Chapter 6
Parallel Robots

This chapter deals with the increasingly popular and high-performing robots that are known as parallel robots. Standard mechanisms of industrial robots possess serial kinematic chains in which links and joints alternate as shown in Fig. 6.1 (left). These are referred to as serial robots. Lately, we have seen a significant advancement of parallel robots. They include closed kinematic chains, an example is shown in Fig. 6.1 (right).

In industry, parallel robots have started to gain ground in the last two decades. However, the initial developments date back to 1962 when Gough and Whitehall developed a parallel robot for testing automobile tires. At about the same time, a similar parallel robot was introduced by Stewart to design a flight simulator. The parallel robot, in which a mobile platform is controlled by six actuated legs, is therefore called the Stewart-Gough platform. The breakthrough of parallel robots was also largely due to the robot developed by Clavel in the eighties. His mechanism was patented in the USA in 1990 under the name of the Delta robot. The parallel mechanisms in robotics had become a subject of systematic scientific research in the early eighties. These activities intensified significantly in the nineties and culminated with some key achievements in robot kinematics in general.

6.1 Characteristics of Parallel Robots

In serial robots, the number of degrees of freedom is identical to the total number of degrees of freedom in joints. Thus, all joints must be actuated, and usually only simple one degree of freedom translational and rotational joints are used. In parallel robots, the number of degrees of freedom is lower than the total number of degrees of freedom in joints so that many joints are passive. Passive joints can be more complex; typical representatives are the universal joint and the spherical joint. The universal joint consists of two perpendicular rotations while three perpendicular

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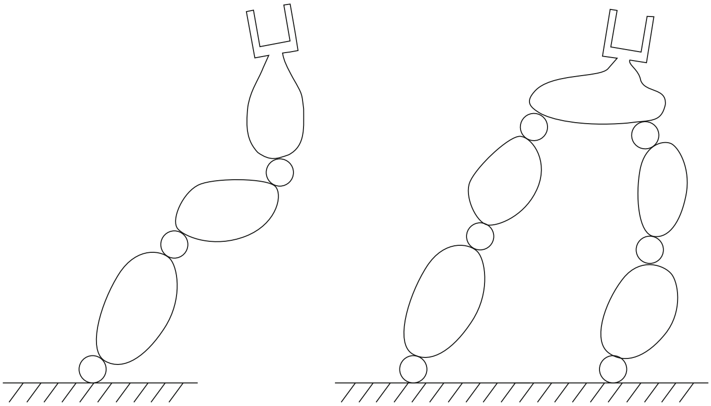

Fig. 6.1 Serial kinematic chain (left) and closed kinematic chain (right)

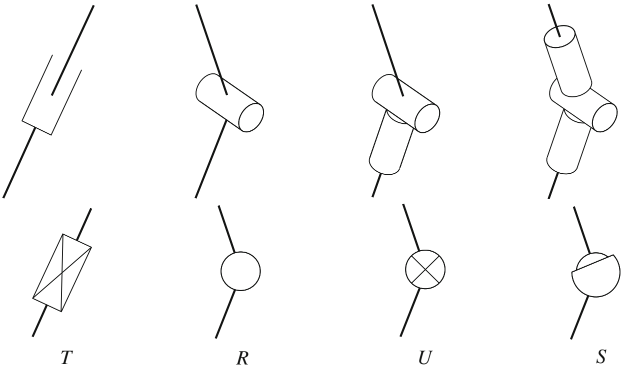

Fig. 6.2 Types of joints often used in parallel mechanisms

rotations compose the spherical joint as shown in Fig. 6.2. Here, letters T, R, U, and S are used to mark the translational joint, the rotational joint, the universal joint, and the spherical joint, respectively.

In parallel robots, the last (top) link of the mechanisms is the so called platform (Fig. 6.3). The platform is the active link to which the end-effector is attached. It is connected to the fixed base by a given number of (usually) serial mechanisms called legs. The whole structure contains at least one closed kinematic chain (minimum two legs). The displacements in the legs produce a displacement of the platform as shown in Fig. 6.3. The motions of the platform and the legs are connected by often very complex trigonometric expressions (direct and inverse kinematics) depending on the geometry of the mechanism, on the type of joints, the number of legs and on their kinematic arrangements.

