# Chapter 9
# Off-Grid System Converters and Controllers


## 9.1 Introduction

It is possible to create an off-grid system consisting of only an energy source, a battery, and a load. These "improvised" or "non-engineered" systems are very common in rural households (see Fig. 9.1). As you might expect, they are limited in size and function and are often short-lived, inefficient, and unreliable. In this book we are considering more formally engineered systems. They involve the use of energy sources, generators, loads, and, pertinent to this chapter, control devices.

In earlier chapters, references have been made to controllers that are used to maintain mini-grids at their desired states of operation. These controllers can be either mechanical or electrical. Many such electrical control systems are categorized as *converters*. Converters are a broad class of power electronic devices that are designed to perform a wide range of functions within DC and AC systems. Converters add cost and complexity to the system but also offer several benefits. They can act as an interface, enabling otherwise incompatible components to work together. For example, converters make it possible for a DC-coupled system to serve an AC load. Converters facilitate the flow of power between the buses in AC–DC coupled systems and regulate battery charging and discharging. Converters are used in maximum power point trackers that increase the production from PV arrays and wind turbines.

The study of converters lies within the subfield of electrical engineering known as power electronics. There are numerous textbooks devoted to the subject [6, 10]. We cannot cover converters in detail in a single chapter. Instead, this chapter presents the basic function of each converter, its principle circuit, and application considerations. Many converters require control circuitry and logic to properly function. We do not dwell on how the control signals are generated. Rather, we consider how the signals affect the operation of the converter.

From the outside, converters appear as metal enclosures with indicator lights and switches on the surface. Inside are the power conditioning and control circuitry, as



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**Fig. 9.1** This improvised solar-powered system has no controllers (courtesy of author)

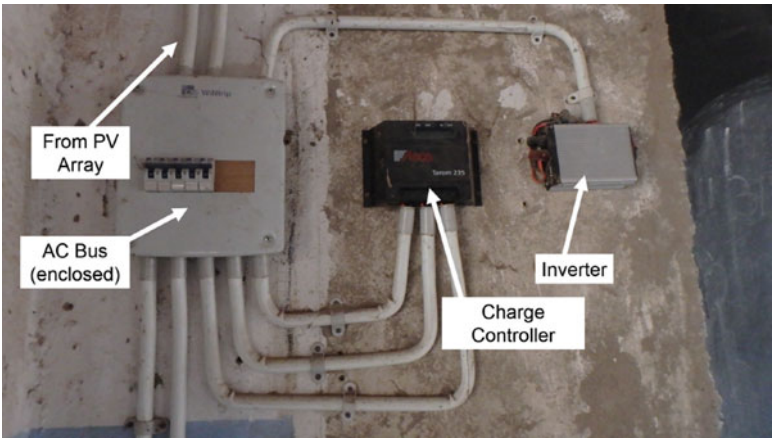


**Fig. 9.2** Converters and controllers of a small PV system installed in a school in Malawi (courtesy of author)

well as fans, fuses, and connection points for other mini-grid components. Most converters are no larger than a microwave oven. A recent trend by manufacturers is to combine several converters into a single unit. For example, one can readily purchase a unit that combines maximum power point tracking, charge control, and inverter functionality. Integrated units can reduce the space requirements and simplify the wiring. We shall generally consider single-purpose converters.

Small systems use only a few converters, as shown in Fig. 9.2. Larger systems will require dedicated control rooms, as in Fig. 9.3.

Table 9.1 shows some of the commonly-available converter types and their applications in off-grid systems. These will be discussed in the following sections.

**Fig. 9.3** The control room of a large mini-grid in Nigeria, consisting of inverters, charge controllers, and system monitoring equipment (courtesy of GVE Projects)

**Table 9.1** Function and applications of converters

| Converter | Basic function | Section |
|---|---|---|
| DC–DC converter | Increases or decreases output voltage | 9.3 |
| Maximum power point tracker | Increases power produced by PV arrays or WECS | 9.4 |
| Solar battery charger | Charges batteries directly from PV sources | 9.5 |
| AC battery charger | Converts AC produced by generators or other sources to DC and manages battery charging | 9.6 |
| Rectifier | Converts AC to DC | 9.6.1–9.6.2 |
| Automatic voltage regulator | Adjusts excitation to synchronous generators | 9.7 |
| Electronic load controller | Controls power to ballast load to regulate frequency | 9.8 |
| Inverter | Converts DC to AC | 9.9 |
| Solar inverter | Converts DC from PV sources to AC | 9.10 |
| Grid-tied inverter | Converts DC to AC and synchronizes with AC bus | 9.11 |
| Bi-directional converter | Allows power to be exchanged between the DC and AC buses | 9.12 |

## 9.2 Basic Concepts

We begin by covering basic concepts. To readers with a background in semiconductor devices or power electronics, much of this will be a review.

### *9.2.1 Solid-State Switching Elements*

The primary circuit elements used in converters are solid-state switches, inductors, and capacitors. Inductors and capacitors provide small amounts of short-term energy storage and are useful as filters. Solid-state switches operate like mechanical switches but at very high speeds, usually tens of thousands of times per second. The circuit symbols for several solid-state switches are shown in Fig. 9.4.

Some switches are operated by a control signal, which we generically denote by the time-dependent binary or logic variable $q(t)$. We use the convention that $q(t) = 1$ is a "close" signal and $q(t) = 0$ is an "open" signal. In practice, the signal must have the appropriate electrical characteristics to initiate conduction, but we will ignore this detail.

*Power MOSFETs*
Power MOSFETs (metal–oxide–semiconductor field-effect transistors) are a type of transistor designed to withstand the high voltages and currents present in power electronic circuits. A MOSFET is a three-terminal device. Conduction between two of the terminals, the drain and the source, is controlled by a control signal $q(t)$ applied to the gate. Conduction continues between the source and the drain as long the signal is applied and the current has positive polarity. If current in the negative direction is required, for example, in a bi-directional converter or in rectifiers with inductive load, an antiparallel diode is placed across the drain and source, as shown in Fig. 9.4. The diode current, however, is not controlled by the gate.

*Thyristors*
Thyristors, also known as silicon-controlled rectifiers (SCRs), are controllable switches. Like MOSFETs, thyristors are three-terminal devices. Conduction begins when the thyristor is forward biased and when the signal $q(t) = 1$ is applied to its

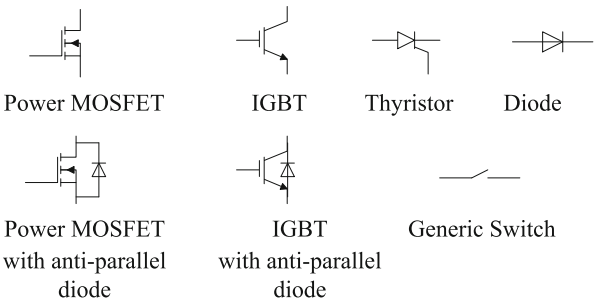


**Fig. 9.4** Circuit symbols for different electronic switch components

gate. Conduction continues even after the close signal to the gate returns to $q(t) = 0$, so long as thyristor is forward biased. Thyristors therefore act like a latched switch. Conduction ends when the current drops below a threshold known as the "holding current," which we will assume to be zero. The thyristor current cannot be negative. In order to allow current to flow in the opposite direction, a diode can be placed in antiparallel. Thyristors are commonly used in phase-controlled rectifiers and some inverters.

*IGBTs*
IGBTs (insulated-gate bipolar transistor) are three-terminal devices designed for use in higher-power applications. IGBTs conduct when $q(t) = 1$. The voltage drop associated with an IGBT when conducting is approximately 2 V. For this reason, it is mostly used in applications with higher voltages, for example, above 96 V.

### 9.2.2 Distortion and Filtering

Many converters have nonlinear characteristics that result in nonconstant, non-sinusoidal current and voltage waveforms. These distorted waveforms can damage or cause malfunction in certain loads. Distortion can also cause generators to overheat, and some devices will emit an annoying humming sound or visible flicker. Obviously, distortion should be minimized.

AC circuit analysis is more complicated when the waveforms are not sinusoidal because phasor-based analysis cannot be directly used. Instead, we use harmonic analysis. The basic idea is to first decompose the distorted waveform into its harmonic components using the Fourier Series [10]. The harmonic components are sinusoids with different amplitudes and phase angles and whose frequencies are at integer multiples of the frequency of the distorted waveform. If the distorted waveform does not have a zero mean, then a constant term is also present in the series. Phasor-based circuit analysis is performed on each harmonic separately (or often only those harmonics that have large enough amplitude to be of consequence), and the results recombined to complete the analysis.

Any periodic zero-average waveform $f(t)$ can be decomposed into its harmonic components by applying the Fourier Series. The Fourier Series is

$$f(t) = \sum_{k=1}^{\infty} F_k \sin(k\omega_0 t + \delta_k) \tag{9.1}$$

where $\omega_0$ is the fundamental frequency of $f(t)$, $k$ is the harmonic number, $k\omega_0$ is the frequency of the $k$th harmonic, and $F_k$ and $\delta_k$ are the magnitude and phase shift associated with the $k$th harmonic. Equation (9.1) applies equally to voltage and current. The magnitude of $F_k$ indicates the "strength" or contribution of a particular harmonic to the total distorted waveform. Waveforms with little distortion will feature a fundamental component $F_1$ much greater than the other components.

The distortion of a waveform is measured by the Total Harmonic Distortion (THD)[7]:

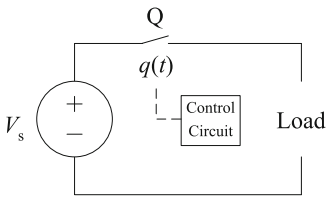


**Fig. 9.5** Pulse width modulation can be used to control the average voltage to the load

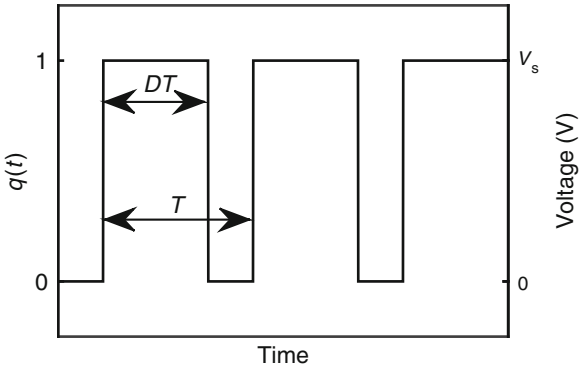


**Fig. 9.6** Pulse train with duty cycle $D$ and period $T$

$$THD = \frac{\sqrt{\sum_{k=2}^{\infty} F_k^2}}{F_1}. \tag{9.2}$$

Keep in mind that distortion is a measure of the signals' deviation from the desired pure sinusoid. The THD is commonly expressed as a percentage. The THD is zero when there is no distortion. In general, we should select gen sets and inverters whose output voltage has low THD. The THD is often reported by manufacturers. THD values less than 5% can often be used, but this depends on the application.

A common way to reduce distortion is by using passive filters at the output or input of device. Such filters attenuate higher harmonics, leaving the fundamental.

### *9.2.3 Pulse Width Modulation (PWM)*

Pulse width modulation (PWM) is a technique in which the width of a series of pulses is adjusted to obtain a desired average value. The signals that control the switching elements in converters often use PWM.

As an example of how PWM can be used in converters, consider the circuit shown in Fig. 9.5. This common circuit is known as a "chopper." The switch Q is controlled by a signal $q(t)$. Let $q(t)$ be the periodic train pulses as shown in Fig. 9.6. The period between pulses is some time $T$. Within each period, the duration when the signal $q(t) = 1$ is $DT$, and the duration when it $q(t) = 0$ is $(1-D)T$. The variable $D$ is the *duty cycle* or *duty ratio*. The duty cycle is often expressed as a percentage, in which case it varies between 0% (constant minimum) and 100% (constant maximum). In PWM, the duty cycle is variable.

When the switch in Fig. 9.5 is closed ($q(t) = 1$), the voltage across the load is $V_s$; otherwise, it is zero. The average value of the voltage across the load $\bar{V}_{\text{Load}}$ is found through integration:

$$\bar{V}_{\text{Load}} = \frac{1}{T}\int_0^T v(t)\mathrm{d}t = \frac{1}{T}\int_0^{DT} V_s \mathrm{d}t = DV_s \tag{9.3}$$

We can therefore adjust the duty cycle to control the average value of the voltage applied to the load.

For a general pulse train whose maximum and minimum values are $V_{\max}$ and $V_{\min}$, the average value is:

$$\bar{V}_{\text{Load}} = \frac{1}{T}\int_0^T v(t)\mathrm{d}t = \frac{1}{T}\left(\int_0^{DT} V_{\max}\mathrm{d}t + \int_{DT}^T V_{\min}\mathrm{d}t\right) = DV_{\max} + (1-D)V_{\min}. \tag{9.4}$$

We are mostly concerned with time-averaged rather than instantaneous values in DC switching circuits. Hereafter, variables in uppercase will denote constant or time-average values.

## 9.3 DC–DC Converters

DC–DC converters are conceptually similar to transformers in AC circuits in that they are used to change voltage levels. This capability is extremely useful. For example, a DC–DC converter can be used to boost the voltage from a nominal 96 V battery bank to 220 V for distribution, and another converter can be used at the consumer's house to reduce the voltage to a nominal 12 V—a voltage that many DC appliances are compatible with.

DC–DC circuits are used within many converters, including battery charge controllers, maximum power point trackers (MPPTs), and inverters. They are also used in solar home systems and solar lanterns, especially those that have USB ports for device charging.

A DC–DC converter has the following property that relates the input and output voltage and current with the power through the converter and its efficiency:

$$P = \eta_{\text{DC-DC}} V_{\text{in}} I_{\text{in}} = V_{\text{out}} I_{\text{out}}. \tag{9.5}$$

Equation (9.5) is nothing more than the conservation of power. Converters that increase the output voltage necessarily decrease the output current and vice versa. This can be done very efficiently. DC–DC converters tend to have efficiencies above 90%.

