

Chapter 6: Dynamic Circuit Elements

Overview

Prerequisites:

- Knowledge of basic circuit theory (Chapters 2 and 3)
- Knowledge of operational amplifiers with negative feedback (Chapter 5)

Objectives of Section 6.1:

- Define types of capacitance encountered in electric circuits
- Define self-inductance and mutual inductance from the first principles
- Define field energy stored in a capacitor/inductor
- Be able to combine capacitances/inductances in series and in parallel
- Understand construction of practical capacitors/inductors
- Understand fringing effect and its use in sensor circuits

Objectives of Section 6.2:

- Derive dynamic equations for capacitance/inductance from the first principles
- Establish how the capacitance may create large transient currents
- Establish how the inductance may create large transient voltages
- Define instantaneous energy and power of dynamic circuit elements
- Establish the behavior of dynamic circuit elements in the DC steady state and at a very high frequency

Objectives of Section 6.3:

- Obtain initial exposure to bypass/blocking capacitor and decoupling inductor
- Obtain initial exposure to amplifier circuits with dynamic circuit elements

Application Examples:

Electrostatic discharge and its effect on integrated circuits

How to design a 1-F capacitor? How to design a 1-mH inductor?

Capacitive touchscreens

Bypassing a DC motor

Keywords:

Capacitance, Capacitance of two conductors, Self-capacitance, Capacitance to ground, Capacitance of two equal conductors separated by large distances, Energy stored in a capacitance, Electrostatic discharge (ESD), ESD effect on integrated circuits, Device under test (DUT), Parallel-plate capacitor (base formulas, fringing effect, fringing fields), Capacitor (absolute dielectric permittivity, relative dielectric permittivity, dielectric strength, normalized breakdown voltage, electrolytic, tantalum, ceramic, marking, set of base values), Capacitive touch screens (self-capacitance method, mutual-capacitance method), Magnetic flux density, Magnetic field, Absolute magnetic permeability, Relative magnetic permeability, Magnetic induction, Magnetic flux, Self-inductance, Inductance, Mutual inductance, Energy stored in an inductance, Solenoid (air core, toroidal magnetic core, straight magnetic core, short, fringing fields), Inductor (marking, set of base values, also see *solenoid*), Dynamic equation for capacitance (definition, derivation, fluid mechanics analogy), Capacitance (instantaneous energy, instantaneous power, behavior in the DC steady state, behavior at very high frequencies), Dynamic equation for inductance (definition, derivation, fluid mechanics analogy), Inductance (instantaneous energy, instantaneous power, behavior in the DC steady state, behavior at very high frequencies), Bypass capacitor, Decoupling capacitor, Shunt capacitor, Snubber RC circuit, Decoupling inductor, Inductor choke, Transient circuit, Amplifier circuits with dynamic circuit elements, Active filters, Miller integrator (circuit, DC gain, compensation, time constant), Analog pulse counter, Analog computer, Differentiator amplifier (circuit, gain at very high frequencies), Active differentiator

Section 6.1 Static Capacitance and Inductance

6.1.1 Capacitance, Self-Capacitance, and Capacitance to Ground

Capacitance reflects the ability of arbitrary conductors to store electric charge and, simultaneously, the store energy of the electric field in the surrounding space. When no dielectric is present, capacitance is determined *entirely* by the geometry of conductors. When a dielectric material is present, its permittivity becomes important. Capacitance definitions will be given with reference to Fig. 6.1.

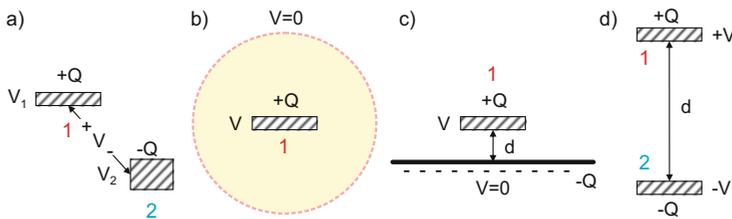


Fig. 6.1. Conductor geometry for capacitance definitions.

1. *Capacitance, C , of Two Conductors.* Two arbitrary insulated conductors near together in Fig. 6.1a constitute a simple *capacitor*. Its capacitance, C , is found with the help of electrostatic theory. Further, it is used in various dynamic models. Capacitance C of two insulated conductors 1 and 2 is defined by the ratio

$$C \equiv \frac{Q}{V} > 0 \quad (6.1a)$$

where $Q > 0$ is the (absolute) net charge of *either* conductor given that the net charge of the system with both conductors is *zero* and V is the potential difference or voltage *between* two conductors 1 and 2, i.e., $V = V_1 - V_2$. This ratio does not depend on V ; it is always taken so as to make the capacitance *positive*.

2. *Self-Capacitance, C_{self} of a Conductor.* When an electric charge Q is added to a single isolated conductor in Fig. 6.1b, its surface will possess a certain absolute voltage V versus 0 V at infinity. The ratio

$$C_{\text{self}} \equiv \frac{Q}{V} > 0 \quad (6.1b)$$

is the *self-capacitance* of the conductor. The self-capacitance is the capacitance when the second conductor is a hollow conducting sphere of infinite radius subject to 0 V.

