Chapter 9
The Three Mechanisms of Heat Transfer: Conduction, Convection, and Radiation

In general, heat flows from here to there by three distinct mechanisms:

- By conduction, or the transfer of energy from matter to adjacent matter by direct contact, without intermixing or flow of any material.
- By convection, or the transfer of energy by the bulk mixing of clumps of material. In natural convection it is the difference in density of hot and cold fluid which causes the mixing. In forced convection a mechanical agitator or an externally imposed pressure difference (by fan or compressor) causes the mixing.
- By radiation such as light, infrared, ultraviolet, and radio waves which emanate from a hot body and are absorbed by a cooler body.

In turn, let us briefly summarize the findings on these three mechanisms of heat transfer.

9.1 Heat Transfer by Conduction

Conduction refers to the transfer of heat from the hotter to the colder part of a body by direct molecular contact, not by gross movement of clumps of hot material to the cold region. At steady state the rate of heat transfer depends on the nature of the material and the temperature differences and is expressed by Fourier's law as

$$\dot{q}_x = -kA\frac{dT}{dx} \qquad \left[\frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}\right] \tag{9.1}$$

where $\dot{q}_x$ is the rate of heat transfer in the x direction [W]; A is the area normal to the direction of heat flow [m^2]; dT/dx is the temperature gradient in the x direction [K/m]; and k is the thermal conductivity, defined as the heat going through a cube of

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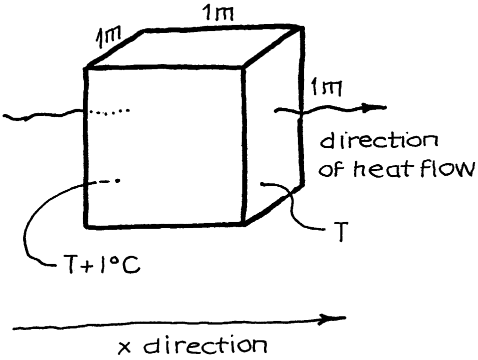
Table 9.1 Short table of thermal conductivities for materials at room temperature[a]

Material	k, W/m K	Material	k, W/m K
Gases		Solids	
SO_2	0.009	Styrofoam	0.036
CO_2, H_2	0.018	Corrugated cardboard	0.064
H_2O	0.025	Paper	0.13
Air	0.026	Sand, dry	0.33
Liquids		Glass	0.35–1.3
Gasoline	0.13	Ice	2.2
Ethanol	0.18	Lead	34
Water	0.61	Steel	45
Mercury	8.4	Aluminum	204
Sodium	85	Copper	380

[a] For additional values, see Appendices A.15 and A.21

the material in question 1 m on a side resulting from a temperature difference on opposite faces of 1 °C. Table 9.1 gives k values for various materials [W/m K].



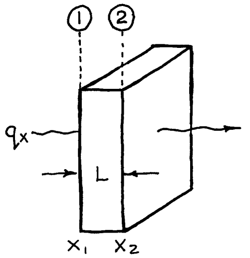
The minus sign in this equation tells that heat flows from regions of higher to lower temperature, not the other way round, and shows that the second law of thermodynamics is at work.

The complete equation for steady-state heat conduction in any arbitrary direction through an isotropic material, without heat generation, is

$$\dot{q} = -kA(\Delta T), \qquad k = \text{constant} \tag{9.2}$$

Fourier's equation has been integrated for various simple geometries. Here are some steady-state solutions:

9.1.1 *Flat Plate, Constant* k



$$\dot{q}_x = -kA\frac{T_2-T_1}{x_2-x_1} = -kA\cdot\frac{T_2-T_1}{L} \tag{9.3}$$

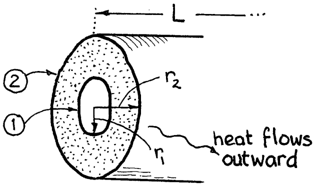
9.1.2 *Flat Plate,* $\mathbf{k} = \mathbf{k}_0\,(1+\beta\mathbf{T})$

$$\dot{q}_x = -k_0A\frac{(T_2-T_1)+(\beta/2)\left(T_2^2-T_1^2\right)}{x_2-x_1} = -k_{\text{mean}}A\frac{T_2-T_1}{L} \tag{9.4}$$

where

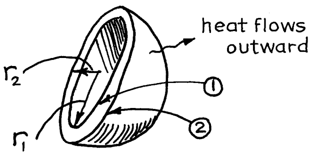
$$k_{\text{mean}} = \frac{k_1+k_2}{2}$$

9.1.3 *Hollow Cylinders, Constant* k



$$\dot{q}_r = -2\pi kL\frac{T_2-T_1}{\ln\left(r_2/r_1\right)} \tag{9.5}$$

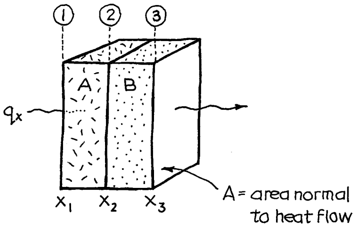
9.1.4 *Hollow Sphere, Constant* k



$$\dot{q}_r = -4\pi kr_1r_2\frac{T_2-T_1}{r_2-r_1} \tag{9.6}$$

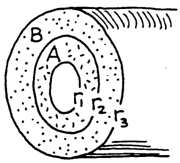
9.1.5 Series of Plane Walls

$$\dot{q}_x = -\frac{A}{\frac{x_2 - x_1}{k_A} + \frac{x_3 - x_2}{k_B}}\left(T_3 - T_1\right) \tag{9.7}$$



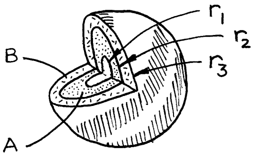
9.1.6 Concentric Cylinders

$$\dot{q}_r = -\frac{2\pi L}{\frac{\ln\left(r_2/r_1\right)}{k_A} + \frac{\ln\left(r_3/r_2\right)}{k_B}}\left(T_3 - T_1\right) \tag{9.8}$$



9.1.7 Concentric Spheres

$$\dot{q}_r = -\frac{4\pi}{\frac{r_2 - r_1}{k_A r_1 r_2} + \frac{r_3 - r_2}{k_B r_2 r_3}}\left(T_3 - T_1\right) \tag{9.9}$$

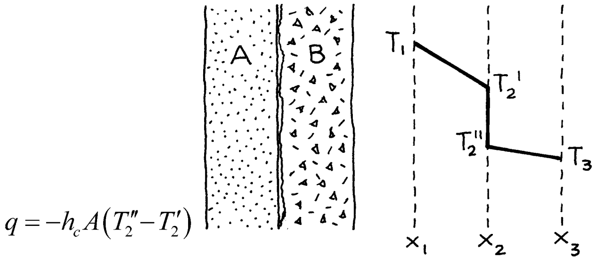


9.1.8 Other Shapes

For nonsimple geometries or for nonuniform temperatures at the boundaries, the heat flow can only be obtained by solving Fourier's equation by numerical or graphical methods [see Welty (1978) or McAdams (1954)].

9.1.9 Contact Resistance

When heat flows across two touching plane walls, an extra resistance normally is found at the interface because the contacting surfaces are not quite smooth. This results in a sharp temperature drop at the surface. The heat flow across the interface can then be related to the temperature drop across the interface by



where h_c is defined as the contact heat transfer coefficient.

