Chapter 4: Circuit Analysis and Power Transfer

Overview

Prerequisites
- Knowledge of major circuit elements, their v-i characteristics, and Ohm's law (Chapter 2)
- Knowledge of basic networking theorems (Chapter 3)

Objectives of Section 4.1:
- Become familiar with the nodal analysis and be able to apply it to solve in arbitrary linear circuits
- Become familiar with the mesh analysis and be able to apply it to solve in arbitrary linear circuits

Objectives of Section 4.2:
- Become familiar with the method of short/open circuit
- Establish and prove the source transformation theorem
- Establish and prove Thévenin's and Norton's theorems

Objectives of Section 4.3:
- Establish the maximum power theorem and become familiar with the power efficiency concept
- Be able to apply the concepts of Thévenin and Norton's equivalents and maximum power theorem in practice

Objectives of Section 4.4:
- Obtain an initial exposure to nonlinear circuit analysis
- Be able to solve in a simple nonlinear circuit

Application examples:
Reading and using data for solar panels
Power radiated by a transmitting antenna
Maximum power extraction from solar panel
Solving the circuit for a generic solar cell

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Keywords:

Nodal analysis, Mesh analysis (mesh-current analysis), Supernode, Supermesh, Method of short/open circuit (definition of, open-circuit network voltage, short-circuit network current), Source transformation theorem, Circuit equivalent (see equivalent circuit), Thévenin's theorem (formulation, proof, special cases), Thévenin equivalent, Norton's theorem, Norton equivalent, R-$2R$ ladder network, Negative equivalent (Thévenin) resistance, Maximum power theorem (principle of maximum power transfer), Power efficiency, Analysis of nonlinear circuits, Load line (definition, method of), Iterative method for nonlinear circuits (definition of, explicit iterative scheme, implicit iterative scheme), Solar cell (c-Si, open-circuit voltage, short-circuit photocurrent density, fill factor, characteristic equation of), Solar panel (series cell connection, open-circuit voltage, short-circuit photocurrent, fill factor, maximum power load voltage, maximum power load current)

Section 4.1 Nodal/Mesh Analysis

4.1.1 Importance of Circuit Simulators

The series and parallel equivalents along with Y and Δ transformations provide a practical tool for solving simple circuits involving typically only a few elements. However, for more elaborate circuits, circuit simulators such as SPICE (Simulation Program with Integrated Circuit Emphasis) and its various modifications become indispensable tools for the professional engineer. SPICE was developed by the Electronics Research Laboratory at the University of California, Berkeley, and first presented in 1973. These circuit simulators are quite general and allow us to model circuits with passive and active elements including semiconductor components such as diodes, transistors, and even solar cells. Since those elements typically exhibit nonlinear current–voltage behaviors, elaborate solution strategies are needed. The circuit simulators use quite interesting algorithms: they often operate in the time domain, even for DC circuits. For example, a solution for a DC circuit is obtained as the steady-state limit of a transient solution, for voltage and/or current sources turned on at a certain time instance. The key of the time-domain approach is its inherent ability to solve nonlinear problems, with passive and active circuit elements. In this section, we are unable to discuss in detail the principles of the numerical circuit simulation. However, we will provide the foundation of the *nodal analysis* (or *node analysis*) and the *mesh analysis* (or the *mesh-current analysis)*, which are two important features of a professional circuit simulator. The nodal and mesh analyses in its pure form do not involve time-domain methods. They are primarily applicable only to *linear circuits*, also referred to as *linear networks*.

4.1.2 Nodal Analysis for Linear Circuits

The *nodal analysis* is a general method of solving linear networks of arbitrary complexity, which is based on KCL and Ohm's law. Let us consider a circuit shown in Fig. 4.1a, which is a resistive bridge circuit with a bridging resistance. This circuit may be solved using Δ to Y conversion; see, for instance, example 3.15 of Chapter 3. Here, we prefer to use the nodal analysis directly. The nodal analysis operates with the *absolute* values of the *node voltages* in the circuit with respect to *ground reference*. It may be divided into a number of distinct steps:

1. A ground reference needs to be assigned first: a node where the voltage is set to 0 V. To this end, we ground the negative terminal of the voltage power supply.
2. Next, we select nontrivial (also called *non-reference*) nodes for which we do not know the voltages. These are nodes 1 and 2 in Fig. 4.1b. The two additional nodes are eliminated from the analysis since the voltages there are already known.
3. We label *absolute* node voltages versus ground reference as V_1, V_2—see Fig. 4.1c.

4. We label currents for every nontrivial node, assuming that all currents are *outflowing*; see Fig. 4.1c. The last condition may be replaced by all inflowing currents.
5. Next, KCL is written for every nontrivial node. We express the currents as the difference of two absolute voltages: the voltage at the beginning of the current arrow (voltage at the master node) minus the voltage at the end of the current arrow (voltage at any other node) and then divide this difference by the appropriate resistance. Hence, we arrive at a system of linear equations for the nodal voltages. Currents are no longer involved.
6. After the resulting system of linear equations is solved, all circuit parameters are determined as necessary.

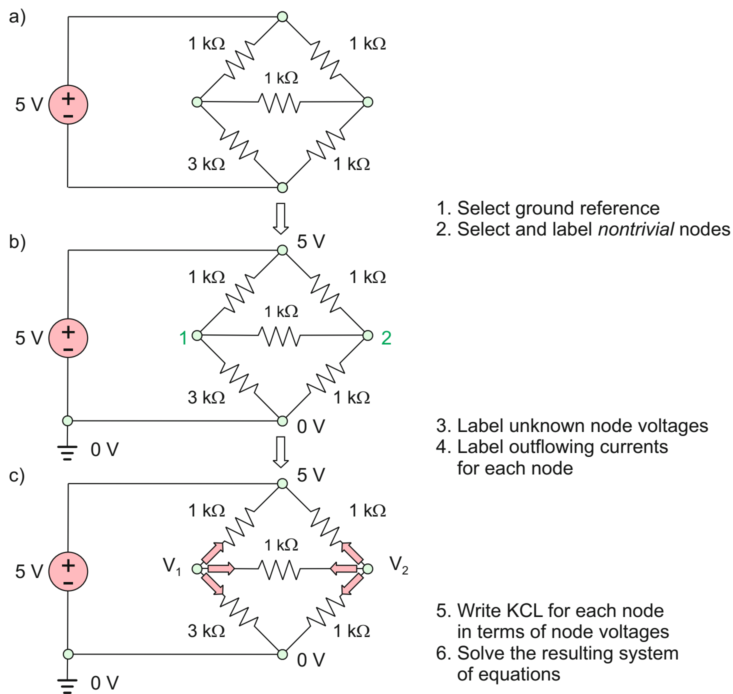


Fig. 4.1. Major steps of the nodal analysis applied to a bridge circuit.

The following two examples will apply the nodal analysis to a circuit with a voltage source.

Example 4.1: Solve the circuit shown in Fig. 4.1a using the nodal analysis—find the supply current.

Solution: Steps 1–4 are indicated in Fig. 4.1b, c. Applying KCL to node 1 and then to node 2 (order is not important), one has

$$\frac{V_1 - 5\ \mathrm{V}}{1\ \mathrm{k\Omega}} + \frac{V_1 - 0\ \mathrm{V}}{3\ \mathrm{k\Omega}} + \frac{V_1 - V_2}{1\ \mathrm{k\Omega}} = 0 \tag{4.1a}$$

$$\frac{V_2 - 5\ \mathrm{V}}{1\ \mathrm{k\Omega}} + \frac{V_2 - 0\ \mathrm{V}}{1\ \mathrm{k\Omega}} + \frac{V_2 - V_1}{1\ \mathrm{k\Omega}} = 0, \tag{4.1b}$$

i.e., a system of the linear equations for two unknown voltages. Its simplification

$$7/3V_1 - V_2 = 5\ \mathrm{V} \tag{4.2a}$$

$$3V_2 - V_1 = 5\ \mathrm{V} \tag{4.2b}$$

is solved via *Gaussian elimination of unknowns*, which yields

$$V_1 = 3.33\ \mathrm{V} \text{ and } V_2 = 2.78\ \mathrm{V} \tag{4.3}$$

The circuit current (current of the voltage source) is

$$(5\ \mathrm{V} - V_1)/1\ \mathrm{k\Omega} + (5\ \mathrm{V} - V_2)/1\ \mathrm{k\Omega} = 3.89\ \mathrm{mA}.$$

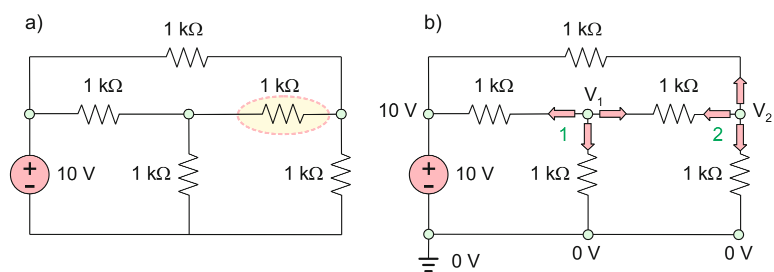


Fig. 4.2. Major steps of the nodal analysis applied to a circuit with a voltage source.

Example 4.2: Solve the circuit shown in Fig. 4.2a using the nodal analysis.

Solution: Steps 1–4 are indicated in Fig. 4.2b. Applying KCL to node 1 and then to node 2, one obtains a system of equations with two unknown voltages:

$$\frac{V_1 - 10\ \mathrm{V}}{1\ \mathrm{k\Omega}} + \frac{V_1 - 0\ \mathrm{V}}{1\ \mathrm{k\Omega}} + \frac{V_1 - V_2}{1\ \mathrm{k\Omega}} = 0 \tag{4.4a}$$

Example 4.2 (cont.):

$$\frac{V_2 - 10\ \text{V}}{1\ \text{k}\Omega} + \frac{V_2 - 0\ \text{V}}{1\ \text{k}\Omega} + \frac{V_2 - V_1}{1\ \text{k}\Omega} = 0 \tag{4.4b}$$

In setting up the equations, it does not matter which sequence of nodes are selected. Simplifying Eq. (4.4) gives

$$3V_1 - V_2 = 10\ \text{V} \tag{4.5a}$$

$$3V_2 - V_1 = 10\ \text{V} \tag{4.5b}$$

The solution is obtained by symmetry, i.e., $V_1 = V_2 = 5\ \text{V}$. The circuit current provided by the power supply is 10 mA. All other branch currents can then be found using Ohm's law. An interesting feature of the circuit shown in Fig. 4.2a is that the marked 1-kΩ resistor can be considered as "dead," since there is no current flowing through it (the voltage difference across this resistor is exactly zero). This resistor can be removed from the circuit without affecting the behavior of the circuit in terms of voltages and currents. It might appear at first sight that the circuits shown in Figs. 4.1 and 4.2 have a different network topology. In fact, they do not. To prove this, attempt to redraw the circuit in Fig. 4.2a; the result will coincide with the circuit in Fig. 4.1a.

Circuits with a Current Source

When a current source is present in a circuit, the solution becomes even simpler: one makes use of the existing current and substitutes its value into KCL equation written for a certain node. For example, KLC for node 1 in Fig. 4.3 includes the outflowing current of −1 mA. The current sign must be taken into account. The same idea may be applied to circuits with multiple current power supplies. When only the current sources are present, the ground may be connected to the incoming terminal of a current source.

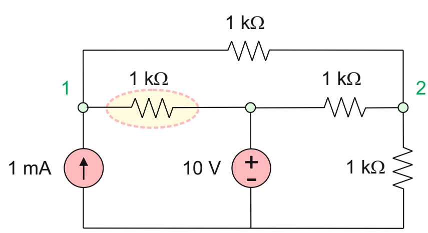


Fig. 4.3. A circuit with a current source solved via nodal analysis.

Exercise 4.1: Determine voltage across the current source in Fig. 4.3 using the nodal analysis.

Answer: 8.6 V.

4.1.3 Supernode

The nodal analysis requires a "good" eye to see possible simplifications when labeling the nodes. Let us examine a particular case and point out a few useful subtleties.[1] Figure 4.4a depicts a network with two voltage sources. The property of the 5 V source is such that it is not fixed to a particular ground connection—we therefore call it a *floating source*. Setting up the node method becomes a little tricky, since we do not know the current through this source. However, a *supernode* may be formed as shown in Fig. 4.4b.

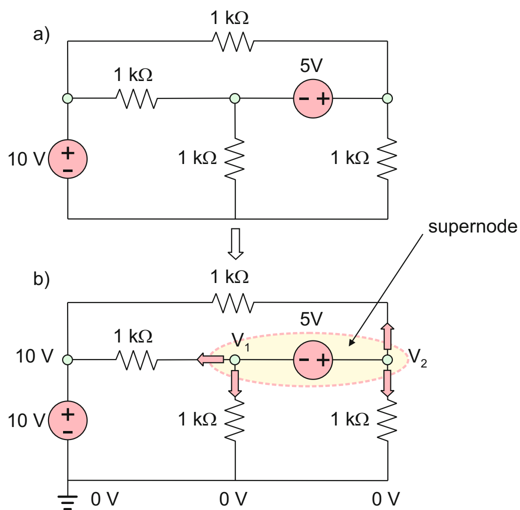


Fig. 4.4. A network with a floating voltage source between nodes 1 and 2.

KCL may be applied to any closed contour around the supernode: the net current must still be zero in such a case. With reference to Fig. 4.4b, this yields

$$\frac{V_1 - 10\ \text{V}}{1\ \text{k}\Omega} + \frac{V_1 - 0\ \text{V}}{1\ \text{k}\Omega} + \frac{V_2 - 0\ \text{V}}{1\ \text{k}\Omega} + \frac{V_2 - 10\ \text{V}}{1\ \text{k}\Omega} = 0 \tag{4.6a}$$

What is the second equation for two unknowns V_1 and V_2 (just the relation between the supernode voltages themselves)? Since $V_2 - V_1$ is the voltage of the power source, one has

$$V_2 = V_1 + 5\ \text{V} \tag{4.6b}$$

Equations (4.6) can now be solved even without a calculator, eliminating one of the unknowns yields

[1] Subtleties are often euphemism for "playing" around with the circuit, like redrawing the wire connections and rearranging the circuit elements. This is done to find simpler solution approaches.

$$V_1 = 2.5 \ \text{V}, \quad V_2 = 7.5 \ \text{V} \tag{4.7}$$

The circuit is solved. All currents are found using the node voltages and Ohm's law.

Example 4.3: Now, solve the circuit shown in Fig. 4.4a using the standard nodal analysis, without the supernode concept.

Solution: We have to specify an unknown current I_x through the 5-V source, which flows, say, from left to right in Fig. 4.4a. It results in the following two nodal equations for the two nodes:

$$\frac{V_1 - 10 \ \text{V}}{1 \ \text{k}\Omega} + \frac{V_1 - 0 \ \text{V}}{1 \ \text{k}\Omega} + I_x = 0 \tag{4.8a}$$

$$\frac{V_2 - 0 \ \text{V}}{1 \ \text{k}\Omega} + \frac{V_2 - 0 \ \text{V}}{1 \ \text{k}\Omega} - I_x = 0 \tag{4.8b}$$

Now, we can add both equations and thereby eliminate I_x. The result is exactly Eq. (4.6a) for the supernode. We must add one more condition to solve this equation. Equation (4.6b) is the only choice, i.e.,

$$V_2 = V_1 + 5 \ \text{V} \tag{4.8c}$$

With this in mind, we arrive at the supernode concept again but in a more complicated way. This is why the supernode approach is a useful tool.

