Chapter 11
Unsteady-State Heating and Cooling of Solid Objects

If a hot object is plunged into cold water, it cools, but not instantaneously. Two factors govern the cooling rate of the object:

- The film resistance at the surface of the object, characterized by the *h* value for that situation.
- The rate of heat flow out of the interior of the object. The governing differential equation for this conduction process is

$$\frac{\partial T_s}{\partial t} = \alpha\left(\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} + \frac{\partial^2 T_s}{\partial z^2}\right) \tag{11.1}$$

where

$$\alpha = \frac{k_s}{\rho_s C_s}, \text{thermal diffusivity} \left[\mathrm{m^2/s}\right] \qquad k_s: \frac{W}{mK}, \text{ thermal conductivity}$$

and

$$T_s = \text{temperature at any point in the object } [\mathrm{K}]$$

A dimensionless measure for conduction, which accounts for both the cooling time and the size of object, is given by the Fourier number

$$\mathrm{Fo} = \frac{\alpha t}{L^2} = \frac{\alpha t}{(V/A)^2} = \frac{k_s}{\rho_s C_s} \cdot \frac{t}{(V/A)^2} \quad [\text{-}] \tag{11.2}$$

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where the characteristic length of the object

$$L\begin{cases} = \dfrac{\text{volume}}{\text{surface}} = \dfrac{V}{A} & \text{in general} \\ = \dfrac{\text{thickness}}{2} & \text{for a slab} \\ = \dfrac{R}{2} & \text{for a cylinder} \\ = \dfrac{R}{3} & \text{for a sphere} \end{cases} \qquad (11.3)$$

The relative importance of the surface and the interior resistance terms is measured by the Biot number, a dimensionless group defined as

$$\text{Bi} = \left(\frac{\begin{matrix}\text{interior}\\ \text{resistance}\end{matrix}}{\begin{matrix}\text{surface}\\ \text{resistance}\end{matrix}} \right) = \frac{h(V/A)}{k_s} = \frac{hL}{k_s} \quad [\text{-}] \qquad (11.4)$$

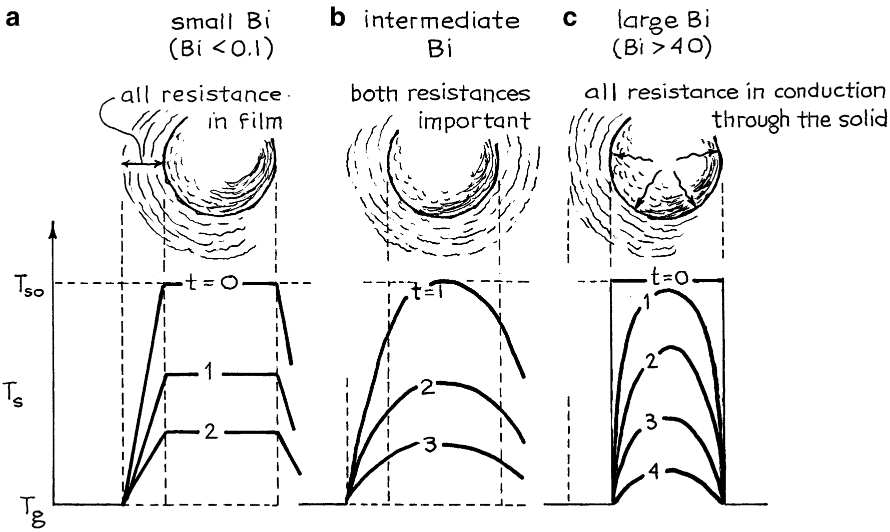


Fig. 11.1 Temperature–time history of a cooling particle for different ranges of Biot numbers

For a small Biot number, the main resistance is in the film; for a large Biot number, the main resistance is conduction of heat out of the body. Figure 11.1 shows the temperature–time history in various regimes for a spherical particle.

We first consider the two extreme cases and then the general case where both resistances are important. Design charts present compactly the whole range of situations for various shapes of particles.

11.1 The Cooling of an Object When All the Resistance Is at Its Surface (Bi = $hL/k_s \to 0$)

This extreme views the object to be isothermal at any time, the whole object cooling (or heating) with time, as illustrated in Figs. 11.1a and 11.2. This type of analysis where the system in question has uniform properties throughout is called a *lumped parameter analysis*. A heat balance about the hot object being cooled then gives

$$-\dot{q} = \begin{pmatrix}\text{heat transfer rate}\\ \text{through the film}\end{pmatrix} = \begin{pmatrix}\text{rate of heat loss}\\ \text{from the object}\end{pmatrix} \quad [\mathrm{W}]$$

$$= hA(T_s - T_g) = -\underbrace{W}_{V\rho_s}C_s\frac{dT_s}{dT} \qquad (11.5)$$

Separating and integrating at constant $hA/V\rho_sC_s$ gives

$$\frac{\Delta T}{\Delta T_{max}} = \frac{T_s - T_g}{T_{s0} - T_g} = e^{-\mathrm{Fo}\cdot\mathrm{Bi}} = e^{-(ht/L\rho_sC_s)} \quad [-] \qquad (11.6)$$

$L = V / A$, characteristic length

The instantaneous rate of heat loss from the object is found by combining equations (11.5) and (11.6), or

$$-\dot{q} = -\rho_sC_sV\frac{dT_s}{dt} = hA(T_{s0} - T_g)e^{-\mathrm{Fo}\cdot\mathrm{Bi}} \quad [\mathrm{W}] \qquad (11.7)$$

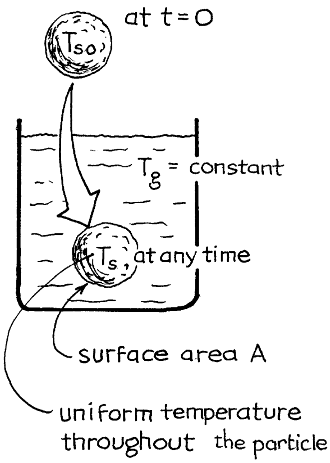


Fig. 11.2 Cooling of particle with all resistance in film

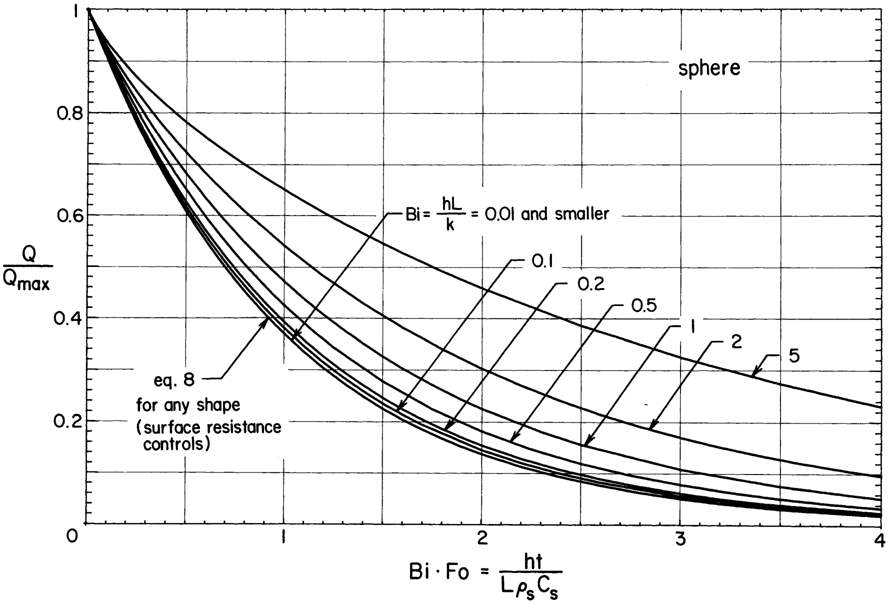


Fig. 11.3 The cooling of any shape of object when surface resistance controls (lowest curve) and the cooling of spheres in general (all the curves). The *y*-axis represents the fraction of heat remaining in the object

Also, the fractional cooling of the object is found either by integrating equation (11.7) or, more simply, by inspection of equation (11.6). Thus,

$$\frac{Q}{Q_{\max}} = \left(\frac{\text{heat left at the object at time } t}{\text{total heat which could be lost}}\right) = \frac{T_s - T_g}{T_{s0} - T_g} = e^{-\mathrm{Fo}\cdot\mathrm{Bi}} = e^{-(ht/L\rho_s C_s)} \tag{11.8}$$

The lowest curve of Fig. 11.3 represents the equation for this extreme. Note that one curve represents all shapes of solids.

In practice, if Bi <0.1 (see Fig. 11.3), then one can reasonably assume that film resistance controls.

