}}}{V} t \tag{4.26}$$

If unknown, start by ignoring this

If C_{tot} is not constant but varies with p (usually S_p varies drastically with pressure), integration gives

$$-\int_{p_{0,\text{start}}}^{p_{0,\text{finish}}} \frac{dp}{C_{\text{tot}}\left(p_0 - p_{0,\infty}\right)} = \frac{1}{V}\int_0^t dt \tag{4.27}$$

This integral can be solved either graphically (see Fig. 4.14) or numerically by taking small slices of p_0; thus, $p_{0,1}, p_{0,2}, \ldots, p_{0,i}, \ldots p_{0,n}$.

$p_{0,i+1}-p_{0,\infty}$	$p_{0,i}-p_{0,\infty}$	$\overline{C_{\text{tot}}}$	$\ln\left(\dfrac{p_{0,i+1}-p_{0,\infty}}{p_{0,i}-p_{0,\infty}}\right)$	t
$p_{0,2}-p_{0,\infty}$	$p_{0,1}-p_{0,\infty}$	—	—	—
$p_{0,3}-p_{0,\infty}$	$p_{0,2}-p_{0,\infty}$	—	—	—
$\vdots$	$\vdots$			
$p_{0,n}-p_{0,\infty}$	$p_{0,n-1}-p_{0,\infty}$	—	—	—

To save effort, take a constant ratio; for example, $\rho = 1, 2, 4, 8, \ldots$

In each interval assume a constant $\overline{C}_{\text{tot}}$

Total time $= \sum t$

These calculations simplify somewhat:

- When there is no leak; thus, when $p_{0,\infty} \to 0$
- When the pump is located right at the vessel to be evacuated, in which case $C_{\text{tot}} \to S_p$

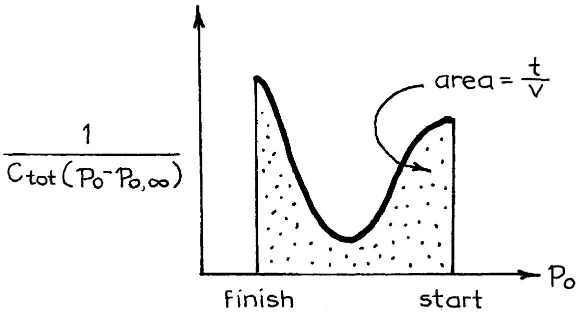


Fig. 4.14 Integrand for equation 4.27

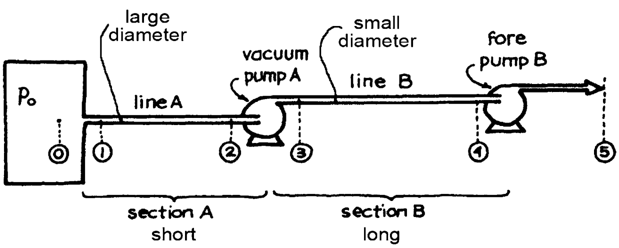


Fig. 4.15 Two-stage pumpdown system

4.4 More Complete Vacuum Systems

Often one employs a two-stage pumping system as shown in Fig. 4.15. Let us see how to treat this situation. For section A of Fig. 4.15 we write

$$Q_A = C_{\text{tot},\ A} p_0 \quad \text{where} \; \frac{1}{C_{\text{tot},\ A}} = \frac{1}{C_{\text{or},\ A}} + \frac{1}{C_{\text{line},\ A}} + \frac{1}{S_{p,\ A}}$$

Similarly for section B of Fig. 4.15

$$Q_B = C_{\text{tot},\ B} p_3 \quad \text{where} \; \frac{1}{C_{\text{tot},B}} = \frac{1}{C_{\text{line},\ B}} + \frac{1}{S_{p,\ B}}$$

Since $Q_A = Q_B$ and $p_3 \gg p_0$ we must make $C_{A,\text{tot}} \gg C_{B,\text{tot}}$. This means that we should design the system so that most of the flow resistance is in line B, not A. Therefore:

- Use a big diameter pipe for line A.
- Keep pump A close to the vessel to be evacuated.
- A long, small-diameter tube can be used for line B with but little harm.

