Chapter 11: AC Power and Power Distribution

Overview

Prerequisites:

- Knowledge of complex arithmetic
- Knowledge of basic circuit analysis (Chapters 3 and 4)
- Knowledge of phasor/impedance method for AC circuit analysis (Chapter 8)

Objectives of Section 11.1:

- Find average AC power for a resistive load and understand the rms values
- Express average power for any AC load in terms of power angle and power factor
- Express average power in terms of phasors/impedances
- Define major AC power types: average power, reactive power, complex power, and apparent power
- Be able to construct the power triangle and classify the load power factor

Objectives of Section 11.2:

- Be able to perform power factor correction of an inductive load (AC motor)
- Learn about maximum power efficiency technique in general
- Derive and test a simple condition for maximum power transfer to a load from an arbitrary AC source

Objectives of Section 11.3:

- Learn the structure of power distribution systems
- Establish the concept of the three-phase power transmission system
- Understand the meaning and realization of three-phase source and three-phase load
- Solve for phase and line voltages and line currents in the three-phase balanced wye-wye system
- Establish the meaning and the role of the neutral conductor in the wye-wye power distribution system

Objectives of Section 11.4:

- Establish that the instantaneous power in balanced three-phase systems is constant
- Extend the concepts of reactive power, complex power, and apparent power to the three-phase systems
- Compare conductor material consumption in single-phase and three-phase systems
- Become familiar with delta-connected three-phase sources and loads
- Establish equivalency between delta and wye topologies with no ground

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Application Examples:

- rms voltages and AC frequencies around the world
- Wattmeter
- Automatic power factor correction system
- Examples of three-phase source and the load
- Conductor material consumption in three-phase systems

Keywords:

Time averaging, Average power, rms voltage, rms current, AC fuse, Root mean square, Sawtooth wave, Triangular wave, Noise signals, Power angle, Power factor, Reactance, Capacitive reactance, Inductive reactance, Active power, True power, Reactive power, Complex power, Apparent power, VAR (volt-amperes reactive), VA (volt-amperes), Power triangle, Lagging power factor, Leading power factor, Wattmeter, Wattmeter current coil, Wattmeter potential coil, AC power conservation laws, Power factor correction, Power factor correction capacitor, PFC capacitor, Principle of maximum power efficiency for AC circuits, Principle of maximum power transfer for AC circuits, Impedance matching, Single-phase two-wire power distribution system, Single-phase three-wire power distribution system, Neutral conductor, Neutral wire, Split-phase distribution system, Polyphase distribution systems, Three-phase four-wire power distribution system, Phase voltages, Line-to-neutral voltages, *abc* phase sequence, Positive phase sequence, *acb* phase sequence, Negative phase sequence, Balanced phase voltages, Wye (or Y) configuration, Balanced three-phase source, Wye-connected source, Wye-connected load, Wye-wye distribution system, Phase impedances, Load impedances per phase, Balanced three-phase load, Synchronous three-phase AC generator, Alternator, Rotor, Stator, Synchronous AC motor, Rotating magnetic field, Line-to-line voltages, Line voltages, Line currents, Superposition principle for three-phase circuits, Per-phase solution, Total instantaneous load power of the three-phase system, Average load power of the balanced three-phase system, Reactive load power of the balanced three-phase system, Complex load power of the balanced three-phase system, Balanced delta-connected load, Balanced delta-connected source, Delta-delta distribution system

Section 11.1 AC Power Types and Their Meaning

The present section studies the basics of AC power. We begin with the root-mean-square (rms) representation of AC voltages and currents. The rms concept enables us to develop a DC equivalent representation, which compares AC to DC conditions in terms of power delivered to the load. It is important to understand that the rms concept is a general power concept; it applies not only to periodic AC circuits but virtually to any circuits, even with nonperiodic power sources, like noise power sources. Further results presented in this section are primarily intended for power electronic circuits; they have an equal applicability to radio-frequency communication circuits.

11.1.1 Instantaneous AC Power

We consider an arbitrary load with resistance R, load current $i(t)$, and load voltage $v(t)$, in the passive reference configuration. The instantaneous power delivered into the load is given by

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R} \tag{11.1}$$

according to Ohm's law. If we use the load voltage in the form $v(t) = V_m \cos \omega t$ [V], then

$$p(t) = v(t)i(t) = \frac{V_m^2}{R} \cos^2 \omega t = \frac{V_m^2}{2R}(1 + \cos 2\omega t) \tag{11.2}$$

where we applied the trigonometric identity $\cos^2 \omega t = 0.5(1 + \cos 2\omega t)$. Interestingly, the load power is not constant; it varies in time, and the behavior is shown in Fig. 11.1 for a load voltage amplitude $V_m = 3$V, frequency $f = 50$ Hz, and load resistance $R = 5\ \Omega$.

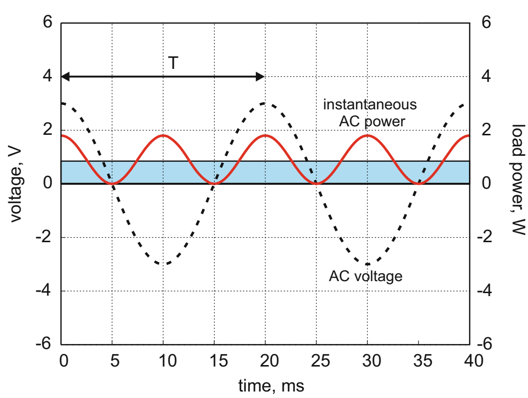


Fig. 11.1. Power (*solid line*) for a load voltage $v(t) = 3\cos(2\pi 50t)$ [V] (*dotted line*).

11.1.2 Time-averaged AC Power

An interesting question arises when we have to determine the bill for the variable AC power in Fig. 11.1. As far as the utility power is concerned, a consumer would prefer to pay for the minimum amount of power. The power minima in Fig. 11.1 occur at $t = 0.25T$, $0.75T,\ 1.25T$, etc.; here T is the period of the AC voltage signal. Since the power is exactly zero at its minima, we would pay nothing. On the other hand, a utility would prefer to charge for the maxima of the power, which occur at $t = 0,\ \ 0.5T,\ 1.0T,\ 1.5T$, etc. A fair solution is clearly somewhere in the middle. It is based on *time averaging* the load power and then charging the consumer for the average (or mean) power as indicated in Fig. 11.1 by the shaded rectangle. Thus, we are interested in the averaged instantaneous power of Eq. (11.2). The time averaging is always done over a full period T of the AC voltage signal. The notation for the time-average value is often denoted by an overbar. Thus, the definition reads:

$$P = \overline{p(t)} \equiv \frac{1}{T}\int_0^T p(t)dt \tag{11.3}$$

where P is now the *average power* delivered to the load. We note that the average power times the period T gives us the energy E (in J or more often in W·h, 1 W·h = 3600 J) delivered to the load per period, i.e., $E = TP$.

rms Voltage and rms Current

Using Eq. (11.2) we obtain from Eq. (11.3)

$$P = \frac{1}{T}\int_0^T \frac{V_{\mathrm{m}}^2}{2R}(1 + \cos 2\omega\, t)\, dt = \frac{1}{T}\frac{V_{\mathrm{m}}^2}{2R}\left(\int_0^T 1 \cdot dt + \int_0^T \cos 2\omega\, tdt\right) \tag{11.4}$$

The first integral yields a nonzero contribution, whereas the second integral is exactly equal to zero, due to fact that the average of the sine or cosine functions over a period, or multiple periods, is zero. Thus,

$$\int_0^T 1 \cdot dt = T, \quad \int_0^T \cos 2\omega tdt = \frac{1}{2\omega}\sin 2\omega t\big|_0^T = \frac{1}{2\omega}\sin(4\pi t/T)\big|_0^T = 0 \tag{11.5}$$

Inserting these values into Eq. (11.4) results in

$$P = \frac{V_{\mathrm{m}}^2}{2R} = \frac{V_{\mathrm{rms}}^2}{R}, \quad V_{\mathrm{rms}} = \frac{V_{\mathrm{m}}}{\sqrt{2}} \tag{11.6}$$

where V_{rms} is the *rms* (*root-mean-square*) value for the load voltage $v(t) = V_m \cos \omega t$ or simply the *rms voltage*. According to Eq. (11.6), the rms voltage is the *equivalent* DC voltage that provides the same power into the load. Once we know the rms voltage, the average power is given by the "DC" formula V_{rms}^2/R. The *rms* voltage is always less than the voltage amplitude by a factor of 0.707 (or 71 %). You should notice that Fig. 11.2 is a replica of Fig. 11.1; additionally, it shows the rms voltage and the averaged power for the load voltage $v(t) = 3\cos(2\pi 50t)$ [V]. If a nonzero phase is present in this expression, the result will not change. The signal will be shifted but all averages over the period will remain the same. The corresponding mathematical proof is suggested as one of the homework problems.

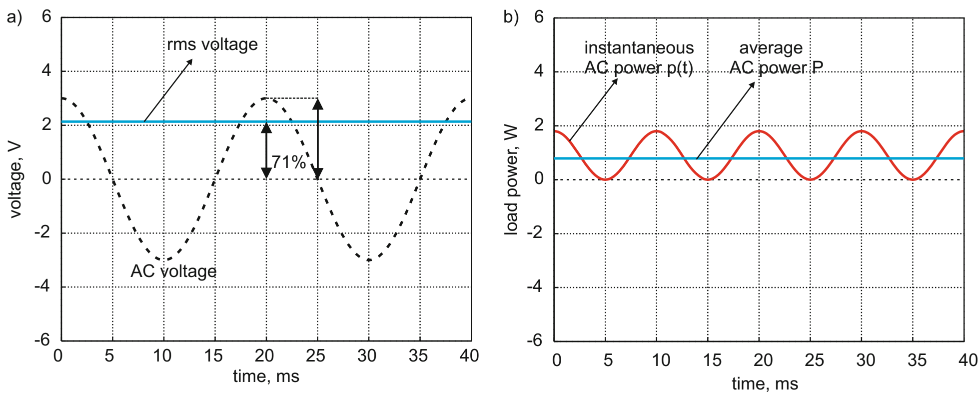


Fig. 11.2. (**a**) Load voltage $v(t) = 3\cos(2\pi 50t)$ [V] (*dotted line*) and its *rms* DC voltage, which delivers the same power into the load and (**b**) instantaneous power and the average power for the load voltage $v(t) = 3\cos(2\pi 50t)$ [V].

A similar expression is obtained for the alternating current $i(t) = I_m \cos \omega t$ across the load. The *rms current* is given by

$$I_{rms} = \frac{I_m}{\sqrt{2}}, \quad V_{rms} = RI_{rms}, \quad P = R\frac{I_m^2}{2} = RI_{rms}^2 \tag{11.7}$$

Example 11.1: Determine average power delivered to a 10-Ω load when the applied AC voltage is given by $v(t) = 170\cos(2\pi 60t)$ [V] (US).

Solution: We find the rms voltage first:

$$V_{rms} = 170/\sqrt{2} \approx 120.21 \text{ V} \tag{11.8}$$

Example 11.1 (cont.): The average power is then given by $P = \frac{V_{rms}^2}{R} = 1.445\ \text{kW}$. Note that the rms current is equal to 11.02 A. Therefore, a 15-A or a 20-A AC *fuse* should be used. The fuse rating is based on the rms electric current value.

Exercise 11.1: Determine average power delivered to a 10-Ω load when the alternating load current is given by $i(t) = 5\cos(2\pi 60t)$ [A].

Answer: 125 W.

11.1.3 Application Example: rms Voltages and AC Frequencies Around the World

The AC voltage reported for the wall plug is an rms voltage. In the USA, the rms voltage typically ranges from 110 V to 127 V. Variations are caused by a specific type of a three-phase secondary distribution system studied later in this chapter. In this text we assume an average nominal value of $V_{rms} = 120$ V, perhaps a safe estimate. Using this value we obtain the voltage amplitude (the peak voltage value) of $V_m = \sqrt{2} \cdot 120\ \text{V} \approx 170\ \text{V}$. This number is considerably greater than the reported 120 V AC. In other countries, the nominal rms wall plug voltage ranges from 220 V to 240 V, depending on the country. Using the nominal rms value of 220 V (People's Republic of China, Russia, France, Argentina, etc.), we obtain a voltage amplitude of $V_m = \sqrt{2} \cdot 220\ \text{V} = 311\ \text{V}$. This number is again greater than the reported 220 V AC. India uses the nominal rms value of 230 V at 50 Hz, as do the European Union and Great Britain. This yields a voltage amplitude of $V_m = \sqrt{2} \cdot 230\ \text{V} = 325\ \text{V}$. Depending on the country you live in, the AC frequency is either 50 Hz or 60 Hz; for instance, it is 60 Hz in the USA.

Historical: *From the IEEE Historical FAQ's and other sources*: The person responsible for adopting 60 Hz was probably Nikola Tesla who figured that for the Westinghouse-designed central stations for incandescent lamps, the efficient distribution was 59 Hz, and it was then rounded to 60 Hz. The German company AEG (Allgemeine Elektrizitäts-Gesellschaft), originally influenced by Thomas Edison, started using 50 Hz as a more "metric" number. Their standard spread to the rest of Europe and to other countries.

Figure 11.3 shows the rms voltage (and frequency) world map. Some countries have a dual distribution system that operates at 120 V and 220 V simultaneously. With the help of an electric transformer, studied in the following text, we can convert the voltages to higher or lower values. However, we cannot convert frequency with a linear circuit or a transformer. Some transformers are designed for both 50 Hz and 60 Hz but unfortunately not all. As time progresses, the frequency difference between the load and the source may have a severe effect on motorized applications and their transformers (power loss, overheating, and even eventual burnout).

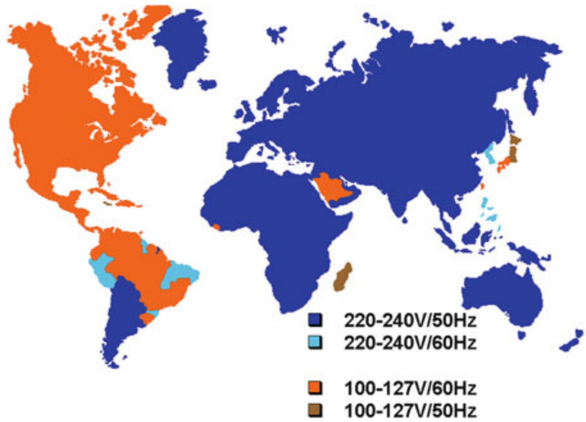


Fig. 11.3. The *rms* voltage world map, courtesy of Mr. Conrad H. McGregor, UK, and reproduced with the author's permission.

Example 11.2: Determine average power delivered to a 100-Ω load when the applied AC voltage has an *rms* value of 220 V (People's Republic of China).

Solution: The average power is given by

$$P = \frac{V_{\text{rms}}^2}{R} = 484 \text{ W} \tag{11.9}$$

Note that the rms current is equal to 2.2 A. Therefore, a 5-A *AC fuse* (but not the 2-A fuse) is sufficient in the present case. The fuse is an overcurrent protective device; a soldered joint within the fuse is melted when the rms current exceeds a threshold.

Exercise 11.2: The load from Example 11.2 is connected to a wall plug in the USA. How would the average load power change?

Answer: The average load power will be 144 W.

11.1.4 rms Voltages for Arbitrary Periodic AC Signals

Analytical expressions given by Eqs. (11.6) and (11.7) are quite sufficient for finding the mean power but only for single-frequency AC signals. In certain cases, the signal may still be periodic with a period of T but may contain multiple frequency components. One such example is the clock signal for which we estimate the average electric power. In this case, we return to the definition Eq. (11.3) and rewrite it in the form, which literally explains the meaning of the *root-mean-square* value:

$$P=\frac{1}{T}\int_0^T p(t)\,dt=\frac{1}{R}\,\frac{1}{T}\int_0^T v^2(t)\,dt=\frac{\left(\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\right)^2}{R} \tag{11.10a}$$

We again wish to define the rms voltage as the DC voltage that gives the same power into the load resistance R. Therefore, it should be

$$V_{\mathrm{rms}}=\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}\Rightarrow P=\frac{V_{\mathrm{rms}}^2}{R} \tag{11.10b}$$

For single-frequency voltage signals, this result reduces to Eq. (11.6). For more complicated voltages or for voltages measured directly, the calculation of the integral in Eq. (11.10b) may constitute some difficulties. At the end of this chapter, we provide a few homework problems tasking you to calculate the integral in Eq. (11.10b) directly. Once the rms voltage is found, the *rms* current across the resistive load is expressed by $I_{\mathrm{rms}}=V_{\mathrm{rms}}/R$, irrespective of the particular signal type.

Example 11.3: Determine the average power delivered to a 100-Ω load when the applied periodic voltage has the form $v(t)=10t/T$ [V] over a period $T=10$ ms. This signal is known as a *sawtooth* or a *triangular wave*.

Solution: We find the rms voltage first:

$$V_{\mathrm{rms}}=\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}=\sqrt{\frac{1}{T}\int_0^T 100t^2/T^2\,dt}=\sqrt{100/3}\approx 5.77\ \mathrm{V} \tag{11.10c}$$

The average power is thus given by $P=V_{\mathrm{rms}}^2/R\approx 333$ mW.

