Chapter 7
Photovoltaic Arrays

7.1 Introduction

Photovoltaic (PV) arrays are commonly used in off-grid systems (see Fig. 7.1) and are becoming the default choice of energy conversion technology in such applications. This is primarily driven by falling costs, and the above average sunlight in Sub-Saharan Africa and South Asia, where electrification rates are the lowest. Whereas relatively few energy-impoverished communities are in locations whose climate and terrain can support wind-, hydro-, or biomass-based generation, most have enough sunlight for PV-based generation to be practical.

7.2 Solar Resource

Each year, 3.8×10^{24} joules of energy in the form of electromagnetic radiation from the sun passes through Earth's atmosphere. This is about 10,000 times our present annual energy consumption.

The amount of solar energy a site receives depends on the location, time of day, and weather conditions. It is important to understand a site's solar resource as this ultimately dictates the required capacity of the PV array. The power provided by sunlight is known as "irradiance", whose units are watts per square meter. Figure 7.2 is a solar resource map that shows the average irradiance around the world. The single measurement that most concisely, but not entirely, describes the solar resource of a location is the *insolation*. This should not be confused with "insulation." Insolation is the energy provided by sunlight per unit area over a period of time. It is commonly expressed as kilowatthours per meter squared per day or kilowatthours per square meter per year.

H. Louie, *Off-Grid Electrical Systems in Developing Countries*,
https://doi.org/10.1007/978-3-319-91890-7_7

Fig. 7.1 A 3 kW PV array split into two strings in Kenya (courtesy E. Patten)

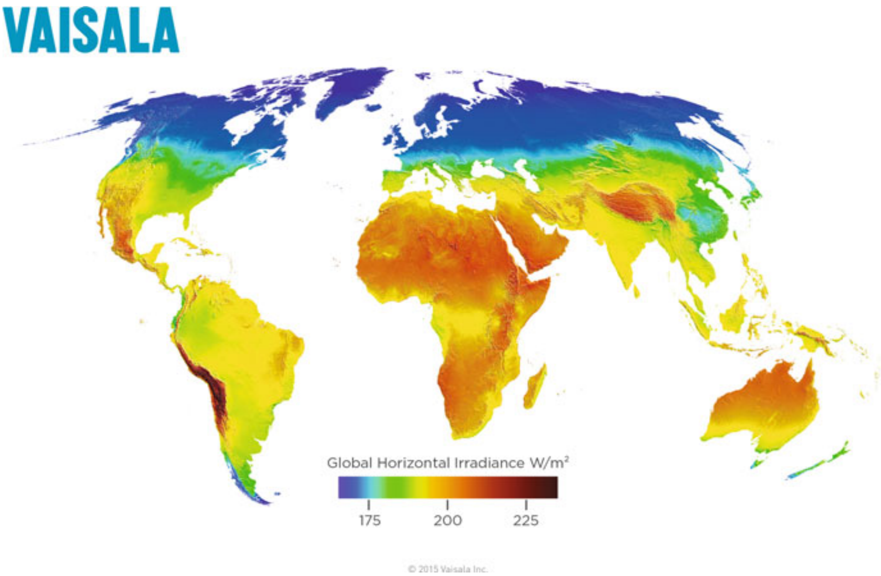


Fig. 7.2 World solar resource map showing average irradiance (courtesy of Viasala, Copyright (c) 2017 Vaisala)

Insolation varies greatly, typically ranging from 3.5 to 6.0 kWh/m^2/day when averaged across a year. The monthly variation in insolation can be substantial. This is especially noticeable in regions with pronounced rainy seasons or are far from the equator.

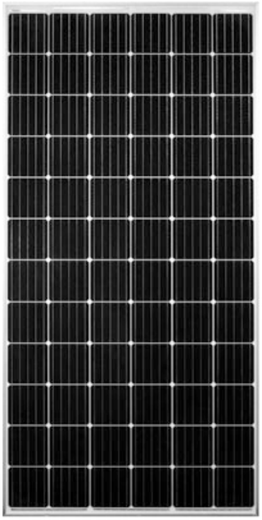


Fig. 7.3 A 350 W monocrystalline PV module with 72 cells in series (courtesy of Itek Energy)

7.3 Physical Description

Solar power has become quite common in recent years. Nearly everyone has seen or is at least familiar with solar panels. One is shown in Fig. 7.3. On the front side are a number of dark rounded square areas. These are photovoltaic cells, the basic building blocks of a PV system. PV cells are typically made from doped silicon crystals that have been cut into thin, flat wafers. PV cells are produced in several standard sizes, for example, 0.125 m × 0.125 m or 0.156 m × 0.156 m.

The current and power output of a PV cell increases in proportion to its area. The voltage, however, is generally unaffected by size. A typical 0.156 m × 0.156 m cell can output approximately 2 to 4 W. To increase the power output, PV cells are connected together to form *modules*. The cells are often connected in series to increase the total voltage. For example, a typically sized 350 W module might contain 72 series-connected cells.

PV modules can be easily obtained all over the world from a variety of manufacturers. When multiple PV modules are connected together they form a *PV array*. The modules can be connected in series, parallel, or combination thereof.

Mini-grid systems tend to use modules that are larger—both in the physical sense and in terms of their power output capability—than used in solar home systems or solar lanterns. The dimensions of a typical 300 W module used in a mini-grid are approximately 1.95 m × 1.00 m × 0.046 m and weigh about 24 kg. PV modules are also called PV panels because they physically resemble flat rectangles.

Figure 7.4 shows the physical layout of a PV module. The cells are all connected in series and are arranged in four columns. The cells in each column are connected to each other using a conductive tabbing ribbon. The PV module is encapsulated within a metal frame to prevent the intrusion of water and pests. The face of the

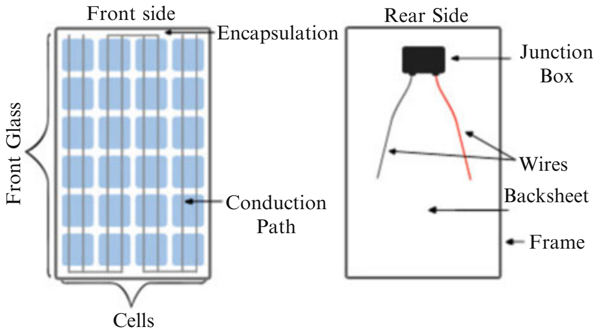


Fig. 7.4 Components of a PV module

module is covered by protective glass with an antireflective coating. On the rear side of the module are a backsheet and a junction box. The positive and negative wires from the string of PV cells extend from the junction box. These wires are used to connect the PV modules together to form an array or to connect the PV module to other components. The junction box often contains by-pass diodes, as discussed later in Sect. 7.12.1.

7.4 Principle of Operation

A PV cell is a solid-state device that converts a portion of the incident sunlight into DC power. The intensity of the sunlight is described by its *irradiance* G. It follows that irradiance is a measure of power density. For context, on a clear sunny summer day in the mid-continental United States, the irradiance will be about 1000 W/m^2. A PV module whose area is 2 m^2 would receive 2000 W of solar power under these irradiance levels. However, only a portion of this power is converted to electricity. The efficiency of commercially available cells is typically between 12 and 18%. This may seem low, but it is improving. Modules with higher efficiency are commercially available but come at a higher cost. In off-grid electrification, cost is often a primary consideration.

