Sparging 7

7.1 Introduction

Sparging is the process of rinsing the sugars away from the grain in the mash. After the mash is completed, we want to separate the dissolved sugar water (sweet wort) from the grain. It is not as simple as just letting the mash liquid drain. There is still a considerable amount of fermentable sugar left in the grain matrix. So, we rinse with hot water. The hotter the water is, the more soluble the sugar is in water, but excessive heat and/or incorrect pH will also remove tannins from the grain husks. This is a condition that must be avoided. The best way to illustrate the flavor of tannins in your beer is to take a tea bag and put it in your mouth. Not pleasant! So we sparge with enough water to extract as much sweet wort as possible, but not to the point where we extract tannins. Also, excessive sparge water will lead to excessively large volumes of more dilute sweet wort.

Sparging is the act of rinsing the grain with hot water. And, a lauter tun is the traditional vessel used for separating the wort from the grain. In this tradition, the brewer would pump the slurry of grist and hot liquor (after it had been mashed and while it was still hot) mash tun (or mash mixer) to the lauter tun. Here, the mixture would be allowed to rest and settle, forcing the solids in the mixture to sink to the bottom of the lauter tun. The process was referred to as lautering. Some breweries have employed the combination mash/lauter tun where the lautering and sparging occur in the same vessel as the mash. We will explore the modern version of the lauter tun later in this chapter.

Regarding sparging, there are several different approaches to how this accomplished. For example, the sweet wort could be drained from the lauter tun; the brewer could refill the tun with sparge water, and drain again, and again. This is the basic method behind *batch sparging*, a method attributed to the English brewing process. Each of the drainings (or *gyles*) could be used separately to make different beers or combined in ratios to ensure a certain specific gravity for a particular beer. This *parti*-*gyle* method of brewing was the “technology” of the day when it was

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first used industrially. Because of its utility in ensuring a consistent preboil gravity, it is slowly seeing resurgence in some brewhouses today.

Alternatively, the brewer could slowly drain the lauter tun and continuously supply fresh sparge water to the top of the grain bed at the same rate that is being grain. This is *fly sparging*, a method attributed to the German brewing process.

In both cases, the first runnings were often returned to the top of the lauter tun to be refiltered through the grain. Initially, the runnings would drip out of the grain bed loaded with insoluble proteins, cellulose, small amounts of insoluble starch that did not get mashed, and even small grain particles. By passing these again through the system, the grain bed would act like a filter and help remove the insoluble material. This process is known as the *vorlauf*, from the German word meaning “the first amount.” Brewers often use the word as both a noun (its intended use) and as a verb.

In this chapter, we will explore a lot of details about the mechanical process of moving liquids around the brewery. This is very important to the brewer, because large quantities of water must be moved from place to place. In home brewing, it is relatively easy to lift 5 gallons of hot water and move it by hand—well, it is “relatively” easy. But brewing on a larger scale requires larger storage vessels, pipes, hoses, and pumps. We certainly do not want to move 50 barrels (1550 gallons, 5800 L) of sweet wort by hand!

Because of that issue alone, fluid engineers design modern breweries. Significant thought goes into the design of pipe size or pumps for the job. This chapter is not meant to train a brewer to design modern piping systems, but it will at least help the brewer understand why certain choices are made and understand the limitations of their installed pumping systems. And understanding all of this requires some basic understanding of the physics of fluid transfer.

7.2 Fluid Physics: Static Case

Before we launch into the more complicated case of moving fluids around the brewery, we will first consider stationary, or “static,” fluids to set some basic definitions and build a knowledge base. Once we have considered the simple static case, we will look at moving fluids, and finally, we will add extra complications such as moving fluids through a grain bed.

7.2.1 Pressure

When considering movement fluids in pipes, we often want to know the force, or the pressure, that a fluid exerts on a container. The words pressure and force are frequently confused and used interchangeably. However, these two terms are very different. By definition, pressure is defined as the force that exerted on a surface divided by the area that the force is distributed across,

$$P = \frac{F}{A}. \tag{7.1}$$

In the SI system, with forces measured in Newtons (N) and area in square meters (m^2), the pressure is measured in N/m^2. This combination of units is known as the Pascal (Pa), named after the French scientist Blaise Pascal (1623–1662). A unit that is sometimes used is known as the bar (bar). One bar is equal to 100,000 Pa. Another common unit for pressure, particularly in fluids engineering in the USA, is pounds per square inch (commonly abbreviated psi, or sometimes as lbs/in^2).

Let us consider the difference between force and pressure using a simple experiment: Place a thumbtack between your index finger and thumb such that the pointy end is pressing on your thumb. Press together gently. You will notice that (ouch!) the pointy end of the tack is digging in and hurting compared to the other, flat end. It is important to note that the force is the same at either end of the tack and thus the same on either finger/thumb. But since each end of the tack has considerably different areas, the pressure is significantly different. You should see that for the same force, a smaller area gives a larger pressure. Likewise, a larger area will give a smaller pressure—again with the same force.

7.2.2 Pascal's Law

Pascal's law (also Pascal's principle) is a principle in fluid physics that states that pressure exerted anywhere in a confined and incompressible fluid (such as water) is transmitted equally in all directions throughout the fluid. This means that, at any given depth of the fluid, the pressure is the same.

Another example here will highlight once more the difference between force and pressure and use Pascal's principle as well. Consider a hydraulic car lift as shown in Fig. 7.1. A car weighting 26.7 kN (about 6000 lbs) is placed on one piston with a radius of 45 cm. The hydraulic fluid is connected to another piston, but with a radius of 1 cm. Since the pressure must be the same in the fluid at identical depths, the required force at the smaller piston to lift the car can be determined from

$$\begin{aligned} P_1 &= P_2 \\ \frac{F_1}{\pi r_1^2} &= \frac{F_2}{\pi r_2^2} \end{aligned} \tag{7.2}$$

or

$$\begin{aligned} F_1 &= F_2 \frac{r_1^2}{r_2^2} = 26{,}700\,\mathrm{N} \frac{1^2}{45^2} \\ F_1 &= 13.2\,\mathrm{N}\ (\text{about } 3\,\text{lbs}) \end{aligned} \tag{7.3}$$

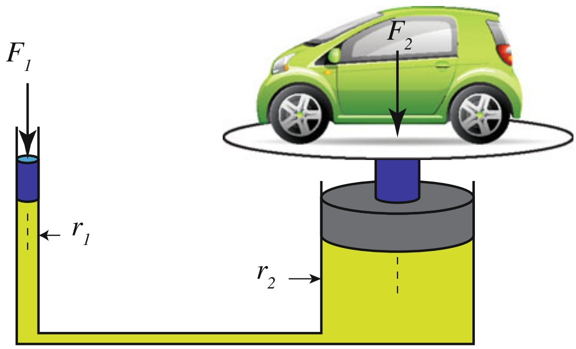


Fig. 7.1 Example illustrating Pascal's principle. The pressure at each piston must be the same

Of course, for a given amount of travel in the large piston, the smaller piston must travel further. The amount, or volume, of the fluid moved must be the same at either piston,

$$V_1 = V_2. \tag{7.4}$$

So for every centimeter that the large piston moves, the smaller one must move

$$\begin{aligned} \pi r_1^2 x &= \pi r_2^2 \,(1\,\text{cm}) \\ x &= (1\,\text{cm})\frac{45^2}{1^2} \\ x &= 2025\,\text{cm} \end{aligned} \tag{7.5}$$

Pascal's law is actually a statement of the weight of the fluid above a certain point. Looking at Fig. 7.2, we now consider a liquid and draw an imaginary cube somewhere in the liquid. The difference in pressure between the top and the bottom of the cube can be determined by finding the weight of this cube of fluid. The weight (a force) of this cube is given as:

$$W = mg = \rho V g \tag{7.6}$$

where ρ is the density of the fluid and V is the volume of the cube. So, the pressure at the bottom of the cube is simply the pressure at the top of the cube *plus* the pressure–"force" due to the weight of the cube.

$$P_{\text{bottom}} = P_{\text{top}} + \frac{\rho V g}{A}. \tag{7.7}$$

Rearranging, Pascal's law is more generally stated as

$$\Delta P = \rho g \Delta h \tag{7.8}$$

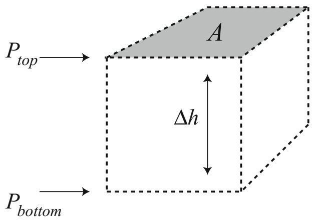


Fig. 7.2 An imaginary cube is drawn around a parcel of fluid to illustrate Pascal's law

with ΔP being the difference between the top and bottom pressures. It just states that the pressure difference between two points depends only on the net difference in height between the two points. So, if two points in a fluid are at the same depth, they will have the same pressure.

Consider another example: A tube, bent into a U-shape, is partially filled with water (ρ = 1 g/cm^3). We add enough water such the bottom part of the U is completely covered. At this point we expect after equilibrium has been reached that the water will be at the same level on either side of the U. Then, on the left side of tube we add oil (ρ = 0.85 g/cm^3) so that the total height of the oil in the tube is 3 cm as shown in Fig. 7.3. Then, the question is: What is the difference in the water level, x, on either side of the tube?

The weight of the oil on the left side will push down on the water below it until the pressures at positions a and b are equal. Since the pressures at the top of the fluids due to the atmosphere are the same, we start with Pascal's law:

$$\begin{aligned} \Delta P_{\text{oil}} &= \Delta P_{\text{water}} \\ \rho_{\text{oil}} g \Delta h_{\text{oil}} &= \rho_{\text{water}} g \Delta h_{\text{water}} \end{aligned} \tag{7.9}$$

The densities are given for both fluids, and the gravitational constant g cancels on both sides. Then substituting,

$$\begin{aligned} &\frac{0.85\,\text{g/cm}^3}{1.0\,\text{g/cm}^3} 3\,\text{cm} = x \\ &x = 2.55\,\text{cm} \end{aligned} \tag{7.10}$$

We can use Pascal's law to illustrate another idea. Let us consider an elevated container of water, such as a hot-liquor tank, as shown in Fig. 7.4. The container has 1.5 m of water, and it is elevated by 3 m. The container has a pipe leading down to the floor where we have a closed valve. It is important to note that the valve is shut, and the fluid is not moving. We can use Pascal's law to calculate the pressure difference between the top of the water and the location of the valve. Since, the density of water is 1000 kg/m^3,

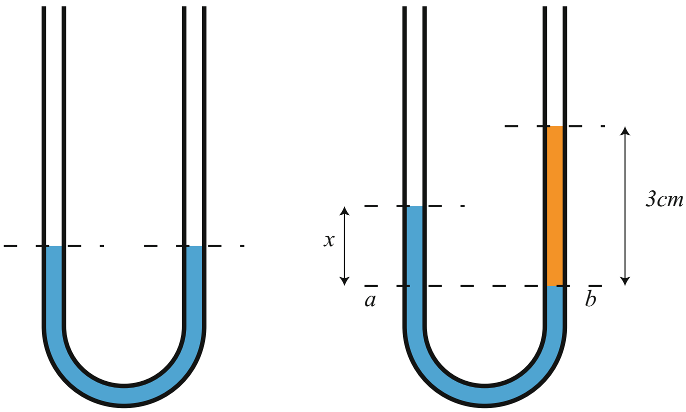


Fig. 7.3 A U-tube is first filled with water and then with oil

$$\begin{aligned} \Delta P &= \rho g \Delta h \\ \Delta P &= 1000\,\text{kg/m}^3 9.8\,\text{m/s}^2 4.5\,\text{m} \\ \Delta P &= 44,100\,\text{N/m}^2 = 44.1\,\text{kPa} \end{aligned} \quad (7.11)$$

where we have used the total height in meters.

In the above example, note that we ignored the atmospheric pressure pressing on the top of the water in the container. Pascal's law only gives the pressure change due to the weight of the fluid. Since there is about 101.3 kPa (14.7 psi) of atmospheric pressure pressing at the top of the water the total, or *absolute pressure,* is 44.1 + 101.3 = 145.4 kPa. We will write this as $P = 145.4\ \text{kPa}_{\text{absolute}}$. In the past, this would have been expressed as 14.7 psia where the "*a*" means absolute. Absolute pressure is measured against a reference pressure of zero, a perfect vacuum.

On the other hand, there is also atmospheric pressure pressing on the outside of the valve. Since it is pressure *differences* that will tend to cause fluids to move (or vessels and pipes to expand), we are frequently only interested in the pressure relative to the outside, or the atmosphere. Thus, the relative pressure, or *gauge pressure,* is always measured relative to the local atmospheric pressure. The pressure measured at the valve in the example will be written as $P = 44.1\ \text{kP}_{\text{gauge}}$. If a pressure measurement does not specify absolute or gauge, then gauge is assumed.

