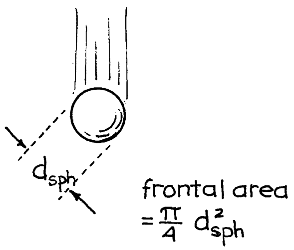
Chapter 8
Solid Particles Falling Through Fluids

8.1 Drag Coefficient of Falling Particles

8.1.1 The Small Sphere

The forces acting on a sphere falling through a fluid (call it gas for convenience) are as follows:

$$\left|\begin{pmatrix}\text{Force causing}\\ \text{particle to}\\ \text{accelerate}\end{pmatrix}\right| = \left|\begin{pmatrix}\text{net weight}\\ \text{of particle}\end{pmatrix}\right| - \left|\begin{pmatrix}\text{drag}\\ \text{force}\end{pmatrix}\right| \qquad \left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right]$$



In symbols

$$|F| = m\frac{du}{dt} = \left(\frac{\pi}{6}d_{\text{sph}}^3\right)\left(\left|\rho_s - \rho_g\right|\right)g - |F_d| \tag{8.1}$$

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where the drag force

$$|F_d| = \underbrace{C_D}_{\text{Drag coefficient}} \cdot \frac{\pi d_{\text{sph}}^2}{4} \frac{\rho_g u^2}{2} \tag{8.2}$$

At the *terminal velocity* $du/dt = 0$, in which case equations (8.1) and (8.2) give

$$u_t = \left[\frac{4 g d_{\text{sph}} \left(|\rho_s - \rho_g| \right)}{3 \rho_g C_D} \right]^{1/2} \quad \text{or} \quad C_D = \frac{4 g d_{\text{sph}} \left(|\rho_s - \rho_g| \right)}{3 \rho_g u_t^2} \tag{8.3}$$

The drag coefficient in equation (8.3) has been found by experiment to be related to the particle Reynolds number at terminal velocity, defined as

$$\text{Re}_{\text{sph},\ t} = \frac{d_{\text{sph}} u_t \rho_g}{\mu} \tag{8.4}$$

For the special case of *viscous flow of a sphere*, occurring when $\text{Re}_t < 1$, Stokes developed the following theoretical expression for the drag force:

$$F_d = 3 \pi d_{\text{sph}} \mu u \qquad \left[\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right] \tag{8.5}$$

Replacing in equation (8.4) then gives

$$\left. \begin{aligned} u_t &= \frac{\left(|\rho_s - \rho_g| \right) g d_{\text{sph}}^2}{18 \mu} \\ C_D &= \frac{24}{\text{Re}_{\text{sph},\ t}} = 24 \left(\frac{\mu}{d_{\text{sph}} u_t \rho_g} \right) \end{aligned} \right\} \quad \text{good only when } \text{Re}_{\text{sph},\ t} < 1 \tag{8.6}$$

The lowest curve on Fig. 8.1 shows the relationship of C_D vs. $\text{Re}_{\text{sph},\ t}$ for spherical particles. The rather long equation for this curve is given by Haider and Levenspiel (1989).

8.1.2 Nonspherical Particles

The drag coefficient for these irregular particles is also shown on Fig. 8.1. Again, see Haider and Levenspiel (1989) for the ugly equation which represents these curves.