There are several types of PV cells. The most common are made from either monocrystalline or polycrystalline silicon (Si) material. Monocrystalline cells are made from silicon that has a single, continuous lattice structure. Polycrystalline cells on the other hand contain several smaller silicon crystal structures. The coloration and shape of the cells allow one to immediately tell if a module uses poly- or monocrystalline cells. The surface of a monocrystalline cell has a consistent coloring, but a polycrystalline cell appears patchy. Polycrystalline cells are typically rectangular; monocrystalline cells are rectangular but with rounded edges as a result of the manufacturing process. Monocrystalline cells are somewhat more efficient than polycrystalline cells but are more expensive to manufacture and hence cost more. Other common PV cell materials include gallium arsenide (GaAs), cadmium telluride (CdTe), and amorphous silicon, which has the advantage of being flexible. We will focus on mono- and polycrystalline silicon cells because they are the most common.

The basic mechanism that allows a PV cell to generate electricity is the photovoltaic effect.[1] The photovoltaic effect is the generation of voltage when light is shined onto a material.

In order to operate as intended, PV cells require light. You may remember from basic physics that light has a wave/particle duality. Most often we think of light as an electromagnetic wave that emanates from a source and varies sinusoidally in time and space. In the visible spectrum oscillations at high frequencies correspond to violet or blue light, lower frequencies correspond to red light.

In the early twentieth century, quantum physicists developed a particle theory of light. The photon theory of light relates well to our understanding of how PV cells operate. In this understanding, light is made up of photons. Photons are characterized by energy. High-energy photons are associated with high-frequency light. Lower-energy photons are associated with lower-frequency light.

Interesting things happen when light shines on a PV cell. If a photon of the right energy comes close to a valence electron of silicon, then there is a high probability that the electron will be excited into the conduction band. Valence electrons are virtually immobile; when an electron is in the conduction band, however, they can move quite freely. In this way light significantly changes the electrical properties of a PV cell.

In order to power an external circuit, PV cells are made from doped silicon—usually with boron and phosphorous—so that they have a *p-n junction*. A p–n junction is a fundamental building block of semiconductor electronic devices, including diodes, transistors, and integrated circuits. A PV cell is conceptually the same as a diode. We will not get into the details of p–n junctions as this is covered in most textbooks on electronics, but it suffices to note that the p–n junction results in a built-in electric field inside the PV cell. The built-in electric field within the PV pushes electrons that are in the conduction band through an external circuit, providing DC current. Energy is transferred from the light to the now-mobile electrons, effectively converting light into electricity.

7.4.1 Unilluminated PV Cell

A PV cell in the absence of light behaves like a diode, which itself is just a p–n junction. The current–voltage (I–V) characteristic of an ideal diode is

$$I_{\mathrm{D}} = I_0 \left(e^{V_{\mathrm{D}}/V_T} - 1 \right) \tag{7.1}$$

[1]Not be confused with the similar, yet different, photoelectric in which the electrons are freed into space.

where I_D is the current through the diode, I_0 is the reverse saturation current, V_D is the voltage across the diode, and V_T is the "thermal voltage." The magnitude of the reverse saturation current is small, around 10^{-9} A, and depends on the physical characteristics of the diode. When a diode is reversed biased, the current through it is near zero. This can be readily observed from (7.1). As V_D becomes negative, the exponential term approaches zero, and the current I_D approaches $-I_0$.

The thermal voltage is:

$$V_T = \frac{qV_D}{nkT} \tag{7.2}$$

where q is the charge of an electron, 1.602×10^{-19} C, T is the temperature, in Kelvin, k is Boltzmann's constant, 1.38×10^{-23} J/K, and n is the ideality factor. The ideality factor is 1 in an ideal diode. For a silicon diode at approximately room temperature (26.85°C), V_T = 25.8 mV.

As stated before, an unilluminated PV cell is simply a p–n junction and so can be modeled as a diode. We will refer to (7.1) as we construct a model for the illuminated PV cell.

7.4.2 *Illuminated PV Cell*

Now assume light is shined on the PV cell. Some of the photons will excite electrons into the conduction band. The built-in electric field will push some of the excited electrons to the n-side of the p–n junction. This concentration of charge on either side of the junction results in a measurable voltage across the cell. We therefore expect an illuminated PV cell to exhibit a non-zero open-circuit voltage.

If an external conduction path is provided, then the electrons on the n-side of the junction will travel through it. The magnitude of the resulting current depends on the resistance of the external circuit. The current under short-circuit conditions is known as the *illumination current* I_G. The illumination current is always positive, and its magnitude is proportional to the irradiance G and the area and efficiency of the PV cell. The illumination current under clear sunny conditions will range from about 0.5 A for small cells to about 9 A for 0.156 m x 0.156 m cells.

We must modify the ideal diode equation (7.1) to account for the presence of the photogenerated current:

$$I_{cell} = I_G - I_0 \left(e^{V_{cell}/V_T} - 1\right) \tag{7.3}$$

$$I_{cell} = I_G - I_D \tag{7.4}$$

where I_{cell} is the current output by the PV cell. We have also replaced the diode voltage with the PV cell voltage V_{cell}. The two voltages are equal if we ignore losses, as we have so far.

Equation (7.3) is known as the *characteristic equation* of the PV cell. Note that when the illumination current is zero (7.3) is equivalent to the ideal diode equation (7.1). We can rearrange (7.3) to solve for the cell voltage:

$$\frac{I_\text{G} - I_\text{cell}}{I_0} + 1 = e^{V_\text{cell}/V_\text{T}} \tag{7.5}$$

$$V_\text{T}\ln\left(\frac{I_\text{G} - I_\text{cell}}{I_0} + 1\right) = V_\text{cell}. \tag{7.6}$$

7.4.3 Open-Circuit Voltage

We can manipulate (7.3) to determine the open-circuit voltage $V_\text{cell,OC}$ when the PV cell is illuminated by setting the cell current to zero:

$$V_\text{cell,OC} = V_\text{T}\ln\left(\frac{I_\text{G}}{I_0} + 1\right). \tag{7.7}$$

From (7.2) to (7.7), we see that the open-circuit voltage depends on the temperature, the ideality factor, the illumination current, and the reverse saturation current. The open-circuit voltage of an illuminated cell is typically 0.6 to 0.7 V. The natural logarithm in (7.7) shows that the open-circuit voltage is relatively insensitive to changes in the illumination current—and therefore it is also relatively insensitive to changes in irradiance. Also note that from (7.2) and (7.7), as temperature decreases, the open-circuit voltage increases.

7.4.4 Short-Circuit Current

Under short-circuit conditions, the PV cell voltage is zero ($V_\text{cell} = 0$), and from (7.3) we see the short-circuit current I_SC is:

$$I_\text{SC} = I_\text{G} - I_0\left(e^{0/V_T} - 1\right) \tag{7.8}$$

$$I_\text{SC} = I_\text{G} - I_0\,(1 - 1) \tag{7.9}$$

$$I_\text{SC} = I_\text{G} \tag{7.10}$$

In other words, the short-circuit current is equal to the illumination current.

Example 7.1 Compute the open-circuit voltage and short-circuit current of a PV cell whose reverse saturation current is 9^{-9} A, illumination current is 8.46 A and whose thermal voltage is 28 mV.