Unfortunately, unique and uniform denominations for parallel robots do not exist. In this work, a parallel robot is denominated by the type of kinematic chains repre-

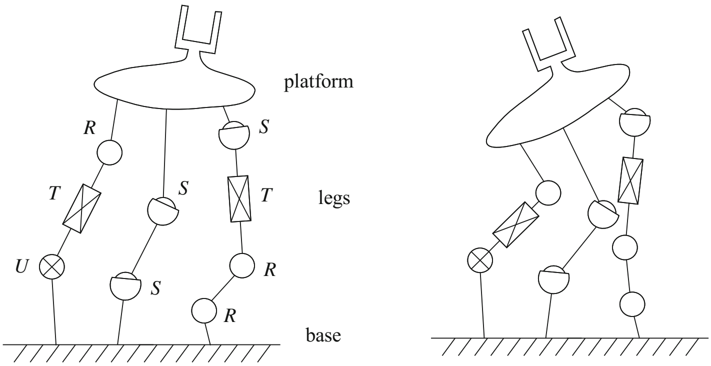

Fig. 6.3 Basic structure of a parallel robot

senting the legs. Thus, the robot in Fig. 6.3 is denominated as UTR-SS-RRTS. When legs of the same type are repeated, for example, in the TRR-TRR-TRR robot, the denomination can be simplified as 3TRR.

Number of degrees of freedom

Each joint contributes to the mobility of the robot by introducing a given number of degrees of freedom or, alternatively, by introducing a corresponding number of constraints, which are defined as follows. Let λ denote the maximum number of degrees of freedom of a freely moving body (in space $\lambda = 6$ and in plane $\lambda = 3$), and let f_i be the number of degrees of freedom of the $i - th$ joint. The corresponding number of constraints is

$$c_i = \lambda - f_i. \tag{6.1}$$

In robotic practice where serial robots dominate, we usually consider joints as elements that add degrees of freedom to the motion of the robot end-effector. In parallel robots, on the contrary, it is more advantageous to consider the movement of the platform (to which the end-effector is attached), taking into account the number of constraints introduced by the joints. Thus, a universal joint U in a space where $\lambda = 6$ introduces $f_i = 2$ degrees of freedom and $c_i = \lambda - f_i = 6 - 2 = 4$ constraints. Or, for example, in a plane where $\lambda = 3$, a rotational joint R introduces $f_i = 1$ degrees of freedom and $c_i = \lambda - f_i = 3 - 1 = 2$ constraints, while the same joint in space introduces $c_i = \lambda - f_i = 6 - 1 = 5$ constraints. Note that rotational and translational joints can operate both in a plane and in space, whereas spherical and universal joints produce only spatial movements and cannot be used in planar robots.

The number of degrees of freedom of a parallel robot is less than the total number of degrees of freedom contributed by the robot joints, unlike in a serial robot where these two numbers are identical. Let N be the number of moving links of the robot and n the number of joints. The joints are referred to as $i = 1, 2, \ldots, n$. Each joint possesses f_i degrees of freedom and c_i constraints. The N free moving links possess $N\lambda$ degrees of freedom. When they are combined into a mechanism, their motion is limited by the constraints introduced by joints, so that the number of degrees of freedom of a robot mechanism is

$$F = N\lambda - \sum_{i=1}^{n} c_i. \tag{6.2}$$

Here, by substituting c_i with $\lambda - f_i$ we obtain the well known Grübler's formula as follows

$$F = \lambda(N - n) + \sum_{i=1}^{n} f_i. \tag{6.3}$$

We must not forget that the number of motors which control the motion of a robot is equal to F.

Note that in serial robots the number of moving links and the number of joints are identical ($N = n$), so that the first part of Grübler's formula is always zero ($\lambda(N - n) = 0$). This explains why the number of degrees of freedom in serial robots is simply

$$F = \sum_{i=1}^{n} f_i. \tag{6.4}$$

A very practical form of Grübler's formula to calculate the degrees of freedom of a parallel robot can be obtained as follows. Suppose that a parallel mechanism includes $k = 1, 2, \ldots, K$ legs, and that each of the legs possesses ν_k degrees of freedom and consequently $\xi_k = \lambda - \nu_k$ constraints. When the platform is not connected to the legs and can freely move in space, it contains λ degrees of freedom. The number of degrees of freedom of a connected platform can thus be computed by subtracting the sum of constraints introduced by the legs

$$F = \lambda - \sum_{k=1}^{K} \xi_k. \tag{6.5}$$

Equations (6.3) and (6.5) are mathematically identical and can be transformed from one to another by simple algebraic operations.