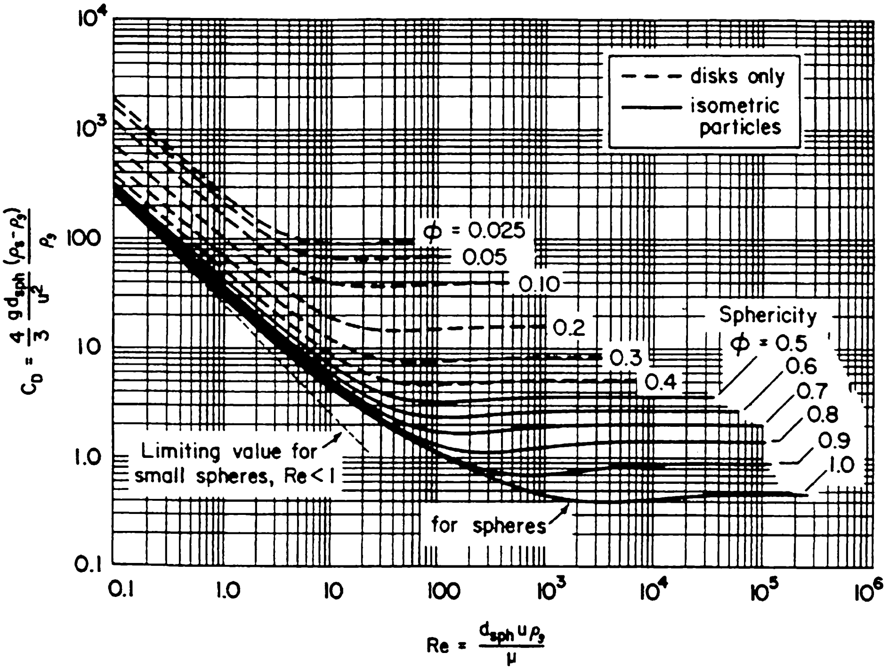


Fig. 8.1 Design chart for drag coefficients of single free-falling particles

8.1.3 Terminal Velocity of Any Shape of Irregular Particles

Larger spheres and other shaped particles generate and are followed by a wake as they fall at their terminal velocity, at $Re_t > 1$. Here no satisfactory theoretical drag force expression has been developed. Consequently, the frictional loss and terminal velocity have to be found by experiment. These findings, by Achenbach (1972), Pettijohn and Christiansen (1948), Schiller (1932), Schlichting (1979), Schmiedel (1928), and Wadell (1934), have been well correlated by Haider and Levenspiel (1989) by the following expressions:

For spheres, ϕ=1

$$u_t^* = \left[\frac{18}{(d^*)^2} + \frac{0.591}{(d^*)^{1/2}} \right]^{-1} \tag{8.7}$$

For nonspherical particles, $0.5 < \phi < 1$

$$u_t^* = \left[\frac{18}{(d^*)^2} + \frac{2.335 - 1.745\,\phi}{(d^*)^{1/2}} \right]^{-1} \tag{8.8}$$

where

$$u_t^* = u_t \left[\frac{\rho_g^2}{g\mu\left(|\rho_s - \rho_g|\right)} \right]^{1/3} \tag{8.9}$$

and

$$d^* = d_{\mathrm{sph}} \left[\frac{g\rho_g\left(|\rho_s - \rho_g|\right)}{\mu^2} \right]^{1/3} \tag{8.10}$$

A three-step procedure is needed to evaluate u_t, given d_{sph} and ϕ.

- First calculate d^* from equation (8.10).
- Then find u_t^* from equation (8.7) or (8.8).
- Finally determine u_t from equation (8.9).

Alternatively, Fig. 8.2 gives the terminal velocity of particles, u_t, directly from the physical properties of the solid and fluid.

Example 8.1. Suing the United States for Its Misbehaving Volcanos
On May 18, 1980, Mount St. Helens on the West Coast of the United States erupted catastrophically, spewing an ash plume to an altitude of 20 km. The winds then carried these millions of tons of particles, consisting mainly of silica (~70 %), across the United States, depositing a 2-cm layer on my campsite high (2 km) in the Rockies and 1,000 km away. It started raining ash just 50 h after the eruption, and although I left for home soon after, I had to breathe this contaminated air.

I am worried because I read on page 19 of the June 9, 1980, issue of *Chemical and Engineering News* that silica particles smaller than 10 μm are respirable and can cause silicosis. No one told me this, I've since developed a cough, and being a normal American, I'm ready to sue the government for gross negligence in not warning me of this danger. But, of course, I will only do this if the particles are in the dangerous size range.

Please estimate the size of particles which settled on me at the start of this ash rain.

Data: Assume that ash particles consist of pure silica for which

$$\rho_s = 2{,}650 \ \mathrm{kg/m^3} \quad \text{and} \quad \phi = 0.6$$

The average atmospheric conditions from 2 to 20 km are $T = -30$ °C, $p = 40$ kPa, at which $\mu_{\mathrm{air}} = 1.5 \times 10^{-5}$ kg/m s.