Frequently encountered voltage signals in microelectronic circuits are *noise signals*, which are neither sinusoidal nor periodic. In this case, Eq. (11.10b) applies again, but only in the limit as $T\to\infty$. Advanced analog electronics deals with certain electronic circuits where such noise sources become important and even critical.

11.1.5 Average AC Power in Terms of Phasors: Power Angle

For arbitrary dynamic circuit elements, the power analysis is carried out in terms of phasors. Consider element A in Fig. 11.4 which has real-valued voltage and current given by

$$v(t) = V_m \cos(\omega t + \varphi) \\ i(t) = I_m \cos(\omega t + \psi) \tag{11.11}$$

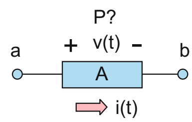


Fig. 11.4. Arbitrary circuit element in the passive reference configuration.

When element A is a resistor, the phases in Eq. (11.11) are the same, and finding the average power is straightforward. However, when element A is an inductor, capacitor, or a combination of resistor and inductor/capacitor, the situation becomes different. In this case, the phases of voltage and current in Eqs. (11.11) do not necessarily coincide. By definition:

$$P = \overline{p(t)} = \frac{1}{T}\int_0^T v(t)i(t)dt = \frac{1}{T}V_m I_m \int_0^T \cos(\omega t + \varphi)\cos(\omega t + \psi)dt \tag{11.12}$$

To manipulate the cosine expression in Eq. (11.12), we can use the trigonometric identity $\cos(\omega t + \varphi)\cos(\omega t + \psi) = 0.5\cos(\varphi - \psi) + 0.5\cos(2\omega t + \varphi + \psi)$. The integral of the second term in Eq. (11.12) will be equal to zero since it is the integral of the plain cosine function over two periods. The result then has the form:

$$P = \overline{p(t)} = \frac{1}{T}\int_0^T v(t)i(t)dt = \frac{V_m I_m}{2}\cos(\varphi - \psi) = \frac{V_m I_m}{2}\cos\theta = V_{rms} I_{rms}\cos\theta \tag{11.13}$$

Equation (11.13) is of great importance for power electronics since it introduces the so-called power angle θ

$$\theta = \varphi - \psi, \quad -90^o \le \theta \le +90^o \tag{11.14}$$

and the power factor

$$PF = \cos(\varphi - \psi) = \cos\theta \tag{11.15}$$

Both of these expressions determine the average power delivered to the circuit element. For any passive load, the power angle must be between −90° and +90°; i.e., the average power delivered to the element must be *nonnegative*! However, for an active load (an amplifier), it is possible that the power angle is no longer within those limits. Then, the element actually delivers power to the circuit rather than absorbing it. Equation (11.13) can now be expressed in terms of phasor voltage $\mathbf{V} = V_{\mathrm{m}}\angle\varphi$ and phasor current $\mathbf{I} = I_{\mathrm{m}}\angle\psi$. The result is simple and elegant:

$$P = \frac{\mathrm{Re}\left(\mathbf{V}\cdot\mathbf{I}^*\right)}{2} = \frac{\mathrm{Re}\left(\mathbf{V}^*\cdot\mathbf{I}\right)}{2} \tag{11.16}$$

where the star denotes the *complex conjugate*, $\left(\mathrm{e}^{j\alpha}\right)^* = \mathrm{e}^{-j\alpha}$, and Re is the real part of a complex number. The proof is based on the phasor substitution, that is,

$$\frac{\mathrm{Re}\left(\mathbf{V}\cdot\mathbf{I}^*\right)}{2} = \frac{\mathrm{Re}(V_{\mathrm{m}}\angle\varphi\cdot I_{\mathrm{m}}\angle-\psi)}{2} = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2}\mathrm{Re}(\angle\varphi-\psi) = \frac{V_{\mathrm{m}}I_{\mathrm{m}}}{2}\cos\left(\varphi-\psi\right) \tag{11.17}$$

Example 11.4: The phasor voltage across a purely resistive load with a resistance $R = 10\ \Omega$ is given by $\mathbf{V} = -3 + j3$ [V]. Find the average power delivered to the load.

Solution: According to Eq. (11.16),

$$P = \frac{\mathrm{Re}\left(\mathbf{V}\cdot\mathbf{I}^*\right)}{2} = \frac{\mathrm{Re}\left(\mathbf{V}\cdot\mathbf{V}^*\right)}{2R} = \frac{|\mathbf{V}|^2}{2R} = \frac{3^2+3^2}{20} = \frac{18}{20} = 900\ \mathrm{mW}$$

Exercise 11.3: The phasor voltage and phasor current for an AC load are given by $\mathbf{V} = -3 + j3$ [V], $\mathbf{I} = +2 + j3$ [A]. Find the average power delivered to the load.

Answer: $P = 1.5$ W.

11.1.6 Average Power for Resistor, Capacitor, and Inductor

For arbitrary passive circuit elements, the phasor voltage **V** and the phasor current **I** are related by Ohm's law in the impedance form, $\mathbf{V} = \mathbf{Z}\mathbf{I}$, where **Z** is the element impedance (or the equivalent impedance of a circuit block). We can substitute this result into Eq. (11.16) and obtain

$$P = \frac{\mathrm{Re}(\mathbf{V}\cdot\mathbf{I}^*)}{2} = \frac{\mathrm{Re}(\mathbf{Z}\cdot\mathbf{I}\cdot\mathbf{I}^*)}{2} = \frac{\mathrm{Re}\left(\mathbf{Z}\cdot|\mathbf{I}|^2\right)}{2} = \frac{\mathrm{Re}(\mathbf{Z})|\mathbf{I}|^2}{2} \tag{11.18}$$

since for any complex number $\mathbf{I}$, the following equality holds: $\mathbf{I}\cdot\mathbf{I}^* = I_m\angle\psi\cdot I_m\angle-\psi = I_m^2 = |\mathbf{I}|^2 > 0$. Equation (11.18) is a remarkable result: if the impedance of an element is purely imaginary, the average power delivered to the circuit element must be zero. Indeed, so are the impedances for the inductor and the capacitor. Therefore, the average power delivered to either the inductor or the capacitor must be zero! The same result may be explained using Eq. (11.13). We put the phase of the current, ψ, equal to zero for simplicity. The voltage phase φ will then be $+90°$ for the inductor and $-90°$ for the capacitor. As the cosine of $\pm 90°$ is zero, Eq. (11.13) will also give zero average power. An additional explanation is related to the phasor diagrams for voltages and currents shown in Fig. 11.5. The average power is half of the dot products of two vectors (phasor voltage and phasor current) in the complex plane. The dot product of two perpendicular vectors is exactly zero.

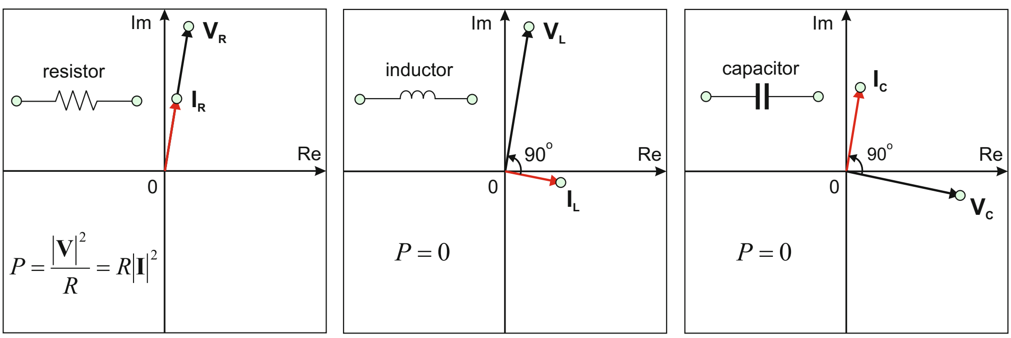


Fig. 11.5. Average power for a resistor, inductor, and capacitor and the related phasor diagrams.

11.1.7 Average Power, Reactive Power, and Apparent Power

We have just seen that the impedance of a load, $\mathbf{Z}$, is most important for the average AC power delivered to the load. If the impedance is a pure resistance, there is no problem. Otherwise, almost no power may be delivered to the load even though large currents can flow in the circuit and large AC voltages are observed. For example, if the load is a pure inductance or capacitance, then no average power will be delivered to the load, no matter which voltages and currents we use. Instead, we will only heat up wires and other circuit components. We write the impedance for an arbitrary load both in rectangular and in polar form:

$$\mathbf{Z} = R + jX \ \ [\Omega], \quad \mathbf{Z} = |\mathbf{Z}|\angle\theta \ \ [\Omega], \quad R = |\mathbf{Z}|\cos\theta, \quad X = |\mathbf{Z}|\sin\theta \tag{11.19a}$$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2} \ [\Omega], \quad \theta = \tan^{-1}\left(\frac{X}{R}\right) \tag{11.19b}$$

The real part of the impedance, R, is the *resistance* of the load, and the imaginary part, X, is the load *reactance*. For example, the inductor and the capacitor are purely reactive loads (have only X but not R), whereas the resistor is purely "resistive". The angle θ is the power angle of the load; the power angle has already been introduced in Eqs. (11.13), (11.14), and (11.15). Thus, the power factor PF is simply the cosine of the angle of the load impedance.

Example 11.5: Determine the resistance and the reactance of an RLC series load shown in Fig. 11.6. The AC angular frequency is 1000 rad/s.

Solution: The three impedances are combined in series (added to each other),

$$\mathbf{Z} = 100 + j\omega L - j\frac{1}{\omega C} = 100 + j1 - j100 = 100 - j99 \ [\Omega] \tag{11.20}$$

The resistance is 100 Ω and the reactance is equal to −99 Ω. The reactance is negative, i.e., *capacitive*. In other words, the capacitive reactance dominates.

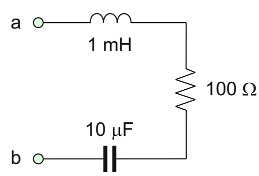


Fig. 11.6. A RLC series load.

Using resistance R and reactance X of the load, we now introduce three *different* AC power types for that load. The first type is the average or *active* power P studied before in this section. The active power is expressed by

$$P = \frac{\mathrm{Re}\left(\mathbf{V} \cdot \mathbf{I}^*\right)}{2} = \frac{R|\mathbf{I}|^2}{2} = \frac{|\mathbf{Z}||\mathbf{I}|^2}{2}\cos\theta = \frac{|\mathbf{V}||\mathbf{I}|}{2}\cos\theta = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos\theta \quad [\mathrm{W}] \tag{11.21a}$$

since $|\mathbf{V}| = V_{\mathrm{m}} = \sqrt{2}V_{\mathrm{rms}}, \quad |\mathbf{I}| = I_{\mathrm{m}} = \sqrt{2}I_{\mathrm{rms}}$. This is *the true or useful power* delivered to the load, with the units of watts. Note the operations with complex magnitudes: $|\mathbf{ab}| = |\mathbf{a}||\mathbf{b}|, \quad |\mathbf{a}/\mathbf{b}| = |\mathbf{a}|/|\mathbf{b}|, \quad |\mathbf{a}^*| = |\mathbf{a}|$, which directly follow from the complex number definition in polar form. The second power type is the *reactive* power Q that is

$$Q = \frac{\mathrm{Im}\left(\mathbf{V}\cdot\mathbf{I}^*\right)}{2} = \frac{X|\mathbf{I}|^2}{2} = \frac{|\mathbf{Z}||\mathbf{I}|^2}{2}\sin\theta = \frac{|\mathbf{V}||\mathbf{I}|}{2}\sin\theta = V_{\mathrm{rms}}I_{\mathrm{rms}}\sin\theta \quad [\mathrm{VAR}] \tag{11.21b}$$

The physical units of the reactive power are also watts. However, to underscore the fact that this power is not an active useful power, the units of VAR *(volt-amperes reactive*) are used. The reactive power flows back and forth from the source to the load, through an electric line but does not do real work. The last power type is the *complex power* **S** that is simply

$$\mathbf{S} = \frac{\mathbf{V}\cdot\mathbf{I}^*}{2} = P + jQ \quad [\mathrm{VA}] \tag{11.21c}$$

The complex power is measured in *volt-amperes* (VA). The magnitude of the complex power $S = |\mathbf{S}|$ is called the *apparent power*. We can see that the apparent power is given by

$$S = |\mathbf{S}| = \frac{|\mathbf{Z}||\mathbf{I}|^2}{2} = \frac{|\mathbf{V}||\mathbf{I}|}{2} = V_{\mathrm{rms}}I_{\mathrm{rms}} \quad [\mathrm{VAR}] \tag{11.21d}$$

The apparent power is the "best possible" load power that can be obtained if one measures current and voltage and ignores the phase shift between them. Equations (11.21) and (11.22) raise the obvious question: why do we need so many AC power types? The answer is that a purely resistive load (the power angle θ equals zero) is merely a dream and not realistic. Any AC load generally has a significant reactive impedance part. So does an electric motor, a small antenna in your cellphone, and even a household electric heater whose heating spiral is a series combination of a resistance and a small, but often visible, inductance. Therefore, we always deal with active and reactive power; the sum of their squares is the square magnitude of the apparent power. The reactive power increases the electric current flowing in the circuit and thus increases the unrecoverable losses in (sometimes very long) power lines, which have a finite resistance. Therefore, our goal is to decrease the percentage of the reactive power and thus decrease the net power loss. The three power definitions show us how to accomplish this task. In the next section, we will need to decrease the power angle θ by modifying the load through adding other circuit components; in other words, we are attempting to *load match* the circuit.

11.1.8 Power Triangle

Since $\cos^2\theta + \sin^2\theta = 1$, the three powers (average, reactive, and apparent) are interconnected by the relation

$$S^2 = P_{\mathrm{avg}}^2 + Q^2 \quad [\mathrm{W}] \tag{11.22}$$

This relation is also called the *power triangle*.

Example 11.6: Determine the average (or true) power and the reactive power for the inductive load shown in Fig 11.7a. Construct the corresponding power triangle. The circuit parameters are as follows:

$$V_m = 170\ \text{V},\quad \omega = 377\ \text{rad/s}, L = 25.7\ \text{mH},\quad R = 9.7\ \Omega.$$

Solution: We need to find active and reactive powers according to Eq. (11.21). To do so, we need the load impedance and the load or circuit current. We convert the circuit to the phasor/impedance form. The equivalent impedance is given by

$$\mathbf{Z} = \mathbf{Z}_R + \mathbf{Z}_L = R + j\omega L = 9.7 + j9.7 = 9.7\sqrt{2}\angle 45^\circ\ [\Omega] \tag{11.23}$$

The power angle (the phase of the complex impedance) is 45°. The phasor voltage $\mathbf{V}$ across the load is equal to $V_m = 170\text{V}$. The load phasor current is given by $\mathbf{I} = V_m/\mathbf{Z} = 170/\left(9.7\sqrt{2}\angle 45^\circ\right) = 12.39\angle -45^\circ\ [\text{A}]$. According to Eq. (11.21),

$$\begin{aligned} P &= 0.5 \times 170 \times 12.39 \times \cos 45^\circ = 745\ [\text{W}] \\ Q &= 0.5 \times 170 \times 12.39 \times \sin 45^\circ = 745\ [\text{VAR}] \end{aligned} \tag{11.24}$$

The corresponding power triangle is plotted in Fig. 11.8a.

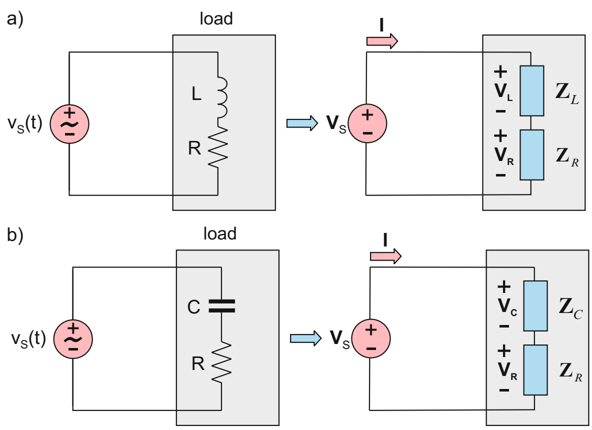


Fig. 11.7. Two circuits for power calculation of (**a**) inductive load and (**b**) capacitive load.

The power angle (the phase of the load impedance) of the power triangle in Fig. 11.8a is +45°. When the power angle θ is positive, as in the present case, the corresponding power factor is said to be *lagging*. The lagging power factor means that the load current lags the load voltage. Thus, the power factor in Fig. 11.8a is 0.707 *lagging* or, which is the same, is 70.7 % *lagging*. Similarly, the power factor in Fig. 11.8b is 44.8 % lagging. However, the power angle (the phase of the load impedance) in Fig. 11.8c is −45°.

When the power angle θ is negative, as is the case here, the corresponding power factor is said to be *leading*. The leading power factor means that the load current leads the load voltage. The power factor in Fig. 11.8c is 70.7 % *leading*. The lagging occurs for a predominantly inductive load, whereas the leading occurs for a predominantly capacitive load; see Fig. 11.7.