4.5 Comments

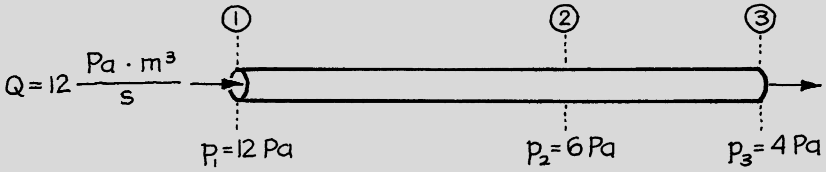
This chapter develops the language of molecular flow in the framework of the tank–line–pump system. The field is much broader than this. Here are some additional areas of study:

- The equations for the tank–line–pump situation only hold well if $L/d \geq 100$. For short pipes we may want to modify these expressions. Luckily these are second-order effects.
- Turbulent flow of gases. This situation occurs only very rarely—for high Δp in large pipes.
- Flow in conduits of other shapes: slits, rectangles, annuli, and triangles.
- Pumping speed of cold traps.
- Degassing problems—to remove adsorbed gases from metal and glass surfaces.
- Design of more complete vacuum systems.
- High-vacuum gauges and pumps.

Example 4.1. High-Vacuum Flow in a Pipe
Find the speeds S_1, S_2, S_3 and the conductances C_{12}, C_{13}, C_{23} for the pipeline below.

(continued)

(continued)



Solution
From the definition of pumping speed, we can write, for locations 1, 2, and 3,

$$Q = S_1 p_1 = S_2 p_2 = S_3 p_3$$

Therefore,

$$S_1 = \frac{Q}{p_1} = 1\ \mathrm{m^3/s}$$
$$S_2 = \frac{Q}{p_2} = 2\ \mathrm{m^3/s}$$
$$S_3 = \frac{Q}{p_3} = 3\ \mathrm{m^3/s}$$

Next consider the conductance of sections 1–2, 2–3, and 1–3 of the pipe. By definition

$$Q = C_{12}(p_1 - p_2) = C_{23}(p_2 - p_3) = C_{13}(p_1 - p_3)$$

Therefore,

$$C_{12} = \frac{Q}{p_1 - p_2} = \frac{12}{12 - 6} = 2\,\mathrm{m^3/s}$$
$$C_{23} = \frac{Q}{p_2 - p_3} = \frac{12}{6 - 4} = 6\,\mathrm{m^3/s}$$
$$C_{13} = \frac{Q}{p_1 - p_3} = \frac{12}{12 - 4} = 1.5\,\mathrm{m^3/s}$$

To check the results:

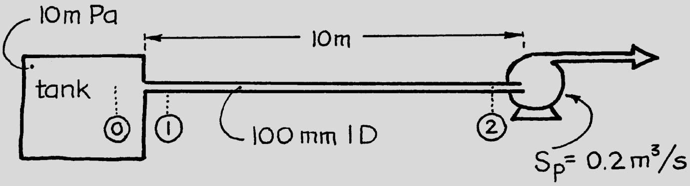
(continued)

(continued)

$$\frac{1}{C_{13}} \stackrel{?}{=} \frac{1}{C_{12}} + \frac{1}{C_{23}} \quad \text{or} \quad \frac{1}{1.5} \stackrel{?}{=} \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \quad \text{(correct)}$$

Example 4.2. Conditions in a Steady-State Vacuum System
A vacuum pump ($S_p = 0.2\ \mathrm{m^3/s}$) is connected by 10 m of 100-mm-i.d. pipe to a large vessel which is to be evacuated of air at room temperature. At a time when the pressure in the tank is 10 mPa:

(a) Calculate the pressure at the mouth of the pump (point 2).
(b) Determine the pumping speed at the tank (point 0). This is the rate at which air at 10 mPa is being removed from the tank.
(c) Locate the major resistance to the evacuation.