Solution From (7.10) the short-circuit current of a PV cell is equal to the illumination current so that $I_{SC} = 8.46$ A. The open-circuit voltage is found by solving (7.7):

$$V_{cell,OC} = V_T \ln\left(\frac{I_G}{I_0} + 1\right) = 0.028 \times \ln\left(\frac{8.46}{9^{-9}} + 1\right) = 0.613 \text{ V}$$

Of course the purpose of a PV panel is to deliver power. Under either short-circuit or open-circuit conditions, the output power is zero. To understand the power delivered under practical conditions, we must first consider the cell's current–voltage (I–V) characteristic.

7.5 *I–V* Curve

A plot of the characteristic equation of a PV cell or module is known as the I–V *curve*. The I–V curve provides information on the quality of the PV cell or module, and its open-circuit voltage and short-circuit current, which are needed to properly design a PV system.

The I–V curve for an ideal PV cell is shown in Fig. 7.5. The I–V curve is unlike that of a voltage source or a current source. It resembles a hybrid of the two. The current is nearly constant for a wide range of terminal voltages, similar to a current source. However, as the knee of the curve is passed, the I–V curve resembles a voltage source, with little change in voltage for a wide range of current. An important characteristic of PV cells is that they are self-limiting. PV cells and modules can be short-circuited or open-circuited without damaging themselves.

7.5.1 Circuit Model

We are now prepared to develop a circuit model of a PV cell. We start with a simplified ideal model that captures the general characteristics. The model consists of an ideal current source in parallel with a diode, as shown in Fig. 7.6. The value of the current source is equal to the illumination current. This is simply the circuit equivalent of (7.3).

Fig. 7.5 *I–V* curve of a single ideal PV cell

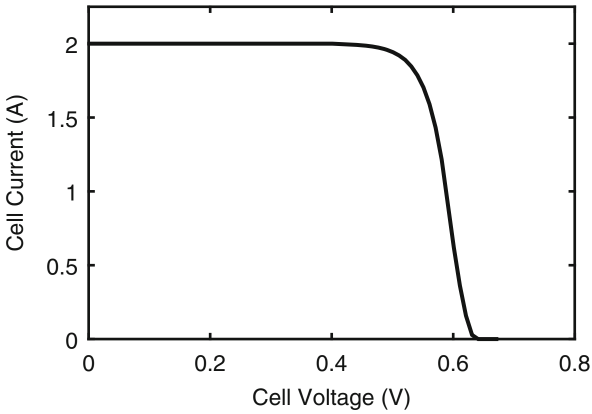


Fig. 7.6 Simplified lossless PV cell circuit model

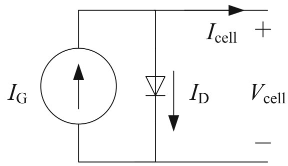
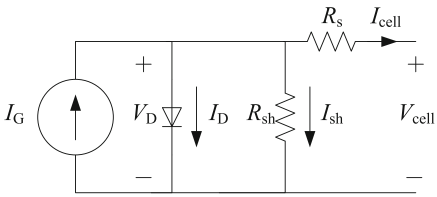


Fig. 7.7 PV cell circuit model including losses



The simplified model ignores losses internal to the cell. These losses can be included in the model as lumped series and shunt resistances shown in Fig. 7.7.

The corresponding current and voltage relationship is found by application of Kirchhoff's Current Law (KCL) at the top node, followed by substitution

$$I_{cell} = I_G - I_D - I_{sh} \tag{7.11}$$

$$I_{cell} = I_G - I_0 \left(e^{(V_{cell} + I_{cell} R_s)/V_T} - 1 \right) - \frac{V_{cell} + I_{cell} R_s}{R_{sh}}. \tag{7.12}$$

The resulting expression is an implicit equation, with the PV cell current I_{cell} appearing on both the left- and right-hand sides. The equation is therefore solved by using numerical means rather than analytically. The power loss associated with shunt resistance is quite small, about 0.1% of the power output; the series resistance loss is larger, perhaps 0.3% [3]. These losses are typically considered to be negligible, and the simplified model is often used.

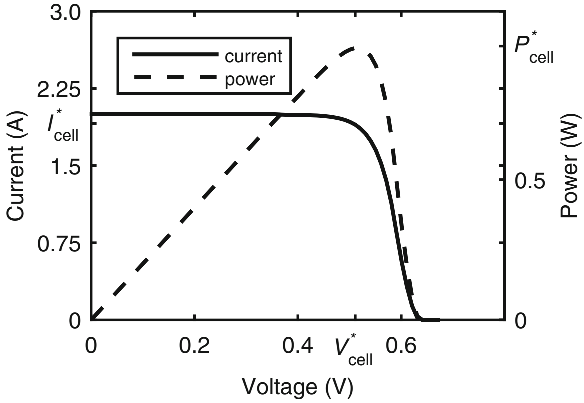


Fig. 7.8 I–V and power curve of a PV cell

7.5.2 *Maximum Power Point*

As with all practical sources, there is a unique operating point that maximizes the power production from a PV cell. Consider a PV cell whose I–V curve for a given irradiance is shown in Fig. 7.8. The power corresponding to each operating point is the product of the voltage and current

$$P_{\text{cell}} = I_{\text{cell}} V_{\text{cell}}. \tag{7.13}$$

The power associated with each voltage is also shown in Fig. 7.8. Importantly, we see that the operating point that maximizes power production is unique. This point is known as the "maximum power point" (MPP). The MPP is located near the knee of the I–V curve. The voltage and current at the MPP are denoted V^*_{cell} and I^*_{cell}, respectively. Here the superscript "*" should not be confused with complex conjugate operator, which has no practical meaning in DC circuit analysis. The corresponding maximum power is P^* so that

$$P^*_{\text{cell}} = V^*_{\text{cell}} I^*_{\text{cell}}. \tag{7.14}$$

A PV cell will only produce its maximum power if it is connected to load whose resistance is $R^* = V^*_{\text{cell}}/I^*_{\text{cell}}$. Alternatively, if the PV cell is used to charge a battery, the battery's terminal voltage must equal V^*_{cell} for maximum power to be produced by the cell. We must not forget that the I–V curve of PV cell depends on several factors, primarily the irradiance. We therefore rewrite (7.14) to show that maximum power and the voltage and current corresponding to the MPP are all functions of irradiance G:

$$P^*_{\text{cell}}(G) = V^*_{\text{cell}}(G) I^*_{\text{cell}}(G). \tag{7.15}$$

Recall that the illumination current is proportional to the irradiance received by the cell. From (7.10), the short-circuit current will then also be proportional

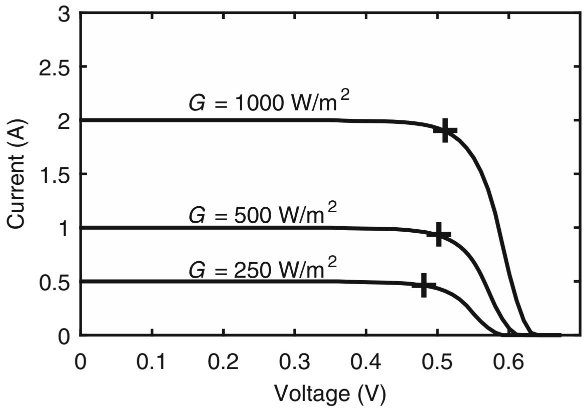


Fig. 7.9 The maximum power point of an example PV cell for different levels irradiance; the "x" mark the maximum power points

to the irradiance. Plotting the characteristic equation for different values of the illumination current results in a family I–V curves, as shown in Fig. 7.9.