Overall, the heat flow across the two walls will then involve three resistances in series:

Across wall A : $\dot{q}_x = -k_A A \frac{T_2' - T_1}{x_2 - x_1}$

Across the interface : $\dot{q}_x = -h_c A\left(T_2' - T_2'\right)$

Across wall B : $\dot{q}_x = -k_B A \frac{T_3 - T_2'}{x_3 - x_2}$

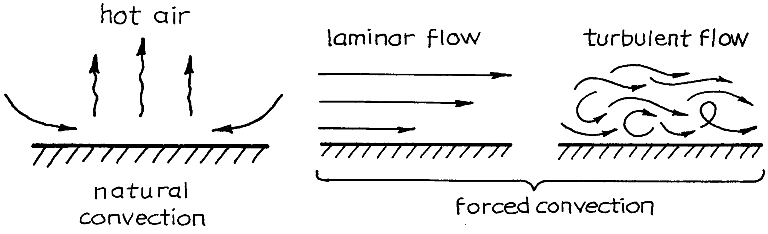
Noting that the $\dot{q}$'s are all equal, we can combine the above equations to eliminate the intermediate temperatures T_2' and T_2'' ending up with

$$\dot{q}_x = -\frac{1}{\frac{x_2 - x_1}{k_A} + \frac{1}{h_c} + \frac{x_3 - x_2}{k_B}} A(T_3 - T_1) \tag{9.10}$$

Equations analogous to the above can be developed for concentric spheres, concentric cylinders, and other shapes.

9.2 Heat Transfer by Convection

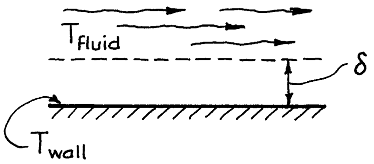
When hot fluid moves past a cool surface, heat goes to the wall at a rate which depends on the properties of the fluid and whether it is moving by natural convection, by laminar flow, or by turbulent flow. To account for



this form of heat transfer, Prandtl, in 1904, invented the concept of a boundary layer in which all the resistance to heat transfer is located. This idealization led to great simplifications and was enthusiastically adopted by practically all workers [see Adiutori (1974) for a vigorous dissenter].

With this way of viewing things and with the thickness of boundary layer δ, we have

$$\dot{q} = -kA\frac{T_{\text{fluid}} - T_{\text{wall}}}{\delta} = -kA\frac{\Delta T}{\delta}$$



Because δ cannot be estimated independently, we combine it with k to give

$$\dot{q} = -(k/\delta)A\Delta T = -hA\Delta T \qquad [\text{W}]$$

where, by definition,

$$h = \text{heat transfer coefficient, } \left[\text{W/m}^2\,\text{K}\right]$$

Note that h incorporates the thickness of an idealized boundary layer which will give the actual heat transfer rate. This quantity h is extremely useful since it is the rate coefficient which allows us to estimate the heat transfer rate in any particular situation.

Values of h have been measured in all sorts of situations, correlated with the properties of the fluid C_p, ρ, μ, k, the flow conditions u, and the system geometry d and compactly summarized in dimensionless form. The sampling of correlations which follows comes from McAdams (1954) or Perry and Chilton (1973) unless otherwise noted.

9.2.1 Turbulent Flow in Pipes

For both heating and cooling of most normal fluids (Pr = 0.7–700) in fully turbulent flow (Re > 10,000), moderate ΔT, and with physical properties measured at bulk conditions,

$$\underbrace{\frac{hd}{k}}_{\substack{\text{Nusselt}\\ \text{number}}} = 0.023\underbrace{\left[1+\left(\frac{d}{L}\right)^{0.7}\right]}_{\substack{\text{Entrance}\\ \text{effect}}}\underbrace{\left[1+3.5\frac{d}{d_{\text{coil}}}\right]}_{\substack{\text{For coiled}\\ \text{pipes}}}\left(\frac{d\overset{\swarrow = u\rho,\ \text{where } u = \text{mean velocity}}{G}}{\mu}\right)^{0.8}\underbrace{\left(\frac{C_p\mu}{k}\right)^{1/3}}_{\substack{\text{Prandtl}\\ \text{number}}}\left(\frac{\mu}{\underset{\nwarrow \text{At wall temperature}}{\mu_w}}\right)^{0.14} \qquad (9.11)$$

A simplified approximation for common gases (error ± 25%):

$$h = 0.0018\frac{C_p G^{0.8}}{d^{0.2}} \quad \left[\mathrm{W/m^2\,K}\right] \tag{9.12}$$

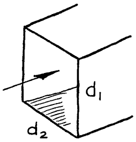
A simplified approximation for cooling or heating of water:

$$h = 91(T + 68)\frac{u^{0.8}}{d^{0.2}} \quad \left[\mathrm{W/m^2\,K}\right] \quad \text{with}\,T\,\text{in}\,{}^\circ\mathrm{C} \tag{9.13}$$

9.2.2 *Turbulent Flow in Noncircular Conduits*

1. *Rectangular cross section*. Use the equation for circular pipes, equation (9.11), with the following two modifications:

$$h_{\mathrm{rect}} \cong 0.76\, h_{\mathrm{pipes}} \tag{9.14}$$



and replace the pipe diameter with an equivalent diameter defined as

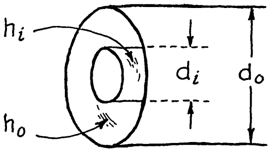
$$d_e = 4\begin{pmatrix}\text{hydraulic}\\ \text{radius}\end{pmatrix} = 4\left(\frac{\begin{matrix}\text{cross-sectional}\\ \text{area}\end{matrix}}{\text{perimeter}}\right) = \frac{2d_1 d_2}{d_1 + d_2} \tag{9.15}$$

2. *Annular passage*. For heat flow to the inner tube wall,

$$\frac{h_i d_e}{k} = 0.02\left(\frac{d_e G}{\mu}\right)^{0.8}\left(\frac{C_p \mu}{k}\right)^{1/3}\left(\frac{d_o}{d_i}\right)^{0.53} \tag{9.16}$$

where

$$d_e = 4\begin{pmatrix}\text{hydraulic}\\ \text{radius}\end{pmatrix} = d_o - d_i \tag{9.17}$$



To the outer tube wall, use equation (9.11) for circular pipes, but with the pipe diameter replaced by de of equation (9.17).

9.2.3 Transition Regime in Flow in Pipes

In the transition regime, $2{,}100 < \mathrm{Re} < 10{,}000$:

$$\frac{hd}{k} = 0.116\left[\left(\frac{dG}{\mu}\right)^{2/3} - 125\right]\left(\frac{C_p\mu}{k}\right)^{1/3}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{9.18}$$

9.2.4 Laminar Flow in Pipes (Perry and Chilton, pg. 168 (1984))

In the laminar flow regime, or $\mathrm{Re} < 2{,}100$, we have, for $\mathrm{Gz} < 100$,

$$\frac{hd}{k} = \left[3.66 + \frac{0.085\ \ \mathrm{Gz}}{1 + 0.047\ \ \mathrm{Gz}^{2/3}}\right]\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{9.19}$$

where the Graetz number is defined as

$$\mathrm{Gz} = \mathrm{Re}\cdot\mathrm{Sc}\cdot\frac{d}{L} = \left(\frac{dG}{\mu}\right)\left(\frac{C_p\mu}{k}\right)\left(\frac{d}{L}\right) \qquad [\text{-}] \tag{9.20}$$

For higher flow rates where $\mathrm{Gz} > 100$,

$$\frac{hd}{k} = 1.86\,\mathrm{Gz}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{9.21}$$

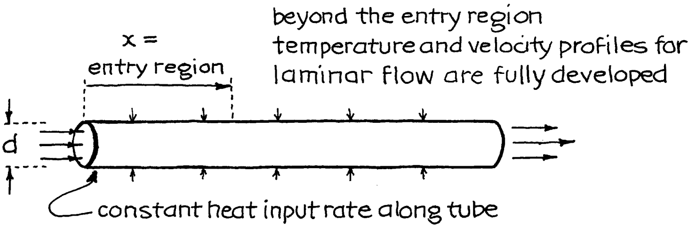
Perry and Chilton (1973) give numerous other expressions.

9.2.5 Laminar Flow in Pipes, Constant Heat Input Rate at the Wall (Kays and Crawford 1980)

When the velocity and temperature profiles are fully developed (away from the entrance region), axial dispersion theory predicts that

$$hd/k = 4.36 \tag{9.22}$$

In evaluating h the term ΔT is defined as the difference in temperature between the wall at position x and the mixing cup temperature of the flowing fluid at the same position. This situation is found when using electrical resistance heating or radiant heating.