Solution
We can see from the figure above that $S_2 = S_p$ and $p_2 = p_p$. Then

Including the pump

$$Q = \underline{S_0 p_0} = C_{01}(p_0 - p_1) = C_{12}(p_1 - p_2) = \underline{S_2 p_2} = \underline{C_{tot}(p_0} - 0) \qquad \text{(i)}$$

Use underlined terms only

Imaginary region beyond the pump

where, from equation (4.22)

(continued)

(continued)

$$\frac{1}{C_{\text{tot}}} = \underbrace{\frac{1}{C_{\text{or, lam}} + C_{\text{or, mol}}}}_{\substack{\text{From} \\ \text{Eq.(4.11)}} \quad \substack{\text{From} \\ \text{Eq.(4.8)}}} + \underbrace{\frac{1}{C_{\text{line, lam}} + C_{\text{line, mol}}}}_{\substack{\text{From} \\ \text{Eq.(4.5)}} \quad \substack{\text{From} \\ \text{Eq.(4.6)}}} + \frac{1}{S_2} \tag{ii}$$

Before we proceed to evaluate all the terms, let us see whether we are completely in one or the other flow regime. If we are this would simplify matters. At the point of highest pressure, at the tank, we have

$$pd = (0.1)(0.01) = 10^{-3}\,\text{Pa m}$$

According to equation (4.1) this condition means that flow is completely in the molecular regime, so we can safely and happily drop the laminar terms in equation (ii). Evaluating the remaining terms gives

$$\frac{1}{C_{\text{tot}}} = \frac{1}{91d^2} + \frac{L}{121.3d^3} + \frac{1}{S_2}$$

and on replacing values we find

$$\frac{1}{C_{\text{tot}}} = 1.1 + 81.5 + 5 = 87.6 \tag{iii}$$

or

$$C_{\text{tot}} = 0.0114\,\text{m}^3/\text{s}$$

Replacing in equation (i) gives

$$p_2 = \frac{C_{\text{tot}} p_0}{S_2} = \frac{(0.0114)(0.01)}{0.2} = 5.7 \times 10^{-4}\ \text{Pa} = 0.571\ \text{mPa} \tag{a}$$

The pumping speed at the tank is also given by equation (i). Thus,

$$S_0 = \frac{C_{\text{tot}} p_0}{p_0} = C_{\text{tot}} = 0.0114\ \text{m}^3/\text{s} \tag{b}$$

Equation (iii) shows that the relative resistances are

(continued)

(continued)

$$\begin{aligned} &\text{Entry orifice :} && \frac{1.1}{87.6} = 1.3\ \% \\ &\text{The line :} && \frac{81.5}{87.6} = 93\ \% \\ &\text{The pump :} && \frac{5}{87.6} = 5.7\ \% \end{aligned} \tag{c}$$

Thus, the line provides the major resistance $(\sim 93\ \%)$.

Note: To speed the evacuation either shorten the pipe or increase the pipe diameter. The latter change is better by far since the pumping speed varies as d^3. Using a bigger pump won't help much. For example, even with the biggest pump in the world, equation (iii) becomes

$$\frac{1}{C_{\text{tot}}} = 1.1 + 81.5 + \frac{1}{\infty} = 82.6 \quad \text{vs.} \quad 87.6$$

Thus, the conductance will only increase by about 6 %.

If we would have included the laminar pipe resistance term in our calculations, our answer would only have changed by about 1 %. This justifies our dropping this term.

Since the molecular orifice resistance is only about 1 % that of the pipe [see equation (iii)], and the laminar orifice resistance can be expected to be so much smaller than either, the latter can well be ignored.

Actually, to evaluate the laminar conductances of pipe and orifice is awkward and requires a trial-and-error procedure. The next example shows how this is done for pipe flow.

Example 4.3. Conditions in Another Vacuum System

Repeat Example 4.2 with just one change; let the pressure in the tank be 10 Pa.

