Chapter 1
Introduction

In this book we introduce the fundamental concepts in our understanding of nature and learn to use them to deepen your understanding of nature. This is a bold and sweeping goal—it is indeed the goal of physics. The tools and concepts from mechanics have a central role in how a physicist thinks about nature. And an important part of learning mechanics is to learn to think like a physicist. Unfortunately there are no short-cuts to acquiring the experience of an expert. The only way to learn physics, and mechanics, is through diligent application of the theory to example and exercises. We will help you by providing hints on how to structure your approach, by introducing well-tested problem solving techniques, and through worked examples, but in the end it is only the amount of work *you* spend on exercises that will determine your success. The examples also provide you with inspirations for what you can do when you master the basic principles of mechanics, and we hope this will indeed show you the power that lies in our knowledge of physics, and the exiting adventure it is to discover how nature works and apply that knowledge to develop technologies for the best of mankind.

1.1 Physics

Physics has several aspects: Physics as a science represents the quest to understand the basic laws of nature. Physics provides the tools to understand the processes occurring in nature on all time and length scales. Physics also provides the conceptual and theoretical background for developing new technologies. The fashionable directions in technological and scientific development change, but they all depend on a solid foundation in physics. Physics as a scientific venture is an interplay between the development of theory and experimental investigations.

How physics is used to understand nature is clearly expressed in the physics of biological processes. If you are interested in how a protein folds—and how it folds is important to understand its functions and interactions—we must understand the

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fundamental physics in the interactions between atoms, between the molecular parts of the protein, and between the protein and the surrounding fluid. Physics provides the tools to develop such an understanding.

Physics provides us with the tools to develop new, better technologies. Technologies that can help solve environmental-or energy-related problems. And physics tempts us with possibilities to develop completely new technologies, based on so-far unknown principles, that may lead to improvements larger than we could have imagined.

There are still unsolved, fundamental problems that are within the reach of physics. But in order to address these problems you must master the tools of the trade, you must develop an ability to understand and address the physics of problems, you must develop knowledge about the laws of physics, since we use this knowledge to guide our intuition when we think of physics, and you must develop your knowledge of mathematical tools so that you can solve real problems. This starts with learning mechanics.

You will learn beautiful laws in physics. Much of the theory you learn will be formulated in nice, mathematical equations, beautiful symmetries. It is elegant, concise, and beautiful. And this is indeed something we want to show you. Nature could have been in so many ways. But, look—it is so simple, and so beautiful.

But try not to be blinded by the beauty. The most beautiful and elegant mathematical formulations are found in the parts of physics that are finished. There is not really anything left to do but to find new decimals in the physical constants. When a field is under development it is often messy, unfinished, unready. It is uncharted territory waiting for someone to make sense of it. There may be many exiting discoveries waiting in the messiness. Such is often the nature of discovery.

1.2 Mechanics

Mechanics is the part of physics that addresses the motion of objects. However, in order to predict motion, we need quantitative tools to describe motion. Our main tool is calculus and associated analytical and numerical methods. The study of motion is traditionally called kinematics, which is in many ways closer to mathematics than to physics. When we approach a problem in physics we first use our physical insight to simplify the problem. We strive to make the problem so simple that we can use simple physical laws to formulate mathematical equations that describe the motion. The first part of this process, finding a good physical model and translating the model into a mathematical problem is what we typical refer to as the "physics of the problem".

When we have formulated a mathematical description of the problem, we find the motion and solve the problem using methods from our mathematical toolbox, which contains both analytical and numerical methods. In practice, there is a significant interplay between finding the right physical formulation and solving the mathematical problem, because our insight in physics, and, in particular, in more general concepts such as conservation laws, often allows us to find short-cuts that lead to an analytical solution. Although, for many problems, and arguably for almost all applied problems, there will never be a simple, analytical solution, and we must depend on our ability to address problems using robust, numerical techniques.

In this book we will take you through this procedure many times. So many times that it becomes deeply rooted in you. And as soon as you have grasped the simplicity of the method, we hope you will keep it a secret—physics is supposed to be difficult, and you are expected to uphold that tradition.

1.3 Integrating Numerical Methods

The most unusual part of this textbook is the integration of numerical and analytical methods into the exposition of theory, examples, and exercises. What do we mean by analytical and numerical methods? Analytical methods are the classical mathematical methods you have learned to use in calculus, giving you an exact analytical solution through derivation, integration, or the solution of differential equations. Numerical methods are a similar set of tools that you may have learned to use to solve the same types of problems on a computer: numerical derivation, numerical integration and numerical solution of differential equations. We have developed this integrated approach because we know that the use of computational methods are going to be important for you—probably more important than the use of analytical techniques; because it allows us to present you with more realistic and inspiring examples and applications; and because it also provides you with a deeper understanding of the underlying mathematics.