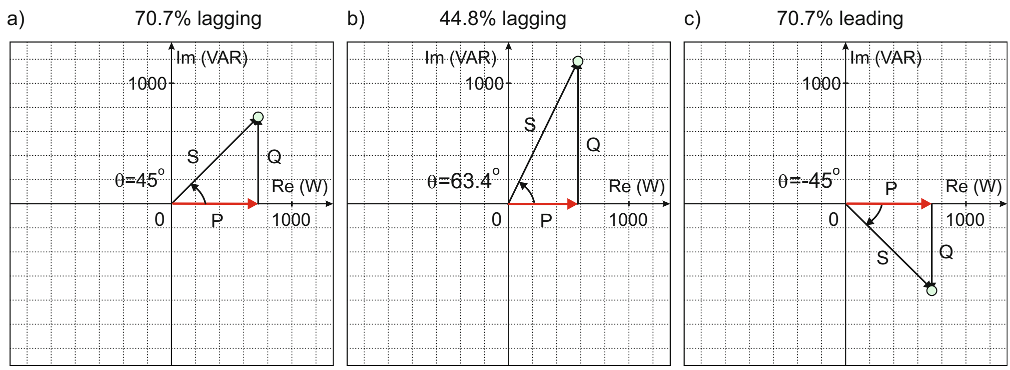


Fig. 11.8. Power triangles for inductive loads (**a**, **b**), and for a capacitive load (**c**). The real axis corresponds to the average (true) power; the imaginary axis is the reactive power.

Exercise 11.4: Determine the average (or true) power and the reactive power for the inductive load shown in Fig 11.7a. Construct the corresponding power triangle. You are given $V_{\mathrm{m}} = 170\ \mathrm{V}$, $\omega = 377\ \mathrm{rad/s}$, $L = 26.5\ \mathrm{mH}$, $R = 5\ \Omega$.

Solution: $P = 578.9\ [\mathrm{W}]$, $Q = 1156.7\ [\mathrm{VAR}]$, $\theta = 63.4°$. The power triangle is plotted in Fig. 11.8b. The power factor is 44.8 % lagging.

Exercise 11.5: Determine the average (or true) power and the reactive power for the capacitive load shown in Fig 11.7b. Construct the corresponding power triangle. The circuit parameters are $V_{\mathrm{m}} = 170\ \mathrm{V}$, $\omega = 377\ \mathrm{rad/s}$, $C = 265\ \mu\mathrm{F}$, $R = 10\ \Omega$.

Answer: $P = 722.5\ [\mathrm{W}]$, $Q = -722.5\ [\mathrm{VAR}]$, $\theta = -45°$. Note that the reactive power becomes negative for the capacitive load. The corresponding power triangle is plotted in Fig. 11.8c. The power factor is 70.7 % leading.

It might be interesting to mention that the circuit in Fig. 11.7b is an equivalent circuit model of a short dipole or monopole *antenna*, the so-called whip antenna; it represents predominantly a capacitive load. Indeed, much higher frequencies are employed, but the concept remains the same. Whip antennas are common on ships, trucks, and other vehicles. Many radio amateurs use whip antennas as well.

11.1.9 Application Example: Wattmeter

AC power is measured with a *wattmeter*. The idea of an analog wattmeter operation is schematically illustrated in Fig. 11.9a. The wattmeter includes at least two coils: the massive immovable *current coil* and the lighter suspended, or pivoted, *voltage* (or *potential*) *coil*. The current coil has a *very low impedance*; it is connected in series with the load in Fig. 11.9b. The voltage coil has a *very high impedance*; it is connected in parallel with the load in Fig. 11.9b. The voltage coil typically has a high-value resistor connected in series to increase the impedance. The voltage coil is constructed of a fine wire, whereas the fixed (current) coil uses a thicker wire to carry the load current. When the current and voltage are in phase, the magnetic fluxes in both coils attempt to align with each other so that the arrow in Fig. 11.9a will move to the right. When the current and voltage are out of phase (phase difference of 180°; the load is in fact an AC source), the arrow in Fig. 11.9a would move to the left. When the phase difference is 90°, the arrow stays at the center. Thus, the power angle could be measured, and the average and reactive powers could be reported in an analog or digital way.

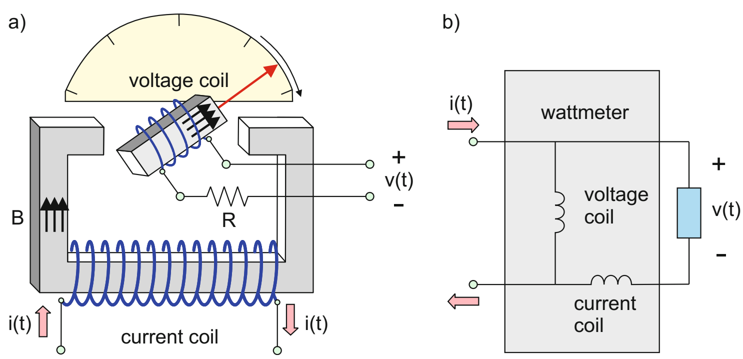


Fig. 11.9. (**a**) Wattmeter concept and (**b**) wattmeter coil connection to the load.

Section 11.2 Power Factor Correction: Maximum Power Efficiency and Maximum Power Transfer

11.2.1 Power Factor Correction

We are about to proceed with the *correction of the power factor* $PF = \cos\theta$ for an arbitrary AC load. The correction has several equivalent definitions make:

1. the power angle exactly equal to zero.
2. the power factor equal to one.
3. the reactive power equal to zero.
4. the load impedance purely resistive.
5. the imaginary part of the load impedance (reactance) equal to zero.

The last definition is perhaps most useful from a practical point of view. We attempt to modify the load by adding extra circuit elements so that the impedance of the *modified* load becomes purely resistive (the imaginary part of the impedance is zero). It is worth noting that the condition of zero reactance is simultaneously the resonance condition for the various RLC tank circuits studied in the previous chapter. It means that we need to make the load "resonant" in order to correct the power factor! This is often achieved by converting the load to an RLC circuit: adding a capacitor to the inductive load or an inductor to a capacitive load. Most residential loads (washer, dryer, air conditioner, refrigerator, etc.) and industrial loads are powered by an induction motor. A simplified equivalent circuit of it is an inductive load shown in Fig. 11.10a. We intend to add a capacitor in parallel with the load as in Fig. 11.10b. We attempt to choose the capacitance value in such a fashion as to make the power factor of the modified load equal to one. The capacitor in Fig. 11.10b is called the *power factor correction capacitor* or the *PFC capacitor*.

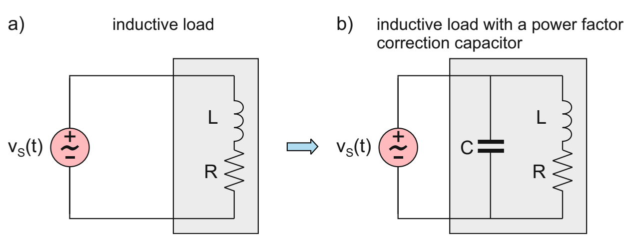


Fig. 11.10. Power correction for an inductive load with the shunt capacitor (capacitor in parallel).

We solve both circuits in Fig. 11.10 in the phasor form. The equivalent impedance (or better its reciprocal, the *admittance*) for the modified load in Fig. 11.10b is found first, that is,

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_C} + \frac{1}{\mathbf{Z}_L + \mathbf{Z}_R} = j\omega C + \frac{R - j\omega L}{R^2 + (\omega L)^2} \tag{11.25}$$

If the impedance is a real number, then the admittance is a real number and vice versa. Therefore, the condition of a real impedance is equivalent to the condition of a real admittance. From Eq. (11.25), one has

$$j\omega C + \frac{-j\omega L}{R^2 + (\omega L)^2} = 0 \Rightarrow R^2 + (\omega L)^2 = \frac{L}{C} \Rightarrow C = \frac{L}{\omega^2 L^2 + R^2} \tag{11.26}$$

The capacitance value is thus found from the equation

$$C = \frac{L}{\omega^2 L^2 + R^2} \tag{11.27}$$

Equation (11.27) is a mathematical statement for the power factor correction capacitor. Its practical value will become apparent from the example that follows. The equivalent impedance of the load with the matching capacitor is then found using the real part of Eq. (11.25), i.e.,

$$\frac{1}{\mathbf{Z}} = \frac{R}{R^2 + (\omega L)^2} \Rightarrow \mathbf{Z} = R + \frac{(\omega L)^2}{R} \tag{11.28}$$

We are interested in the phasor circuit current **I** with and without the PFC capacitor. Given the voltage source $v_S(t) = V_m \cos \omega t$, one obtains for the circuits in Fig. 11.10:

$$\mathbf{I} = \overbrace{\underbrace{\frac{RV_m}{R^2 + (\omega L)^2}}_{\text{with capacitor}} - \frac{j\omega L V_m}{R^2 + (\omega L)^2}}^{\text{without capacitor}} \tag{11.29}$$

As you can see, the two terms of the expression without capacitor are reduced to the first term when the power correction capacitor is included. This completes the analysis of the circuits in Fig. 11.10.

Example 11.7: For the circuit in Fig. 11.10, find the average (or true) load power and the reactive load power with and without the power correction factor capacitor. You are given $V_m = 170\ \text{V}, \quad \omega = 377\ \text{rad/s}, \quad L = 25.7\ \text{mH}, R = 9.7\ \Omega$.

Example 11.7 (cont.):

Solution: To find the power expressions, we need the phasor voltage across the load, which is simply V_m. We also need the phasor current, which is given by Eq. (11.29) in either case. Plugging in the numbers we obtain,

$$\mathbf{I} = 8.77 \text{ A} \quad \text{or} \quad \mathbf{I} = 8.77 - j8.77 \text{ A} \tag{11.30}$$

with and without the PFC capacitor, respectively. Now, we use the power definitions:

$$P = \frac{\text{Re}(\mathbf{V} \cdot \mathbf{I}^*)}{2} \Rightarrow P = 745 \text{ W} \quad \text{or} \quad P = 745 \text{ W} \tag{11.31}$$

$$Q = \frac{\text{Im}(\mathbf{V} \cdot \mathbf{I}^*)}{2} \Rightarrow Q = 0 \text{ VAR} \quad \text{or} \quad Q = 745 \text{ VAR} \tag{11.32}$$

with and without the PFC capacitor, respectively.

In summary, we have found the following information from this example:

No power factor correction:

- Average (active or true) power: $P = 745$ W
- Reactive power: $Q = 745$ VAR
- Amplitude of the circuit current: 12.41 A

Power factor correction:

- Average (active or true) power: $P = 745$ W
- Reactive power: 0
- Amplitude of the circuit current: 8.77 A

By correcting the load power factor with the capacitor in parallel, we did *not* change the average power (power delivered to the load), but we eliminated the reactive power and decreased the amplitude of the circuit current by 70.7 % ($1/\sqrt{2}$). This means that the ohmic losses in the electric line connecting load and generator will decrease by 50 %, since these losses are proportional to the square of the current amplitude. In other examples, the loss reduction factor may be even more significant. Is it worth doing a power factor correction? Well, if the electric power line is long enough or the initial power factor is not high enough, it is definitely a very useful and professional task. To support this conclusion, we mention a citation from *IEEE Transactions on Power Electronics*: "Everyone knows that correcting power factor is the easiest and fastest way to save energy dollars."

Exercise 11.6: Find the value of the power factor correction capacitor in Example 11.7.

Answer: $C = 136.73\ \mu\text{F}$.

Exercise 11.7: Find the value of the load impedance in Example 11.7 with and without the power factor correction capacitor. Express your result in polar form.

Answer: $\mathbf{Z} = 13.7\angle 45^\circ\ \Omega$ and $\mathbf{Z} = 19.4\angle 0^\circ\ \Omega$, respectively.

11.2.2 Application Example: Automatic Power Factor Correction System

The power factor correction capacitors are frequently seen on residential power poles in the form of pole-mounted capacitor banks. Figure 11.11 shows an automatically switched power factor correction system that measures all three power types (active, reactive, and apparent power) using the same wattmeter principle described in the previous section. Based on the recorded measurements, the required capacitor value is selected, which assures the targeted power factor.

Fig. 11.11. Automatically switched power factor correction systems for low-voltage applications. Six capacitor cells are seen on the bottom. Technical Data TD02607001E Cutler-Hammer.

11.2.3 Principle of Maximum Power Efficiency for AC Circuits

Why is the power correction capacitor placed in parallel, not in series with the load? To answer this question, we should establish and understand the *principle of maximum power efficiency for AC circuits*. Consider a generic source-load AC circuit depicted in Fig. 11.12a in phasor form. It is based on a Thévenin equivalent circuit for an AC source with the source impedance $\mathbf{Z}_\text{T}$ connected to the load impedance $\mathbf{Z}_\text{L}$ (note: load is subscribed as L, not L). The source impedance will also include the loss resistance of power lines. Figure 11.12b shows the corresponding DC counterpart, which is useful for the subsequent analysis. The source resistance R_T will include the loss resistance of power lines as well.

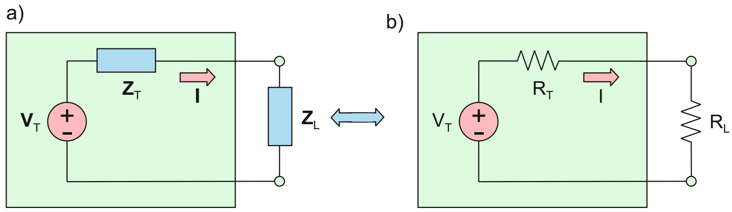


Fig. 11.12. Thévenin equivalent circuit for an AC source with the load impedance $\mathbf{Z}_L$ and its DC counterpart with the load resistance R_L.

In the DC case, the source-load configuration of Fig. 11.12b is *maximally efficient* when the load resistance is much greater than the source (loss) resistance. This fact has been established in Chapter 4. When $R_L \gg R_T$, the useful power delivered to the load resistance R_L is much larger than the power loss in R_T. We emphasize that the load power itself could be relatively small in this case as compared to the maximum available load power at $R_L = R_T$. The same situation occurs for the AC system shown in Fig. 11.12a. Do we wish to deliver the maximum available power of a remote megawatt AC source to the household? No, this is not our goal. We would rather deliver a reasonable amount of power but in a most efficient way. It means that not only do we need to make $\mathbf{Z}_L$ real but also *as high as possible*. This operation would further reduce the circuit current and the associated loss. Exactly this goal is accomplished by the shunt PFC capacitor in Fig. 11.10. If we consider the series-connected PFC capacitor as an alternative, we will obtain

$$\mathbf{Z}_L = R \quad \text{for series connection} \quad \text{versus} \quad \mathbf{Z}_L = R + \frac{(\omega L)^2}{R} \quad \text{for shunt connection.} \tag{11.33}$$

Both impedances in Eq. (11.33) are real; there is no reactive power in either case. However, the second impedance is considerably greater than the first one for poor power factors, i.e., for $\omega L \gg R$! Hence, considerably smaller circuit currents and considerably better efficiencies are achieved. Moreover, the parallel connection is easier to accomplish in practice—we remember how easy it is to connect a voltmeter as opposed to an ammeter

Exercise 11.8: Find the amplitude of the circuit current in Example 11.7 if the power correction capacitor were in series.

Answer: $I_m = 17.53$ A.

11.2.4 Principle of Maximum Power Transfer for AC Circuits

The *principle of maximum power transfer* is perhaps less important for residential power distribution systems where efficiency counts. However, it is critical for radio-frequency and communication circuits, which are conceptually the same AC circuits but operating at much higher frequencies. With reference to Fig. 11.12a, the following question should now be asked: at which value of the load impedance $\mathbf{Z}_\mathrm{L} = R_\mathrm{L} + jX_\mathrm{L}$ is the average (true) power delivered to the load maximized? The phasor current in Fig. 11.12a is given by

$$\mathbf{I} = \frac{\mathbf{V}_\mathrm{T}}{\mathbf{Z}_\mathrm{L} + \mathbf{Z}_\mathrm{T}} \ [\mathbf{A}] \tag{11.34}$$

so that the average power delivered to the load becomes

$$P = \frac{R_\mathrm{L}|\mathbf{I}|^2}{2} = \frac{R_\mathrm{L}|\mathbf{V}_\mathrm{T}|^2}{2|\mathbf{Z}_\mathrm{L} + \mathbf{Z}_\mathrm{T}|^2} = \frac{0.5R_\mathrm{L}|\mathbf{V}_\mathrm{T}|^2}{(R_\mathrm{L} + R_\mathrm{T})^2 + (X_\mathrm{L} + X_\mathrm{T})^2} \quad [\mathrm{W}] \tag{11.35}$$

Let us take a closer look at Eq. (11.35); in order to reach the maximum true power, the load reactance X_L should be equal to the generator reactance X_T taken with the opposite sign so that $X_\mathrm{L} + X_\mathrm{T} = 0$. This yields for the average load power

$$P = \frac{0.5R_\mathrm{L}|\mathbf{V}_\mathrm{T}|^2}{(R_\mathrm{L} + R_\mathrm{T})^2} \quad [\mathrm{W}] \tag{11.36}$$

Consequently, the problem reduces to the maximum power transfer of a DC circuit as studied in Chapter 2. The corresponding condition for the maximum load power is

$$R_\mathrm{L} = R_\mathrm{T} \tag{11.37}$$

This condition, augmented by the equality for the reactances

$$X_\mathrm{L} = -X_\mathrm{T} \tag{11.38}$$

leads to a simple, yet very useful result for the maximum power transfer to the load:

$$\mathbf{Z}_\mathrm{L} = \mathbf{Z}_\mathrm{T}^* \Rightarrow P_\mathrm{max} = \frac{1}{8}\frac{|\mathbf{V}_\mathrm{T}|^2}{R_\mathrm{T}} \quad [\mathrm{W}] \tag{11.39}$$

We note that the load impedance should be the complex conjugate of the generator impedance. Along with the maximum power transfer, Eq. (11.39) assures that there is no reflection of radio-frequency waves propagating along the circuit transmission lines from the source to the load, which may even be a more important factor. A process of modifying the load impedance in order to satisfy Eq. (11.39) is called *impedance matching*.