4.1.4 Mesh Analysis for Linear Circuits

The *mesh analysis* (or the *mesh-current analysis*) is using loops instead of nodes. Only loops that do not contain any other loops—*the meshes*—are employed. The meshes as elements of the networking topology were defined in Section 3.1. Accordingly, instead of KCL, the mesh analysis makes use of KVL. Hence, we need to choose *mesh* currents for every mesh. Figure 4.5 depicts the concept for a circuit with three meshes. Note that this circuit is identical to the circuit from Fig. 4.1. A ground connection does not have to be introduced for the mesh method. Let us denote the mesh current for mesh 1 in Fig. 4.5 by I_1, the mesh current for mesh 2 by I_2, and the mesh current for mesh 3 by I_3.

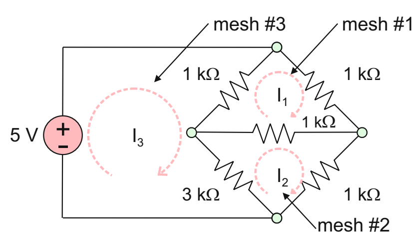


Fig. 4.5. Circuit solution using the mesh analysis. Circuits in Figs. 4.5 and 4.1 coincide.

KVL equations for the three meshes are based on Ohm's law for this passive reference configuration. We do not need the fourth (large) loop encompassing the entire circuit. For resistances that are shared by two adjacent meshes, we combine either the difference or the sum of the two adjacent-mesh currents. The mesh equations become

$$\text{Mesh 1}: \ 1\ \text{k}\Omega \cdot (I_1 - I_3) + 1\ \text{k}\Omega \cdot I_1 + 1\ \text{k}\Omega \cdot (I_1 - I_2) = 0 \quad (4.9a)$$

$$\text{Mesh 2}: \ 3\ \text{k}\Omega \cdot (I_2 - I_3) + 1\ \text{k}\Omega \cdot (I_2 - I_1) + 1\ \text{k}\Omega \cdot I_2 = 0 \quad (4.9b)$$

$$\text{Mesh 3}: \ -5\text{V} + 1\ \text{k}\Omega \cdot (I_3 - I_1) + 3\ \text{k}\Omega \cdot (I_3 - I_2) = 0 \quad (4.9c)$$

We have arrived at a system of *three* equations for the three unknown mesh currents I_1, I_2, and I_3. It is simplified to (after division by 1 kΩ and combining similar terms)

$$\text{Mesh 1}: \ +3I_1 - I_2 - I_3 = 0 \quad (4.10a)$$

$$\text{Mesh 2}: \ -I_1 + 5I_2 - 3I_3 = 0 \quad (4.10b)$$

$$\text{Mesh 3}: \ -I_1 - 3I_2 + 4I_3 = 5\ \text{mA} \quad (4.10c)$$

In contrast, the nodal analysis applied to the same circuit requires only *two* equations for two unknown node voltages; see Example 4.1. The final solution is indeed the same. Thus, the nodal analysis is more beneficial for small networks when a voltage source or sources are present. If, however, a current source were present in Fig. 4.5 instead of the voltage source, the nodal analysis would require three equations. At the same time, the mesh analysis would require only two equations, because I_3 is defined by the current source. Reasoning like this gives us clues which method is most suitable. When mixed power supplies like voltage and current sources are involved, there is usually no real difference between the two methods. The choice often becomes a matter of taste.

Exercise 4.2: Determine mesh currents for the circuit in Fig. 4.5.

Answer: $I_1 = -0.833\ \text{mA}$, $I_2 = +0.833\ \text{mA}$, $I_3 = +1.667\ \text{mA}$.

4.1.5 Supermesh

Consider a circuit shown in Fig. 4.6. The straightforward mesh analysis should use KVL written for meshes 1 and 2. However, KVL cannot be formulated directly since we do not know the voltage across the current source. A solution is to combine meshes 1 and 2 into a *supermesh* and write KVL around its periphery. Mesh equations become

$$\text{Supermesh}: 1\ \text{k}\Omega \cdot (I_1 - I_3) + 1\ \text{k}\Omega \cdot I_1 + 1\ \text{k}\Omega \cdot I_2 + 3\ \text{k}\Omega \cdot (I_2 - I_3) = 0 \quad (4.11a)$$

$$\text{Mesh 3}: \ -5\ \text{V} + 1\ \text{k}\Omega \cdot (I_3 - I_1) + 3\ \text{k}\Omega \cdot (I_3 - I_2) = 0 \quad (4.11b)$$

KCL for the central branch of the source:

$$I_1 - I_2 = 1 \;\; \text{mA} \tag{4.11c}$$

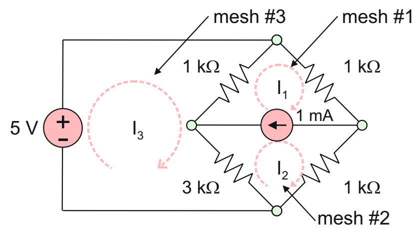


Fig. 4.6. Circuit solved with the supermesh method.

After division by 1 kΩ and combining similar terms, the system of equations (4.11) is simplified to

$$\begin{aligned} &+2I_1 + 4I_2 - 4I_3 = 0 \\ &-I_1 - 3I_2 + 4I_3 = 5 \;\; \text{mA} \\ &+I_1 - I_2 = 1 \;\; \text{mA} \end{aligned} \tag{4.12}$$

Exercise 4.3: Determine mesh currents for the circuit in Fig. 4.6.

Answer: $I_1 = 2.5$ mA, $I_2 = +2.5$ mA, $I_3 = 3.75$ mA.

Example 4.4: Outline the solution approach for the circuit shown in Fig. 4.6 using the standard mesh analysis, without the supermesh concept.

Solution: The voltage across the current source is introduced as an extra unknown, V_x. Then, we write *three* KVL equations for *three* meshes in Fig. 4.6, which will contain *four* unknowns: I_1, I_2, I_3, and V_x. An extra equation is needed, which is KCL for the central branch: $I_1 - I_2 = 1$ mA. Now, we need to solve a system of four simultaneous equations. This is considerably more work than in the previous case. This is why the supermesh approach is a useful tool for the mesh analysis.

Section 4.2 Generator Theorems

4.2.1 Equivalence of Active One-Port Networks: Method of Short/Open Circuit

In Chapter 3, we considered *passive linear networks* with only resistances, and we have transformed them into equivalent circuits. *Active linear networks*, which include sources and resistances simultaneously, can undergo similar transformations. We know that two electric single-port networks are equivalent when their terminal v-i characteristics are identical. For passive resistive networks studied in Chapter 3, we connected *arbitrary source(s)* across the network terminals and checked the resulting v-i characteristics. For active networks with sources and resistances, we can use the same method. Alternatively, we could connect arbitrary *resistance(s)* across the network terminals and check either the resulting voltage or current. A test resistance to be connected will be denoted here by R. If for two networks the voltages across the resistance R (or currents through it) coincide for *all* values of R, the networks are equivalents.

Method of Short/Open Circuit

In general, testing all possible values of resistance R connected to terminals a and b of a network in Fig. 4.7 is not necessary. Note that an active linear network may ultimately have only two elements: a source and a resistance. To uniquely determine the two elements (their values), only *two* equations are necessary. It is therefore customary to check only *two* (limiting) values of the test resistance:

$$R \to \infty \text{ and } R = 0 \qquad (4.13)$$

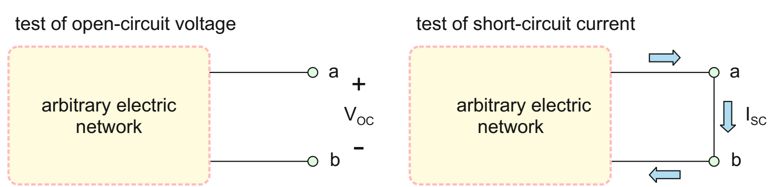


Fig. 4.7. Method of short/open circuit for an active, one-port network.

Conveniently, this corresponds to open- and short-circuit conditions. In the first case, the voltage between terminals a and b is the *open-circuit network voltage* V_{OC}. In the second case, the current flowing from terminal a to terminal b is the *short-circuit network current* I_{SC}. The pair V_{OC}, I_{SC} is key for the *method of short/open circuit*. This method states that two active linear circuits are equivalent when their V_{OC} and I_{SC} coincide. Network equivalency relates not only to the linear active networks with two components, but, as will be shown soon, it is valid for *all* active linear networks.

4.2.2 Application Example: Reading and Using Data for Solar Panels

The method of short/open circuit is also very useful for *active nonlinear networks*, including nonlinear sources. An example is a *solar cell* or a combination thereof, a *solar panel*. Every solar panel has the measured data for V_{OC} and I_{SC} listed on its backside. The short-circuit current is simultaneously the *photocurrent* of the solar cell. Table 4.1 collects this data for common *crystalline silicon* (or c-Si) solar panels. It is organized in such way that V_{OC} is given per cell in the panel and I_{SC} is given in terms of photocurrent density, J_P, per unit cell area. The cells in the panel are connected *in series*.

Table 4.1. Manufacturers' specified parameters for different c-Si solar panels from five different manufacturers (1–230 W output power range). The cell area is either measured directly or extracted from the datasheet.

Solar panel	Cells, N	V_{OC}/N, V	Cell area A, cm^2	$J_P = I_{SC}/A$ A/cm^2
1-W BSPI-12 Power Up c-Si panel	36	0.59	2.36	0.030
10-W BSP-1012 Power Up c-Si panel	36	0.59	~22.0	0.030
65-W BSP-1012 Power Up c-Si panel	36	0.61	121.7	0.032
230-W Sharp ND-U230C1 c-Si panel	60	0.62	241.0	0.034
175-W BP Solar SX3175 c-Si panel	72	0.61	156.25	0.033
6-W Global Solar GSE-6 c-Si panel	44	0.52	16.6	0.027
200-W GE Energy GEPVp-200 c-Si panel	54	0.61	249.3	0.032
Average	NA	0.593	NA	0.0311

Table 4.1 demonstrates that c-Si solar cells have approximately the same *open-circuit voltage* of 0.6 V per cell. The open-circuit voltage does not depend on the area of the cell. The *short-circuit photocurrent density* is also approximately the same for c-Si solar cells from different manufacturers. On average, it is given by $J_P = 0.03\ A/cm^2$. These values correspond to an incident light intensity of 1000 W/m^2 at $T = 25\,°C$. The photocurrent density does not depend on the area of the cell. However, the total photocurrent does.

Exercise 4.4: A c-Si solar panel (or *solar module*) has the open-circuit voltage of 23.4 V? How many individual solar cells does it have?

Answer: Approximately 39.

Exercise 4.5: A c-Si solar panel is needed with the open-circuit voltage of 12 V and the short-circuit current of 3 A. Design the panel: find the number of cells to be connected in series and the required unit cell area.

Answer: 20 cells with the area of 100 cm^2 (10×10 cm) each.

4.2.3 Source Transformation Theorem

The most fundamental transformation of active linear networks is the subject of the source transformation theorem. The *source transformation theorem* is a substitution of an independent voltage source V_T in series with resistance R_T for an independent current source I_N with resistance R_N and vice versa; see Fig. 4.8a, b. The meaning of indexes N and T will become apparent soon. The *identical* theorem applies to the dependent sources shown in Fig. 4.8c, d.

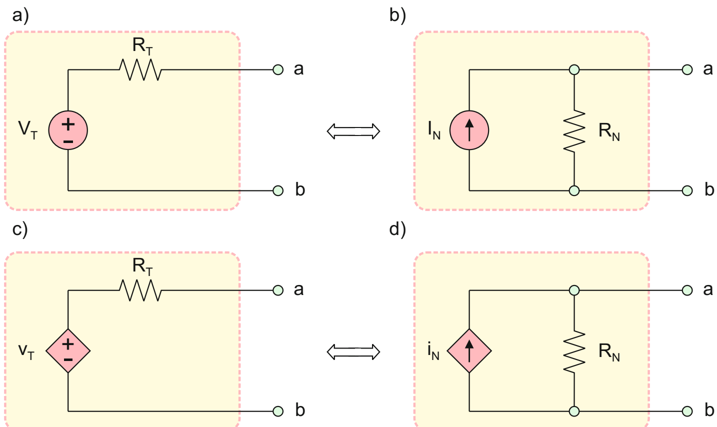


Fig. 4.8. Transformation of dependent and independent sources.

Let us prove this theorem by establishing the circuit equivalence. The pair V_{OC}, I_{SC} is to be found for every network. For the two networks in Fig. 4.8a, b, we have

$$V_{OC} = V_T, I_{SC} = \frac{V_T}{R_T} \tag{4.14a}$$

$$V_{OC} = R_N I_N, I_{SC} = I_N \tag{4.14b}$$

Equation (4.14) has a unique solution in the form of the source transformation theorem

$$R_N = R_T,\ I_N = \frac{V_T}{R_T} \tag{4.15}$$

If Eq. (4.15) is satisfied, both networks in 4.8a, b have equal V_{OC} and I_{SC}. This ensures that their entire v-i characteristics are also the same. To confirm this fact, an arbitrary resistance R could be connected across the port. The resulting voltages may be found directly, by solving the voltage divider and the current divider circuits, respectively. Both voltages are equal to $V_T R/(R + R_T)$. Thus, the source transformation theorem is proved.

Exercise 4.6: A network has a 10-V voltage source in series with a 20-Ω resistance. It is replaced by a current source I_N in parallel with resistance R_N. Find I_N and R_N.

Answer: $I_N = 0.5\ \text{A}$, $R_N = 20\ \Omega$.

Exercise 4.7: A linear active circuit measures the open-circuit voltage of 5 V and the short-circuit current of 1 mA. Determine its equivalents in the form of a voltage source in series with a resistance and in the form of a current source in parallel with a resistance.

Answer: $V_T = 5\ \text{V}$, $R_T = 5\ \text{k}\Omega$ and $I_N = 1\ \text{mA}$, $R_N = 5\ \text{k}\Omega$.

Often, the source transformation theorem allows us to simplify the circuit analysis through network manipulations.

Example 4.5: Find current I_1 in the circuit shown in Fig. 4.9a.

Solution: The circuit may be solved using the superposition theorem. Another way is to use the source transformation theorem. The corresponding steps are outlined in Fig. 4.9b, c. We use the source transformation three times and end up with the parallel combination of two current sources and three resistances. The three resistances in parallel are equivalent to the $0.75\ \text{k}\Omega$ resistance; the voltage across every element in parallel is then $0.75\ \text{k}\Omega \times 2\ \text{mA} = 1.5\ \text{V}$. Therefore, $I_1 = 0.75\ \text{mA}$.

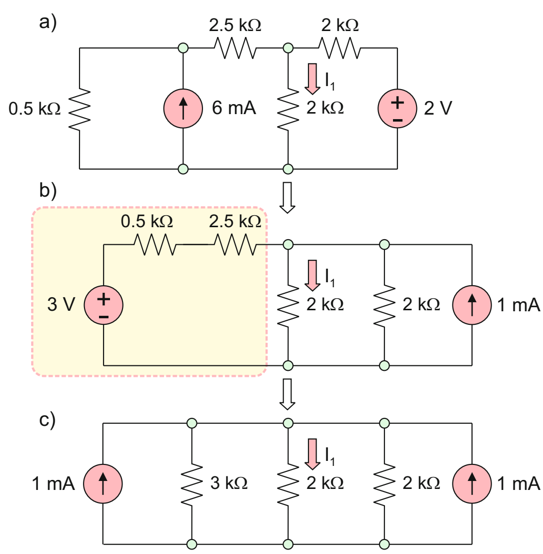


Fig. 4.9. Circuit modifications using the source transformation theorem.

Example 4.6: The circuit in Fig. 4.10a includes a current-controlled voltage source with the strength of $4000i_x$ [V]. Find current i_x using the source transformation.