Theory shows that the laminar velocity profile is fully developed at about

$$x/d = 0.05\ \mathrm{Re}$$

and that the thermal profile is fully developed at about

$$x/d = 0.05\ \mathrm{Re} \cdot \mathrm{Pr}$$

Thus, equation (9.22) only applies in tubes much longer than the larger of the above two entry lengths. Let us look at a few typical entry lengths at Re = 100:

Entry length, x/d				
Fluid	Pr	From velocity profile	From temperature profile	Slower developing profile
Liquid metal	0.01	5	0.05	Velocity
Water	1	5	5	Same for both
Oil	100	5	500	Temperatures

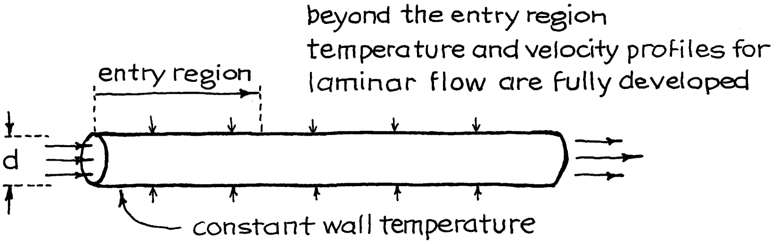
These values show that for liquid metals or ordinary aqueous fluids, the entry length is rather short. However, if oil or some other high Prandtl number fluid is flowing through the pipe, then the entry length may become substantial, and the value of *h* predicted by equation (9.22) will be too low. See Kays and Crawford, pg. 114 (1980), for *h* values for short pipes, and see Perry and Chilton (1973) for *h* values in other shaped ducts.

9.2.6 Laminar Flow in Pipes, Constant Wall Temperature (Kays and Crawford 1980)

This situation is approached when a process with high *h* occurs on the outside of the tubes (boiling, condensation, transfer to finned tubes). Here theory says that in the region of fully developed laminar velocity and temperature profiles,

$$\frac{hd}{k} = 3.66 \quad [\text{-}] \qquad (9.23)$$

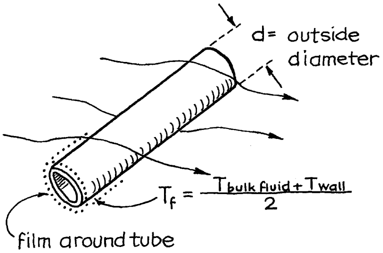
Again, this equation only applies when the pipe is much longer than the above two entry lengths. For shorter pipes the h value predicted by equation (9.23) will be too low. Kays and Crawford, pg. 128 (1980), give h values for short pipes, and Perry and Chilton (1973) give h values for other shaped ducts.



9.2.7 Flow of Gases Normal to a Single Cylinder

Over a very wide range of Reynolds numbers, experimental results can be correlated by

$$\frac{hd}{k_f} = A\left(\frac{dG}{\mu_f}\right)^n \left(\frac{C_p\mu}{k_f}\right)^{0.3} \qquad (9.24)$$



where subscript f refers to properties of the gas at the film temperature estimated as

$$T_f = \frac{T_{\text{bulk fluid}} + T_{\text{wall}}}{2}$$

and where the constants A and n are given in Table 9.2. For air at 93 °C and Re = 1,000–50,000, we have the following simplified equation:

$$h = 0.0018\frac{C_p G^{0.6}}{d^{0.4}} \quad [\text{W/m}^2\,\text{K}] \qquad (9.25)$$

Table 9.2 Constants in equation (9.24) for flow normal to single cylinders

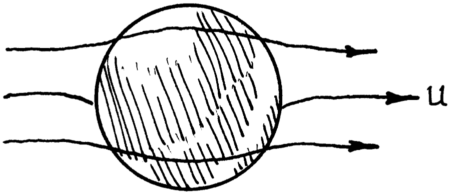
$\frac{du\rho_f}{\mu_f}$	A	n	$\frac{hd}{k_f}$ for air, from equation (9.24)
1–4	0.960	0.330	0.890–1.42
4–40	0.885	0.385	1.40–3.40
40–4,000	0.663	0.466	3.43–29.6
4,000–40,000	0.174	0.618	29.5–121.
40,000–250,000	0.257	0.805	121.–528.

9.2.8 Flow of Liquids Normal to a Single Cylinder

For Re = 0.1–300, the data are correlated by

$$\frac{hd}{k_f} = \left[0.35 + 0.56\left(\frac{dG}{\mu_f}\right)^{0.52}\right]\left(\frac{C_p\mu}{k_f}\right)^{0.3} \tag{9.26}$$

9.2.9 Flow of Gases Past a Sphere



$$\frac{hd}{k_f} = 2 + 0.6\left(\frac{dG}{\mu_f}\right)^{0.5}\left(\frac{C_p\mu}{k}\right)_f^{1/3} \quad \text{for}\left(\frac{dG}{\mu_f}\right) < 325 \tag{9.27}$$

$$\frac{hd}{k_f} = 0.4\left(\frac{dG}{\mu_f}\right)^{0.6}\left(\frac{C_p\mu}{k}\right)_f^{1/3} \quad \text{for}\left(\frac{dG}{\mu_f}\right) = 325 - 70{,}000 \tag{9.28}$$

9.2.10 Flow of Liquids Past a Sphere

$$\frac{hd}{k_f} = \left[0.97 + 0.68\left(\frac{dG}{\mu_f}\right)^{0.52}\right]\left(\frac{C_p\mu}{k}\right)_f^{0.3} \tag{9.29}$$

9.2.11 Other Geometries

For tube banks, h values can be up to 50 % higher than for single tubes, the actual value depending on the number of rows and the geometry used. For tube banks, coiled tubes, tubes of noncircular cross section, finned tubes, and many other situations, see McAdams (1954), Chap. 10.

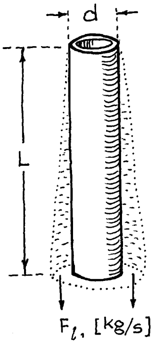
9.2.12 Condensation on Vertical Tubes

The theoretical equation derived by Nusselt in 1916 is still recommended today:

$$\frac{hL}{k_l} = 0.925\left(\frac{L^3\rho_l^2 g}{\mu_l \Gamma}\right)^{1/3} = 0.943\left(\frac{L^2\rho_l^2 g\lambda}{k_l \mu_l \Delta T}\right)^{1/4} \tag{9.30}$$

where

$$\Gamma = \left(\frac{\text{flow rate of condensate from the tube}}{\text{circumference}}\right) = \frac{F_l}{\pi d} \quad [\text{kg/s m}] \tag{9.31}$$



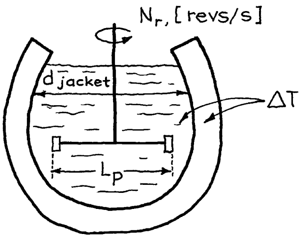
For steam condensing at atmospheric conditions, this equation reduces to

$$h = 0.97\left(\tfrac{d}{F_l}\right)^{1/3} \quad [\text{W/m}^2\,\text{K}] \tag{9.32}$$

9.2.13 Agitated Vessels to Jacketed Walls

For various types of agitators, we have the general expression

$$\frac{hd_{\text{jacket}}}{k} = a\underbrace{\left(\frac{L_p^2 N_r \rho}{\mu}\right)}_{\text{Reynolds number for agitated vessels}}^{b}\left(\frac{C_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^m \tag{9.33}$$



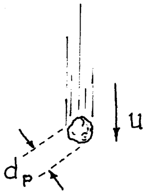
where the constants a, b, and m are given in Table 9.3.