Example 11.8: A generator impedance is 50 Ω. The load impedance is $10 + j100$ Ω. What percentage of maximum available power (at a load impedance of 50 Ω) is transferred to the load?

Solution: According to Eq. (11.36), when the load impedance is exactly 50 Ω,

$$P = \frac{0.5 \times 50|\mathbf{V}_\mathrm{T}|^2}{(50+50)^2} = \frac{|\mathbf{V}_\mathrm{T}|^2}{8 \times 50} = 0.0025|\mathbf{V}_\mathrm{T}|^2 \quad [\mathrm{W}] \tag{11.40a}$$

When the load impedance is $10 + j100\Omega$, the same equation gives

$$P = \frac{0.5 \times 10|\mathbf{V}_\mathrm{T}|^2}{(50+10)^2 + 100^2} = \frac{|\mathbf{V}_\mathrm{T}|^2}{2720} = 0.00037|\mathbf{V}_\mathrm{T}|^2 \quad [\mathrm{W}] \tag{11.40b}$$

The ratio of the two power expressions is 0.147, or 14.7%. In other words, 85.3% of available power is lost!

Exercise 11.9: A generator's impedance is $50 - j100$ Ω. What should the load impedance be for maximum power transfer?

Answer: $50 + j100$ Ω.

Exercise 11.10: Solve Example 11.8 when the load impedance is $10 - j100$ Ω.

Answer: The same result of 14.7 % is obtained.

Section 11.3 AC Power Distribution: Balanced Three-Phase Power Distribution System

11.3.1 AC Power Distribution Systems

Representative AC power distribution systems are shown in Fig. 11.13. A *single-phase two-wire power distribution system* is depicted in Fig. 11.13a. It consists of a generator with a voltage amplitude of V_m, an rms value of $V_{rms} = V_m/\sqrt{2}$, and a phase φ connected through two conductors to a load with impedance **Z**. The previous analysis of AC power was solely restricted to this configuration. An extension is the *single-phase three-wire power distribution system* shown in Fig. 11.13b. Such a system contains two identical AC sources of the same amplitude and phase connected to two ($\mathbf{Z}_1$, $\mathbf{Z}_2$) loads or to one (**Z**) load through two outer conductors and the *neutral conductor* (or *neutral wire*). This system is the common household distribution system. It allows us to connect both 120-V and 240-V appliances as shown in Fig. 11.13b; we sometimes called it the *split-phase distribution system*. The neutral wire is usually physically grounded. In contrast to those two cases, the power distribution systems shown in Fig. 11.13c, d are the *polyphase distribution systems* in the sense that they use AC sources with *different* phases. For example, Fig. 11.13c illustrates a *two-phase three-wire distribution system* with two voltage sources; the second one lags the former by 90°. Finally, Fig. 11.13d shows the most important and practical *three-phase four-wire power distribution system* with three sources and three load impedances $\mathbf{Z}_1$, $\mathbf{Z}_2$, and $\mathbf{Z}_3$. Generally, the three-phase system also uses a (grounded) neutral wire. We will show that this wire may be omitted for balanced power distribution circuits, with the earth itself acting as the neutral conductor. This is important for long-distance, high-power transmissions. Power systems designed in this way are grounded at critical points to ensure safety.

Today, a vast majority of electric power is generated and distributed via the three-phase power systems. Why is this so? You will soon learn that in contrast to the single-phase systems, the instantaneous power in balanced three-phase systems is constant or independent of time rather than pulsating. This circumstance results in more uniform power transmission and less vibration of electric machines. Furthermore, three-phase AC motors have a nonzero starting torque in contrast to the single-phase motors. Last but not least, it will be shown that the three-phase system surprisingly requires a *lesser* amount of wire compared to the single-phase system.

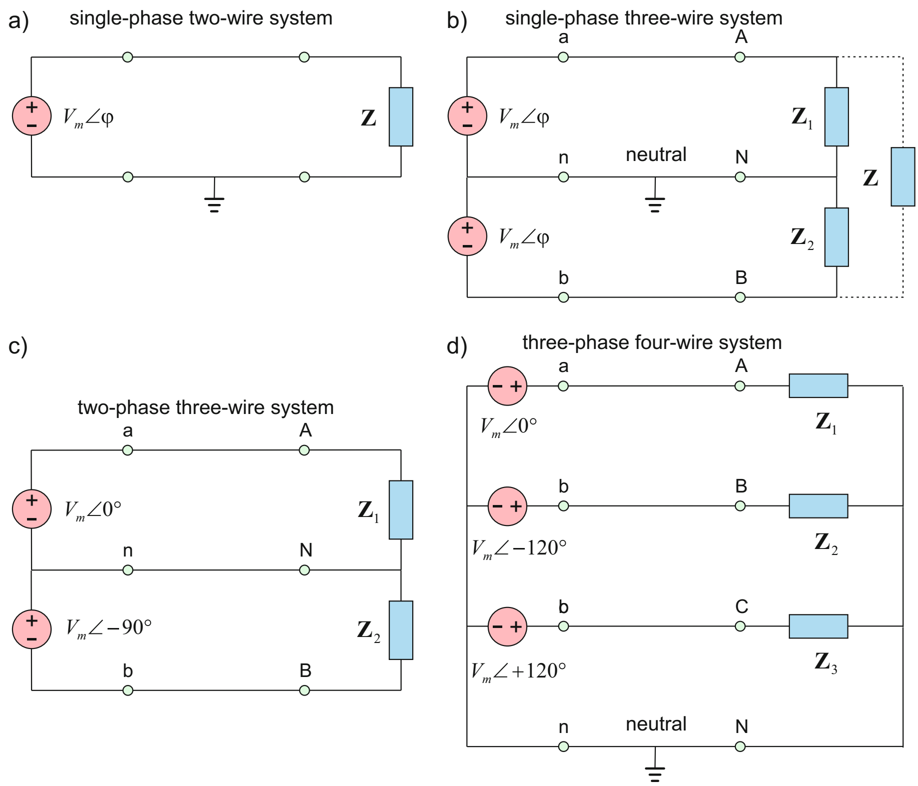


Fig. 11.13. Various AC power distribution systems. N or n indicates the neutral line.

11.3.2 Phase Voltages: Phase Sequence

The voltage sources in the three-phase system in Fig. 11.13d are set between lines a, b, c and the neutral line n. Those voltages are called *phase voltages* or *line-to-neutral voltages*. The phase voltages are 120° out of phase. One possible scenario for the real-valued phase voltages is

$$v_{\mathrm{an}}(t) = V_{\mathrm{m}} \cos(\omega t), \quad v_{\mathrm{bn}}(t) = V_{\mathrm{m}} \cos(\omega t - 120°), \quad v_{\mathrm{cn}}(t) = V_{\mathrm{m}} \cos(\omega t + 120°) \tag{11.41a}$$

$$\mathbf{V}_{\mathrm{an}} = V_{\mathrm{m}}, \quad \mathbf{V}_{\mathrm{bn}} = V_{\mathrm{m}}\angle - 120°, \quad \mathbf{V}_{\mathrm{cn}} = V_{\mathrm{m}}\angle + 120° \tag{11.41b}$$

Phase voltage v_{an} leads phase voltage v_{bn}, which in turn leads v_{cn}. This set of voltages is shown in Fig. 11.14. It has a *positive* or *abc phase sequence* since the voltages reach their peak values in the order *abc* as seen in Fig. 11.14. Simultaneously, the phasor voltages are obtained from each other by clockwise rotation in the phasor diagram. This is shown in Fig. 11.15a.

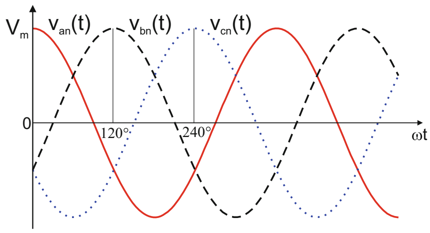


Fig. 11.14. Balanced phase voltages in positive phase sequence.

An alternative is the *negative* or *acb* phase sequence, which corresponds to $0, +120°, \; -120°$ phases in Eqs. (11.41a, b). In this case, the phasor voltages are obtained from each other by counterclockwise rotation in the phasor diagram of Fig. 11.15a. Thus, the *balanced phase voltages* are those which have *equal* amplitudes and are out of phase with each other by 120° (either in positive $0, \; -120°, \; +120°$ or in negative $0, +120°, \; -120°$ phase sequence). An example of balanced voltages is given by Eqs. (11.41a, b). The concept of balanced phase voltages is critical for the subsequent analysis.

Example 11.9: Determine whether the phase voltages

$$v_{\mathrm{an}}(t) = 3\cos(\omega t - 90°), \quad v_{\mathrm{bn}}(t) = 3\cos(\omega t + 150°), \quad v_{\mathrm{cn}}(t) = 3\cos(\omega t + 30°) \qquad (11.42)$$

of a three-phase system are balanced or not. If yes, determine the corresponding phase sequence.

Solution: The amplitudes of the phase voltages are equal, which is the first necessary condition of the balanced sources. To analyze the phases, we plot the voltages in the phasor diagram and obtain Fig. 11.15b. Despite the common phase shift of $-90°$ as compared to Eqs. (11.41a, b), the phase voltages are still out of phase with each other by 120° and form the same positive phase sequence; see Fig. 11.15b.

Exercise 11.11: The phase voltage $\mathbf{V}_{\mathrm{bn}}$ is given by $V_{\mathrm{m}}\angle + 45°$. Determine the remaining phase voltages $\mathbf{V}_{\mathrm{an}}$, $\mathbf{V}_{\mathrm{cn}}$ of the balanced three-phase system for the positive phase sequence. Express your result in phasor form.

Answer: $\mathbf{V}_{\mathrm{an}} = V_{\mathrm{m}}\angle 165°, \quad \mathbf{V}_{\mathrm{cn}} = V_{\mathrm{m}}\angle -75°$.

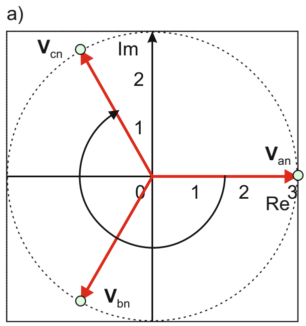


Fig. 11.15. (**a**) Phasor diagram for the phase sequence $0, \quad +120°, \quad -120°$ and (**b**) phasor diagram for individual phase voltages from Eq. (11.42). Both three-phase sources are equivalent.

11.3.3 Wye (Y) Source and Load Configurations for Three-Phase Circuits

The voltage sources in the three-phase system in Fig. 11.13d are now rearranged as shown in Fig. 11.16a. This configuration is indeed equivalent to the original one; it is known as the *wye* (or *Y*) *configuration*. Accordingly, the *balanced three-phase source* in Fig. 11.16a is the *wye-connected source,* and the load in Fig. 11.16b is the *wye-connected load*.

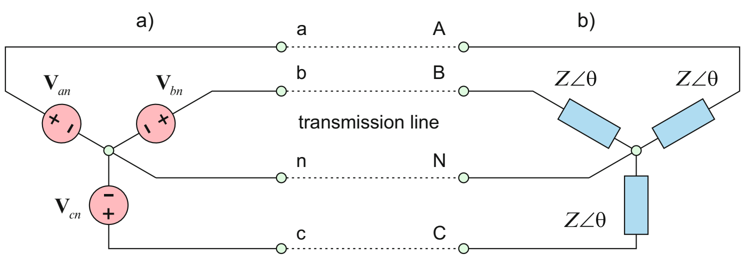


Fig. 11.16. Wye configuration for the three-phase source connected to a three-phase load.

The AC voltage source in Fig. 11.16a has four terminals. The corresponding load should also have four terminals. The concept is shown in Fig. 11.16b. This load assembly is also identical with the topology of Fig. 11.16d. The load includes three impedance elements *(phase impedances* or *load impedances per phase*) $\mathbf{Z} = Z\angle\theta$ with impedance magnitude Z and phasor angle (power angle) θ each. The load so assembled is the *balanced three-phase load*. In the balanced load, the phase impedances are equal in magnitude and phase. The source and the load are typically connected by (long) wire transmission lines. When necessary, the wire resistance may be added to each individual load impedance.

11.3.4 Application: Examples of Three-Phase Source and the Load

Synchronous Three-Phase AC Generator

Despite the apparent complexity, the three-phase source and the three-phase loads are relatively simple to realize in practice. Figure 11.17 shows *a synchronous three-phase AC generator* (or *alternator*), which is equivalent to the three-phase source in Fig. 11.16a.

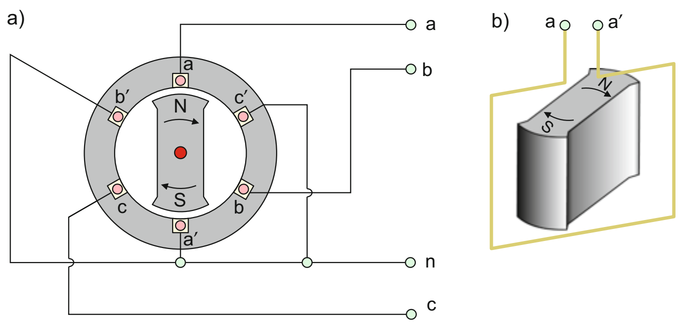


Fig. 11.17. Structure of the three-phase AC generator. (**a**) Cross-section view. (**b**) Simplified outline of one of the three windings.

Consider first the generator cross section shown in Fig. 11.17a. The generator's *rotor* is a permanent magnet (small scale) or an electromagnet (industrial scale) rotated by a mechanical torque (a turbine). Three individual coil windings aa', bb', and cc' in the *stator* are spaced exactly 120° apart around the stator. When the rotor moves, an induced emf (induced voltage) will be created in every individual winding according to Faraday's law of induction—see Fig. 11.17b. From the geometry considerations, the induced voltages are equal in magnitude and out of phase by 120°. When the coil terminals a', b', and c' are all connected to the neutral wire, see Fig. 11.17a, we obtain exactly the three-phase source with the neutral wire in Fig. 11.16a.

Automotive Alternator

The automotive alternator operates based on the same principle. However, the resulting three-phase voltage is further converted to the DC voltage (rectified, see Chapter 16).

Synchronous Three-Phase AC Motor

The counterpart of the three-phase generator is the three-phase AC motor (*the synchronous AC motor*). The stator, which is subject to the three-phase voltage source, creates a *rotating magnetic field*; the rotor magnet is aligned with this field at every time moment and rotates accordingly. The stator's circuit model is similar to the three-phase load model in Fig. 11.16b where each load impedance **Z** includes resistance and inductance of the individual (identical) coil windings. The induced emf should be included into our consideration as well. Why is the phase sequence important for power distribution? There is a simple answer to this question. Assume that the machine in Fig. 11.17 operates in the motor mode. Changing the phase sequence from *abc* to *acb* will reverse the direction of the magnetic field rotation and thus reverse the direction of the motor rotation! This method is used in practice since it requires interchanging only two connections.

Residential Household

Another example of the load impedance per phase is related to a typical residential household in the USA. A single phase of a three-phase residential distribution system is normally used to power them up; see Fig. 11.18. This single phase still has a high rms voltage (4800 V or 7200 V). A step-down center-tap transformer is used to decrease this voltage level to the desired level of 120–240 V and provide the neutral contact necessary for the three-wire single-phase residential system shown in Fig. 11.13b. This transformer case is also seen in Fig. 11.18. In the USA, a pole-mounted transformer in a suburban setting may supply one to three houses.

Fig. 11.18. Three-phase to three-wire residential system connected via a step-down transformer. From the pole transformer, the residential power system serving two houses is run down the pole underground. Cape Cod, MA.

11.3.5 Solution for the Balanced Three-Phase Wye-Wye Circuit

Phase Voltages and Line Voltages

A three-phase balanced circuit (*wye-wye configuration*) which includes the source and the load is shown in Fig. 11.19. We place the nodes *n* and *N* at the originally anticipated center positions. The positive phase sequence of 0, $-120°$, $+120°$ is assumed. The sum of phase voltages is to be found first. In the phasor form,

$$\begin{aligned}\mathbf{V}_{\mathrm{an}} + \mathbf{V}_{\mathrm{bn}} + \mathbf{V}_{\mathrm{cn}} &= V_{\mathrm{m}}(1 + 1\angle{-120°} + 1\angle{+120°}) = V_{\mathrm{m}}(1 + 2\cos 120°) \\ &= V_{\mathrm{m}}(1 - 1) = 0\end{aligned} \tag{11.43}$$

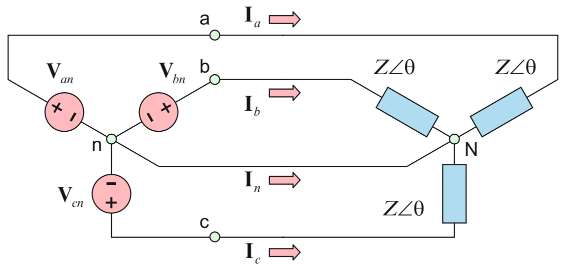


Fig. 11.19. Three-phase, four-wire balanced *wye-wye distribution system*. Ground connection is implied for the neutral wire.