DC–DC converters incorporate high-frequency solid-state switching and energy storage components—inductors and capacitors—to change the output voltage. The difference between the input and the output voltage is controlled by adjusting the duty cycle of the switch. A controller is used to adjust the duty cycle to achieve a targeted output voltage.

**Table 9.2** DC–DC converter input/output relationships

| Converter | Relationship |
|---|---|
| Boost | $V_{\text{out}} = \frac{1}{1-D} V_{\text{in}}$ |
| Buck | $V_{\text{out}} = D V_{\text{in}}$ |
| Buck–boost | $V_{\text{out}} = \frac{-D}{1-D} V_{\text{in}}$ |

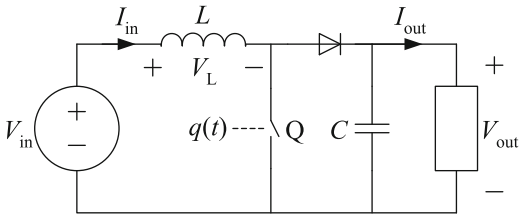


**Fig. 9.7** Boost converter circuit

There are several types of DC–DC converters. The most popular are the boost, buck, and buck–boost. Boost converters increase the output voltage; buck converters decrease the output voltage; and buck–boost either increase or decrease the output voltage. Note that the polarity of the output voltage of a buck-boost converter is negative. The circuit topology of each converter is different, as is the effect of adjusting the duty cycle, as shown in Table 9.2.

To understand the basic principles of how DC–DC converters function, we will briefly discuss the boost converter. The basic circuit is provided in Fig. 9.7. When the switch signal is $q(t) = 1$, the switch Q conducts. The diode is not forward biased and so it does not conduct. The voltage across the inductor $V_{\text{L}}$ is then equal to $V_{\text{in}}$. The current through the inductor consequentially increases according to

$$I_{\text{in}}(t) = \frac{1}{L} \int V_{\text{L}} \mathrm{d}t. \tag{9.6}$$

As the current increases, so does the energy stored in the magnetic field of the inductor.

When $q(t) = 0$, Q opens. The inductor current cannot instantly change, and so the diode must be forward biased to provide a pathway for the current. The inductor current is therefore supplied to the load. Applying Kirchhoff's Voltage Law and ignoring the often-negligible voltage drop across the diode:

$$V_{\text{in}} = V_{\text{L}} + V_{\text{out}}. \tag{9.7}$$

The inductor current decreases as energy is transferred to the load. From (9.6), when the inductor current decreases, the voltage across it is negative. Therefore, from (9.7), the output voltage is *greater* than the input, and the basic function of the boost converter has been accomplished.

The input and output voltages are related by the duty cycle, which is derived as follows. The average voltage across the inductor during a switching period must be zero. If it were non-zero, then the inductor current would increase or decrease from one period to the next—this cannot continue indefinitely.

When Q is closed, the voltage across the inductor is $V_{\text{in}}$. When Q is open, from (9.6), the voltage is $V_{\text{L}} = V_{\text{in}} - V_{\text{out}}$. The switch is closed for the period $D$ and open for $1 - D$. Therefore, setting the average voltage to zero shows that

$$DV_{\text{in}} + (1 - D)(V_{\text{in}} - V_{\text{out}}) = 0 \tag{9.8}$$

where theoretically $0 \leq D \leq 1$. In practice $D$ must be somewhat lower than 1 so that the output voltage cannot be arbitrarily large. Rearranging (9.8):

$$V_{\text{out}} = \frac{1}{1 - D} V_{\text{in}}. \tag{9.9}$$

This equation, as well as those in Table 9.2, is valid so long as the inductor and switching frequency are such that the inductor current does not drop to zero.

A few notes. A capacitor is usually placed in parallel to the output so that the voltage is stable. The capacitor will charge when Q is open and discharge when Q is closed so that the load current is nearly constant. The inductor on the input side is usually sized so that energy is supplied by the input source, regardless of the state of the switch, and therefore makes good use of the power supply capability of the input voltage source.

## 9.4 Maximum Power Point Tracker

Maximum power point trackers (MPPTs) are used to increase the power produced by a PV module or array. Although controlled differently, MPPTs can also be used with WECS to provide the same function. However, most MPPTs are used with PV arrays, and so we focus our discussion on this application.

The basic premise of an MPPT is to decouple the load voltage from the PV array voltage, allowing the PV array to operate at $V^*(G)$—the voltage corresponding to its maximum power point. The basic scheme is shown in Fig. 9.8. The PV array is connected to a DC–DC converter; the output of the converter is connected directly to the load, battery, or charge controller. The type of DC–DC converter used depends on the maximum power point voltage of the array relative to the battery or load voltage. In any case, the duty cycle of the converter is controlled to track $V^*(G)$ as irradiance, temperature, and shading changes. Most MPPT uses digital control, but analog control is also possible. The algorithms used to track the maximum power point (MPP) are discussed in the next chapter.

As an example, consider a PV module connected to a battery through a MPPT, as shown in Fig. 9.9. The $I$–$V$ curve of the module is shown in Fig. 9.10. The MPP is at "A." Without an MPPT, the PV module voltage is equal to the battery terminal voltage $V_{\text{B}}$ since they would be directly connected to each other. The operating point of the PV module is therefore point "B." The voltage is lower than $V^*(G)$, and the power corresponding to point B is less than A.

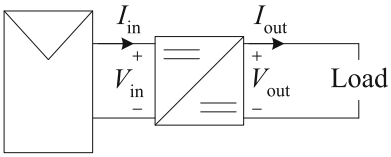


**Fig. 9.8** Connection of a PV module to a load through an MPPT

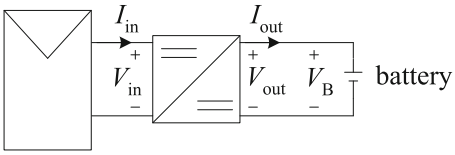


**Fig. 9.9** Connection of a PV module to a battery through an MPPT

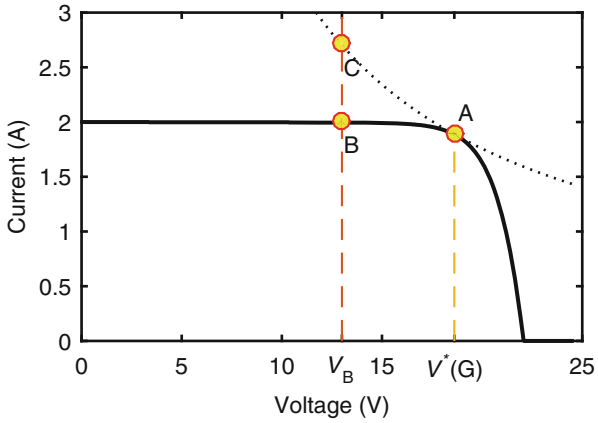


**Fig. 9.10** Operating points of the PV module and battery when an MPPT is used. The dashed line is the equi-power curve of the maximum power point. The product of voltage and power at each point along this curve is equal to $P^*(G)$

When an MPPT is used, the module's voltage is increased so that it operates at $V^*(G)$ (point "A") and therefore produces the maximum power $P^*(G)$. The battery's operating voltage remains at $V_B$. However, assuming the MPPT is lossless, the power to the battery must also be $P^*(G)$. The current corresponding to this operating point is $P^*(G)/V_B$, so that the operating point of the battery is point "C." This point lies along the same equi-power curve as the maximum power point. The MPPT is operating as a buck converter since the module voltage is greater than the battery voltage.

MPPTs can increase the total energy production by about 10 to 15%. This increase must be weighed against the cost of the MPPT unit. MPPTs are almost always used in larger-capacity off-grid systems, for example, above 2 kW. MPPT units are now commonly integrated with solar battery chargers. Manufacturers tend to brand units without MPPT functionality as "PWM" charge controllers; but this is confusing, as MPPT charge controllers also use PWM to control their internal DC–DC converter.

*Example 9.1* An off-grid house has an improvised system consisting of a PV module that is directly connected to a battery. The $I$–$V$ curve of a PV module under the present irradiance and temperature conditions is shown in Fig. 9.10. The PV module is used to charge a battery whose terminal voltage is 13 V. Estimate the power produced by the PV module. Next, consider the scenario in which the house has an MPPT (buck converter) that is connected between the module and the battery. The voltage and current corresponding to the maximum power point are 18.7 V and 1.89 A. Compute the duty cycle so that the PV module operates at its maximum power point and the corresponding power and current into the battery.

**Solution** By inspection of Fig. 9.10, the current when the PV module voltage is 13 V is approximately 2 A. Therefore the PV module produces $13 \times 2 = 26$ W when connected directly to the battery.

The duty cycle of the buck converter is found by arranging the corresponding equation in Table 9.2:

$$D = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{13}{18.7} = 0.695.$$

The corresponding power and current are:

$$P = V^* I^* = 18.7 \times 1.89 = 35.34 \text{ W}$$

$$I_{\text{out}} = \frac{P}{V_{\text{out}}} = \frac{P}{V_{\text{B}}} = \frac{35.34}{13.0} = 2.72 \text{ A}$$

The MPPT increased the PV power by 35% (35.34 W versus 26 W).

## 9.5 Solar Battery Charger

Solar battery chargers, also known as solar charge controllers, regulate the charging of a battery from a PV array or module. They are sometimes integrated with an MPPT, but we will not consider this scenario for the sake of clarity. They are used to prevent the battery from being damaged or degraded during charging. They are used in mini-grids and most solar home systems and solar lanterns. There are three general types of solar charge controllers: shunt, series, and pulse width modulation (PWM) [12].

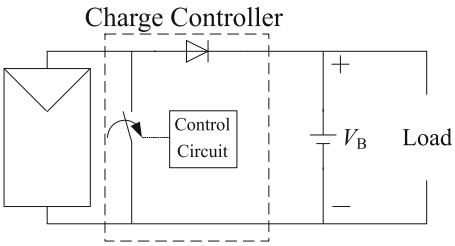


**Fig. 9.11** Shunt-type battery charger

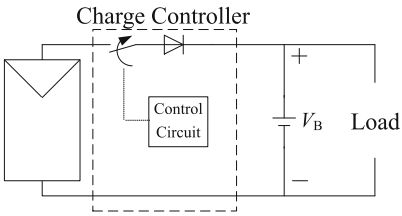


**Fig. 9.12** Series-type battery charger

### *9.5.1 Shunt-Type Solar Battery Charger*

Shunt-type charge controllers are the simplest type and the first that were developed. A schematic is shown in Fig. 9.11. The controller functions by operating a solid-state switch or relay that is in parallel (shunt) with the PV module. When closed, the switch short-circuits the module and prevents the battery from being charged. The diode between the switch and the battery prevents the battery from discharging back through the shunt. The simplicity and low cost of the design make shunt controllers viable for low-end off-grid systems. Shunt controllers can also be used with other energy conversion technologies such as a wind energy conversion system (WECS) that can be shorted without damage to the source.

### *9.5.2 Series-Type Solar Battery Charger*

A series-type solar battery charger is a chopper whose input is a PV module, as shown in Fig. 9.12. When the switch is opened, the energy source is open-circuited, isolating it from the battery. A series-type charge controller can also be used with energy conversion technologies that can be safely open-circuited: fossil fuel or biomass gen sets and micro hydro power systems with speed control.

### *9.5.3 Series-Type Solar Battery Charger with PWM*

The series-type solar battery charger can be improved by using pulse width modulation to control the solid-state switch. The duty cycle is adjusted to vary

the average current to the battery. The duty cycle is increased as greater current is desired. When the battery voltage reaches a predefined upper limit, the duty cycle is set to zero, thus open-circuiting the PV module. The greater control of the current in PWM type controllers allows more sophisticated charging algorithms to be used such as the three-stage algorithm discussed in Sect. 10.2.1.

*Example 9.2* Consider a series-type charger with PWM. The PV array is modeled as a constant voltage source $V_{PV}$ whose value is 16 V. The battery state-of-charge (SoC) voltage is $V_{SoC} = 12.8$ V and its resistance $R_B = 0.1\ \Omega$. Determine the duty cycle $D$ needed for the battery current to have an average value of 1.5 A.

**Solution** The current to the battery at any time $t$ is:

$$i_B(t) = \left(\frac{V_{PV} - V_{SOC}}{R_B}\right) q(t).$$

Applying (9.4) to the battery current pulse train:

$$I_B = D i_{max}(t) + (1 - D) i_{min}(t) = \left(\frac{V_{PV} - V_{SOC}}{R_B}\right) D + (1 - D)0$$
$$= \left(\frac{V_{PV} - V_{SOC}}{R_B}\right) D.$$

Solving for $D$ yields

$$D = I_B \left(\frac{R_B}{V_{PV} - V_{SOC}}\right) = 1.5 \left(\frac{0.1}{16 - 12.8}\right) = 0.0469.$$

## 9.6 AC Battery Charger

Batteries can also be charged from AC sources such as gen sets, WECS, MHP systems, and, in general, an AC bus. The basic principles of charging the large stationary batteries in mini-grids also apply to the smaller-capacity batteries found in solar home systems and other electronic devices such as mobile phones and laptop computers. Within the charging circuit of all of these devices is a rectifier, which converts the input AC voltage to the DC voltage used to charge a battery.

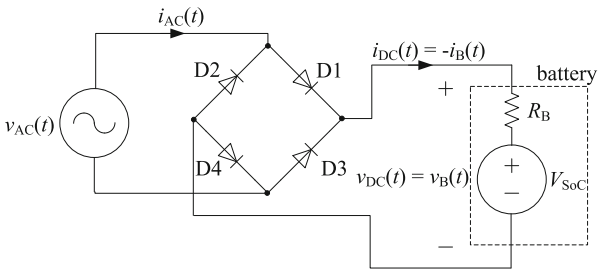


**Fig. 9.13** Single-phase full-bridge rectifier connected to a battery

### *9.6.1 Single-Phase Full-Bridge Rectifier*

A battery charger whose input is single-phase AC uses a single-phase full-bridge rectifier to convert AC to DC. The circuit consists of four diodes as shown in Fig. 9.13.