3. *Capacitance to Ground, C , of a Conductor.* For conductor 1 in Fig. 6.1c with charge $+Q$, its capacitance to ground is the capacitance when the second conductor is an infinite conducting ground plane in Fig. 6.1c subject to 0 V (and charged to $-Q$).

Capacitance to ground, C , is always *greater* than the self-capacitance, C_{self} ; their ratio becomes quite large when the separation distance d from the plane is small. On the other hand,

$$C \rightarrow C_{\text{self}} \text{ when } d \rightarrow \infty \text{ in Fig. 6.1c.} \quad (6.1c)$$

4. *Capacitance, C , of Two Equal Conductors Separated by Large Distances.* For conductors 1 and 2 in Fig. 6.1d, the capacitance approaches

$$C \rightarrow \frac{1}{2}C_{\text{self}} \text{ when } d \rightarrow \infty \text{ in Fig. 6.1d.} \quad (6.1d)$$

The separation distance d must be large compared to the conductor's size. Equation (6.1d) will be proved shortly.

The capacitance is recorded in units of farads or F. This unit is named in honor of Michael Faraday (1791–1867), a British physicist and chemist, who was known to many as “the best experimentalist in the history of science.” Typical capacitance values in electronics are pF (picofarad or simply *pdf*), nF (nanofarad), and μF (microfarad). In power electronic circuits, larger capacitances might be used. The capacitance unit is linked to other MKS units as follows:

$$1 \text{ F} = 1 \frac{\text{A} \cdot \text{s}}{\text{V}} = 1 \frac{\text{J}}{\text{V}^2} = 1 \frac{\text{C}}{\text{V}} \quad (6.2)$$

where C is the unit of coulomb. The total *electric field energy* stored between two conductors and in the surrounding space is given by

$$E = \frac{1}{2}CV^2 \quad (6.3)$$

This result can be derived from the definition of the electric potential (voltage). The energy is equal to work, which is necessary to put all charges of the capacitor in place. Equation (6.3) is valid for any configuration shown in Fig. 6.1.

Example 6.1: Prove Eq. (6.1d).

Solution: For a conductor in Fig. 6.1b, the stored electric energy is given by

$$E_{\text{self}} = \frac{1}{2}C_{\text{self}}V^2 \quad (6.4a)$$

For two conductors in Fig. 6.1d separated by a very large distance d , the stored electric energy is approximately given by

Example 6.1 (cont.):

$$E = 2E_{\text{self}} = \frac{1}{2}C(2V)^2 \quad (6.4b)$$

Comparing Eqs. (6.4a) and (6.4b) we obtain the necessary result.

Exercise 6.1: A metal circle of radius $r = 0.1$ m has the self-capacitance $C_{\text{self}} = 8\epsilon_0 r$ where $\epsilon_0 = 8.85419 \times 10^{-12}$ F/m is the permittivity of vacuum. Estimate capacitance of a capacitor formed by two coaxial circles separated by 1 m.

Answer: ~ 3.54 pF from Eq. (6.1d). A precise numerical solution predicts 3.77 pF.

Exercise 6.2: How large is the stored energy in a 100- μ F laboratory capacitor at 10 V?

Answer: 0.005 J or, which is the same on the power basis, 5 mW of power delivered during one second. However, this power will not be delivered uniformly.

Exercise 6.3: How large is the stored energy in a 20-F *ultracapacitor* charged to 25 V?

Answer: 6250 J or 6.25 kJ. This is certainly a significant value. At the same time, the discharge rate (available current or power) is much less in this case than the current or power delivered by laboratory electrolytic capacitors.

6.1.2 Application Example: ESD

Self-capacitance results may be applied for the prediction of ESD (*electrostatic discharge*) effects on *integrated circuits* (ICs). One of the most common causes of electrostatic damage is the direct transfer of electrostatic charge through a significant series resistor from the human body or from a charged material to the electrostatic discharge-sensitive (ESDS) device. The concept is shown in Fig. 6.2a. A metal ground plane and the highly conducting human body naturally form a capacitor. The *body capacitance* C in Fig. 6.2b is defined as the capacitance between the body, assumed to be a conductor, and the large (ideally infinite) ground plane. Its value depends significantly on the posture of the body with respect to the ground surface. The typical separation distance is 2 cm. It may be shown that, at such distances, $C \approx 2C_{\text{self}}$. Therefore, instead of calculating C directly, we can find the self-capacitance of the human body, C_{self} , and then multiply it by 2.

Typical self-capacitances of a 177-cm-tall male student are indicated in Fig. 6.3. These values were obtained by the method described in Chapter 1.

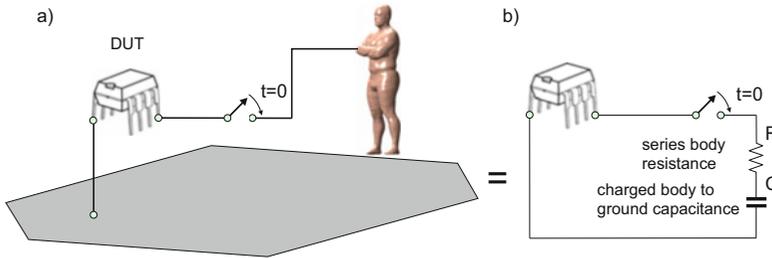


Fig 6.2. Equivalent circuit for understanding ESD and its effect on a *device under test* (DUT).