Solution

At the pipe entry $pd = 1$ Pa m; thus, we are in the regime of laminar flow and should use equation (4.11). However, from Example 4.2 we find that the resistance of the entry is negligible (about 1 % of the total), so let us ignore it. Next, following the procedure of Example 4.2, we write

(continued)

(continued)

$$Q = S_0 p_0 = S_2 p_2 = C_{tot}(p_0 - 0) \tag{i}$$

where

$$\frac{1}{C_{tot}} = \cancel{\frac{1}{C_{or}}} + \frac{1}{\dfrac{1364 d^4 \bar{p}}{L} + \dfrac{121.3 d^3}{L}} + \frac{1}{S_2} \tag{ii}$$

Neglect

$\bar{p} = \frac{p_1 + p_2}{2}$ where p_2 is unknown

Since p_2 is unknown guess that $p_2 = p_1 = 10$ Pa. Then replacing all values in equation (ii) gives

$$\frac{1}{C_{tot}} = \frac{1}{0.1364 + 0.0121} + \frac{1}{0.2} = 6.7326 + 5 = 11.7326$$

or

$$C_{tot} = 0.0852 \text{ m}^2/\text{s}$$

Then from equation (i)

$$p_2 = \frac{C_{tot} p_0}{S_2} = \frac{(0.0852)(10)}{0.2} = 4.2616 \text{ Pa}$$

This value for p_2 does not agree with our guess. So try again. With the help of an astrologer, let us guess that $p_2 = 3.4$ Pa. Then equation (ii) gives

$$\frac{1}{C_{tot}} = \frac{1}{\dfrac{1364(0.1)^4(10+3.4)/2}{10} + \dfrac{121.3\left(10^{-3}\right)}{10}} + \frac{1}{0.2}$$
$$= \frac{1}{0.0914 + 0.01213} + 5 = 9.66 + 5 = 14.66$$

or

$$C_{tot} = 0.0682 \text{ m}^3/\text{s}$$

Then equation (4.1) gives

(continued)

(continued)

$$p_2 = \frac{C_{\text{tot}}p_0}{S_2} = \frac{(0.0682)(10)}{0.2} = 3.41\,\text{Pa}$$

Our guess was right, so this is the right pressure for p_2, or

$$p_2 = 3.41\text{ Pa} \tag{a}$$

The pumping speed at the tank is also given by equation (i). Thus

$$S_0 = \frac{C_{\text{tot}}p_0}{p_0} = C_{\text{tot}} = 0.0682\,\text{m}^3/\text{s} \tag{b}$$

Again the line provides the major resistance, or

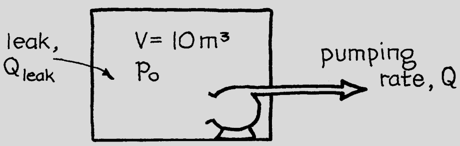
$$\frac{9.66}{14.66} \cong 66\% \tag{c}$$

NOTE: Laminar flow is the main mechanism causing the movement of fluid in the line. In fact it accounts for

$$\frac{0.0914}{0.0914 + 0.01213} = 88\%$$

of the total flow. Molecular flow accounts for just 12 % in the conditions of this problem.

Example 4.4. Pumping Down a Leaky Vacuum System
A pump ($S_p = 1$ m^3/s) is placed within a vessel ($V = 10$ m^3) and is pumping it out. However, because of leaks into the vessel, the pressure within the vessel decreases to a limiting value $p_{0,\infty} = 1$ Pa. Find the leak rate into the vessel.



Solution
When the system reaches steady state,

$$Q_{\text{leak}} = Q = C_{\text{tot}} p_0$$

But since there is no line present,

$$\frac{1}{C_{\text{tot}}} = \underbrace{\cancel{\frac{1}{C_{\text{or}}}}}_{=0} + \underbrace{\cancel{\frac{1}{C_{\text{line}}}}}_{=0} + \frac{1}{S_p} = \frac{1}{S_p}.$$

Therefore,

$$Q_{\text{leak}} = S_p p_0 = \left(1\,\text{m}^3/\text{s}\right)(1\,\text{Pa}) = 1\,\text{Pa}\,\text{m}^3/\text{s}$$

Thus, 1 m^3 of air measured at 1 Pa, or 10 cm^3 of air measured at 1 atm, leaks into the vessel each second.