The use of computations to solve problems in mathematics and physics is not new. For example, when the famous physicist Richard Feynman introduced planetary motion in his classic lectures at CalTech in 1961, he used a simple numerical scheme to calculate the motion of the planets. However, with the advent of the computer we now have the possibility to do billions of computations per second with ease, and this completely changes the game. We can now solve very complicated problems on any computer—if we only know how. The use of computational methods is becoming increasingly important in most areas in science and engineering, in academia and in industry. Since the ambition of any education is to prepare you for a 40 year working life, we know that you need to master the use of computational methods just as well, if not even better, than you master classical analytical methods—since this is what you will be using to solve problems.

This text is based on the principle that you learn best what you do every day. That is why we have integrated the use of numerical methods into every part of the text—it is part of how we explain the theory, it is part of the examples, and it is part of the exercises you do. However, such an integration requires you to learn a particular programming language. This text comes in two versions, one version based on Python and one version based on Matlab. The text is identical, it is only the parts describing specifics of the programming languages that are different in the two cases. It is an advantage to know some basic scientific programming before reading this text, but it is not necessary—many students have become proficient at programming simply by reading this text, solving the exercises, and discussing with students and tutors.

1.4 Problems and Exercises

This book consists of several types of problems and exercises that have various functions:

Discussion questions: A classical type of problems in physics are called "Fermi" problems named after Enrico Fermi. They are mainly estimation problems of complex questions with many unknowns. The main point of such a problem is not to identify the correct answer—there may be none known—the point is the process of reasoning to find an order of magnitude estimation of the answer. Such problems are well suited for a group discussion. Similar questions have recently become very popular as part of job interviews—since they test how the applicant think and apply her knowledge and reasoning power to address an unknown problem.

Closed, structured problems: This is the classical physics problem. We call the problem "closed" if all the necessary data is given in the problem, and "structured" if the steps to go from the initial problem to the solution are given as sub exercises. These problems are popular because the teach problem solving by example and practice by following a structured approach. The idea is that you will learn to do this automatically for yourself if you have done it a sufficient number of times in the exercises.

Open, unstructured problems: When you have practice in solving structured and closed problems, you should be ready for open and unstructured problems. In "open" problems, not all necessary details are given—you have to figure out or decide several key facts yourself. This is the type of problems you will meet in your professional life. Students may initially find these problems frustrating, in particular since they have to introduce many approximations themselves and evaluate whether they are appropriate. However, such problems may also be inspiring, since they allow for more creativity and for more discussions.

Examples of open, closed, structured, and unstructured problems: An open, unstructured problem could be: "A tank is filled with water from a faucet. How long does it take to fill the tank?". The corresponding closed, unstructured problem would be: "A cylindrical tank of diameter 10 cm and height 20 cm is filled with water at a rate of 0.1 dm^3/s. How long does it take to fill the tank?". The corresponding closed, structured problem would be: "A cylindrical tank of diameter 10 cm and height 20 cm is filled with water at a rate of 0.1 dm^3/s. (a) What is the area of the base of the tank? (b) What is the volume of the tank? (c) How long time does it take to fill this volume?".

Projects: We provide long, structured problems that require the application of both analytical and numerical methods. In addition, we focus on discussion and evaluation of the results, and evaluation of the approaches and approximation. The idea is that you will learn the work-flow used in actual research through these project. Solving the projects is considered a major objective of this book. Indeed, the text is meant to give you the background to solve these problems.

1.5 How to Learn Physics

We actually know quite a lot about how to learn and how to teach physics. The research area known as Physics Education Research (PER) is well developed, and provide teaching institutions good insights into what methods work and how they work. This knowledge is important for you, since it gives you a research based insight into how you can optimize your time and learn physics as efficiently as possible.

Almost all pedagogical research (on how to learn physics) can be boiled down to a single result: Students learn best when they are deeply involved in doing physics (reading, discussing, solving problems) with material that is adapted to the student, receiving immediate, individualized feedback.

Learning physics is an active, mental process where *you* have to construct the knowledge in your own mind. Reflecting on how you learn is therefore important for your own learning process. One aim of your education in physics is to be able to think like an expert. Experts think about physics using a few general principles that are organized in a hierarchy so that they cover all of physics. Novices tend to think of physics as many, independent results—one for each situation, one for each problem—which makes learning physics an hopeless endeavor in memorizing formulas.