The simplifying assumption $C \approx 2C_{\text{self}}$ and Fig. 6.3 predict body capacitances in the range 86–95 pF. These values are in a good agreement with the generally accepted *human body model* (HBM), which, with reference to Fig. 6.2b, uses

$$R = 1.5 \text{ k}\Omega, \quad C = 100 \text{ pF} \tag{6.5}$$

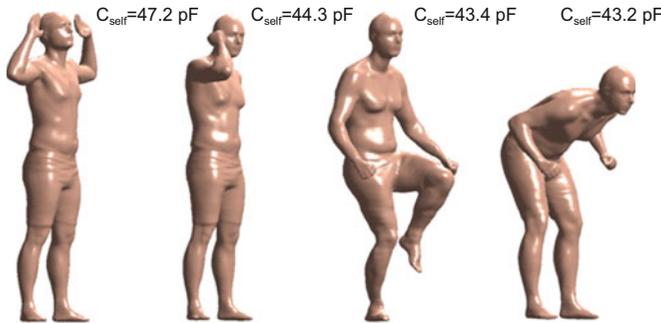


Fig 6.3. Typical self-capacitance values for a 177-cm-tall male person. Note how the self-capacitance changes when the body poses change.

6.1.3 Parallel-Plate Capacitor

Consider a parallel-plate capacitor shown in Fig. 6.4a, b. Both infinitely thin conducting square plates with the side a and area $A = a^2$ are separated by distance d . The upper plate has a total charge $+Q$; the lower plate has the opposite charge $-Q$; the net charge of the capacitor is zero. Feeding conductors are implied to be disconnected; they are excluded from consideration. Assuming that the entire electric field is concentrated *only* within the capacitor and that it is uniform in space (equal to V/d), the approximate capacitance is established as

$$C = \frac{\epsilon_0 A}{d} \quad (6.6)$$

where ϵ_0 is the *dielectric permittivity of vacuum* if the capacitor is situated in vacuum. For Eq. (6.6) to hold, the plates do not have to be square. If the capacitor does not have a high- ϵ dielectric inside, Eq. (6.6) is a good approximation only if d is *very small* compared to the dimensions of the plates. Otherwise, the *fringing effect* must be taken into account. The fringing effect is illustrated in Fig. 6.4a, b. Fringing means that the electric field extends outside the physical capacitor. The electric field outside the capacitor possesses certain extra energy. Therefore, according to Eq. (6.3) where voltage V is fixed, the capacitance must *increase* compared to the non-fringing case. Figure 6.4c in the summary of this chapter present numerically found capacitance values C_{exact} for the parallel-plate capacitor with fringing. These values have been accurately computed using a rigorous numerical adaptive procedure. Figure 6.4c predicts a nearly linear increase of the ratio C_{exact}/C as a function of the separation distance. Therefore, the wrong result, $C \rightarrow 0$ when $d \rightarrow \infty$, which is predicted by Eq. (6.6), is corrected. Instead, one will have $C \rightarrow 0.5 C_{\text{self}}$ when $d \rightarrow \infty$.

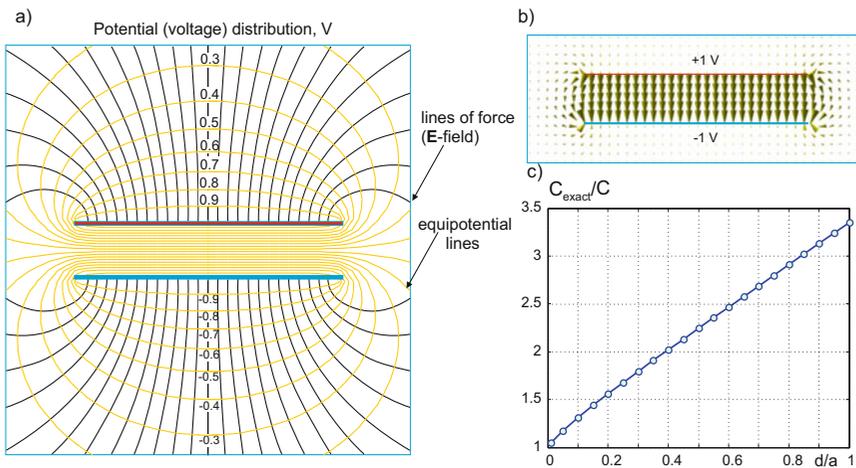


Fig. 6.4. (a) Equipotential lines and lines of force for a capacitor with $d/a = 0.2$ in the central cross-sectional plane (the plates are at ± 1 V). (b) Fringing electric field for the same capacitor observed in the central cross-sectional plane (the plates are at ± 1 V). (c) Ratio of the accurate capacitance values (found numerically) to the values predicted by Eq. (6.6).

The fringing field of capacitors is utilized in capacitive *touch screens*. In this case, the significant fringing field is a desired effect. Therefore, configurations other than the parallel-plate capacitor are used. These configurations will be studied later in this section.