The diodes conduct when they are forward biased. This occurs in pairs, so that D1 and D4 are conducting together while D2 and D3 are blocking and vice versa. The state of the diodes changes automatically, without the need for external control. For this reason, diode-based rectifiers are known as "uncontrolled" rectifiers.

For simplicity, the input is modeled as an AC voltage source. We will assume that the rectifier output is connected to a battery. We will sometimes refer to the input side of the rectifier as the AC side and the output as the DC side. It should be noted that the output side is not a constant voltage, but it does have a positive average value. The voltage at the AC side and DC side is denoted as $v_{AC}(t)$ and $v_{DC}(t)$, respectively. Since the DC side of the rectifier is connected to the battery

$$v_{DC}(t) = v_B(t) = R_B i_{DC}(t) + V_{SoC} \tag{9.10}$$

where $i_{DC}(t)$ is the current output by the rectifier, which is also the negative of the battery current. (Recall from Chap. 8 that the battery current is defined as positive when discharging.)

Let the voltage on the AC side of the rectifier have amplitude $v_{AC}^{max}$ and operate at a frequency of $\omega$ so that

$$v_{AC}(t) = v_{AC}^{max} \sin(\omega t). \tag{9.11}$$

When the diodes are not conducting, $i_{DC}(t) = 0$ and from (9.10) $V_{DC}(t) = V_{SoC}$. A pair of diodes will conduct when they are forward biased. This occurs when $|v_{AC}(t) - 2V_D| > V_{SoC}$ where $V_D$ is the voltage drop associated with a single diode. The phase angle at which conduction begins varies with the battery voltage. The conduction angle $\theta_{on}$ is

$$\theta_{on} = \sin^{-1}\left(\frac{V_{SoC} + 2V_D}{v_{AC}^{max}}\right). \tag{9.12}$$

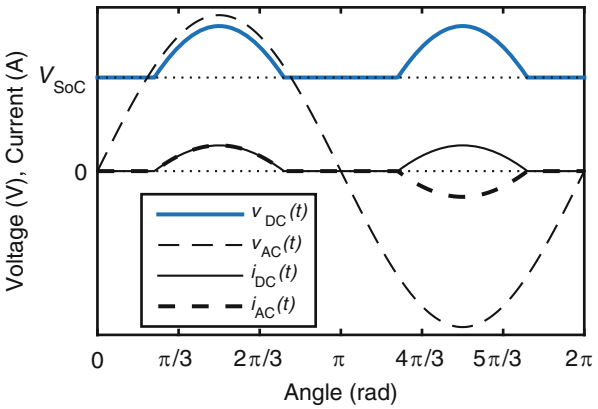


**Fig. 9.14** Waveforms of a single-phase full-bridge rectifier connected to a battery

Due to symmetry, the angle at which conduction ends for each half cycle is

$$\theta_{\text{off}} = \pi - \theta_{\text{on}}. \tag{9.13}$$

The voltage on the DC side is therefore

$$v_{\text{DC}}(t) = \begin{cases} v_{\text{AC}}^{\max} \sin(\omega t) - 2V_{\text{D}} & : \theta_{\text{on}} < \theta < \theta_{\text{off}} \\ V_{\text{SoC}} & : \text{else} \end{cases} \tag{9.14}$$

The current to the battery is found by rearranging (9.10)

$$i_{\text{DC}}(t) = \frac{v_{\text{DC}}(t) - V_{\text{SoC}}}{R_{\text{B}}}. \tag{9.15}$$

The resulting waveforms are shown in Fig. 9.14. The reader should confirm that the waveforms are as expected. The battery voltage pulsates at twice the frequency of the AC source. When the diodes are not conducting, the battery voltage is $V_{\text{SoC}}$. The current from the AC source is only non-zero when the diodes are conducting. The AC current is not sinusoidal and so there is some level of harmonic distortion. We note that the voltage and current from the AC source peak at the same time (angle), and both are symmetric around the peak for each half of the waveform. They are in phase.

The instantaneous power to the battery $p(t)$ is the product of the instantaneous voltage and current on the DC side of the rectifier:

$$p(t) = v_{\text{DC}}(t) i_{\text{DC}}(t) \tag{9.16}$$

The real power $P$ is defined as the average of the instantaneous power. The power delivered to the battery is found by substituting (9.14) and (9.15) into (9.16) and averaging over one half cycle:

$$P_{\text{B}} = \frac{1}{\pi} \int_{\theta_{\text{on}}}^{\theta_{\text{off}}} \left( v_{\text{AC}}^{\max} \sin(\theta) - 2V_{\text{D}} \right) \frac{v_{\text{AC}}^{\max} \sin(\theta) - 2V_{\text{D}} - V_{\text{SoC}}}{R_{\text{B}}} \mathrm{d}\theta. \tag{9.17}$$

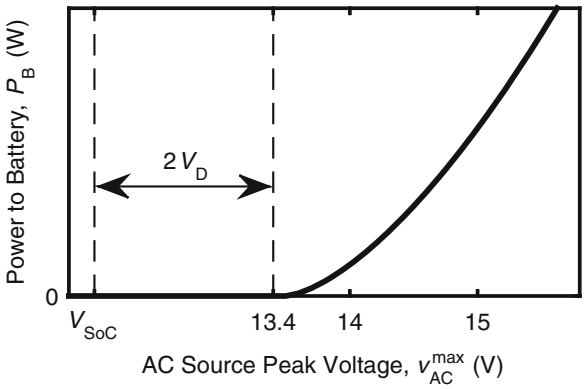


**Fig. 9.15** The power to the battery increases nonlinearly with the voltage of the AC source. The battery begins charging once the AC voltage is sufficient to bias the diodes in the rectifier

From this we see that the power delivered to the battery is dependent on the amplitude of the voltage on the AC side of the rectifier, the battery SoC (which affects $V_{SOC}$, $\theta_{on}$ and $\theta_{off}$ ), and the battery resistance. Figure 9.15 shows an example of how the power to the battery changes with the voltage of the AC source. This could, for example, be a WECS. Notice that no current is delivered to the battery until the AC-side voltage exceeds the battery voltage by $2V_D$. If the battery terminal voltage is much greater than the diode voltage drops, they can be ignored, and (9.17) is reduced to

$$P_B = \frac{v_{AC}^{max}}{\pi R_B}\left(v_{AC}^{max}\int_{\theta_{on}}^{\theta_{off}} \sin^2(\theta)\,d\theta - V_{SoC}\int_{\theta_{on}}^{\theta_{off}} \sin\theta d\theta\right) \tag{9.18}$$

where $\theta_{on}$ and $\theta_{off}$ are calculated from (9.12) and (9.13) with $V_D = 0$.

### *9.6.2 Three-Phase Rectifier*

The basic principles of a single-phase rectifier apply to a three-phase rectifier. The principle circuit is shown in Fig. 9.16. There are now six diodes instead of two. Hereafter, we assume that the AC voltage is such that there is always a conduction path between the AC side and DC side, and that the voltage drop associated with the diodes is negligible. In other words, the battery is always being charged. In this case, the voltage on the DC side of the rectifier is as shown in Fig. 9.17. The peak of $v_{DC}(t)$ is the peak of line–line voltage on the AC-side $V_{\ell\ell}^{max}$. The output voltage stays close to this average value, but pulsates with a period of $\pi/3$ radians (60°). By symmetry, we can analyze a single 60°segment and extend the results to the entire period. For convenience, we will select a period centered around the peak of one of the humps:

$$v_{DC}(t) = V_{\ell\ell}^{max}\cos(\theta) : -\pi/6 \le \theta \le \pi/6 \tag{9.19}$$

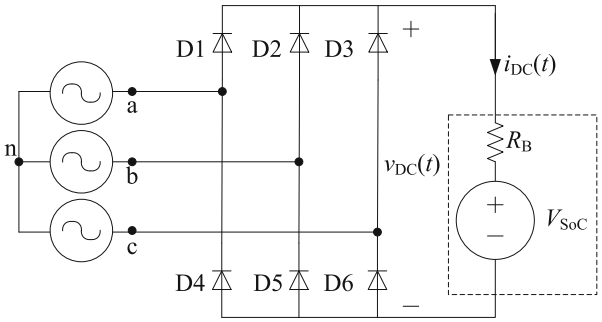


**Fig. 9.16** Three-phase full-bridge rectifier

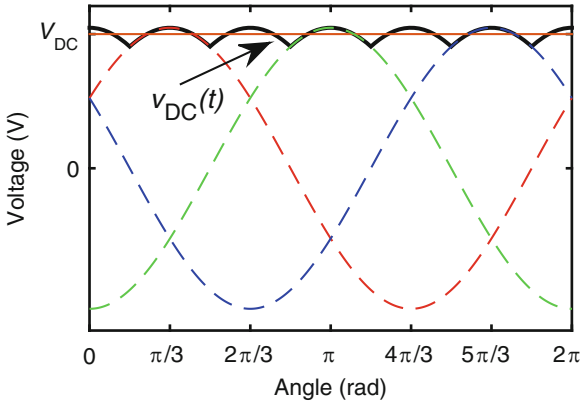


**Fig. 9.17** Voltage output waveform of an ideal three-phase rectifier. The dashed lines are the line–line voltages of the AC source

The DC-side current during this period is:

$$i_{DC}(t) = \frac{V_{DC}(t) - V_{SoC}}{R_B} = \frac{V_{\ell\ell}^{max}\cos(\theta) - V_{SoC}}{R_B}. \tag{9.20}$$

The *average* value of the voltage and current on the DC side of the rectifier is

$$V_{DC} = \frac{3}{\pi}\int_{-\pi/6}^{\pi/6} V_{\ell\ell}^{max}\cos(\theta)\mathrm{d}\theta = \frac{3}{\pi}V_{\ell\ell}^{max} \approx 0.955 V_{\ell\ell}^{max} \tag{9.21}$$

$$I_{DC} = \frac{V_{DC} - V_{SoC}}{R_B}. \tag{9.22}$$

In other words, the average voltage on the DC side of the rectifier is approximately 95.5% of the peak line–line voltage on the AC side. We cannot simply multiply the DC-side average voltage and current to find the power to the battery because the voltage and current are not constant. Instead, we must take the average of the

instantaneous power, which repeats with a period $\pi/3$. The instantaneous power into the battery during this period is found by multiplying the voltage (9.19) and current (9.20):

$$p(t) = v_{\mathrm{DC}}(t) i_{\mathrm{DC}}(t) = \frac{V_{\ell\ell}^{\max} \cos(\theta) \left( V_{\ell\ell}^{\max} \cos(\theta) - V_{\mathrm{SoC}} \right)}{R_{\mathrm{B}}} \tag{9.23}$$

$$p(t) = \frac{1}{R_{\mathrm{B}}} \left( \left( V_{\ell\ell}^{\max} \right)^2 \cos^2(\theta) - V_{\ell\ell}^{\max} V_{\mathrm{SoC}} \cos(\theta) \right) \tag{9.24}$$

The real power into the battery is found by averaging $p(t)$ over the period from $-\pi/6$ to $\pi/6$:

$$P_{\mathrm{B}} = \frac{3}{\pi} \frac{1}{R_{\mathrm{B}}} \int_{-\pi/6}^{\pi/6} \left( V_{\ell\ell}^{\max} \right)^2 \cos^2(\theta) - V_{\ell\ell}^{\max} V_{\mathrm{SoC}} \cos(\theta) \mathrm{d}\theta \tag{9.25}$$

$$= \frac{3}{\pi} \frac{1}{R_{\mathrm{B}}} \left( \left( V_{\ell\ell}^{\max} \right)^2 \left( \frac{\pi}{6} + \frac{\sqrt{3}}{4} \right) - V_{\ell\ell}^{\max} V_{\mathrm{SoC}} \right) \tag{9.26}$$

$$= \frac{3}{\pi} \frac{1}{R_{\mathrm{B}}} \left( 0.957 \left( V_{\ell\ell}^{\max} \right)^2 - V_{\ell\ell}^{\max} V_{\mathrm{SoC}} \right). \tag{9.27}$$

Equation (9.27) shows that the power to the battery increases as the voltage on the AC side increases. It decreases as the battery state-of-charge increases. The current and voltage associated with a single phase of the AC source are shown in Fig. 9.18. As with a single-phase rectifier, the current is distorted, but it is in phase with the voltage.

The assumption that the voltage on the AC side is unchanging in steady state is not always valid. It is a reasonable approximation for systems where the AC bus voltage is formed and controlled by an inverter or synchronous generator with an automatic voltage regulator. However, in some mini-grids this will not be the case.

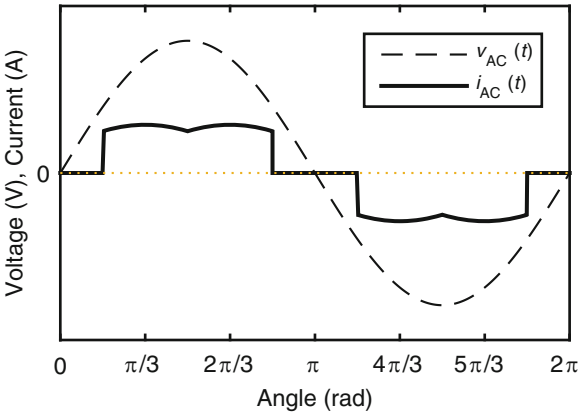


**Fig. 9.18** Phase voltage (line-to-neutral) and current from a single phase of a three-phase AC source charging a battery through a rectifier

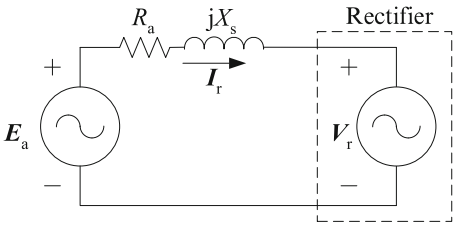


**Fig. 9.19** Single-phase phasor-domain model of a three-phase AC generator charging a battery through a three-phase rectifier

An example is a mini-grid powered only by WECS. In this case, terminal voltage of the WECS and hence the voltage on the AC side of the rectifier will vary with current due to the voltage drop associated with the winding impedance. It will also vary with rotational speed under changing wind conditions.