Problems on Vacuum Flow

4.1. A molecular still, to be kept at 0.01 Pa, is connected to an oil diffusion pump by a 0.1-m-i.d. line. Data on the pump indicates that it has a pumping speed of 250 lit/s at low pressure. Assume that air at 20 °C is being pumped.

(a) What is the speed factor of the pump?
(b) What length of this line can be used and still not reduce the pumping speed at the still to less than 50 lit/s?

4.2. A large vacuum system is connected by 1 m of 0.1-m-i.d. pipe to a 0.4 m^3/s vacuum pump. After pumpdown and with the pump working at full speed, the pressure in the system is 1 mPa.

(a) What is the pressure at the pump intake?
(b) What is the leak rate of room air into the system? Give this as lit/h of air at 1 atm.
(c) If the leak rate can be halved, what will be the pressure in the system?

4.3. At one end of a tube (10 m long, 0.1 mm i.d.), the pressure is 1 atm, and at the other end the pressure is 1 Pa. The temperature is 20 °C.

(a) Find the flow rate of air (measured at 1 atm) through this tube.
(b) Plot the pressure 1/4, 1/2, and 3/4 of the way along the tube.
(c) How will the flow rate change if the tube length is halved?
(d) How will the flow rate change if the tube diameter is doubled? Ignore the entrance (or orifice) effects.

4.4. An apparatus is connected by 1 m of 4-cm-i.d. glass tubing, free from leaks, to a combined mercury diffusion and mechanical pump (speed = 40 lit/s). Because of small unavoidable leaks in the apparatus itself, the minimum pressure attainable in the apparatus is 10 mPa.

(a) What pressure can be maintained in the apparatus if the tubing connecting the pumps to apparatus were shortened to 0.1 m?
(b) What pressure can be maintained in the original apparatus if the pumps were replaced by new ones 10 times as fast?

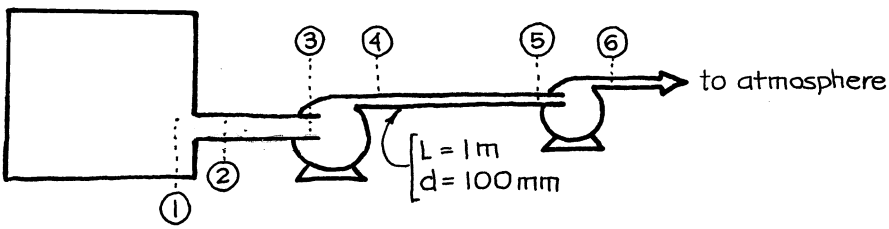
4.5. During the pumpdown of our vacuum system, I noted that it took 1 day for the pressure to drop from 0.3 to 0.2 Pa. Further pumping reduced the pressure down to 0.1 Pa, but no lower. Yesterday I put extra sealant on what I thought was a leaky joint. Sure enough the pressure in the system started to fall and after 24 h had gone from 0.1 to 0.06 Pa. Estimate the new limiting low pressure of the system. Since the pump characteristics are not given, assume a constant pumping speed at all pressures.

4.6. At present an apparatus is kept at 1 mPa by a vacuum pump and backup pump running at a speed of 10 1it/s and connected to the apparatus by 1 m of 20-mm-i.d. tubing. We wish to lower the pressure in the apparatus, and three choices come to mind.

(i) Double the tube diameter.
(ii) Shorten the connecting pipe from 1 to 0.1 m.
(iii) Replace the present pump with a larger one having a pumping speed of 30 lit/s.

Rate these alternatives and present calculations to back your rating.

4.7. A vacuum system consists of a vessel connected by pipe (2–3) to a vacuum pump, more pipe (4–5) to a forepump, and then to the atmosphere as shown.