When a dielectric material of relative permittivity $\epsilon_r > 1$ is inserted between the capacitor plates, a substitution $\epsilon_0 \rightarrow \epsilon_r \epsilon_0$ has to be made in Eq. (6.6). The fringing effect is less apparent for higher values of ϵ_r .

Exercise 6.4: Estimate the static capacitance of a parallel-plate capacitor with $a = 1 \text{ cm}^2$ and $d = 1.57 \text{ mm}$ using the basic formula. The substrate material is Rogers 4003 laminate with $\epsilon_r = 3.55$.

Answer: 2.00 pF.

6.1.4 Circuit Symbol: Capacitances in Parallel and in Series

Figure 6.5 shows the capacitances in parallel and in series, along with the capacitance circuit symbol. This symbol is reserved for the capacitance as a *circuit element*. Such an element is an *ideal* capacitor excluding manufacturing imperfections (parasitic resistance and inductance). In the following text, we will frequently employ both words—capacitance and capacitor—to denote the same ideal circuit element. The parallel and series connections of capacitances are *opposite* when compared to the resistances. To establish this fact we consider two combinations in Fig. 6.5.

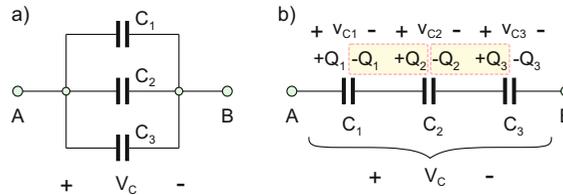


Fig. 6.5. Capacitances in parallel and in series: (a) capacitances in parallel are added; they behave similarly to resistances in series and (b) capacitances in series are combined in the same way as resistances in parallel.

For the parallel configuration in Fig. 6.5a, the same voltage V_C is applied to every capacitance. One has for the charges on the capacitor plates,

$$\begin{aligned} Q_1 &= C_1 V_C, \quad Q_2 = C_2 V_C, \quad Q_3 = C_3 V_C \Rightarrow Q_1 + Q_2 + Q_3 = Q_{\text{total}} \\ &= (C_1 + C_2 + C_3) V_C \end{aligned} \quad (6.7)$$

Thus, the capacitances connected in parallel behave like a single capacitance C_{eq} ,

$$C_{\text{eq}} = C_1 + C_2 + C_3 \quad (6.8)$$

Equation (6.8) also makes intuitive sense if we take a closer look at Fig. 6.5a where the three individual capacitors visually form a bigger capacitor comprised of larger plate areas. This clearly increases the capacitance accordingly. In Fig. 6.5b, however, the situation is different. The thicknesses of each capacitor add together, which decreases the overall capacitance since thickness varies inversely with capacitance. Assume that every capacitor was initially uncharged and apply voltage V_C between terminals A and B . Since each pair of inner conductors in Fig. 6.5b has remained insulated, $Q_1 = Q_2 = Q_3 = Q$. Next, by KVL,

$$V_C = V_1 + V_2 + V_3 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) Q = \frac{Q}{C_{\text{eq}}} \quad (6.9)$$

and for the series combination of the capacitances, one has

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (6.10)$$

Exercise 6.5: Find the equivalent capacitance of the circuit shown in Fig. 6.6.

Answer: $C_{\text{eq}} = 44 \mu\text{F}$

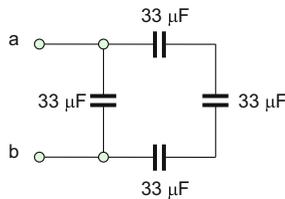


Fig. 6.6. A capacitive network which includes series and parallel combinations of capacitances.

6.1.5 Application Example: How to Design a 1- μF Capacitor?

Let us to design a capacitor. Our modest goal is a 1- μF capacitor. We consider two aluminum plates separated by 1 mm. Equation (6.6) then allows us to predict the required plate area:

$$A = \frac{dC}{\epsilon_0} = \frac{10^{-3} \times 10^{-6}}{8.854 \times 10^{-12}} = 113 \text{ m}^2! \quad (6.11a)$$

Well, such a capacitor will certainly occupy a significant fraction of a lecture hall and is hardly practical. How, then, do manufactures design a capacitor of 1 μF ? The first step is to use a dielectric material sandwiched within the capacitor. A dielectric medium increases charges stored on the two metal capacitor plates depending on $\epsilon_r \geq 1$, the relative dielectric permittivity of the dielectric medium. Table 6.1 gives us a list of permittivities for a number of dielectric materials. For each material, a *dielectric strength*, or *normalized breakdown voltage*, is also given. This is actually the maximum electric field (notice the unit of V/m) that the capacitor can handle. It is for this reason that capacitors carry a voltage rating that you should not exceed in your circuit. From a practical point of view, the higher the capacitance, the lower the voltage rating. The well-known dilemma with the capacitor is that a decrease in the separation distance increases the capacitance and the stored energy. However, as already mentioned, it simultaneously decreases the maximum applied voltage due to the *dielectric breakdown effect*. For our capacitor, we will again use the mica dielectric material listed in Table 6.1. Equation (6.6) now transforms to

$$A = \frac{dC}{\epsilon_0 \epsilon_r} = \frac{10^{-3} \times 10^{-6}}{8.854 \times 10^{-12} \times 7} = 16 \text{ m}^2 \quad (6.11b)$$

Table 6.1. Relative dielectric permittivity and dielectric strength of some common materials.