Thus, the sum of the balanced phase voltages is exactly *zero*, either in the phasor form or in the time domain. Now, along with the phase (line-to-neutral) voltages, we define *line-to-line voltages* (or just *line voltages*) $\mathbf{V}_{\rm ab}$, $\mathbf{V}_{\rm bc}$, $\mathbf{V}_{\rm ca}$ between nodes *a*–*b*, *b*–*c*, and *c*–*a*, as indicated in Fig. 11.19. These voltages are expressed through the phase voltages using KVL. Using the trigonometric identity $1 - 1\angle{-120°} = \sqrt{3}\angle 30°$ three times, it can be shown that

$$\begin{aligned}
\mathbf{V}_{\rm ab} &= \mathbf{V}_{\rm an} - \mathbf{V}_{\rm bn} = \sqrt{3}\mathbf{V}_{\rm an}\angle 30°, \\
\mathbf{V}_{\rm bc} &= \mathbf{V}_{\rm bn} - \mathbf{V}_{\rm cn} = \sqrt{3}\mathbf{V}_{\rm bn}\angle 30°, \\
\mathbf{V}_{\rm ca} &= \mathbf{V}_{\rm cn} - \mathbf{V}_{\rm an} = \sqrt{3}\mathbf{V}_{\rm cn}\angle 30°
\end{aligned} \tag{11.44}$$

It is seen that the line voltages are higher in amplitude than the phasor voltages by a factor of $\sqrt{3} \approx 1.73$. Furthermore, they lead their corresponding phase voltages by 30°. According to Eqs. (11.43) and (11.44), the sum of the line voltages is also equal to zero. Both the phase voltages and the line voltages may be used in the three-phase system.

Exercise 11.12: Is Eq. (11.44) also valid for the negative phase sequence?

Answer: Not exactly. A substitution $30° \rightarrow -30°$ has to be made.

Example 11.10: The electric service for commercial buildings (university campus buildings) in the USA is a three-phase, four-wire wye system schematically shown in Fig. 11.19. Determine rms phase voltages if the line voltages are all equal to 208 V rms.

Solution: According to Eq. (11.44), we should divide the line voltage of 208 V rms by $\sqrt{3}$. This gives us exactly 120 V rms voltage per phase. Thus, the present wye system is also powering common 120 V wall plugs with any of the line-to-neutral voltages. Note that the source in Fig. 11.19 typically models an output of a three-phase transformer.

Line Currents: Per-Phase Solution

The currents $\mathbf{I}_{a,b,c}$ in Fig. 11.19 are called *line currents*. To find the line currents, the circuit may be solved separately for every phase using the *superposition principle*. The superposition principle implies shorting out two of the three voltage sources at a time. This method applies to both balanced and unbalanced circuits. Shorting out voltage sources $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$ leads to a single-phase equivalent circuit, shown in Fig. 11.20, since the two remaining source impedances will be shorted out by the neutral wire. As long as the system is balanced, the same equivalent circuit will be derived for every other phase.

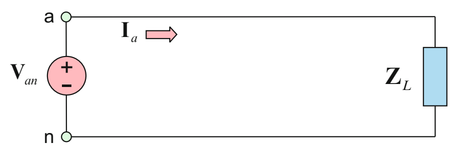


Fig. 11.20. A single-phase equivalent circuit by shorting out $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$.

Applying this method to every phase, we obtain

$$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z} = I_m\angle-\theta, \quad \mathbf{I}_b = \mathbf{V}_{bn}/\mathbf{Z} = I_m\angle{-120^\circ}-\theta, \\ \mathbf{I}_c = \mathbf{V}_{cn}/\mathbf{Z} = I_m\angle 120^\circ{-\theta} \tag{11.45}$$

where $I_m = V_m/Z$. The sum of the line currents is given by

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = (\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn})/\mathbf{Z} = 0 \tag{11.46}$$

according to Eq. (11.43). Thus, the sum of the balanced line currents is also exactly *zero*, either in phasor form or in the time domain. Equations (11.43), (11.44), (11.45), and (11.46) hold for any phase sequence, with or without the common phase shift.

11.3.6 Removing the Neutral Wire in Long-Distance Power Transmission

Equation (11.46) for the line currents has an important implication. By taking into account Eq. (11.46), KCL for node *n* in Fig. 11.19 yields

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \tag{11.47}$$

Equation (11.47) states that the neutral conductor in the *balanced circuit* carries *no* current. Such a wire could in principle be *removed* from the balanced circuit *without* affecting the rest of it. Removing the neutral conductor is economically beneficial in long-distance high-voltage power transmission, which utilizes the balanced circuits. In high-voltage power lines, the conductors in multiples of three are used; see Fig. 11.21.

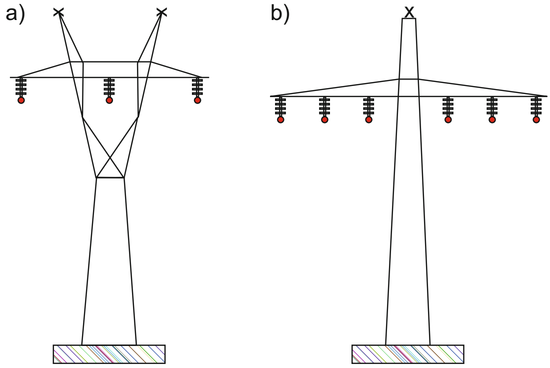


Fig. 11.21. (**a**) Three-phase single-circuit high-voltage overhead power transmission line and (**b**) three-phase double-circuit, high-voltage overhead line. Both lines include (thinner) shielding wire (s) on top of it to protect against lightning strikes (F. Kiessling, et al., "*Overhead Power Lines: Planning, Design, Construction*," Springer 2003).

In fact, the neutral wire is not removed entirely since the earth ground itself plays the role of the neutral conductor. We will not draw the neutral wire in the three-phase *balanced* wye-wye circuit; in Fig. 11.22 only three wires are drawn. However, the meaning of the phase voltage or the phase-to-neutral voltage still remains unchanged—this voltage is simply defined with regard to the reference node n in Fig. 11.22.

If a three-phase circuit is unbalanced, like having different individual loads in Fig. 11.19, then a significant current may flow in the neutral wire. The neutral wire is thus meant to carry unbalanced currents in the electrical system. It should be kept in place for potentially unbalanced systems.

Example 11.11: Determine line currents in the balanced three-phase wye-wye circuit shown in Fig. 11.22 given the *acb* sequence of phase voltages $\mathbf{V}_{an} = 325\angle 0°$, $\mathbf{V}_{bn} = 325\angle 120°$, $\mathbf{V}_{cn} = 325\angle -120°$ [V] and load impedance per phase $\mathbf{Z} = 8.333 + j14.434\ \Omega$.

Solution: The three-phase circuit in Fig. 11.22 is balanced; hence, the single-phase circuit in Fig. 11.20 applies to *every* phase (to visualize the per-phase method, we can still imagine the neural wire present). We convert the load impedance to polar form first, i.e., $\mathbf{Z} = 16.667\angle 60°\ \Omega$. Then, we solve the circuit in Fig. 11.20 for every phase and obtain $\mathbf{I}_a = 19.5\angle -60°$, $\mathbf{I}_b = 19.5\angle 60°$, $\mathbf{I}_c = 19.5\angle 180°$ [A]. The solution is shown in the phasor diagram in Fig. 11.23. Note that the phasor voltages/currents are obtained from each other by counter clockwise rotation in the phasor diagram, which corresponds to the negative or *acb* phase sequence. Also note that the rms values for the phase voltages in this example are 230 V, which corresponds to the European residential power distribution system.

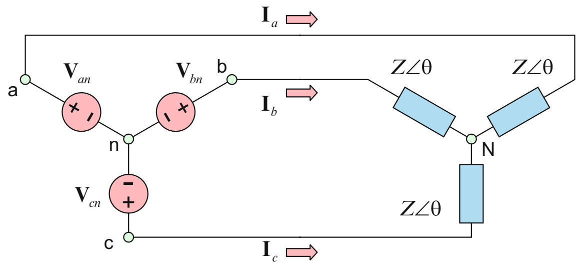


Fig. 11.22. Three-phase balanced wye-wye system with the neutral conductor removed. The neutral conductor may still be implied for the solution using the per-phase method.

The wye-wye circuit in Fig. 11.22, along with the similar circuits in Figs. 11.13 and 11.19, may contain extra impedances. Those are *line impedance*, which characterizes transmission line loss and inductance, and *source impedances*, which are present for nonideal voltage sources. Fortunately, all those (equal) impedances are combined in series along the line into one impedance **Z** which is called the *total load impedance per phase*. In this sense, Fig. 11.22 represents this general case as well.

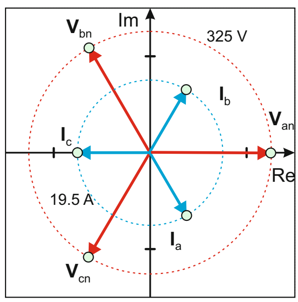


Fig. 11.23. Phasor diagram for the three-phase circuit of Example 11.11. Note the separate scales for the phasors of voltages and phasors of currents.

Section 11.4 Power in Balanced Three-Phase Systems: Delta-connected Three-Phase Circuits

11.4.1 Instantaneous Power

The analysis of the instantaneous power requires a source-load circuit in terms of real-value expressions of voltages and currents. This is shown in Fig. 11.24 for the wye-wye configuration. We assume a balanced source and a balanced load. This means that individual loads *a*, *b*, and *c* in Fig. 11.24 are identical. Each of them can be a mixed RLC load, with an arbitrary impedance $\mathbf{Z} = Z\angle\theta$.

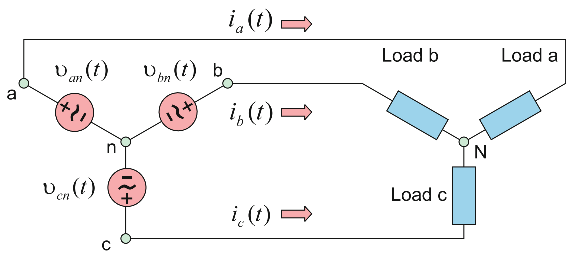


Fig. 11.24. Three-phase balanced wye-wye circuit in the time domain. Three individual loads are identical. Each of them is characterized by the impedance $\mathbf{Z} = Z\angle\theta$ in the frequency domain.

We consider the positive phase sequence. According to Eqs. (11.41) and (11.45) of the previous section, the phase voltages and line currents in Fig. 11.24 are given by

$$\upsilon_{\rm an}(t) = V_{\rm m}\cos(\omega t), \quad \upsilon_{\rm bn}(t) = V_{\rm m}\cos(\omega t - 120^\circ), \quad \upsilon_{\rm cn}(t) = V_{\rm m}\cos(\omega t + 120^\circ) \tag{11.48a}$$

$$i_{\rm a}(t) = I_{\rm m}\cos(\omega t - \theta), \quad i_{\rm b}(t) = I_{\rm m}\cos(\omega t - 120^\circ - \theta), \quad i_{\rm c}(t) = I_{\rm m}\cos(\omega t + 120^\circ - \theta) \tag{11.48b}$$

The *total instantaneous load power of the three-phase system* is the sum of the three power contributions for each phase voltage, that is,

$$p(t) = \upsilon_{\rm an}(t)i_{\rm a}(t) + \upsilon_{\rm bn}(t)i_{\rm b}(t) + \upsilon_{\rm cn}(t)i_{\rm c}(t) \tag{11.49}$$

Every summand on the right-hand side of Eq. (11.49) is the product of two cosines. To transform this product back to cosines, we use the trigonometric identity $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$ and obtain

$$p(t) = \frac{3}{2}V_{\rm m}I_{\rm m}\cos\theta + \frac{1}{2}V_{\rm m}I_{\rm m}\left[\cos(2\omega t - \theta) + \underbrace{\cos(2\omega t - \theta + 120^\circ) + \cos(2\omega t - \theta - 120^\circ)}\right] \tag{11.50}$$

We then use the above trigonometric identity again to convert the underlined term to $2\cos(2\omega t-\theta)\cos(120°) = -\cos(2\omega t-\theta)$. Consequently, the entire term in the square brackets in Eq. (11.50) is equal to zero, and the final result for the instantaneous power is

$$p(t) = \frac{3}{2}V_{\mathrm{m}}I_{\mathrm{m}}\cos\theta = 3V_{\mathrm{rms}}I_{\mathrm{rms}}\cos\theta = \mathrm{const}\,! \tag{11.51}$$

where the rms values of phase voltages and line currents are indeed related to the amplitudes by $V_{\mathrm{m}} = \sqrt{2}V_{\mathrm{rms}}, \quad I_{\mathrm{m}} = \sqrt{2}I_{\mathrm{rms}}$.

Example 11.12: A balanced wye-wye three-phase system in Fig. 11.24 operates at 60 Hz. The line-to-neutral voltages have the amplitudes of 170 V, $V_{\mathrm{m}} = 170$ V. Every phase impedance is a 77.2-mH inductance in series with a 29.1-Ω resistance. Find the instantaneous load power.

Solution: The first step is to find the impedance for every phase of the load. We have

$$\mathbf{Z} = R + j\omega L = 29.1 + j29.1\ \ \Omega = 29.1\sqrt{2}\angle 45°\ \ \Omega \tag{11.52}$$

Next, we find the line currents. Since the circuit is balanced, the per-phase solution applies, with the equivalent circuit shown in Fig. 11.20 of the previous section. It yields

$$I_{\mathrm{m}} = \frac{V_{\mathrm{m}}}{|\mathbf{Z}|} = \frac{V_{\mathrm{m}}}{Z} = \frac{170}{41.154} = 4.1309\ \ \mathrm{A} \tag{11.53}$$

The instantaneous load power follows Eq. (11.51) with the power angle, $\theta = 45°$. Therefore, we obtain

$$p(t) = \frac{3}{2}V_{\mathrm{m}}I_{\mathrm{m}}\cos\theta = 745\ \ \mathrm{W} = \mathrm{const} \tag{11.54}$$

Equation (11.51) is critical for three-phase systems. It tells us that the total instantaneous power delivered to the load remains constant at any instance in time. This is in contrast to the instantaneous power of every individual single phase, which is still pulsating in time. Equation (11.51) implies that a three-phase load (e.g., an induction motor) as well as the three-phase generator introduced in the previous section should generate or require a *constant* torque. Thus, they undergo less vibration since the net power transfer is uniform.

11.4.2 Average Power, Reactive Power, and Apparent Power

The AC power types defined for the single-phase power distribution in Section 11.1 of this chapter also apply for the three-phase circuits. The *average* or active load power *P*, the *reactive* load power *Q*, and the *complex* load power **S** *of the balanced three-phase system* are given by

$$P = 3V_{\text{rms}}I_{\text{rms}}\cos\theta\ \ [\text{W}],\quad Q = 3V_{\text{rms}}I_{\text{rms}}\sin\theta\ \ [\text{VAR}],\quad \mathbf{S} = 3V_{\text{rms}}I_{\text{rms}}\angle\theta\ \ [\text{VA}] \tag{11.55a}$$

While the instantaneous powers *per phase* are pulsating, their average values labeled with indexes *a*, *b*, *c* are exactly one third of the load powers. One has

$$P_{\text{a,b,c}} = V_{\text{rms}}I_{\text{rms}}\cos\theta\ \ [\text{W}],\quad Q_{\text{a,b,c}} = V_{\text{rms}}I_{\text{rms}}\sin\theta\ \ [\text{VAR}],\quad \mathbf{S}_{\text{a,b,c}} = V_{\text{rms}}I_{\text{rms}}\angle\theta\ \ [\text{VA}] \tag{11.55b}$$

per phase. Equation (11.55) uses the rms values of phase voltages and line currents.

Example 11.13: For the previous example, determine the load average power, reactive power, and the apparent power. Do these powers coincide with the corresponding source measures?

Solution: The average power is simply the load instantaneous power, $P = 745\ \text{W}$. The reactive power is $Q = \frac{3}{2}V_{\text{m}}I_{\text{m}}\sin\theta = 745\ \text{VAR}$. The apparent power is $S = |\mathbf{S}| = \frac{3}{2}V_{\text{m}}I_{\text{m}} = 1053\ \text{VA}$. And the apparent power can be also found from the power triangle. All load powers coincide with the corresponding source powers since the transmission lines in Fig. 11.24 are assumed to be ideal conductors.

Exercise 11.13: A three-phase induction motor is modeled by a balanced wye load in Fig. 11.24. The motor (active) power is 6 kW; the line current is 20 A rms, and the line voltage is 208 V rms. Determine the power factor of the motor, which is the ratio of the active load power *P* to the magnitude of the total apparent power, |**S**|.

Answer: $PF \approx 5/6 = 0.833$.