Now we can calculate the degrees of freedom for the robot shown in Fig. 6.3. This robot possesses $N = 7$ moving links and $n = 9$ joints. The total number of degrees of freedom in joints is 16 (3 rotational joints, 2 translational joints, 1 universal and 3 spherical joints). Using the standard Grübler's formula given in Eq. (6.3), we get

$$F = 6(7 - 9) + 16 = 4.$$

If we now use the modified form of Grübler's formula we need to calculate the constraints introduced by each leg. This is rather simple because we only need to subtract the number of degrees of freedom of each leg from λ. For the given robot (legs are counted from left to right) we have $\xi_1 = 2$, $\xi_2 = 0$, and $\xi_3 = 0$. By introducing these values in Eq. (6.5), as expected, we obtain

$$F = 6 - 2 = 4.$$

Advantages and disadvantages of parallel robots

The introduction of parallel robots in industry is motivated by the number of significant advantages that parallel robots have in comparison to serial robots. The most evident are the following:

Load capacity, rigidity, and accuracy. The load carrying capacity of parallel robots is considerably larger than that of serial robots. Parallel robots are also more rigid, and their accuracy in positioning and orienting an end-effector is several times better than with serial robots.

Excellent dynamic properties. The platform can achieve high velocities and accelerations. Furthermore, the resonant frequency of a parallel robot is orders of magnitude higher.

Simple construction. Several passive joints in parallel robots enable less expensive and simple mechanical construction. When building parallel robots standard bearings, spindles, and other machine elements can be used.

The use of parallel robots is, nevertheless, limited. Because of the tangled legs, parallel robots can have difficulties in avoiding obstacles in their workspace. Other significant disadvantages are:

Small workspace. Parallel robots have considerably smaller workspaces than serial robots of comparable size. Their workspace may be further reduced since during motion of the platform the legs may interfere with each other.

Complex kinematics. The computation of kinematics of parallel robots is complex and lengthy. In contrast to serial robots, where the difficulty arises when solving the inverse kinematics problem, in parallel mechanisms the difficulty arises in solving the direct kinematics problem.

Fatal kinematic singularities. Serial robots in kinematically singular poses lose mobility. Parallel robots in singular poses gain degrees of freedom, which cannot be controlled. This is a fatal situation because it cannot be resolved.

6.2 Kinematic Arrangements of Parallel Robots

We can create an immense number of kinematic arrangements of parallel robots. In industrial practice, however, only few of these are used. The most popular and general in the kinematic sense is the Stewart-Gough platform as shown in Fig. 6.4.

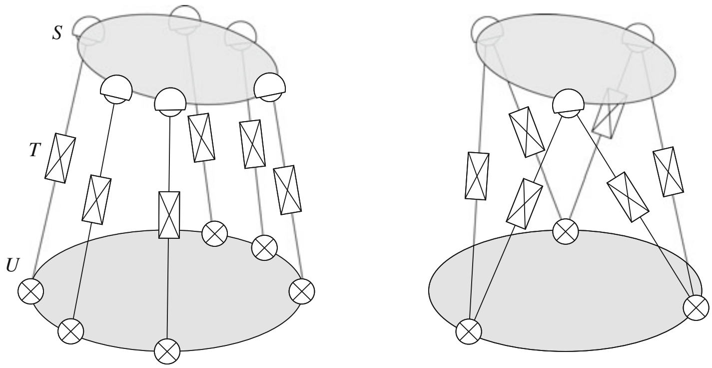

Fig. 6.4 The Stewart-Gough platform

Stewart-Gough platform

A general Stewart-Gough platform is shown on the left side of Fig. 6.4. According to our denomination, the mechanism is of type 6UTS. The robot contains $n = 18$ joints, $N = 13$ moving links and the sum of $f_i, i = 1, 2, \ldots n$ is 36. This gives the expected result

$$F = 6(13 - 18) + 36 = 6$$

degrees of freedom. The platform of this robot can be spatially positioned and oriented under the control of six motors, which are typically the six translations. By shortening or expanding the legs (changing the lengths of the legs) the platform can be moved into a desired pose (position and orientation). A special advantage of the Stewart-Gough platform with the UTS legs is that loads acting on the platform are transferred to each particular leg in the form of a longitudinal force in the direction of the leg and there is no transverse loading on the legs. This peculiarity allows excellent dynamic performances.