The resistance corresponding to the MPP will also depend on the irradiance $R^*(G)$. Therefore, as the irradiance changes, so must the resistance in order to maximize the power. The practical consequence of this is that any device powered by the PV cell must continually adjust its equivalent input resistance to track the MPP; otherwise, the power production will not be maximized. We shall see in Chap. 10 that specialized devices called "maximum power point trackers" are used to operate a PV module at its maximum power point.

7.6 PV Modules

PV cells are usually connected in series inside a module. The result is a stretching of the I–V curve along the voltage axis, as shown in Fig. 7.10. The short-circuit current is not affected, and the open-circuit voltage increases in proportion to the number of cells. The characteristic equation becomes:

$$I_{\text{module}} = I_{\text{G}} - I_0 \left(e^{V_{\text{module}}/(V_{\text{T}} N_{\text{ser}})} - 1 \right) \tag{7.16}$$

where I_{module} is the module current and V_{module} is the module voltage. The module voltage is simply the cell voltage multiplied by the number of cells in series

$$V_{\text{module}} = N_{\text{ser}} V_{\text{cell}}. \tag{7.17}$$

PV cells can be connected in parallel to increase the current, although it is somewhat uncommon. This stretches the I–V curve along the current axis as shown in Fig. 7.10. The characteristic equation for the module becomes:

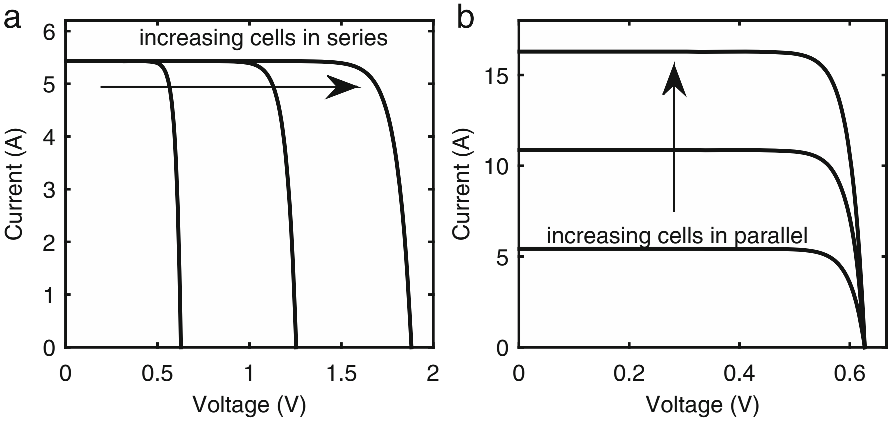


Fig. 7.10 (**a**) The I–V curve is changed as PV cells are connected in series and (**b**) parallel

Table 7.1 Example electrical characteristics of PV module

Characteristic at STC	Value
Maximum power	185.3 W
Optimum operating voltage (V^*)	36.4 V
Optimum operating current (I^*)	5.08 A
Open-circuit voltage (V_{OC})	45.0 V
Short-circuit current (I_{SC})	5.43 A
Short-circuit current temp. coeff. (α_i)	0.055 %/K
Open-circuit temp. coeff. (α_v)	−0.37%/K
Max. power temp. coeff. (α_P)	−0.48%/K
NOCT	45°C
Number of Cells	72 (series)

$$\frac{I_{\text{module}}}{N_{\text{par}}} = I_{\text{G}} - I_0 \left(e^{V_{\text{module}}/(V_{\text{T}} N_{\text{ser}})} - 1 \right) \tag{7.18}$$

where N_{par} is the number of parallel-connected cells.

The I–V curve of a PV module resembles that of a cell. The open-circuit voltage and short-circuit current are simply N_{ser} and N_{par} times greater than that of a single cell. Like a cell, a PV module has a unique maximum power point. In general, we are more interested in the behavior of a module or array than of a single cell. Hereafter, we will suppress the "module" subscript so that V, I, and P refer to the voltage, current, and power of a module.

Manufacturers provide information about the characteristics of a PV module in their data sheets. See Table 7.1 for example.

Example 7.2 Compute the open-circuit voltage if 60 of the cells described in the Example 7.1 are connected in series.

Solution Rearranging (7.16) and setting the module current to zero shows that

$$V_{OC} = N_s V_T \ln\left(\frac{I_G}{I_0} + 1\right) = 60 \times 0.028 \times \ln\left(\frac{8.46}{9^{-9}} + 1\right) = 36.81 \text{ V}.$$

More generally if the PV cells are connected in series within a module, the voltage of the module is equally divided among the cells. This holds under any loading conditions, including open and short circuit. Similarly, for parallel-connected cells, the module current is equally divided among the cells.

7.7 Standard Test Conditions

The power output of a PV module is dependent on both the irradiance and the resistance of the load. Given that both can vary, the power rating of a PV module must correspond to a set of specific operating conditions. The power rating, or *capacity,* of a PV module is the maximum power that can be produced under "standard test conditions" (STC). Standard test conditions are defined as:

- Irradiance of 1000 W/m^2
- Cell temperature of 25°C

In addition, STC also correspond to a specific distribution or spectrum of the photon wavelengths known as AM (Air Mass) 1.5.

The meaning of STC and how it relates to the rated power of a PV is especially important. For example, a PV panel rated at 100 W will produce 100 W if the panel operates under STC and is connected to a load whose input resistance results in maximum power point operation. In practice, PV modules deployed in actual power systems almost never operate at STC. It would not be surprising for a module to never produce its rated power.

We will use the subscript "STC" to refer to current, voltage, and power values referenced to STC. For example, P^*_{STC} is the maximum power output under STC—this is equivalent to the rated power of the PV module. The irradiance corresponding to STC, 1000 W/m^2, is hereafter denoted G_{STC}.

7.8 Correcting for Temperature

In practice, the temperature of a PV module T_C is rarely 25°C as specified in STC. In many locations, the ambient temperature routinely exceeds 25°C, and the exposure to direct sunlight further increases the module temperature. Figure 7.11 shows how the I–V curve varies with temperature for a certain module.

The I–V curve of a PV module, like that of a diode, is sensitive to temperature as expressed by the variable V_T in (7.2). From (7.3), as temperature increases, the current of the PV cell increases, but the voltage decreases. The decrease in voltage dominates the increase in current. Therefore, the maximum power from a PV cell is reduced as the temperature increases. PV module manufacturers provide information on the sensitivity of the PV module parameters to temperature. The temperature coefficients for the short-circuit current α_i, open-circuit voltage α_v, and maximum power α_p are used to correct for changes in temperature as follows:

$$I_{SC}(T_C) = I_{SC}(25^\circ\ C)\left(1 + \frac{\alpha_i}{100} \times (T_C - 25)\right) \tag{7.19}$$

$$V_{OC}(T_C) = V_{OC}(25^\circ\ C)\left(1 + \frac{\alpha_v}{100} \times (T_C - 25)\right) \tag{7.20}$$

$$P^*(T_C) = P^*(25^\circ\ C)\left(1 + \frac{\alpha_p}{100} \times (T_C - 25)\right). \tag{7.21}$$

We note that if the cell temperature is 25°C, which corresponds to STC, then no correction is made. The coefficients are expressed as percent per Kelvin. The open-circuit coefficient is sometimes expressed as millivolt per Kelvin. To convert this to percent per Kelvin, divide the coefficient by $V_{OC,STC}$.