Calculating the power to the battery under these conditions requires reconciling the AC (generator side) and DC (battery side) circuits. Our approach is to model the DC side as an equivalent AC load and use phasors to analyze the circuit [11]. This is useful in understanding how the variables affect the complete system. A more accurate, but less insightful, analysis can be obtained using a circuit model-based simulation.

We begin by making the assumption that the voltage drop across the diodes is negligible compared to the amplitude of the AC input. We next develop a model that is useful in analyzing the AC side of the circuit. The model, which represents one of the three phases of a synchronous AC generator connected to the rectifier, is shown in Fig. 9.19. The generator and its internal impedance are on the left side of the circuit. The rectifier and battery cannot be represented in the phasor domain. However, we can approximate them as a voltage source $\boldsymbol{V}_r$ if we are careful in understanding the limitations and conditions of doing so. The model is only valid when the battery is continuously being charged; therefore, the current $\boldsymbol{I}_r$ must be positive in the direction shown. We noted that in Fig. 9.18, the phase voltage and current are in phase with each other. Therefore, the power supplied to the battery must only have a real component. This means that $\boldsymbol{I}_r$ and $\boldsymbol{V}_r$ must be in phase. For convenience, we set the phase angle of both to zero (reference) so that

$$\boldsymbol{V}_r = |\boldsymbol{V}_r| \angle 0^\circ \tag{9.28}$$

$$\boldsymbol{I}_r = |\boldsymbol{I}_r| \angle 0^\circ. \tag{9.29}$$

Lastly, since the rectifier is modeled as a lossless component, the power into the AC side must equal the power into the battery:

$$P_B = 3\text{Re}\left\{\boldsymbol{V}_r \boldsymbol{I}_r^*\right\} = 3|\boldsymbol{V}_r||\boldsymbol{I}_r|. \tag{9.30}$$

The factor of three is needed because the model pertains to a single phase of the three-phase source.

What value should $|\boldsymbol{V}_\mathrm{r}|$ be? It is the line-to-neutral voltage on the AC side of the rectifier. It can be computed from the maximum line–line voltage:

$$|\boldsymbol{V}_\mathrm{r}| = \frac{V_\phi^{\max}}{\sqrt{2}} = \frac{V_{\ell\ell}^{\max}}{\sqrt{2}\sqrt{3}} = \frac{V_{\ell\ell}^{\max}}{\sqrt{6}} \tag{9.31}$$

where $V_\phi^{\max}$ is the peak line-to-neutral voltage.

The magnitude of the current $\boldsymbol{I}_\mathrm{r}$ is found by rearranging (9.30) using (9.27) and using (9.31) to replace $V_{\ell\ell}^{\max}$ with $|\boldsymbol{V}_\mathrm{r}|$:

$$|\boldsymbol{I}_\mathrm{r}| = \frac{P_\mathrm{B}}{3|\boldsymbol{V}_\mathrm{r}|} = \frac{6}{\pi}\frac{1}{R_\mathrm{B}}\left(0.957|\boldsymbol{V}_\mathrm{r}| - \frac{1}{\sqrt{6}}V_\mathrm{SoC}\right) \tag{9.32}$$

Simplifying further:

$$|\boldsymbol{I}_\mathrm{r}| = \frac{1.8270}{R_\mathrm{B}}\left(|\boldsymbol{V}_\mathrm{r}| - 0.4268 V_\mathrm{SoC}\right) \tag{9.33}$$

Assuming the generator resistance to be negligible ($R_\mathrm{a} = 0$), and noting that $\boldsymbol{I}_\mathrm{r}$ and $\boldsymbol{V}_\mathrm{r}$ have an angle of zero degrees, we have:

$$\boldsymbol{E} = \mathrm{j}X_\mathrm{s}\boldsymbol{I}_\mathrm{r} + \boldsymbol{V}_\mathrm{r} = \mathrm{j}X_\mathrm{s}|\boldsymbol{I}_\mathrm{r}| + |\boldsymbol{V}_\mathrm{r}| \tag{9.34}$$

Equation (9.33) and (9.34) are sufficient to analyze the circuit, as shown in the following example.

*Example 9.3* A three-phase WECS is used to charge a battery through a rectifier. The per-phase RMS induced voltage at a certain rotational speed is 14 V at 10 Hz. The per-phase synchronous reactance at this frequency is 0.19 Ω. The battery internal SoC voltage is 12.3 V, and the corresponding battery resistance is 0.25 Ω. Determine the power to the battery.

**Solution** To determine the power to the battery, we must first compute $\boldsymbol{I}_\mathrm{r}$ and $\boldsymbol{V}_\mathrm{r}$. Only the magnitude is needed as the angle of each of these phasors is set to zero degrees. Rewriting (9.34)

$$|\boldsymbol{E}|\angle\delta = \mathrm{j}X_\mathrm{s}|\boldsymbol{I}_\mathrm{r}| + |\boldsymbol{V}_\mathrm{r}|.$$

Next, splitting this into real and imaginary parts using Euler's Identity:

$$|\boldsymbol{E}|\cos\delta = 14\cos\delta = |\boldsymbol{V}_\mathrm{r}| \tag{9.35}$$

$$|\boldsymbol{E}|\sin\delta = 14\sin\delta = 0.19|\boldsymbol{I}_\mathrm{r}| \tag{9.36}$$

(continued)

The current $|\boldsymbol{I}_\mathrm{r}|$ is related to the voltage $|\boldsymbol{V}_\mathrm{r}|$ from (9.33)

$$|\boldsymbol{I}_\mathrm{r}| = \frac{1.8270}{0.25}\left(|\boldsymbol{V}_\mathrm{r}| - 0.4268 V_\mathrm{SOC}\right) \tag{9.37}$$

$$= 7.308\left(|\boldsymbol{V}_\mathrm{r}| - 5.250\right) \tag{9.38}$$

$$= 7.308|\boldsymbol{V}_\mathrm{r}| - 38.364. \tag{9.39}$$

Using

$$\sin^2\delta + \cos^2\delta = 1$$

on (9.35) and (9.36) and substituting for $|\boldsymbol{I}_\mathrm{r}|$ using (9.37) yield

$$0.19^2\frac{(7.308|\boldsymbol{V}_\mathrm{r}| - 38.364)^2}{14^2} + \frac{|\boldsymbol{V}_\mathrm{r}|^2}{14^2} = 1$$

This can be solved through the quadratic equation to yield $|\boldsymbol{V}_\mathrm{r}| = 11.25$V. Using (9.33), the current is $|\boldsymbol{I}_\mathrm{r}| = 43.85$ A. The power to the battery is found from (9.30):

$$P_\mathrm{B} = 3|\boldsymbol{V}_\mathrm{r}||\boldsymbol{I}_\mathrm{r}| = 1480.1\ \mathrm{W}$$

Properly understanding $\boldsymbol{I}_\mathrm{r}$ is not completely straightforward. Although $\boldsymbol{I}_\mathrm{r}$ is modeled as a phasor in the circuit in Fig. 9.20, its waveform is not sinusoidal, as shown in Fig. 9.18. However, the calculated value of $\boldsymbol{I}_\mathrm{r}$ is close, within 5% of the RMS value of the single-phase current.

The relationship between the generator speed and power delivered to the battery is as follows. The battery does not charge until the voltage is sufficient to bias the diodes. As speed increases, the induced voltage in the generator $\boldsymbol{E}_\mathrm{a}$ increases in proportion. Once the diodes are biased, the power to the battery increases approximately linearly. This continues over a range of speeds and induced voltages. However, as the speed further increases, the power levels off. This is evident in Fig. 9.20. The nonlinearity is attributed to the inductive reactance, which increases with speed. The voltage drop across $\mathrm{j}X_\mathrm{s}$ then increases, reducing the terminal voltage of the generator.

### *9.6.3 Charging Circuit*

A rectifier is a necessary component in an AC battery charger, but it is usually only part of the total circuit. Most AC battery chargers use a series of stages to

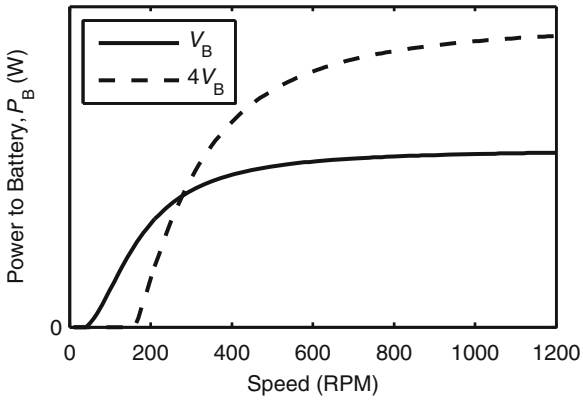


**Fig. 9.20** Power to the battery from a certain three-phase WECS using a three-phase rectifier as computed using a phasor-domain model

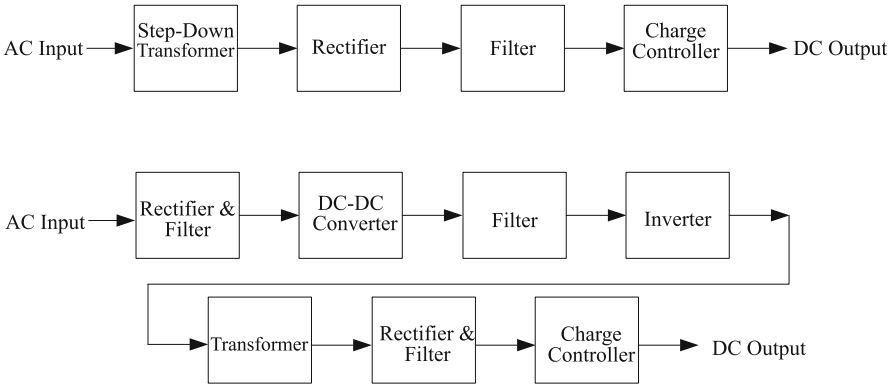


**Fig. 9.21** Battery charging circuit block diagram

convert the input AC to a stable DC voltage. Two such approaches are shown in Fig. 9.21 [4, 13, 15]. The different stages are usually combined into a single unit. The approaches assume the input AC has a higher peak voltage than the DC voltage required to charge the battery, as is often the case.

In the first approach, a transformer steps down the AC voltage to slightly above the maximum voltage the battery should be charged at. This lower-voltage AC is rectified and filtered so that it is a stable DC. The DC is input into the charge controller stage which is often a chopper circuit.

Another approach to producing DC is used in switch-mode power supplies. Using this approach the input voltage undergoes several conversions. The first stage is rectification and filtering to produce a stable DC voltage. The voltage is likely higher than needed. The next stages are a buck converter and filter, which reduce the voltage to the required magnitude. An inverter is then used to convert the DC to high-frequency, low-amplitude AC. The details of inverter circuits will be provided later. The high-frequency AC then is input into a transformer. Although this may further adjust the voltage magnitude, its primary purpose is to provide isolation

between the input and output. An output rectifier and filter are used once again to produce stable DC voltage. This is fed into the charge controller stage.

## 9.7 Automatic Voltage Regulator

Automatic voltage regulators (AVR) perform the important function of holding the output voltage of a generator steady and at the desired value. Voltage control is needed in any generator forming the AC bus voltage in an AC-coupled system. Most synchronous generators, even those in small-capacity portable gen sets, have an AVR. Without an AVR, the terminal voltage will vary with the load and power factor, as discussed in Sect. 5.2.3. AVRs are also used to synchronize and share the reactive power load among generators in systems with multiple gen sets, as will be discussed in the next chapter.

AVRs function by controlling the generator's excitation system, adjusting the field current to the rotor of a synchronous generator. A control system continually monitors the output voltage and compares it against a target value, increasing or decreasing the field current accordingly. This can be done either with an analog circuit or with a digital controller.

Recall that a field winding requires DC current. One way of supplying the DC current is to rectify the AC output of generator. However, the current output by a diode-based rectifier is not controllable. A simple chopper circuit is not used due to the large inductance of field winding, which would result in a large voltage each time the switch was opened. Instead, a buck converter can be placed at the output of the rectifier.