The most effective way to learn physics is to have a private, competent tutor who can adapt the material, provide feedback, individualize explanations, give you problems that are at the appropriate level of difficulty, monitor your thinking by discussions, and help you correct your thinking to learn fruitful mental models. This is actually the way physics is taught at the graduate level.

The second best way is to learn physics in a social setting where you immerse yourself in physics discussions and thinking throughout the day. Discussions with your co-students as well as your teachers will guide you, give you feedback on your thinking, and provide you with direction on how to work. But you will have to provide important parts of the individualization yourself. You have to choose how much of the textbook you should read, you may have to select problems to solve and use solution manuals in useful ways, and you are responsible for your own progress. This is the environment we typically try to create at a college or university.

Advice for How to Succeed

- Everybody can learn physics. Do not believe otherwise. Experiments with one-to-one mentoring shows that large improvements can be gained by attending a proper learning regimen. You can do it—but it will require effort. Take responsibility for your own learning.

- You learn efficiently by trying to frame your understanding of a problem in words—you learn by discussing physics with other students and your teachers. It turns out that it is the one who does the explaining who learns the most. Asking another student about something is therefore an important part of learning, but it is actually the student you ask who benefits the most—even if it does not feel this way. You should therefore seek environments where you participate actively in discussing physics problems: Attend workshops and teaching groups, find a good group of students to work with, use web-forums actively to discuss and formulate your understanding. If you participate actively in making a good learning environment, you will benefit from this yourself.
- You learn from getting feedback on your work. You should therefore grab all chances of getting feedback constructively: Hand in written exercises, demand relevant feedback on the exercises, and use this feedback to guide your own teaching process: Read based on the feedback and choose problems based on the feedback. Try again if you fail.
- Act as your own coach. Set your own targets and measure how well you are doing in reaching them. Prepare for the teaching you participate in. Reading even just 10 min about a subject before attending class will greatly improve your learning outcome. Think about how you learn: What parts of the teaching activities do you find most useful for your learning? What environments help you learn better? How do you learn physics?
- Multitasking does not work—it is a myth. Your mind cannot pay attention to more than one thing at a time. Trying to do more things at once decreases your overall productivity. You should therefore try to work uninterrupted when as you learn new stuff.
- Learning new things "hurts"—you should be able to feel the mental strain as you push yourself to the limits. This text and many others provide solution manuals. Do not use them recklessly! You only fool yourself. Even just glancing at a sketch of a problem can significantly reduce the learning outcome from this problem. You will not have access to this on the exam, and you will definitely not have access to solutions when you solve real world problems. However, you will have access to other examples. Learn to use examples in a constructive manner—initially as templates for how to think and solve a problem, eventually as an automated problem-solving approach that you can apply to any problem.
- A textbook is a perfect adaptive learning tool, since you can choose for yourself what parts to read, when to read it, and how to read it. You can also choose whether you want to start by reading, start from the example, or start from the problems. Use this flexibility wisely, and reflect on how you use it.

1.6 How to Use This Book

This textbook is meant to be used as a stand-alone textbook in physics or as a supplementary text on numerical methods in introductory physics. The book is intended to be read linearly: first you read the text, then your read the examples, and then you solve the problems. However, I realize and even encourage you to choose your own learning strategies. The most important part is what you do in the form of tutorials and exercises, and not what you read. The text can therefore be seen as supporting material for the projects: In order to be able to do the projects, you need to read the text and study the examples. Still, the book has a certain organization, which is based on the knowledge that you may take several paths through the text:

Background: This text requires a course in calculus. It does not require a course in numerical methods and programming, but experience shows that programming requires maturity to master—and an additional course in programming is therefore useful to ensure that the methods learned here become integrated into the toolbox of each student.

Numerical methods: The main exposition of the material, the theoretical explanation, includes both numerical and analytical methods where they are apt. We do not separate them, since they are equally important. However, in addition to the use of numerical methods in the main text, we have added addition material on numerical methods, which is meant to provide a more solid mathematical foundation for the use of numerical methods.

Problem-solving strategies: We introduce a few, robust problem-solving strategies. These are meant to be general templates that you should become so used to, that you use them without thinking about it. Initially, we therefore suggest that you follow these strategies as closely as possible, but as you get more experienced you may take short cuts or automate larger parts of the solution strategies. However, if you are baffled by a problem, you always have the problem-solving strategies to fall back on.

Proofs: Most of the more advanced material and many of the longer derivations and proofs are left out of the text flow. However, you can find relevant derivations and associated mathematical theory at the end of the book or online.

Examples: Each concept can be explained by key worked examples. These are the central examples that may serve as templates for how to address a particular class of physical systems. They are often extended and provide the best background for solving the projects.