The number of degrees of freedom of a UTS leg is six and the number of constraints is zero. If we consider Grübler's formula (6.5) it is easy to verify that the number of UTS legs does not affect the number of degrees of freedom of the robot and that the mobility of the Stewart-Gough platform does not depend on the number of legs. A robot with only one UTS leg, which is a serial robot, possesses six degrees of freedom, the same as the fully parallel original six-legs Stewart-Gough robot.

The six-legged mechanism on the right side of Fig. 6.4 schematically represents the original Stewart-Gough platform which has a central-symmetrical star shape. In this arrangement, two by two legs are clamped in one point in which two overlapping coincident spherical (or universal) joints are placed. Therefore, the number of independent spherical joints is six and the same is the number of universal joints. The overlapping joints not only simplify the construction but also allow easier computations of the robot kinematics and dynamics.

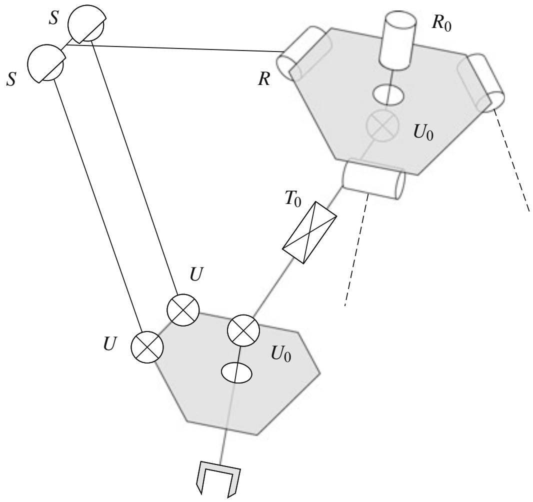

Fig. 6.5 The Delta robot

Delta robot

Due to its specific motion characteristics and its numerous applications in industry, the Delta robot found its place among robot manufacturers (see Fig. 6.5). The kinematics of this robot is very sophisticated. The main objective of its creator was to create a lightweight robot with extreme dynamic performances.

The fixed base of the robot is the upper hexagon while the lower hexagon represents the moving platform. The robot has three lateral legs. Only one is presented in the figure, with one R joint, two S joints and two U joints; the other two legs are symbolically drawn with a dotted line. There is also an independent middle leg $R_0U_0T_0U_0$ which has no influence on the motion of the platform. There is a parallelogram mechanism between the middle of the leg and the base, which consists of two spherical joints S and two universal joints U. Each leg, therefore, has 3 links and 5 joints. Without considering the middle leg, the number of degrees of freedom of the mechanism is

$$F = 6(10 - 15) + 33 = 3.$$

The pose of the platform is determined by only three variables. In the original version of the Delta robot the three rotation angles R in lateral legs are controlled by motors. Due to the parallelogram structure of the legs, the platform executes only translation and is always parallel to the base.

The purpose of the middle leg is to transfer the rotation R_0 across the platform to the gripper at the end-point of the robot. It acts as a telescoping driveshaft for rotating the gripper. This leg is a cardan joint with two universal joints U_0 separated by a translational joint T_0. In all, the mechanism has four degrees of freedom: three translational, enabling the spatial position of the gripper and one rotational, enabling rotation of the gripper about an axis perpendicular to the platform. All actuators of the Delta mechanism are attached to the base and do not move. Therefore the mechanism is extremely lightweight and the platform can move with high velocities and accelerations.

Planar parallel robots

The following examples are planar parallel robots which operate in a given plane where $\lambda = 3$. The first example is given in Fig. 6.6 left. The robot contains three legs of the type RTR-RRR-RRR. As a result we have $N = 7$ and $n = 9$ and the total number of degrees of freedom in joints is 9. According to Eq. (6.3), the number of degrees of freedom of this robot is

$$F = 3(7 - 9) + 9 = 3.$$

The result is expected since all legs introduce zero constraints (6.5). Consequently, the platform can achieve any desired pose inside the workspace. Note that in plane two degrees of freedom are needed for the position (translations in x–y plane) and one degree of freedom for the orientation (a rotation about z axis). To activate this robot three motors are needed. To attach the motors, we can select any of the nine joints. Usually we prefer the joints attached to the base so that the motors are not moving and their weight does not influence the robot dynamics. In a specific case, the translational joint can also be motorized using an electric spindle or a hydraulic cylinder.