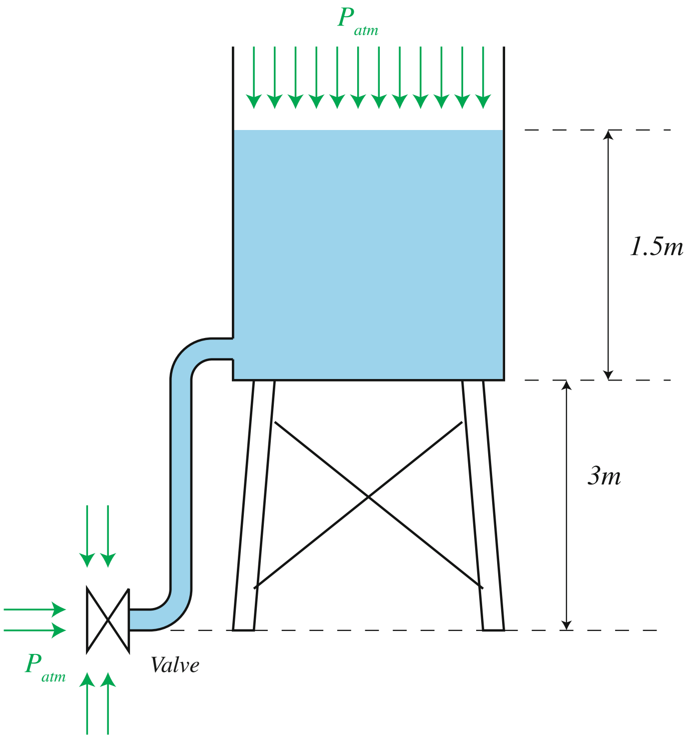


Fig. 7.4 An elevated container of water is connected to a valve

CHECKPOINT 7.1

A famous experiment is attributed to Pascal in which it is said that he inserted a long tube into a wooden barrel. It is reported that the barrel busted after he filled the tube with water. While the accuracy of this tale has not been verified, it is educational to calculate the **force** of the water on the bottom of the barrel. Why calculate force and not just pressure? Because the staves of the barrel must resist this total force to hold the bottom in place, in the same way the piston held up the total weight of the car in Fig. 7.1. Consider a standard-sized bourbon barrel; the ends have diameters of about 50 cm and are 1 m tall. It is said that Pascal used a 10-m tube. Calculate the total force on the bottom of the barrel once the tube is filled with water.

7.3 Fluid Physics: Dynamic Case

Fluid dynamics deals with fluids in motion. The motion of fluids can change the apparent pressure exerted by the fluid. If this were not the case, airplanes would not fly. The physics behind fluid dynamics sets limits on pumping speeds in the brewery, so it is important to understand the underlying physical laws.

In fluid dynamics, there are four basic assumptions made from physics: (i) that mass is conserved when an incompressible fluid encounters a junction or a change in pipe diameter, (ii) that both potential energy and kinetic energy are conserved, (iii) that momentum is conserved, and (iv) that the fluid is a continuum. This last condition basically ignores the fact that a fluid is made of discrete atoms and molecules. For example, liquid water is actually composed of discrete molecules of H_2O separated by empty space. Stating that water is a continuous fluid ignores this extreme microscopic view. Said in yet another way, the mean free path between collisions of neighboring particles in the fluid is very small compared to the size of the pipe or container. Liquids can almost always be treated as continuous; however, this condition fails under certain situations with vapors.

7.3.1 Conservation of Mass: The Continuity Equation

One important law in physics is the conservation of mass. This law states that for a given mass entering a region, we must have a commensurate mass exiting the same region - assuming that the fluid is incompressible. The vernacular "conservation" is borrowed from physics and basically means that it does not change. The flow of a fluid is usually expressed as mass per unit of time, so conservation of mass takes the form algebraically as:

$$\frac{\Delta m_{\text{in}}}{t} = \frac{\Delta m_{\text{out}}}{t}. \tag{7.12}$$

As a simple example, consider a fluid flowing through a pipe which narrows as shown in Fig. 7.5. Since the mass flowing in must equal the mass flowing out, and since mass is density times volume $m = \rho(Ax)$, we can rewrite Eq. 7.12 as:

$$\frac{\rho A_{\text{in}} \Delta x_{\text{in}}}{t} = \frac{\rho A_{\text{out}} \Delta x_{\text{out}}}{t}. \tag{7.13}$$

Recognizing that $\Delta x/t$ is a velocity, the mass flow rate is then $\Delta m/t = \rho A v$. If the density of the fluid does not change, conservation of mass implies

$$A_{\text{in}} v_{\text{in}} = A_{\text{out}} v_{\text{out}} = Q \tag{7.14}$$

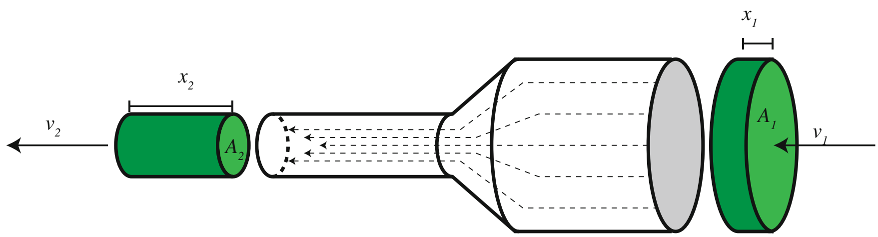


Fig. 7.5 A horizontal section of pipe that changes diameter. The mass flowing into the pipe must equal the mass flowing out of the pipe

where v is the velocity of the fluid through the appropriate cross-sectional area of pipe. The product of an area with velocity gives a volumetric flow rate, customarily symbolized as Q.

For example, consider a level section of 15-cm-diameter pipe that carries water at a rate of 45 L per minute. The pipe tapers to another pipe, 10 cm in diameter. (A) What is the flow rate out of the pipe? (B) What is the velocity of the water at the entrance? and (C) What is velocity of the water at the exit?

(A) Since there are 45 L per minute (L/min) flowing into the pipe, there must also be 45 L per minute flowing out—assuming that the density of the water did not change. It really is as simple as that: *amount in* must equal *amount out*. Note that we expressed this flow rate as a volume per unit of time. The most accurate flow rate should be mass per unit of time (kg/s in SI units), but sometimes weight per unit of time is used in imperial units (such as pounds/minute). If the density does not change, then volumetric flow rate (as in L/min) is acceptable.

(B) Recall the volumetric flow rate is $Q = Av$. We are given this (45 L/min), but we need to be careful with units,

$$Q = 45\,\text{L/min} \cdot \frac{1\,\text{m}^3}{1000\,\text{L}} \cdot \frac{1\,\text{min}}{60\,\text{s}} = 0.00075\,\text{m}^3/\text{s}. \tag{7.15}$$

We need to find the cross-sectional area of the larger pipe before we find velocity. So, since the area of a circle is πr^2, we can determine the area (A) if we know the radius:

$$A = \pi r^2 = \pi (0.075\ \text{m})^2 = 0.0177\,\text{m}^2. \tag{7.16}$$

Then,

$$Q = Av$$
$$0.00075\,\text{m}^3/\text{s} = 0.0177\text{m}^2 \cdot v \tag{7.17}$$
$$v = 0.042\,\text{m/s}$$

(C) The area of the smaller pipe,

$$A = \pi r^2 = \pi(0.05\,\text{m})^2 = 0.00785\,\text{m}^2. \tag{7.18}$$

Thus,

$$A_{\text{in}} v_{\text{in}} = A_{\text{out}} v_{\text{out}}$$
$$0.00075\,\text{m}^3/\text{s} = 0.00785\,\text{m}^2 \cdot v_{\text{out}} \tag{7.19}$$
$$0.096\,\text{m/s} = v_{\text{out}}$$

Notice that the velocity in the smaller pipe is $(15/10)^2 = 2.25$ times the velocity in the larger pipe (without rounding errors).

7.3.2 Bernoulli's Principle and Laminar Flow

Bernoulli's principle is a statement of conservation of energy applied to fluid flow. It considers both potential energy of an elevated static fluid and the kinetic energy of a moving fluid. Bernoulli's principle, named after Daniel Bernoulli a Swiss mathematician and physicist that lived in the 1700s, states that for an increase in speed of a fluid there will be a simultaneous decrease in pressure.

Bernoulli's principle can be derived by applying conservation of energy to a streamline of fluid moving through a system. A streamline is the path that a mass element of the fluid will follow as it moves through the piping system. In this analysis, we make two very important assumptions. First, we assume that friction caused by viscous forces is small. This is an important consideration to keep in mind. As piping systems neck-down to smaller sizes or we are moving viscous fluids such as sweet wort, this assumption may not hold. Secondly, we assume that the density does not change along a streamline. This assumption is more likely to hold for water-based fluids in the brewery since the only thing that will cause a density change is a significant change in temperature.

From physics, the conservation of energy states that work done must equal the change in kinetic and potential energies,

$$W_{\text{NET}} = \Delta K + \Delta U. \tag{7.20}$$

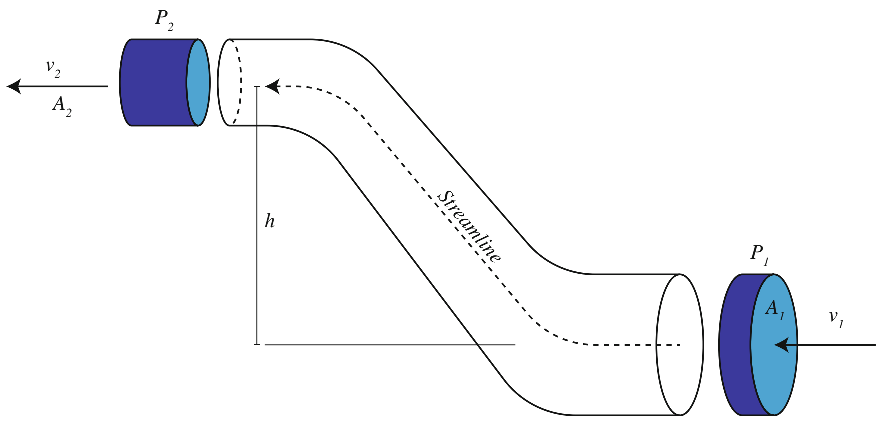


Fig. 7.6 General configuration for Bernoulli's principle

Work done is defined as force times a distance that a certain mass is moved. We will consider a small bolus of mass entering a streamline at point 1 in Fig. 7.6. Thus, the work done moving this mass from point 1 to point 2 can be expressed in terms of the difference in pressure,

$$W = (F_1 \Delta x_1 - F_1 \Delta x_1) = (P_1 A_1 \Delta x_1 - P_1 A_1 \Delta x_1) = (P_1 - P_2)V. \qquad (7.21)$$

The change in kinetic energy is related to the change in speed of the fluid. Recall that kinetic energy is a measure of "energy associated with motion." So,

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho V\left(v_2^2 - v_1^2\right). \qquad (7.22)$$

And, the change in potential energy is given as

$$\Delta U = mgy_2 - mgy_1. \qquad (7.23)$$

Putting these together and rearranging, Bernoulli's principal can be summarized in equation form by

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = \text{constant}. \qquad (7.24)$$

First, notice that each term has units of pressure. Also, since it is only the differences in elevation that matter, we frequently set $h = (y_2 - y_1)$, or alternately let $y_1 = 0$ and $y_2 = h$, the height above the starting point. Now, let us identify different terms in Bernoulli's equation. If we set the velocity to zero at point 1 and point 2, Bernoulli's equation simply reduces to Pascal's law. So the terms $P + \rho\, g\, h$ can be called "static pressure." The other term involving velocity, $\frac{1}{2}\rho v^2$, is called the "dynamic pressure." So in words, Bernoulli's principle simply states that the total pressure is the static pressure, caused by an applied pressure and the weight of the fluid, plus the dynamic pressure caused by a moving fluid.