For the vacuum pump, $S_p = 0.1\ m^3/s$, $p_3 = 0.2$ Pa, $p_4 = 100$ Pa. Determine the size of forepump which is being used.

4.8. Repeat the previous problem with just one change: the 1 m of 100-mm-i.d. line is to be replaced by 1 m of 1-mm-i.d. line.

4.9. The pumpdown of a 7.5-m^3 leak-free vacuum system from 1 to 0.1 Pa takes 2 h. The pump is connected to the vessel by 1 m of 30-mm-i.d. pipe. Estimate the speed of the pump in this pressure range.

4.10. Our laboratory has a monstrous white elephant, a good-for-nothing three-story-high 12-m^3 distillation column. We tried to sell it, to give it away, and finally in desperation we offered $1,000 to anyone who'd remove it. Metal dealers who looked it over agreed that there was a good bit of metal there, but it was too bulky for them.

Well, I think I can help with that. I will have our machinist connect the diffusion pump to the column and evacuate it. From my *Strength of Materials* text, I calculate that the column will collapse with a mighty bang into a compact easily moved ball of metal when its interior pressure just reaches 0.1 Pa.

Since our department head is due to make his annual whirlwind visit of our laboratory about 2 pm next Friday, wouldn't it be a nice surprise if he were there, may be even leaning on the tower when it collapsed? What a delicious thought. When should we start evacuating the column so that our honored chief will forever remember his visit?

Data: The pump is connected directly to the tower, and from the manufacturer's data sheet, the pump speed is as follows:

Pressure (Pa)	0.8	0.2	0.5	1	2	5	10	20	50	100	1 atm
Pump speed (l/s)	18	25.5	30.5	32.5	32	19	11	5.5	2.5	1.5	1

One of the key steps in a local company's manufacture of integrated circuits is the low-pressure chemical vapor deposition (LPCVD) of exotic materials. This operation takes place in a battery of special reaction chambers, or furnaces, kept at 1 mPa by using an oil diffusion pump followed by a conventional vacuum pump. Without reaction the chamber is capable of maintaining 0.36 mPa.

The operating pressure is reached in a two-step pumpdown:

Step 1. A relatively rapid pumpdown to 100 mPa limited by the conventional vacuum pump. Ignore this time.

Step 2. A longer pumpdown limited by the oil diffusion pump which drops the pressure from 100 to 1 mPa.

Some of the reaction chambers are connected to their pairs of pumps by 1 m of 5-cm line and have a pumpdown time of 42 min; others are connected with 2 m of 5-cm line and have a pumpdown time of 63 min.

Operations are expanding, the maintenance isle is too crowded, and so the staff is thinking of relocating some of the pumps on the next floor. This would require using 8-m lines between chamber and pump pairs.

4.11. How would this affect the pumpdown time?

4.12. What diameter of connecting line should be used to keep the pumpdown time at 42 min? [Problem prepared by Jim McDaniel]

4.13. A simple way of detecting small leaks in heat exchangers is as follows. Pressurize the unit with air, say to 2 atm absolute; immerse it in hot water containing a sprinkle of detergent to reduce the surface tension; and carefully look for bubbles. This technique is sufficiently sensitive to detect very small leak rates roughly equivalent to forming a 1-mm-diameter bubble each minute. How big a hole do you estimate this to represent? Consider the hole to be a pinch point or orifice.
Note: This problem involves material from both Chaps. 3 and 4

At the beginning of the week, my bicycle tire (wall thickness = 1.7 mm) contained 1 lit of air at 700 kPa and 20 °C. But after 5 days the pressure was down to 690 kPa, and I am sure that the air leaked out of just one hole—which was made by *you* when you kicked the tire. Yes, I saw you do it! Estimate the size of the hole.

4.14. Assuming that the hole is a "pinch point," an orifice

4.15. Assuming that the hole is tubular in form
Note: These problems involve material from both Chaps. 3 and 4.

References and Further Readings

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