Solution: The corresponding circuit transformation is shown in Fig. 4.10b. The circuit with the current-controlled current source in Fig. 4.10b is solved using KCL and KVL. KCL written for the bottom node states that the current of $3\text{ mA} + 3i_x$ flows through the rightmost 1-kΩ resistance (directed down). Since, by KVL, the voltages across both resistances must be equal, one has

$$3\,\text{mA} + 3i_x = i_x \Rightarrow i_x = -1.5\,\text{mA} \tag{4.16}$$

Alternatively, one might convert the independent current source to the independent voltage source. However, this method would hide i_x.

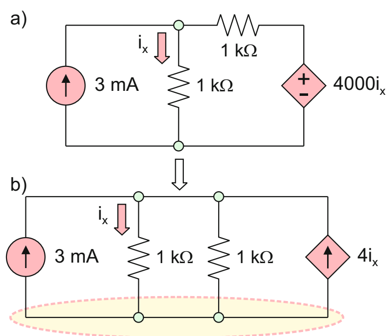


Fig. 4.10. Using source transformation for a circuit with dependent sources.

4.2.4 Thévenin's and Norton's Theorems: Proof Without Dependent Sources

The origin of *Thévenin's theorem* is due to Léon Charles Thévenin, a French engineer (1857–1926). The theorem is illustrated in Fig. 4.11a, b and can be expressed in the following form:

1. Any linear network with independent voltage and current sources, dependent linear sources, and resistances, as shown in Fig. 4.11a, can be replaced by a simple equivalent network: a voltage source V_T in series with resistance R_T.
2. The equivalent network in Fig. 4.11b is called the *Thévenin equivalent*.
3. Voltage V_T is the open-circuit voltage V_{OC} of the original network.
4. When dependent sources are *not* present, Thévenin resistance R_T is an equivalent resistance R_{eq} of the original network with *all* independent sources *turned off* (voltage sources are replaced by short circuits and current sources by open circuits).
5. When *both* dependent and independent sources are present, the independent sources are not turned off. Resistance R_T is given by $R_T = V_{OC}/I_{SC}$, where I_{SC} is the short-circuit current of the original network.

6. When *only* the dependent sources are present, a current (or voltage) source is connected to the network terminals. Thévenin or equivalent resistance is given by $R_T = V/I$ where I is the source current and V is the voltage across the source. Thévenin voltage is, strictly speaking, not defined in this case.

The *Norton's theorem* is *dual* to the Thévenin's theorem. It was named in honor of Edward L. Norton (1898–1983), an engineer at Bell Labs in New Jersey.[2] Norton's theorem is illustrated in Fig. 4.11c, d. The equivalent circuit (*Norton's equivalent*) is now the current source in parallel with the resistance as shown in Fig. 4.11d. Since the equivalence of both networks in Fig. 4.11b, d has already been established, the Norton's theorem will follow from the Thévenin's theorem and vice versa.

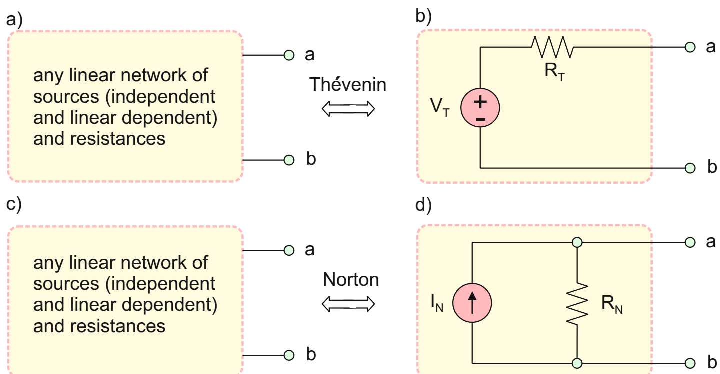


Fig. 4.11. Thévenin's and Norton's theorems: replacing linear active circuits by its Thévenin and Norton equivalents.

Proof of Thévenin's Theorem for Active Networks Without Dependent Sources

The proof is based on circuit linearity. The *v-i* characteristics of both networks will be established using a current source of strength I connected as shown in Fig. 4.12.

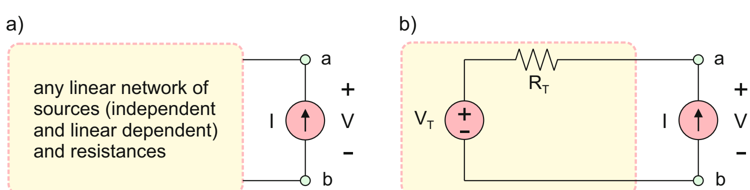


Fig. 4.12. Derivation of Thévenin's theorem by establishing the *v-i* characteristics.

[2] The first publication that discusses this equivalent circuit concept is actually due to Hans F. Mayer (1895–1980) who made the discovery in 1926 while a researcher at Siemens Company.

Since the entire circuit is still linear, the v-i characteristic of the current source in Fig. 4.12a must have the form of a *linear function*,

$$V = AI + B, \tag{4.17}$$

where V is the voltage across the current source. A and B are some "constant" coefficients, which do *not* depend on I, but do depend on the network parameters. Our goal is to find A and B, respectively. First, we check the value $I = 0$ when the external current source is turned off, i.e. replaced by an open circuit. From Eq. (4.17), voltage V equals B. On the other hand, it equals of V_{OC} the original network. Therefore,

$$B = V_{OC} \tag{4.18}$$

Now, let us turn off all the internal sources. The network becomes an equivalent resistance R_{eq}. The constant B (its open-circuit voltage) is zero. Equation (4.17) therefore yields $V = AI$, for any value of I. On the other hand, for the current source I connected to the resistance R_{eq}, it must be $V = R_{eq}I$. Comparing the two expressions, we obtain

$$A = R_{eq} \tag{4.19}$$

The simpler network in Fig. 4.12b is also described by the v-i characteristic in the form of Eq. (4.17). In this case, $B = V_{OC} = V_T$, $A = R_{eq} = R_T$. We finally compare two v-i characteristics,

$$\begin{aligned} &V = R_{eq}I + V_{OC} \text{ for linear active network and} \\ &V = R_T I + V_T \text{ for Thévenin equivalent,} \end{aligned} \tag{4.20}$$

and establish the Thévenin's theorem. A test voltage source could be used in place of the current source in Fig. 4.12, with the same result obtained. The physical background of the Thévenin's theorem is thus the fact that the terminal response of any linear network is a linear v-i characteristic—a linear function with only *two* independent coefficients, A and B. A simpler network with exactly *two* independent parameters—V_T and R_T – is just right to model this response.

***Equivalence of Arbitrary Linear Networks with Identical* V_{OC}, I_{SC}**

On one hand, linear active networks with only two elements (a source and a resistance) are equivalent when their V_{OC}, I_{SC} coincide. On the other hand, any active linear network is equivalent to a linear network with only two elements. Therefore, we conclude that two arbitrary linear networks are equivalent when their V_{OC} and I_{SC} coincide.

Exercise 4.8: Establish Thévenin equivalent circuits for the two networks shown in Fig. 4.13. In the first case, $R_1 = R_2 = R_3 = 1\ \text{k}\Omega$ and $V_S = 10\ \text{V}$. The second network is a battery bank—a network of series-connected practical voltage sources—with $R_{B1} = R_{B2} = R_{B3} = 1\ \Omega$, $V_{B1} = V_{B2} = V_{B3} = 6\ \text{V}$.

Answer: $V_T = 5\ \text{V}$, $R_T = 0.5\ \text{k}\Omega$ and $V_T = 18\ \text{V}$, $R_T = 3\ \Omega$, respectively.

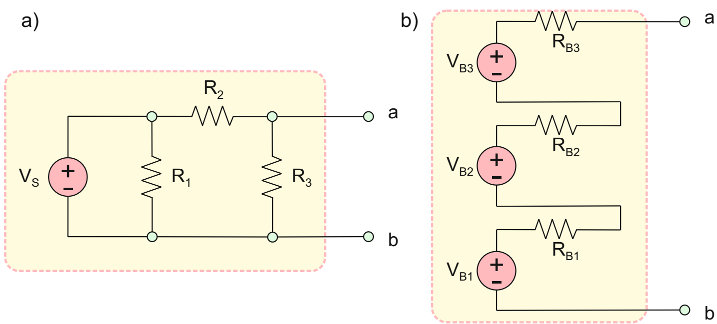


Fig. 4.13. Two active linear networks.

Example 4.7: A two-terminal network shown in Fig. 4.14a is a two-bit *R*-2*R ladder network* used for digital-to-analog conversion. Express

1. Thévenin (or equivalent) voltage V_T
2. Thévenin (or equivalent) resistance R_T

in terms of (digital) voltages D_0, D_1 and resistance R.

Solution: One way to solve this problem is to find V_T and R_T directly from the circuit in Fig. 4.14a. While the solution for R_T is straightforward, finding V_T requires more work. Yet another method is to apply the Thévenin equivalent to the leftmost section of the ladder network first. The result is the circuit shown in Fig. 4.14b. The final Thévenin equivalent has the form $R_T = R, \quad V_T = \frac{D_1}{2} + \frac{D_0}{4}$.
This method may be applied to ladder networks with multiple sections.

Exercise 4.9: Repeat the previous example for the ladder shown in Fig. 4.14c.

Answer: $R_T = R, \quad V_T = \frac{D_2}{2} + \frac{D_1}{4} + \frac{D_0}{8}$.

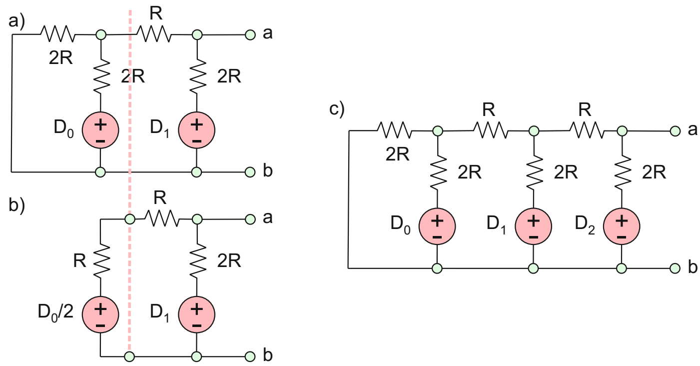


Fig. 4.14. Two-bit and three-bit *R*-2*R* ladder networks.

Example 4.8: Find Thévenin and Norton equivalent circuits for the network in Fig. 4.15a.

Solution: The network includes a voltage-controlled voltage source. Therefore, its analysis should be performed in a general form, by finding the pair V_{OC}, I_{SC}. The short-circuit current I_{SC} is found straightforwardly. Since the rightmost resistance is shorted out, $v_x = 0$, and $I_{SC} = 10$ mA. To find the open-circuit voltage, which is equal to v_x, we use the source transformation theorem and arrive at the equivalent circuit in Fig. 4.15b. Next, we solve this circuit. By KVL, the voltage across the leftmost resistance is equal to $10\,\text{V} - 4v_x$. By KCL, the currents through both resistances must be the same. Since the resistances are equal, we obtain the equality $10\,\text{V} - 4v_x = v_x$ so that $v_x = 2\,\text{V}$. The open-circuit voltage has the same value. Thévenin and Norton equivalents are

$$V_T = V_{OC} = 2\ \text{V},\ R_T = \frac{V_{OC}}{I_{SC}} = 200\ \Omega \tag{4.21a}$$

$$I_N = I_{SC} = 10\ \text{mA},\ R_N = \frac{V_{OC}}{I_{SC}} = 200\ \Omega \tag{4.21b}$$

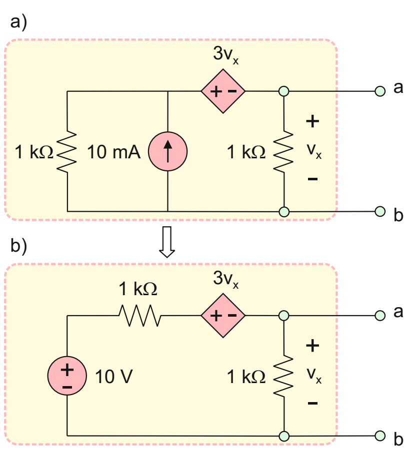


Fig. 4.15. A network with dependent sources to be converted to its equivalent forms.

4.2.5 Application Example: Generating Negative Equivalent Resistance

Consider a circuit shown in Fig. 4.16a. It includes only a dependent source: the voltage-controlled voltage source; it does not have independent sources. Therefore, the circuit analysis has to be done by connecting a current (or voltage) source between terminals *a* and *b* as shown in Fig. 4.16b. Quantity of interest is Thévenin (or equivalent) resistance. KVL for the circuit in Fig. 4.16b gives $-Av_x - RI + v_x = 0$. Therefore, by definition,

$$R_T \equiv \frac{v_x}{I} = \frac{R}{1-A} \tag{4.22}$$

As long as the open-circuit voltage gain of the dependent source, A, is greater than one, Eq. (4.22) states the *negative equivalent or Thévenin resistance*. Physically, this means that the Thévenin equivalent circuit is delivering power instead of absorbing it.

Construction and Use of Negative Equivalent Resistance

A circuit block, which is equivalent to the negative resistance, may be constructed using the operational amplifier studied in the next chapter. This block may be used for different purposes including signal generation. The difference between the negative resistance and the power source is that the negative resistance may supply power of any type (DC, AC, or an arbitrary waveform), i.e., support the *self-oscillating circuits*. Figure 4.16c summarizes Thévenin resistances generated by the basic networks with the only dependent sources. The same method of analysis (simultaneous use of KCL and KVL) has been applied to every network. Although all of the networks may in principle generate negative equivalent resistance values, the realization of some particular circuits may be difficult.

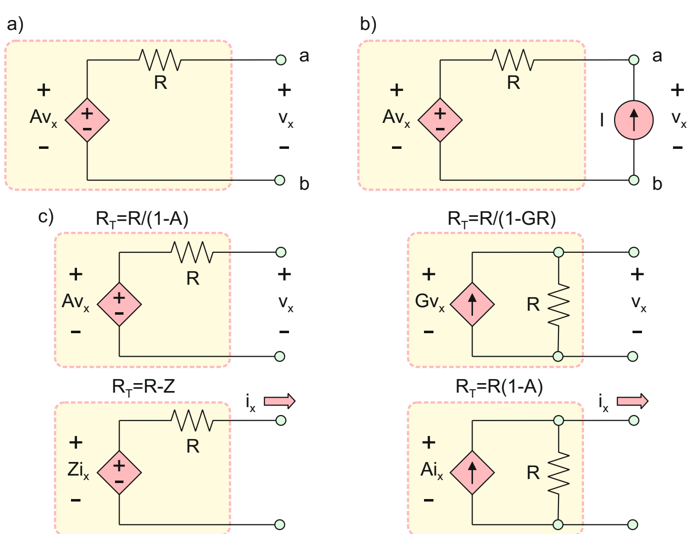


Fig. 4.16. Thévenin equivalent circuits for basic networks with dependent sources.

4.2.6 Summary of Circuit Analysis Methods

In summary, a linear circuit can be solved using any of the methods studied in this and in the previous chapters:

- Superposition theorem
- Nodal/mesh analysis
- Source transformation theorem
- Thévenin and Norton equivalent circuits

or a combination of those. While the nodal/mesh analysis is always applicable, other methods may even be more useful since they often provide physical insight into the circuit behavior.

Exercise 4.10: How could you find the open-circuit voltage V_{ab} in Fig. 4.14a?

Answer:

A. When the superposition theorem is applied, shorting out D_0 gives $V_{ab} = D_1/2$. Shorting out D_1 gives $V_{ab} = D_0/4$.
B. When the nodal analysis is applied, we ground negative terminals of both sources and find the unknown voltage of the upper left node via the KCL. Only one equation needs to be solved. This is perhaps the simplest solution method.
C. The source transformation theorem can hardly be applied.
D. The method of Thévenin equivalent circuits has been described in Example 4.7.