As an example, consider a Venturi tube in which water (density ρ = 1000 kg/m^3) is flowing through a pipe that gradually tapers to a smaller size. Let us say that the larger pipe has a radius of 3 cm and the smaller pipe has a radius of 1 cm and that the fluid is forced through the larger pipe at a velocity v_1 = 10 m/s (this is quite fast, ~22 mph). We wish to calculate the pressure difference between the two regions. Given the statement of conservation of mass, the fluid *must* travel faster through the smaller region. Since we need it, we first calculate the speed in the smaller pipe. Following the example above, the speed can be found from

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ \pi\, 3^2 \cdot 10\,\text{m/s} &= \pi\, 1^2\, v_2 \\ 90\,\text{m/s} &= v_2. \end{aligned} \tag{7.25}$$

Since the two sections are at the same level, $h = (y_2 - y_1) = 0$, we do not need to worry about those terms in Bernoulli's equation. We start with

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2. \tag{7.26}$$

Rearranging,

$$\begin{aligned} P_1 - P_2 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \\ P_1 - P_2 &= \frac{1}{2}1000\,\text{kg/m}^3\left(90^2 - 10^2\right)(\text{m/s})^2 \\ P_1 - P_2 &= 4{,}000{,}000\,\text{Pa} \approx 580\,\text{psi} \end{aligned} \tag{7.27}$$

This calculation means that the pressure at the smaller pipe is about 4000 kPa *less* than the pressure at the entrance. Said in another way, the kinetic energy of the fluid increases at the expense of the pressure. Note that if the working fluid was air (ρ = 1 kg/m^3) instead of liquid, the pressure difference would be a thousand times less than that for water, but P_1 is still greater than P_2.

CHECKPOINT 7.2

A *pitot tube* (named after Henri Pitot, pronounced "pee-toe") is a device for measuring velocity of a fluid. These devices are generally used in aircraft to measure airspeed but can be adapted to measure liquid flow in the brewery. Air, with density ρ_{air} = 1 kg/m^3, is made to flow past two points of a tube. Often, these points on the tube are housed in a larger, main tube as shown in Fig. 7.7. One point of the tube is directed into the air stream; thus, the air stagnates at this point and its velocity is zero. This point registers pressure P_2. At the other point, the air stream is allowed to flow past the tube opening unobstructed. This point registers pressure P_1.

(A) If the airspeed is 35 m/s, find the pressure difference $P_2 - P_1$.
(B) These two points are connected to a U-tube filled with water, $\rho_{\text{water}} = 1000\ \text{kg/m}^3$. Based on the pressure difference found in part A, find the height difference, h, of the water.
(C) Now let us work this backward. If the height difference is $h = 5$ cm, determine the air velocity in the main tube.

7.3.3 Pressure and Hydraulic Head

Bernoulli's equations suggest that there is a flow speed for which pressure could be zero or even negative. In normal situations, it is not possible for fluids to have negative pressure so it is apparent that Bernoulli's equations are not valid in this regime. Yet, it is instructive to think about this situation. Recall the water tower in Fig. 7.4. With the water level used in that example, we found that the static pressure at the valve is about $P = 44.1\ \text{kPa}_{\text{gauge}}$ or $P = 145.4\ \text{kPa}_{\text{absolute}}$ given an approximate atmospheric pressure of $101.3\ \text{kPa}_{\text{absolute}}$. Now let us open the valve and consider the flow velocity with different back-pressures on the other side of the valve. To do this using Bernoulli's equation, we will assume that the volume of

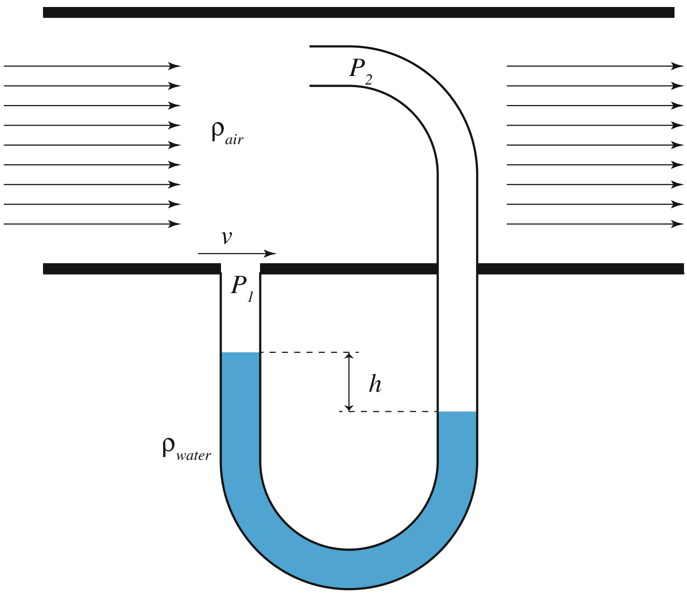


Fig. 7.7 A typical pitot tube

water in the tank is so large that the water level does not change—the velocity of the water at the top of the tank is zero. Inserting this situation into Bernoulli's equation,

$$\begin{aligned} P_{\text{top}} + \rho g h &= P_{\text{valve}} + \frac{1}{2}\rho v_{\text{valve}}^2 \\ 101.3\,\text{kPa} + 44.1\,\text{kPa} &= P_{\text{valve}} + \frac{1}{2}\rho v_{\text{valve}}^2. \end{aligned} \tag{7.28}$$

So, if the back-pressure at the valve is P_{valve} = 145.4 $\text{kPa}_{\text{absolute}}$, then the flow velocity at the valve is zero. If we lower the back-pressure to, say, P_{valve} = 80 $\text{kPa}_{\text{absolute}}$, then the velocity is given as

$$\begin{aligned} &101.3\,\text{kPa} + 44.1\,\text{kPa} = 80\,\text{kPa} + \tfrac{1}{2}1000\,\text{kg/m}^3 v^2 \\ &65.4\,\text{kPa} = 500\,\text{kg/m}^3\, v^2 \\ &11.44\,\text{m/s} = v \end{aligned} \tag{7.29}$$

This relationship also suggests that there is an upper limit to the velocity through the valve, since there is a lower limit on the pressure at this point. Setting P_{valve} = 0, we get an upper limit for a fluid velocity of v_{valve} = 17.1 m/s. Installing a pump at the valve and attempting to pump faster than this will lead to cavitation at the pump. *Cavitation* and its deleterious effects are discussed later, but note in this example that pipe sizes and restrictions were not considered. Considering real pipes with real restrictions will only serve to lower the maximum velocity and exacerbate the problem.

So, how could we increase the *speed* (not to be confused with volumetric flow rate) of the fluid in this example? The only possible way is to increase "*hydraulic head"* at the valve by either increasing the height of the water level, and/or increasing the pressure on top of the water. In fluid dynamics, it is customary to rewrite Bernoulli's equation as

$$q + \rho g h' = \text{constant} \tag{7.30}$$

where $q = \frac{1}{2}\rho v^2$ is the dynamic pressure, and

$$h' = h + \frac{P}{\rho g} \tag{7.31}$$

is the hydraulic head which is due to the height, h, of the fluid above a given point plus the pressure head on the fluid. In this example, it is very important to distinguish between speed of the fluid (in m/s) and volumetric flow rate (m^3/s). Hydraulic head will place an upper limit on the speed of a fluid through a pipe. But, if we want a greater volumetric flow rate, all we have to do is to make the pipe larger.

7.3.4 Head and Pump Dynamics

Pumps are used to transfer fluids around the brewery. The example surrounding the water tank used gravity to transfer liquid to a lower level. But many times we are trying to lift fluids to a higher elevation, such as transferring hot water from the hot-liquor tank to a sparging vessel. Here, we will explore some of the relevant issues surrounding pumping system design. Much of what is presented here will draw on and apply the fundamental fluid dynamics discussed above. We will first consider the model system as shown in Fig. 7.8 where we are drawing fluid from below the pump, and transferring to a higher tank. In this discussion, we will introduce some of the vernacular associated with pump installations.

Suction head refers to the distance below the pump from where we are drawing fluid. This distance will have special meaning, which will be explained later. For the moment, let us apply Bernoulli's principle to this section of the pumping system and explore some of the physical constraints of this system.

The two points in our system that we consider in applying Bernoulli's equations are (1) the top of the liquid in the lower tank, and (2) the pump inlet. As we have done before, we will assume that the tank is large enough that the velocity of the fluid in the tank is very small. So, the only relevant term in Bernoulli's equation is the atmospheric pressure pressing on the top of the fluid. The pump inlet is of some height h_1 about the liquid level and we will assume some arbitrary pressure and fluid velocity here. So, set up Bernoulli's equation as

$$101.3\,\text{kPa} = P_{\text{inlet}} + \rho g h_1 + \frac{1}{2}\rho v_{\text{inlet}}^2. \tag{7.32}$$

Now we will consider some extremes in this situation. First consider the condition with the lowest possible pump inlet pressure $P_{\text{inlet}} = 0$ and a fluid velocity $v_{\text{inlet}} = 0$. Granted, this is not a very useful situation in practice. With zero velocity, we are not transferring any fluid; however, this example gives us the maximum height, h_1 (i.e., maximum suction head), that is possible with normal atmospheric pressure. Inserting the numbers,

$$\begin{aligned} 101.3\,\text{kPa} &= 1000\,\text{kg/m}^3\ 9.8\,\text{m/s} \cdot h_1 \\ 10.34\,\text{m} &= h_1 \end{aligned} \tag{7.33}$$

The above example is an important illustration of basic fluid physics. The movement of fluid is caused by a pressure difference. The pump lowers the pressure at its inlet, so that the atmospheric pressure can push it in. It is technically incorrect to say that the pumps suck the fluid into the pump, although we may say this in informal discussion.

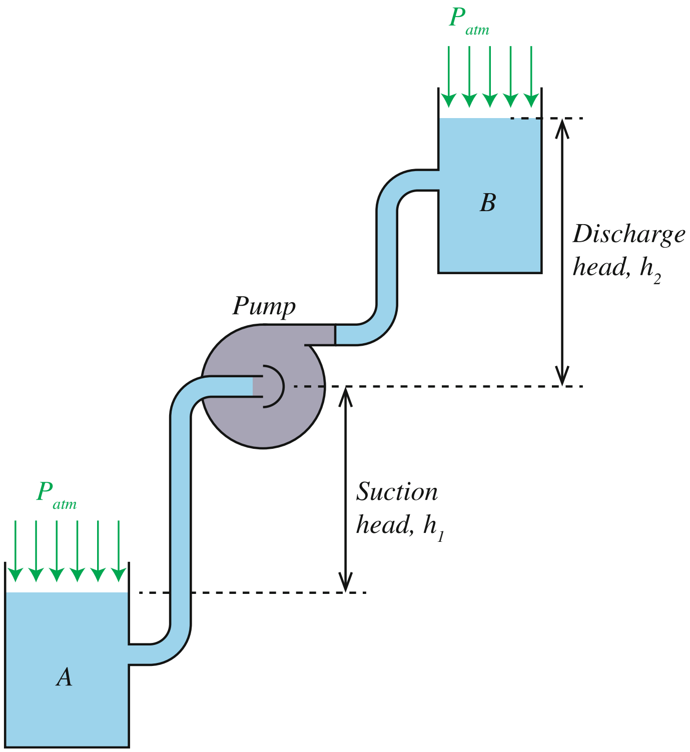


Fig. 7.8 Model pump system to illustrate pump dynamics

> **CHECKPOINT 7.3**
> Determine the suction head and pump inlet pressure in Fig. 7.8 that gives the maximum possible fluid velocity at the pump. What is this velocity?

Discharge head, h_2 in the figure, is the distance above the pump that we are discharging the fluid. Note that this distance is from the pump to the actual, upper fluid level—not just where the pipe ends. Again, we will discuss discharge head in a greater context, but for now we will use this distance to again explore what Bernoulli's equations imply. Here we look at the two points: (1) pump outlet and (2) the top of the highest point of the fluid at discharge. Using arbitrary pressures and velocities at the pump as before

$$P_{\text{outlet}} + \frac{1}{2}\rho v_{\text{outlet}}^2 = 101.3\,\text{kPa} + \rho g h_2. \tag{7.34}$$

As we will see in a moment, it somewhat misleading to ask what minimum pressure at the pump outlet is required to move the fluid. Inserting a velocity

$$\begin{aligned} v_{\text{outlet}} &= 0, \\ P_{\text{outlet}} &= 101.3\,\text{kPa} + \rho g h_2, \end{aligned} \tag{7.35}$$

which is essentially Pascal's law. The minimum pressure depends on the final height of the elevated fluid. So, for example, if the height is h_2 = 4.5 m as in our previous examples, then P_{outlet} = 145.4 kPa. What is the pressure at the pump outlet that will give maximum velocity? We will get our maximum velocity when the pressure at the pump outlet is zero! Inserting the numbers

$$\begin{aligned} P_{\text{outlet}} + \frac{1}{2}\rho v_{\text{outlet}}^2 &= 101.3\,\text{kPa} + \rho g h_2 \\ 0 + \frac{1}{2} \cdot 1000\,\text{kg/m}^3 \cdot v_{\text{outlet}}^2 &= 145.4\,\text{kPa} \\ v_{\text{outlet}} &= 17.1\,\text{m/s} \end{aligned} \tag{7.36}$$

This example illustrates what Bernoulli's equation really means. Recall that Bernoulli's equation was derived based on energy. In this example, the kinetic energy of the fluid is converted to potential energy at the final fluid height. When fluid is moving, the pressure must necessarily drop. This example **does not** imply that if we put a (static) zero pressure at the outlet that we will get maximum velocity. It is important to realize here that the pump is doing work and adding energy to the system. This work shows up as either kinetic energy of the fluid, or as an incremental work (related to pressure), or a mixture of the two. In the end, however, the final energy of the fluid is simply potential energy.