11.2 The Cooling of an Object Having Negligible Surface Resistance (Bi = $hL/k_s \to \infty$)

In this extreme, when the hot object is plunged into cold fluid, its surface immediately drops to the temperature of the fluid, and conduction within the object is all important. This is illustrated in Fig. 11.1c and in Fig. 11.4. Solving equation (11.1)

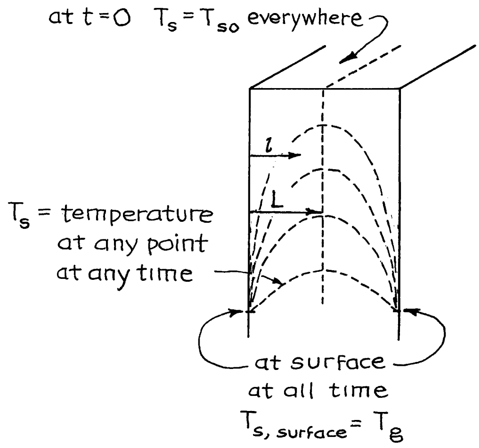


Fig. 11.4 The cooling of an object of negligible surface resistance

for a slab gives a rapidly converging infinite series for the temperature T_s at any location and any time:

$$\frac{\Delta T}{\Delta T_{\max}} = \frac{T_s - T_g}{T_{s0} - T_g} = \frac{4}{\pi}\left(e^{-a}\sin b + \frac{1}{3}e^{-9a}\sin 3b + \frac{1}{5}e^{-25a}\sin 5b + \cdots\right) \quad (11.9)$$

where

$$a = \frac{\pi\alpha t}{4L^2}$$

and

$$b = \frac{\pi l}{2L}$$

The fraction of total heat remaining in the slab is then given by

$$\frac{Q}{Q_{\max}} = \left(\frac{\text{heat remaining in slab}}{\text{total heat initially in slab}}\right) = \frac{8}{\pi^2}\left(e^{-a} + \frac{1}{9}e^{-9a} + \frac{1}{25}e^{-25a} + \cdots\right) \quad (11.10)$$

Similar equations have been derived for infinite cylinders and for spheres. Figure 11.5 shows both the fraction of heat remaining and the fraction of heat lost from these regular solids. By interpolation between these curves, one can estimate the extent of heating or cooling of any irregular solid.

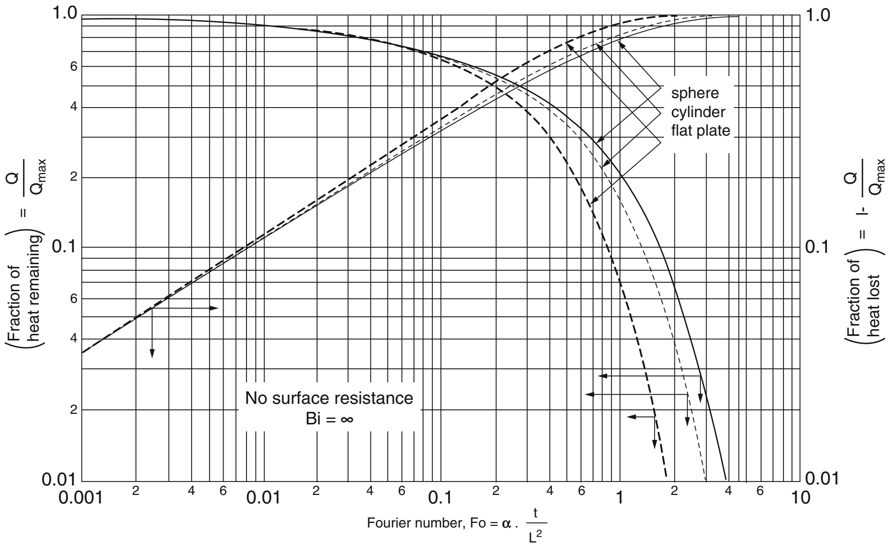


Fig. 11.5 Heat lost and heat remaining in a cooling object for negligible surface resistance [Prepared by Mator (1982)]

11.3 The Cooling of an Object Where Both Surface and Internal Resistances to Heat Flow Are Important (0.1 < Bi < 40)

Here (see Fig. 11.1b) we have the conduction equations of the just treated case B with the boundary condition at any time:

$$\begin{pmatrix} \text{Rate of heat flow out} \\ \text{from surface} \end{pmatrix} = -\dot{q} = hA\left(T_{s,\,l=L} - T_g\right) = k_s A \left(\frac{\partial T_s}{\partial l}\right)_{l=L} \qquad (11.11)$$

The solutions to these equations have been derived for a number of shapes and are available in many heat transfer texts [e.g., see Gröber et al. (1961) and Boelter et al. (1956)]. In all cases these solutions involve slowly converging infinite series, which are tedious to evaluate. However, convenient graphical representations of these solutions have been prepared and are reproduced in Figs. 11.6, 11.7, 11.8, 11.9, 11.10, 11.11, 11.12, 11.13, and 11.14 in terms of the following dimensionless groups:

- An unaccomplished temperature change: $\frac{\Delta T}{\Delta T_{\max}} = \frac{T_s - T_g}{T_{s0} - T_g}$
- The fraction of heat remaining in the solids: $\frac{Q}{Q_{\max}}$
- A relative time: $\text{Fo} = \frac{\alpha t}{L^2}$
- A resistance ratio: $\text{Bi} = \frac{hL}{k_s}$
- A radius or distance ratio: $\frac{r}{R}$ or $\frac{l}{L}$

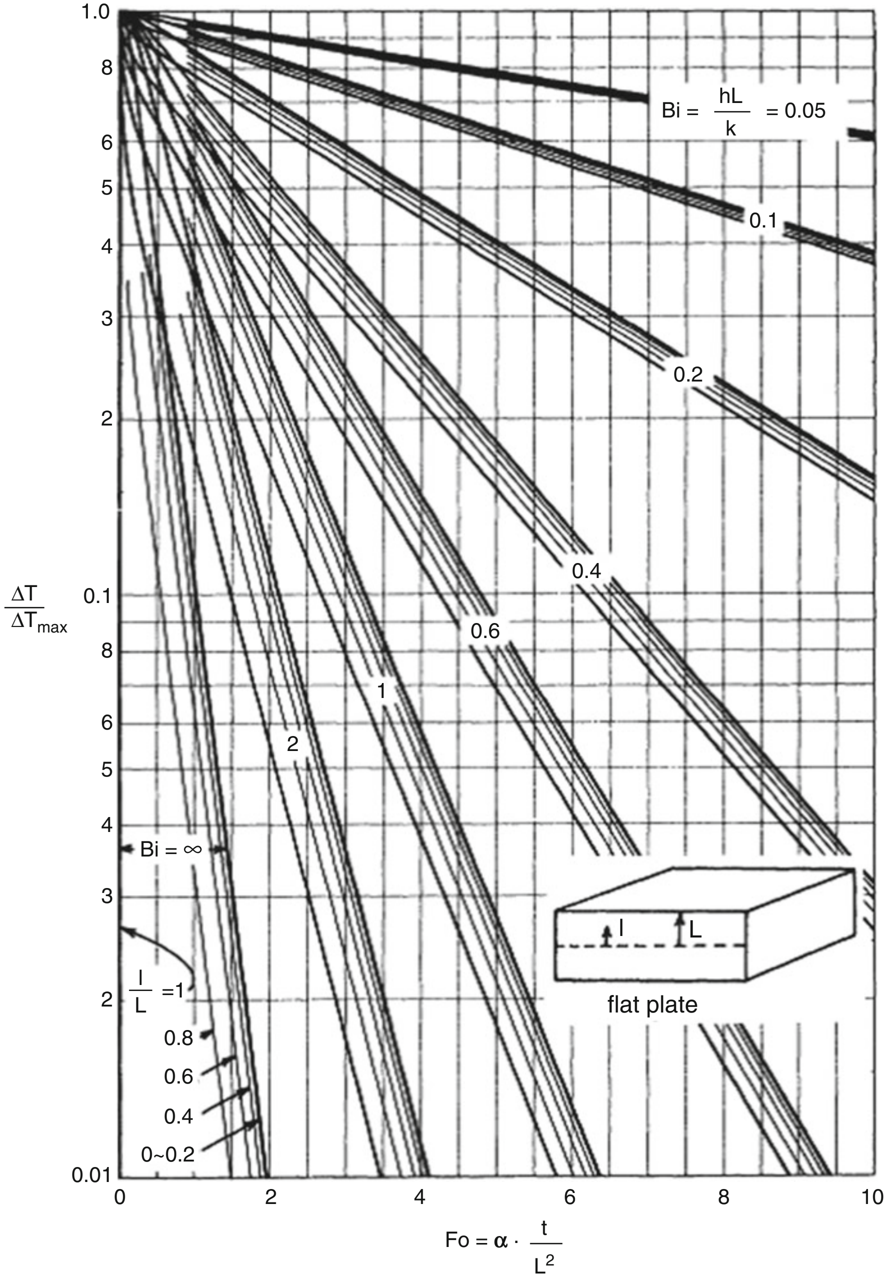


Fig. 11.6 Temperature distribution within cooling infinite flat plates, general case