Section 4.3 Power Transfer

4.3.1 Maximum Power Transfer

The *principle of maximum power transfer* from a source to a load will now be quantified. This principle is also known as a *maximum power theorem*. The circuit under study is shown in Fig. 4.17. It involves an arbitrary linear source (a battery, generator, etc.), which is represented by its Thévenin equivalent, and a load, which is characterized by its equivalent resistance R_L. All other load parameters (dynamic, mechanical, and thermal) are implicitly included in the load's resistance.

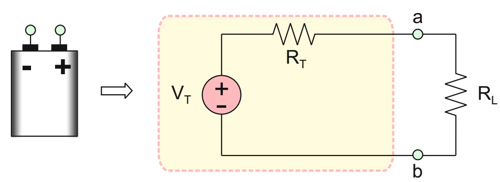


Fig. 4.17. A battery (or another practical voltage source) connected to a load.

The key question you have to ask yourself is this: for a *given* ideal voltage source V_T and a *given* internal resistance R_T, can the electric power delivered to the load be maximized, and at which value of R_L does the maximum occur? The answer is found by solving the circuit in Fig. 4.17. First, the current is determined from the given voltage source V_T and the total resistance using the series equivalent,

$$I = \frac{V_T}{R_T + R_L} \tag{4.23}$$

This allows us to compute the power at the load based on

$$P_L = R_L I^2 = \frac{R_L V_T^2}{(R_T + R_L)^2} \tag{4.24}$$

When V_T and R_T are fixed, the magnitude of the load resistance determines the delivered power P_L. This power tends to zero when $R_L \to 0$ or $R_L \to \infty$; moreover, it is always positive. Therefore, according to Rolle's theorem of calculus, the power must have a maximum at a certain value of R_L. For example, Fig. 4.18 shows a plot of the load power as a function of R_L when V_T=9 V and R_T=5 Ω.

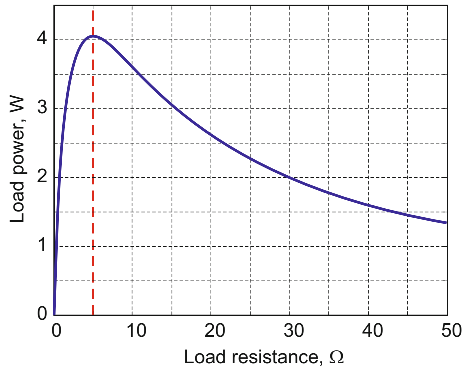


Fig. 4.18. Load power as a function of the load resistance for fixed $V_T = 9\ \text{V},\ R_T = 5\ \Omega$.

We will find the maximum of the load power analytically. We treat P_L in Eq. (4.24) as a function of R_L, i.e., $P_L = P_L(R_L)$. It is known that a function has a maximum when its first derivative is zero. Consequently, differentiating P_L with respect to R_L gives

$$\frac{dP_L}{dR_L} = V_T^2\left[\frac{1}{(R_T + R_L)^2} - 2\frac{R_L}{(R_T + R_L)^3}\right] = V_T^2\left[\frac{R_T - R_L}{(R_T + R_L)^3}\right] = 0 \tag{4.25}$$

The necessary and sufficient condition for Eq. (4.25) to hold is

$$R_L = R_T \Rightarrow P_L = 0.25V_T^2/R_T \tag{4.26}$$

This result is of significant practical value despite, or maybe thanks to, its simplicity. The maximum output power is achieved when the load resistance is equal to the internal resistance of the power source. In other words, the load is *matched* to the source; it is called the *matched load*. In power engineering and in RF and microwave engineering, the problem of load matching is very important. However, it must be clearly stated that *no more* than 50 % of the *total* circuit power can be extracted even in the best case. This statement makes sense if we again examine the circuit in Fig. 4.17 with two equal resistances. The power is divided equally; half of the total power is spent to heat up the power source. The power maximum in Fig. 4.18 is relatively flat over the domain $R_L > R_T$; however, the power drops sharply when $R_L < R_T$. This last condition should be avoided if at all possible.

Example 4.9: An audio amplifier produces an rms output of 20 V. Amplifier's output resistance is rated at 4 Ω. You are given four 4-Ω speakers. How should you connect the speakers for the maximum acoustic power—in series, parallel, or a single speaker only?

Example 4.9 (cont.):

Solution: The rms voltage simply means the equivalent DC voltage that provides the same power to the load as the average power of the primary AC voltage. Hence, the sophisticated AC audio amplifier circuit is essentially replaced by its DC Thévenin equivalent with $V_T = 20$ V and $R_T = 4\ \Omega$. Similarly, the dynamic speakers are replaced by a DC load with $R_L = 16\ \Omega$ if connected in series combination or with $R_L = 1\ \Omega$ if connected in parallel or with $R_L = 4\ \Omega$ if only a single speaker is employed. The output (audio) powers are as follows:

$$P_L = \frac{1 \times 400}{(4+1)^2} = 16\ \text{W four speakers in parallel} \tag{4.27a}$$

$$P_L = \frac{4 \times 400}{(4+4)^2} = 25\ \text{W single speaker} \tag{4.27b}$$

$$P_L = \frac{16 \times 400}{(4+16)^2} = 16\ \text{W four speakers in series} \tag{4.27c}$$

The best (loudest) choice would be surprisingly one single speaker.

4.3.2 Maximum Power Efficiency

A power analysis would be incomplete without discussing the efficiency of the power transfer. Consider an electric boat driven by a marine battery. The optimization of the battery-motor system for maximum power transfer implies that we will move fast but perhaps not very far. Another optimization is possible for maximum power efficiency. In this case, we could tolerate a smaller speed in order to travel a longer distance. The circuit to be analyzed is again shown in Fig. 4.17. The useful power delivered to the load is given by Eq. (4.24). The total power delivered by the source is

$$P = (R_T + R_L)I^2 = \frac{V_T^2}{R_T + R_L} \tag{4.28}$$

The *power efficiency E* is defined as the ratio of the useful power to the total power:

$$E = \frac{P_L}{P} = (R_T + R_L)I^2 = \frac{R_L}{R_T + R_L} \tag{4.29}$$

Thus, the power efficiency is a simple function of the load resistance and the source resistance. It does not depend on the source voltage. The efficiency is zero when the load resistance is zero. It monotonically increases and approaches maximum (the maximum value is unity, which corresponds to an efficiency of 100 %) when the load resistance becomes large enough when compared to the source resistance. For example,

Fig. 4.19 augments the load power graph from Fig. 4.18 with the corresponding efficiency curve.

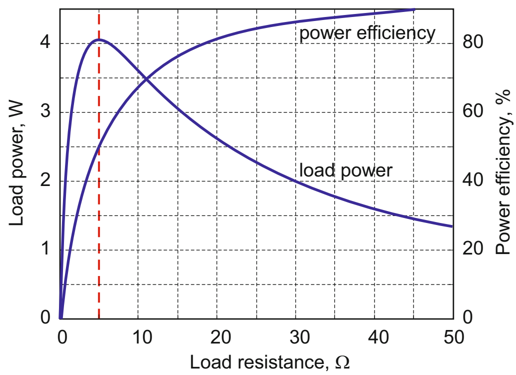


Fig. 4.19. Load power and power efficiency for fixed $V_T = 9V, \; R_T = 5\,\Omega$.

Example 4.10: A battery with the stored energy of $E_B = 0.1$ MJ, $V_T = 12$ V, and $R_T = 5\ \Omega$ delivers its entire energy during the time period $0 \le t \le T$ and discharges with a constant output voltage/current. Two loads are used: $R_L = 5\ \Omega$ and $R_L = 50\ \Omega$. Determine discharge time T and total energy delivered to the load in each case.

Solution: The discharge time, $T = E_B/P$, is determined first where the total power P follows Eq. (4.28). Assuming constant battery discharge rate, we obtain

$$\begin{aligned} &T \approx 1.9 \text{ h} \quad \text{for the } 5\ \Omega \text{ load and} \\ &T \approx 10.6 \text{ h} \quad \text{for the } 50\ \Omega \text{ load} \end{aligned} \tag{4.30}$$

The total energy delivered to the load, $E = TP_L$, in each case is given by

$$\begin{aligned} &E_{5\,\Omega} = 50 \text{ kJ} \quad \text{for the } 5\ \Omega \text{ load and} \\ &E_{50\,\Omega} = 91 \text{ kJ} \quad \text{for the } 50\ \Omega \text{ load} \end{aligned} \tag{4.31}$$

Thus, the total energy extracted from the battery is nearly *twice* as high in the second case. However, it takes about *five* times longer to extract this energy.

4.3.3 Application Example: Power Radiated by a Transmitting Antenna

A transmitting antenna in a radio handset features a monopole antenna. It is connected to a source that has the same basic form as in Fig. 4.17 but with an AC generator instead of the DC source and with an internal (generator) resistance of 50 Ω. The antenna as a load also has a "radiation" resistance of 50 Ω. This resistance describes power loss in terms of electromagnetic radiation from the antenna. Thus, the antenna, if properly matched to the power source, will radiate 50 % of the total power as electromagnetic waves into space. Now a young electrical engineer decides to "modify" the handset by cutting the monopole antenna and leaving only one third of its length. In this case, the antenna's radiation resistance is reduced to one ninth of its original value. How does this affect the radiated signal? To answer this question, we find the instantaneous load power, which also follows Eq. (4.24), i.e.,

$$P_L(t) = \frac{R_L V_T^2(t)}{(R_T + R_L)^2} \tag{4.32}$$

The ratio of the power levels for the two antenna configurations does not depend on time:

$$\frac{P_{L-\text{short}}}{P_{L-\text{original}}} = \frac{50/9}{(50 + 50/9)^2} / \frac{50}{(50 + 50)^2} = \frac{0.0018}{0.0050} = 0.36 \tag{4.33}$$

Thus, for the shorter antenna, we will only achieve about 36 % of the radiated power compared to the original handset. In practice, this estimate becomes even lower due to the appearance of a very significant antenna capacitance.

4.3.4 Application Example: Maximum Power Extraction from Solar Panel

Every solar panel has the measured data for V_{OC} and I_{SC} listed on its backside. For linear circuits, $V_T = V_{OC}, \quad R_T = V_{OC}/I_{SC}$. If the solar panel were a linear circuit, the maximum extracted power would be exactly equal to $0.25 V_{OC} I_{CS}$ according to Eq. (4.26). Fortunately, this is not the case. The maximum extracted power is significantly greater than this value. However, it is still less than the "best" possible value of $V_{OC} I_{CS}$. To quantify the maximum power output, every solar panel has another set of measured data, V_{MP} and I_{MP}, also listed on its backside. V_{MP} stands for *maximum power load voltage* and I_{MP} stands for the *maximum power load current*. The maximum extracted power is the product $V_{MP} I_{MP}$, which is always less than $V_{OC} I_{CS}$. The ratio of these two powers,

$$F = \frac{V_{MP} I_{MP}}{V_{OC} I_{SC}} < 1, \tag{4.34}$$

is known as the *fill factor of the solar panel* (or solar module). We will derive the theoretical value of the fill factor in the next section. Table 4.2 lists some experimental data for crystalline (c-Si) solar panels. The experimental fill factor not only accounts for the nonlinear physics of the cell, but it also includes some resistive losses in an individual cell and in the entire solar module. Equation (4.34) approximates the *fill factor of a cell* too.

Exercise 4.11: A REC SCM220 220 Watt c-Si solar panel has the following readings on the back: the short-circuit current of 8.20 A, the open-circuit voltage V_{OC} of 36.0 V, the maximum power voltage of 28.7 V, and the maximum power current of 7.70 A. Estimate the load resistance required for the maximum power transfer to the load.

Answer: 3.72 Ω.

Table 4.2. Manufacturer-provided circuit parameters for twelve different c-Si solar panels from five different manufacturers (1-W to 230-W output power range).

Solar panel	Cells (series)	V_{MP}/V_{OC}	I_{MP}/I_{SC}	F	\$/Watt (2010)
1-W BSPI-12 Power Up c-Si panel	36	0.81	0.86	0.70	24.00
10-W BSP-1012 Power Up c-Si panel	36	0.81	0.88	0.71	8.80
65-W BSP-1012 Power Up c-Si panel	36	0.80	0.94	0.75	6.35
80-W Sharp NE-80EJEA c-Si panel	36	0.80	0.88	0.70	4.29
176-W Sharp ND-176U1Y c-Si panel	48	0.80	0.91	0.73	4.68
230-W Sharp ND-U230C1 c-Si panel	60	0.82	0.92	0.75	3.51
5-W BP Solar SX-305M c-Si panel	36	0.80	0.90	0.72	15.00
20-W BP Solar SX-320M c-Si panel	36	0.80	0.92	0.74	8.30
175-W BP Solar SX3175N c-Si panel	72	0.83	0.92	0.76	4.19
65-W Kyocera KC65T c-Si panel	36	0.80	0.94	0.75	5.22
165-W SolarWorld SW230 c-Si panel	72	0.80	0.90	0.72	4.72
230-W SolarWorld SW230 c-Si panel	60	0.80	0.92	0.74	3.18
Average		0.806	0.908	0.73	NA

Table 4.2 demonstrates that different c-Si solar cells have approximately the *same* values of V_{MP}/V_{OC}, I_{MP}/I_{SC}, and the same fill factor. Using the photocurrent density estimate and the open-circuit cell voltage estimate given at the beginning of this section, we may assume approximate generic values for c-Si solar *cells* at normal irradiation conditions:

$$V_{MP} = 0.8V_{OC}, \quad I_{MP} = 0.9I_{SC}, \quad F = 0.72, \quad V_{OC} = 0.6 \ \ \mathrm{V(cell)}, \quad J_P = 0.03 \ \ \mathrm{A/cm^2} \tag{4.35}$$

These values are not exact; they are meant as a convenient tool for engineering estimates. Equation (4.35) may be used to address an important task: identify the proper panel configuration and its approximate size in order to provide enough power for a given load.

Example 4.11: A 3-Ω load (for instance, a hot plate in a camp) is rated at 23 V and is to be powered by a solar panel. A c-Si photovoltaic sheet material is your material of choice. Outline parameters of a solar module that is capable of powering the load and estimate the overall module size.

Example 4.11 (cont.):

Solution: First, we need to find the required load current. It is given by $I = 23\ \text{V}/3\ \Omega = 7.67\ \text{A}$. Thus, the maximum power parameters of the module must be equal to $V_{MP} = 23\ \text{V}$ and $I_{MP} = 6.67\ \text{A}$. Next, we the find the measurable parameters, V_{OC}, I_{SC} of the module. According to Eq. (4.35),

$$V_{OC} = \frac{V_{MP}}{0.8} = 28.75\,\text{V}, \quad I_{SC} = \frac{I_{MP}}{0.9} = 8.52\,\text{A} \tag{4.36}$$

Then we find the number of cells N and the area of an individual cell A, assuming a *series* combination of individual cells:

$$N = \frac{V_{OC}}{0.6\text{V}} = \text{round}\,(47.9) = 48, \quad A = \frac{I_{SC}}{0.03\,\text{A/cm}^2} = 284\,\text{cm}^2 \tag{4.37}$$

The overall module (panel) size for closely spaced cells is then 1.36 m^2.

Example 4.12: Compare the theoretical design of Example 4.11 with a real solar module having nearly the same output power (176 W) and nearly the same maximum power voltage (23 V).

Solution: We choose a Sharp ND-176U1Y, 176-watt solar panel from Table 4.2 for comparison. Its maximum power voltage is 23.4 V. Table 4.3 lists the parameters of both panels. The designs agree with the number of cells and with the size of the unit cell. The overall panel size for closely spaced cells is also quite similar: 1.36 m^2 versus 1.32 m^2.