Table 9.3 Constants in equation (9.33) for heat transfer to the walls of agitated vessels

Type of agitator	*a*	*b*	*m*	Range of Re
Paddle	0.36	2/3	0.21	$300–3 \times 10^5$
Pitched blade turbine	0.53	2/3	0.24	80–200
Disk, flat blade turbine	0.54	2/3	0.14	$40–3 \times 10^5$
Propeller	0.54	2/3	0.14	2×10^3
Anchor	1.0	1/2	0.18	10–300
Anchor	0.36	2/3	0.18	300–40,000
Helical ribbon	0.633	1/2	0.18	$8–10^5$

9.2.14 Single Particles Falling Through Gases and Liquids (Ranz and Marshall 1952)

$$\frac{hd_p}{k} = 2 + 0.6\left(\frac{d_p u \rho}{\mu}\right)^{1/2}\left(\frac{C_p \mu}{k}\right)^{1/3} \tag{9.34}$$



9.2.15 Fluid to Particles in Fixed Beds (Kunii and Levenspiel, 1991)

(a) For beds of fine solids

$$\text{with gases}: \quad \frac{hd_p}{k} = 0.012\,\text{Re}_p^{1.6}\text{Pr}^{1/3} \quad \text{for Re}_p < 100 \tag{9.35}$$

$$\text{with liquids}: \quad \frac{hd_p}{k} = 0.16\,\text{Re}_p^{1.6}\text{Pr}^{1/3} \quad \text{for Re}_p < 10 \tag{9.36}$$

(b) For coarse solids with both gases and liquids

$$\frac{hd_p}{k} = 2 + 1.8\,\text{Re}_p^{1/2}\text{Pr}^{1/3} \quad \begin{cases} \text{for Re}_p > 100, \text{ gases} \\ \text{for Re}_p > 10, \text{ liquids} \end{cases} \tag{9.37}$$

where $\text{Re}_p = (dpu_0\rho/\mu)$ and u_0 = superficial velocity (upstream velocity or in vessel with no solids).

9.2.16 Gas to Fluidized Particles

The heat transfer coefficient is difficult to measure in this situation, so until reliable data becomes available, the following equation is suggested as a conservative estimate of h:

$$\frac{hd_p}{k} = 2 + 0.6\left(\frac{d_p u_0 \rho}{\mu}\right)^{1/2}\left(\frac{C_p \mu}{k}\right)^{1/3} \tag{9.38}$$

9.2.17 Fluidized Beds to Immersed Tubes

For beds of fine particles, or $\mathrm{Re}_{mf} < 12.5$, Botterill (1983) recommends the following simple dimensional expression (in SI units):

$$\frac{hd_p}{k_g} = 25\frac{d_p^{0.64}\rho_s^{0.2}}{\left[k_g(\text{at bed temperature})\right]^{0.4}} \tag{9.39}$$

For beds of large particles, or $\mathrm{Re}_{mf} > 12.5$, Botterill suggests using

$$\frac{hd_p}{k_g} = 0.7\left[d_p^{0.5}\left(d_p^*\right)^{1.17} + \left(d_p^*\right)^{0.45}\right] \tag{9.40}$$

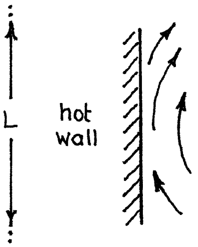
where d_p^* is defined by equation (8.10).

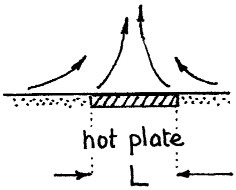
9.2.18 Fixed and Fluidized Particles to Bed Surfaces

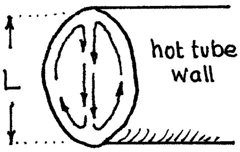
See Kunii and Levenspiel (1991), Chap. 13, equations (16)–(20).

9.2.19 Natural Convection

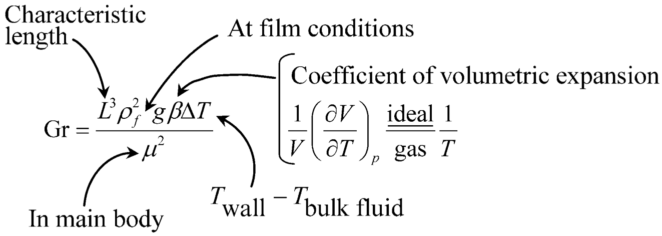
Slow-moving fluids passing by hot surfaces give larger than expected h values. This is because of natural convection. The particular variables which







characterize natural convection are combined into a dimensionless group, the Grashof number, defined as



Correlations for natural convection are often of the form

$$\left.\begin{aligned} &\mathrm{Nu} = A[\mathrm{Gr} \cdot \mathrm{Pr}]^B \\ \text{or} \quad &\frac{hL}{k} = A\Bigg[\underbrace{\left(\frac{L^3 \rho_f^2 g \beta \Delta T}{\mu_f^2}\right)\left(\frac{C_p \mu_f}{k_f}\right)}_{X}\Bigg]^B \\ \text{or} \quad &Y = AX^B \end{aligned}\right\} \tag{9.41}$$

9.2.20 *Natural Convection: Vertical Plates and Cylinders,* L > *1 m*

$$\text{Laminar}: \quad Y = 1.36\, X^{1/5} \quad \text{for}\, X < 10^4 \tag{9.42}$$

$$\text{Laminar}: \quad Y = 0.55\, X^{1/4} \quad \text{for}\, X = 10^4 - 10^9 \tag{9.43}$$

$$\text{Turbulent}: \quad Y = 0.13\, X^{1/3} \quad \text{for}\, X > 10^9 \tag{9.44}$$

Simplified equations for air at room conditions:

$$h = 1.4\left(\frac{\Delta T}{L}\right)^{1/4} \quad [\mathrm{W/m^2\,K}] \quad \text{for laminar regime} \tag{9.45}$$

$$h = 1.3(\Delta T)^{1/3} \quad [\mathrm{W/m^2\,K}] \quad \text{for turbulent regime} \tag{9.46}$$

and for water at room conditions:

$$h = 120(\Delta T)^{1/3} \quad [\mathrm{W/m^2\,K}] \quad \text{for} X > 10^9 \tag{9.47}$$

9.2.21 Natural Convection: Spheres and Horizontal Cylinders, d < 0.2 m

$$\text{Laminar}: \quad Y = 0.53 \ X^{1/4} \qquad \text{for} X = 10^3 - 10^9 \tag{9.48}$$

$$\text{Turbulent}: \quad Y = 0.13\, X^{1/3} \qquad \text{for}\, X > 10^9 \tag{9.49}$$

For $X < 10^4$, see Perry and Chilton (1973).

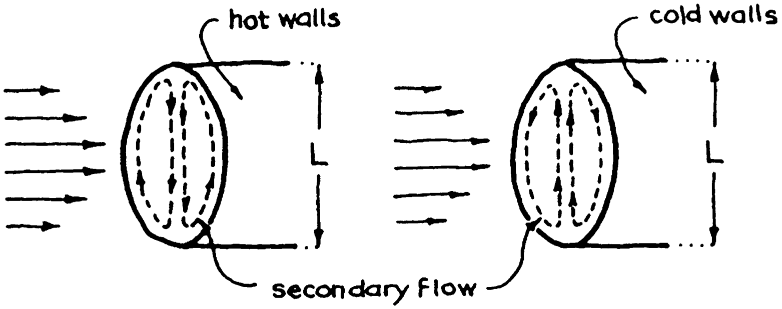
Simplified equations for air at room conditions:

$$h = 1.3(\Delta T/L)^{1/4} \quad [\mathrm{W/m^2\,K}] \quad \text{for laminar regime} \tag{9.50}$$

$$h = 1.2(\Delta T)^{1/3} \quad [\mathrm{W/m^2\,K}] \quad \text{for turbulent regime, the usual case for pipes} \tag{9.51}$$

9.2.22 Natural Convection for Fluids in Laminar Flow Inside Pipes

In laminar flow, when Gr > 1,000, natural convection sets up an appreciable secondary flow of fluid in the pipe which in turn increases the heat transfer coefficient. In this situation equations (9.19), (9.20), and (9.23) for laminar flow should include the additional multiplying factor:



$$0.87\left(1 + 0.015\,\mathrm{Gr}^{1/3}\right) \tag{9.52}$$

For turbulent flow no such correction is needed because the tendency to set up a secondary flow pattern is effectively overwhelmed by the vigorous turbulent eddies.