11.4.3 Application Example: Material Consumption in Three-Phase Systems

A comparison is made between the conductor material consumption in a single-phase two-wire transmission system (shown in Fig. 11.25a) and the three-phase, three-wire transmission system shown in Fig. 11.25b. Both systems have the identical distance from the source to the load, the same average power *P* distributed to a purely resistive load, and the same rms line voltages *close* to the load. They all use the same conductor material. The distributed resistance per wire is modeled by a lumped resistor R for the single-phase line and by a lumped resistor R' for the three-phase line. Given the same load power and line voltage, the rms line currents are expressed as

$$I_{\text{single phase}} = \frac{P}{V}, \quad I_{\text{three phase}} = \frac{P}{\sqrt{3}\,V} \tag{11.56}$$

Equating power loss in the wire conductors, we obtain

$$2RI^2_{\text{single phase}} = 3R'I^2_{\text{three phase}} \Rightarrow 2R\left(\frac{P}{V}\right)^2 = 3R'\left(\frac{P}{\sqrt{3}\,V}\right)^2 \Rightarrow R' = 2R \tag{11.57}$$

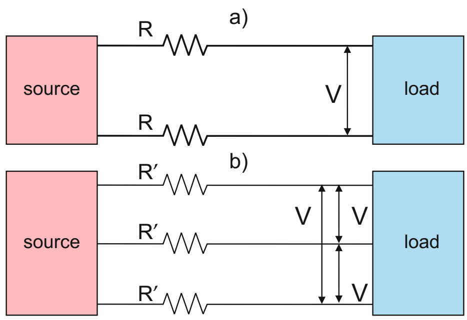


Fig. 11.25. Comparison of single-phase and three-phase transmission systems.

Since resistance R is twice as large as resistance R', the cross section of the corresponding cylindrical conductor is smaller by a factor of two in the three-phase configuration. Hence, its radius is $1/\sqrt{2}$ times less than the radius, r, of the single-phase line. Figure 11.26 depicts the radii of the equivalent conductors.

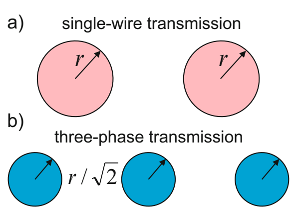


Fig. 11.26. Radii of equivalent conductors for the two systems in Fig. 11.25.

For the single-phase line, the total conductor cross section is $2\pi r^2$; for the three-phase line, the total cross section is $1.5\pi r^2$. Given the same length, the ratio of conductor material required is exactly the cross-section ratio, that is, $1.5/2 = 0.75$. In other words, the three-phase system consumes 75 % less conductor material compared to the single-phase system. The key is the absence of the neutral wire (or, possibly, using a much thinner neutral wire). Other examples for particular loads might result in even more dramatic savings.

11.4.4 Balanced Delta-Connected Load

Along with the wye-connected load, an important example of the three-phase load is the *delta-connected load*, which is shown in Fig. 11.27a in the *balanced configuration*. The balanced delta-connected load is common, along with a balanced wye-connected load. The delta-connected load inherently does not have a neutral port. This load may be converted to the wye-connected load shown in Fig. 11.27b by using the Y-Δ transformation algorithm established in Chapter 3. This algorithm equally applies to the impedance circuits. The algorithm considerably simplifies when the loads are balanced (load resistances or impedances are equal). With reference to Fig. 11.27, one has

$$\mathbf{Z}_\mathrm{Y} = \frac{1}{3}\mathbf{Z}_\Delta \leftrightarrow \mathbf{Z}_\Delta = 3\mathbf{Z}_\mathrm{Y} \tag{11.58}$$

for phase impedance transformation. Here, indexes Y and Δ refer to the wye-connected and delta-connected loads, respectively.

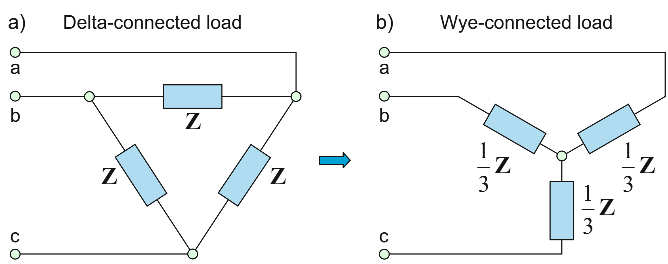


Fig. 11.27. Delta-connected load versus ad.

11.4.5 Balanced Delta-Connected Source

The *balanced delta-connected source* is shown in Fig. 11.28b. In its original configuration, it is not using the ground terminal or a neutral conductor. The delta-connected source so wound is generally less common and less safe than the wye-connected source. It may be created by the three-phase generator shown in Fig. 11.17 of the previous section, assuming the three individual coil windings aa', bb', and cc' are interconnected in a closed loop.

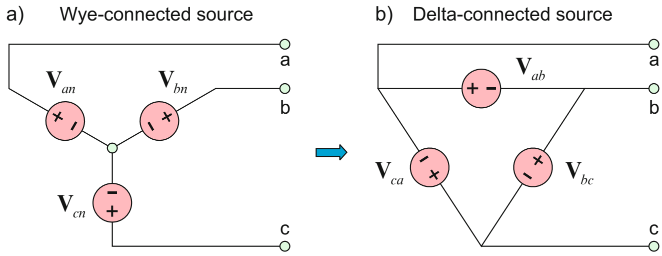


Fig. 11.28. Wye-connected source versus delta-connected source.

The balanced wye-connected source *without* a neutral or ground conductor can be easily converted to the balanced delta source and vice versa. The concept is shown in Fig. 11.28. The line voltages $\mathbf{V}_{\mathrm{ab}}$, $\mathbf{V}_{\mathrm{bc}}$, $\mathbf{V}_{\mathrm{ca}}$ between nodes *a*–*b*, *b*–*c*, and *c*–*a* of the wye source become the phase voltages of the delta source. The relation between two voltage types is given by Eq. (11.44) of the previous section, that is (positive phase sequence),

$$\mathbf{V}_{\mathrm{ab}} = \sqrt{3}\mathbf{V}_{\mathrm{an}}\angle 30^\circ, \quad \mathbf{V}_{\mathrm{bc}} = \sqrt{3}\mathbf{V}_{\mathrm{bn}}\angle 30^\circ, \quad \mathbf{V}_{\mathrm{ca}} = \sqrt{3}\mathbf{V}_{\mathrm{cn}}\angle 30^\circ \tag{11.59}$$

Thus, according to Eq. (11.59) and Fig. 11.28, the phase voltages of the *equivalent* delta-connected source $\mathbf{V}_{\mathrm{ab}}$, $\mathbf{V}_{\mathrm{bc}}$, $\mathbf{V}_{\mathrm{ca}}$ are greater in amplitude by a factor of $\sqrt{3} \approx 1.73$ as compared to the phase voltages $\mathbf{V}_{\mathrm{an}}$, $\mathbf{V}_{\mathrm{bn}}$, $\mathbf{V}_{\mathrm{cn}}$ of the equivalent wye-connected source in Fig. 11.28. The line voltages of the delta-connected source *coincide* with its phase voltages given lossless conductors and *coincide* with the line voltages of the wye-wye source; all of them are simply $\mathbf{V}_{\mathrm{ab}}$, $\mathbf{V}_{\mathrm{bc}}$, $\mathbf{V}_{\mathrm{ca}}$. Indeed, the sum of the phase voltages for the delta-connected source is still equal to zero according to Eq. (11.43) of the previous section. Hence, there is no current circulation in the (ideal) delta loop in Fig. 11.28b. Transformations given by Eqs. (11.58) and (11.59) allow us to consider four distinct source-load configurations in the three-phase systems: wye-wye, wye-delta, delta-wye, and delta-delta. All of them may be reduced to the wye-wye circuit or solved independently. Figure 11.29 shows one such configuration: a balanced *delta-delta distribution system*. In the delta-delta system, the line voltages coincide with the phase voltages, whereas the line currents $\mathbf{I}_{\mathrm{a}}$, $\mathbf{I}_{\mathrm{b}}$, and $\mathbf{I}_{\mathrm{c}}$ are different from the load (or phase) currents $\mathbf{I}_{\mathrm{AB}}$, $\mathbf{I}_{\mathrm{BC}}$, and $\mathbf{I}_{\mathrm{CA}}$. This is in contrast to the wye-wye system where the line and phase voltages are different, but the line and load currents remain the same.

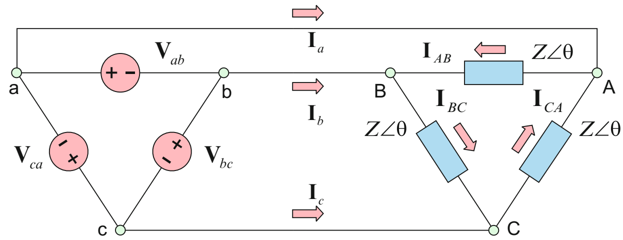


Fig. 11.29. Three-phase balanced *delta-delta distribution system*. Note the load (phase) currents circulating in the delta loop.

Example 11.14: A balanced delta-delta system in Fig. 11.29 operates at 60 Hz. The phase voltages of the delta source, $\mathbf{V}_{\mathrm{ab}}$, $\mathbf{V}_{\mathrm{bc}}$, $\mathbf{V}_{\mathrm{ca}}$, have amplitudes of $V_{\mathrm{m}} = 294.5$ V each. Moreover, each phase impedance is a 0.2315 H inductance in series with a 87.3 Ω resistance. Find the average load power.

Solution: We find the impedance for each phase of the load first. One has

$$\mathbf{Z} = R + j\omega L = 87.3 + j87.3\ \ \Omega = 87.3\sqrt{2}\angle 45^\circ\ \ \Omega \tag{11.60}$$

The power angle is thus given by $\theta = 45°$. Next, we find the load (phase) currents $\mathbf{I}_{\mathrm{AB}}$, $\mathbf{I}_{\mathrm{BC}}$, $\mathbf{I}_{\mathrm{CA}}$ circulating in the delta-connected load. Since the individual voltage sources in Fig. 11.6 are now directly connected to the individual load phases, one has for the amplitude of the phase current $\mathbf{I}_{\mathrm{AB}}$:

$$\mathbf{I}_{\mathrm{AB}} = \frac{\mathbf{V}_{\mathrm{ab}}}{\mathbf{Z}} \Rightarrow I_{\mathrm{m}} = \frac{V_{\mathrm{m}}}{|\mathbf{Z}|} = \frac{294.5}{Z} = \frac{294.5}{123.46} = 2.385\ \ \mathrm{A} \tag{11.61}$$

The remaining phases have the same amplitudes: the per-phase method is used again. Both the average load power and the instantaneous load powers are the sum of three individual contributions, that is,

$$P = p(t) = 3 \times \left(\frac{1}{2} V_{\mathrm{m}} I_{\mathrm{m}} \cos\theta\right) = \frac{3}{2} 294.5 \times 2.385 \times 0.707 = 745\ \ \mathrm{W} \tag{11.62}$$

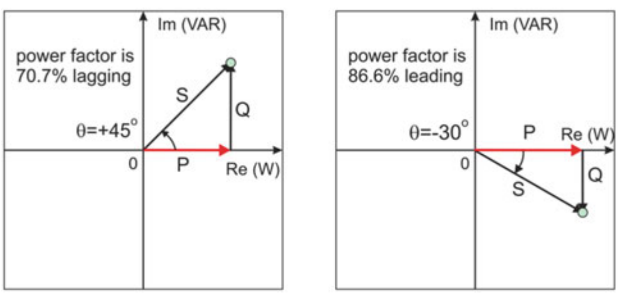
The instantaneous power may be calculated; it is constant and equals 745 W. Note that the rms line voltages in this example are 208 V.

Example 11.15: Solve the previous example by converting the delta-delta system to the equivalent wye-wye system.

Solution: First, the phase impedance of the wye load should be three times less than the phase impedance of the delta load, that is, $\mathbf{Z} = 29.1\sqrt{2}\angle 45^\circ\ \Omega$. Second, the amplitude of the wye phase source should be $1/\sqrt{3}$ times less than the amplitude of the delta phase source, that is, $V_\mathrm{m} = 170\ \ \mathrm{V}$. These numbers have been used in Examples 11.12 and 11.13 for the wye-wye system, which gave us exactly the same value of 745 W (one horsepower) for the average and instantaneous powers, respectively.

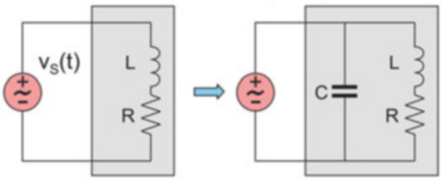
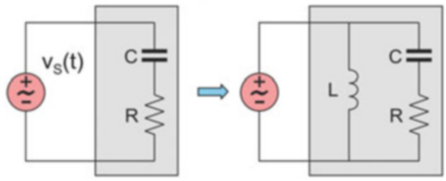
Apart from the circuit equivalence, one may look at Fig. 11.28 from a slightly different perspective. What if the voltage sources in Fig. 11.28 are all the same (the same windings of the three-phase generator just connected differently)? In this case, the wye connection gives us a line voltage $\sqrt{3}$ times greater than the delta connection. Hence, the line current, which is required for the same power transfer, will be $1/\sqrt{3}$ times *less*. Reducing line currents reduces line losses. This explains why the wye source connection is preferable for long-distance power transmission.

Summary

rms Voltages and currents in terms of sine/cosine amplitudes and in the general case
For sinusoidal signals: $V_{\text{rms}} = \frac{V_m}{\sqrt{2}}, I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$ General periodic case: $V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$
Average power for resistive load: $P = \frac{1}{2}V_m I_m = V_{\text{rms}} I_{\text{rms}},\ V_{\text{rms}} = R I_{\text{rms}}$
Power angle θ and power factor PF
$v(t) = V_m \cos(\omega t + \varphi)$, $i(t) = I_m \cos(\omega t + \psi)$ $\Rightarrow \theta = \varphi - \psi, \quad -90^\circ \le \theta \le +90^\circ$ $PF = \cos(\varphi - \psi) = \cos\theta$
Average power for arbitrary load: $P = \frac{V_m I_m}{2}\cos\theta = V_{\text{rms}} I_{\text{rms}}\cos\theta$ (zero for L and C)
Average power and power angle in terms of phasors: $P = \frac{\text{Re}(\mathbf{V}\cdot\mathbf{I}^*)}{2}$, $\mathbf{Z} = \lvert\mathbf{Z}\rvert\angle\theta$
Average power P, reactive power Q, complex power $\mathbf{S}$, and apparent power S
$P = \frac{\text{Re}(\mathbf{V}\cdot\mathbf{I}^*)}{2} = \frac{R\lvert\mathbf{I}\rvert^2}{2} = \frac{\lvert\mathbf{Z}\rvert\lvert\mathbf{I}\rvert^2}{2}\cos\theta = \frac{\lvert\mathbf{V}\rvert\lvert\mathbf{I}\rvert}{2}\cos\theta = V_{\text{rms}} I_{\text{rms}}\cos\theta\,[\text{W}]$ $Q = \frac{\text{Im}(\mathbf{V}\cdot\mathbf{I}^*)}{2} = \frac{X\lvert\mathbf{I}\rvert^2}{2} = \frac{\lvert\mathbf{Z}\rvert\lvert\mathbf{I}\rvert^2}{2}\sin\theta = \frac{\lvert\mathbf{V}\rvert\lvert\mathbf{I}\rvert}{2}\sin\theta = V_{\text{rms}} I_{\text{rms}}\sin\theta\ [\text{VAR}]$ $\mathbf{S} = \frac{\mathbf{V}\cdot\mathbf{I}^*}{2} = P + jQ\ [\text{VA}]$ $S = \lvert\mathbf{S}\rvert = \frac{\lvert\mathbf{Z}\rvert\lvert\mathbf{I}\rvert^2}{2} = \frac{\lvert\mathbf{V}\rvert\lvert\mathbf{I}\rvert}{2} = V_{\text{rms}} I_{\text{rms}}\ [\text{VAR}]$
Power triangle (lagging/leading power factor)

AC power conservation laws For any network of N loads connected in series, parallel, or in general: $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \ldots \mathbf{S}_N,\ P = P_1 + P_2 + \ldots P_N,\ Q = Q_1 + Q_2 + \ldots Q_N$

(continued)

Power factor correction

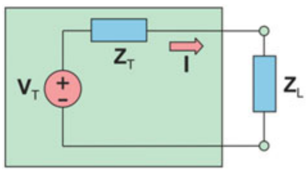
for most common inductive (motor) load	for capacitive load
	

$$C = \frac{L}{\omega^2 L^2 + R^2} \Rightarrow \mathbf{Z} = R + \frac{(\omega L)^2}{R} \Rightarrow \qquad L = \frac{1+\omega^2 R^2 C^2}{\omega^2 C} \Rightarrow \mathbf{Z} = R + \frac{1}{R(\omega C)^2} \Rightarrow$$

$$\mathbf{I} = \overbrace{\underbrace{\frac{RV_m}{R^2+(\omega L)^2}}_{\text{with capacitor}} - \frac{j\omega L V_m}{R^2+(\omega L)^2}}^{\text{without capacitor}} \qquad \mathbf{I} = \overbrace{\underbrace{\frac{R(\omega C)^2 V_m}{1+(\omega R C)^2}}_{\text{with inductor}} + \frac{j\omega C V_m}{1+(\omega R C)^2}}^{\text{without inductor}}$$