In Fig. 6.6 right a similar planar parallel robot is presented, its structure is RTR-RR-RR. Here, we can see that each of the two RR legs introduce one constraint. According to Eq. (6.5), the number of degrees of freedom of this parallel robot is

$$F = 3 - 2 = 1.$$

The robot is controlled using one motor. The platform has limited mobility and can only move along a curve in plane x–y. We can, for example, either position the

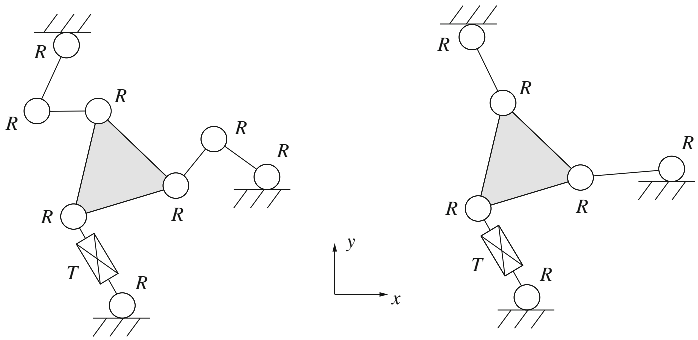

Fig. 6.6 Planar parallel robots

platform along x axis without having control over y and the platform's orientation or, alternatively, orient the platform without having control over its position in x and y.

Parallel humanoid shoulder

Parallel mechanisms are very common in nature, in the human body or in animals. It is, therefore, no surprise that the models of parallel robots can be efficiently used in simulating biomechanical properties of humans where muscles and ligaments stretched over the joints form various parallel kinematic structures. For instance, the shoulder complex can be represented by two basic compositions, the so-called inner joint, which includes the motion of the clavicle and the scapula with respect to the trunk, and the so-called outer joint, which is associated with the glenohumeral joint. In today's humanoid robotics, the motion of the inner joint is typically neglected because of its mechanical complexity. Nevertheless, its contribution to human motion, reachability of the arm and dynamics is crucial.

A parallel shoulder mechanism representing the inner shoulder was proposed in the literature. Its motion is shown in Fig. 6.7. The proposed structure is TS-3UTS. There is a central leg T_0S_0 with four degrees of freedom and two constraints. Around the axis of the central leg three UTS lateral legs are attached possessing six degrees of freedom each, their number of constraints is zero. According to Eq. (6.5), the number of degrees of freedom of the robot is

$$F = 6 - 2 = 4.$$

The robot can produce a complete orientation of the platform (about three principal orientation angles), and can expand or shrink similarly to the human shoulder. The arm is attached to this platform through the glenohumeral joint. The inner shoulder joints, as it is proposed, precisely mimic the motion of the arm, including shrugging

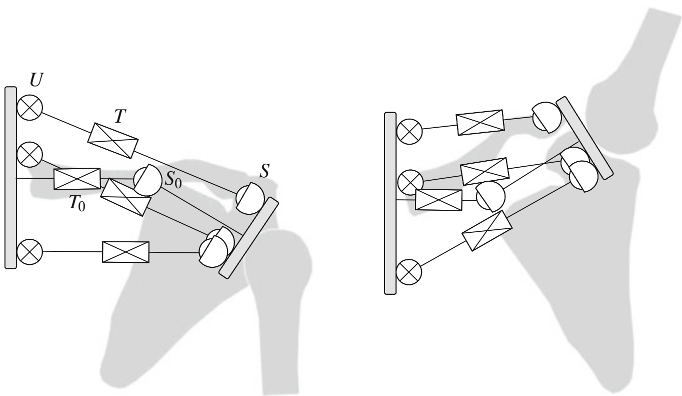

Fig. 6.7 Parallel robot mimicking the inner shoulder mechanism

and avoiding collisions with the body, and provides excellent static load capacity and dynamic capabilities.

6.3 Modelling and Design of Parallel Robots

The majority of parallel robots which appear in industry or in research laboratories possess symmetrical kinematic arrangements. From the point of view of their construction, it is useful that they are composed of the same mechanical elements. Symmetry also contributes to making their mathematical treatment simpler.