9.2.23 Natural Convection: Horizontal Plates

(a) For heated plates facing up or cooled plates facing down:

$$\text{Laminar}: \quad Y = 0.54\,X^{1/4} \quad \text{for}\,X = 10^5 - 2 \times 10^7 \tag{9.53}$$

$$\text{Turbulent}: \quad Y = 0.14\,X^{1/3} \quad \text{for}\,X = 2 \times 10^7 - 3 \times 10^{10} \tag{9.54}$$

(b) For heated plates facing down or cooled plates facing up:

$$\text{Laminar}: \quad Y = 0.27\,X^{1/4} \quad \text{for}\ X = 3 \times 10^5 - 3 \times 10^{10} \tag{9.55}$$

(c) The three corresponding simplified equations for air at room conditions:

$$h = 1.3\left(\tfrac{\Delta T}{L}\right)^{1/4} \quad [\text{W/m}^2\,\text{K}] \quad \text{for laminar regime} \tag{9.56}$$

$$h = 1.5\Delta T^{1/3} \quad [\text{W/m}^2\,\text{K}] \quad \text{for turbulent regime} \tag{9.57}$$

$$h = 0.64\left(\tfrac{\Delta T}{L}\right)^{1/4} \quad [\text{W/m}^2\,\text{K}] \quad \text{for laminar regime} \tag{9.58}$$

9.2.24 Other Situations

Heat transfer coefficients for boiling, condensation, high-velocity gas flow (compressibility effects and supersonic flow), high-vacuum flow, and many other situations have been studied and reported in the vast heat transfer literature and are well condensed in McAdams (1954), in Perry and Chilton (1973), and in Cavaseno (1979).

9.3 Heat Transfer by Radiation

All materials emit, absorb, and transmit radiation to an extent which is strongly dependent on their temperature. Let

$$\alpha_{1\leftarrow 2} = \frac{\text{energy absorbed by a surface at } T_1}{\text{energy incident coming from a source } T_2}, \quad \text{the absorptivity} \tag{9.59}$$

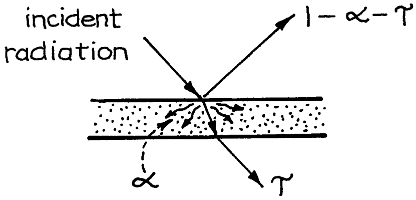
The absorptivity varies from 0 to 1. The perfect absorber has $\alpha = 1$ and is called a blackbody. Next, let

$$\varepsilon_1 = \frac{\text{energy emitted by a surface at } T_1}{\text{energy emitted by an ideal emitter, a blackbody, at} T_1}, \quad \text{emissivity} \tag{9.60}$$

and

$$\tau_1 = \frac{\text{energy transmitted through the body at } T_1}{\text{energy incident}}, \quad \text{transmittance} \tag{9.61}$$

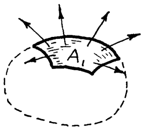
Then, the fraction of energy reflected is $1 - \alpha - \tau$.



9.3.1 Radiation from a Body

The energy emitted from surface A_1 of a body is strongly dependent on the temperature and nature of the surface and is given by

$$\dot{q}_{1\rightarrow} = \sigma A_1 \varepsilon_1 T_1^4 \quad [\text{W}] \tag{9.62}$$



where the radiation constant

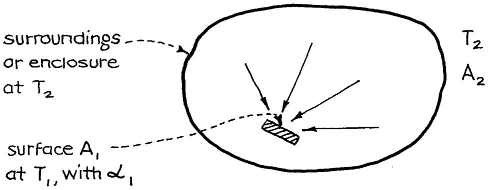
$$\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\ \text{K}^4 \tag{9.63}$$

is called the Stefan–Boltzmann constant. Equation (9.62) is called the Stefan–Boltzmann law of radiation, and the fourth power of temperature is a consequence of the second law of thermodynamics.

9.3.2 Radiation onto a Body

The energy absorbed by a surface A_1 which is at T_1 from blackbody surroundings at T_2 is given by

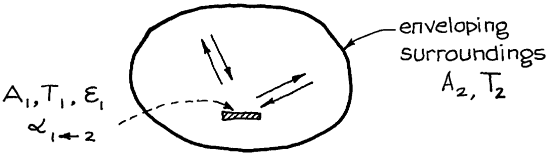
$$\dot{q}_{1\leftarrow 2} = \sigma A_1 \alpha_{1\leftarrow 2} T_2^4 \quad [\mathrm{W}] \tag{9.64}$$



9.3.3 Energy Interchange Between a Body and Its Enveloping Surroundings

The energy interchange between a surface A_1 at T_1 and any kind of surroundings at T_2, from equations (9.62) and (9.64), is then

$$\dot{q}_{12} = \sigma A_1 \left[\varepsilon_1 T_1^4 - \alpha_{1\leftarrow 2} T_2^4 \right] \tag{9.65}$$



9.3.4 Absorptivity and Emissivity

If an object and its surroundings are both at T_1, then the object does not gain or lose heat. Thus, equation (9.65) becomes

$$\dot{q}_{12} = 0 = \sigma A_1 \left[\varepsilon_1 T_1^4 - \alpha_{1\leftarrow 1} T_1^4 \right]$$

Now the value of α and of ε can vary greatly with the type of surface and with temperature, as shown in Table 9.4. However, at any particular temperature T_1, the above expression shows that

$$\varepsilon_1 = \alpha_{1\leftarrow 1} \tag{9.66}$$

Table 9.4 Short table of absorptivities and emissivities of various materials[a]

Material	For solar radiation (~ 5,000 K) onto a surface at room temperature, $\alpha_{\mathrm{room} \leftarrow \mathrm{solar}}$	For room temperature radiation, $\varepsilon_{\mathrm{room}} = \alpha_{\mathrm{room} \leftarrow \mathrm{room}}$
Ag, polished	0.07	0.01
Al, bright foil or polished	0.1–0.3	0.04–0.09
Cu, polished	0.18	0.02–0.04
Galvanized iron, weathered	0.89	0.23–0.28
Hg, clean	—	0.09
Stainless steel # 301, polished	0.37	0.16
White paint, gloss	0.18	0.92–0.96
Black paint, flat	0.97	0.96–0.98
Aluminum paint	0.55	0.51–0.67
Asphalt pavement, clean	0.93	—
Concrete, rough	—	0.94
Earth, plowed field	0.75	—
Grass	0.75–0.80	—
Gravel	0.29	—
Red brick, rough	0.7–0.75	0.93
Roofing paper, black	—	0.91
White paper	0.28	0.95
Wood	—	0.90–0.04
Snow, clean	0.2–0.35	0.82
Ice	—	0.97
Water, deep	—	0.96

[a] Taken from references in this chapter

This means that the absorptivity of a surface for T_1 radiation equals the emissivity of that surface when it is at T_1.

9.3.5 Greybodies

An object whose absorptivity is the same for all temperature radiation is called a greybody. So for a greybody

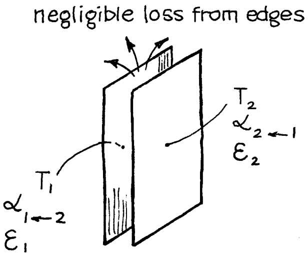
$$\alpha = \varepsilon = \text{const, at all temperatures}$$

The greybody approximation is often used since it greatly simplifies difficult analyses.