At the end of the day, the pump's job is to transfer fluid. In the simplest of terms using the smallest amount of energy, the fluid starts at rest, moved to a higher elevation, and ends at rest. The energy put into the system, or the work done by the pump, simply changes the potential energy of the fluid. The total change in potential energy depends only on the net change in height. So, we define the *total static head* as the sum of the suction head plus the discharge head

$$\text{Total static head} = \text{suction head} + \text{discharge head}. \tag{7.37}$$

The total change in potential energy of the fluid can be expressed as

$$\Delta PE = mgh_1 + mgh_2 \tag{7.38}$$

where h_1 and h_2 are the suction and discharge head, respectively. Note also that the term mg is the weight of the fluid we are moving. It is customary in fluid engineering to express this in terms of the specific weight,

$$\gamma = \frac{mg}{V} \tag{7.39}$$

the weight per unit volume; $\gamma_{\text{water}} = 9800\,\text{N/m}^3$. So, the change in potential energy will occasionally be reorganized to look like

$$\Delta\text{PE} = \gamma V(h_1 + h_2). \tag{7.40}$$

Also, if the level of the suction head is above the pump, we would subtract h_1; the only thing that matters is the net difference in elevation between the starting and ending points.

As an example, consider how much energy is required to lift one cubic meter of water. Let us assume we are using a pump with a suction head of 1.2 m and a discharge head of 3.2 m. So, the total static head is 4.4 m. Recalling the definition of work and potential energy, the work done is

$$\begin{aligned} W &= mgh \\ &= \rho V g h \end{aligned} \tag{7.41}$$

Inserting numbers

$$\begin{aligned} W &= 1000\,\text{kg/m}^3 \cdot 1\,\text{m}^3 \cdot 9.8\,\text{m/s}^2 \cdot 4.4\,\text{m} \\ W &= 43,120\,\text{kg m/s}^2 = 43,120\,\text{J}. \end{aligned} \tag{7.42}$$

where a kg m/s^2 is the unit known as a Joule (J).

Now, if we use a pump that is rated at one horsepower, how much time will this take? The horsepower (h.p.) is traditional measure of work per unit of time for pumps and has the definition 1 h.p. = 745.7 W. To solve this, we use the definition of power (work divided by time),

$$P = \frac{W}{t} = \frac{43,120\,\text{J}}{t} = 745.7\,\text{W}, \tag{7.43}$$

and then solve for time, t = 57.8 s. So, in conclusion, a one horsepower pump can transfer one cubic meter of water vertically up by 4.4 m (about 14 ft) in about a minute—this is the minimum energy required.

This example is somewhat misleading in that it assumes the fluid starts at rest and ends at rest. However, if we are actually trying to fill a container in a given amount of time, we need to consider giving the fluid a bit more energy in the form of kinetic energy. Kinetic energy is the energy associated with motion and expressed as

$$\text{KE} = \frac{1}{2} m v^2 \tag{7.44}$$

Rather than specifying the final velocity, or the energy put into the system, it is customary to express the kinetic energy as a "velocity head," in units of height. Essentially, this means expressing the height that gives an equivalent amount of potential energy as the kinetic energy

$$\frac{1}{2}mv^2 = mgh_\mathrm{v}. \tag{7.45}$$

where h_v is the "*velocity head.*" Expressing energy in this way simplifies calculations since changes in elevation are already expressed in height.

Another customary simplification is to approximate friction losses as equivalent losses in head. Friction losses are often tabulated this way for different types and sizes of fittings, and even for the pipes themselves. So, one might consult a table for a given set of fittings in the application. These tables are easily found in the Internet by searching for "friction head table."

The total amount of energy required to move fluid from one point to another includes changes in potential energy due to elevation, the final kinetic energy of the fluid at the end point (assuming the fluid starts from rest), and any friction losses along the way. So, *Total Dynamic Head* captures all of this information

$$\begin{aligned}&\text{Total Dynamic Head}\\&\quad = \text{suction head} + \text{discharge head} + \text{friction head} + \text{velocity head}\\&\quad = (mgh_1) + (mgh_2) + (mgh_\mathrm{f}) + (mgh_\mathrm{v})\end{aligned} \tag{7.46}$$

This can be viewed as the total energy required to move the fluid. And by rearranging the equation, we get:

$$E_\mathrm{total} = mg(h_1 + h_2 + h_\mathrm{f} + h_\mathrm{v}) = \lambda V(h_1 + h_2 + h_\mathrm{f} + h_\mathrm{v}). \tag{7.47}$$

As a final example, consider the model system shown in Fig. 7.9; note that the discharge arrangement is somewhat different than in Fig. 7.8. Let us assume that we are moving water and that the suction head is h_1 = 3.5 m, the discharge head is h_2 = 5 m, and we want to move water at a rate of 3.2 L/s = 0.0032 m^3/s (about 51 gallons per minute) into the upper tank. It is a simple system using 5-cm ($\sim$2 in)-diameter pipe, and both the suction piping and the discharge piping have one 90° elbow. Accounting friction losses, the goal is to determine the minimum horsepower rating for a pump.

The first step in this analysis is to determine the velocity of the water. Again, the flow rate is equal to the product of the cross-sectional area of the pipe and the velocity, so since we know the flow rate (Q = 0.0032 m^3/s) and the radius of the pipe (diameter = 5 cm; radius (r) = 2.5 cm = 0.025 m), we can solve for the velocity of the fluid in the pipe:

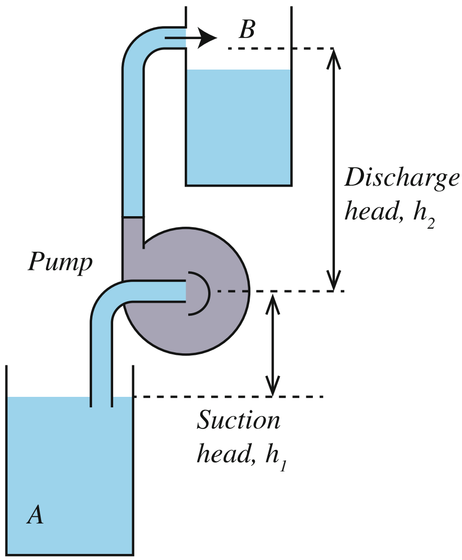


Fig. 7.9 Final example system that uses all concepts

$$
\begin{aligned}
Q &= Av \\
0.0032\,\mathrm{m}^3/\mathrm{s} &= \left(\pi(0.025\,\mathrm{m})^2\right)\cdot v \\
1.63\,\mathrm{m/s} &= v
\end{aligned}
\tag{7.48}
$$

This means that we need a velocity head of

$$
\begin{aligned}
\frac{1}{2}mv^2 &= mgh_\mathrm{v} \\
\frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}g} = \frac{v^2}{2g} &= h_\mathrm{v} \\
\frac{(1.63\,\mathrm{m/s})^2}{2\cdot 9.8\,\mathrm{m/s}^2} &= h_\mathrm{v} \\
0.136\,\mathrm{m} &= h_\mathrm{v}
\end{aligned}
\tag{7.49}
$$

To calculate the friction losses, we consult Tables 7.1 and 7.2. Each elbow will add 2.7 m of equivalent pipe for friction purposes. So, adding the suction head and discharge head with this yields 13.9 m of pipe. Table 7.1 indicates that there will be a 0.394 m loss of head for every 10 m of pipe at 3 L/s (the closest entry to our problem). So we simply scale our result to find the friction head

$$
h_\mathrm{f} = \frac{0.394\,\mathrm{m}}{10\,\mathrm{m}}\cdot 13.9\,\mathrm{m} = 0.548\,\mathrm{m}. \tag{7.50}
$$

Table 7.1 Head loss (in meters) per 10 m of 5 cm diameter (~2 in) high-density polyethylene (HDPE) pipe moving clear water

Liters per second	Head loss (m)
0.1	0.002
0.5	0.016
1.0	0.051
1.5	0.107
2.0	0.182
3.0	0.394
5.0	1.056
10	4.112

Table 7.2 Equivalent length of pipe in meters caused by a 5 cm (~2 in) joint

90° Elbow	2.7
T Branch	5.2

The total dynamic head is then the sum of the suction, discharge, velocity, and friction head. Then, the total dynamic head for this problem is: Total Dynamic Head = 5 m + 3.5 m + 0.548 m + 0.136 m = 9.184 m. We can then find the power required by multiplying by the specific weight. Note that we also recognize that the volumetric flow rate is volume per unit of time $Q = V/t$,

$$\begin{aligned} P &= \frac{E}{t} = \frac{\lambda V(h_1 + h_2 + h_{\mathrm{f}} + h_{\mathrm{v}})}{t}. \\ P &= 9800\,\mathrm{N/m^3} \cdot 0.0032\,\mathrm{m^3/s} \cdot (9.184\,\mathrm{m}) = 288\,\mathrm{W} \\ P &= 288\,\mathrm{W} \cdot \left(\frac{1\,\mathrm{h.p.}}{745.7\,\mathrm{W}}\right) = 0.386\,\mathrm{h.p.} \end{aligned} \tag{7.51}$$

Earlier, we briefly touched on the fact that the pressure at the inlet of a pump cannot be less than zero; otherwise, a condition called *cavitation* will occur. Let us look at this again armed with more background knowledge. When the total pressure, or more correctly the total energy, of the fluid drops below a certain point, it will flash into a vapor—usually little bubbles of vapor. This will most likely happen at the pump inlet where the pressure is the lowest. As the vapor travels through the pump and then encounters higher pressures, the bubbles implode with enough force to pit the metal of the pump impeller. To prevent this, we want the pressure at the pump inlet plus the dynamic pressure (the kinetic energy, velocity term in Bernoulli's equation) to be greater than the vapor pressure of the liquid. The vapor pressure of a liquid is the minimum pressure to keep it in a liquid state. So, we want

$$P_{\mathrm{inlet}} + \frac{1}{2}\rho v_{\mathrm{inlet}}^2 \geq P_{\mathrm{vp}}. \tag{7.52}$$

We can rearrange this to set the variables on the left-hand side of the greater than or equal to sign

$$P_{\text{inlet}} + \frac{1}{2}\rho v_{\text{inlet}}^2 - P_{\text{vp}} \geq 0. \tag{7.53}$$

The *Net Positive Suction Head* (NPSH) is the equivalent height (i.e., head) of liquid above the inlet at the pump required to avoid cavitation. It is defined as

$$\text{NPSH} = \frac{P_{\text{inlet}} + \frac{1}{2}\rho v_{\text{inlet}}^2 - P_{\text{vp}}}{\rho g}. \tag{7.54}$$

NPSH is a characteristic of a particular pump—each pump has requirements for the amount of liquid that must be fed into it in order to run without cavitating. If the liquid comes into the pump too slowly or without enough pressure, the pump will not work. Since the pressure at the inlet depends on the suction head, the atmospheric pressure pushing the liquid from the suction tank, and friction losses for the pipe leading to the pump, NPSH can be expressed as

$$\text{NPSH} = h_{\text{atm}} \pm h_{\text{s}} + h_{\text{v}} - h_{\text{f}} - h_{\text{vp}}, \tag{7.55}$$

where

- h_{atm} is the equivalent liquid height for the atmospheric pressure (10.34 m of water in the earlier example),
- h_{s} is the suction head above (use the +) or below (use the −) the pump inlet,
- h_{v} is the velocity head,
- h_{f} is the friction head, and
- h_{vp} is the equivalent liquid height for the vapor pressure.