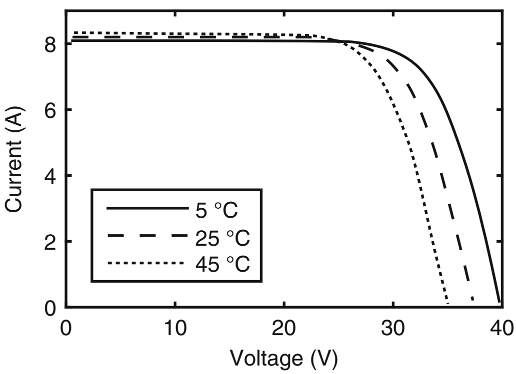


Fig. 7.11 The effect of temperature on a PV I–V curve for a typical 285 W module

Example 7.3 The temperature of a PV module with characteristics in Table 7.1 is 47°C. Compute the short-circuit current, open-circuit voltage, and maximum power.

Solution We assume that the module is operating at STC with the exception of the temperature. We then apply (7.19)–(7.21) to compute the temperature-corrected short-circuit current, open-circuit voltage, and maximum power.

$$I_{SC}(T_C) = 5.43\,(1 + 0.00055 \times (47 - 25)) = 5.496 \text{ A}$$

$$V_{OC}(T_C) = 45.0\,(1 - 0.00370 \times (47 - 25)) = 41.337 \text{ V}$$

$$P^*(T_C) = 185.3\,(1 - 0.0048 \times (47 - 25)) = 165.732 \text{ W}$$

Notice that the magnitude of the percentage change in V_{OC} is greater than that of I_{SC} and the percent change in power is greater still.

7.8.1 Nominal Operating Cell Temperature

In order to apply the temperature corrections, we must know the temperature of the PV module. Similar to our discussion on distribution line thermal ratings, the temperature of the module depends on several factors, including the ambient temperature, wind speed, and irradiance. Because these conditions vary, manufacturers report the cell temperature, known as the Nominal Operating Cell Temperature (NOCT), when exposed to the following standard operating conditions (SOC):

- Irradiance: 800 W/m^2 (0.8 G_{STC})
- Ambient temperature: 20°C
- Wind speed: 1 m/s
- Spectral distribution: AM 1.5
- Power output: 0 W (no load)

Be careful not to confuse STC—the conditions in which PV electrical characteristics are referenced to—and SOC, the conditions in which the cell temperature are referenced to. Typical values of NOCT are 42°C to 50°C. Like STC, PV modules often operate in an environment that deviates from SOC. Most often, adjustments are only made for deviations in ambient temperature T_a and irradiance. In this case cell or module temperature is approximated as

$$T_C = T_a + (NOCT - 20)\,\frac{G}{800}. \tag{7.22}$$

It is common to assume that all cells in a module have the same temperature, so that (7.22) also applies to the module.

Example 7.4 Compute the module temperature of a PV module whose reported NOCT is 45°C when exposed to irradiance of 600 W/m^2 and an ambient temperature of 34° C.

Solution The PV module is not operating under SOC, and so (7.22) must be applied.

$$T_C = 34 + (45 - 20)\,\frac{600}{800} = 52.75°\text{C}.$$

7.9 Correcting for Irradiance

We now consider the common situation in which the irradiance does not correspond to G_{STC} (1000 W/m^2). A simple method of correcting for irradiance is to assume the maximum power changes in proportion to irradiance

$$P^*(G) = P^*_{STC} \times \frac{G}{G_{STC}}. \tag{7.23}$$

Using this method, the maximum power produced by a 100 W PV module exposed to irradiance of 500W/m^2 is 50 W. Underlying this model are the assumptions that

$$V^*(G) = V^*_{STC} \tag{7.24}$$

$$I^*(G) = I^*_{STC} \times \frac{G}{G_{STC}}. \tag{7.25}$$

That is, the voltage at the maximum power point is insensitive to changes in irradiance, and the current at the maximum power point scales proportionally to the irradiance. These are reasonable approximations in must circumstances.

Example 7.5 Compute the maximum power of the PV panel whose characteristics are described in Table 7.1 assuming the irradiance is 600 W/m^2.

Solution From (7.23):

$$P^*(G) = 185.3 \times \frac{600}{1000} = 111.18\text{W}.$$

7.10 Correcting for Temperature and Irradiance

We next consider the case in which both the irradiance and temperature deviate from STC. Several methods have been proposed for adjusting the maximum power output for varying temperature and irradiance [2]. One method that produces a reasonable balance of accuracy and simplicity is Osterwald's method [1]:

$$P = P^{*}_{\text{STC}} \times \frac{G}{1000} \times \left(1 + \frac{\alpha_{\text{P}}}{100} \times (T_{\text{C}} - 25)\right). \tag{7.26}$$

Osterwald's method corrects for irradiance using (7.23), and (7.21) for temperature.

Example 7.6 Compute the maximum power of the PV module whose characteristics are described in Table 7.1 assuming the irradiance is 600 W/m^2, and the ambient temperature is 34°C, using Osterwald's method.

Solution Osterwald's method requires the cell temperature to be computed, as done in Example 7.4. From (7.26), Osterwald's method yields a maximum power output of:

$$P = 185.3 \times \frac{600}{1000} \times \left(1 + \frac{-0.48}{100} \times (52.75 - 25)\right) = 96.37 \text{ W}.$$

In summary, the conditions most favorable for PV power production are when the PV modules are at a low temperature but high irradiance. These conditions do not often exist simultaneously, especially in regions of the world with low electricity access, with the exception of certain high-altitude locations.

In designing off-grid PV systems, we must be aware that the open-circuit voltage, as well as the short-circuit current provided by the manufacture can be exceeded when conditions deviate from STC, as they typically do.

7.11 PV Arrays

Commercially available PV panels rarely exceed 350 W in rated capacity. For mini-grids that require more capacity, multiple modules are combined to form a PV array. Modules can be combined in series, parallel, or a combination thereof, as shown in Fig. 7.12.

Series-connected PV modules are called "strings." If the modules are physically identical and exposed to the same conditions, then the output voltage of string is the voltage of an individual module multiplied by the number of modules. Strings with

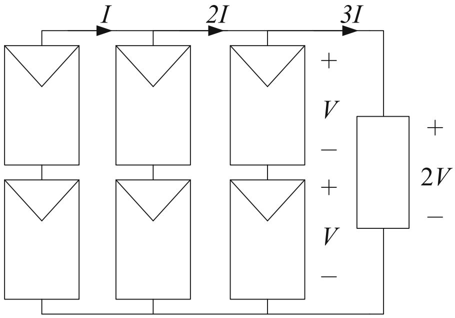


Fig. 7.12 A PV array consisting of three strings with two modules in each string

the same number of modules can be connected in parallel. The current to the load increases in proportion to the number of strings. Mathematically

$$V_{\text{array}} = N_{\text{ser,str}} \times V \tag{7.27}$$

$$I_{\text{array}} = N_{\text{par,str}} \times I \tag{7.28}$$

where $N_{\text{ser,str}}$ is the number of series-connected modules in a string and $N_{\text{par,str}}$ is the number of strings. The subscript "array" has been used to identify variables associated with the array.