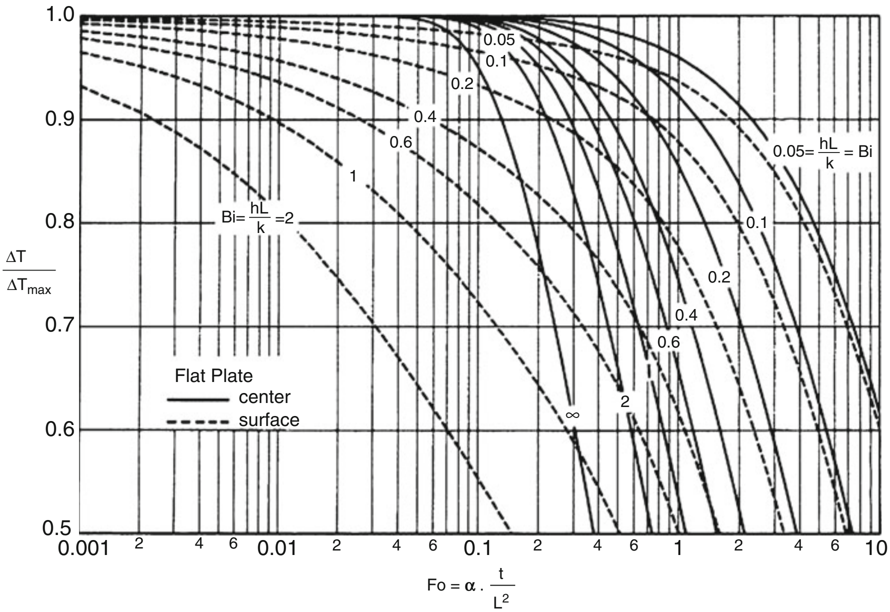


Fig. 11.7 Top left-hand corner of Fig. 11.6

Figures 11.6, 11.7, 11.8, 11.9, 11.10, 11.11, and 11.12 are the Gurney–Lurie (1923) charts accurately redrawn by Colakyan et al. (1984) to represent the changing temperature distribution within the cooling solids. Unfortunately, there is no characteristic length which will allow these charts to collapse into one graph. Hence, for irregular solids one must interpolate between these graphs.

Figure 11.13 represents the changing heat content of cooling particles whose sizes are measured by their characteristic lengths. Note that in a wide range of conditions, the curves for spheres, cylinders, and flat plates all collapse to a single curve. Thus, the cooling rate of any irregularly shaped particle can be evaluated directly from this chart.

The curves on the left side of this graph approach the extreme where surface resistance is negligible (see Fig. 11.5), while the curves on the right side of the graph approach the extreme where surface resistance dominates (see Fig. 11.3). The sketch of Fig. 11.14 displays the relationship between these figures.

11.4 The Cooling of a Semi-infinite Solid for Negligible Surface Resistance ($\mathbf{Bi} = hL/k_s \rightarrow \infty$)

When a hot body at temperature T_{s0} is placed in contact with cold fluid at temperature T_g, the surface immediately drops to T_g, heat flows from the body, and it progressively cools as shown in Fig. 11.15. The governing differential

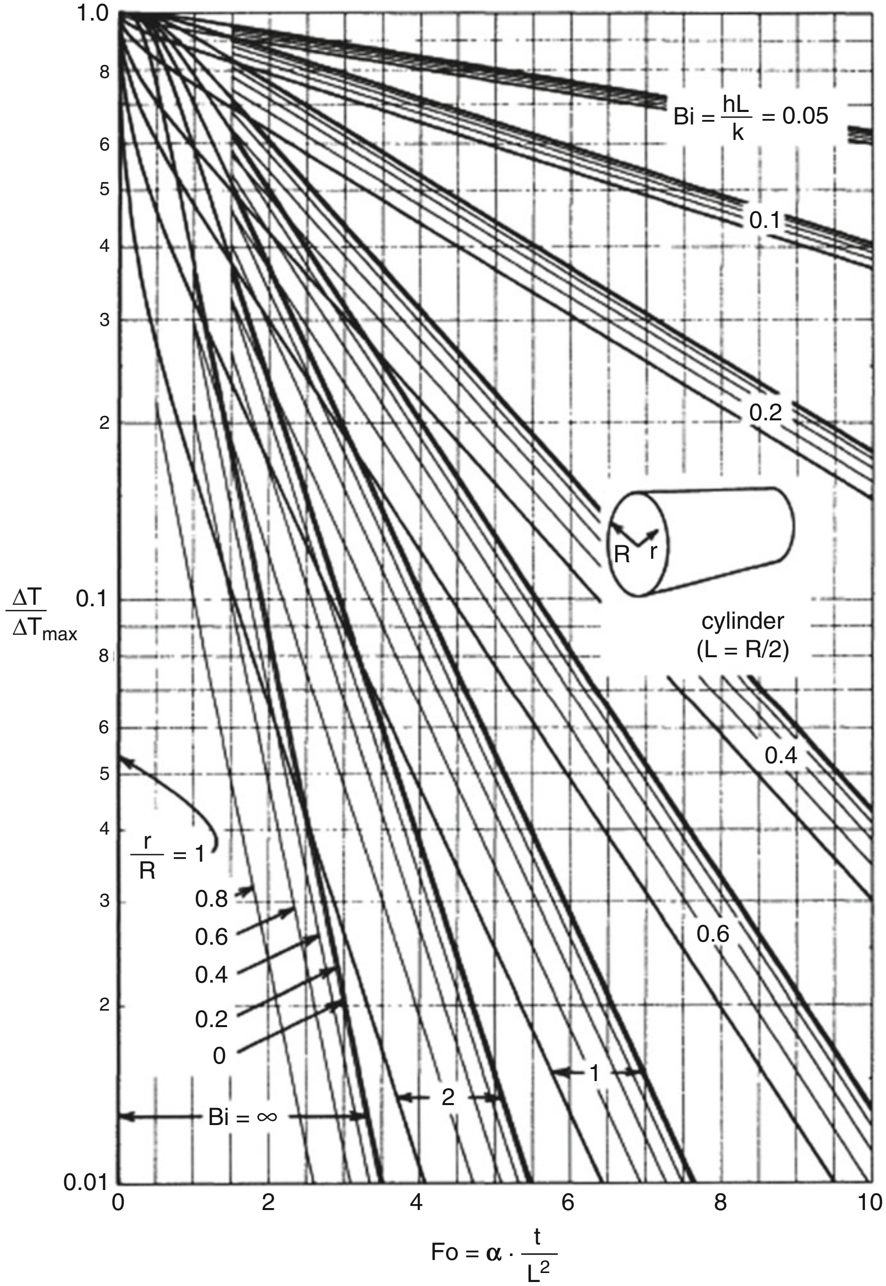


Fig. 11.8 Temperature distribution within cooling infinite cylinders, general case

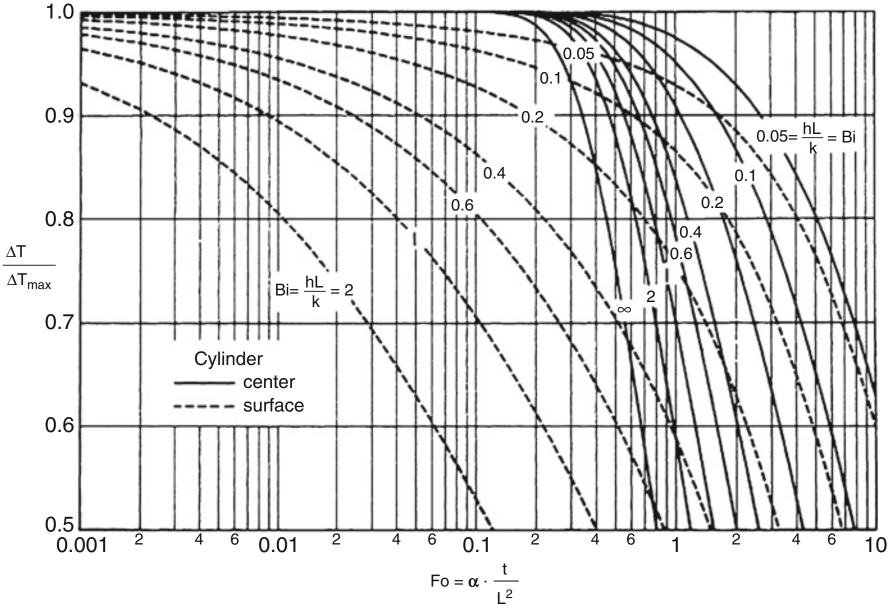


Fig. 11.9 Top left-hand corner of Fig. 11.8

equation for conduction, equation (11.1), when integrated for the boundary conditions of this situation, gives the temperature at any time and at any point in the object in terms of the Gaussian error function, as follows [see Welty (1974)]:

$$\frac{\Delta T}{\Delta T_{\max}} = \frac{T_s - T_g}{T_{s0} - T_g} = \operatorname{erf}\left(\frac{1}{\sqrt{4\alpha t}}\right) = \operatorname{erf}(y) \tag{11.12}$$

where the error function is defined as

$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx \tag{11.13}$$

Numerical values for the error function are given in Table 11.1. From this the temperature of the solid can be evaluated directly at any position and at any time, as shown in the lowest curve of Fig. 11.16.