One common group of kinematic arrangements is represented by the previously described shoulder robot. This group contains a central leg with ν_1 degrees of freedom around which there are symmetrically placed lateral legs, which are often of type UTS possessing $\nu_2, \nu_3, ..., \nu_K = \lambda$ degrees of freedom (and zero constraints). The central leg is therefore crucial to determine the kinematic properties of the whole robot, as the number of degrees of freedom of the robot is $F = \nu_1$.

The second group of kinematic arrangements are represented by the Stewart-Gough platform in which all the legs are identical and are usually of type UTS so that $\nu_1, \nu_2, ..., \nu_K = \lambda$. When $\nu_1, \nu_2, ..., \nu_K < \lambda$ only a small number of such robots are movable, most of their structures are with zero or negative degrees of freedom. Robots with a negative number of degrees of freedom are referred to as overconstrained.

Consider the second group of robots (Gough-Stewart-like kinematic structure) with a single motor in each leg. Such a robot must have $K = F$ legs, as a robot with $K < F$ cannot be controlled. It is easy to verify that only the following robots can exist in space (where $\lambda = 6$)

$$\begin{aligned} &K = 1,\ \nu_1 = 1 \\ &K = 2,\ \nu_1 = \nu_2 = 4 \\ &K = 3,\ \nu_1 = \nu_2 = \nu_3 = 5 \\ &K = 6,\ \nu_1 = \nu_2 = ... = \nu_6 = 6 \end{aligned}$$

Robots in this group with four and five legs do not exist. In plane, where $\lambda = 3$, only the following robots can exist

$$\begin{aligned} &K = 1,\ \nu_1 = 1 \\ &K = 3,\ \nu_1 = \nu_2 = \nu_3 = 3 \end{aligned}$$

In the planar case, robots with two legs do not exist.

Kinematic parameters and coordinates of parallel robots

In Fig. 6.8 the coordinate frame x–y–z is attached to the moving platform, while x_0–y_0–z_0 is fixed to the base. The position of the platform is given with respect to the fixed coordinate frame by vector $\mathbf{r}$; its components are r_x, r_y, r_z. The orientation

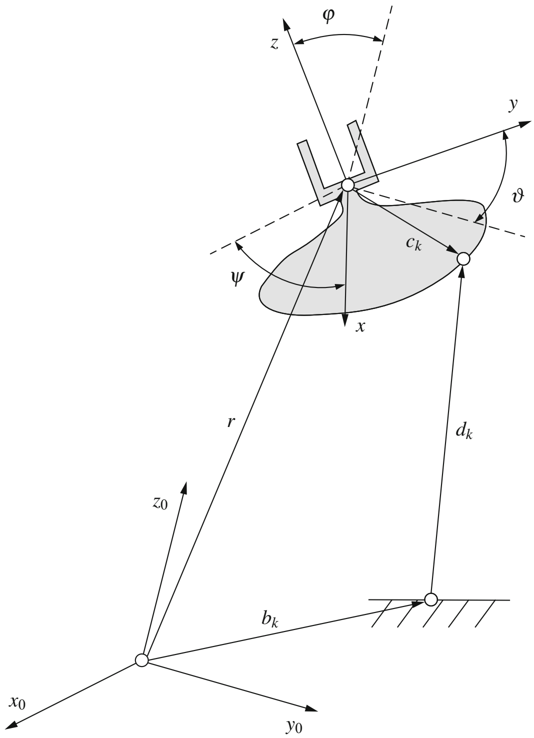

Fig. 6.8 Kinematic parameters of a parallel robot

of the platform can be described by a chosen triplet of orientation angles ψ, ϑ, φ occurring between both coordinate frames (see Chap. 4 for details).