The power produced by the array is:

$$P_{\text{array}} = V_{\text{array}} \times I_{\text{array}} = N_{\text{ser,str}} \times V \times N_{\text{par,str}} \times I = N_{\text{ser,str}} \times N_{\text{par,str}} \times P. \tag{7.29}$$

Note that $N_{\text{ser,str}} \times N_{\text{par,str}}$ is the total number of modules in the array and P is the power output of one module. This tells us that the maximum power from an array depends only on the total number of modules, not how they are connected. The power from an array can be corrected for non-STC conditions in the same manner as for individual modules. Hereafter, we will drop the "array" subscript. The voltage V, current I, and power P for the remainder of this chapter will refer either to the cell, module, or array quantities. This should be obvious by the context of the derivation or problem.

The electrical characteristics of an array can be calculated from the characteristics of its constituent modules. Voltage characteristics are multiplied by the number of modules in a string, and the current characteristics are multiplied by the number of strings. Table 7.2 shows an example for an array with three strings and two modules per string.

For a given number of modules, the selection of the number of strings and number of modules per string is not arbitrary. Increasing the number of strings increases the required ampacity of the cables connecting the array to the rest of the system. However, increasing the number of modules per string increases the operating voltage, which has safety and insulation consequences. Usually the

Table 7.2 Example electrical characteristics of a six-module array with $N_{ser,str} = 2$, $N_{par,str} = 3$

Characteristic at STC	Module	Array
Maximum power	290.16 W	1740.96 W
Optimum operating voltage (V^*)	36 V	72 V
Optimum operating current (I^*)	8.06 A	24.18 A
Open circuit voltage (V_{OC})	45.5 V	91.0 V
Short circuit current (I_{SC})	8.56 A	25.68 A

voltage capability of a storage battery or charge controller is used to determine the maximum voltage rating of a string.

Example 7.7 A mini-grid requires a PV array rated at least 2.5 kW. Each PV module is rated at 190 W. Each has an open-circuit voltage of 43.2 V and short-circuit current of 5.98 A (all under standard test conditions). The array is connected to a battery charge controller. The input voltage of the charge controller cannot exceed 150 V, and input current cannot exceed 85 A. The maximum array power through the charge controller cannot exceed 3.2 kW.

Design a PV array that is compatible with the charge controller under STC. Sketch the layout of the array.

Solution The minimum number of modules required is found from

$$\frac{\text{required array capacity}}{\text{capacity per module}} = \frac{2.5\ \text{kW}}{0.190\ \text{kW}} = 13.15$$

so at least 14 modules are needed.

The controller's input voltage limit is 150 V. The maximum number of modules that can be placed in a string is found from

$$\frac{\text{max. string voltage}}{\text{module open} - \text{circuit voltage}} = \frac{150\ \text{V}}{43.2\ \text{V}} = 3.47$$

so at most three modules can be connected in a string.

Each string must have the same number of modules. Therefore, we can select five strings of three modules each, for a total of 15 modules. The maximum power produced by this array is 15 x 190 = 2850 W, which is under the charge controller's power limit.

The current from each string during short circuit is 5.98 A, giving a total current of $5 \times 5.98 = 29.9$ A, well below the charge controller's current limit.

The corresponding array is shown in Fig. 7.13. Note that a more economical solution would be to use seven strings of two modules, for a total of 14 modules. There are also other viable designs.

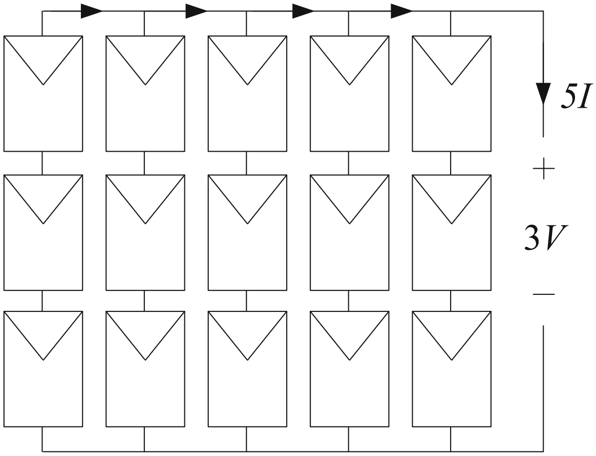


Fig. 7.13 A PV array with five strings of three modules per string

7.12 Effects of Shading

We have seen that the maximum power output from a PV array is approximately proportional to the irradiance it receives. What happens when a portion of the array is shaded? In many mini-grids, the PV array is affixed to an existing roof line. Trees and other building might shade the array during certain hours of the day or days of the year. In larger systems, there is often sufficient budget to clear trees and obstructions from the array, so this is less of a challenge.

Figure 7.14 shows the lossless circuit model of three series-connected ideal PV cells. Assume that the middle cell is completely shaded so that the illumination current is zero. Applying Kirchhoff's Current Law at the node above the shaded cell yields

$$I = I_{G,2} - I_0 \left(e^{V_2/V_T} - 1 \right) \tag{7.30}$$

$$I - I_{G,2} = -I_0 \left(e^{V_2/V_T} - 1 \right) \tag{7.31}$$

$$I = -I_0 \left(e^{V_2/V_T} - 1 \right). \tag{7.32}$$

The current I and I_0 are positive. The exponential term must be less than one for the right-hand side of (7.32) to be positive. The voltage of PV cell 2, V_2, must therefore be negative. The diode is therefore reverse biased, and the current I cannot exceed the reverse saturation current I_0. Recall that I_0 is typically several orders of magnitude smaller than the illumination current of the unshaded cells. We can therefore expect the power supplied by the PV module to be substantially decreased.

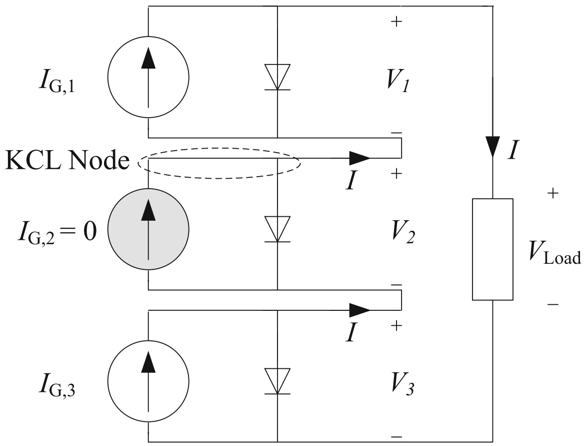


Fig. 7.14 Circuit model of PV module with one cell shaded

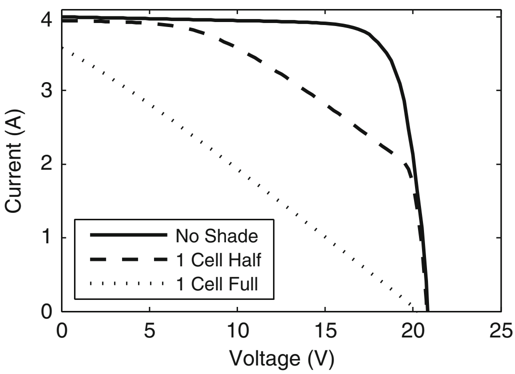


Fig. 7.15 I–V curve of a PV module showing the effects of shading

A memorable, but not entirely true, adage is that the current supplied by a PV module with series-connected cells is limited to the current produced by its least productive cell.