The instantaneous rate of heat loss is found from equation (11.12) by evaluating the temperature gradient at the surface (at $l = 0$). From the mathematics this gives

$$-\dot{q} = k_s A \frac{\partial T_s}{\partial l}\bigg|_{l=0} = k_s A \left(\frac{T_{s0} - T_g}{\sqrt{\pi \alpha t}}\right) \quad [\mathrm{W}] \tag{11.14}$$

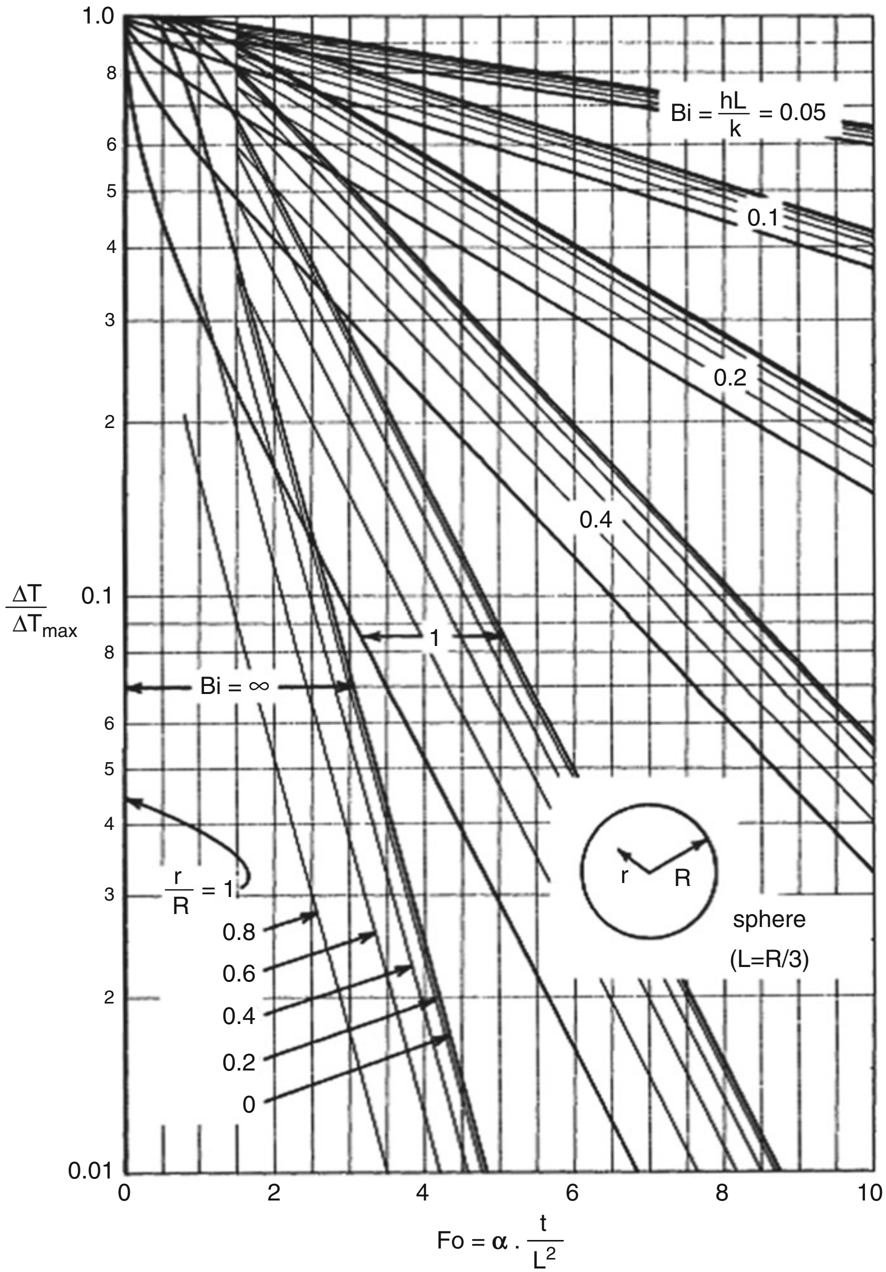


Fig. 11.10 Temperature distribution within cooling spheres, general case (also see Fig. 11.11)

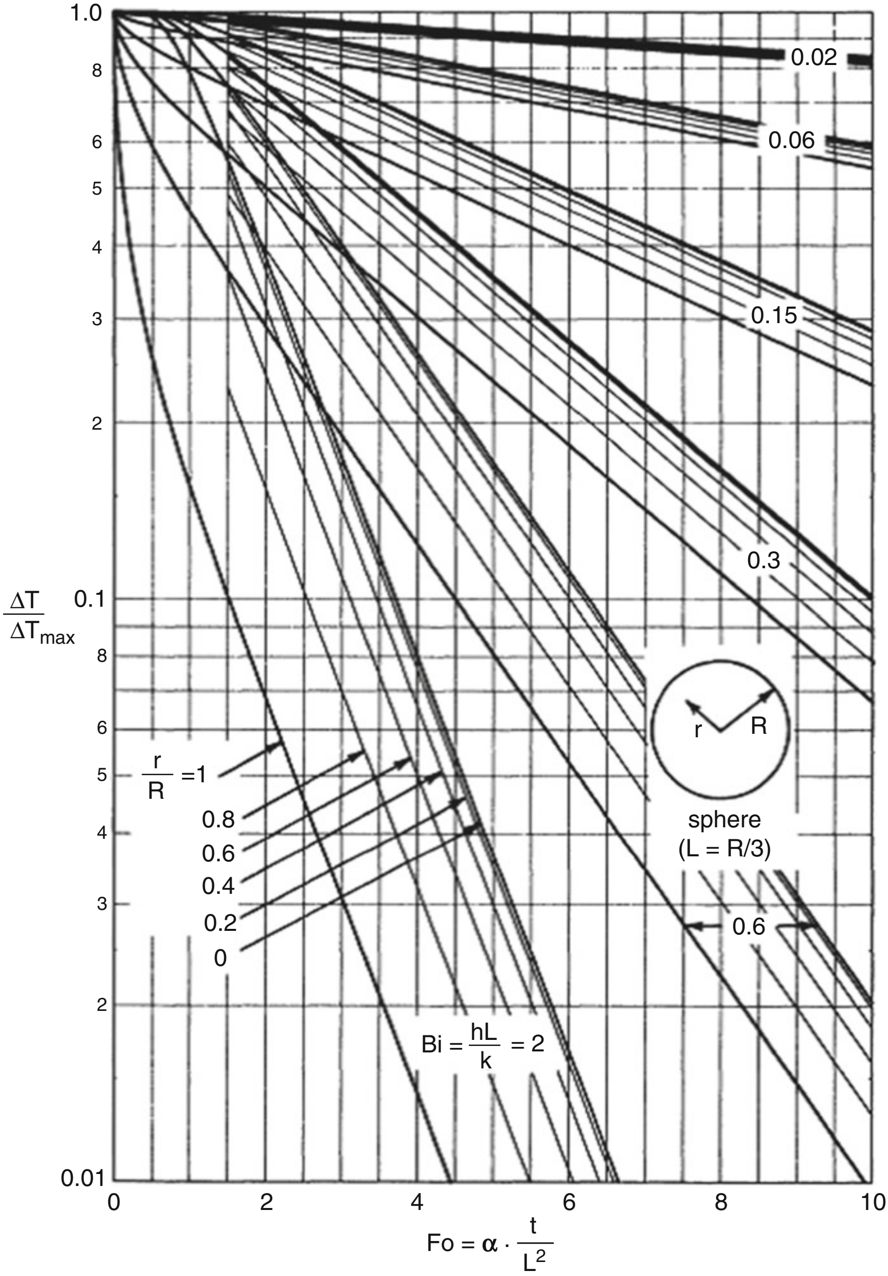


Fig. 11.11 Temperature distribution within cooling spheres, general case (also see Fig. 11.10)