P remains exactly the same, Q becomes zero, PF becomes 100 %

Maximum power transfer

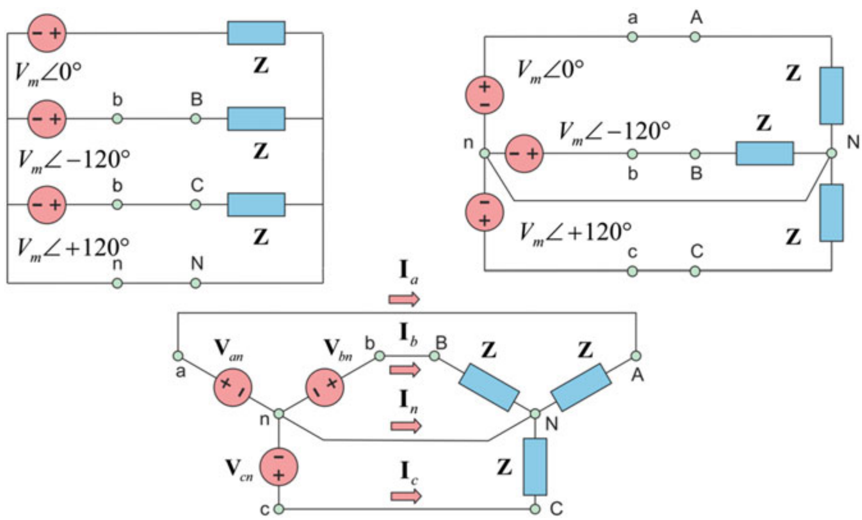


$$P = \frac{0.5 R_L |\mathbf{V}_T|^2}{(R_L + R_T)^2 + (X_L + X_T)^2} \text{ [W]}$$

$$P_{max} = \frac{1}{8}\frac{|\mathbf{V}_T|^2}{R_T} \text{ [W]}$$

at $\mathbf{Z}_L = \mathbf{Z}_T^*$

Some equivalent drawings of the *same* balanced *three-phase four-wire wye–wye* power distribution system



(continued)

Major parameters of the balanced three-phase four-wire wye–wye power distribution system

Positive phase sequence $\mathbf{V}_{an} = V_m, \ \ \mathbf{V}_{bn} = V_m\angle -120°, \ \ \mathbf{V}_{cn} = V_m\angle +120°$

Negative phase sequence $\mathbf{V}_{an} = V_m, \ \ \mathbf{V}_{bn} = V_m\angle +120°, \ \ \mathbf{V}_{cn} = V_m\angle -120°$

Current in the neutral wire: $\mathbf{I}_n = 0$

Per phase solution: $\mathbf{I}_a = I_m\angle -\theta, \ \mathbf{I}_b = I_m\angle -120° - \theta, \ \mathbf{I}_c = I_m\angle 120° - \theta$

$I_m = V_m/Z, \ \mathbf{Z} = Z\angle\theta$

Line voltages (positive phase sequence):

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3}\mathbf{V}_{an}\angle 30°,$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}\mathbf{V}_{bn}\angle 30°,$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}\mathbf{V}_{cn}\angle 30°$$

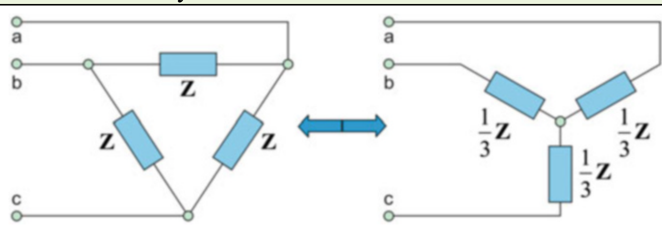
Instantaneous/average load power: $p(t) = P = \frac{3}{2}V_m I_m\cos\theta = 3V_{\rm rms}I_{\rm rms}\cos\theta = \text{const}$

Apparent load power: $S = \frac{3}{2}V_m I_m = 3V_{\rm rms}I_{\rm rms}$

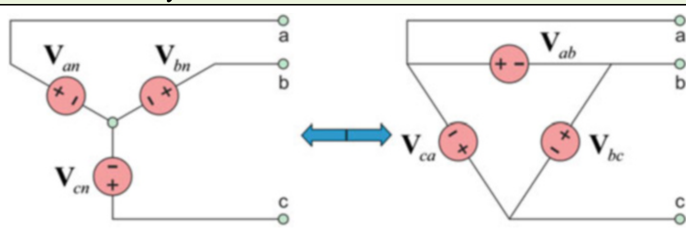
Some common wye distribution systems

3-Phase, 4-Wire 208Y/120 V (US)	Line :	$V_{\rm rms} = 208$ V, $V_{\rm m} = 294$ V
	Phase :	$V_{\rm rms} = 120$ V, $V_{\rm m} = 170$ V
3-Phase, 4-Wire 400Y/230 V (EU, Others)	Line :	$V_{\rm rms} = 400$ V, $V_{\rm m} = 566$ V
	Phase :	$V_{\rm rms} = 230$ V, $V_{\rm m} = 325$ V

Wye load to delta load conversion



Wye source to delta source conversion



$\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30°, \quad \mathbf{V}_{bc} = \sqrt{3}\mathbf{V}_{bn}\angle 30°, \quad \mathbf{V}_{ca} = \sqrt{3}\mathbf{V}_{cn}\angle 30°$ (positive phase sequence)

Problems

11.1 AC Power Types and Their Meaning

11.1.1 Instantaneous AC Power

11.1.2 Time-Averaged AC Power

Problem 11.1. An AC voltage signal across a resistive load with $R = 10\ \Omega$ is given by:

A. $v(t) = V_m \cos 1000t$ [V]
B. $v(t) = V_m \sin 60t$ [V]
C. $v(t) = V_m \cos(60t + 45°)$ [V]

where $V_m = 10$ V. Determine the average AC power into the load in every case.

Problem 11.2. An alternating current through a resistive load with $R = 100\ \Omega$ is given by:

A. $i(t) = I_m \cos 10^6 t$ [V]
B. $i(t) = I_m \cos 37t$ [V]
C. $i(t) = I_m \sin(2011t + 45°)$ [V]

where $I_m = 1$ A. Determine the average AC power into the load in every case.

Problem 11.3. An rms voltage across a resistive load with $R = 100\ \Omega$ is given by:

A. $V_{rms} = 5$ V
B. $V_{rms} = 100$ V
C. $V_{rms} = 0$ V

Determine the average power into the load in every case.

Problem 11.4. An rms current through a resistive load with $R = 1\ k\Omega$ is given by:

A. $I_{rms} = 1$ A
B. $I_{rms} = 100\ \mu A$
C. $I_{rms} = 0$ A

Determine the average power into the load in every case.

Problem 11.5. An AC voltage signal is given by:

A. $v(t) = V_m \cos(\omega t + \varphi)$ [V]
B. $v(t) = 1\,V + V_m \cos(\omega t + \varphi)$ [V]
C. $v(t) = 1\ V - V_m \sin(\omega t + \varphi)$ [V]

where $V_m = 1$ V, $\omega = 100$ rad/s, and $\varphi = \pi/2$ rad. Find the time-average voltage $\overline{v(t)}$ in every case.

Problem 11.6. Present a mathematical proof of the fact that the expression for the average power, $P = \frac{V_m^2}{2R}$, holds for an AC voltage signal given by $v(t) = V_m \cos(\omega t + \varphi)$ [V] where φ is an arbitrary phase.

11.1.3 Application Example: rms Voltages and AC Frequencies Around the World

Problem 11.7. A 100 Ω resistive load is connected to an AC wall plug in:

A. Peoples Republic of China
B. India
C. USA
D. Germany

Determine the average power delivered to the load in every case. Also determine the rms load current in every case.

Problem 11.8. What do you think is a major

A. Advantage
B. Disadvantage

of having a higher AC voltage?

11.1.4 rms Voltages for Arbitrary Periodic AC Signals

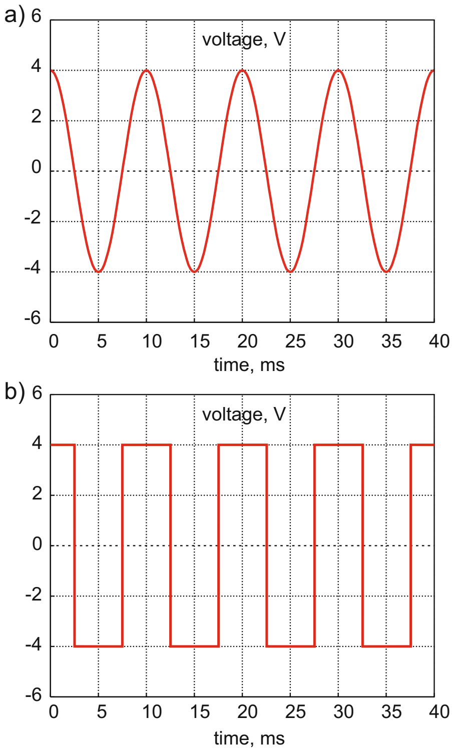
Problem 11.9. Determine the average power delivered to a 100 Ω resistive load when the applied periodic voltage signal has the form $v(t) = (5t + 0.01)/T$ [V] over one period $T = 0.01$ s. This signal is known as the sawtooth or the triangular wave:

A. Use the analytical calculation of the rms voltage.
B. Use the rms voltage found numerically, based on a MATLAB script or any software of your choice.

Problem 11.10. Determine the average power delivered to a 100 Ω resistive load when the applied periodic voltage has the form $v(t) = \sqrt{t}/T$ [V] over one period $T = 0.01$ s:

A. Use the analytical calculation of the rms voltage.
B. Use the rms voltage found numerically, based on a MATLAB script or any software of your choice.

Problem 11.11. Of the two periodic voltage signals shown in the figures below,

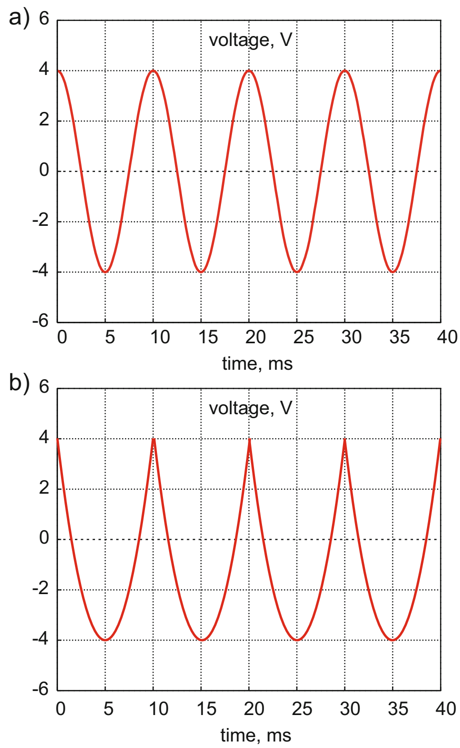


which signal delivers more average power into a resistive load? The periodic voltage on the top graph is the cosine function. Explain your answer and provide an analytical proof (find the *rms* voltages and the average power in every case).

Problem 11.12. Of the two periodic signals shown in the figures that follow, which signal delivers more average power into a resistive load? Explain your answer and provide:

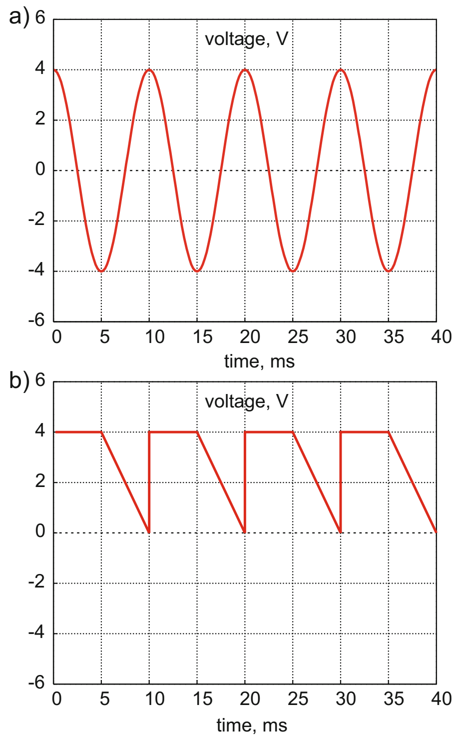
A. An analytical proof—find the *rms* voltage and the average power in every case
B. A numerical proof (use MATLAB or any software of your choice).

The periodic voltage on the top graph is the cosine function. The periodic voltage on the bottom graph is given by $v(t) = 3.2 \times 10^5 (t - 0.005)^2 - 4$ [V] over the time interval from 0 to T.



Problem 11.13. Of the two periodic signals shown in the figure that follows, which signal delivers more average power into a resistive load? The periodic voltage on the top graph is the cosine function. Explain your answer and provide:

A. An analytical proof—find the *rms* voltage and the average power in every case
B. A numerical proof (use MATLAB or any software of your choice).



11.1.5 Average AC Power in Terms of Phasors: Power Angle

11.1.6 Average Power for the Resistor, Capacitor, and Inductor

Problem 11.14. The phasor voltage across a purely resistive load with the resistance $R = 100\ \Omega$ is given by $\mathbf{V} = -2 - j1.5$ [V]. Find the average power delivered to the load.

Problem 11.15. The phasor current through a purely resistive load with the resistance $R = 100\ \Omega$ is given by $\mathbf{I} = -1 - j0.5$ [A]. Find the average power delivered to the load.

Problem 11.16. The phasor voltage across an AC load and the phasor current through the same AC load are given by:

$$\mathbf{V} = -3 + j3 \quad [\text{V}]$$
$$\mathbf{I} = +j0.1 \quad [\text{A}]$$

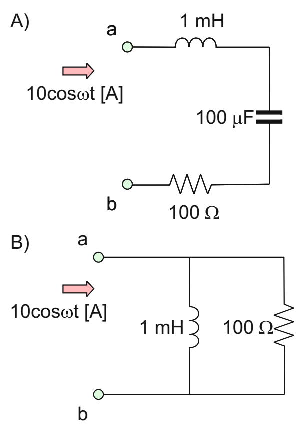
A. Find the average power delivered to the load analytically.
B. Find the average power delivered to the load numerically using MATLAB or any software of your choice.

Problem 11.17. Repeat the previous problem for phasor voltage and phasor current in the form:

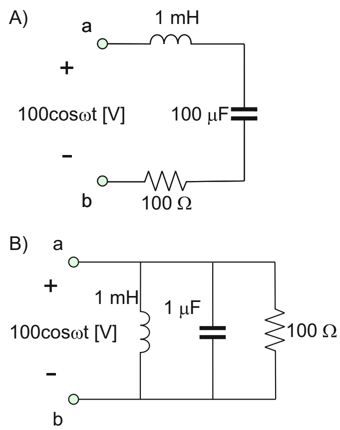
$$\mathbf{V} = 2 + j2 \quad [\text{V}]$$
$$\mathbf{I} = 1 - j1 \quad [\text{A}]$$

Problem 11.18. Express the average power given by $P = \frac{\mathrm{Re}(\mathbf{V}\cdot\mathbf{I}^*)}{2}$ in terms of the following three quantities: magnitude of the phasor voltage, $|\mathbf{V}|$; the impedance magnitude, $|\mathbf{Z}|$; and the real part of the impedance, $\mathrm{Re}(\mathbf{Z})$.

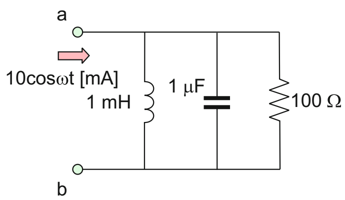
Problem 11.19. Determine the average power delivered to the load circuit between terminals *a* and *b* shown in the figure that follows. The AC angular frequency is 100 rad/s.



Problem 11.20. Determine the average power delivered to the load circuit between terminals *a* and *b* shown in the figure that follows. The AC angular frequency is 1000 rad/s.



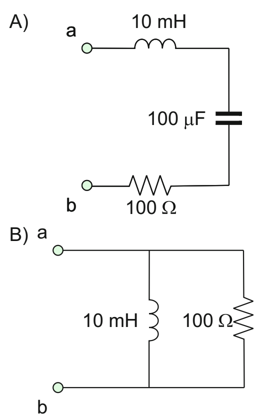
Problem 11.21. Determine the average power delivered to the load circuit shown in the figure below. The AC signal frequency is 10^6 Hz.



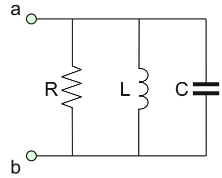
11.1.7. Average Power, Reactive Power, and Apparent Power

11.1.8. Power Triangle

Problem 11.22. Determine the resistance and the reactance of the circuit blocks (the load) shown in the figure. The AC angular frequency is 1000 rad/s.



Problem 11.23. Determine the resistance and the reactance of the circuit block (the load) shown in the figure in terms of R, L, and C in a general form. The AC angular frequency is ω.

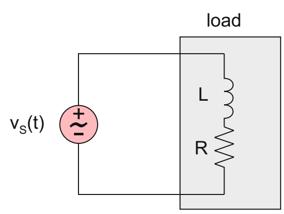


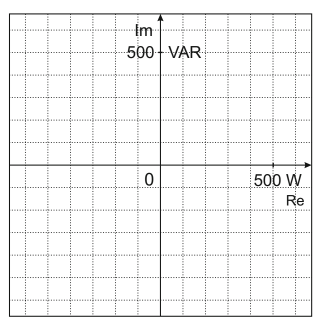
Problem 11.24. Write the expressions (and show units) for the average power P and the reactive power Q in terms of:

A. Phasor current **I** through the load and the load resistance R and the reactance X
B. Phasor voltage **V** across the load, the load impedance magnitude |**Z**|, and the impedance phase (or the power angle) θ

Problem 11.25. For the circuit shown in the figure with the parameters $V_m = 170$ V, $\omega = 377$ rad/s, $L = 26.5$ mH, $R = 25$ Ω:

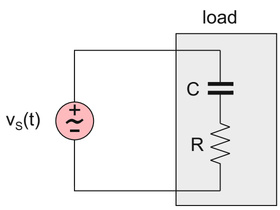
A. Determine the power angle and the power factor.
B. Determine the average (or true) power and the reactive power for the inductive load shown in the figure.
C. Construct the corresponding power triangle.