Vector $\mathbf{b}_k$ defines the attachment of leg k to the base expressed in frame x_0–y_0–z_0, while vector $\mathbf{c}_k$ defines the attachment of the same leg to the platform in frame x–y–z. The vectors

$$\mathbf{d}_k = \mathbf{r} + \mathbf{R}\mathbf{c}_k - \mathbf{b}_k,\ k = 1, 2, ..., K, \tag{6.6}$$

describe the geometry of the robot legs expressed in coordinate frame x_0–y_0–z_0. Here, $\mathbf{R} = \mathbf{R}(\psi, \vartheta, \varphi)$ is the 3×3 rotation matrix which transforms the coordinate frame x–y–z into x_0–y_0–z_0. Equation (6.6) can also be formulated in a homogeneous form as follows

$$\mathbf{d}_k = \mathbf{H}\mathbf{c}_k,\ k = 1, 2, \ldots, K, \tag{6.7}$$

where the homogeneous transformation matrix is

$$\mathbf{H} = \begin{bmatrix} & \mathbf{R} & & \mathbf{r} - \mathbf{b}_k \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6.8}$$

We assume that the leg lengths are the joint coordinates of the robot

$$q_k = \|\mathbf{d}_k\|, \ k = 1, 2, \ldots, K, \tag{6.9}$$

where $\| \cdot \|$ indicates vector norm. They are elements of the vector of joint coordinates

$$\mathbf{q} = (q_1, q_2, ..., q_K)^{\mathrm{T}}.$$

The robot kinematic parameters are vectors $\mathbf{b}_k$, $k = 1, 2, \ldots, K$ expressed in frame x_0–y_0–z_0 and vectors $\mathbf{c}_k$ expressed in frame x–y–z.

Once we have defined the internal coordinates, let's look at what the robot's external coordinates are. In parallel robots they usually represent some characteristics in the motion of the platform to which the end-effector is attached. In most cases, the chosen external coordinates are the position and orientation of the platform, the so-called Cartesian coordinates. In space where $\lambda = 6$ they include the three components $r_{\mathrm{x}}, r_{\mathrm{y}}, r_{\mathrm{z}}$ of the position vector in Fig. 6.8, and the three orientation angles ψ, ϑ, φ, so that the vector of external coordinates is defined as follows

$$\mathbf{p} = (r_{\mathrm{x}}, r_{\mathrm{y}}, r_{\mathrm{z}}, \psi, \vartheta, \varphi)^{\mathrm{T}}.$$

Inverse and direct kinematics of parallel robots

From the control point of view, the relation between the external and internal coordinates is of utmost importance. Their relationship is, similarly to serial robots, determined by very complicated algebraic trigonometric equations.

The inverse kinematics problem of parallel robots requires determining the internal coordinates $\mathbf{q}$, which are the leg lengths, from a given set of external coordinates $\mathbf{p}$, which represent the position and orientation of the platform. For a given set of external coordinates $\mathbf{p}$ the internal coordinates can be obtained by simply solving Eq. (6.7). Here, unlike in serial robots, it is important to recognize that the values of the external coordinates uniquely define the leg lengths of the parallel robot and the computation is straightforward.

The direct kinematics problem of parallel robots requires determining the external coordinates $\mathbf{p}$ from a given set of joint coordinates $\mathbf{q}$ (Fig. 6.9). This problem is extremely complicated in mathematical terms and the computation procedures are cumbersome. In general, it is not possible to express the external coordinates as explicit functions of the internal coordinates, whereas with serial robots this is quite straightforward. Usually, these are coupled trigonometric and quadratic equations which can be solved in closed-form only in special cases. There exist no rules as how to approach symbolic solutions. The following difficulties are common:

Nonexistence of a real solution. For some values of internal coordinates real solutions for the external coordinates do not exist. Intervals of internal coordinates when this can happen cannot be foreseen in advance.

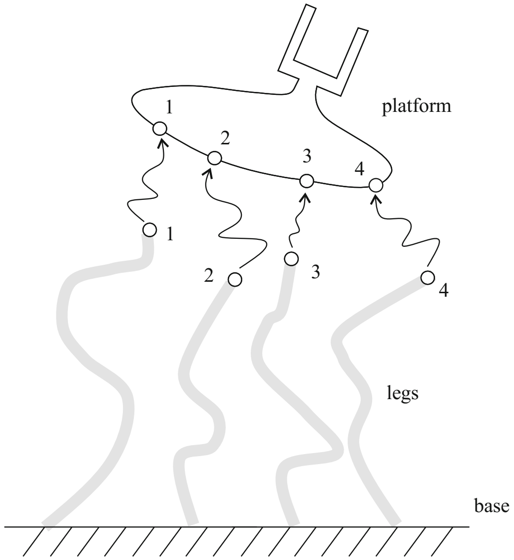

Fig. 6.9 The direct kinematics problem consists of finding the pose of the platform corresponding to the length of the legs. Leg end-points need to match corresponding points on the platform (e.g., 1 − 1)