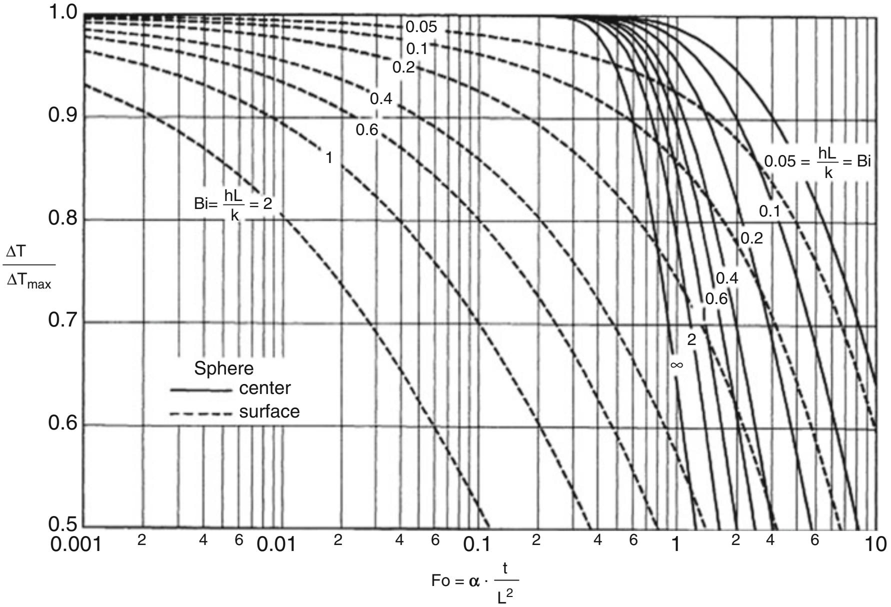


Fig. 11.12 Top left-hand corner of Figs. 11.10 and 11.11

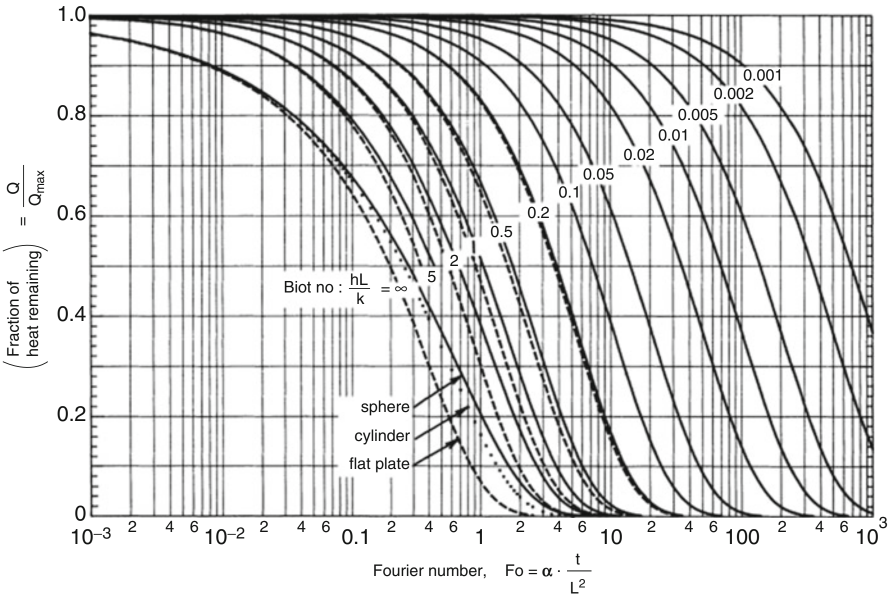


Fig. 11.13 General representation of the heat lost within a cooling sphere, infinite cylinder, and infinite flat plate [Prepared by Colakyan and Turton (1983)]

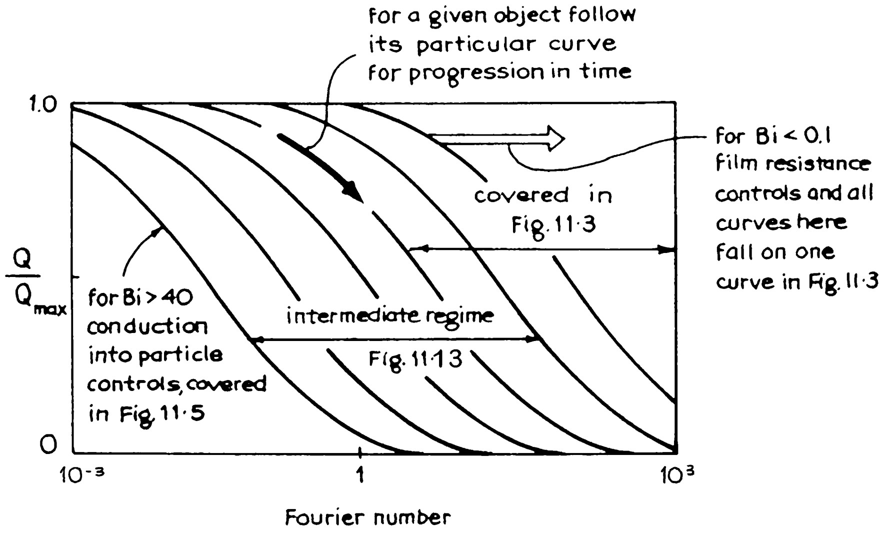


Fig. 11.14 Sketch of the relationship of the curves of Figs. 11.3, 11.5, and 11.13

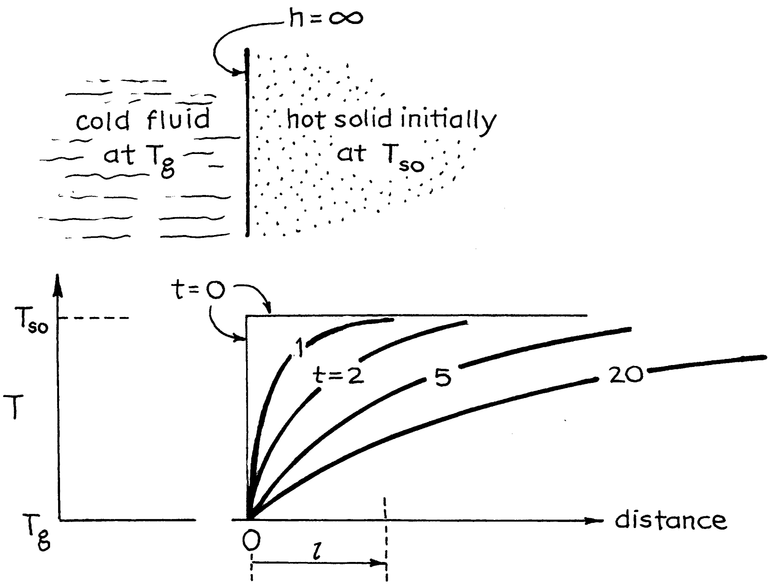


Fig. 11.15 The cooling of a semi-infinite object with negligible surface resistance

Finally, the total amount of heat loss from time $t = 0$ to time t is found by integrating the instantaneous rate of heat loss, equation (11.14), giving

$$Q_{\text{lost}} = \int_0^t (-\dot{q})dt = 2k_s A\left(T_{s0} - T_g\right)\sqrt{\frac{t}{\pi\alpha}} \quad [\text{J}] \tag{11.15}$$

Table 11.1 Values of the error function[a] [This gives the solution to equation (11.12)]

$y = \frac{1}{\sqrt{4\alpha t}}$	$\mathrm{erf}(y) = \frac{\Delta T}{\Delta T_{\max}}$	$y = \frac{1}{\sqrt{4\alpha t}}$	$\mathrm{erf}(y) = \frac{\Delta T}{\Delta T_{\max}}$
0.0	0.0	0.70	0.678
0.05	0.056	0.75	0.711
0.10	0.112	0.80	0.742
0.15	0.168	0.85	0.771
0.20	0.223	0.90	0.797
0.25	0.276	0.95	0.821
0.30	0.329	1.0	0.843
0.35	0.379	1.2	0.910
0.40	0.428	1.4	0.952
0.45	0.475	1.6	0.976
0.50	0.520	1.8	0.989
0.55	0.563	2.0	0.995
0.60	0.604	2.5	0.9996
0.65	0.642	∞	1

[a]From *Tables of the Error Function and Its Derivative*. National Bureau of Standards. Applied Mathematics Series 41. Washington. D.C. (1954)

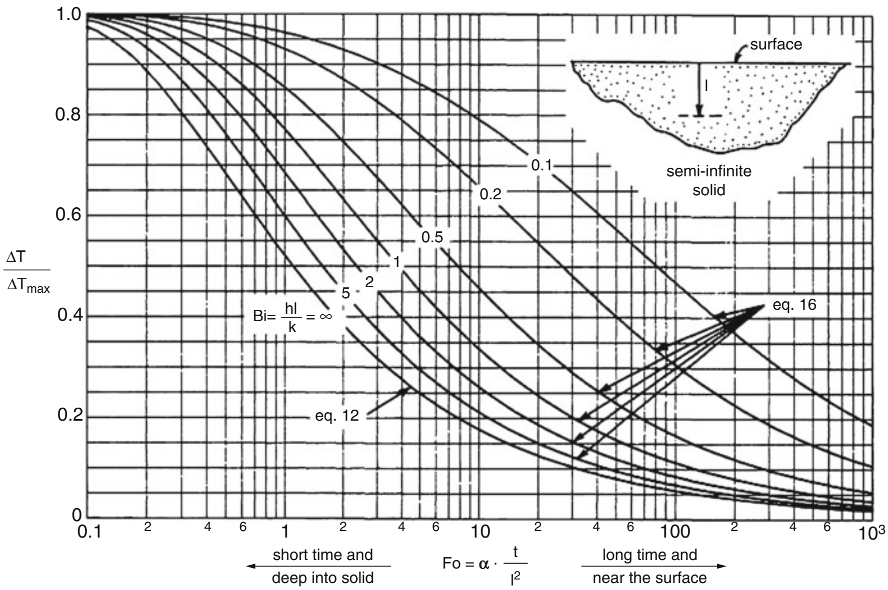


Fig. 11.16 Temperature distribution at any time and any position in a cooling semi-infinite object, general case [Equation (11.16)], and for no surface resistance [Equation (11.12)]

11.5 The Cooling of a Semi-infinite Body Including a Surface Resistance

This situation is similar to the previous case treated, but with an added surface resistance. Thus, we have a cooling behavior somewhat as sketched in Fig. 11.17. Integration of the conduction equation for this situation [see Sucec (1975)] gives the temperature of the solid at any position *l* from the surface at any time *t* as

$$\frac{\Delta T}{\Delta T_{\max}} = \operatorname{erf}\left(\frac{1}{2\sqrt{\mathrm{Fo}}}\right) + \left[1 - \operatorname{erf}\left(\frac{1}{2\sqrt{\mathrm{Fo}}} + \mathrm{Bi}\sqrt{\mathrm{Fo}}\right)\right] \exp\left(\mathrm{Bi} + \mathrm{Bi}^2 \cdot \mathrm{Fo}\right) \tag{11.16}$$

By putting $\mathrm{Bi} = \infty$, we see that equation (11.16) reduces directly to the special case expression of equation (11.12).