Material	Relative permittivity	Dielectric strength in V/m
Air	1.0	$0.4\text{--}3.0 \times 10^6$
Aluminum oxide	8.5	Up to 1000×10^6
Fused silica (glass)	3.8	$470\text{--}670 \times 10^6$ (or lower)
Gallium arsenide (GaAs)	13	
Germanium (Ge) crystal	16	$\sim 10 \times 10^6$
Mica	7.0	Up to 400×10^6
Nylon	3.8	$\sim 20 \times 10^6$
Plexiglas	3.4	$\sim 30 \times 10^6$
Polyester	3.4	
Quartz	4.3	8×10^6 (fused quartz)
Rutile (titanium dioxide)	100–200	$10\text{--}25 \times 10^6$
Silicon (Si) crystal	12	$\sim 30 \times 10^6$
Styrofoam	1.03–1.05	
Teflon	2.2	$87\text{--}173 \times 10^6$
Water (distilled, deionized)	~ 80	$65\text{--}70 \times 10^6$

Even though the result looks a bit better, it is still far from practical. However, what if we try to make the dielectric layer very thin? An oxide is a dielectric, so could we just oxidize one top aluminum plate with a very thin (i.e., $d = 10 \mu\text{m}$) oxide layer and press-fit it to

the second plate? The result becomes (the relative dielectric constant of 8.5 is now that for aluminum oxide from Table 6.1)

$$A = \frac{hC}{\epsilon_0 \epsilon_r} = \frac{10^{-5} \times 10^{-6}}{8.854 \times 10^{-12} \times 8.5} = 13 \text{ cm}^2 \quad (6.11c)$$

Electrolytic Capacitors

Once such a thin film is rolled into a cylinder, it will clearly become a compact design, similar in size to a 1- μF electrolytic capacitor routinely used in the laboratory. Unfortunately, one problem still remains: the permanent oxide layer is fragile and rough in shape. A better idea is to chemically grow such a layer using a so-called anodization process. This process occurs when the aluminum foil is in contact with an electrolyte as a second conductor and an appropriate *voltage* is applied between them. This is the smart idea behind an electrolytic capacitor. And this is also the reason why an electrolytic capacitor is *polarized*. The term *electrolytic capacitor* is applied to any capacitor in which the dielectric material is formed by an electrolytic method; the capacitor itself does not necessarily contain an electrolyte. Along with aluminum capacitors, *tantalum capacitors* (both *wet* and *dry*) are also electrolytic capacitors.

Ceramic Capacitors

A competitor to the electrolytic capacitor is a *non-polarized ceramic* capacitor. Ceramic capacitors consist of a sandwich of conductor sheets alternated with ceramic material. In these capacitors the dielectric material is a ceramic agglomerate whose relative static dielectric permittivity, ϵ_r , can be changed over a very wide range from 10 to 10,000 by dedicated compositions. The ceramic capacitors with lower ϵ_r values have a stable capacitance and very low losses, so they are preferred in high-precision circuits and in high-frequency and RF electronic circuits. Typically, these “fast” ceramic capacitors have very small capacitances, on the order of pF and nF, and they can hold a high voltage. At the same time, the “slow” ceramic capacitors may have values as high as 1 μF . Therefore, the task of the above example can be solved with the ceramic capacitor as well.

Capacitor Marking

Figure 6.7 shows two examples of ceramic capacitors, with 100-pF and 1.0- μF capacitance from two different companies. To read the capacitance in the figure, we use the following rule: 101 = $10 \times 10^1 \text{ pF} = 100 \text{ pF}$, and 105 = $10 \times 10^5 \text{ pF} = 1 \mu\text{F}$. Indeed, 473 = $47 \times 10^3 \text{ pF} = 47 \text{ nF}$, and so forth. The tolerance letters may be present: $F = 1 \%$, $G = 2 \%$, $J = 5\%$, $K = 10 \%$, and $M = 20 \%$. Also, the voltage rating should be given.

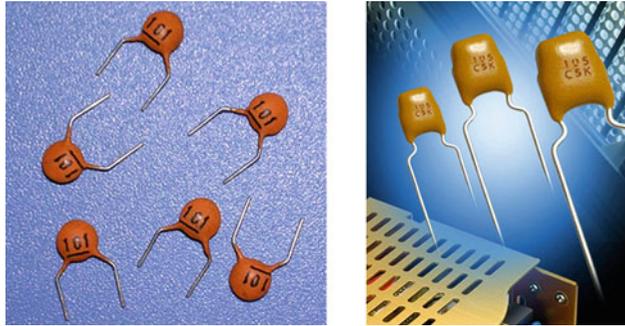


Fig. 6.7. *Left*, ceramic capacitors of 100 pF. *Right*, radial leaded ceramic capacitors of 1.0 μF .