Figure 7.15 shows the effects of shading a single cell of a 66 W PV module with 40 series-connected cells. The shunt resistance of each cell has been modeled. The maximum power drops to 42 W when one cell is shaded by 50% and to 19 W when it is shaded by 100%. Shading is to be avoided not only because of the decreased power output but also because the shaded cells dissipate power, which can lead to local hot spots that permanently damage the module.

7.12.1 By-pass and Blocking Diodes

The substantial reduction in power and the potentially damaging effects of shading can be mitigated by using by-pass diodes. If a diode is installed in parallel but in the opposite direction as the diode in the PV cell model, then it will conduct whenever

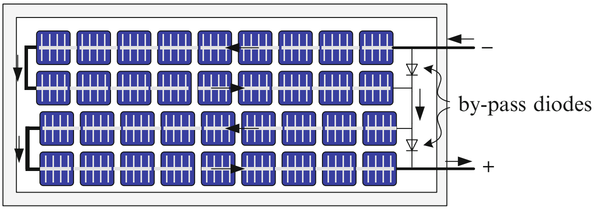
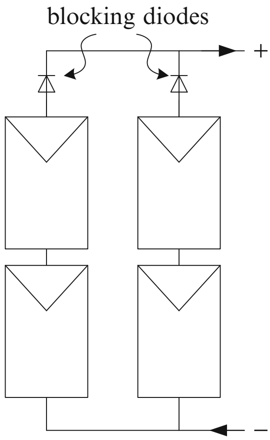


Fig. 7.16 PV modules usually contain several by-pass diodes

Fig. 7.17 Blocking diodes of a PV array



the PV cell voltage becomes negative (when the cell is shaded). The shaded cell does not meaningfully contribute to the power production, but it also does not substantially reduce it.

Adding by-pass diodes as described above to each cell is generally not done for economic reasons. Rather, a single diode is used to by-pass several cells, as shown in Fig. 7.16. In this figure, each diode serves as a by-pass for 18 of the 36 cells. Should one of the cells be shaded, then one half of the cells are by-passed. This strikes a reasonable balance between the cost of diodes and mitigating the effects of any shading. In larger arrays, external diodes are often used to by-pass entire modules in strings.

Figure 7.17 shows blocking diodes. The purpose of a blocking diode is to ensure that current only flows one way through the PV module or string. Blocking diodes should be included whenever a PV module or array is connected directly to a battery (in other words, in cases in which there is no charge controller). Otherwise, after sunset—or any time the module voltage is lower than the battery voltage—the battery will discharge through the array, draining the battery and potentially damaging the PV array.

7.13 Energy Production

The irradiance received by a PV array varies throughout the day. This is primarily caused by changing atmospheric conditions and the angle of incidence between the sun's rays and the face of the PV array. We can re-write (7.26) to show the dependency of irradiance and temperature on time:

$$P(t) = P^*_{\mathrm{STC}} \times \frac{G(t)}{1000} \times \left(1 + \frac{\alpha_{\mathrm{P}}}{100} \times (T_{\mathrm{C}}(t) - 25)\right). \tag{7.33}$$

The plot of $P(t)$ depends on several factors, including the location and orientation of the PV array. A typical plot for an un-tilted (horizontal) 1 kW PV array located at the equator is shown in Fig. 7.18. Production begins in the morning, peaks at midday, and ends around sunset.

The production in Fig. 7.18a might appear low. After all, the PV array is rated at 1 kW, but its maximum production is less than 0.8 kW. Keep in mind that the rated power is based on STC, which rarely occurs in real-world conditions.

The energy produced by a PV array is the area under the curve in Fig. 7.18. It can be computed by integrating (7.33) between the time that irradiance is first and last received by the array t_{rise} and t_{set}

$$E = \int_{t_{\mathrm{rise}}}^{t_{\mathrm{set}}} P(t)\mathrm{d}t. \tag{7.34}$$

The energy production corresponding to Fig. 7.18 is 5.1 kWh. During certain times of the year, it will be more, and other times it will be less. The factors influencing PV array energy production are discussed further in Chap. 12.

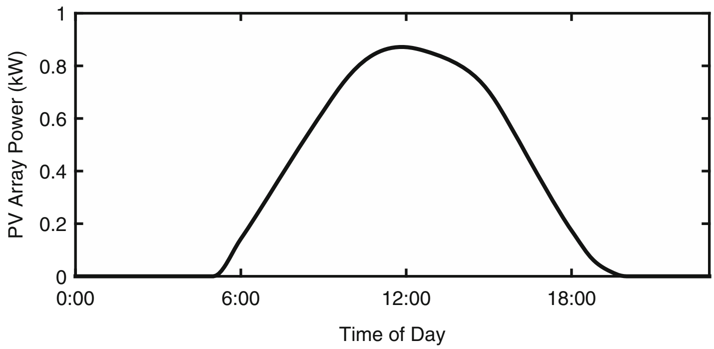


Fig. 7.18 Theoretical power production for a 1 kW PV array

Example 7.8 The irradiance received by a PV array can be modeled as

$$G(t) = 900 \sin\left(\frac{2\pi(t-6)}{24}\right)$$

where t is in hours. The model is valid between sunrise $t = 6$ and sunset $t = 18$. The energy that the PV array is to provide each day to a mini-grid is 16 kWh. Compute the minimum capacity of the PV array to supply this energy. Ignore the effects of cell temperature. Compute the minimum capacity of a gen set needed to supply the required energy.

Solution From (7.33) and (7.34), the energy output of the PV array is expressed as:

$$E = \int_{t_{\text{rise}}}^{t_{\text{set}}} P(t)\text{d}t = P^*_{\text{STC}} \frac{900}{1000} \int_6^{18} \sin\left(\frac{2\pi(t-6)}{24}\right) \text{d}t.$$

Setting the energy produced to the required 16 kWh and integrating using u substitution shows:

$$16 = P^*_{\text{STC}} \frac{900}{1000} \frac{24}{2\pi} 2 = 6.876 P^*_{\text{STC}}$$

and therefore the rating of the PV array must be at least 16/6.876 = 2.327 kW.

$$P^*_{\text{STC}} = \frac{16}{6.876} = 2.327 \text{ kW}.$$

Assuming a gen set can be operated continuously, then the minimum required capacity is:

$$\frac{16 \text{ kWh/day}}{24\text{h/day}} = 0.67 \text{ kw}.$$

For this simple scenario, the energy produced by a 2.327 kW PV array can be replaced by a 0.667 kW gen set. This example highlights the importance in considering energy production, not just power ratings, when comparing different energy conversion technologies.