The cooling curves for both finite and infinite values of the Biot number are shown in Fig. 11.16. Note that at any point in the object, the approach to equilibrium (or the final temperature) takes a longer time for larger surface resistance (smaller values of the Biot number).

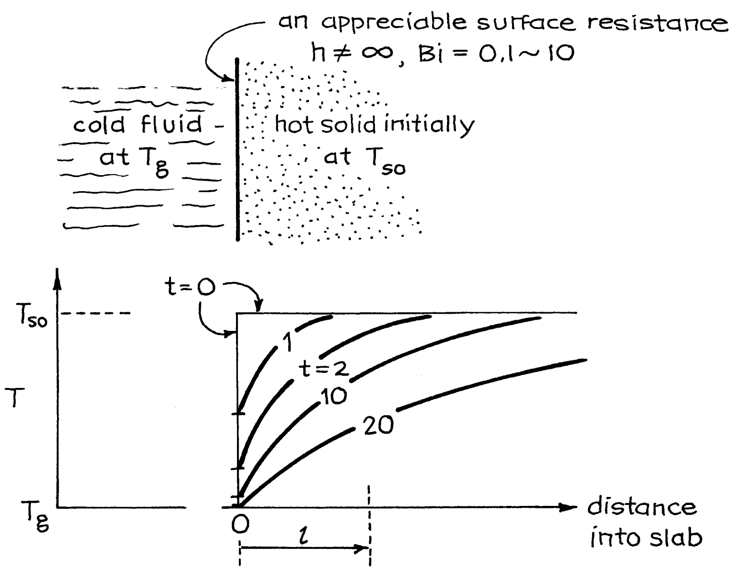


Fig. 11.17 The cooling of a semi-infinite object with surface resistance, the general case

11.6 Heat Loss in Objects of Size *L* for Short Cooling Times

Short cooling times mean that heat is only lost in the outer layer of the solid and that cooling has not yet penetrated deep into the object. In these situations the object can be treated as a semi-infinite solid, and its cooling behavior, whether it be slab, sphere, or what, only depends on the amount of surface in contact with the fluid, not its shape.

Thus, equations (11.12) and (11.16) and Fig. 11.16 represent the cooling in this time period. Boelter et al. (1956) and Schack (1965) both find that these simple "short time" solutions apply in the time period

$$\mathrm{Fo} = \frac{\alpha t}{L^2} < 0.077$$

where L is the characteristic length of the particle.

11.7 The Cooling of Finite Objects Such as Cubes, Short Cylinders, Rectangular Parallelepipeds, and So On

The temperature at any point in these finite objects is related to the corresponding temperature in the three mutually perpendicular infinite bodies whose intersections produce the object in question. This relationship is a simple one and is given by Sucec (1975) as

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_{\text{object}} = \left(\frac{\Delta T}{\Delta T_{\max}}\right)_x \left(\frac{\Delta T}{\Delta T_{\max}}\right)_y \left(\frac{\Delta T}{\Delta T_{\max}}\right)_z \tag{11.17}$$

where the terms on the right are evaluated from Figs. 11.6, 11.7, and/or 11.8, 11.9 for the point in question.

Similarly, the total heat loss from a finite object is related to the heat loss from the bounding infinite slabs and cylinders by

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{object}} = \left(\frac{Q}{Q_{\max}}\right)_x \left(\frac{Q}{Q_{\max}}\right)_y \left(\frac{Q}{Q_{\max}}\right)_z \tag{11.18}$$

where the terms on the right are evaluated from Fig. 11.13.

Example 11.2 shows how to use these equations.

11.8 Intrusion of Radiation Effects

When heat enters or leaves a body by both convection and radiation, the coefficient h used in the Biot numbers throughout this chapter should be the overall coefficient accounting for both these mechanisms of heat transfer, or

$$h_{\text{overall}} = h_{\text{convection}} + h_{\text{radiation}}$$

This radiation coefficient may change considerably as the surface temperature of the particle changes. To determine whether the radiation contribution is appreciable compared to convection and thus needs to be considered, use Fig. 9.6 with a correction for view factor and emissivity.

11.9 Note on the Use of the Biot and Fourier Numbers

In most texts the Fourier and the Biot numbers for spheres and cylinders are defined in terms of the radius of the object, rather than the characteristic size of the object, *V*/*A*. Be careful not to confuse these measures in the charts from other books:

$$\text{for flat plates}: \text{Fo}_{\text{here}} = \text{Fo}_{\text{other}};\ \text{Bi}_{\text{here}} = \text{Bi}_{\text{other}}$$
$$\text{for cylinders}: \text{Fo}_{\text{here}} = 4\text{Fo}_{\text{other}};\ \text{Bi}_{\text{here}} = \frac{1}{2}\,\text{Bi}_{\text{other}}$$
$$\text{for spheres}: \quad \text{Fo}_{\text{here}} = 9\text{Fo}_{\text{other}};\ \text{Bi}_{\text{here}} = \frac{1}{3}\,\text{Bi}_{\text{other}}$$

The advantage of using *V*/*A* over *R* is that the curves for various shapes of objects often lie close to each other or collapse to a single curve. In addition, with the definition used here, the Biot and Fourier numbers are the actual ratios of resistances as defined in equations (11.3) and (11.4).

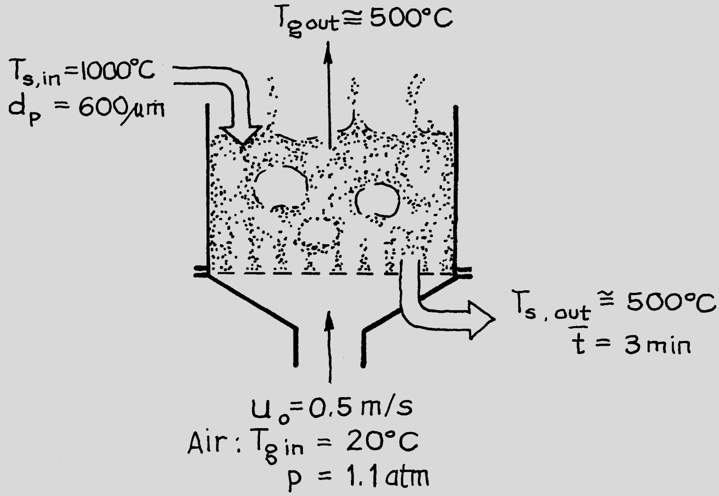
Example 11.1 Verifying a Key Assumption in the Analysis of Fluidized Bed Heat Exchangers

When a stream of hot solids is contacted by cold gas in a fluidized bed heat exchanger, the simple treatment of Chap. 14 assumes that:

(a) Cold entering gas heats up instantaneously to the bed temperature.
(b) Each particle of entering solid cools down instantaneously to the bed temperature.
(c) Both gas and solid leave the bed at the same temperature.

(continued)

(continued)



The term "instantaneous" as used above is reasonably approximated when the time needed for the two feed streams to get close to the bed temperature is much shorter than the time of stay of those streams in the exchanger. Preliminary considerations in Chap. 14 showed that assumption (a) is reasonably well met in practice. Let us look at assumption (b) here.

Suppose that hot sand ($d_p = 600\ \mu$m) at 1,000 °C flows continuously into a fluidized bed exchanger where it is cooled by air at room temperature and that gas and solid leave at about 500 °C. For a fluidizing velocity of $u_0 = 0.5$ m/s at 1.1 atm, find how long it takes for an incoming particle to cool to 550 °C (90 % approach).

(a) Assume that conduction within the particles controls.
(b) Assume that film resistance at the surface of the particles controls.
(c) Account for both resistances.

Compare these times to the mean residence time of solids in the exchanger, about 3 min.