Another option is to use a phase-controlled rectifier. The basic circuit for a phase-controlled rectifier is shown in Fig. 9.22. The topology is the same as for the 3-phase rectifier in Fig. 9.16, except that thyristors have replaced the diodes. Control signals are sent to the thyristors to control the angle that conduction begins. More specifically, the conduction angle is delayed by some angle $\alpha$, known as the "firing angle." The resulting waveforms are shown in Fig. 9.23 for a firing angle of 30°. For simplicity, the load is assumed to be purely resistive. Delaying the conduction angle reduces the average voltage appearing on the DC side of the inverter. The average DC-side voltage can be derived from (9.21), accounting for the delay introduced by the firing angle:

$$V_{\mathrm{DC}} = \frac{3V_{\ell\ell}^{\max}}{\pi}\left(\sin(\pi/6+\alpha) - \sin(-\pi/6+\alpha)\right) \tag{9.40}$$

$$\begin{aligned} V_{\mathrm{DC}}\frac{\pi}{3V_{\ell\ell}^{\max}} &= \sin(\pi/6)\cos(\alpha) + \cos(\pi/6)\sin(\alpha) \\ &\quad - \sin(-\pi/6)\cos(\alpha) - \cos(-\pi/6)\sin(\alpha) \end{aligned} \tag{9.41}$$

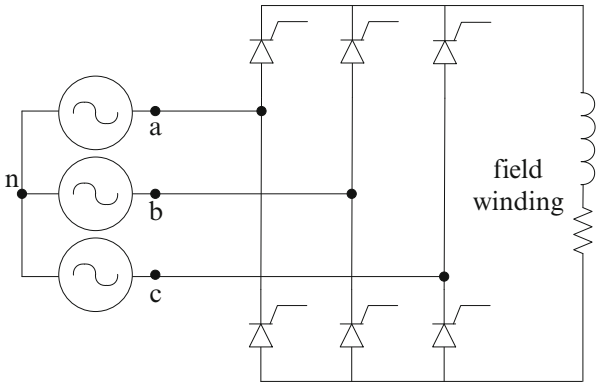


**Fig. 9.22** Circuit of a three-phase phase-controlled rectifier connected to the field winding of a synchronous generator

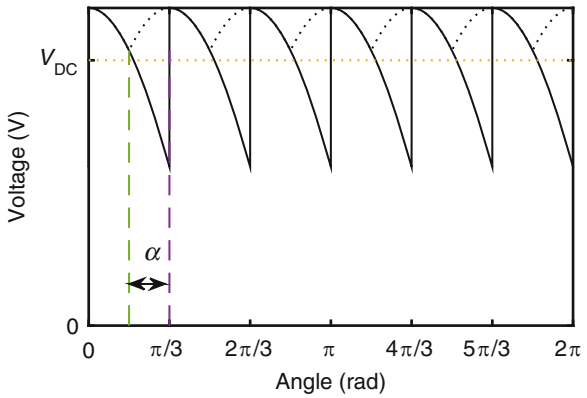


**Fig. 9.23** Voltage output waveform of a phase-controlled rectifier

$$V_{\text{DC}} = \frac{3V_{\ell\ell}^{\max}}{\pi}\left(0.5\cos(\alpha) + \frac{\sqrt{3}}{2}\sin(\alpha) + 0.5\cos(\alpha) - \frac{\sqrt{3}}{2}\sin(\alpha)\right) \quad (9.42)$$

$$V_{\text{DC}} \approx 0.955 V_{\ell\ell}^{\max}\cos(\alpha) \quad (9.43)$$

We see that (9.43) reconciles with (9.21) when the firing angle is zero. The firing angle should not exceed 90°as the voltage would become negative. The average current to the field winding is simply the average voltage divided by the field winding resistance.

Figure 9.24 also shows the relationship between firing angle and power to a resistive load, which decreases as the firing angle increases. The values are in reference to $\alpha = 0°$.

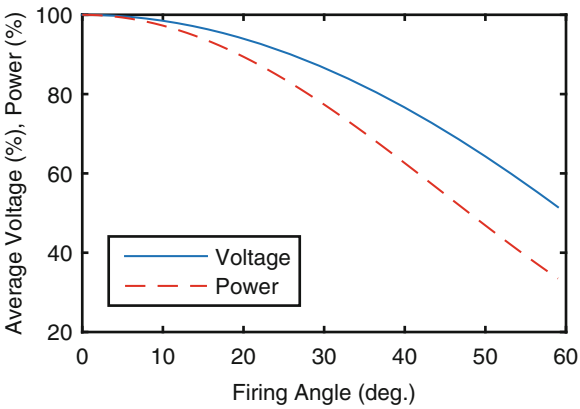


**Fig. 9.24** Voltage and power output of a phase-controlled rectifier as a function of the firing angle

## 9.8 Electronic Load Controller

Recall from Sect. 6.3.7.3 that electronic load controllers (ELCs) are used in MHP systems as a method of demand-side frequency control on the AC bus. They allow the generator to operate at a constant speed [8, 14]. More generally, electronic load control is a method of balancing the net torque on the turbine shaft by adjusting the electromagnetic torque. ELCs are sometimes referred to as "electric load governors."

ELC's are preferred over mechanical speed control systems such as spear valves and deflectors because:

- they are able to quickly respond to rapid changes in load;
- they are less complex;
- the dissipated power can be put to productive use, for example, heating water;
- they require little maintenance;
- they can be less expensive.

An ELC maintains constant torque by adjusting the real power to a diversion load (also referred to as a "ballast load") so that the real power supplied by the generator is constant. We will consider the ELC and ballast load as a collective unit. Keep in mind that when we refer to the "ELC power" the power is actually being consumed by the ballast load. The ballast load itself is simply a resistor with a high power rating.

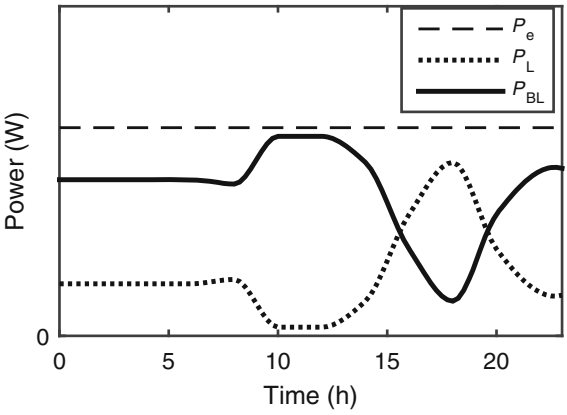


**Fig. 9.25** Power consumed by the ELC $P_{BL}$ is adjusted to maintain constant generator power $P_e$ as the load $P_L$ changes

The operation of an ELC is illustrated in Fig. 9.25. The real power from the generator $P_e$ is the sum of the power to the load $P_L$ and, to the ELC (ballast load), $P_{BL}$

$$P_e = P_L + P_{BL}. \tag{9.44}$$

To keep the generator power constant, the power to the ELC must equally balance any change in load. As the load increases, the ELC power decreases and vice versa. A frequency-based control scheme is usually used by the ELC. Any increase in electrical frequency above the targeted 50 Hz or 60 Hz leads to a decrease in $P_{BL}$ and vice versa. The ballast load is typically sized to match the generator's power rating so that even if $P_L$ drops to zero, constant speed can be maintained.

Electronic load control can be accomplished in several ways, including binary-weighted resistor network, phase-angle-controlled rectification, and impedance control.

### *9.8.1 Binary-Weighted Resistor Network*

In the binary-weighted approach, the ELC controls relays that connect and disconnect fixed-value parallel resistors to the generator to balance changes in user load. The resistors are "binary weighted" in that their values follow the pattern of $2^n$, for example, $R$, $2R$, $4R$, $8R$, and so on. An example of a single phase of a binary-weighted resistor network is shown in Fig. 9.26. The switches can be closed in different combinations to achieve several discrete steps of equivalent resistance and power consumption. The resistors should be sized so that when all are connected, the power consumed equals the rated power from the generator. The binary-weighted method is conceptually simple and easy to implement, but the power to the diversion load cannot be finely controlled, and so the frequency is not tightly regulated.

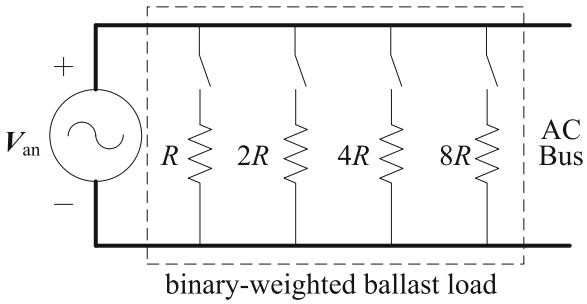


**Fig. 9.26** A single phase of an ELC ballast load using binary-weighted resistor network

### *9.8.2 Phase-Angle-Controlled Rectification: Revisited*

An alternative approach to controlling the power to the ballast load is to use a phase-angle-controlled rectifier. This allows for more precise control than a binary-weighted resistor network. However, as the firing angle increases, the current from the AC source begins to lag the voltage. When the current lags voltage, reactive power is consumed. This might seem strange, since the load is purely resistive. However, the power factor associated with any periodic but not necessarily sinusoidal current $i(t)$ and voltage $v(t)$, both with period $T$, is

$$PF = \frac{P}{I_{\text{RMS}} V_{\text{RMS}}} = \frac{\frac{1}{T}\int_0^T i(t)v(t)\mathrm{d}t}{\sqrt{\frac{1}{T}\int_0^T i^2(t)\mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T v^2(t)\mathrm{d}t}} \tag{9.45}$$

where $I_{\text{RMS}}$ and $V_{\text{RMS}}$ are the RMS values of the current and voltage. There is nothing about this definition that requires resistive loads to have unity power factor. The notion that resistive loads operate at unity power factor is only valid if the circuit consists entirely of linear elements. This is a common assumption and one that makes phasor-based circuit analysis possible. However, it is not valid when power electronic converters with nonlinear characteristics such as a phase-controlled rectifier are in the circuit.

As an example, the current and voltage of a single phase of a three-phase AC source connected to a phase-angle-controlled rectifier are shown in Fig. 9.27. The firing angle is 30°. The waveforms are not symmetric in phase as they were for uncontrolled rectifiers (see Fig. 9.18). It can be shown that the power factor of a three-phase phase-controlled rectifier is

$$PF = \frac{I_{\text{RMS},1}}{I_{\text{RMS}}} \cos(\alpha) \tag{9.46}$$

where $I_{\text{RMS},1}$ is the RMS value of the first harmonic of the current from a phase of the generator. From (9.46), we see that the power factor is affected by both the firing angle and the distortion of the current waveform.

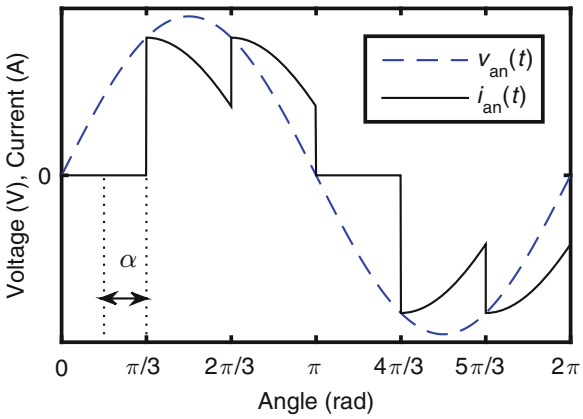


**Fig. 9.27** Phase voltage (line-to-neutral) and current waveforms of a three-phase phase-controlled rectifier

The firing angle therefore affects both the real power and reactive power consumed by the ELC. One cannot be controlled independent of the other. This can be a problem. Reactive power consumption increases the overall losses in a system and can cause the terminal voltage of the generators to be reduced, particularly with induction generators.

*Example 9.4* A mini-grid consists of an AC-coupled MHP system with a phase-angle-controlled ELC. The mini-grid supplies power to a three-phase AC load. The peak line–line voltage is 560 V. The MHP system produces 18 kW of real power. At a certain moment, the power to the load and ELC are 15 kW and 3 kW, respectively. If the load decreases to 13 kW, determine the power that should be consumed by the ELC and the corresponding firing angle assuming the ballast load resistance is 15 Ω.

**Solution** The purpose of the ELC is to maintain the power output by the MHP generator at a constant value, in this case 18 kW. Therefore, if the load decreases by $15 - 13 = 2$ kW, then $P_{\mathrm{BL}}$ should increase to $3 + 2 = 5$ kW. The average voltage required to consume this power is

$$V_{\mathrm{DC}} = \sqrt{R_{\mathrm{BL}} P_{\mathrm{BL}}} = \sqrt{15 \times 5000} = 273.9 \text{ V}.$$

The corresponding firing angle is computed by rearranging (9.43)

$$\begin{aligned} \bar{V}_{\mathrm{DC}} &\approx 0.9546 V_{\mathrm{ll}}^{\max} \cos(\alpha) \\ 273.9 &\approx 0.9546 \times 560 \times \cos(\alpha) \\ \alpha &= \cos^{-1}(0.5121) = 1.033 \text{ rad} = 59.20°. \end{aligned}$$

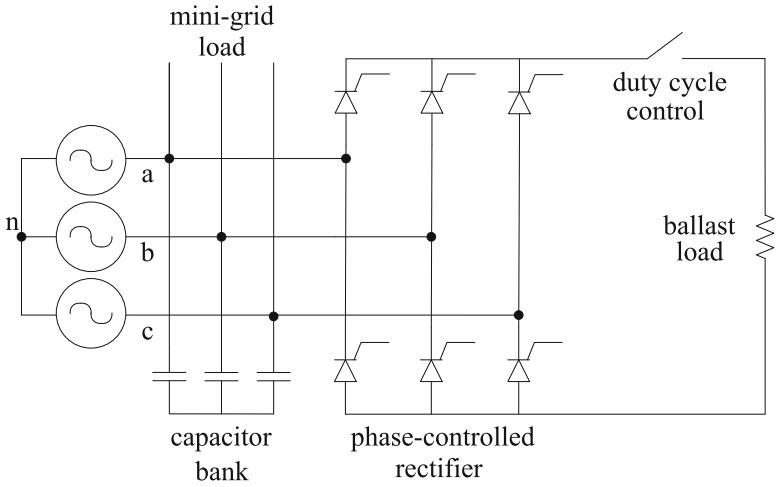


**Fig. 9.28** Impedance controller circuit for electronic load control with capacitor bank

### *9.8.3 Impedance Controller*

The phase-angle-controlled rectifier can be modified to allow greater control of the real and reactive power consumed by the ELC. The DC output of the phase-angle-controlled rectifier is connected to a chopper and then to the load. In parallel with the ballast load are capacitors, as shown in Fig. 9.28. The combined circuit is known as an ELC with impedance control or simply as an "impedance controller" [1, 2]. The capacitors can be omitted if the ability of the impedance controller to supply reactive power is not necessary. The impedance controller has two control variables: the firing angle and the duty cycle of the chopper. This allows the ELC to adjust the reactive power and real power consumed. Although the real and reactive power are not entirely independent of each other, better control is offered than from a phase-controlled rectifier. The capacitors allow the controller to supply reactive power as well as consume it.

An example waveform of the current of a single phase of a three-phase ELC with impedance control is shown in Fig. 9.29. The phase voltage is shown for reference. The reader should compare the current waveform with that in Fig. 9.27. Figure 9.30 shows the real and reactive power for different firing angles and duty cycles. The effect of the capacitors has been excluded so that the effects of the firing angle and duty cycle are highlighted.