A standardized set of *capacitance base values* is defined in the industry. The capacitance of any (electrolytic or not) capacitor can then be derived by multiplying one of the base numbers **1.0**, **1.5**, **2.2**, **3.3**, **4.7**, or **6.8** by powers of ten. Therefore, it is common to find capacitors with capacitances of 10, 15, 22, 33, 47, 68, 100, 220 μF , and so on. Using this method, values ranging from 0.01 to 4700 μF are customary in most applications. The value of the capacitance and the allowed maximum voltage are prominently written on the case of the electrolytic capacitor so reading those does not constitute any difficulties.

6.1.6 Application Example: Capacitive Touchscreens

Capacitive touchscreens use the fringing field of a capacitor studied previously. Many small capacitors with a significant fringing field are involved. If a conducting finger (an extra conductor) is placed in the fringing field, the corresponding capacitance changes. There are two possible solutions called the *self-capacitance method* and the *mutual-capacitance method*, respectively. The difference is in the measurement nodes for the capacitance. In the first case, the capacitance is measured between the touch pad electrode and a ground. In the second case, the capacitance is measured between two pad electrodes, neither of which is grounded. Both methods may be combined.

Self-Capacitance Method

Consider a human finger in the proximity of a touchscreen as shown in Fig. 6.8a. The touchscreen itself may be a lattice of circular touch pads surrounded by a ground plane and separated from it by an air-gap ring—see Fig. 6.8b. When the finger is not present, each pad has capacitance C_P to ground, which is called a *parasitic capacitance*. When the (grounded) finger appears in the vicinity of the touchpad, there appears another capacitance, C_F , which is called the *finger capacitance*. Figure 6.8a indicates that both capacitances are in parallel so that the resulting ground capacitance *increases* as

$$C_P \rightarrow C_P + C_F > C_P \quad (6.12)$$

This change in capacitance is recorded. Physically, the presence of the finger (or hand) increases the size of the ground conductor and thus increases the resulting capacitance. Typical values of C_P are on the order of 100 pF; C_F is on the order of 1–0.1 pF. Now assume that the desired resolution along one dimension of the screen is N . Then, N^2 individual touch pads are needed including the corresponding sensing circuitry. This may be a significant disadvantage of the self-capacitance method.

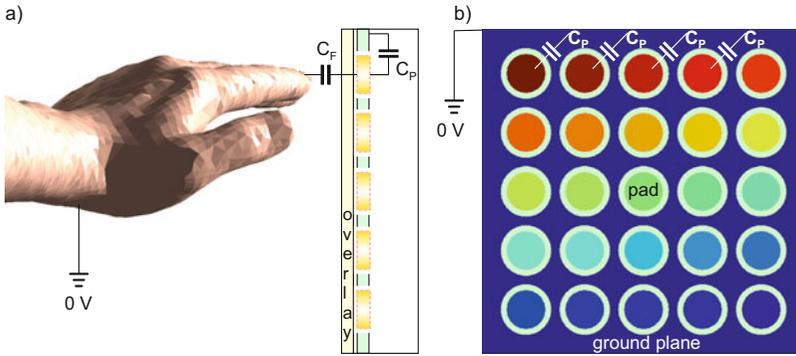


Fig. 6.8. Self-capacitance method for a capacitive touchscreen. The touchscreen is enlarged.

Mutual-Capacitance Method

The electrodes are typically interleaving rows and columns of interconnected square patches, which are shown in Fig. 6.9. Neither of them is connected to circuit ground (the third conductor) or to each other. When a finger touches the panel, the mutual capacitance C_M between the row and column, which mostly concentrates at the intersection, *decreases*, in contrast to the previous case. This change in capacitance is recorded. Assume again that the desired resolution along one dimension of the screen is N . Then, only $2N$ individual touch pads (electrodes) are needed including the corresponding sensing circuitry. This is a significant advantage of the mutual-capacitance method.

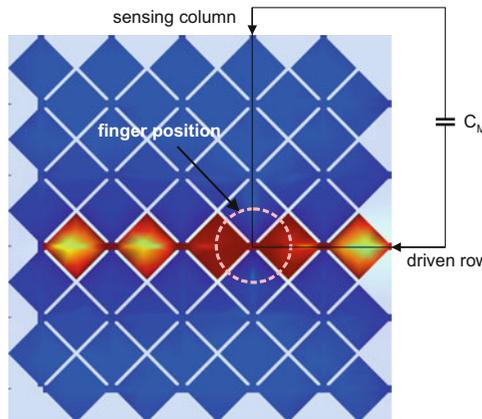


Fig. 6.9. Mutual-capacitance method for a capacitive touchscreen. Surface charge distribution is illustrated when the driven row is subject to an applied voltage. Finger projection is a circle.