Solution

The problem is to find how long it takes for a particle to cool such that $Q/Q_{\max} = 0.1$. Let us tabulate all the physical properties needed to answer this question. For sand at 500 ° C, Appendix A.21 gives

$$\rho_s = \frac{\rho_{s,\,\text{bulk}}}{1-\varepsilon} = \frac{1{,}500}{1-0.42} = 2{,}600\ \text{kg/m}^3$$
$$k_s = 0.33\ \text{W/m K}$$
$$C_s = 800\ \text{J/kg K}$$

Thus,

$$\alpha = k_s/\rho_s C_s = 1.59 \times 10^{-7}\text{m}^2/\text{s}$$

For air at 1.1 atm and 500 °C, from Appendix A.21

$$\mu_g = 36.19 \times 10^{-6}\ \text{kg/m s}$$
$$k_g = 57.45 \times 10^{-3}\ \text{W/m K}$$
$$C_g = 1{,}093\ \text{J/kg k}$$

and from Appendix A.12

$$\rho_g = \frac{p_A(mv)}{RT} = \frac{(101{,}325)(1.1)(0.0289)}{(8.314)(773)} = 0.50\,\text{kg/m}^3$$

The heat transfer coefficient between a fluidized particle and its surroundings is given by equation (9.38). Replacing values gives

$$\begin{aligned} h &= \frac{k_g}{d_p}\left[2 + 0.6\left(\frac{d_p u_0 \rho_g}{\mu_g}\right)^{1/2}\left(\frac{C_g \mu_g}{k_g}\right)^{1/3}\right] \\ &= \frac{57.45 \times 10^{-3}}{6 \times 10^{-4}}\left\{2 + 0.6\left[\frac{(6 \times 10^{-4})(0.5)(0.5)}{36.19 \times 10^{-6}}\right]^{1/2}\left[\frac{(1{,}093)(36.19 \times 10^{-6})}{57.45 \times 10^{-3}}\right]^{1/3}\right\} \\ &= 295\ \text{W/m}^2\,\text{K} \end{aligned}$$

We are now ready to solve for the cooling time based on the three different assumptions.

11.9.1 Assumption A. Particle Conduction Controls: Bi → ∞

For spherical particles ($L = d_p/6 = 10^{-4}$ m) and $Q/Q_{\max} = 0.1$, Fig. 11.5 gives

$$\mathrm{Fo} = \alpha \frac{t}{L^2} = 1.7$$

from which

$$t = 1.7 \frac{L^2}{\alpha}$$

Replacing values gives

$$t = \frac{(1.7)\left(10^{-4}\right)^2}{1.59 \times 10^{-7}} = 0.11\,\mathrm{s}$$

11.9.2 Assumption B. Film Resistance Controls: Bi → 0

Method A. Use equation (11.8). From equation (11.8) rearranged, we have

$$\begin{aligned} t &= \frac{L\rho_s C_s}{h} \ln \frac{Q_{\max}}{Q} \\ &= \frac{\left(10^{-4}\right)(2{,}600)(800)}{295} \ln 10 = 1.62\ \mathrm{s} \end{aligned}$$

Method B. Use Fig. 11.3. From the lowest curve of this figure, we have

$$\mathrm{Bi} \cdot \mathrm{Fo} = \frac{ht}{L\rho_s C_s} = 2.32$$

Thus,

$$\begin{aligned} t &= 2.32 \left(\frac{L\rho_s C_s}{h} \right) \\ &= 2.32 \left[\frac{\left(10^{-4}\right)(2{,}600)(800)}{295} \right] = 1.64\,\mathrm{s} \end{aligned}$$

11.9.3 Assumption C. Accounting for Both Resistances

For this we must first evaluate the Biot number for the cooling particles. Thus,

$$\mathrm{Bi} = \frac{hL}{k_s} = \frac{(295)\left(10^{-4}\right)}{0.33} = 0.0893$$

Then, Fig. 11.13 shows that

$$\mathrm{Fo} = \alpha \frac{t}{L^2} = 28$$

from which

$$t = 28 \frac{L^2}{\alpha} = \frac{28\left(10^{-4}\right)^2}{1.59 \times 10^{-7}} = 1.76\,\mathrm{s}$$

Note that Fig. 11.3 can be used for this solution in place of Fig. 11.13. However, if you try using the design charts of Figs. 11.10, 11.11, and 11.12, you will have problems.

Comments

As expected, the correct solution which accounts for both resistances gives a longer time, 1.76 s, than either of the solutions which only considers one or the other of the two resistances.

Comparing the two resistances, we see that the film dominates (1.64 s vs. 0.11 s) and just about controls (1.64 s vs. 1.76 s). Now both Figs. 11.1 and 11.14 point out that when $\mathrm{Bi} < 0.1$, then one can assume that film resistance controls. In this problem, $\mathrm{Bi} = 0.089$, which just meets this condition, and so the results are as expected.

The cooling time found here (1.76 s) is very much shorter than the mean residence time in the exchanger (3 min); hence, the assumption that the hot incoming particles cool instantaneously to the bed temperature is a reasonable idealization of the state of affairs.

Table 14.1 gives the relaxation times for a whole range of materials and particle sizes, calculated by the method of this example. Note where the particles of this problem fit into Table 14.1.

Example 11.2 Deep-Fried Fish Sticks

A cod fillet, about 6 × 1 × 2 cm, is taken from a cooler at 0 °C and slipped in hot oil at 180 °C.

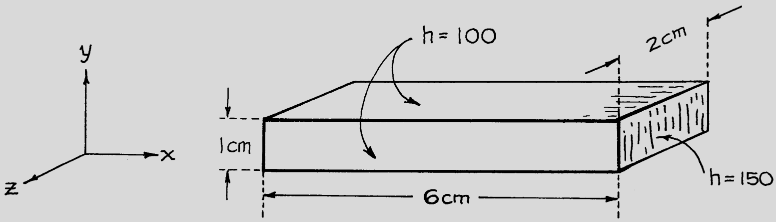
(a) What is the center point temperature of the fillet after 5 min?
(b) How much heat has been taken up by the fillet during this time?

Data: For cod,

$$k = 0.5\ \mathrm{W/m\,K}$$
$$\alpha = 0.17 \times 10^{-6}\ \mathrm{m^2/s}$$

(continued)

(continued)



For the fillet in the deep-fat fryer,

$$h = 150\ \text{W/m}^2\ \text{K for the two small end faces}$$
$$h = 100\ \text{W/m}^2\ \text{K for the four long faces}$$

Solution

The fillet can be represented by the intersection of three mutually perpendicular planes 6 cm, 1 cm, and 2 cm thick. Thus,

$$L_x = 0.06/2 = 0.03\ \text{m}$$
$$L_y = 0.01/2 = 0.005\ \text{m}$$
$$L_z = 0.02/2 = 0.01\ \text{m}$$

The individual Biot numbers are

$$\text{Bi}_x = \frac{h_x L_x}{k} = \frac{150(0.03)}{0.5} = 9$$
$$\text{Bi}_y = \frac{100(0.005)}{0.5} = 1$$
$$\text{Bi}_z = \frac{100(0.01)}{0.5} = 2$$

The individual Fourier numbers are

(continued)

(continued)

$$\mathrm{Fo}_x = \frac{\alpha t}{L_x^2} = \frac{\left(0.17 \times 10^{-6}\right)(300)}{(0.03)^2} = 0.055$$

$$\mathrm{Fo}_y = \frac{\left(0.17 \times 10^{-6}\right)(300)}{(0.005)^2} = 2.0$$

$$\mathrm{Fo}_z = \frac{\left(0.17 \times 10^{-6}\right)(300)}{(0.01)^2} = 0.50$$

A. Center Point Temperature. From Figs. 11.6 and 11.7, for the three midplanes, we have

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_x \cong 1$$

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_y = 0.25$$

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_z = 0.65$$

We are now ready to replace in equation (11.17). Thus, at the intersection of the three midplanes, in effect the center point, we have

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_{\text{centerpoint}} = \frac{T_{\text{oil}} - T_{\text{center}}}{T_{\text{oil}} - T_{\text{cooler}}} = \left(\frac{\Delta T}{\Delta T_{\max}}\right)_x \left(\frac{\Delta T}{\Delta T_{\max}}\right)_y \left(\frac{\Delta T}{\Delta T_{\max}}\right)_z \quad \text{(i)}$$

$$= 1(0.25)(0.65) = 0.1625$$

but

$$\left(\frac{\Delta T}{\Delta T_{\max}}\right)_{\text{centerpoint}} = \frac{T_{\text{oil}} - T_{\text{center}}}{T_{\text{oil}} - T_{\text{cooler}}} = \frac{180 - T_{\text{conter}}}{180 - 0} \quad \text{(ii)}$$

Combining equations (i) and (ii) gives

$$T_{\text{center}} = 151\ ^\circ\text{C}$$

B. Heat Absorbed. First of all, for cod fish fillets

$$\rho = 1{,}050\,\text{kg/m}^3 \text{(approximately)}$$

$$C_p = \frac{k}{\rho \alpha} = \frac{0.5}{(1{,}050)\left(0.17 \times 10^{-6}\right)} = 2{,}801\ \text{J/kg K}$$

(continued)

(continued)

The maximum amount of heat which can be absorbed

$$Q_{\max} = WC_p(T_{\text{oil}} - T_{\text{cooler}})$$
$$= [(1{,}050)(0.06 \times 0.01 \times 0.02)](2{,}801)(180 - 0) = 6{,}353 \text{ J} \quad \text{(iii)}$$

For the three intersecting infinite slabs, each with its own Biot and Fourier number, Fig. 11.13 shows that

$$\left(\frac{Q}{Q_{\max}}\right)_x = 0.76 \quad \text{at Bi} = 9 \text{ and Fo} = 0.055$$
$$\left(\frac{Q}{Q_{\max}}\right)_y = 0.23 \quad \text{at Bi} = 1 \text{ and Fo} = 2$$
$$\left(\frac{Q}{Q_{\max}}\right)_z = 0.54 \quad \text{at Bi} = 9 \text{ and Fo} = 0.5$$

Thus, for the fish fillet, equation (11.18) becomes

$$\left(\frac{Q}{Q_{\max}}\right) = (0.76)(0.23)(0.54) = 0.094 \quad \text{(iv)}$$

and with equation (iii) the heat remaining to be absorbed is

$$Q = (0.094)(6{,}353) = 997$$

So the heat which has been absorbed is

$$Q = 6{,}353 - 997 = 5{,}756 \text{ J}$$

Note: One can find the average temperature of the fillet directly from these Q values.