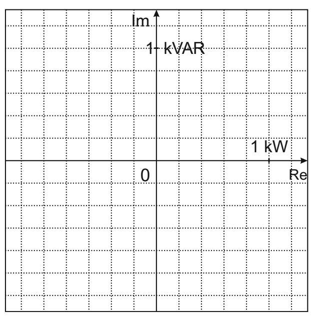




Problem 11.26. For the circuit shown in the figure with the parameters $V_m = 170$ V, $\omega = 377$ rad/s, $C = 500$ µF, $R = 10$ Ω:

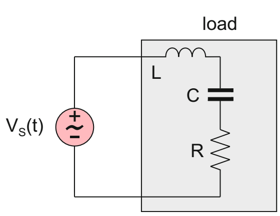
A. Determine the power angle and the power factor.
B. Determine the average (or true) power and the reactive power for the capacitive load shown in the figure.
C. Construct the corresponding power triangle.

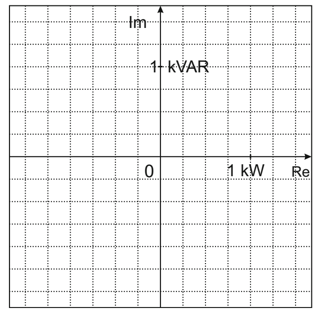




Problem 11.27. For the circuit shown in the figure with the parameters $V_m = 170$ V, $\omega = 377$ rad/s, $L = 14.07$ mH, $C = 500$ µF, $R = 10$ Ω:

A. Determine the power angle and the power factor.
B. Determine the average (or true) power and the reactive power for the complex load shown in the figure.
C. Construct the corresponding power triangle.





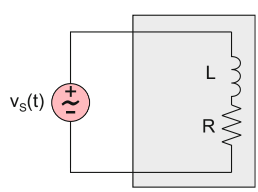
11.2 Power Factor Correction: Maximum Power Efficiency and Maximum Power Transfer

11.2.1 Power Factor Correction

11.2.3 Principle of Maximum Power Efficiency for AC Circuits

Problem 11.28. Correct the power factor for the inductive load shown in the figure below. The circuit parameters are $V_m = 170$ V, $\omega = 377$ rad/s and $L = 53$ mH, $R = 10$ Ω:

A. Present the circuit diagram of the modified load and determine the required capacitance.
B. Determine average (true) power, reactive power, power factor, and amplitude of the circuit current before the power factor correction.
C. Determine average (true) power, reactive power, power factor, and amplitude of the circuit current after the power factor correction.

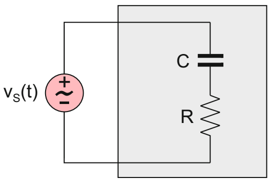


Problem 11.29. Correct the power factor for the capacitive load shown in the figure that

follows. The circuit parameters are $V_m = 170$ V, $\omega = 377$ rad/s and $C = 265$ μF, $R = 10$ Ω:

A. Present the circuit diagram of the modified load and determine the required inductance.

B. Determine average (true) power, reactive power, power factor, and amplitude of the circuit current before the power factor correction.



Problem 11.30. A whip monopole antenna used in US Coast Guard ships has an equivalent electric circuit shown in the figure of the previous problem. Its (radiation) resistance is 1 Ω, and the reactance is $-j1000$ Ω. By modifying the antenna circuit with a lumped inductor, it is required to make the antenna impedance real and as large as possible:

A. Present the circuit diagram of the modified load

B. Determine the required impedance of the inductor.

11.2.4 Principle of Maximum Power Transfer for AC Circuits

Problem 11.31. Describe in your own words the difference between the concepts of maximum power efficiency and maximum power transfer for AC circuits.

Problem 11.32. A generator's impedance is $50\angle 30^\circ$ [Ω]. What should the load impedance be to allow the maximum power transfer to the load?

Problem 11.33

A. A generator's impedance is 50 Ω. The load impedance is $1+j50$ [Ω]. What percentage of the maximum available power (at the load impedance of 50 Ω) is transferred to the load?

B. Repeat the same task for the load impedance of $1-j50$ [Ω].

C. Repeat the same task for the load impedance of $5+j50$ [Ω].

Hint: Derive the general expression for the power ratio first and then plug in the numbers.

11.3 AC Power Distribution: Balanced Three-Phase Power Distribution System

11.3.1 AC Power Distribution Systems

11.3.2 Phase Voltages: Phase Sequence

Problem 11.34. Draw generic circuits for the following representative AC power distribution systems:

A. Single-phase two-wire system

B. Single-phase three-wire system

C. Two-phase three-wire system

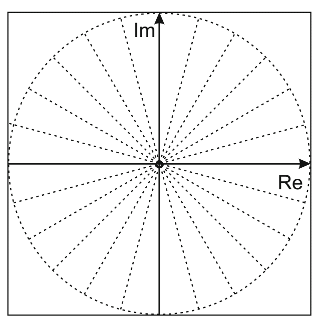
D. Three-phase four-wire system

Show loads and phasor voltages with the corresponding phases.

Problem 11.35. Determine the phase sequence for the phase voltages given by:

$$v_{an}(t) = 240\cos(314t + 75^\circ)\ \ [\mathrm{V}],$$
$$v_{bn}(t) = 240\cos(314t - 165^\circ)\ \ [\mathrm{V}],$$
$$v_{cn}(t) = 240\cos(314t - 45^\circ)\ \ [\mathrm{V}].$$

To simplify the solution, construct the corresponding phasor diagram in the figure below:



Problem 11.36. Given $\mathbf{V}_{\mathrm{bn}} = 120\ \angle 45°[\mathrm{V}]$, find $\mathbf{V}_{\mathrm{an}}$ and $\mathbf{V}_{\mathrm{cn}}$ assuming:

A. The positive *abc* phase sequence
B. The negative *acb* phase sequence

Express your result in phasor form. Make sure that the phase ranges from $-180°$ to $+180°$. To check the solution, you may want to use the corresponding phasor diagram shown in the figure for the previous problem.

11.3.3 Wye (Y) Source and Load Configurations for Three-phase Circuits

11.3.4 Application: Examples of Three-phase Source and the Load

11.3.5 Solution for the Balanced Three-Phase Wye-Wye Circuit

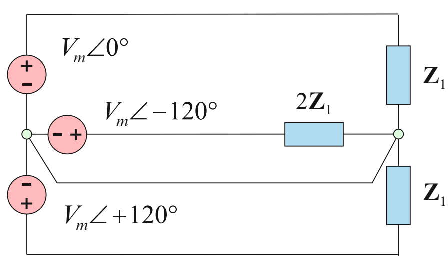
11.3.6 Removing the Neutral Wire in Long-Distance Balanced High-power Transmission

Problem 11.37

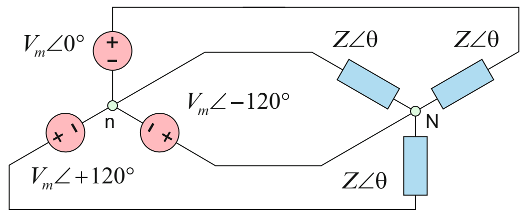
A. Draw the circuit diagram for a generic three-phase four-wire balanced wye-wye power distribution system.
B. Label phase voltages and phase impedances (load impedances per phase).
C. Label line currents.

Problem 11.38. A three-phase circuit is shown in the figure that follows:

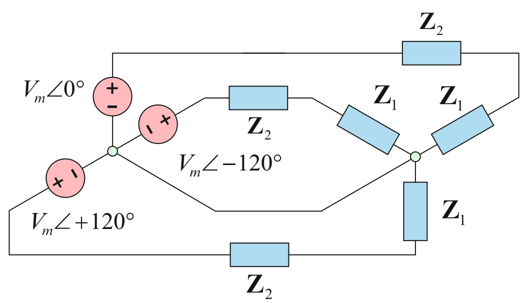
A. Is it a balanced wye-wye circuit?
B. If not, show your corrections on the figure.



Problem 11.39. Repeat the previous problem for the circuit shown in the figure below:



Problem 11.40. Repeat Problem 11.38 for the circuits shown in the figure that follows:



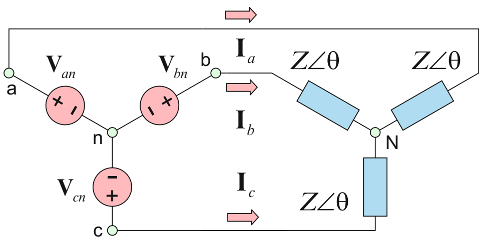
Problem 11.41. Prove that Eq. (11.28) of this chapter for line voltages also holds for the negative phase sequence to within the substitution $30° \rightarrow -30°$.

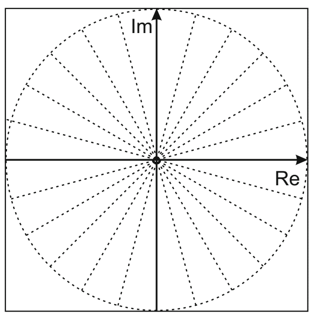
Problem 11.42. The local electric service in the European Union is provided by a three-phase four-wire *abcn* wye system with the line voltages equal to 400 V rms each (so-called Niederspannungsnetz):

A. Determine the rms phase voltages.
B. By connecting terminals *abcn* in any sequence of your choice, could you in principle obtain the rms voltages higher than 400 V?

Problem 11.43. Determine line currents in the balanced three-phase wye-wye circuit shown in the figure that follows. You are given the *acb* sequence of phase voltages $\mathbf{V}_{\mathrm{an}} = 170\angle 0°\ [\mathrm{V}]$,

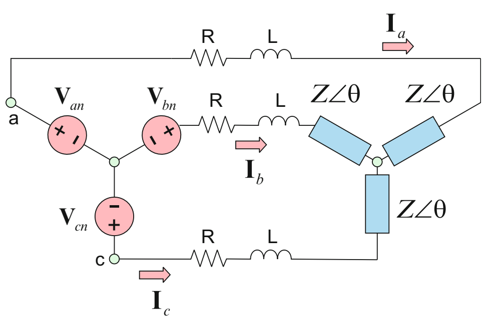
$\mathbf{V}_{bn} = 170\angle 120^\circ$ [V], $\mathbf{V}_{bn} = 170\angle{-120^\circ}$ [V], and load impedance per phase, $\mathbf{Z} = 8 + j30\ \Omega$.
Plot phasor currents on the phasor diagram that follows.





Problem 11.44. In the balanced three-phase wye-wye circuit shown in the figure that follows, the power line resistance and inductance are additionally included into consideration. The three-phase source operates at 60 Hz; $R = 2\ \Omega$, $L = 9.6$ mH. You are given the *abc* sequence of phase voltages $\mathbf{V}_{an} = 170\angle 0^\circ$ [V], $\mathbf{V}_{bn} = 170\angle - 120^\circ$ [V], $\mathbf{V}_{bn} = 170\angle 120^\circ$ [V], and load impedance per phase, $\mathbf{Z} = 7 + j30\ \Omega$.

A. Determine line currents.
B. Plot phasor line currents on the phasor diagram to the previous problem.



11.4 Power in Balanced Three-Phase Systems: Delta-connected Three-Phase Circuits

11.4.1 Intantaneous Power

11.4.2 Average Power, Reactive Power, and Apparent Power

Problem 11.45. In a three-phase balanced wye-wye system, the rms phase voltages are 120 V, and the rms line currents are 10 A. The impedance has the power angle of $\theta = 75^\circ$. Find:

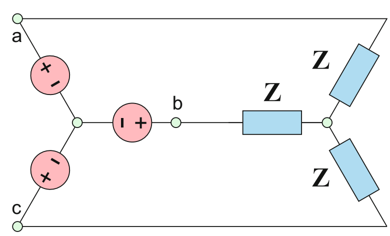
1. The instantaneous load power
2. The average load power

Problem 11.46. In a three-phase balanced wye-wye system, the rms line voltages are 400 V, and the rms line currents are 10 A. The impedance has the power angle of $\theta = 60^\circ$. Find:

1. The instantaneous load power
2. The average load power

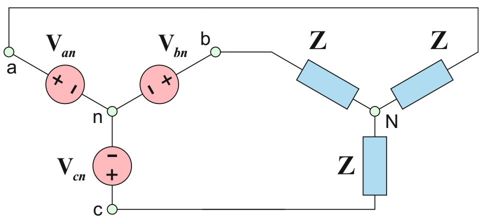
Problem 11.47. In the three-phase system shown in the figure that follows, $\mathbf{Z} = 40\angle 60^\circ$. The sources have the relative phases $0,\ -120^\circ,\ +120^\circ$. The rms *line* voltages are 208 V. Determine:

A. The type of the three-phase system
B. Instantaneous power delivered to the three-phase load
C. Average power delivered to the three-phase load



Problem 11.48. A balanced wye-wye three-phase system in the figure that follows uses lossless transmission lines and operates at 60 Hz.

The line-to-neutral voltages have the amplitudes of 170 V, $V_m = 170$ V. Every phase impedance is a 92-mH inductance in series with a 20 Ω resistance. Find the instantaneous load power.



Problem 11.49. In the previous problem:

A. Determine the load average power, reactive power, and the apparent power.

B. Do these powers coincide with the corresponding source measures?

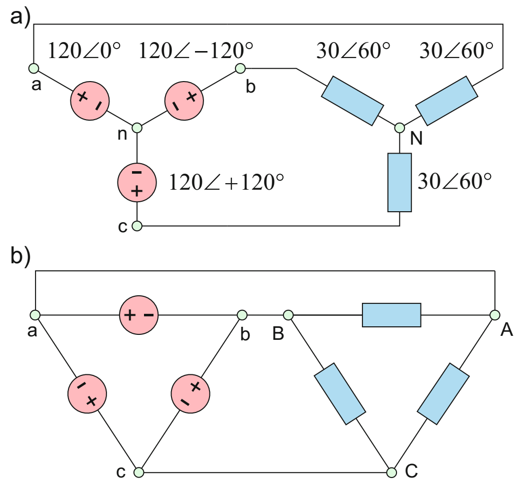
Problem 11.50. A three-phase induction motor is modeled by a balanced wye load. The motor (active) power is 2.5 kW; the line current is 10 A rms, and the phase voltage of a three-phase wye source is 120 V rms. Determine the power factor of the motor.

Problem 11.51. In the previous problem, the motor (active) power is 9 kW; the line current is 15 A rms, and the line voltage of a three-phase wye source is 400 V rms. Determine the power factor of the motor.

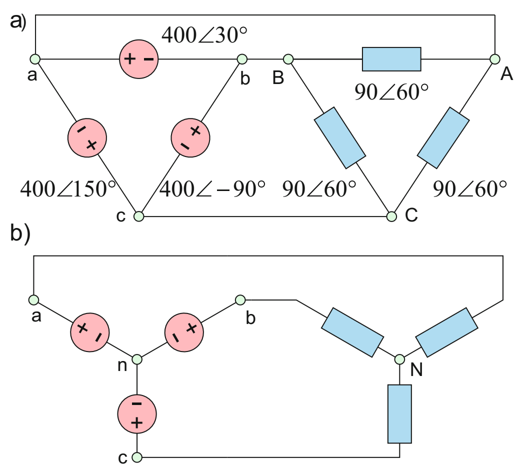
11.4.4 Balanced Delta-Connected Load

11.4.5 Balanced Delta-Connected Source

Problem 11.52. A three-phase balanced wye-wye system is shown in the figure that follows. Its delta-delta equivalent is sought, which is shown in the same figure. For the delta-delta system, write the corresponding voltage and impedance values in the phasor form close to every circuit element.

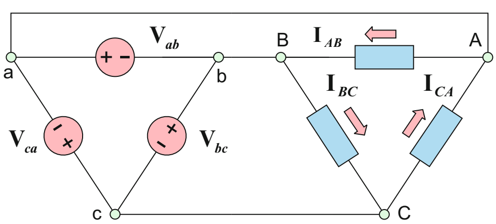


Problem 11.53. A three-phase balanced delta-delta system is shown in the figure that follows. Its wye-wye equivalent is sought, which is shown in the same figure. For the wye-wye system, write the corresponding voltage and impedance values in the phasor form close to every circuit element.



Problem 11.54. A balanced delta-delta system shown in the figure below operates at 50 Hz. The phase voltages of the delta source, $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$, have the amplitudes of $V_m = 563.4$ V each. Every phase impedance is a 0.21 H inductance in series with a 38-Ω resistance:

A. Find the average load power,
B. Find the instantaneous load power,
C. Find the apparent load power.



Problem 11.55. A balanced wye-delta system shown in the figure below operates at 60 Hz. The phase voltages of the wye source, $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$, have the amplitudes of $V_m = 170$ V each. Each phase impedance is a 0.18 H inductance in series with a 90 Ω resistance:

A. Find the average load power,
B. Find the instantaneous load power,
C. Find the apparent load power.