6.1.7 Self-Inductance (Inductance) and Mutual Inductance

As long as the physical capacitor stores energy of the electric field, a physical *inductor* stores energy of the magnetic field. The inductor stores magnetic-field energy only when an electric current I flows through it. This is in contrast to a capacitor, which, once charged, stores the electric field energy in vacuum indefinitely, even when disconnected from the charging circuit. To use the inductor as an energy-storage element, one therefore needs to maintain a current in the circuit. The primary quantity is the *magnetic flux density*, \vec{B} , which is measured in *webers per m²*, or *tesla* ($1 \text{ T} = 1 \text{ Wb/m}^2 = 1 \text{ V} \cdot \text{s/m}^2$). The magnetic flux density is related to the *magnetic field*, \vec{H} , by $\vec{B} = \mu\vec{H}$ where μ is *magnetic permeability*. In older power electronics texts, \vec{B} , may be called *magnetic induction*. In Fig. 6.10, the magnetic flux density \vec{B} is created by circuit #1 (a closed loop of current I). Instead of the vector field \vec{B} , it is convenient to use a simpler scalar quantity known as *magnetic flux* or simply *flux*, Φ . For a constant \vec{B} , which is strictly perpendicular to the plane of circuit #1 with area A , the magnetic flux would be equal to

$$\Phi = AB \quad (6.13)$$

where B is the magnitude (length) of vector \vec{B} . The flux is measured in *webers* or in $\text{V} \cdot \text{s}$ ($1 \text{ Wb} = 1 \text{ V} \cdot \text{s}$).

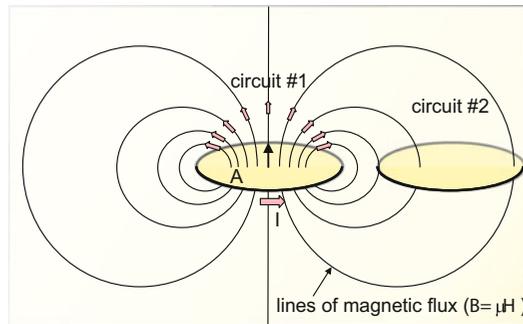


Fig. 6.10. Magnetic flux density generated by circuit #1.

Generally, Eq. (6.13) is only approximately valid for circuit #1 in Fig. 6.10. The exact flux is given by a surface integral over the area of the circuit,

$$\Phi = \iint_A \vec{B} \cdot \vec{n} da \quad (6.14)$$

where \vec{n} is a unit normal to the surface A . Flux is an algebraic quantity and hence could be positive or negative depending on the chosen direction of \vec{n} . We assume that *positive current in the circuit produces positive flux*—see Fig. 6.10. The *self-inductance* of circuit

#1 in Fig. 6.1 is its *inductance*; both terms have the same meaning. The inductance, L , of circuit #1 is given by

$$L \equiv \frac{\Phi}{I} > 0 \quad (6.15)$$

Thus, the inductance is the magnetic flux through circuit #1 produced by a unit current in the same circuit. The *mutual inductance*, M , between circuits #2 and #1 in Fig. 6.10 is the magnetic flux, Φ' , through circuit #2 produced by unit current in circuit #1, i.e.,

$$M \equiv \frac{\Phi'}{I} \quad (6.16)$$

Both L and M have the units of henry, or H. This unit is named in honor of Joseph Henry (1797–1878), an American scientist. Typical inductance values in electronics are nH (nanohenries) and μH (microhenries). In power electronics, larger inductances may be used. Henry is converted to V, A, and energy, J, as follows:

$$1 \text{ H} = 1 \frac{\text{V} \cdot \text{s}}{\text{A}} = 1 \frac{\text{J}}{\text{A}^2} \quad (6.17)$$

One may observe a close similarity between Eqs. (6.17) and (6.2). Both equations become identical if we interchange V and A. Equation (6.17) also has a number of simple and important implications related to energy and power. Total *magnetic-field energy* stored in space surrounding circuit #1 in Fig. 6.10 is given by

$$E = \frac{1}{2}LI^2 \quad (6.18)$$

Equation (6.18) may be considered as another definition of self-inductance (or inductance). As such, it is frequently used in practice.

Exercise 6.6: A flux linking the circuit is 0.1 Wb. Find the circuit's inductance and magnetic-field energy stored if the circuit current is 1 A.

Answer: $L = 100 \text{ mH}$, $E = 0.05 \text{ J}$.

6.1.8 Inductance of a Solenoid With and Without Magnetic Core

Consider a *solenoid* (a long helical coil of length l) with applied current I shown in Fig. 6.11. The case of air-filled coil in Fig. 6.11a is studied first. Magnetic flux density \vec{B} within the solenoid is nearly uniform and is directed along its axis. Therefore, the flux through one turn of the coil (one loop) is given by Eq. (6.13). It is equal to AB , where A is the loop area. The net flux Φ through the entire solenoid is AB times the number of turns comprising the coil, N . The inductance is therefore obtained from Eq. (6.15) as

$$L = N \frac{AB}{l} \tag{6.19}$$

The magnetic flux, B , within the solenoid is found in physics courses:

$$B = \frac{\mu_0 N I}{l} \tag{6.20}$$

where the *natural constant* $\mu_0 = 4\pi \times 10^{-7}$ H/m is the *magnetic permeability of vacuum* (or air, which is very close to a vacuum with regard to magnetic properties). Substitution of Eq. (6.20) into Eq. (6.19) yields a simple equation for the inductance

$$L = \frac{\mu_0 A N^2}{l} \quad [\text{H}] \tag{6.21}$$

Thus, strong inductances can be created by a large number of turns (a quadratic dependence), a large coil cross section, and a smaller coil length. Equation (6.21) also holds for various bent solenoids (such as toroidal coils). Equation (6.21) makes clear that the inductance, like capacitance and resistance, is independent of externally applied circuit conditions.