Table 4.3. Parameters of a theoretically designed 176 W solar panel versus the corresponding 176 W hardware prototype.

Example 4.11 (theory estimates)	176-watt Sharp ND-176U1Y panel
$V_{MP} = 23.0\,\text{V}, P_L = 176\,\text{W}$	$V_{MP} = 23.4\,\text{V}, P_L = 176\,\text{W}$
No. of cells: 48	No. of cells: 48
Unit cell area: 284 cm^2	Unit cell area: 275 cm^2

Exercise 4.12: A 9.6 W DC motor in an autonomous robot is rated at 17 V and is to be powered by a solar panel. A c-Si photovoltaic sheet material is your material of choice. Outline parameters of a solar module that is capable of powering the load and estimate the overall module size.

Answer: The module should include 36 cells in series, with the area of $A = 21.0\,\text{cm}^2$ each. The overall module (panel) size for closely spaced cells is then 0.0756 m^2.

Section 4.4 Analysis of Nonlinear Circuits: Generic Solar Cell

4.4.1 Analysis of Nonlinear Circuits: Load Line Method

Consider a nonlinear passive circuit element which possesses a particular v-i characteristic. It is shown in Fig. 4.20 by a rectangle. Examples of such elements were given in Chapter 2. Element's polarity (direction of current inflow for passive reference configuration) is labeled by a plus sign. Figure 4.20 presents four basic nonlinear circuits (networks) encountered in practice: a linear active network given by its Thévenin equivalent and connected to a nonlinear load, a practical nonlinear voltage source connected to a linear load, a linear active network in the form of the Norton equivalent connected to a nonlinear load, and a practical nonlinear current source connected to a linear load. Interchanging the place of the nonlinear element and resistance if necessary and using the source transformation theorem, we can state that all four circuits in Fig. 4.20 are topologically *equivalent*. Therefore, only one of them will be studied, for example, the network shown in Fig. 4.20a. The v-i characteristic of the practical voltage source between terminals a and b in Fig. 4.20a is given, according to KVL, by

$$I = \frac{V_T - V}{R_T} \tag{4.38}$$

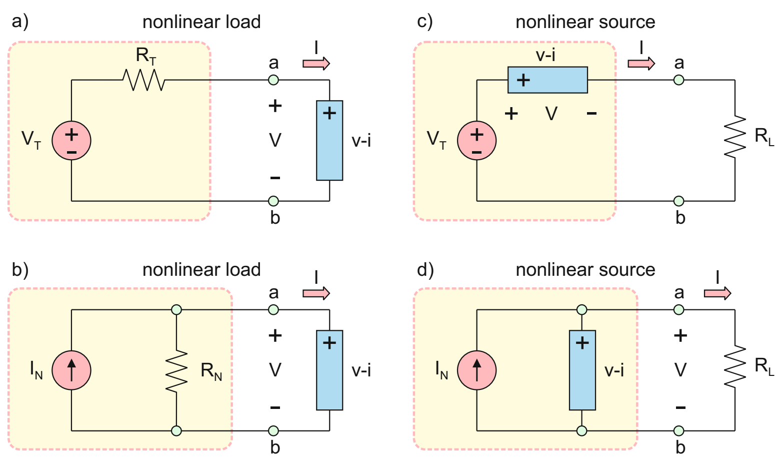


Fig. 4.20. Four basic nonlinear circuits.

This linear function given by Eq. (4.38) is known as the *load line*. It is plotted in Fig. 4.21 and intersects the voltage axis at $V = V_T$ and the current axis at $I = V_T/R_T$. The v-i characteristic of a nonlinear element is plotted in the same figure. Both v-i characteristics must give the identical values of voltage and current. Thus, the intersection of the load line with the v-i characteristic is the circuit solution. This is the essence of the *load line method*.

Though primarily graphical, the load line method provides a great insight into the problem under study.

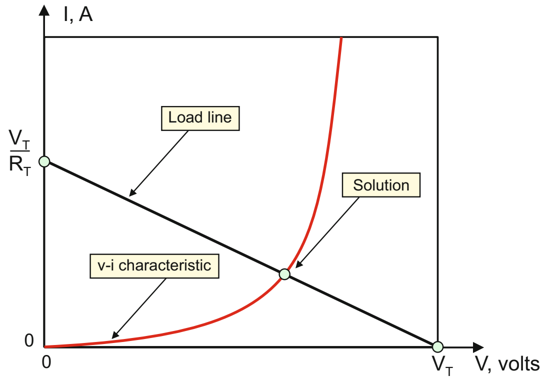


Fig. 4.21. Load line method for a nonlinear circuit.

Example 4.13: The circuit in Fig. 4.20a is characterized by $V_\mathrm{T} = 3\ \ \mathrm{V}$, $R_\mathrm{T} = 1\ \ \mathrm{k\Omega}$. The υ-i characteristic of the nonlinear element (the ideal Shockley diode) is $I = 1 \times 10^{-9}\left[\exp\left(\frac{V}{0.0257\ \ \mathrm{V}}\right) - 1\right]\ \ [\mathrm{A}]$. The goal is to solve the circuit using the load line method.

Solution: Figure 4.22 plots two dependencies: the load line of Eq. (4.38) and the υ-i characteristic of the ideal diode specified by the present example. Using visual inspection, the intersection is evaluated as $I \approx 2.6\ \ \mathrm{mA}$, $V \approx 0.4\ \ \mathrm{V}$. This is the solution for the circuit current and for the load voltage, respectively. The solution accuracy improves when the scale of the plot is adjusted. In particular, we usually do not have to extend the voltage axis all the way from 0 V to the supply voltage; only a small interval may be sufficient.

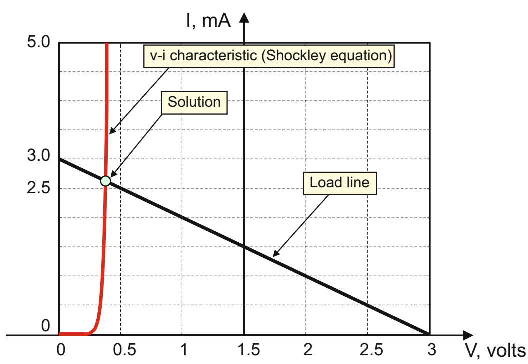


Fig. 4.22. Load line method applied to a nonlinear circuit with an ideal diode.

4.4.2 Iterative Method for Nonlinear Circuits

Assume that the nonlinear circuit element is characterized by an explicit function $I = I(V)$. Then, according to Eq. (4.38), the load line method is equivalent to the graphical solution of the transcendental algebraic equation in the form

$$\frac{V_T - V}{R_T} = I(V) \Rightarrow V = V_T - R_T I(V) \tag{4.39}$$

An alternative is to solve this equation iteratively, starting with some initial guess $V = V^0$. The *iterative method for nonlinear circuits* may be formulated as follows. Two iterative schemes (*explicit* and *implicit*) may formally be used. The first (explicit) scheme follows from the second Eq. (4.39), and the second (implicit) scheme from the first Eq. (4.39):

$$\begin{aligned} V^{n+1} &= V_T - R_T I(V^n), \quad n = 0,\ 1,\ 2,\ \ldots \text{ or} \\ V^{n+1} &= I^{-1}\left(\frac{V_T - V^n}{R_T}\right), \quad n = 0,\ 1,\ 2,\ \ldots \end{aligned} \tag{4.40}$$

where I^{-1} denotes the inverse function of $I(V)$. The first (explicit) scheme is simpler when $I(V)$ is given. However, *only* the second scheme is recommended in practice since the first scheme may not converge for typical nonlinear circuit elements, which model semiconductor devices.

Example 4.14: Solve the previous example using the iterative solution of the transcendental circuit equation.

Solution: We find the inverse *v*-*i* characteristic of the nonlinear element first. It is

$$V = 0.0257 \times \ln\left[\frac{I}{1 \times 10^{-9}\ \text{A}} + 1\right]\ [\text{V}], \quad I = \frac{3\ \text{V} - V}{1\ \text{k}\Omega} \tag{4.41}$$

The iterative scheme has the form (the second method of Eq. (4.40) is used)

$$V^{n+1} = 0.0257 \times \ln\left[\frac{3\ \text{V} - V^n}{1 \times 10^{-6}\ \text{V}} + 1\right], \quad n = 0,\ 1,\ 2,\ \ldots \tag{4.42}$$

with the initial guess $V^0 = 0$ V. It converges very fast; the corresponding iterations are $V^0 = 0$ V, $V^1 = 0.3833$ V, $V^2 = 0.3798$ V, and $V^3 = 0.3798$ V.

Therefore, only a few iterations are usually sufficient. The final result is $I = 2.62$ mA, $V = 0.380$ V, which improves the solution obtained with the load line method—see the previous example. The initial guess of the iterative solution may vary widely, but it should *not* exceed the source voltage.

4.4.3 Application Example: Solving the Circuit for a Generic Solar Cell

Figure 4.23a shows a simplified physical composition of the solar cell in the form of a pn-junction (a junction of two semiconductor materials), which essentially forms a semiconductor diode.

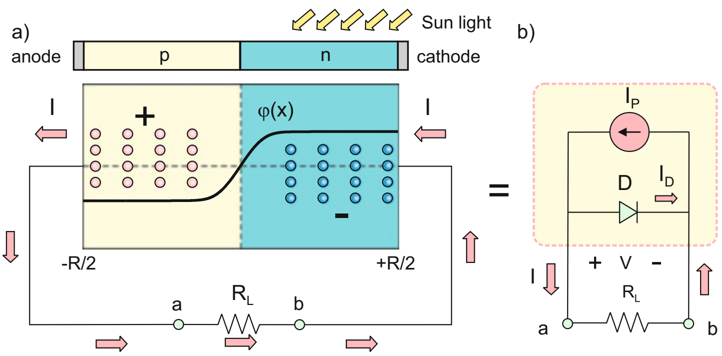


Fig. 4.23. (**a**) Simplified physical composition of the solar cell in the form of a pn-junction—a semiconductor diode. (**b**) Simplified (lossless single-diode) equivalent circuit.

Free charge carries generated by sunlight are separated by a built-in voltage or potential $\varphi(x)$ within the diode, which is the cause of an equivalent current source—the photocurrent of the solar cell I_P. The photocurrent mostly flows through the load. At the same time, a certain portion of it, I_D, could still flow through the pn-junction diode itself as a *forward diode current*. Therefore, the load current I in Fig. 4.23 is less than the photocurrent. Figure 4.23b shows a simplified equivalent circuit of a solar cell. This circuit coincides with the nonlinear circuit in Fig. 4.20d and can be solved in the same way once the v-i characteristic of the equivalent diode is known. It is often given by

$$I_D = I_S\left[\exp\left(\frac{V}{nV_T}\right) - 1\right], \quad V_T = 0.0257 \text{ V} \tag{4.43}$$

with an *effective* ideality factor n and an *effective* saturation current I_S of the corresponding diode. The *characteristic equation of the cell* is the KCL in Fig. 4.23b:

$$I = I_P - I_S\left[\exp\left(\frac{V}{nV_T}\right) - 1\right] \tag{4.44}$$

Figure 4.24 plots the $I(V)$ dependence of the characteristic equation (4.44). The horizontal straight asymptote is the photocurrent I_P or the short-circuit current I_{SC}. The vertical straight asymptote is the open-circuit voltage V_{OC}, $V_{OC} \approx V_T n \ln(I_P/I_S)$ when $I_P/I_S >> 1$. The area of the shaded rectangle is the load power; it is clearly maximized at a certain operating point Q, where $V = V_{MP}$, $I = I_{MP}$.

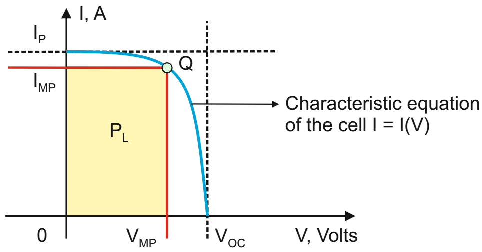


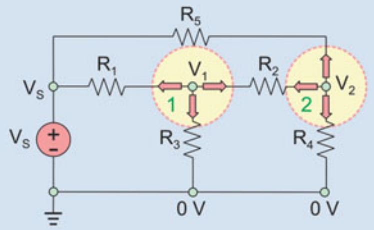
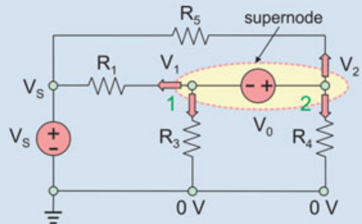
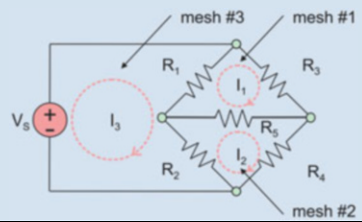
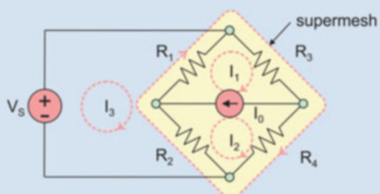
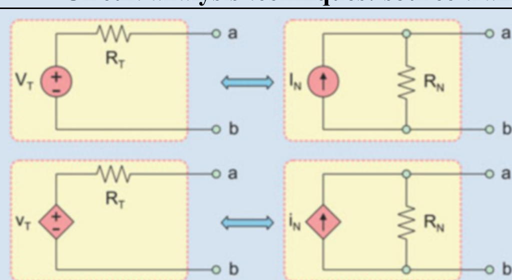
Fig. 4.24. Finding operating point Q of the solar cell for the maximum power transfer.

Finding maximum power parameters is a straightforward but lengthy procedure. The load power is found as a function of V; its derivative must be equal to zero to maximize the load power. This is essentially the maximum power theorem for nonlinear circuits. The final expressions are

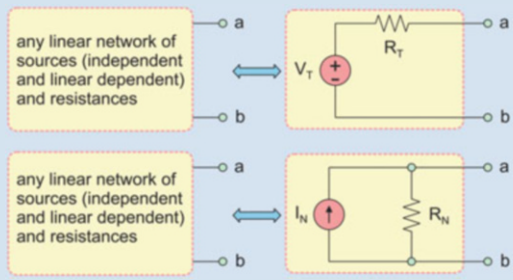
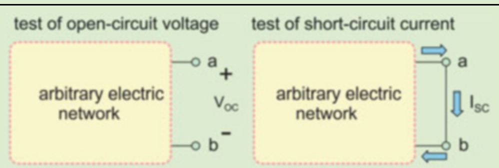
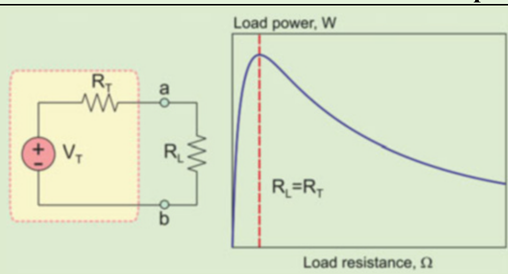
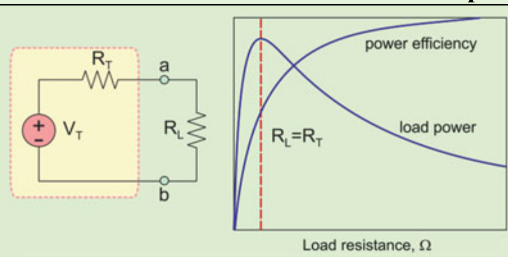
$$V_{MP} = V_{OC} - nV_T \ln\left(1 + \frac{V_{OC}}{nV_T}\right),\ I_{MP} = I_{SC}\left(1 - \frac{nV_T}{V_{MP}}\right) \approx 90\,\% \text{ of } I_{SC} \tag{4.45}$$

For $n = 1.75$ and $V_{OC} = 0.6\,\text{V}$, $V_{MP} \approx 80\,\%$ of V_{OC}, $I_{MP} \approx 90\,\%$ of I_{SC} which is close to the data from Table 4.2 given that the fill factors for the cell and the panel are approximately the same. Note that the load resistance is finally found as $R_L = V_{MP}/I_{MP}$.