Multiple solutions. For a given set of internal coordinates, there exist multiple solutions for the external coordinates. The number of solutions for a given combination of leg lengths depends on the kinematic structure of the mechanism. The general Stewart-Gough platform has forty possible solutions of the direct kinematics problem. For a selected combination of leg lengths there exist forty different poses of the platform. In addition, sometimes two poses of the platform cannot be transitioned between as the legs get entangled. In such cases, the platform could transit from one pose into another only by dismantling the legs in the first pose and reassembling them in the new pose.

Nonexistence of closed-form solutions. Generally for a given set of joint coordinates, it is not possible to find an exact solution to the direct kinematics problem, even if a real solution exists. In such cases we use numerical techniques which may not necessarily converge and may not find all the solutions.

Design of parallel robots

The design of parallel robots depends on desired performance, flexibility, mobility, and load capacity as well as the actual workspace.

In considering the workspaces for both parallel and serial robots, we are referring to the reachable workspace and the dexterous workspace. One of the main drawbacks of parallel robots is their small workspace. The main goal in workspace analysis is, therefore, to determine if a desired trajectory lies inside the robot workspace. The size of the workspace in parallel robots is limited by the ranges in the displacements of the legs, displacements of passive joints, and, particularly, by interference between the legs of the robot. Even with small movements, the legs can collide with each other. The interlacing of legs is in practice a major obstacle in a robot's motion and its reachability. The determination and analysis of robot workspace is in general a tedious process. In parallel robots it is usually even more complex, depending on the number of degrees of freedom and the mechanism's architecture.

The effect of load in serial robots is usually seen in terms of dynamics, which to a large extent includes the inertia of the links. The contribution of an external force is typically smaller and in many cases can be neglected. In parallel robots with a large number of legs, where the links are very light and the motors typically attached to the fixed base, the robot statics plays an important role. The computation of robot statics is related to the well-known Jacobian matrix which represents the transformation between the external and the internal coordinates. This goes beyond the scope of our book, but considerable literature, articles, and textbooks are available to the interested reader.

In practice, we can often see a Stewart-Gough platform that has a structure as presented in Fig. 6.10. The robot contains (instead of six legs of type UTS) six legs of type STS. Kinematically, this architecture is quite unusual and redundant. The robot has too many degrees of freedom. Each leg possesses 7 degrees of freedom

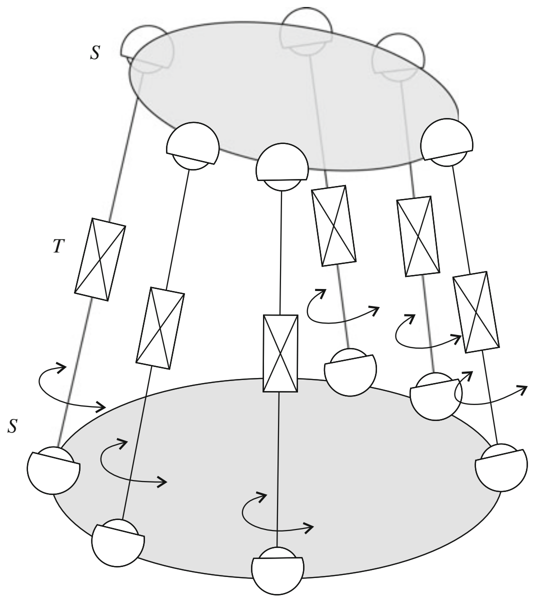

Fig. 6.10 A modification of the Stewart-Gough platform

which corresponds to -1 constraints. According to Grübler's formula (6.5), the number of degrees of freedom of the robot is

$$F = 6 - (-6) = 12.$$

It is important to note that six of the twelve degrees of freedom are manifested as rotations of the legs around their own axes. These rotations have no influence on the movement of the platform. Thus, the robot can still be motorized by only six motors that change the length of the legs, affecting the translation T, while the rotations around the leg axes can be left passive and can freely change. The advantages of this construction are that S joints are easier to build than U joints (and therefore cheaper), and that the passive rotations around the leg axes enable more flexibility when connecting power and signal cables, as these are often arranged along the legs from the base to the robot platform.