It is important to realize that the vapor pressure is dependent on temperature of the fluid. For example, water at 10 °C (50 °F) h_{vp} is just 0.12 m. But at 70 °C (160 °F) a head of h_{vp} = 3.4 m is required to prevent the water from flashing into vapor.

Cavitation leads to excessive noise and vibration in the pump. If the pump operates in this condition for an extended length of time, the pump impeller will begin to degrade and pits will form in the metal parts of the impeller. The violent collapse of vapor bubbles forces liquid at high velocity into the pores of the metal. The forces due to the imploding bubble can exceed the tensile strength of the metal and actually blasts out bits of metal giving the impeller a pitted appearance. This looks like corrosion to the naked eye and is often talked about at the same time as corrosion, but the pits in the metal result from a very different process. Cavitation also causes excessive vibration leading to bearing failure, shaft breakage, and other fatigue-related failures in the pump. And let us not forget the key product of cavitation, the liquid itself. Excessive cavitation can heat up the liquid and potentially cause wort to brown via Maillard reactions, or result in degassing beer entirely.

CHECKPOINT 7.4
Calculate the NPSH for the example shown in Fig. 7.9 assuming 70 °C water.

7.3.5 Darcy's Law and Laminar Flow in Porous Media

Darcy's law was originally developed for hydrology problems, for example, groundwater moving through sand. This law addresses the resistance that a sand bed offers to the flow of water through it. It can also be loosely applied to the flow of sparge water through a grain bed. In other words, Darcy's law gives a very good approximation of the sparging process. The law basically relates the flow rate Q to the physical *dimensions* of the bed, the physical *properties of the bed*, and the physical *properties of the fluid* moving through it. The main idea behind Darcy's law is virtually identical to the idea behind resistance in electricity (Ohm's law) and heat conduction through materials (Fourier's Law).

Before we get into the details of Darcy's law, it is useful to consider a mechanical analogy to the problem: the *Plinko* game. In the game of *Plinko*, users drop "chips" onto a board filled with regularly space pegs. Gravity provides the driving force pulling the chips down, but regularly spaced pegs momentarily stop the chips. Through collisions with the pegs, the chips will take a more-or-less random path through the array of pegs. The angle of board will vary the gravitational force pulling the chips. (A *Plinko* game with a horizontal board would not work.) The maximum possible "flow rate," i.e., number of chips per second through the board, will depend on how wide the board is, how long (how many total pegs), and the density of pegs.

The flow of the *Plinko* chips in this analogy accurately models: (1) flow of water through a permeable substance such as sand, (2) flow of charges through a conductor, and (3) flow of energy through a heat-conductor such as metals. In each case, there is a driving force trying to move something. But due to constant collisions with the bulk material, the flow is restricted. These three examples also have something else common: They can be modeled as a diffusive system. If we put a *source* of water, or a *source* of charges, or a *source* of heat in a system, then the water, or charges, or energy will tend to diffuse away from this source. This is one aspect that is a bit more difficult to see with the *Plinko* analogy. Image dumping a large number of *Plinko* chips at the top of the board in one single location. Due to random collisions with the pegs and other chips, the chips will tend to spread out as they make their way through the board: diffusion.

In diffusion problems such as these, it is customary to discuss the dynamics of the system in terms of a *potential*, rather than a driving force. A "potential" is closely related to potential energy, but certain material properties of the system under study have been separated. In electrostatics, the potential energy of a charge in an electric field is given by

$$\mathrm{PE} = qV, \tag{7.56}$$

where q is the charge, and V is the potential in volts. We say that charges tend to "flow" from high potential to low potential. In hydrodynamics, the potential energy of a bolus of liquid is given by

$$\mathrm{PE} = mgh, \tag{7.57}$$

where m is the mass of the liquid, g is the gravitation constant, and h is now called the "potential." We have already been introduced to this earlier—this is simply the head. So, using the "potential" vernacular, water flows from higher potential (i.e., head) to lower potential (lower head) (Fig. 7.10).

Ohm's law relates the flow of charges through a conductor and is the electrical analog to Darcy's law. The conduction of charges through a conductor is virtually identical to the *Plinko* analogy. The nuclei of the bulk material represent the pegs that the charges (chips) continuously collide. The average velocity, "drift velocity," of the charges is very small, and flow rate is largely determined by the "chip-peg" collisions. An applied electric field will try to move the charges in the same way as gravity tries to move the chips, but collisions with nuclei limit the velocity to a very small value. Overall, and from a macroscopic point of view, the flow *rate* of the charges can be expressed, in words, as

$$\frac{\text{charge}}{\text{time} \cdot \text{area}} = \text{constant}\left(-\frac{\text{difference in potential}}{\text{difference in distance}}\right) \tag{7.58}$$

The minus sign is there indicating that flow is from higher potential (volts) to lower potential. For those that are interested, the full form of Ohm's law is given as:

$$\frac{I}{A} = -\sigma \frac{\Delta V}{\Delta x}. \tag{7.59}$$

Here, I is the current in Amps, A is the cross-sectional area that the current is going through, σ is the conductivity of the material, and $\Delta V/\Delta x$ is the change in potential (in volts) per distance applied across the conductor.

Darcy's law is conceptually identical to Ohm's law. We can use the *Plinko* game, and lessons learned from electrostatics to model fluid flow through a resistive media, such as water through a sand bed or grain bed. The driving force for fluid through a bed will either be gravity or a pressure difference. In either case, we can say that fluid flows from higher potential to a lower potential—i.e., head.

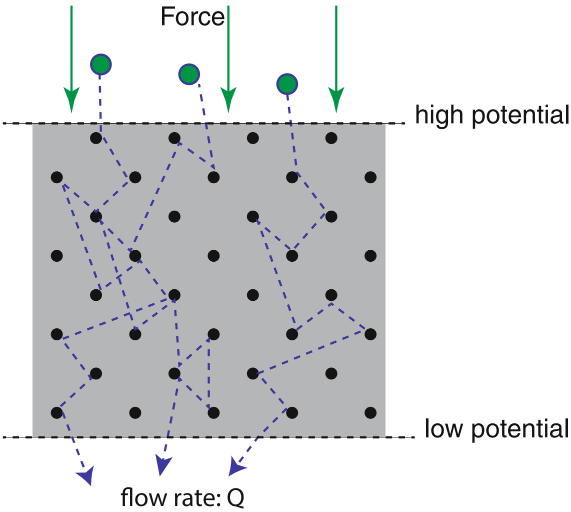


Fig. 7.10 "Flow" of *Plinko* disks through a peg board

Furthermore, we can say, like Ohm's law, that the fluid flow rate is proportional to material-specific properties, pressure differences, and geometry of object. The only difference in concept between Darcy's law and Ohm's law is that we are moving volumes of liquid instead of charges. Thus,

$$\frac{Q}{A} = -\frac{\kappa}{\mu}\frac{\Delta h}{\Delta x}. \tag{7.60}$$

where

Q	is the flow rate (volumes per second)
A	is the cross-sectional area, and
$\Delta h/\Delta x$	is the change in potential (head) applied across a sand or grain bed,
κ	is the intrinsic permeability of the bed (similar to σ in Ohm's law), and
μ	is the viscosity of the fluid through the bed.

It can be argued that modeling the flow of water through a sand bed, as in an aquifer, is not the same thing as modeling flow of sparge water through a grain bed. First, Darcy's law and the hydrodynamics behind it as applied to permeable materials assume that the bulk material is not compressible. It assumes that the permeability of the bed does not change. This is certainly not absolutely true for a grain bed during sparging. Secondly, Darcy's law assumes that the viscosity is also a constant throughout the flow—also not true in a sparging situation. As water rinses away sugars from the grain bed, the viscosity of the fluid decreases.

With all of this in mind, this does not mean that Darcy's law is useless in the sparging problem. As with virtually every scientific law or principle, there are limits of applicability. The laws and principles, however, are very useful in determining overall behavior and approximating the problem at hand. In the case of sparging, the changing bed permeability and wort viscosity would make this problem virtually impossible to solve accurately without a huge number of sensors in the grain bed and a computer program. But, we can still learn about the overall "big-picture" behavior by exploring Darcy's law and the associated hydrodynamics.

Let us examine Darcy's law with the use of a spreadsheet on a computer. Doing so will allow us to approximate the flow of liquid through a grain bed to explore changes to the bed and to the flow. The setup will require a little bit of math to show how to build the spreadsheet. But, once we have set it up, we can play with different situations and see how the sparging system behaves quite easily.

To start, we introduce two new variables as a form of shorthand. First, the specific flow rate (q),

$$q = \frac{Q}{A}, \tag{7.61}$$

is the volumetric flow rate normalized to the area that it goes through. Second, we replace the ratio of permeability to viscosity with the variable k,

$$\frac{\kappa}{\mu} = k \tag{7.62}$$

Finally, we extend Darcy's law to two dimensions as

$$q_x = -k\frac{\Delta h}{\Delta x} \quad q_y = -k\frac{\Delta h}{\Delta y}. \tag{7.63}$$

In words, q_x is the specific flow rate in the x dimension and q_y is the specific flow rate in the y dimension. These flow rates are related to the degree of the potential (head) in different directions. In other words, if the flow goes through the bed at an angle, we break the flow down into the movement in the x and y dimensions (Fig. 7.11). Extending the third dimension is straightforward, but we will limit our discussion to two dimensions.

Now let us consider a small "square" of bulk material: the grains in our lauter tun. Darcy's law will tell us the flow rate through this square, both in the x and y directions and how it depends on some arbitrarily applied potential difference. Applying the principles of the conservation of volume flow rate implies

$$\frac{\Delta q_x}{\Delta x} + \frac{\Delta q_y}{\Delta y} = 0 \tag{7.64}$$

where $\Delta q_x = q_{x,\text{out}} - q_{x,\text{in}}$ is the difference between what flows in and out of the square in the x direction, and a similar definition for Δq_y. In other words, if there is no source of q in the square (if no liquid is being added at that point), then what flows in must flow out. Inserting Darcy's law into Eq. 7.64 gives

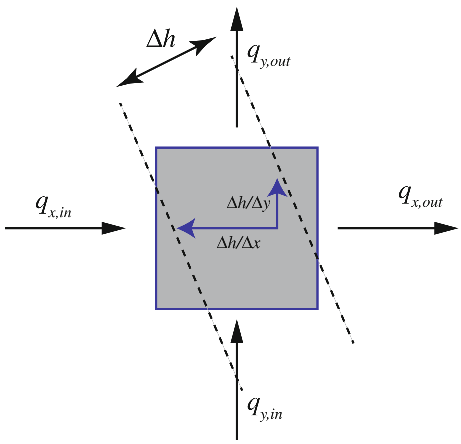


Fig. 7.11 Model square "volume" of the bulk material with an applied potential difference

$$\frac{\Delta}{\Delta x}\left(k\frac{\Delta h}{\Delta x}\right)+\frac{\Delta}{\Delta y}\left(k\frac{\Delta h}{\Delta y}\right)=0 \tag{7.65}$$

If there is a source S of specific flow into our little square, then we can modify this to[1]

$$-\frac{\Delta}{\Delta x}\left(k\frac{\Delta h}{\Delta x}\right)-\frac{\Delta}{\Delta y}\left(k\frac{\Delta h}{\Delta y}\right)=S \tag{7.66}$$

While that is a LOT of Δ's, we are now in a position to build a spreadsheet that will calculate the flow through a bed of grain. Consider the four nearest neighbors to our little square, to the North, South, East, and West as shown in Fig. 7.12. Each nearest neighbor square will be allowed to have its own potential (head) and constant k. This second allowance will let us play with the possibility of "channeling" through the grain bed. What we need to do now is approximate the differences as differences in values in neighboring cells.

CHECKPOINT 7.5

Approximate the difference in potential on the right-hand side of our square as

$$\left(\frac{\Delta h}{\Delta x}\right)_{\text{right}}=\frac{(h_{\text{E}}-h)}{\Delta x},$$

with h_{E} the potential in the East cell and similar definitions for the West, North, and South cells. Insert these into

[1]For those who enjoy a good calculus problem, the full, time-independent expression in three dimensions will look like $-k\nabla^2 h = S$.

$$\frac{\Delta}{\Delta x}\left(k\frac{\Delta h}{\Delta x}\right)+\frac{\Delta}{\Delta y}\left(k\frac{\Delta h}{\Delta y}\right)=0$$

and solve for the potential h in our square. You should show that the potential in our square is simply the weighted average

$$h=\frac{k_\mathrm{N}h_\mathrm{N}+k_\mathrm{S}h_\mathrm{S}+k_\mathrm{E}h_\mathrm{E}+k_\mathrm{W}h_\mathrm{W}}{k_\mathrm{N}+k_\mathrm{S}+k_\mathrm{E}+k_\mathrm{W}}.$$

What assumptions did you have to make? Repeat the derivation, but now consider the possibility of a source S in our square. What happens if all of the k's are the same everywhere?