7.14 Practical Considerations

PV arrays offer the following advantages in off-grid applications:

- Although the capital cost is higher than for gen sets, there are no fuel costs. In some cases, a PV system can provide energy at a lower cost than one using a gen set.
- Energy-impoverished communities are often in areas with abundant sunlight. Moreover, solar resource databases can be consulted to determine if a location is suitable for a PV-based system. Wind or hydro resources require very specific local conditions to be feasible, and on-site local measurements are typically required.
- The energy produced by PV modules correlates with daytime load.
- PV arrays are modular, which means that individual modules can easily be added (or removed) to achieve an appropriate capacity.
- PV arrays have no moving parts, which reduces maintenance (only periodic cleaning and inspection are required) and eliminates noise.
- PV arrays are considered to be environmentally benign as there is no particulate or carbon dioxide emissions associated with their operation.
- PV arrays and associated equipment such as the rackings upon which they are mounted, charge controllers, and batteries are widely available. The supply chain is maturing: licensed installers, original and replacement parts, and warranty service can be found in many developing countries.
- PV arrays do not directly consume water, although periodic washing is needed.

Balancing these advantages are the following considerations.

- The energy produced by PV arrays is variable and uncertain. PV array power production is driven by sunlight, which varies throughout the day and year. Cloud coverage is difficult to forecast, and production might be severely limited during rainy seasons. This adds uncertainty to the design process, leading to arrays that are larger than needed and consequentially more expensive, or smaller than needed causing the system to be unreliable.
- In certain locations, particularly those with perennial cloud coverage or at polar latitudes, the solar resource is inadequate for a PV array to be an economic and practical solution.
- Although PV array prices have fallen globally to less than US$1/W, energy storage, charge controllers, and other components are needed, increasing the cost and complexity.
- PV arrays have low power density, and so a large amount of roof space or land is needed. For example, a 5 kW system requires approximately 40 m^2 of surface area for the PV array. Further, the PV array must be tilted and oriented in a specific way to maximize power production. This often necessitates custom-made racking structures.

Despite these drawbacks, given the substantial solar resource in much of Africa and South Asia, many see PV-based mini-grids as a sustainable and scalable solution to off-grid electricity access.

7.15 Summary

This chapter provided the basic technical principles and characteristics of photovoltaic (PV) cells, modules, and arrays. PV modules are becoming the default choice for energy conversion technology in locations with suitable insolation. PV cells rely on the photovoltaic effect to produce DC current when illuminated. They can be modeled by a current source, representing the sunlight-driven current, in antiparallel with a diode, which models the p–n junction of the PV cell.

The I–V characteristic of a PV array is important in understanding its operation when connected to a load. In particular, there is a unique point of maximum power for a given irradiance and temperature. Maximum power production requires the load to be "matched" to the PV array's maximum power point.

PV modules are rated under Standard Test Conditions. These conditions rarely occur outside a laboratory setting. The electrical characteristics of a PV array are sensitive to irradiance and temperature. Osterwald's method can be used to estimate the power production for different conditions. Osterwald's method assumes the power is proportional to the irradiance, with the temperature effects modeled using the PV modules' temperature coefficient. Complete or partial shading is especially detrimental to PV power production. Shading of a single cell can dramatically reduce power produced by an entire module.

When PV modules are connected together, they form an array. The array voltage is proportional to the number of modules connected in series; the current is proportional to the number of parallel-connected module strings. The maximum power that can be produced by an array scales in proportion to the number of modules, regardless of how they are connected.

Problems

7.1 Consider PV module A whose characteristics are shown in Table 7.3. Compute:

- The cell temperature if the irradiance is 1000 W/m^2 and the ambient temperature is 31°C
- The short-circuit current, open-circuit voltage, and maximum power if the irradiance is 1000 W/m^2 and the cell temperature is as computed in the previous part of this problem

Table 7.3 Electrical characteristics of PV module A and module B

Characteristic at STC	Module A	Module B
Maximum power	350 W	20 W
Optimum operating voltage (V^*)	38.54 V	18 V
Optimum operating current (I^*)	9.08 A	1.11 A
Open-circuit voltage (V_{OC})	47.43 V	22.5 V
Short-circuit current (I_{SC})	9.49 A	1.23 A
Short-circuit current temp. coeff. (α_i)	0.040 %/K	0.045%/K
Open-circuit voltage temp. coeff. (α_v)	−0.29 %/K	−0.34 %/K
Max. power temp. coeff. (α_P)	−0.38 %/K	−0.47 %/K
NOCT	45°C	47°C
Number of cells	72 (series)	36 (series)

- The short-circuit current if the irradiance is 500 W/m^2
- The power produced using Osterwald's method if the irradiance is 650 W/m^2 and the ambient temperature is 20°C

7.2 Repeat Problem 7.1 but consider module B.

7.3 A mini-grid is supplied by a PV array consisting of 12 PV modules arranged in three strings. The module's parameters are provided in Table 7.3 (PV module A). Compute the total power output by the array under STC. Compute the corresponding voltage and current of the array under STC.

7.4 Consider the PV array in the previous problem. Compute the open-circuit voltage, short-circuit current and maximum power of the array if the irradiance is 1050 W/m^2, and the ambient temperature is 31°C using Osterwald's method. Assume the short-circuit current is proportional to the irradiance and the open-circuit voltage is only affected by the ambient temperature, not the irradiance (but its value must be adjusted according to the cell temperature).

7.5 A mini-grid is supplied by a PV array consisting of eight PV modules arranged in two strings. The module's parameters are provided in Table 7.3 (PV module B). Compute the total power output by the array under STC. Compute the corresponding voltage and current of the array under STC.

7.6 When designing a PV array for a mini-grid, the maximum open-circuit voltage of the array must be within the limits of the charge controller it is connected to. The maximum open-circuit voltage is usually based on the lowest expected daytime cell temperature, assuming the irradiance is also 1000 W/m^2. What is the maximum open-circuit voltage of the array in the previous problem if the minimum expected daytime cell temperature is 10°C?

7.7 A mini-grid is to be implemented for the community of Ababju. The community has 500 households. Each household is expected to consume 600 Wh per day. Assume the power consumption is constant throughout the day. Let the daily irradiance of Ababju be

$$G(t) = 700 \sin \left(\frac{2\pi(t-6)}{24} \right)$$

where t is in hours (valid between sunrise $t = 6$ and sunset $t = 18$). Determine how many of the modules described in Table 7.3 are needed to power Ababju. Assume that the mini-grid contains sufficient and lossless energy storage so that the excess energy produced during the day can be used to power the load in the evening.

7.8 The PV module of a solar home system is placed in a position such that one of its 36 cells is completely shaded. Explain, qualitatively, how this will affect the power output of the module.

7.9 Describe a condition in which a PV module rated at 30 W will produce 30 W of power.

7.10 A solar home system is powered by PV module B, whose parameters are provided in Table 7.3. Estimate the maximum power that can be supplied by the module when the irradiance is 600 W/m^2 and the ambient temperature is 24°C.

7.11 A small PV array is formed by connecting two PV modules in parallel. The modules' parameters are provided in Table 7.3 (module B). Compute the short-circuit current if the irradiance is 1250 W/m^2. Ignore the effects of temperature.

7.12 Use the typical I–V curve of a PV module to explain why PV modules can be short-circuited without being damaged.

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