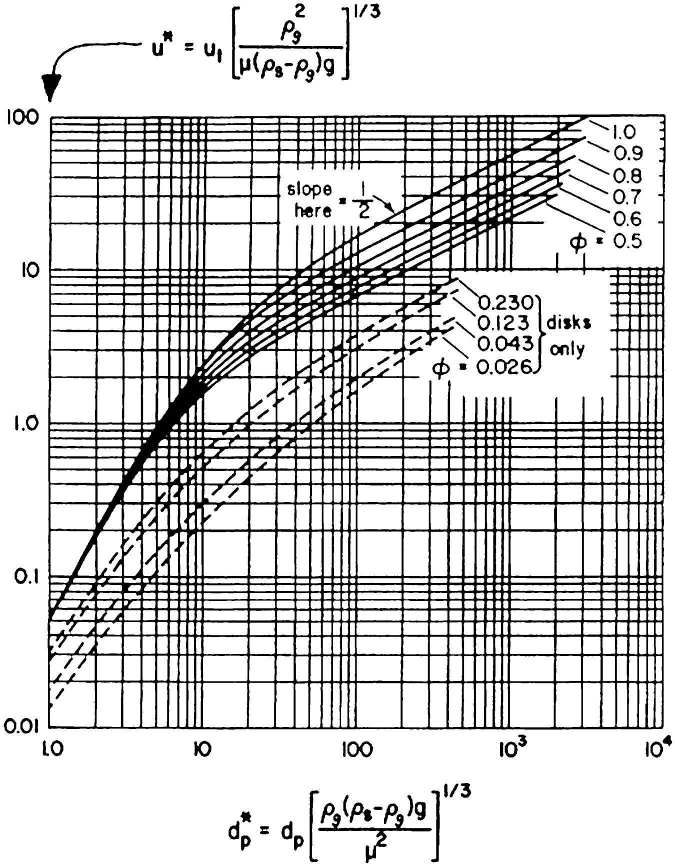


Fig. 8.2 Design chart for finding the terminal velocity of single free-falling particles

Solution

Method A. Use the C_D and Re_p equations with Fig. 8.1.

The problem statement does not specify what is meant by the words "particle size"—silica is quite nonspherical—so let us determine both the screen size and the equivalent spherical size. First we find

$$\rho_g = \frac{(mw)p}{RT} = \frac{(0.0289)(40{,}000)}{(8.314)(243)} = 0.5722\ \mathrm{kg/m^3}$$

$$u_t = \frac{\text{distance fallen}}{\text{time}} = \frac{20{,}000 - 2{,}000}{50 \times 3{,}600} = 0.1\ \mathrm{m/s}$$

(continued)

(continued)

Then,

$$\mathrm{Re}_{\mathrm{sph},\ t} = \frac{d_{\mathrm{sph}} u_t \rho_g}{\mu} = \frac{d_{\mathrm{sph}}(0.1) \times (0.5722)}{1.5 \times 10^{-5}} = 3{,}815\ d_{\mathrm{sph}} \tag{i}$$

and from equation (8.3)

$$C_D = \frac{4 g d_{\mathrm{sph}} \left(\rho_s - \rho_g\right)}{3 \rho_g u_t^2} = \frac{4(9.8) d_{\mathrm{sph}} (2{,}650 - 0.57)}{3(0.5722)(0.1)^2} \tag{ii}$$
$$= 6.05 \times 10^6 d_{\mathrm{sph}}$$

Now solve by trial and error using Fig. 8.1.