After working the above checkpoint, we see that the head at a given location is just the weighted average of the heads at the nearest neighboring cells. This will be easy to build a spreadsheet to make these calculations. The only issue is deciding on what to do at the boundaries of the model system.

To illustrate the process, we start by modeling a very simple system where a constant head of 10 (arbitrary units) is applied to the top of a container of permeable material. This container has one exit, and we assume that a constant head of 0 is applied here. This will model our lauter tun, filled with a constant layer of liquid above the grain bed with a single pipe exiting to the atmosphere. An example spreadsheet, with a very coarse grid, is shown in Fig. 7.13. The positions of constant head are identified in yellow and these cell values are fixed. In the bulk of the

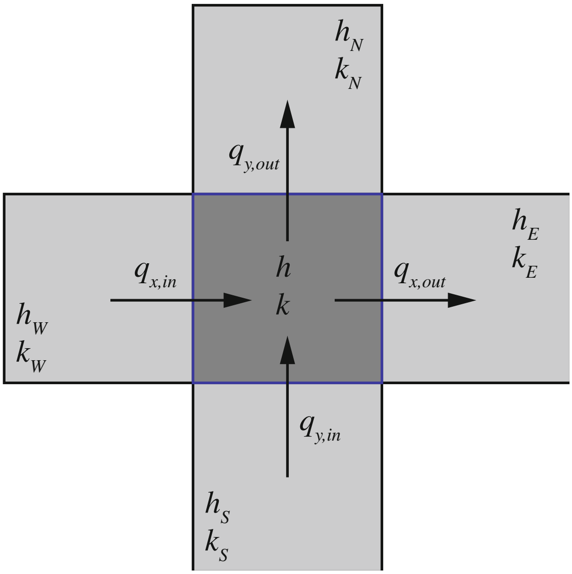


Fig. 7.12 A system of discrete squares to approximate fluid dynamics in a permeable media

	A	B	C	D	E
1	10	10	10	10	10
2	=B2	=(B1+C2+A2+B3)/4	=(C1+D2+B2+C3)/4	=(D1+E2+C2+D3)/4	=D2
3	=B3	=(B2+C3+A3+B4)/4	=(C2+D3+B3+C4)/4	=(D2+E3+C3+D4)/4	=D3
4	=B4	=(B3+C4+A4+B5)/4	=(C3+D4+B4+C5)/4	=(D3+E4+C4+D5)/4	=D4
5	=B5	=(B4+C5+A5+B6)/4	=(C4+D5+B5+C6)/4	=(D4+E5+C5+D6)/4	=D5
6	=B6	=B5	0	=D5	=D6

Fig. 7.13 Spreadsheet formulas to model a simple lauter tun with (i) constant head applied at the top—row 1 contains the same values, (ii) a single exit to the atmosphere at the bottom—cell C6 with a value of 0, (iii) constant permeability and viscosity everywhere, and (iv) no flow out of the boundaries of the container

permeable bed, the head calculation is just the average of heads around each cell—we are assuming that the k value (permeability and viscosity) is the same everywhere. The container walls are boundaries where there should be zero flow out of the system. Setting the head at these cells (green) equal to their interior, neighboring cells will ensure flow *out* of the container boundaries is zero.

One final note is in order. Since each cell in the spreadsheet references another, this is by definition a circular reference. Most spreadsheet software can be set to ignore this "error." Also, the spreadsheet must "recalculate" until there is very little change observed in the cell values. Again, modern spreadsheet packages can automatically iterate until the changes are small.

Using a finer grid, that is to say, using more cells will give us a reasonable picture of what is happening in our simple lauter tun. Extending the formulas to a larger array of spreadsheet cells is straightforward; however, excessively large arrays are not necessary to give a reasonable approximation. Figure 7.14 shows the resulting potential (head) map inside our simple lauter tun using a 14 × 15 spreadsheet grid. You can imagine a little ball bearing rolling down the surface of this map to mimic the flow of fluid—with one important difference. A ball bearing will roll down the surface and *gain* speed. Due to the restrictive nature of the grain bed, the flow is limited; this is the *Plinko* game again. So, in this type of map the slope indicates the flow rate—the steeper the surface, the greater the (constant) rate. If the map is nearly "flat," then the flow rate is nearly zero. We use this potential

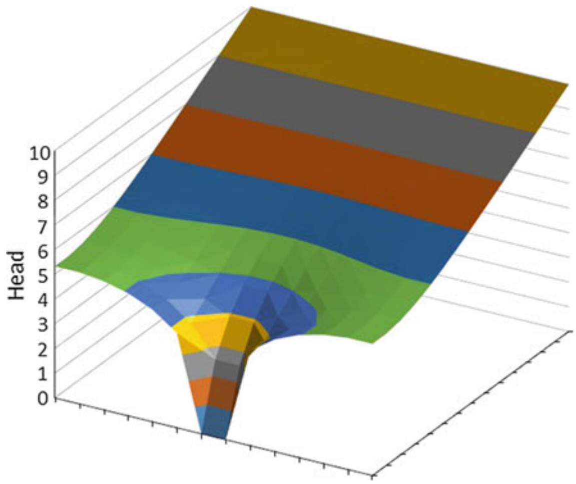


Fig. 7.14 Potential map for simple lauter tun with constant head and single exit

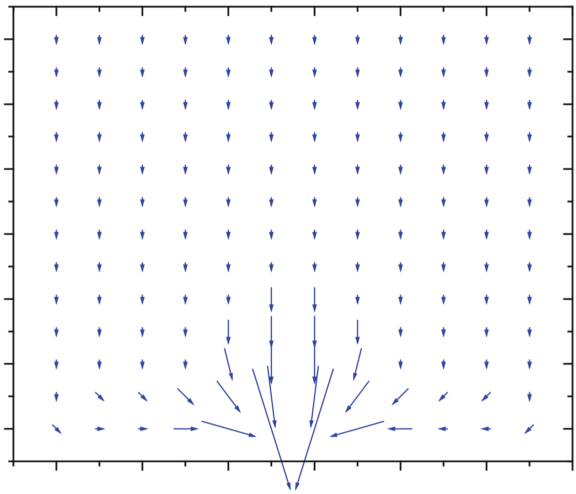


Fig. 7.15 Velocity vector field for a simple lauter tun with constant head and single exit

map and Eq. 7.63 to produce a velocity vector map as shown in Fig. 7.15. This map is like the weather forecaster's map showing wind speed and direction.

Our model shows that the flow is uniform at the top of our simple lauter tun, but near the exit the flow is greater and not uniform. We are now able to reach a very important conclusion. With the greatly increased flow rate near the single exit, we might expect this area of the grain bed to be oversparged and regions far from the exit undersparged. This would give us two issues in the brewery. First, we run the risk of extracting tannins from the portion of the grain bed that is oversparged, and secondly, we are compromising our brewhouse efficiency.

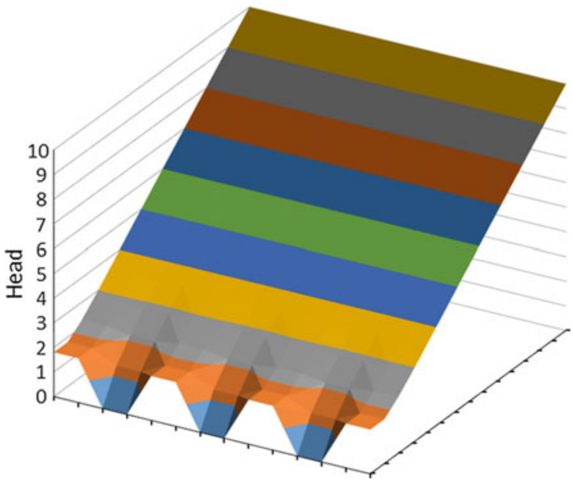


Fig. 7.16 Potential map for a simple lauter tun with constant head and multiple exits

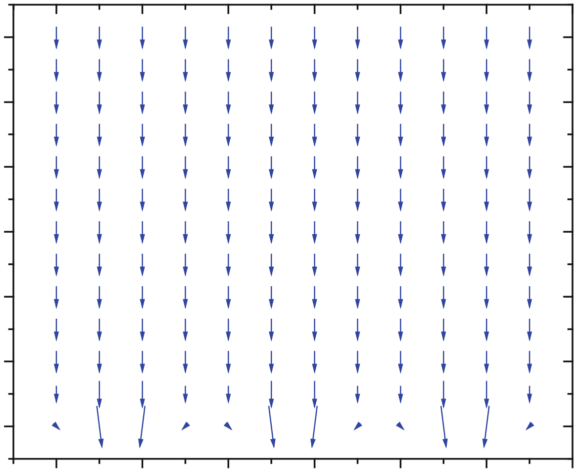


Fig. 7.17 Velocity vector field for a simple lauter tun with constant head and multiple exits

We can improve our simple lauter tun by adding more exits, perhaps making the bottom a "false bottom" with many small exits. Figures 7.16 and 7.17 show that this improvement will lead to more uniform flow over the grain bed near the exit.

We now want to define a new boundary condition that allows for a source of flow. Note that before we just fixed a constant head and let the flow rate be determined from this. To do this, we will eliminate the "constant head" boundary condition at the top of the lauter tun by setting each cell to the average of those around it like the interior of the vessel, but with one small change. Since there is not a cell above the top layer of cells, we count the cell below a given cell twice, as illustrated in the blue cells of Fig. 7.18. Finally, we simply add a constant term to

	A	B	C	D	E
1	=B1	=(C1+A1+2*B2)/4	=(D1+B1+2*C2)/4+2.3	=(E1+C1+2*D2)/4	=D1
2	=B2	=(B1+C2+A2+B3)/4	=(C1+D2+B2+C3)/4	=(D1+E2+C2+D3)/4	=D2
3	=B3	=(B2+C3+A3+B4)/4	=(C2+D3+B3+C4)/4	=(D2+E3+C3+D4)/4	=D3
4	=B4	=(B3+C4+A4+B5)/4	=(C3+D4+B4+C5)/4	=(D3+E4+C4+D5)/4	=D4
5	=B5	=B4	0	=D4	=D5

Fig. 7.18 Spreadsheet formulas to model a simple lauter tun with a constant source (2.3 arbitrary units) at the *top*

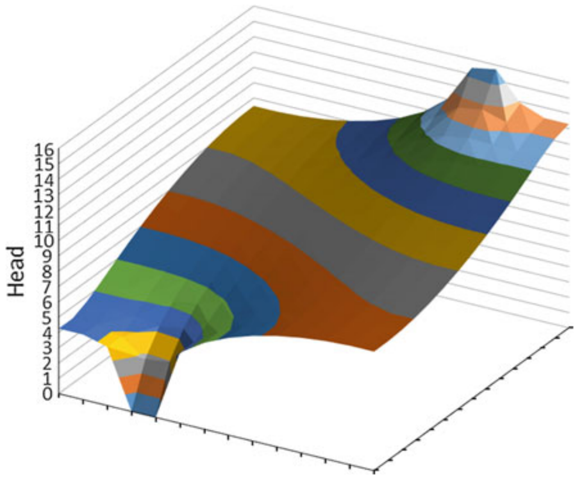


Fig. 7.19 Potential map for a lauter tun with a single constant fluid source and a single drain

the cells that we wish to have a constant source of flow into the system: the single orange cell in the figure.

By adding a single source term into the spreadsheet, we can approximate a stuck sparge arm where the source of rinse water is stationary. We further exacerbate the problem in our model by considering a single exit which is offset from the source. A contour and flow map for this model is shown in Figs. 7.19 and 7.20. Again notice that the flow is not uniform through the grain bed, which will lead to very uneven sparging.

Uneven sparging such as this, or a grain bed that is not constantly stirred, will lead to excessive rinsing along certain "channels." The excessive rinsing will tend to reduce the viscosity of the liquid in this area, making the problem worse. This will make recovery of sugars from the other parts of the grain bed more difficult. Finally, this will lead to poor brewhouse efficiency or weaker worts.