9.3.6 Radiation Between Two Adjacent Surfaces

If the facing surfaces are close enough so that all radiation leaving one surface hits the other, then the heat interchange is

$$\dot{q}_{12} = \frac{\sigma A\left[\alpha_{2\leftarrow 1}\varepsilon_1 T_1^4 - \alpha_{1\leftarrow 2}\varepsilon_2 T_2^4\right]}{\alpha_{1\leftarrow 2} + \alpha_{2\leftarrow 1} - \alpha_{2\leftarrow 1}\alpha_{1\leftarrow 2}} \tag{9.67}$$

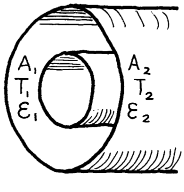


1. For *two facing grey surfaces*, $\alpha_{2\leftarrow 1} = \varepsilon_2$, $\alpha_{1\leftarrow 2} = \varepsilon_1$, so the above expression reduces to

$$\dot{q}_{12} = \frac{\sigma A\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \tag{9.68}$$

2. For *concentric grey cylinders*, we obtain, similarly,

$$\dot{q}_{12} = \frac{\sigma \overset{\text{Always the smaller area}}{A_1}\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \tag{9.69}$$



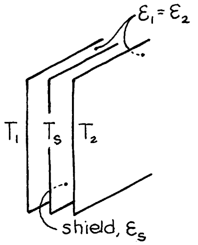
9.3.7 Radiation Between Nearby Surfaces with Intercepting Shields

If two facing surfaces are separated by a very thin opaque shield, then if $\varepsilon_1 = \varepsilon_2$, while ε_s can be any value, we find

$$T_s^4 = \frac{T_1^4 + T_2^4}{2} \tag{9.70}$$

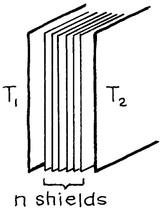
from which

$$\dot{q}_{12} = \frac{1}{2}\frac{\sigma A\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_s} - 1} \tag{9.71}$$



Thus, a shield of material similar to the two radiating surfaces will halve the radiation transfer between two closely facing surfaces. Extending this kind of analysis, we find for n shields of identical emissivity that

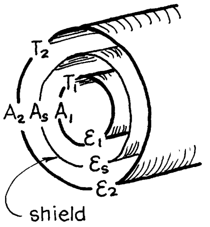
$$\dot{q}_{12} = \frac{1}{n+1}\frac{\sigma A\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_s} - 1} \tag{9.72}$$



Similarly, for a shield between pipes or around a sphere,

Always the inside pipe

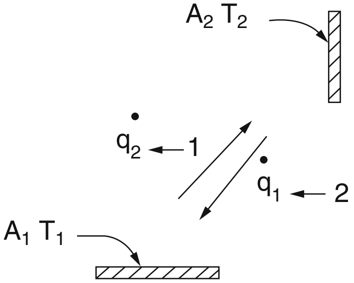
$$\dot{q}_{12} = \frac{\sigma A_1\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{A_1}{A_s}\left(\frac{2}{\varepsilon_s} - 1\right)} \tag{9.73}$$



In all cases radiation shields reduce the radiative heat interchange between bodies.

9.3.8 View Factors for Blackbodies

If both surfaces are black and not close together, then only a portion of the radiation leaving surface 1 is intercepted by surface 2. We call this the



view factor F_{12}, and so the radiation leaving 1 which is intercepted by 2 is

$$\dot{q}_{2\leftarrow1} = \sigma A_1 F_{12} T_1^4 \tag{9.74}$$

Similarly, the radiation leaving 2 which is intercepted by 1 is

$$\dot{q}_{1\leftarrow2} = \sigma A_2 F_{21} T_2^4 \tag{9.75}$$

If both temperatures are equal, there can be no net transfer of heat between 1 and 2. Thus, we find

$$A_1 F_{12} = A_2 F_{21} \tag{9.76}$$

The net interchange of heat between these two surfaces is then

$$\dot{q}_{12} = \sigma A_1 F_{12}\left[T_1^4 - T_2^4\right] \quad \text{with } A_1 F_{12} = A_2 F_{21} \tag{9.77}$$

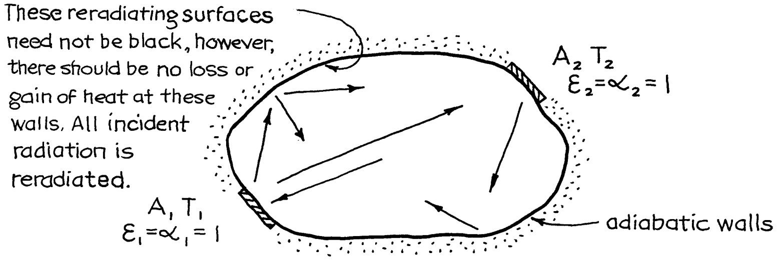
If the surfaces are grey, not black, the heat exchange is approximated by

$$\dot{q}_{12} = \sigma A F'_{12}\left[T_1^4 - T_2^4\right] \tag{9.78}$$

where

$$F'_{12} = \frac{1}{\frac{1}{F_{12}} + \left(\frac{1}{\varepsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \tag{9.79}$$

9.3.9 View Factor for Two Blackbodies (or GreyBodies) Plus Reradiating Surfaces



The net heat interchange between black surfaces 1 and 2 in the presence of adiabatic reradiating surfaces is given by

$$\dot{q}_{12} = \sigma A_1 \overline{F}_{12}\left(T_1^4 - T_2^4\right) \tag{9.80}$$

where $\overline{F}_{12}$ depends on F_{12} and the geometry of the reradiating surfaces. After making simplifying assumptions that surfaces 1 and 2 cannot see themselves and that the reradiating surfaces are all at one temperature, we find that

$$\overline{F}_{12} = \frac{A_2 - A_1 F_{12}^2}{A_1 + A_2 - 2A_1 F_{12}} \tag{9.81}$$

Because these reradiating surfaces return some of the radiation which would otherwise be lost suggests that $\overline{F}_{12}$ is always larger than F_{12}, and this is so.

If surfaces 1 and 2 are grey, then

$$\dot{q}_{12} = \sigma A_1 \mathscr{F}_{12}\left(T_1^4 - T_2^4\right) \tag{9.82}$$

where 1_{12} is the greybody view factor for systems with reradiating surfaces and is approximated by

$$\mathscr{F}_{12} = \frac{1}{\frac{1}{\overline{F}_{12}} + \left(\frac{1}{\varepsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \tag{9.83}$$

This is the most general of view factors.

The appendix in Siegel and Howell (1981) refers to view factors for over 200 different kinds of geometries and gives equations for 38 of these geometries. Figures 9.1, 9.2, 9.3, 9.4, and 9.5, from Jakob (1957), show the view factors for five simple geometries.