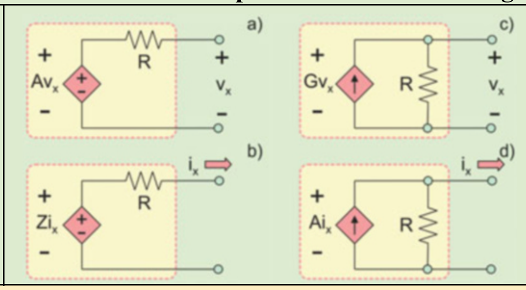
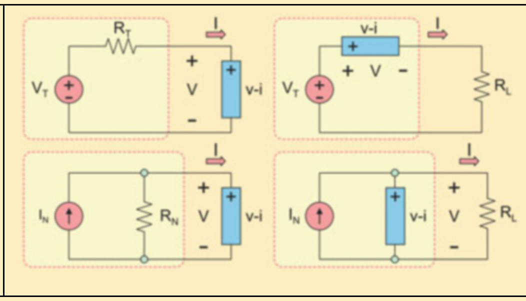
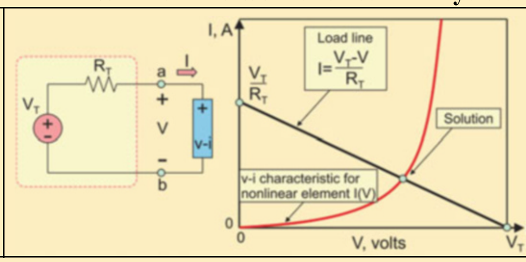
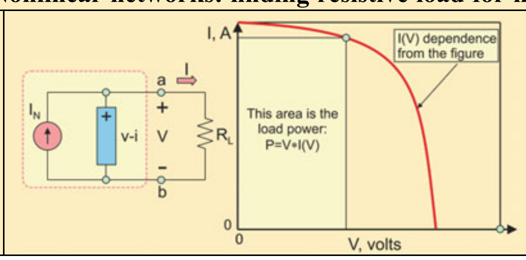
Summary

Circuit analysis techniques: nodal/mesh analysis		
Nodal analysis		Based on KCL and Ohm's law: $\frac{V_1-V_S}{R_1}+\frac{V_1-0\ \text{V}}{R_3}+\frac{V_1-V_2}{R_2}=0$ $\frac{V_2-V_S}{R_5}+\frac{V_2-0\ \text{V}}{R_4}+\frac{V_2-V_1}{R_2}=0$
Supernode		KCL for the supernode: $\frac{V_1-V_S}{R_1}+\frac{V_1}{R_3}$ $+\frac{V_2}{R_4}+\frac{V_2-V_S}{R_5}=0$ plus KVL: $V_2=V_1+V_0$
Mesh analysis		Based on KVL and Ohm's law: $R_1(I_1-I_3)+R_3I_1+R_5(I_1-I_2)=0$ $R_2(I_2-I_3)+R_5(I_2-I_1)+R_4I_2=0$ $-V_S+R_1(I_3-I_1)+R_2(I_3-I_2)=0$ for meshes 1, 2, and 3
Supermesh		KVL for the supermesh: $R_1(I_1-I_3)+R_3I_1+R_4I_2+$ $R_2(I_2-I_3)=0$ plus Eq. for mesh 3 and the KCL: $I_1-I_2=I_S$
Circuit analysis techniques: source transformation theorem		
Source transformation theorem		Substitution of voltage source V_T in series with resistance R_T for current source I_N with resistance R_N: $R_N=R_T\ ,\ I_N=\frac{V_T}{R_T}$ $V_{OC}=V_T\ ,\ I_{SC}=\frac{V_T}{R_T}$

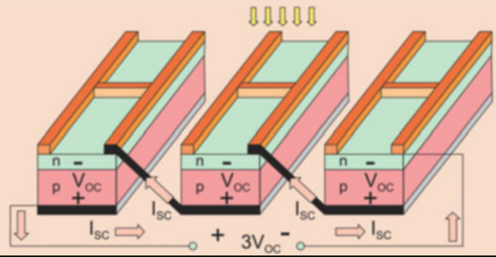
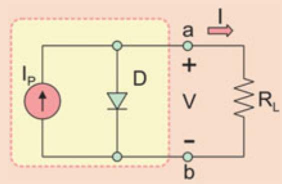
(continued)

<table>
<tr><th colspan="3">Circuit analysis techniques: Thévenin/Norton theorems/equivalents</th></tr>
<tr><td>Thévenin and Norton theorems</td><td>
</td><td>Any linear network with independent sources, dependent linear sources, and resistances can be replaced by a simple equivalent network in the form:
i. a voltage source V_T in series with resistance R_T;
ii. a current source I_N in parallel with resistance R_N</td></tr>
<tr><th colspan="3">Summary of major circuit analysis methods (linear circuits)</th></tr>
<tr><td colspan="3">– Superposition theorem (previous chapter);
– Nodal/mesh analysis (this chapter);
– Source transformation theorem (this chapter);
– Thévenin and Norton equivalent circuits (this chapter)</td></tr>
<tr><th colspan="3">Linear networks: measurements/equivalence</th></tr>
<tr><td>Method of short/ open circuit</td><td>
</td><td>– Two arbitrary linear networks are equivalent when their V_{OC} and I_{SC} coincide.
– This method is also used for nonlinear circuits</td></tr>
<tr><th colspan="3">Linear networks: maximum power theorem</th></tr>
<tr><td>Maximum power theorem (load matching)</td><td>
</td><td>– Power delivered to the load is maximized when $R_L = R_T$;
– For high-frequency circuits it also means no “voltage/current wave reflection” from the load</td></tr>
<tr><th colspan="3">Linear networks: maximum power efficiency</th></tr>
<tr><td>Power efficiency is maximized when the load resistance is very high (load bridging)</td><td>
</td><td>Power transfer efficiency:
$$E = \frac{P_L}{P} = \frac{R_L}{R_T + R_L}$$</td></tr>
</table>

(continued)

Linear networks: dependent sources and negative equivalent resistance		
Equivalent resistances of basic linear networks with dependent sources		Case a): $R_T = \frac{R}{1-A}$ Case b): $R_T = R - Z$ Case c): $R_T = \frac{R}{1-GR}$ Case d): $R_T = R(1-A)$ Equivalent resistance may become negative
Nonlinear networks: four basic topologies		
Basic nonlinear circuits		– Linear voltage source connected to a nonlinear load (diode circuit); – Linear current source connected to a nonlinear load; – Nonlinear voltage source connected to a linear load; – Nonlinear current source connected to a linear load (photovoltaic circuit)
Nonlinear networks: circuit analysis via load line method		
Load line method		Solution: The load line (v-i characteristic of the linear source, which is $I = \frac{V_T - V}{R_T}$) intersects the v-i characteristic of the nonlinear load, $I(V)$
Nonlinear networks: iterative solution		
Finding the intersection point iteratively	For the circuit in the previous row: $$I(V) = \frac{V_T - V}{R_T} \Rightarrow V^{n+1} = I^{-1}\left(\frac{V_T - V^n}{R_T}\right), \quad n = 0,\ 1,\ \ldots$$	Implicit scheme: initial guess V^0 may be 0 V
Nonlinear networks: finding resistive load for maximum power extraction		
Finding maximum load power for the equivalent model of a solar cell		Load power is computed as $P_L = V \times I(V)$. Then, it is maximized, which is equivalent to solving equation: $$\frac{dP_L}{dV}(V) = 0$$ for unknown voltage V

(continued)

Some useful facts about power extraction from solar cells/modules	
Typical values of open-circuit voltage V_{OC} and photocurrent density J_P of a c-Si cell	Crystalline silicon or c-Si cell: $V_{OC} \approx 0.6\ \text{V}; J_P \approx 0.03\ \text{A/cm}^2$
– Open-circuit module voltage is N times the cell voltage, $N \cdot V_{OC}$ – Short-circuit module current I_{SC} is the cell short-circuit current $I_{SC} = AJ_P$ where A is cell area	
Typical values of maximum-power parameters and fill factor for c-Si cells/modules; the load resistance must be $R_L = V_{MP}/I_{MP}$	$F\|_{module} = \frac{V_{MP}I_{MP}}{V_{OC}I_{SC}} \approx F\|_{cell}$ (for low-loss modules) $V_{MP} \approx 0.8\,V_{OC}, \quad I_{MP} \approx 0.9 I_{SC}, \quad F \approx 0.72$
Lossless single-diode model of a solar cell: $I_P = AJ_P$—photocurrent (A); A—cell area (cm^2); V_T—thermal volt. (0.0257 V); n—ideality factor ($1 < n < 2$); $I_S \approx I_P \exp\left(-\frac{V_{OC}}{nV_T}\right)$ (A)	 $I = I_P - I_S\left[\exp\left(\frac{V}{nV_T}\right) - 1\right]$
Maximum-power analytical solution for loss-less single-diode model of a solar cell	$V_{MP} \approx V_{OC} - nV_T \ln\left(1 + \frac{V_{OC}}{nV_T}\right),$ $I_{MP} = I_{SC}\left(1 - \frac{nV_T}{V_{MP}}\right)$

Problems

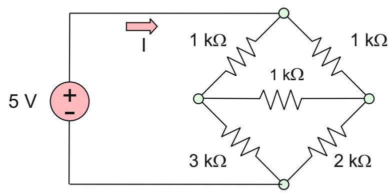
4.1 Nodal/Mesh Analysis

4.1.2 Nodal analysis

Problem 4.1.

A. Using the nodal analysis, determine the supply current I for the circuit shown in the figure.

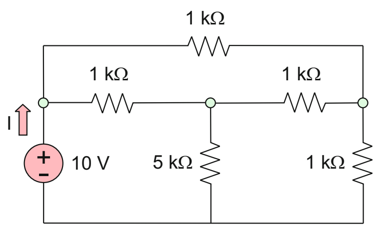
B. Show the current directions for every resistance in the circuit.



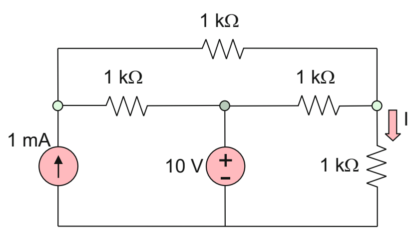
Problem 4.2.

A. Using the nodal analysis, determine the total circuit current I for the circuit shown in the following figure.

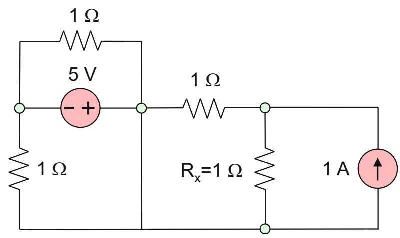
B. Show the current directions for every resistance in the circuit.



Problem 4.3. Introduce the ground termination, write the nodal equations, and solve for the node voltages for the circuit shown in the following figure. Calculate the current I shown in the figure.

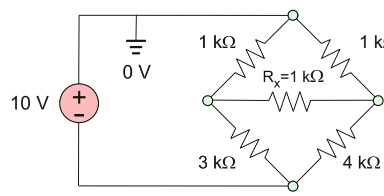


Problem 4.4. Introduce the ground termination, write the nodal equations, and solve for the node voltages for the circuit shown in the figure. Calculate the current through the resistance R_x and show its direction in the figure.



Problem 4.5. For the circuit shown in the following figure,

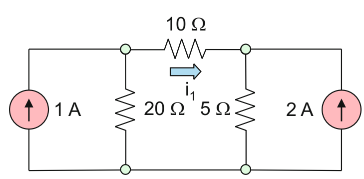
1. Determine current i_x through resistance R_x.
2. Show its direction on the figure.



Problem 4.6. For the circuit shown in the following figure,

A. Write nodal equations and solve for the node voltages. Then, find the value of i_1.

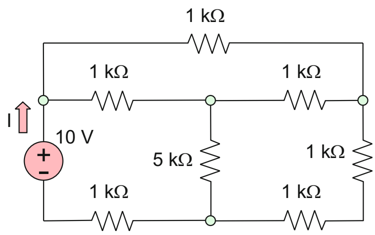
B. Could this problem be solved in another (simpler) way?



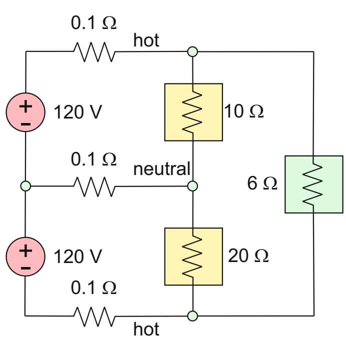
Problem 4.7.

A. Write the nodal equations and solve for the node voltages for the circuit shown in the following figure. Then, find the value of i_1.

B. Use MATLAB or other software of your choice for the solution of the system of linear equations; attach the code to the solution.

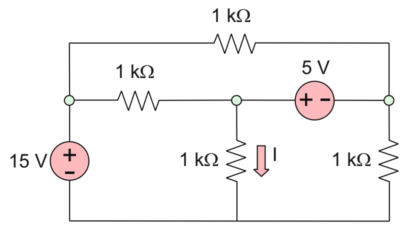


Problem 4.8. The following figure shows the DC equivalent of a residential *three-wire* system, which operates at 240 V rms (do not confuse it with the three-phase system, which carries a higher current). Two 120-V rms power supplies are connected as one dual-polarity power supply, i.e., in series. The 10-Ω load and the 20-Ω load are those driven by the two-wire (and one ground) standard wall plug with 120 V rms—the lights, a TV, etc. The 6-Ω load consumes more power and it is driven with 240 V rms using a separate bigger wall plug (+/– and neutral (not shown))—the stove, washer, dryer, etc. Determine the power delivered to each load. *Hint:* Use a calculator or software of your choice for the solution of the system of linear equations (MATLAB is recommended).

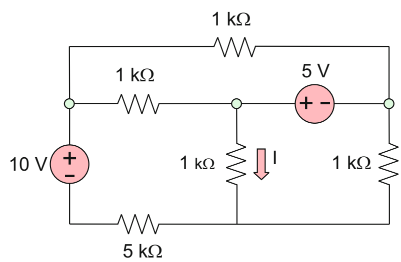


4.1.3 Supernode

Problem 4.9. Introduce the ground terminal, write the nodal equations, and solve for the node voltages for the circuit shown in the figure. Calculate the current I shown in the figure.



Problem 4.10. Introduce the ground terminal, write the nodal equations, and solve for the node voltages for the circuit shown in the figure. Calculate the current I shown in the figure.



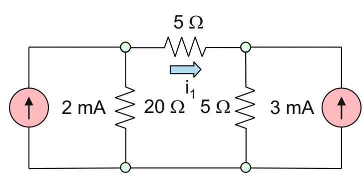
4.1.4 Mesh analysis

4.1.5 Supermesh

Problem 4.11. For the circuit shown in the figure, determine current i_1

A. Using the mesh analysis
B. Using the nodal analysis

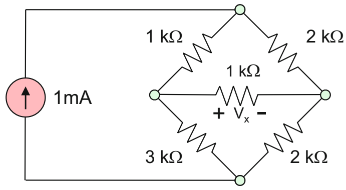
Which method is simpler?