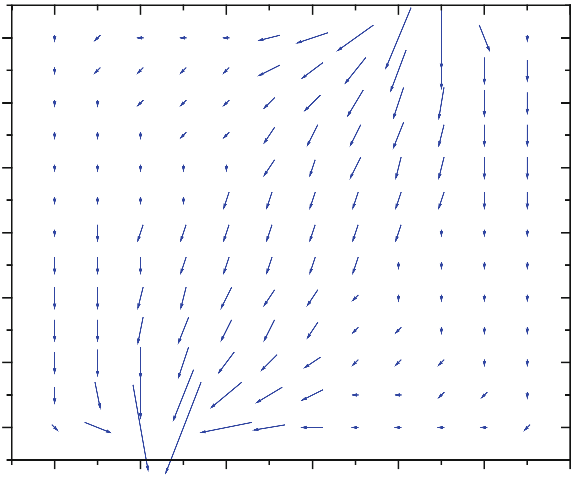


Fig. 7.20 Velocity vector field for a single source and single exit

> **CHECKPOINT 7.6**
> Given the information that has just been discussed about sparging, what inferences can you draw about the flow of water at the top and bottom of the bed of grain?

7.4 Equipment Used in Sparging and Lautering

As we have discussed in this chapter, after completion of the mash, the liquid (also known as the sweet wort) is separated from the spent grains by filtration. Traditionally, draining the liquid from the grains using a process known as batch sparging did this. While the batch sparge process is not used as much anymore, some brewers recognize the usefulness of the parti-gyle method for beer production. Modern techniques include the use of fly sparging and the *mash filter*.

The purpose of these different techniques is to completely remove the fermentable sugars from the residual grains. The efficiency of the process is of paramount concern to the brewer, because any loss of fermentable sugars results in a decrease in the potential quality of the finished beer (and a concomitant decrease in profit).

7.4.1 Batch Sparging

Batch sparging is typically accomplished by simply draining the liquid from the grains. Some brewers continue to use this process today, but it was much more common in the early days of brewing. In this process, the first batch of liquid drains slowly from the mash and is then returned to the top of the mash. This process is continued until the wort is clear (known as the vorlauf). After the liquid draining from the mash is clear, the liquid is collected in the underback (also known as the grant). This collection vessel is typically located underneath the tun and has multiple drain holes from the mash directly into the underback. The grant is a specialized version of this piece of equipment that fills with the drained liquid. The volume of the liquid in the grant is maintained to ensure that the pressure of the filtration can be monitored. If the difference in the height of the liquid in the grant and the height of the liquid in the mash tun gets too great, the brewer can slow down the removal of the liquid from the grant or turn on the mash rakes to increase the flow rate.

At this point, after all of the liquid has been removed (the first runnings), the flow of liquid across the grain bed is stopped and another batch of hot liquor is added to the mash tun. Often the entire bed is mixed, stirred, and then allowed to resettle. Mixing is important because it allows the new hot liquor to rinse the grains and extract any sugars from the grains. Then, the process of running the liquid through the underback or grant is repeated. This second batch of wort (the second runnings) can either be added to the first or treated separately as a different beer. The process is often repeated a third time. By the end of the third runnings, there are very little sugars left in the grains.

The equipment used for the batch sparge is often the same as that used for the mash. In other words, the mash tun can double as the lauter tun when the batch sparge is performed. Alternatively, the entire mash can be pumped (grain and liquor) into a lauter tun to perform the batch sparge. The benefit is the same as that which is described in the subsequent section on fly sparging.

Each of the drainings (or runnings) in a batch sparge operation is known as a *gyle*. These worts were evaluated for the concentration of sugars and then mixed to provide different beers. The mixing and the overall process of doing so were referred to as *parti-gyle* brewing. The strongest (and first draining of the wort) has the greatest maltose content and can be used to make export beer, barleywine, or strong ale. The weakest and last drainings of the wort can be used (historically this was the case) to make *small beer*—a lower alcohol beer that was consumed regularly by everyone in the family.

Parti-gyle brewing is not performed much anymore, but there are some commercial brewers who have reconsidered this process as a viable method for creating a wide variety of beers from a single wort stream.

7.4.2 Fly Sparging

In this process, which we discussed at length in the bulk of this chapter, hot liquor is added to the mash as the wort is removed from the mash. Again, the goal is to remove all of the fermentable sugars. A special sparge arm is employed to add the additional hot liquor without disturbing the bed of grains. Again, the first runnings are returned to the top of the mash (the vorlauf) in order to use the grain as a filter and clarify the wort.

There are two forms of fly sparging that are used commercially. In the simplest system, the mash tun serves as the location for the process. If the mash tun is equipped with stir paddles, these are stopped and held in place during the sparge. A layer of water at least 2–5 cm (1–2 in.) thick is placed on top of the spent grains and the liquid slowly drained out of the system. Additional hot liquor is added at the same rate that it is removed so that the pressure differential across the bed of grain is kept small. The first of the sparge water that is added during the sparge must be at about the same temperature as the mash during mash-out. After about half of the wort has been removed, the sparge water can be traded out for room temperature water. Studies have shown that no benefit is obtained by sparging with hot liquor during the entire sparge.

In more efficient processes, the entire mash is pumped into a much wider tun that resembles the mash tun, but is often fitted with rakes rather than stir paddles (see Fig. 7.21). The wider lauter tun allows the thickness of the grain bed to be significantly reduced. This allows the sparging process to take place faster (with a higher flow rate) while keeping the pressure differential across the bed the same as it would be in the mash tun. Design of the mash rakes often used in the lauter tun indicates that their shape and dimensions can seriously impact the extraction efficiency during the sparge. Often the rakes have metal triangles placed at angles along the rake. The rakes are also not simply straight; they are bent into a zigzag pattern from top to bottom. The design of the rakes allows maximum effect in increasing the bed permeability. These mash rakes are employed if the pressure differential begins to increase at the flow rates employed. This is simply done by rotating the rakes around the grain bed. In some systems, the rakes can be raised

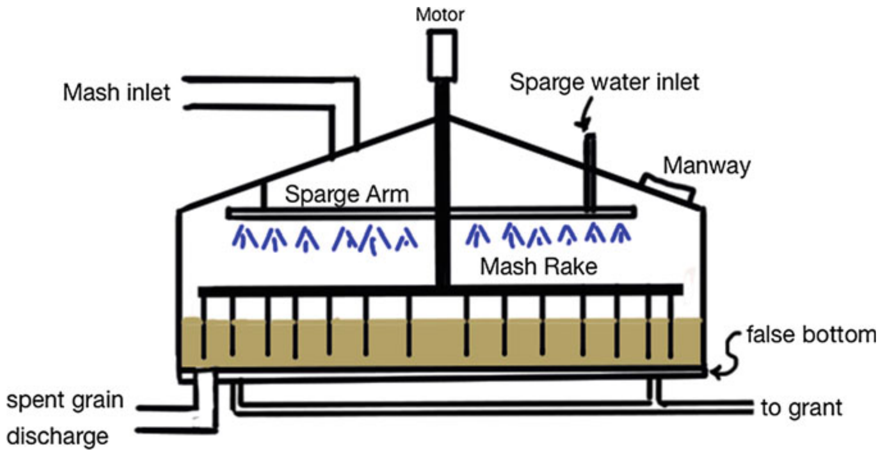


Fig. 7.21 Lauter tun versus the mash tun for sparging

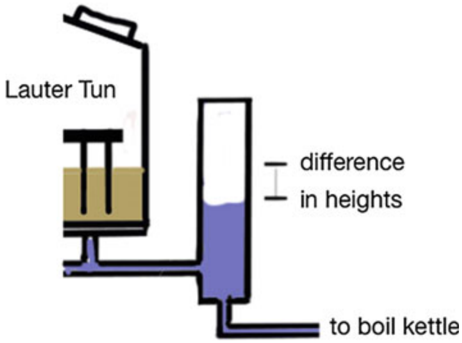


Fig. 7.22 Visual comparison of the wort levels in the lauter tun and grant can be used to determine the pressure of the system. If the height is measured, the head (in m) or the pressure (in kPa) can be determined

and lowered allowing even more control over the effect of the rakes on the process of sparging.

The sweet wort that exits the fly sparging setup is usually emptied immediately into the underback or grant. As we noted before, the grant collects all of the wort across the entire bottom of the lauter tun or mash tun and fills slowly. Visually, the difference in the heights of the liquid in the grant and the level of the liquid in the mash/lauter tun can be compared (see Fig. 7.22). In doing so, the pressure across the grain bed can be either estimated or calculated (if the difference in heights is measured.) This is done to verify that the pressure does not exceed the capabilities of the false bottom. Too high and the false bottom can buckle under the pressure. In addition, the difference can be used to verify the flow rate of the wort being removed from the vessel.

7.4.3 Mash Filter

The most efficient extraction of fermentable sugars occurs with the use of the mash filter. This process also requires that the grist is much more finely ground. In fact, it is often ground to the point where it resembles flour. In these systems, the mash is much thicker and first runnings of the wort after removal from the mash can be greater than 24 °P. The efficiency of the mash filter drives its usage in the brewing industry.

When a mash filter is used, the entire mash resembles a thick porridge. Malt flour is added to the hot liquor in a mash mixer. The flour can be added directly to the hot liquor, or as is done with the ground grist using a lauter tun, a premash mixer is often employed. Constant mixing during the mash aids in the conversion of starches into fermentable sugars. Just as with the other systems, care must be taken to avoid high shear forces that can destroy the proteins and other compounds in the mixture. Once the mash is complete, the entire mixture is pumped into the mash filter

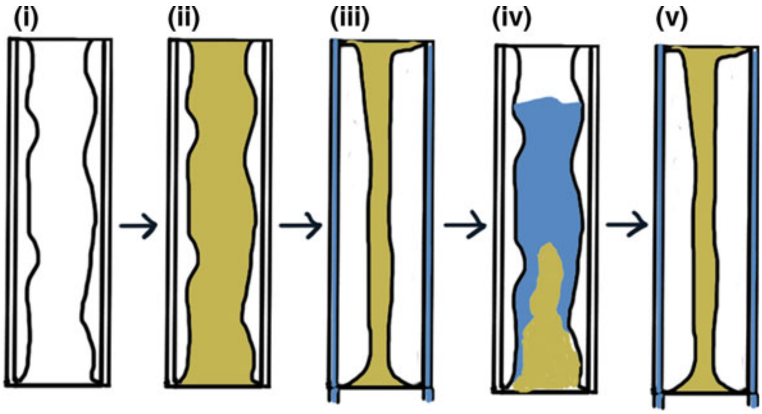


Fig. 7.23 The Mash filter. The mash is added to the apparatus *(ii)*, and the bladders are then inflated *(iii)*. The bladders are deflated *(iv)* and rinse water is added. The mash and sparge water are resqueezed *(v)* to remove the 2nd runnings. The process is repeated multiple times to remove all fermentable sugars from the mash

apparatus and then squeezed with bladders to push the wort away from the flour (Fig. 7.23). Hot liquor is added to the pressed flour and the mash is resqueezed. The wort is collected and passed along to the boil kettle.

Use of the mash filter allows extraction efficiencies to approach or even be better than theoretical yields. After rinsing the flour and resqueezing, the apparatus is opened and the almost "dry" spent grains are released. This process, too, can be automated, dropping the spent grains onto a conveyor belt and removing them from the system.

While this seems like it might be a rather new technology in mashing, the Meura mash filter, one of the hallmark versions of the mash filter, was actually invented in 1901 by Phillippe Meura. The success of the apparatus has ensured its place in the brewing industry. Slight improvements since then have been made, but the basic apparatus has been used since then.

The apparatus looks like a typical plate-and-frame filter (which we will discuss later in this text). Basically, a series of bladders are bolted together in a tall rectangular shape. The success of the filter in improving efficiency of extraction has resulted in its use across the world. In fact, this system is responsible for the production of more than one-third of all the beer consumed annually.

7.5 When Do We Stop Sparging?

Hot liquor used in sparging should be hot (about 80 °C or 175 °F). The temperature is extremely important in the initial stages of this process. Keeping the temperature hot continues to denature the proteins and destroy the enzymatic activity. In addition, the hot water aids in the extraction of the fermentable sugars from the remnants of the grains. As we noted earlier, some research has been done on the use of sparge water with different temperatures. Those studies have shown that only about one-third volume of hot liquor is needed. Any remaining sparge water need not be heated to continue and conclude the sparging process. In spite of this information, it is very common for the brewer to maintain the hot temperatures during the entire process. In most cases, the addition energy required to heat the volume of water to that temperature does not seriously impact the brewer's budget. However, as the volume of sparging grows, this can be a significant cost.