Guess d_{sph}	$\mathrm{Re}_{\mathrm{sph},\ t}$ from equation (i)	C_D from equation (ii)	C_D from Fig. 8.1
1×20^{-5} m	0.038	60	632
10×10^{-5}	0.38	600	67
3.4×10^{-5}	0.13	206	190 (close enough)

Thus, $d_{\mathrm{sph}} = 34\ \mu$m, and for irregular particles with no particularly short or long dimension, equation (6.4) gives

$$d_{\mathrm{scr}} = \frac{d_p}{\phi} = d_{\mathrm{sph}} = 34 \mu m$$

Method B. Use of u_t^* and d^* equations, either alone or with Fig. 8.2

In this problem u_t^* is known, and d^* is to be found. So from equation (8.9)

$$u_t^* = 0.1 \left[\frac{(0.5722)^2}{9.8\left(1.5 \times 10^{-5}\right)(2{,}650)} \right]^{1/3} = 0.084\,05 \tag{iii}$$

We next find d^* either directly from Fig. 8.2 or from equation (8.8).

From Fig. 8.2 we find

$$d^* = 0.13$$

Alternatively, from equation (8.8)

$$u_t^* = 0.084\,05 = \left[\frac{18}{(d^*)^2} + \frac{2.335 - 1.745(0.6)}{d^{1/2}} \right]^{-1}$$

(continued)

(continued)

Rearranging gives

$$\frac{18}{(d^*)^2} + \frac{1.288}{(d^*)^{1/2}} = 11.8977 \tag{iv}$$

Now solve equation (iv) by trial and error.

Guess d^*	LHS of equation (ii)
1.0	19.28
1.3	11.78
1.2931	11.8975 (close enough to the RHS of equation (iv))

So from equation (8.10)

$$d_{sph} = d_{scr} = 1.293\left[\frac{\left(1.5 \times 10^{-5}\right)^2}{9.8(0.5722)(2{,}650)}\right]^{1/3} = 32 \times 10^{-6}\text{m} \quad = \quad 32\ \mu\text{m}$$

Comment Method B does not require pulling a value from a chart and overall is simpler to use than Method A.

Conclusion. Either way you define particle size, you'd better not sue.

Problems on Falling Objects

8.1. *Skydiving*. Indoor skydiving has come to Saint Simon, near Montreal, in the form of the "Aerodium," a squat vertical cylinder 12 m high and 6 m i.d., with safety nets at top and bottom. A DC-3 propeller which is driven by a 300-kW diesel engine blasts air upward through the Aerodium at close to 150 km/h, while the "jumper" dressed in an air inflated jumpsuit floats, tumbles, and enjoys artificial free fall in this rush of air without the danger of the real thing.

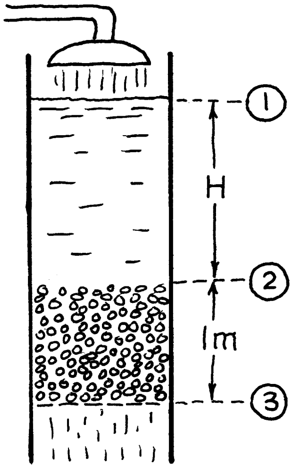
If an 80-kg, suited adult (density = 500 kg/m^3) in the spread-eagled position can hang suspended when the air velocity is 130 km/h, what is his sphericity in this position? (information from *Parachutist*, pg. 17, August 1981)

8.2. The free-fall velocity of a very tiny spherical copper particle in 20 °C water is measured by a microscope and found to be 1 mm/s. What is the size of the particle?

8.3. Who else could it happen to but "Bad Luck" Joe? He goes hunting, gets lost, fires three quick shots into the air, and gets hit squarely on the head by all three bullets as they come down. How fast were the bullets going when they hit him?

Data: Each bullet has a mass of 180 grains or 0.0117 kg, $\phi = 0.806$, and $\rho_{\text{bullet}} = 9{,}500\ \text{kg/m}^3$.

8.4. Water at 20 °C flows downward through a 1-m deep packed bed ($\varepsilon = 0.4$) of 1-mm plastic spheres ($\rho_s = 500\ \text{kg/m}^3$). What head of water is needed to keep the spheres from floating upward?



8.5. Rutherford Arlington, a freshman who weighs 80 kg stripped, yens to streak with a difference, and stepping out of a balloon at 3,000 m directly above Central Square during a noon revival meeting appeals to him. Imagine the impact—a real heavenly body entering their midst. At what speed would he join the faithful:

(a) If he curls up into a perfect sphere?
(b) If he is spread eagled? In this orientation, $\phi = 0.22$.