Problem 4.12. For the circuit shown in the figure, determine voltage V_x

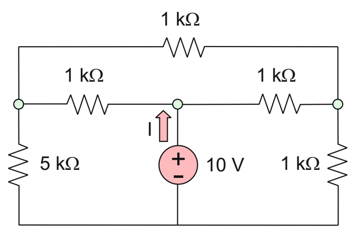
A. Using the mesh analysis
B. Using the nodal analysis

Which method appears to be simpler?

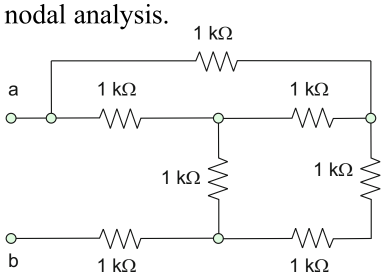


Problem 4.13. For the circuit shown in the figure

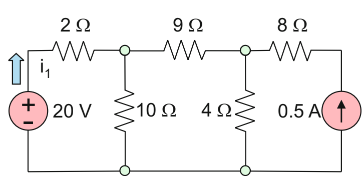
A. Determine the circuit current I using either the nodal analysis or the mesh analysis.
B. Explain your choice for the selected method.



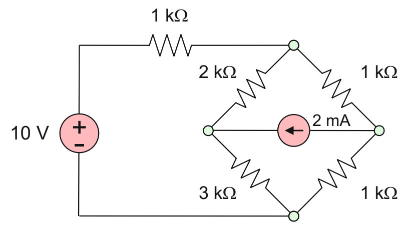
Problem 4.14. For the circuit shown in the figure, determine its equivalent resistance between terminals *a* and *b*. *Hint:* Connect a power source and use a mesh-current analysis or the nodal analysis.



Problem 4.15. For the circuit shown in the figure, determine the current i_1 of the 20-V voltage source.



Problem 4.16. Determine voltage across the current source for the circuit shown in the figure that follows using the mesh analysis and the supermesh concept.

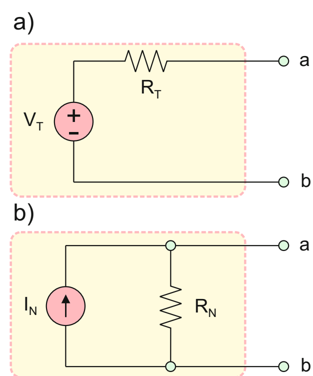


4.2 Generator Theorems

4.2.1 Equivalence of Active One-Port Networks. Method of Short/Open Circuit

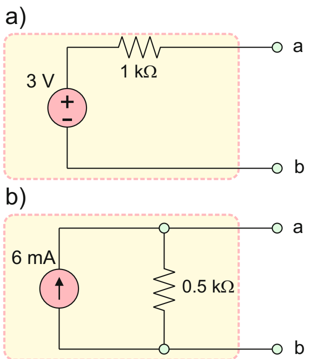
Problem 4.17. A linear active network with two unknown circuit elements measures $V_{OC} = 5\ \text{V}, I_{SC} = 10\ \text{mA}$.

A. Determine parameters V_T, R_T and I_N, R_N of two equivalent networks shown in the figures below.
B. Could you identify which exactly network is it?



Problem 4.18. Given two networks shown in the figure that follows:

A. Determine their open-circuit voltage V_{OC} and short-circuit current I_{SC} for each of them.
B. Are the networks equivalent?



4.2.2 Application example: reading and using data for solar panels

Problem 4.19. The area of a single cell in the 10-W BSP-1012 Power Up c-Si panel is ~22 cm^2. Predict:

A. Open-circuit voltage of the solar cell, V_{OC}
B. Photocurrent density of the cell, J_P
C. Short-circuit current of the cell, I_{SC}

Compare the above value with the value I_{SC} = 0.66 A reported by the manufacturer.

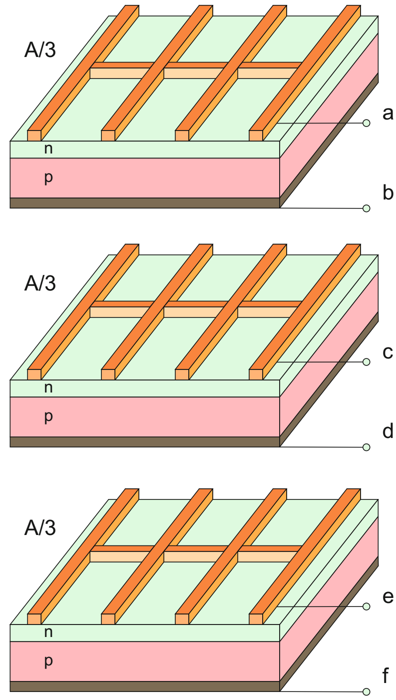
Problem 4.20. The area of a single cell in the 175-W BP Solar SX3175 c-Si panel is 156.25 cm^2. Predict:

A. Open-circuit voltage of the solar cell, V_{OC}
B. Photocurrent density of the cell, J_P
C. Short-circuit current of the cell, I_{SC}

Compare the above value with the value I_{SC} = 5.1 A reported by the manufacturer. What value should the photocurrent density have in order to exactly match the short-circuit current reported by the manufacturer?

Problem 4.21. Are the solar cells in the solar module connected in parallel or in series? Why is one particular connection preferred?

Problem 4.22. You are given three c-Si solar cells shown in the figure that follows, each of area $A/3$. Draw wire connections for a cell bank, which has the performance equivalent to that of a large solar cell with the area A.



Problem 4.23. Individual solar cells in the figure to the previous problem are to be connected into a standard solar module. Draw the corresponding wire connections.

Problem 4.24. A 10-W BSP1012 PV c-Si module shown in the figure has 36 unit cells connected in series, the short-circuit current of 0.66 A, and the open-circuit voltage of 21.3 V.

Power Up
10 Watt Module

A. Estimate the area of the single solar cell using the common photocurrent density value for c-Si solar cells.
B. Estimate the open-circuit voltage for the single cell.

Problem 4.25. A 20-W BSP2012 PV c-Si module shown in the figure has 36 unit cells connected in series, the short-circuit current of 1.30 A, and the open-circuit voltage of 21.7 V.

Power Up
20 Watt Module

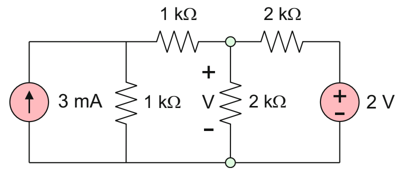
A. Estimate the area of the single solar cell using the common photocurrent density value for c-Si solar cells.
B. Estimate the open-circuit voltage for the single cell.

Problem 4.26. A c-Si solar module is needed with the open-circuit voltage of 10 V and the short-circuit current of 1.0 A. A number of individual solar cells are available; each has the area of 34 cm^2, the open-circuit voltage of 0.5 V, and the short-circuit current of 1.0 A. Identify the proper module configuration (number of cells) and estimate the module's approximate size.

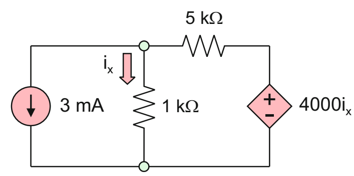
Problem 4.27. A c-Si solar module is needed with the open-circuit voltage of 12 V and the short-circuit current of 3.0 A. A number of individual solar cells are available; each has the area of 34 cm^2, the open-circuit voltage of 0.5 V, and the short-circuit current of 1.0 A. Identify the proper module configuration (number of cells) and estimate the module's approximate size.

4.2.3 Source Transformation Theorem

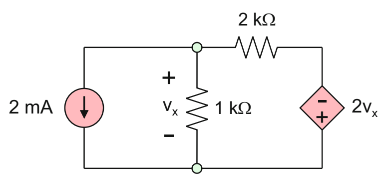
Problem 4.28. Find voltage V in the circuit shown in the figure that follows using source transformation.



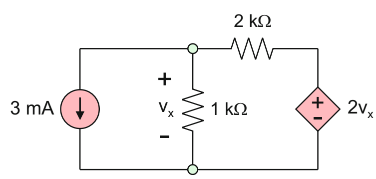
Problem 4.29. The circuit shown in the following figure includes a current-controlled voltage source. Find current i_x using source transformation.



Problem 4.30. The circuit shown in the following figure includes a voltage-controlled voltage source. Find voltage v_x using source transformation.



Problem 4.31. Repeat the previous problem for the circuit shown in the figure below.

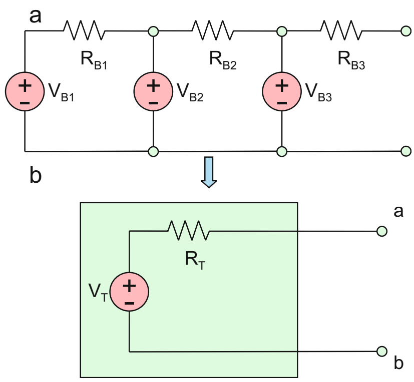


4.2.4 Thévenin's and Norton's Theorems

Problem 4.32. Find:

1. Thévenin (or equivalent) voltage
2. Thévenin (or equivalent) resistance

for the two-terminal network shown in the figure that follows. Assume three 9-V sources separated by resistances of 1 Ω each.

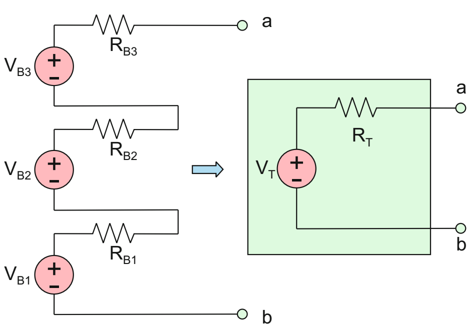


Problem 4.33. Find:

1. Thévenin (or equivalent) voltage
2. Thévenin (or equivalent) resistance

for the two-terminal network shown in the figure that follows (three practical voltage sources in series) when

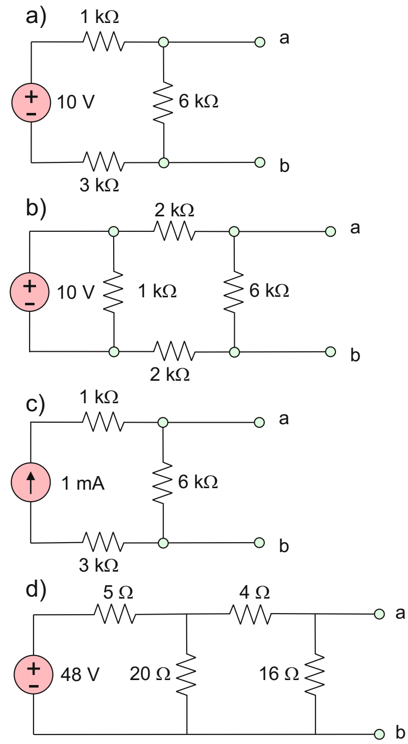
$$R_{B1} = 2\Omega, R_{B2} = 3\ \ \Omega, R_{B3} = 0.5\Omega,$$
$$V_{B1} = 3\ \ \text{V},\ \ V_{B2} = 6\ \ \text{V}, V_{B3} = 3\ \ \text{V}.$$



Problem 4.34. Find:

1. Thévenin (or equivalent) voltage;
2. Thévenin (or equivalent) resistance

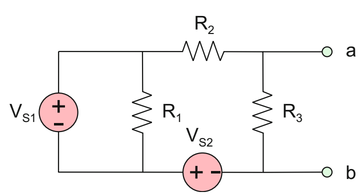
for the two-terminal networks shown in the following figures.



Problem 4.35. Find:

1. Thévenin voltage
2. Thévenin resistance

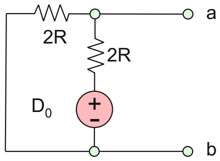
for the two-terminal network shown in the following figure when
$R_1 = R_2 = R_3 = 1\text{k}\Omega, V_{S1} = V_{S2} = 10$ V.



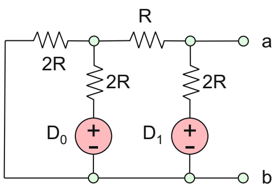
Problem 4.36. A two-terminal network shown in the figure is a starting section of a ladder network used for digital-to-analog conversion. Express:

1. Thévenin (or equivalent) voltage
2. Thévenin (or equivalent) resistance

in terms of (digital) voltage D_0 and resistance R.



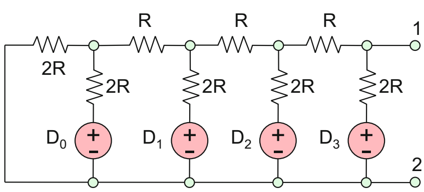
Problem 4.37. A two-terminal circuit network in the figure is a two-bit ladder network used for digital-to-analog conversion. Express parameters of the corresponding Norton equivalent circuit in terms of (digital) voltages D_0, D_1 and resistance R.



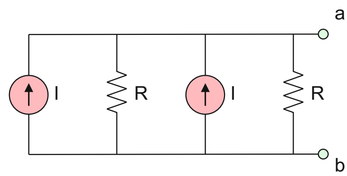
Problem 4.38. A two-terminal network shown in the figure is a four-bit ladder network used for digital-to-analog conversion. Express:

1. Thévenin (or equivalent) voltage
2. Thévenin (or equivalent) resistance

in terms of four (digital) voltages D_0, D_1, D_2, D_3 and resistance R.

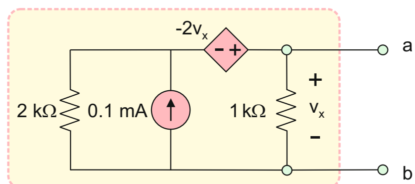


Problem 4.39. Determine the Norton equivalent for the circuit shown in the figure. Express your result in terms of I and R.



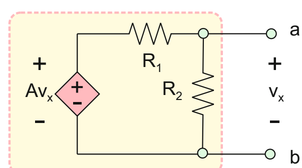
Problem 4.40. Each of three identical batteries is characterized by its Thévenin equivalent circuit with $V_T = 9$ V and $R_T = 1\ \Omega$. The batteries are connected in parallel. The parallel battery bank is again replaced by its Thévenin equivalent circuit with unknown V_{TBank} and R_{TBank}. Find those parameters (show units). *Hint:* This is a tricky problem; double-check the connecting nodes when drawing the circuit diagram.

Problem 4.41. Establish Thévenin and Norton equivalent circuits for the network shown in the following figure.

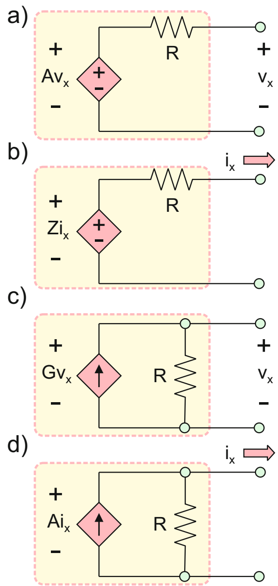


4.2.5 Application Example: Generating Negative Equivalent Resistance

Problem 4.42. Establish the equivalent (Thévenin) resistance for the network shown in the following figure. Carefully examine the sign of the equivalent resistance.



Problem 4.43. Derive the equivalent (Thévenin) resistance for the networks shown in the figure that follows (confirm Fig. 4.16 of the main text).