The capacitors have a fixed value, and so the reactive power they supply is also fixed. The total reactive power consumed by the impedance controller $Q_{\mathrm{IC}}$ is:

$$Q_{\mathrm{IC}} = Q_{\mathrm{BL}}(\alpha, D) - Q_{\mathrm{cap}} \tag{9.47}$$

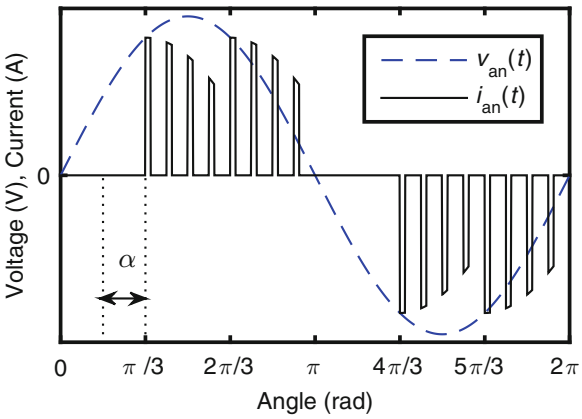


**Fig. 9.29** Phase voltage (line-to-neutral) and current waveforms of a three-phase phase-controlled rectifier with firing angle of 30°and a duty cycle of 25%

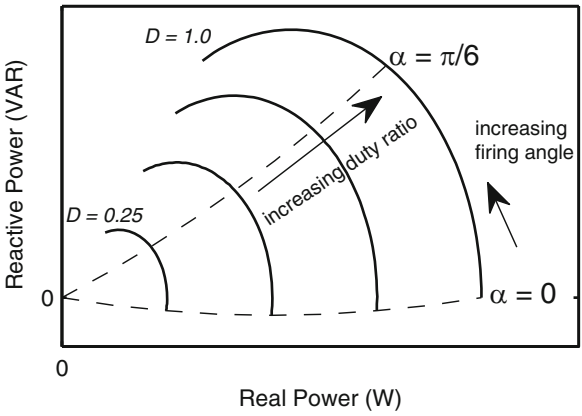


**Fig. 9.30** Family of $P$–$Q$ curves of an impedance controller without capacitors. Increasing the duty cycle increases the real power, and increasing the firing angle increases the reactive power consumption. The duty cycle and firing angle must be coordinated to achieve the desired operating point

where $Q_{BL}(\alpha, D)$ is the reactive power consumed by the ballast load portion of the impedance controller. The real power of the impedance controller is the same as the ELC without fixed capacitance: $P_{IC} = P_{BL}$. The impedance controller reactive power can be positive or negative. When it is negative, it is supplying reactive power. Impedance controllers are often used when in MHP systems with induction rather than synchronous generators.

Induction generators do not have a controllable excitation system, and so an AVR cannot be used. The generator voltage therefore changes with the current it provides. It varies with the load and the power factor of the load. This is problematic. An impedance controller can be used to set the speed of the generator by adjusting $P_{ELC}$ and the voltage by controlling $Q_{ELC}$.

## 9.9 Inverters

Inverters perform the important function of converting DC voltage to AC voltage [5, 6]. They are used in DC-coupled systems to supply an AC load and in AC–DC systems to facilitate the flow of power from the DC bus to the AC bus. An example of inverters used in a mini-grid is shown in Fig. 9.31. Some inverters are capable of forming the AC bus voltage and synchronizing with other AC sources. Inverters are a critical component in many off-grid systems, including mini-grids and some solar home systems. Keep in mind though that inverters are not necessary in systems that either do not have a DC bus or only have DC loads.

### *9.9.1 Principle Circuit*

The basic premise of an inverter is to use solid-state switching to alternate the polarity of the voltage applied to a load. A schematic of a simple inverter is shown in Fig. 9.32. The control circuitry is not shown. A DC voltage source is used to represent the battery, but this could also be a large charged capacitor. For inverters rated up to 5 kVA and 96 V, power MOSFETs are most often used as the switching elements. Above these ratings IGBTs are used.

A controller, not shown, is used to control the state of the switches. The switches are operated in pairs: Q1 and Q3 are always in the same state, as are Q2 and Q4. When Q1 and Q3 are conducting, Q2 and Q4 are not conducting. As the switching alternates, so does the polarity of the applied voltage and current. The resulting voltage waveform is shown in Fig. 9.33. The time each switch spends conducting and not conducting is the same so that the applied voltage waveform has an average of zero. The basic goal of achieving an alternating voltage output has been realized.

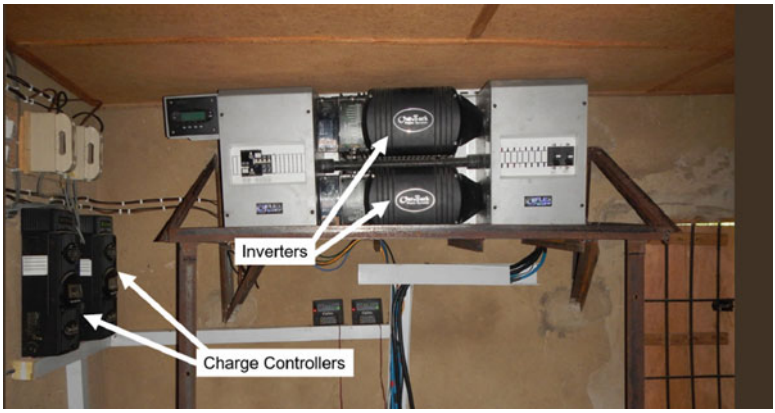


**Fig. 9.31** A pair of inverters provide AC to a mini-grid in Nigeria (courtesy GVE Projects)

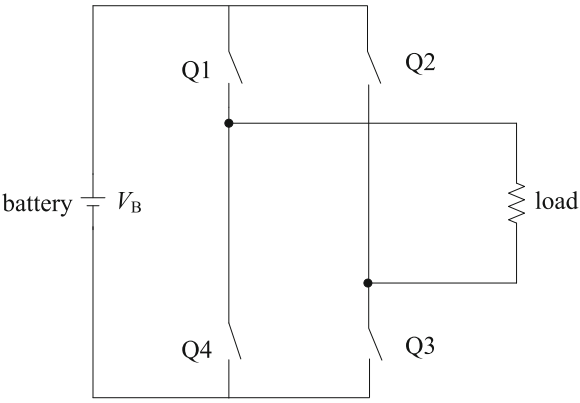


**Fig. 9.32** Basic circuit of single-phase inverter

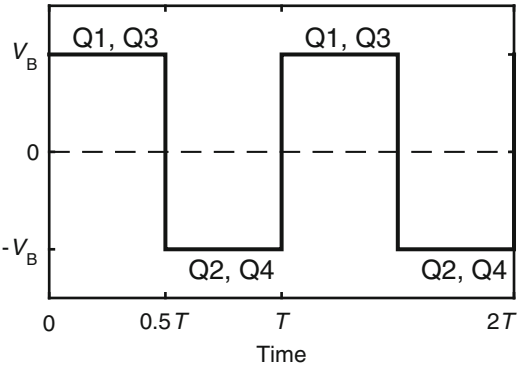


**Fig. 9.33** Voltage output of a simple inverter circuit

The waveform, although no longer DC, is also not sinusoidal. It is a square wave. Square waves have high harmonic distortion relative to the desired pure sinusoidal waveform. A low-pass filter can be placed at the output to reduce the distortion. However, because the fundamental frequency of the square wave is either 50 or 60 Hz, the filter capacitor must be large and therefore expensive. The output voltage varies between $\pm V_B$, which is typically lower than the 120 VAC or 230 VAC (RMS) that is desired. An output transformer can be used to increase the voltage to the desired level. Alternatively, a boost DC–DC converter can be used to increase the DC voltage input to the inverter.

### *9.9.2 Sinusoidal Pulse Width Modulation Switching*

We can reduce the distortion of the output voltage by using pulse width modulation. However, rather than using a fixed duty cycle, the duty cycle varies over the course of the desired period of the output voltage.

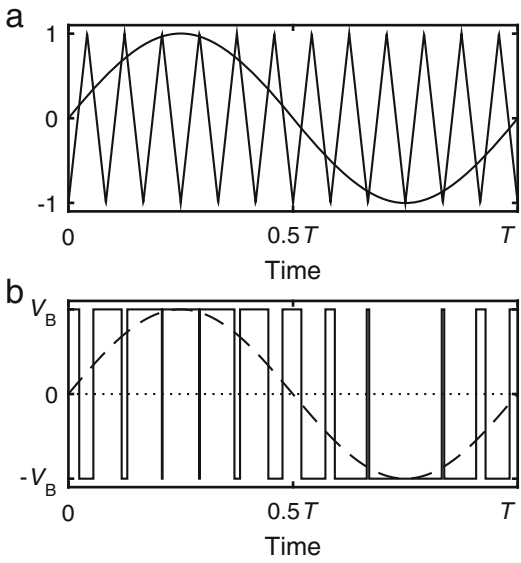


**Fig. 9.34** (**a**) Carrier (triangular) and modulation (sinusoidal) signals used in sine wave pulse width modulation. The carrier frequency shown is much lower than in practice. (**b**) Output voltage form and its fundamental component (dashed)

The switching pattern is realized using a carrier-based PWM technique known as sinusoidal PWM (SPWM). In SPWM, the modulating signal is a sinusoid $f_{\mathrm{m}}(t)$ with frequency $\omega_{\mathrm{m}}$ and amplitude $a_{\mathrm{m}}$. The carrier is a triangle-shaped waveform $f_{\Delta}(t)$with amplitude $a_{\Delta}$ and frequency $\omega_{\Delta}$. The modulating frequency is much lower than the carrier $\omega_{\mathrm{m}} << \omega_{\Delta}$. For example, the modular frequency is 50 or 60 Hz, but the carrier frequency is several kilohertz. The switches are controlled such that when $f_{\mathrm{m}}(t) > f_{\Delta}(t)$, Q1 and Q3 are on and Q2 and Q4 are off; when $f_{\mathrm{m}}(t) < f_{\Delta}(t)$, Q1 and Q3 are off and Q2 and Q4 are on. The resulting waveforms are shown in Fig. 9.34.

Although the output waveform in Fig. 9.34b (the variable duty cycle waveform) does not appear to be sinusoidal, it in fact has a large Fourier Series component at the fundamental frequency. This is shown as the dashed line in Fig. 9.34b. The distortion is largely due to harmonics near the carrier frequency. This higher frequency distortion can be filtered using a small, inexpensive capacitor so that the output is nearly sinusoidal.

SPWM offers a way to control the amplitude of the filtered output voltage. The amplitude is changed by adjusting the modulation index. The modulation index is defined as the ratio of the amplitude of the modulating signal to the carrier signal.

$$m_{\mathrm{a}} = \frac{a_{\mathrm{m}}}{a_{\Delta}}. \tag{9.48}$$

We can show that the output voltage at the fundamental frequency is:

$$V_{\mathrm{AC}}^{\mathrm{max}} = m_{\mathrm{a}} V_{\mathrm{DC}} \tag{9.49}$$

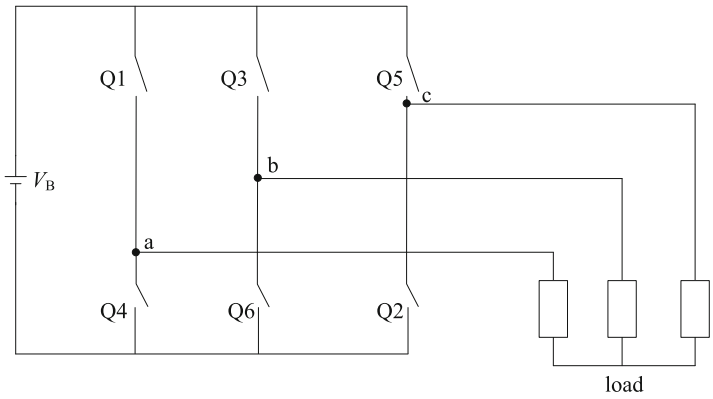


**Fig. 9.35** Basic circuit of a three-phase inverter

for modulation indexes between zero and one. The ability to control the voltage magnitude is important as it allows the output voltage to be tightly regulated, even if the load or input voltage source changes. It also allows the reactive power to be shared between multiple synchronized inverters, which is discussed in Sect. 10.10.2. The voltage can be boosted using a transformer at the output or a boost converter at the inverter's input.

### *9.9.3 Three-Phase Inverters*

Some mini-grid users, especially those with large motors or high-power loads, require three-phase power. It is technically possible for three single-phase inverters to be connected together to create a three-phase supply. However, not all inverters have this functionality. Instead, a three-phase inverter can be used.

Three-phase inverters rely on the same principles as a single-phase inverter with SPWM. The circuit however now contains six switches, as shown in Fig. 9.35. The output line–line voltage waveforms are shown in Fig. 9.36 for a single period $T$. The switches that are conducting for each sixth of a cycle are shown in Fig. 9.36a. At any moment, three of the MOSFETS are conducting. The line–line voltage waveforms are each shifted by $0.33T$ (120°), as in a three-phase supply. The waveforms approximate sine waves but are heavily distorted but can be filtered.

Some mini-grids might require the use of more than one inverter. The implementation is straightforward if each inverter supplies separate, isolated AC circuits. However, if the inverters are to be connected in parallel anywhere in the system, most likely the AC breaker box, then care must be taken to synchronize their outputs.