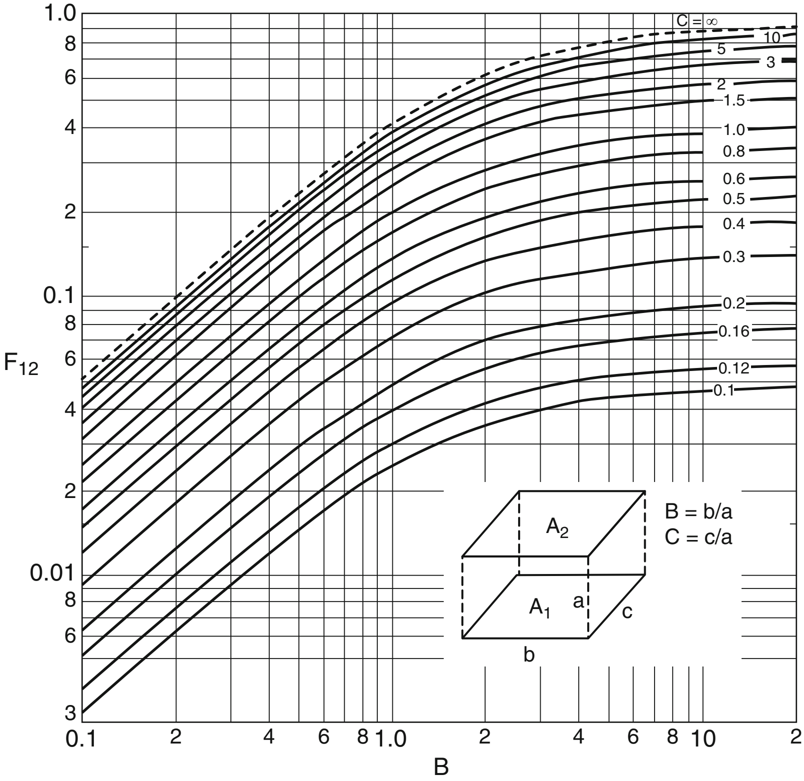


Fig. 9.1 View factor for two facing equal-sized rectangular surfaces

$$F_{12}=\frac{1}{\pi}\left[\frac{1}{BC}\ln\frac{1+B^2+C^2+B^2C^2}{1+B^2+C^2}+\frac{2}{C}\left(1+C^2\right)^{1/2}\tan^{-1}\frac{B}{\left(1+C^2\right)^{1/2}}\right.$$
$$\left.-\frac{2}{B}\tan^{-1}C-\frac{2}{C}\tan^{-1}B+\frac{2}{B}\left(1+B^2\right)^{1/2}\tan^{-1}\frac{C}{\left(1+B^2\right)^{1/2}}\right]$$

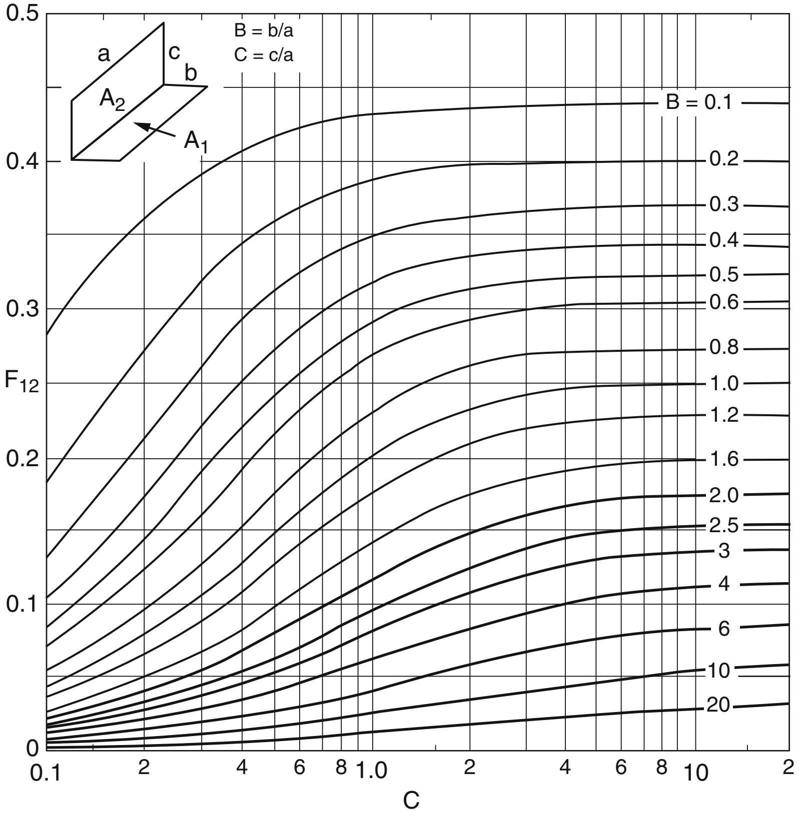


Fig. 9.2 View factor for two perpendicular rectangular surfaces having a common edge

$$
\begin{aligned}
F_{12} = \frac{1}{\pi B}\Bigg\{ & \frac{1}{4}\ln\Big[\left(1+B^2+C^2\right)^{B^2+C^2-1}\left(1+B^2\right)^{1-B^2} \\
& \times \left(1+C^2\right)^{1-C^2}\left(B^2\right)^{B^2}\left(C^2\right)^{C^2}\Big] \\
& - \frac{1}{4}\ln\left(B^2+C^2\right)^{B^2+C^2} + B\,\tan^{-1}\frac{1}{B} + C\tan^{-1}\frac{1}{C} \\
& - \left(B^2+C^2\right)^{1/2}\tan^{-1}\frac{1}{\left(B^2+C^2\right)^{1/2}}\Bigg\}
\end{aligned}
$$

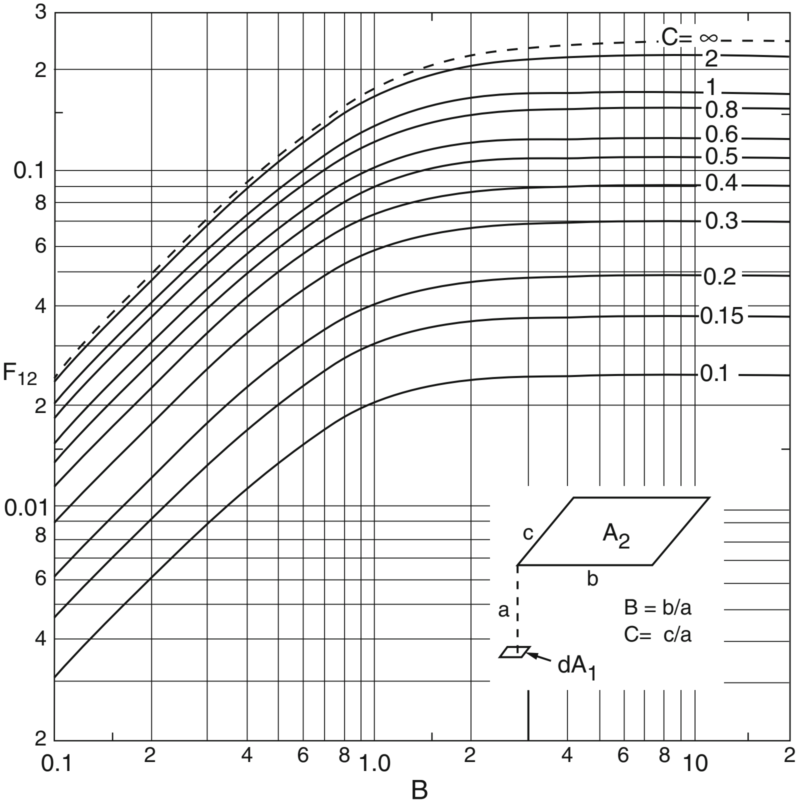


Fig. 9.3 Fraction of radiation leaving a differential surface which is intercepted by a facing rectangular surface, located as shown

$$F_{12} = \frac{1}{2\pi}\left(\frac{B}{\left(1+B^2\right)^{1/2}}\sin^{-1}\frac{C}{\left(1+B^2+C^2\right)^{1/2}} + \frac{C}{\left(1+C^2\right)^{1/2}}\sin^{-1}\frac{B}{\left(1+B^2+C^2\right)^{1/2}}\right)$$