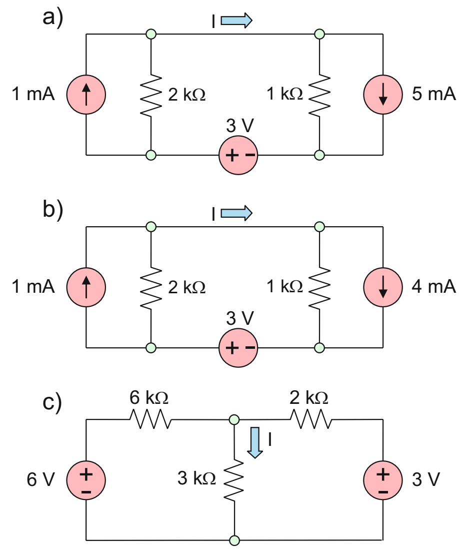


4.2.6 Summary of Circuit Analysis Methods

Problem 4.44. Solve the circuits shown in the following figure—determine current I (show units). You can use any of the methods studied in class:

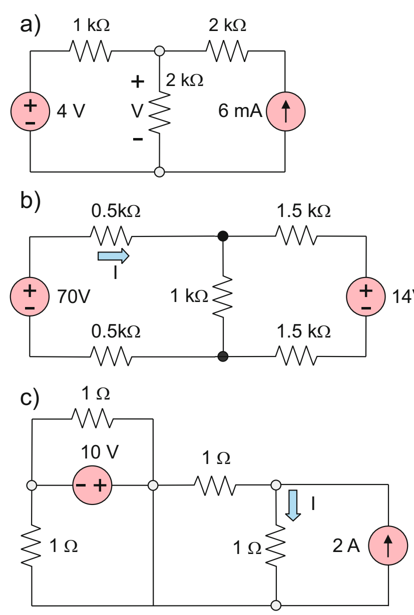
- Superposition theorem
- Nodal/mesh analysis
- Source transformation theorem
- Thévenin and Norton equivalent circuits

Attempt to apply two different methods of your choice to every circuit.



Problem 4.45. Solve the circuits shown in the following figure—determine unknown voltage V or current I (show units). You can use any of the methods studied in class:

- Superposition theorem
- Nodal/mesh analysis
- Source transformation theorem
- Thévenin and Norton equivalent circuits

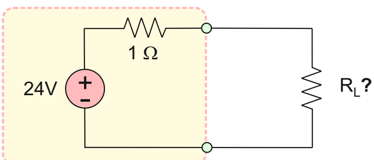


4.3 Power Transfer

4.3.1 Maximum Power Transfer

4.3.2 Maximum Power Efficiency

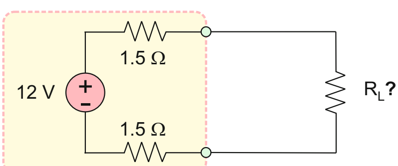
Problem 4.46. A deep-cycle marine battery is modeled by an ideal voltage source of 24 V in series with a 1-Ω resistance shown in the figure that follows. The battery is connected to a load, and the load's resistance, R_L, needs to be optimized. Power delivered to the load has a *maximum* at exactly one value of the load resistance. Find that value and prove your answer graphically using software of your choice (MATLAB is recommended).



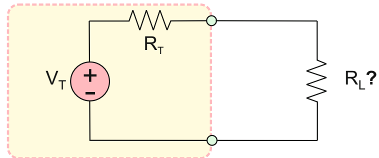
Problem 4.47. A power supply for an electric heater can be modeled by an ideal voltage source of unknown voltage in series with the internal resistance $R_T = 4\ \Omega$.

A. Can you still determine when the power delivered to a load (a heating spiral with resistance R_L) is maximized?
B. Does the answer depend on the source voltage?

Problem 4.48. For the circuit shown in the figure, when is the power delivered to the load maximized?

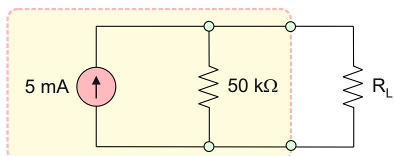


Problem 4.49. A battery can be modeled by an ideal voltage source V_T in series with a resistance R_T.



1. For what value of the load resistance R_L is the power delivered to the load maximized?
2. What percentage of the power taken from the voltage source V_T is actually delivered to a load (assuming R_L is chosen to maximize the power delivered)?
3. What percentage of the power taken from the voltage source V_T is delivered to a load when $R_L = 0.1R_T$?

Problem 4.50. A micro-power photovoltaic device can be modeled under certain conditions as an ideal current power source and a resistance in parallel—see the figure that follows. At which value of the load resistance, R_L, is the power delivered to the load maximized?



Problem 4.51. A low-cost polycrystalline Power Up BSP1-12 1-W solar panel lists ratings for the output voltage and current, which give maximum load power: $V_{L\,max} = 17.28$ V, $I_{L\,max} = 0.06$ A. Based on these cell specifications, which value of the equivalent resistance should the load to be connected to the solar cell have for maximum power output?

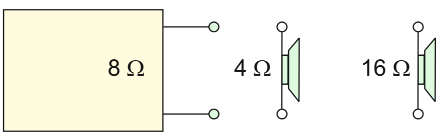
Problem 4.52. The solar panel from the previous problem generates a significant voltage of ~13 V in a classroom without direct sunlight, but the resulting current is small, ~1 mA.

A. Now, what value of the equivalent resistance should the load have for maximum power output?

B. How is the maximum load power different, compared to the previous problem?

Problem 4.53. The heating element of an electric cooktop has two resistive elements, $R_1 = 50\ \Omega$ and $R_2 = 100\ \Omega$, that can be operated separately, in series, or in parallel from a certain voltage source that has a Thévenin (rms) voltage of 120 V and internal (Thévenin) resistance of 30 Ω. For the highest power output, how should the elements be operated? Select and explain one of the following: 50 Ω only, 100 Ω only, series, or parallel.

Problem 4.54. You are given two speakers (rated at 4 Ω and 16 Ω, respectively) and an audio amplifier with the output resistance (impedance) equal to 8 Ω.



A. Sketch the circuit diagram that gives the maximum acoustic output with the available components. Explain your choice.
B. Sketch the circuit diagram for the maximum power efficiency. Explain your choice.

4.3.4 Application Example: Maximum Power Extraction from Solar Panel

Problem 4.55.

A. Describe in your own words the meaning of the fill factor of a solar cell (and solar module).
B. A 200-W GE Energy GEPVp-200 c-Si panel has the following reading on its back: $V_{OC} = 32.9$ V, $I_{SC} = 8.1$ A, $V_{MP} = 26.3$ V, and $I_{MP} = 7.6$ A. What is the module fill factor? What is approximately the fill factor of the individual cell?

Problem 4.56. Using two Web links
http://powerupco.com/site/
http://www.affordable-solar.com/,
identify the solar panel that has the greatest fill factor to date.

Problem 4.57. A 10-W BSP1012 PV c-Si module shown in the figure has 36 unit cells connected in series, the short-circuit current of 0.66 A, and the open-circuit voltage of 21.3 V. The maximum power parameters are $V_{MP} = 17.3$ V and $I_{MP} = 0.58$ A.

Power Up
10 Watt Module

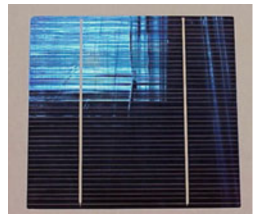
A. Estimate the area of the single solar cell using the common photocurrent density value for c-Si solar cells (show units).
B. Estimate the open-circuit voltage for the single cell.
C. Estimate the fill factor of the module and of the cell.
D. Estimate the value of the equivalent load resistance R required for the maximum power transfer from the module to the load.

Problem 4.58. A 20-W BSP2012 PV c-Si module shown in the figure has 36 unit cells connected in series, the short-circuit current of 1.30 A, the open-circuit voltage of 21.7 V, the maximum power voltage V_{MP} of 17.3 V, and the maximum power current I_{MP} of 1.20 A. Repeat the four tasks of the previous problem.

Power Up
20 Watt Module

Problem 4.59. A REC SCM220 220-watt 20V c-Si solar panel shown in the figure has the following readings on the back: the short-circuit current I_{SC} of ~8.20 A, the open-circuit voltage V_{OC} of ~36.0 V, the maximum power voltage V_{MP} of ~28.7 V, and the maximum power current I_{MP} of ~7.70 A. Repeat the four tasks of problem 4.57.

Problem 4.60. A 14.4-W load (a DC motor) rated at 12 V is to be driven by a solar panel. A c-Si photovoltaic sheet material is given, which has the open-circuit voltage of 0.6 V and the photocurrent density of $J_P = 0.03\ \text{A/cm}^2$.



Outline parameters of a solar module (number of cells, cell area, and overall area) which is capable of driving the motor at the above conditions and estimate the overall panel size.

Problem 4.61. A custom 100-W load (a DC motor) rated at 24 V is to be driven by a solar panel. A c-Si photovoltaic sheet material is given, which has the open-circuit voltage of 0.6 V and the photocurrent density of $J_P = 0.03\ \text{A/cm}^2$. Outline parameters of a solar module (number of cells, cell area, and overall area) which is capable of driving the motor at the above conditions and estimate the overall panel size.

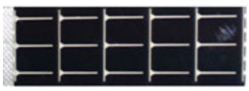
Problem 4.62. You are given: the generic fill factor $F = 0.72$ for a c-Si solar panels, the generic open-circuit voltage $V_{OC} = 0.6\text{V}$ of a c-Si cell, and the generic photocurrent density $J_P = 0.03\ \text{W/cm}^2$.

A. Derive an analytical formula that expresses the total area A_{module} in cm^2 of a solar panel, which is needed to power a load, in terms of the required load power P_L.
B. Test your result by applying it to the previous problem.

Problem 4.63. You are given a low-cost low-power flexible (with the thickness of 0.2 mm) a-Si laminate from PowerFilm, Inc., with the following parameters: a fill factor of $F = 0.61$, a single-cell open-circuit voltage of $V_{OC} = 0.82$ V, and a photocurrent density of $J_P = 0.0081\ \text{A/cm}^2$.

A. Derive an analytical formula that expresses the total module area A_{module} in cm^2, which is needed to power a load, in terms of the required load power P_L.

B. Compare your solution with the solution to the previous problem.

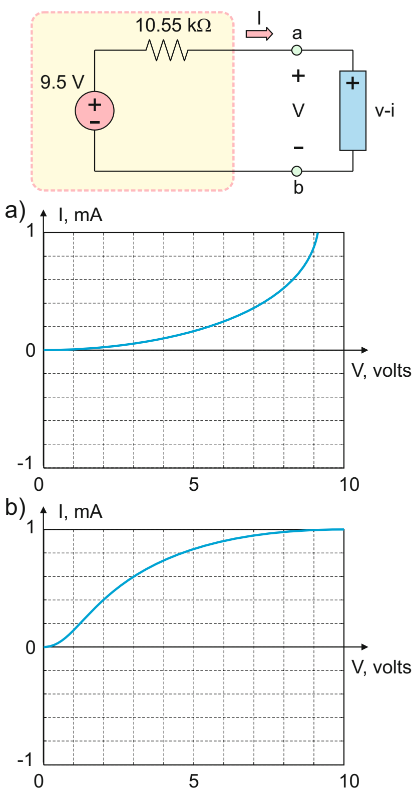


4.4 Analysis of Nonlinear Circuits. Generic Solar Cell

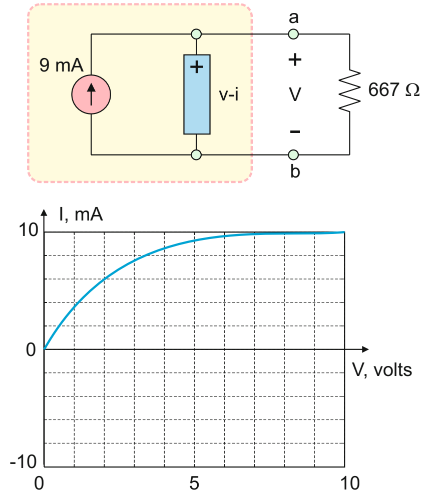
4.4.1 Analysis of Nonlinear Circuits: Load Line Method

4.4.2 Iterative Solution for Nonlinear Circuits

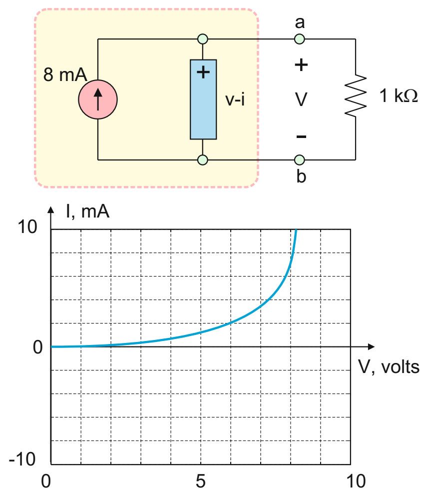
Problem 4.64. A circuit shown in the following figure contains a nonlinear passive element. Using the load line method, approximately determine the voltage across the element and the current through it for the two types of the *υ-i* characteristic, respectively.



Problem 4.65. A circuit shown in the following figure contains a nonlinear passive element as part of a current source. Using the load line method, approximately determine the voltage across the element and the current through it for the *υ-i* characteristic of the nonlinear element shown in the same figure.



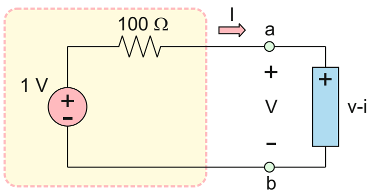
Problem 4.66. Repeat the previous problem for the circuit shown in the following figure.



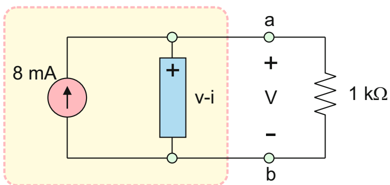
Problem 4.67. A circuit shown in the following figure contains a nonlinear passive element. The *υ-i* characteristic of the nonlinear element (the ideal Shockley diode) is

$$I = 5 \times 10^{-10}\left[\exp\left(\frac{V}{0.025\ \mathrm{V}}\right) - 1\right]\ [\mathrm{A}].$$

Using the iterative solution, determine the voltage across the element and the current through it.

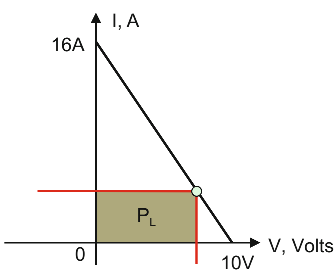


Problem 4.68. Repeat the previous problem for the circuit shown in the following figure. The *v-i* characteristic of the nonlinear element is the same.



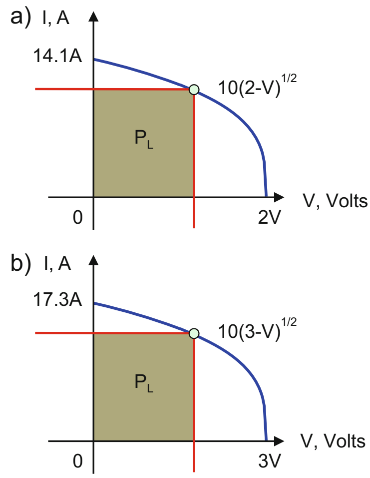
4.4.3 Application Example: Solving the Circuit for a Generic Solar Cell

Problem 4.69. The $I(V)$ dependence for a resistive load in a circuit is shown in the figure that follows.



A. At which value of the load voltage is the power delivered to the load maximized?
B. What is the related value of load resistance?

Problem 4.70. A hypothetic thermoelectric engine developed by the US Navy has the $I(V)$ dependence shown in the figure that follows.



1. At which value of the load voltage is the power, P_L, delivered to the load maximized?
2. What is the related value of load resistance for maximum power transfer?

Problem 4.71. Estimate the values of V_{MP} and I_{MP} versus V_{OC} and I_{SC} for a set of generic c-Si solar cells. Every cell has $V_T = 0.026\,\mathrm{V}$ (room temperature of 25 °C) and $V_{OC} = 0.6$ V. The ideality factor n in Eqs. (4.43)–(4.45) is allowed to vary over its entire range as shown in the table that follows.

n	V_{MP}/V_{OC},%	I_{MP}/I_{OC},%	F
1.00			
1.25			
1.50			
1.75			
2.00			