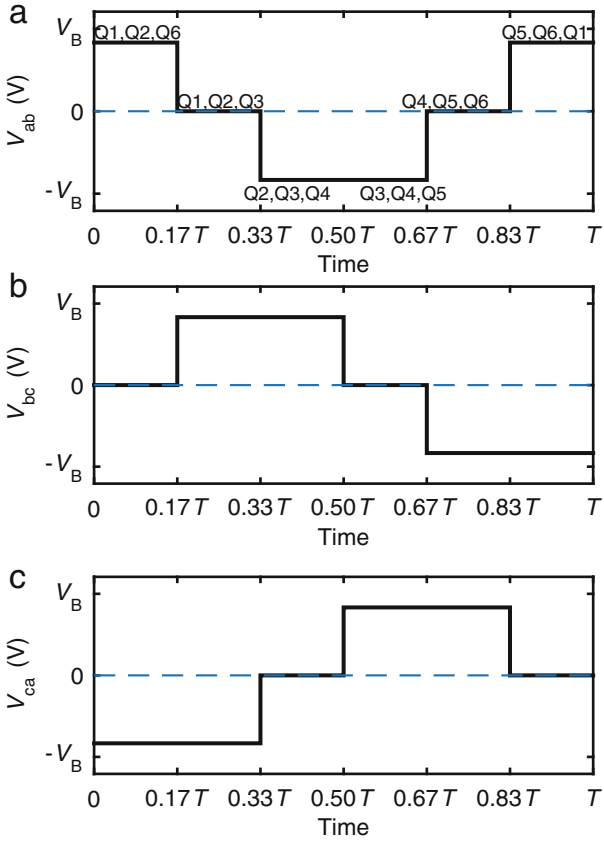


**Fig. 9.36** Output line–line voltage for a three-phase inverter

### *9.9.4 Modified Sine Wave Inverters*

In applications in which extreme affordability is a requirement, *modified sine wave* inverters can be considered. As the name implies, the output voltage waveform is not sinusoidal. Most often it is a square wave.

Modified sine wave inverters are less expensive and so are popular in improvised systems often found in rural areas. The distortion is high. The THD for a square wave is about 48%. Modified sine wave inverters should only be used to supply loads that can tolerate heavily distorted voltage, such as heaters, power tools, and incandescent bulbs. Modified sine wave inverters typically have poor voltage regulation—as the battery voltage or load varies, the output voltage changes in proportion. This means that the output voltage can change by more than 10% as the a battery discharges. As a general rule, "pure sine wave" inverters—those with low harmonic distortion—should be used.

### 9.9.5 *Efficiency*

In mini-grids without AC generators, all of the AC load passes through an inverter. The efficiency of the inverter then affects the energy that must be supplied by the system. For example, if the inverter is operating at 80% efficiency, then for every 1 kWh of load, 1.25 kWh must be supplied to the inverter. This means that the battery bank, PV array, or other energy sources must be larger than if a more efficient inverter were used.

If the inverter is modeled as lossless, then power is conserved and the input power equals the output power.

$$P_{\text{inv,in}} = V_{\text{DC}} I_{\text{DC}} = P_{\text{inv,out}} = |\boldsymbol{V}_{\text{inv}}||\boldsymbol{I}_{\text{inv}}| \cos(\Phi) \tag{9.50}$$

where $V_{\text{DC}}$ and $I_{\text{DC}}$ are the inverter input voltage and current and $\Phi$ is the phase angle between the voltage phasor $\boldsymbol{V}_{\text{inv}}$ and current phasors $\boldsymbol{I}_{\text{inv}}$. The voltage $\boldsymbol{V}_{\text{inv}}$ is applied to the AC bus or the load. In actuality, inverters have losses associated with switching, copper losses, and controller losses. Some inverters have fans that actuate to reduce the internal temperature but consume power in doing so. Including the inverter losses:

$$P_{\text{inv,out}} = \eta_{\text{inv}} P_{\text{inv,in}}. \tag{9.51}$$

The efficiency curve of an inverter is nonlinear, as shown in Fig. 9.37. Under no load, inverters consume standby power, and so the efficiency is low. Standby power is consumed even when there is no load. As the output power increases, the inverter efficiency increases and then slowly tapers. Peak efficiency is typically between 92 % and 95%, but we should not assume the inverter always operates at this efficiency.

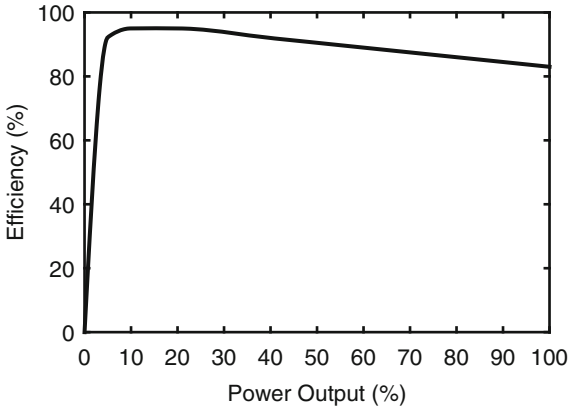


**Fig. 9.37** Typical inverter efficiency curve

The daytime load of a mini-grid that primarily serves households is low, which means the inverter tends to operate far below its peak capacity. An average efficiency over the course of the day could be about 70%. One strategy to improve efficiency is to disconnect the inverter when the load is zero or near zero. This of course reduces the availability of the electricity. Some inverters have sleep modes which automatically reduce the standby consumption when the load is low.

To account for the variation in efficiency, some manufacturers report the "European" efficiency, which is the weighted average efficiency of the inverter at different loading levels, $X$, $\eta_{\text{inv}}(X)$:

$$\begin{aligned}\eta_{\text{inv}} = {} & 0.03\eta_{\text{inv}}(0.05) + 0.06\eta_{\text{inv}}(0.10) + 0.13\eta_{\text{inv}}(0.20) \\ & +0.10\eta_{\text{inv}}(0.30) + 0.48\eta_{\text{inv}}(0.50) + 0.20\eta_{\text{inv}}(1.00)\end{aligned} \tag{9.52}$$

where the loading level is $P_{\text{inv,out}}$ divided by the rated power of the inverter.

## 9.10 Solar Inverters

Solar inverters, also known as "PV inverters," are designed to couple a PV array directly to the AC bus, bypassing or eliminating the DC bus. Solar inverters are commonly used in applications where there is no need for energy storage or a DC bus. However, solar inverters are not able to form the AC bus voltage. Thus, they must be used with another source, for example, a gen set, in an AC-coupled architecture.

The basic premise of a solar inverter is shown in Fig. 9.38. Shown is a two-stage solar inverter. The first stage is a DC–DC converter. Maximum power point tracking is performed in this stage. The converter is typically a buck–boost converter.

A large capacitor is placed either at the input or the output of the DC–DC converter to stabilize the voltage. This conceptually replaces the voltage source (battery) on the DC side of the inverter as previously discussed. Although the two-stage inverter is conceptually simple, greater efficiencies can be achieved using a single-stage inverter [3]. These inverters combine the DC–DC and AC–DC conversion in a single stage.

Solar inverters can be arranged in four architectures as shown in Fig. 9.39 [9]. The centralized architecture uses just one inverter, whereas in the module architecture, there is one inverter per PV module. The primary advantage of using more inverters is that the MPPT tend to be more effective. The disadvantage is primarily the cost of using more inverters.

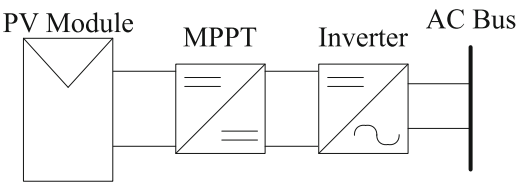


**Fig. 9.38** Two-stage solar inverter

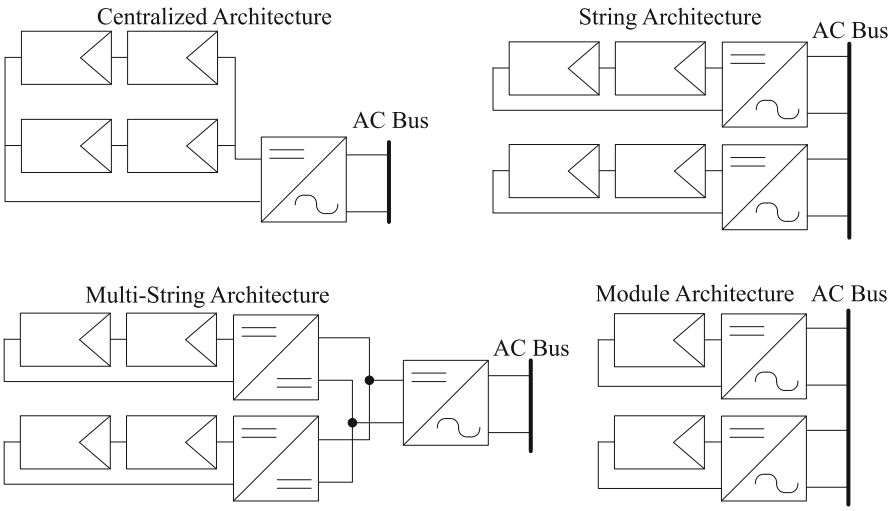


**Fig. 9.39** Solar inverters can be arranged in different architectures

*Centralized Architecture*

In the centralized architecture, one or more PV strings are combined and input into a single solar inverter. This is the lowest cost-per-kilowatt architecture because there is a single inverter. However, there are several drawbacks. The MPPT is performed on the entire array. That is, all strings in the array operate at the same voltage. Most likely, the modules have slightly different irradiance, temperatures, and perhaps shading. The optimal point for the array is likely not the optimal point for each string. There is also a single point of failure. If the inverter fails, no load can be served. In addition, because the power rating is larger than in the other architectures, a larger capacitor must be used in the filter. Electrolytic capacitors are often used because they can offer high values of capacitance. However, they are often the first component to fail in an inverter. Their lifespan, perhaps 10–15 years, quickly reduces as temperature increases [3]. This can be of concern in off-grid systems where the inverter is not located in a temperature-controlled environment. Electrolytic capacitors are not just used in solar inverters, but in any inverter with a larger power rating.

*Multi-string Architecture*

Multi-string inverters use a single DC–AC stage, but each string is connected to its own DC–DC converter. This allows the MPP to be tracked on a per-string basis, likely improving the power production. There is no need for blocking diodes and so the associated cost and losses are avoided. The multi-string architecture is more expensive than the centralized architecture, and a single point of failure remains.

*String Architecture*
String solar inverters are similar to the centralized architecture, but each string is connected to a solar inverter. The costs are increased, but the there is no longer a single point of failure, and the power is maximized at the string level. Blocking diodes are also omitted.

*Module Architecture*
Module inverters (also known as "micro-inverters") are solar inverters that are connected to an individual PV module. DC–DC conversion, MPPT, and DC–AC are performed in a single unit. The units are typically compact and can be integrated into the module's junction box. The maximum power point is tracked at the module level, yielding the highest power output of any of the architectures. Because the PV modules are only connected at the AC bus, they can be added or removed (in case of failure) easily. The ratings are typically less than 300 W, which means that electrolytic capacitors can be avoided. This can substantially increase the lifespan of the inverter. Module inverters are more expensive, ranging from about US$0.40 to 0.50/W, but this is decreasing as the industry matures. Module inverters are almost always single phase. Most are not capable of forming the AC bus voltage.

## 9.11 Grid-Tied Inverters

Inverters can be classified as being stand-alone or grid-tied. We have discussed stand-alone inverters. The output of a stand-alone inverter is controlled so that it approximates an ideal voltage source, establishing the voltage magnitude and frequency of the AC bus. When an inverter is "grid-tied," it does not necessarily mean it is connected to the national grid. Rather, it means that it is capable of being coupled to an AC bus where another source—a gen set or another inverter—has formed the AC bus voltage. An example is shown in Fig. 9.40.

Grid-tied inverters (GTIs) operate on the same principles as stand-alone inverters. However, they must be controlled so that their output synchronizes with the frequency of the AC bus voltage. If they do not, large, damaging current could be exchanged between the inverter and the AC Bus. They are controlled so that they provide the desired current or power to the AC bus. Not all inverters are capable of synchronizing to the AC bus. Serious damage can result if an inverter capable only of stand-alone operation is connected to an AC bus whose voltage is established by another source.

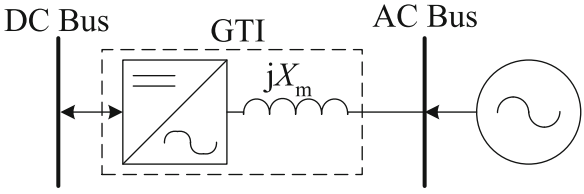


**Fig. 9.40** Grid-tied inverter connected to the AC bus through an inductance. Other sources and loads on the DC and AC bus are not shown

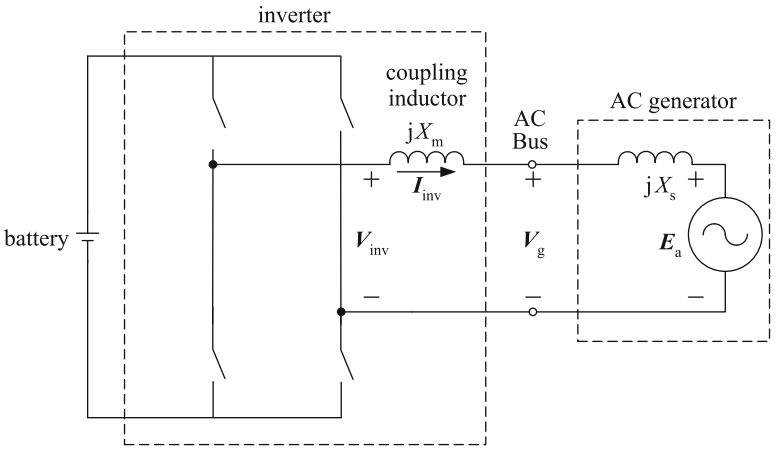


**Fig. 9.41** Example of an off-grid system with grid-tied inverter; a simple model of the inverter is shown

Internal to a GTI is an output inductance. A more detailed view is shown in Fig. 9.41. The inductance allows the internal inverter voltage $\boldsymbol{V}_{\text{inv}}$ to be somewhat different than the voltage at the AC bus. This is conceptually similar to how the synchronous reactance $X_{\text{s}}$ in a generator decouples the induced voltage from the terminal voltage. The output inductance allows the inverter to control the real and reactive power it supplies to the AC bus.