Problems on Unsteady-State Heating and Cooling of Solid Objects

11.1. Table 14.1 states that a 1-cm PVC plastic sphere moving through air at 1 m/s would have a thermal relaxation time of 170 s. Verify this figure.

11.2. A cold (0 °C), long, instrumented copper cylinder 5 cm o.d. is quickly plunged into a fluidized bed maintained at 100 °C, and the cylinder's center point temperature reads 40 °C, 60 °C, and 80 °C after 60, 110, and 200 s. Find the heat transfer coefficient between cylinder and bed.

11.3. *Roasting peanuts*. One method of preparing fat-free roasted peanuts involves lowering a wire basket of raw shelled peanuts into a vat of molten mannitol and sorbitol (nonsweet sugars) instead of into hot oil. When the peanuts are well roasted, they are removed, drained, lightly salted, and ready for packing. If peanuts, originally at 15 °C, are lowered into the 165 °C roasting medium:

(a) Find the time needed for their centers to reach 105 °C.
(b) How hot does the surface temperature of the peanuts become?

Data and assumptions: Assume that the peanuts are close to spherical with diameters of 7.5 mm and have the following properties:

$$\begin{aligned} k_s &= 0.5\ \mathrm{W/m\ K} \\ \rho_s &= 1{,}150\ \mathrm{kg/m^3} \\ C_s &= 1{,}700\ \mathrm{J/kg\ K} \end{aligned}$$

Between peanuts and molten sugar, take $h = 80$ W/m^2 K.

11.4. *More on roasting peanuts.* Another way to dry-roast peanuts is to dump a batch of peanuts into a fluidized bed of mannitol particles kept at 140 °C (melting point of mannitol = 160 °C) and then remove them when their centers reach 105 °C. This process does not leave a coating of hexose on the peanuts (see previous problem).

(a) Find the time needed to roast a batch of peanuts this way.
(b) Estimate the surface temperature of the just roasted peanuts.

Data: See previous problem for the thermal properties of peanuts. Between peanuts and fluidized bed, take

$$h = 200\ \mathrm{W/m^2\ K}$$

11.5. The thermal properties of a sand pile are to be evaluated by quickly pouring a bucketful of hot sand into a long 12-cm-i.d. aluminum pipe which is immersed in water at 20 °C. An axial thermocouple reads 140 °C just after the sand is poured in, 32 °C after 100 min, and 25 °C after 135 min.

From this information, evaluate the thermal conductivity and thermal diffusivity of the sand pile.

11.6. *Restarting a fluidized bed incinerator*. If the airflow to a fluidized bed incinerator stops because of a power outage or some such reason, the solids will collapse to form a "slumped" bed, which then will cool slowly. If the bed is still hot enough when the flow of cold fluidizing air is resumed, it will reignite spontaneously. However, if the bed temperature drops below the ignition temperature, then a long involved procedure must be followed to reignite it.

Let us estimate how long the bed can remain slumped and still restart spontaneously when airflow is resumed. In restarting:

(a) Assume that only a little air is fed to the bed at the beginning so that the bed remains slumped while the hot center region reignites and then spreads. Then, airflow is turned up and the bed refluidizes.
(b) Assume that full airflow is used immediately so that the slumped bed refluidizes right away.

Can you think of any of the advantages and drawbacks of these alternatives? *Data:* The slumped bed is 1 m deep and 4 m in diameter. The hot operating fluidized bed is at 850 °C. The ignition temperature is 600 °C. The thermal diffusivity of the slumped bed is 10^{-6} m^2/s. Assume that the top and bottom surfaces of the slumped bed drop to room temperature in about an hour and that the bed walls are well insulated.

11.7. *Heating of carbon particles*. Hot spherical carbon particles ($d_p = 3$ mm) are needed for an experiment. To prepare this feed material, carbon spheres at 0 °C are allowed to fall one at a time through a large diameter heated pipe containing nitrogen at 1 atm. Wall and gas are kept at 500 °C.

(a) Determine the length of pipe needed for the leaving particles to be at an average temperature of 300 °C.
(b) What are the surface temperature and the midpoint temperature of these leaving particles?

Data: For carbon particles,

$$\begin{aligned} \rho_s &= 550\ \mathrm{kg/m^3} \\ C_s &= 1{,}415\ \mathrm{J/kg\ K} \\ k_s &= 0.18\ \mathrm{W/m\ K} \end{aligned}$$

Assume that the particles are falling at their terminal velocity when they enter the heating chamber.

11.8. A very viscous chocolate syrup is to be heated by forcing it through a scraped wall heat exchanger. If the blades wipe the walls clean and bring fresh syrup to the surface twice a second, estimate the heat transfer coefficient at the walls of the exchanger.
Data: Thermal properties of chocolate syrup are estimated to be

$$\begin{aligned} k_s &= 0.5\ \mathrm{W/mK} \\ \rho_s &= 1{,}200\ \mathrm{kg/m^3} \\ C_s &= 3{,}600\ \mathrm{J/kg\ K} \end{aligned}$$

11.9. *From hot dogs to knackwurst*. Why make hot dogs when it costs about the same to make knackwurst, which sells for about twice the price? The process is nearly the same—a few different spices and ingredients and minor adjustments to the slicers and dicers and mashers and smashers. The only problem that I see is with the sterilizer.

Sanitary standards require that every part of the product be heated to 105 °C for proper sterilization. For hot dogs this is done by forcing the paste-like mass in plug flow through a long 18-mm tube whose wall is kept at 120 °C by outside condensing steam. Knackwursts are fatter than hot dogs, so we have to replace this 18-mm tube with a 28-mm tube. For the same processing rate of product (tons/day), how long should this tube be?

11.10. *Liver paste for sandwiches.* Chubby liver sausages (assume cylindrical, 5 cm across, 10 cm long), originally at 20 °C, are to be processed in an autoclave kept at 115 °C, and we want every part of the sausage to reach 105 °C. Estimate the lowest temperature of the sausage after 3 h in the autoclave.

Data: For food products containing a water fraction x, we have the following estimates:

$$C_p = 4{,}184x + 800(1 - x),\ \mathrm{J/kg \cdot K}$$
$$k = 0.56x + 0.25(1 - x),\ \mathrm{W/m \cdot K}$$

Liver sausage has about the same water content as canned dog food or 73 %; its density is about 1,050 kg/m^3, and in the autoclave, $h = 7.6$ W/m^2 K.

11.11. *Saving money at the university.* Our university's steam heating plant keeps our campus buildings at a comfortable 22 °C, day and night, 7 days a week. To save on the heating bill, it is proposed that steam to the buildings be shut off every afternoon at 6:00 p.m. and be turned on again the following morning at 6:00 a.m. However, irrespective of whether the heat is off or on, the forced air circulation fans will be kept running in all the buildings to keep the temperature in the buildings uniform throughout.

Test runs at various seasons of the year show that when the heat is turned off, the buildings all cool about halfway to the temperature of the surroundings by 6:00 a.m. How much heating steam would the university save if it followed this on/off procedure?

11.12. *The age of the Earth.* It is well known that the ground becomes hotter and hotter as one digs 1, 2, 5, or more kilometers down into the Earth. The thermal gradient dT/dl varies from place to place, under continent and ocean, in tunnel and bore hole, but on the average it has been found that the temperature increases by about 1 °C for each 24 m of depth. From this information Fourier (in 1820) and later Kelvin (in 1864) made rough estimates on when the Earth began to cool from its molten state. Let us try to do this too.

(a) Taking the temperature of the Earth's surface to be 20 °C and the freezing temperature of rock as 1,200 °C, estimate when the Earth began to solidify. Ignore surface resistance.

(b) Check whether it is reasonable to ignore surface resistance to heat transfer.
(c) This calculation ignores any heat that may be generated in the Earth's interior by radioactivity. If we accounted for this added factor, would it increase or decrease our time estimate?

For references and discussions, see H. S. Carslaw and J. C. Jaeger, *The Conduction of Heat in Solids*, 2nd ed., p. 85, Oxford, 1959.

11.13. A hot granular material ($\alpha_{\mathrm{bulk}} = 2 \times 10^{-7}$ $\mathrm{m^2/s}$) is to be cooled by having it slide down a vertical tube 8 m long and 40 mm i.d. whose walls are kept at 100 °C. The hot solids will be fed to the cooler at 300 °C and are expected to slide in plug flow down the tube at 16 mm/s. At what mean temperature will the solids leave the cooler?

References and Notes

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