Another feature of the sparge water that must be maintained is the pH. If the pH of the sparge water becomes too alkaline (above pH 6) while still being hot, the extraction of tannins becomes much greater. Thus, care must be taken to make sure that all hot liquor in the brewery is slightly acidic. The risk of tannin extraction may one day cause brewers to consider colder water for the sparge.

Chapter Summary

Section 7.1

Fly sparging and batch sparging are two common methods for removing the sweet wort from the spent grains.

The mash tun can also be used for separating the liquid from the spent grains.

Section 7.2

Pascal's law states that the pressure difference between two points in a fluid depends only on the height, or elevation, difference. This pressure difference is due to the weight of the fluid contained between the elevation differences, $\Delta P = \rho g \Delta h$.

Section 7.3

Bernoulli's equation, $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = \text{constant}$, allows us to understand fluid flow and pumping systems in the brewery.

Darcy's law explains the relationship between bed permeability, flow rate, and pressure across a grain bed during filtration.

Modeling the lauter tun allows us the opportunity to consider modifications to the inlet and outlet and the resulting flow patterns.

Section 7.4

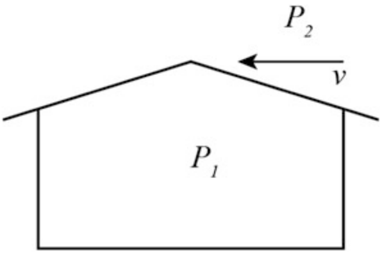
The lauter tun is much wider than the mash tun, allowing the grain bed to be significantly less deep.

The lauter tun employs rakes that can be used to increase bed permeability.

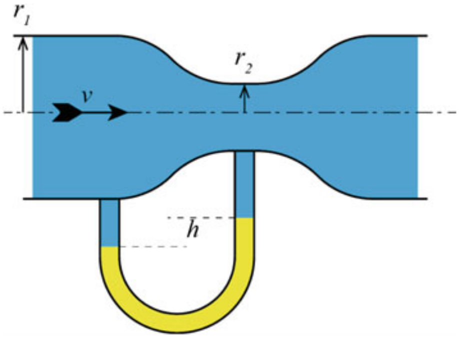
The grant can be used to estimate or calculate the pressure during the sparge.

Questions to Consider

1. High winds, such as from severe thunderstorms and tornados, have the potential to remove roofs from houses. Assuming that air has a density of 2.3×10^{-3} slugs/ft^3 and that the wind speed across the top of a roof is 80 mph (117 ft/s), use Bernoulli's principle to find the pressure difference $P_1 - P_2$ on either side of the roof. If the roof has an area of 1300 ft^2, find the total force on the roof.



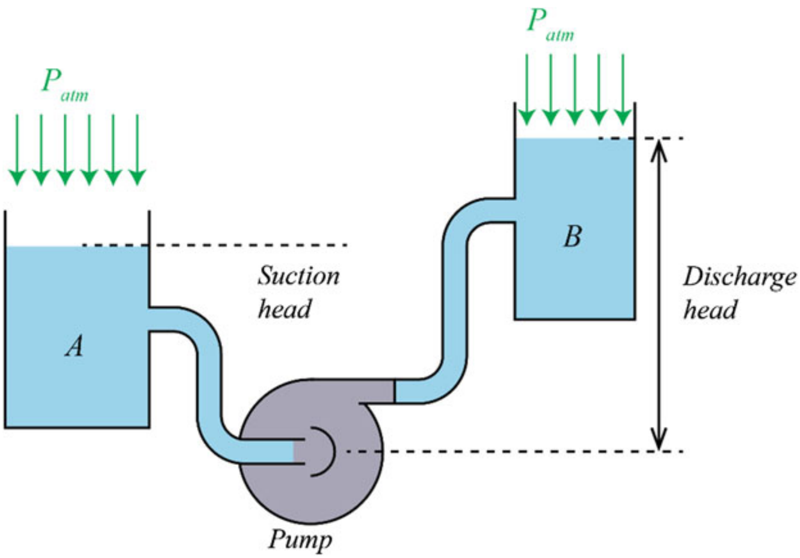
2. Water with density ρ = 1000 kg/m^3 is passed along a pipe of radius r_1 = 6 cm. The pipe tapers to a smaller size, radius r_2 = 4 cm. A U-tube, partially filled with an unknown fluid that is immiscible with water of density ρ = 1390 kg/m^3, is connected to the two sections. If the height difference h is 1 cm, determine (A) the velocity of the fluid in the narrowest part of the device and (B) the volumetric flow rate.



3. A person with advanced arteriosclerosis has an accumulation of plaque on the inner walls. Due to the Bernoulli's principle, the increased velocity of blood flow through the restriction, and the subsequent reduction in pressure, the artery will collapse and momentarily stop blood flow. When the flow stops, the artery opens again and the process repeats. Such a condition leads to vascular flutter. Assume that diameter of an artery is 6 mm, and that at some point it is restricted to 3 mm. If blood flows through the larger part of the artery at 5 cm/s at a pressure of 100 mmHg (13 kPa), what is the pressure in the restricted portion.
4. The continuity equation also applies to junction of pipes as well as reduction of sizes. Consider a 3″ diameter pipe which "T's" into a 1″ diameter pipe and a 2″ diameter pipe. If the flow rate through the 3″ pipe is 4 gallons per minute, what

is the flow rate in the other two pipes assuming that the velocity of the fluid is the same in all three pipes?

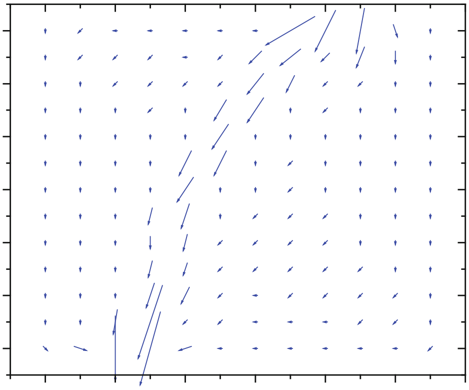
5. Graph the data presented in Table 7.1 and extrapolate head loss values for flow rates of 4 and 8 L/s. Looking at your graph, what can you conclude about the effects of friction as the flow rate increases? If we double the flow rate, do we also double the friction loss?



6. Consider the pumping system illustrated above. All piping connections use 2″ HDPE pipe. Note that there are two 90° elbows in both the suction piping and the discharge piping. The suction head is 3 ft above the pump inlet and we are moving 160 °F water with a vapor pressure h_{vp} = 11.2 ft. What is the maximum volumetric flow rate in ft^3/s keeping the NPSH greater than zero?
7. Repeat problem 6 and determine the maximum volumetric flow rate when the water is 20 °C, assuming the vapor pressure is h_{vp} = 1.2 ft.
8. In your own words, describe the relationship of flow rate to pressure across a grain bed. What would happen to the pressure and flow rate if the bed was half of the original depth and if the bed was 2× as compacted and if the bed was raked periodically with a mash rake?
9. Outline the pros and cons for the use of a mash filter. Be sure to include at least three pros and three cons.
10. What is the head, in meters, if the pressure is observed to be 4200 kPa? What is the pressure in bar and in psi?
11. Which has a greater pressure, the bottom of a column of air that is 20 miles deep or the bottom of a column of water that is 2 m deep? (ρ_{air} = 1.225 kg/m^3; ρ_{water} = 1000 kg/m^3)
12. Adapt your Darcy's law—sparging spreadsheet to handle situations where the permeability and the viscosity are different. This is easiest by referencing

another area of the spreadsheet that will contain the k values, such as the example shown. You will model a situation where the viscosity is lower. This means that the k value in Eq. 7.63 is larger. Make your grid 14 × 15 as in the above examples. Assign most of the k values to 1, but make a "channel" from the source to the exit with $k = 4$. How does the flow through the grain bed compare to the situation where $k = 1$ everywhere?

	A	B	C	D	E	F	G	H	I	J	K
1			Potential map						k values		
2	=B2	=(I2*C2·	=(J2*D2+H2*B2+2*I3*C3)/(J2+H2+2*I3) +2.3	=(K2*E2	=D2		1	1	4	1	1
3	=B3	=(H2*B2	=(I2*C2+J3*D3+H3*B3+I4*C4)/(H3+I2+J3+I4)	=(J2*D2	=D3		1	4	1	1	1
4	=B4	=(H3*B3	=(I3*C3+J4*D4+H4*B4+I5*C5)/(H4+I3+J4+I5)	=(J3*D3	=D4		1	4	1	1	1
5	=B5	=(H4*B4	=(I4*C4+J5*D5+H5*B5+I6*C6)/(H5+I4+J5+I6)	=(J4*D4	=D5		1	4	1	1	1
6	=B6	=B5	0	=D5	=D6		1	1	4	1	1



13. Describe the differences between a mash tun and a lauter tun in terms of their use in filtering the hot wort away from the spent grains.
14. A liquid must be moved from a storage vessel to another. The bottom of the first vessel is 3 m below the bottom of the second vessel. The pump is 3 m below the first vessel. If the pipes attach to the bottom of the vessels, what is the suction head on the pump? What is the required delivery head for the pump?
15. In Fig. 7.9, the inlet pipe to the pump is placed near the top of the supply vessel. What would be the effect on the calculation of the horsepower required for the pump of extending the inlet pipe so that it reached the bottom of the vessel?
16. Repeat question 12, but consider that a dough-ball has been allowed to set up in the middle of the lauter tun. Set the value of k to 0 in the middle-most cell of the table. What is the effect of this dough-ball?
17. What is the effect of having exit points at every location on the bottom of the grain bed during sparging?

18. What would you qualitatively expect to determine for the gravity of the first wort, second wort, and third wort in a batch sparge process? How would this compare to a fly sparge that collected the same volumes?
19. Describe the operation of a grant in your own words.
20. Examine the figure below. What is the pressure across the grain bed at the given flow rate? What would happen to the pressure if the viscosity of the wort dropped from 1050 to 1030 kg/m^3?
21. Assume a pump has a suction head of 1.5 m and a discharge head of 4.0 m. If the density of the liquid being pumped is 1040 kg/m^3, what is the horsepower required by the pump?
22. What is the volumetric flow rate of a sweet wort (r = 1060 kg/m^3) flowing at 20 L/min? What would be the volumetric flow rate in a pipe that is 2 cm versus one in a pipe that is 5 cm in diameter?
23. In this chapter, we noted that Darcy's law is only an approximation of the flow across a grain bed, but that it works fairly well for sand. What differences exist between grain and sand that might result in deviations from Darcy's law? For each, describe the effect that would result.

Laboratory Exercises

Exploring Darcy's Law

In this experiment, we will explore some of the parameters of Darcy's law by modeling it in the laboratory. Our "grain" will be pebbles, rocks, and sand, and our "wort" will be water.

Equipment Needed

50-mL funnel with a stopcock—at least 1 cm in diameter
ring stand and clamp to fit the funnel
cotton ball
thin glass rod
masking tape
100-mL graduated cylinder
100-mL beaker
clock with sweep hand or stopwatch
sand, pea-sized gravel, marble-sized gravel
water

Experiment

The setup is to clamp the 50-mL funnel in an upright position. Place a 100-mL graduated cylinder underneath the setup so that it collects everything that drips out of the end of the cylinder.

Tear a cotton ball apart to obtain a small piece of cotton. Place this into the funnel and using a glass rod, push it into the small neck at the bottom of the funnel. The point of the cotton is to stop sand and gravel from becoming lodged into the stopcock when it is opened. Be careful when using the thin glass rod as any lateral force could cause it to snap, resulting in cuts to the hand.
Obtain a portion of sand and add it to the funnel with the stopcock closed so that the funnel is filled 1/4 to 1/3 of the way. Use a small piece of tape to mark the top of the sand.
Then, use the 100-mL beaker to fill the entire funnel with water. Make sure that no air bubbles exist in the funnel by holding your hand over the top of the funnel, inverting it, and then reclamping it in place.
Then, at the same time, open the stop cock and start the timer. Collect at least 20 mL of water into the graduated cylinder and stop the timer when that occurs. Note that more accurate readings are obtained when the volume collected is increased.
Repeat the experiment at least three times by closing the stopcock and refilling the water. Record the average of all times for sand. Then, repeat the experiment by washing out the funnel to make sure all of the sand has been removed, and replacing the sand with pea-sized gravel, and then repeat again with marble-sized gravel. Record your results and make a statement about the size of the particles versus flow rate (in these cases, the pressure across the bed will be relatively constant).
Repeat the experiment by placing one marble-sized stone in the middle of a batch of sand and determining the time. Then, repeat the process with 3 marble-sized stones. Finally, write conclusions about the application of this experiment to Darcy's law.