Here the gen set line-to-neutral output voltage is $\boldsymbol{V}_{\text{g}}$, and the internal inverter line-to-neutral voltage is $\boldsymbol{V}_{\text{inv}}$. Both are represented as phasors. The inverter has a control system that synchronizes its frequency with that of the gen set.

We set $\boldsymbol{V}_{\text{g}} = |\boldsymbol{V}_g|\angle 0$ so that it is the reference, and let $\boldsymbol{V}_{\text{inv}} = |\boldsymbol{V}_{\text{inv}}|\angle\delta$ where $\delta$ is the angle between the inverter voltage and generator voltage.

The current from the inverter to the AC bus is

$$\boldsymbol{I}_{\text{inv}} = \frac{\boldsymbol{V}_{\text{inv}} - \boldsymbol{V}_{\text{g}}}{\text{j}X_{\text{m}}}. \tag{9.53}$$

The real power output by the inverter through the coupling inductance is:

$$P_{\text{inv}} = \text{Re}\left\{\boldsymbol{V}_{\text{inv}}\boldsymbol{I}_{\text{inv}}^{*}\right\} = \text{Re}\left\{\boldsymbol{V}_{\text{inv}}\left(\frac{\boldsymbol{V}_{\text{inv}} - \boldsymbol{V}_{\text{g}}}{\text{j}X_{\text{m}}}\right)^{*}\right\} \tag{9.54}$$

$$= \text{Re}\left\{\frac{|\boldsymbol{V}_{\text{inv}}|^2}{-\text{j}X_{\text{m}}} - \frac{\boldsymbol{V}_{\text{inv}}\boldsymbol{V}_{\text{g}}}{-\text{j}X_{\text{m}}}\right\} \tag{9.55}$$

$$= \text{Re}\left\{\frac{|\boldsymbol{V}_{\text{inv}}|^2}{-\text{j}X_{\text{m}}} - \frac{|\boldsymbol{V}_{\text{inv}}||\boldsymbol{V}_{\text{g}}|\cos(\delta) + \text{j}|V_{\text{inv}}||\boldsymbol{V}_{\text{g}}|\sin(\delta)}{-\text{j}X_{\text{m}}}\right\} \tag{9.56}$$

$$= \frac{|\boldsymbol{V}_{\text{inv}}||\boldsymbol{V}_{\text{g}}|\sin(\delta)}{X_{\text{m}}}. \tag{9.57}$$

The angle $\delta$ and magnitude $|\boldsymbol{V}_{\text{inv}}|$ are controlled by the inverter. Adjusting the magnitude of $\delta$ controls the magnitude of the real power exchanged between the buses. Adjusting the magnitude of the inverter voltage also affects the real power, but to a lesser extent. Because $\delta$ is usually small, the real power is more sensitive to changes in $\delta$ than $|\boldsymbol{V}_{\text{inv}}|$. The inverter can also supply reactive power. It can be shown that the reactive power produced by the inverter is

$$Q_{\text{inv}} = \frac{|\boldsymbol{V}_{\text{inv}}|^2}{X_{\text{m}}} - \frac{|\boldsymbol{V}_{\text{inv}}||\boldsymbol{V}_{\text{g}}|\cos\delta}{X_{\text{m}}}. \tag{9.58}$$

Note that some of this reactive power is absorbed by the inductance $X_{\text{m}}$. The reactive power is more sensitive to the inverter voltage than the angle. This conveniently establishes a basic strategy for the control of the inverter: adjust $\delta$ to control the real power; adjust $|\boldsymbol{V}_{\text{inv}}|$ to control the reactive power. Since there is not a complete de-coupling of these variables in (9.57) and (9.58), the control of the real and reactive power cannot be completely independent. A phase-locked loop is used to determine the firing angle of the MOSFET in relation to the AC bus voltage, and the modulation index (9.48) can be used to adjust the magnitude. The formulations in (9.57) and (9.58) are for single-phase inverters. As usual, they are multiplied by three if the three-phase real and reactive power are of interest.

## 9.12 Bi-directional Converters

There is nothing about (9.57) that restricts the real or reactive power supplied by the inverter to be positive. For example, controlling the inverter such that $\delta$ is negative results causes power to flow from the AC bus to the DC bus. The reactive power can also flow from AC bus to the DC bus. Some AC–DC-coupled mini-grids benefit from the flexibility of bi-directional conversion. For example, a hybrid solar/gen set mini-grid may require power to flow from the DC bus to AC bus when the battery is discharging. But when the gen set charges the battery, power flows from the AC bus to the DC bus. When power can flow in both directions through a converter, it is known as a "bi-directional converter." Most bi-directional converters encountered in mini-grids are inverters whose switches allow bi-directional conduction, for example, an IGBT with antiparallel diode.

*Example 9.5* A lossless single-phase bi-directional inverter is connected to an AC bus. A single-phase gen set is also connected to the AC bus. The real portion of the AC load is 13 kW. The gen set supplies 10 kW. Compute the required power from the inverter and the associated angle $\delta$. Compute the

(continued)

reactive power from the inverter. The decoupling reactance is 0.5 Ω; let the generator voltage be 230 V and the inverter voltage be 232 V.

**Solution** The real power required from the inverter is $P_{\text{inv}} = 13 - 10 = 3\text{kW}$. From (9.57), the angle $\delta$ is computed as

$$\sin(\delta) = P_{\text{inv}} \frac{X_{\text{m}}}{|\boldsymbol{V}_{\text{inv}}||\boldsymbol{V}_g|} = 3000 \frac{0.5}{230 \times 232} = 0.0281$$

$$\sin^{-1}(0.0281) = 0.0281 \text{ rad} = 1.61°.$$

The reactive power of the inverter is found using (9.58)

$$Q_{\text{inv}} = \frac{|\boldsymbol{V}_{\text{inv}}|^2}{X_{\text{m}}} - \frac{|\boldsymbol{V}_{\text{inv}}||\boldsymbol{V}_{\text{g}}| \cos\delta}{X_{\text{m}}} = \frac{232^2}{0.5} - \frac{232 \times 230 \times \cos 1.61°}{0.5} = 970 \text{ VAR}.$$

## 9.13 Inverter Practical Considerations

Commercially available inverters are typically compatible with nominal DC-side voltages of 12, 24, 48, or 96 V. Some inverters are compatible with several nominal voltages. Inverters with higher-power ratings tend to be compatible with higher DC voltages.

The rating of an inverter is based on the power it can continuously supply without overheating. Many inverters "peak" rating, which is the power the inverter can supply for short intervals. This is an important consideration for loads that draw additional power at start-up or draw power irregularly. Inverter ratings can be specified in watts or volt–amps, with the watt rating always being less than the volt–amps rating. If the units of the rating are overlooked, the inverter might be over- or under-designed. If the power factor of the load is unknown, the designer must make an assumption to convert between watts and volt–amps; a power factor of 0.85 is often reasonable.

The power an inverter can continuously supply is limited by the temperature increase associated with its internal losses as it supplies power. The ambient temperature also affects this—the higher the ambient temperature, the lower the power the inverter can supply. Many manufactures provide temperature-dependent ratings. A 10% de-rating of the rated power is appropriate for an inverter operating at 40°C instead of 25°C. For example, if the continuous load is expected to be 3.0 kW, an inverter rated at 3.3 kW can be selected if the inverter is expected to operate in a high-temperature environment.

There are many types and sizes of commercially available inverters. High-quality inverters feature the following:

- output sinusoidal voltage with little distortion and constant frequency and magnitude;
- good voltage regulation;
- high efficiency at low loading;
- insensitivity to changes in input voltage;
- short-term increased surge capacity;
- low-voltage disconnect capability;
- can be configured in the field;
- include data logging and diagnostic features.

## 9.14 Summary

This chapter covered the basic principles and functions of the converters found in off-grid systems. A brief description of each follows:

- DC–DC converters: these devices can increase or decrease the input DC voltage by varying their duty cycle. They are also used in maximum power point trackers and battery chargers.
- Maximum power point trackers (MPPT): MPPTs allow PV arrays to operate at or near their point of maximum power by decoupling the array voltage from the battery or load voltage.
- Solar battery charger: these devices are found in solar home systems, solar lanterns, and some PV-based mini-grids. They are designed to prevent the battery from being overcharged, and those controlled using PWM are able to employ three-stage charging.
- AC battery charger: AC battery chargers are common in AC-coupled mini-grids that also have a DC bus. Single-phase and three-phase chargers use diode-based rectifiers that convert AC to DC. Rectifiers are also used to connect variable speed AC generators like WECS to the DC bus for battery charging applications.
- Automatic voltage regulator (AVR): AVRs often use phase-angle-controlled rectifiers to control the field current of synchronous generators, allowing the terminal voltage to be regulated.
- Electronic load controller (ELC): ELCs are used for speed control in MHP systems. The power to a ballast load—a resistor with high-power rating—is controlled to electrically govern the system frequency. There are several types of ELCs, including those that use binary-weighted resistor networks, phase-angle control or impedance control. Impedance control is achieved by a chopper in series with the output of phase-angle-controlled rectifier. This allows greater control over the real and reactive power consumed by the ballast load. Fixed capacitors can also be included, depending on whether or not consumption of reactive power is an issue.

- Inverter: inverters convert DC voltage to AC. They are required when a DC-coupled architecture supplies an AC load. There are several types of inverters.
- Solar inverter: solar inverters convert the DC output by PV modules, strings, or arrays to AC directly, without a dedicated DC bus. Most solar inverters are unable to form the AC bus voltage.
- Grid-tied inverter (GTI): GTIs transfer power from a DC bus to an AC bus whose voltage is formed by another generator. The angle and magnitude of the inverter voltage are controlled to achieve the desired injection of real and reactive power to the AC bus.
- Bi-directional inverter: bi-directional inverters allow power to flow from the DC Bus to the AC bus and vice versa. This capability is useful in AC–DC-coupled mini-grids with AC and DC loads or AC loads and a battery bank.

## Problems

**9.1** A square wave like that produced by a modified sinewave inverter only has odd harmonics. The magnitude of the first, third, fifth, and seventh harmonics are 1.273, 0.424, 0.255, and 0.182, respectively. Compute the THD associated with these harmonics.

**9.2** A DC mini-grid incorporates a battery bank whose nominal voltage is 49.5 V. A boost converter is used to increase the voltage to 230 V to the distribution system. At the user's home, the voltage is reduced to 12.8 V using a buck converter. Compute duty cycle of the boost converter and the buck converter. Assume the voltage drop of the distribution system is 5%.

**9.3** An MPPT is used to connect a PV module to a load whose resistance is 10 Ω. The MPP of the module under the present conditions is 350 W corresponding to a voltage of 38.54 V. The MPPT uses a buck–boost converter. Compute the duty cycle of the converter for maximum power supplied to the resistor. Compute the current into the MPPT and to the load.

**9.4** An MPPT is used to connect a PV array to a battery whose voltage is 49.5 V. The PV array consists of two PV modules connected in series. Under the present conditions, the maximum power of each modules is 185 W corresponding to a voltage of 36.4 V. Compute the duty cycle of the MPPT if a buck converter is used. Compute the current to the battery. Ignore the resistance of the battery.

**9.5** A single-phase AC source is used to charge a nominal 24 V battery bank through a full-bridge rectifier. The battery resistance is 0.3 Ω, and $V_{\mathrm{SoC}}$ is 25.6 V. The RMS value of the AC source is 25 V. Compute the power to the battery. Ignore the voltage drop of the diodes. Plot the current into the battery and the battery terminal voltage $V_{\mathrm{DC}}$ for one period.

**9.6** A three-phase source with line–line RMS voltage of 43 V is used to charge a battery through a three-phase rectifier. The battery resistance is 0.15 Ω and $V_{\mathrm{SoC}}$

is 50.9 V. Compute the power to the battery. What is the average battery terminal voltage and average battery current? Ignore the voltage drop of the diodes.

**9.7** A three-phase MHP is used to charge a battery bank through a three-phase rectifier. The synchronous reactance of the MHP's generator $X_s$ is 1.0 Ω. The magnitude of the induced phase voltage $|\boldsymbol{E}_a| = 40$V. The battery resistance is 0.13 Ω and $V_{SoC}$ is 25.0 V. Use the phasor model of the AC charger to determine the power to the battery and the current $\boldsymbol{I}_r$, voltage $\boldsymbol{V}_r$, and angle of the induced phase voltage $\delta$.

**9.8** Compute the required firing angle $\alpha$ of a three-phased controllable rectifier used in an AVR to achieve an average DC field winding voltage of 63 V. The RMS value of the line–line voltage input to the AVR is 220 V.

**9.9** A single-phase inverter supplies 1350 VA to the AC bus. The DC bus voltage is 25.2 V. Compute the current into the inverter, assuming the inverter efficiency at this operating point is 74%.

**9.10** A single-phase bi-directional grid-tied inverter is connected to an AC bus. The inverter voltage is $\boldsymbol{V}_{inv} = 121\angle 5°$ V. The voltage of the AC bus is $\boldsymbol{V}_g = 120\angle 0°$ V. The inductance $X_m$ is 2.6 Ω. Compute the inverter current and the real and reactive power supplied by the inverter.

**9.11** A three-phase bi-directional grid-tied inverter is connected to an AC bus. The inverter line-to-neutral voltage is $\boldsymbol{V}_{inv} = 234\angle 0°$ V. The line-to-neutral voltage of the AC bus is $\boldsymbol{V}_g = 230\angle 0°$ V. The inductance $X_m$ is 2.6 Ω. Compute the current $\boldsymbol{I}_r$ and the real and reactive power supplied by the inverter.

**9.12** A three-phase bi-directional grid-tied inverter is connected to an AC bus. The inverter line-to-neutral voltage is $\boldsymbol{V}_{inv} = 230\angle 5°$ V. The line-to-neutral voltage of the AC bus is $\boldsymbol{V}_g = 230\angle 0°$ V. The inductance $X_m$ is 2.6 Ω. Compute the current $\boldsymbol{I}_r$ and the real and reactive power supplied by the inverter.

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