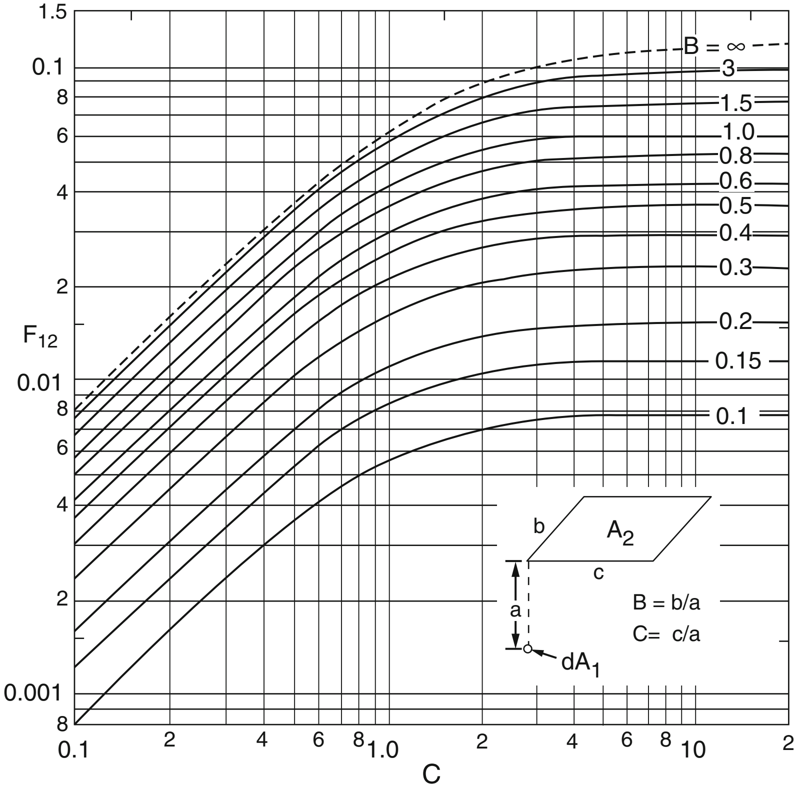


Fig. 9.4 Fraction of radiation leaving a differential sphere which is intercepted by a rectangular surface, located as shown

$$F_{12} = \frac{1}{4\pi} \sin^{-1} \frac{BC}{\left(1 + B^2 + C^2 + B^2 C^2\right)^{1/2}}$$

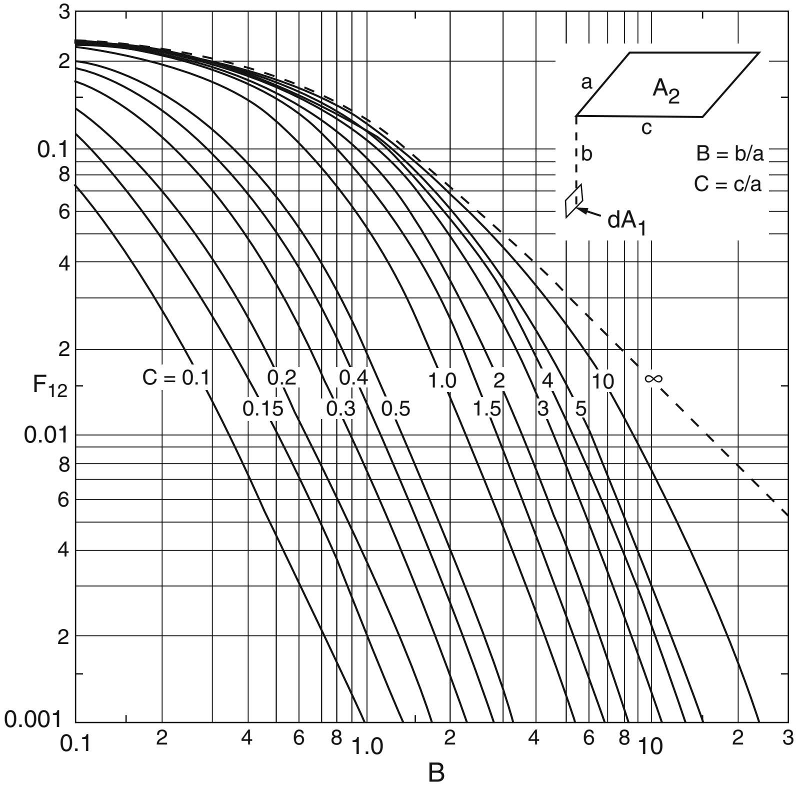


Fig. 9.5 Fraction of radiation leaving a differential surface which is intercepted by a rectangular surface perpendicular to it and located as shown

$$F_{12} = \frac{1}{2\pi}\left[\sin^{-1}\frac{1}{\left(1+B^2\right)^{1/2}} - \frac{B}{\left(B^2+C^2\right)^{1/2}}\sin^{-1}\frac{1}{\left(1+B^2+C^2\right)^{1/2}}\right]$$

9.3.10 Extensions

Gases consisting of molecules which are not symmetrical about all three principal axes (NH_3, CO, CO_2, H_2O, HCl, etc.) absorb and emit significant amounts of radiation at high temperature. Symmetrical molecules (O_2, N_2, H_2, etc.) do not absorb or emit significantly in the temperature range of practical interest.

Heat interactions between absorbing gases and surfaces are accounted for by a characteristic emissivity and a characteristic view factor, somewhat like two-surface systems. Clouds of fine particles, soot, luminous flames, etc., are treated in the same manner.

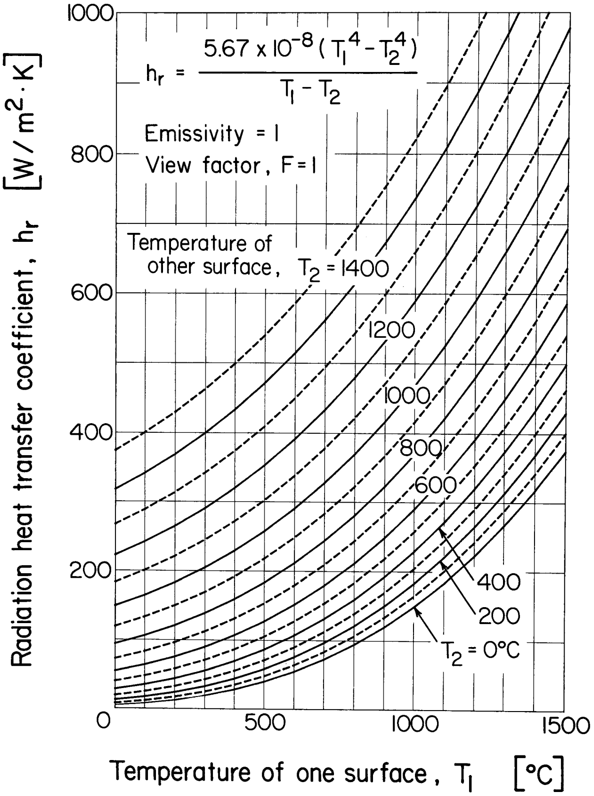


Fig. 9.6 Radiation between two surfaces in terms of a heat transfer coefficient

We do not take up these subjects here. The reader is referred to the references for further readings on these subjects.

9.3.11 Estimating the Magnitude of $\mathbf{h_r}$

For design we want to know whether radiative transfer is appreciable compared to the other competing mechanisms of heat transfer and whether it need be considered at all in any analysis. Figure 9.6 is helpful for this purpose. It gives the radiation heat transfer coefficient between two closely facing ($F = 1$) black surfaces ($\varepsilon = \alpha = 1$). To find $h_{\mathrm{r,actual}}$ for a particular situation, lower the h_r value given in this figure to account for $1 < 1$ and $\varepsilon < 1$. Thus,

$$h_{r,\,\mathrm{actual}} = \varepsilon \mathscr{F} h_{r,\,\mathrm{figure}}$$

If $h_r \ll h_{\mathrm{convection}}$, ignore the radiation contribution to the overall heat transfer. A look at this figure shows that h_r becomes very large at high temperature.

Problems on Conduction, Convection, and Radiation

9.1. Which paint (black, white, or aluminum) can be roughly considered to be a blackbody, a greybody, or neither?

9.2. A pottery oven with 10-cm-thick walls has a number of 6 cm × 5 cm peep holes for monitoring the progress of the firing. How much heat will be lost from the 1,500 °C oven when a peep hole plug is removed?

9.3. A hot black painted pipe passes through a 20 °C room to heat it. At what pipe temperature is its convective heat loss and its radiative heat loss equal?

9.4. The Sun, a blackbody 1,392,000 km in diameter and 148×10^4 km away from the Earth, emits 6,150 K radiation. With the Sun directly overhead, what is the Sun's energy flux at Earth's surface (W/m_2)? Ignore absorption of radiation by clouds and atmosphere. Earth's diameter is 12,732 km.

References and Related Readings

Conduction

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