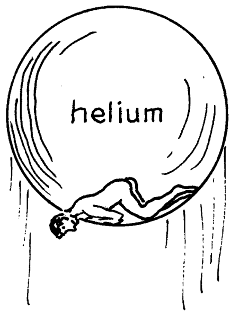
8.6. Find the upward velocity of air at 20 °C which will just float a ping pong ball.
Data: Nittaku 3-Star ping pong balls, used for the 37th World Championships in Tokyo in 1983, have a diameter of 37.5 mm and a mass of 2.50 g.

8.7. Lower Slobovia recently entered the space race with its own innovative designs. For example, the touchdown parachute of their lunar space probe, the "Lunik," was ingeniously stored in the mouth of the braking rocket to save space. Unfortunately, instead of releasing the parachute, then firing the rocket, our intrepid spaceman, the "lunatic," first fired the rocket, which then used all its fuel to burn up the parachute—all this 150 km above the Earth. When the Slob finally returned to Earth, he had a somewhat rough landing. At what speed do you estimate that he hit the ground?
Information on the rocket: Volume = 5 m^3, mass = 2.5 t, and surface area = 20 m^2.

8.8. Rutherford Arlington, famed streaker, plans to use a helium balloon for the ascent prior to his spectacular free fall (see Problem 8.5). To reach a height of 1,000 m in 10 min, what size of balloon would he need?

Data: The combined mass of Ruthy and his balloon = 120 kg, $\phi \cong 1$, $T = 20$ °C, $\pi = 100$ kPa.

(problem prepared by Dan Griffith)



8.9. Referring to the proposed ride called "Typhoon" for the Tokyo Disneyland (Problem 7.10) in which children are fluidized in a large Plexiglas cylinder, the only worry is that some small child may rise above his screaming fellows to get away from it all. To see if this is likely to occur, calculate the terminal velocity of a little Japanese child. See Problem 7.10 for additional data.

8.10. Referring to the data of Example 8.1, how long would it take for 1-μm ash particles (equivalent spherical diameter) disgorged to an altitude of 20 km to settle out of the atmosphere down to sea level? How far around the world would particles of this size go in this time?

Data: Average westerly wind speed at 45° latitude (close to the location of the volcano) is 800 km/day. Ignore updrafts and downdrafts; in the long run they should cancel out.

8.11. The *Official Baseball Rules* states, in part:

"1.09 The ball should be a sphere formed by yarn wound round a small sphere of cork, rubber, or similar material covered with two strips of white horsehide or cowhide, tightly stitched together. It shall weigh not less than 5 nor more than 5¼ ounces avoirdupois and measure no less than 9 nor more than 9¼ inches in circumference."

(a) What should be the terminal velocity of a *smooth* baseball?

(b) Baseball aficionados know that the two cover pieces of a regulation baseball are hand-stitched together with exactly 216 raised cotton stitches. The seam and stitches add roughness to the surface and this lowers the drag coefficient by about 44 %, according to Adair (1990). What would this do to your calculated u_t?

How do your calculations compare with wind tunnel tests which give $u_t = 95$ miles/h?

8.12. *Baseball trivia*. Goose Gossage grumbled:

"Shucks, t' would have been over the 406 ft fence it it'd been warmer," as Willie Mays speared Goose's 400 ft drive on a wintry 0 °C day.

Common lore in baseball has it that a ball flies farther on a warmer day. To check this, estimate whether the batter would have gotten a home run had the day been warmer, say at 20 °C, instead of 0 °C.

Data: From the table at the back of this book, for air,

$$\text{At } 20\,^\circ\text{C} \quad \rho = 1.205\,\text{kg/m}^3,\ \mu = 1.81 \times 10^{-5}\text{kg/m}\cdot\text{s}$$
$$\text{At } 0\,^\circ\text{C} \quad \rho = 1.293\,\text{kg/m}^3,\ \mu = 1.72 \times 10^{-5}\text{kg/m}\cdot\text{s}$$

A hit ball travels at roughly 100 mph.

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