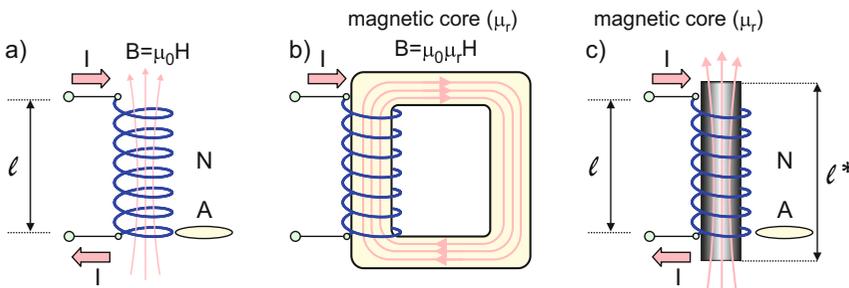


Fig. 6.11. Three types of a solenoid.

The above derivation is only valid for a solenoid that is long compared to its diameter. When this is not the case, a modification to Eq. (6.22) is made, namely,

$$L = \frac{\mu_0 A N^2}{l} \left(1 - \frac{8w}{3\pi} + \frac{w^2}{2} - \frac{w^4}{4} \right), \quad w = \frac{r}{l} < 1 \tag{6.22}$$

where r is the radius of the coil. Other handy formulas for *short solenoids* exist. Equation (6.22) describes the *fringing effect* in practical inductors, which may also be used for sensor purposes, similar to the fringing fields for the capacitor. The inductance of the solenoid increases when it has a core with a magnetic material within the coil, as shown in Fig. 6.11b, c. This material is called a *magnetic core* and inserting it into the coil increases the magnetic-field energy stored in the inductor. When the magnetic core is closed, i.e., has the form of a toroid as in Fig. 6.11b, Eq. (6.21) is transformed to

$$L = \frac{\mu_0 \mu_r AN^2}{l} \quad (6.23)$$

where $\mu_r \geq 1$ is known as the *relative magnetic permeability* of a magnetic material. Emphasize that Eq. (6.23) is not valid for the straight cylindrical core in Fig. 6.11c. The calculation of inductance for the straight core becomes a nontrivial theoretical exercise. We present here a useful theoretical result, which is only valid for a high-permeability magnetic core, with approximately $\mu_r \geq 100$. The resulting inductance has the form

$$L \approx \frac{0.5\pi\mu_0 l^* N^2}{\ln\left[\frac{l^*}{r} - 1\right]} \left(1 - \frac{l}{2l^*}\right) \quad [\text{H}] \quad (6.24)$$

where l, l^* are indicated in Fig. 6.11c and r is the radius of the coil (core). The resulting inductance does not explicitly depend on the specific value of μ_r as long as this value is sufficiently large. Equation (6.24) holds only for the situations where the core length-to-diameter ratio is considerably *smaller* than the relative magnetic permeability, μ_r . It predicts inductances that are much lower than those found by using Eq. (6.23).

Exercise 6.7: A solenoid coil in Fig. 6.11a, b, and c has $r = 0.45$ cm, $N = 110$, $l = 2.15$ cm. Determine the coil inductance in all three cases. In cases b and c, respectively, $\mu_r = 100$. Furthermore, $l^* = 8.90$ cm in Fig. 6.11c.

Answer: $L = 45 \mu\text{H}$ (air-core coil); $L = 4500 \mu\text{H}$ (toroidal coil); $L = 640 \mu\text{H}$ (straight-magnetic-core coil).

6.1.9 Circuit Symbol: Inductances in Series and in Parallel

Figure 6.12 shows the inductances in series and in parallel, along with inductance circuit symbol. This symbol is reserved for the inductance as a *circuit element*. Such an element is an *ideal* inductor excluding manufacturing imperfections (parasitic resistance and capacitance). In the following text, we will frequently employ both words—inductance and inductor—to denote the same ideal circuit element. The series and parallel connections of inductances are *identical* when compared to the connections of resistances. To establish this fact we consider two combinations shown in Fig. 6.12.

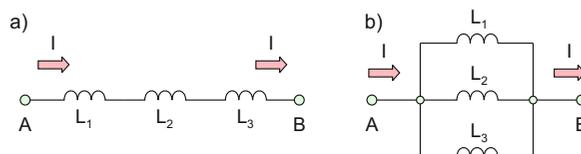


Fig. 6.12. Inductances in series and in parallel.

For the series configuration in Fig. 6.12a, the same current I is applied to every inductance. Given that the equivalent inductance is also subject to current I and must possess the same magnetic-field energy defined by Eq. (6.18), one has

$$E = \frac{1}{2}L_{\text{eq}}I^2 = \frac{1}{2}L_1I^2 + \frac{1}{2}L_2I^2 + \frac{1}{2}L_3I^2 \Rightarrow L_{\text{eq}} = L_1 + L_2 + L_3 \quad (6.25)$$

The parallel configuration in Fig. 6.12b may be analyzed given the condition of equal magnetic flux through each inductance. Since this condition is related to Faraday's law of induction, we postpone the corresponding discussion until the next section.

6.1.10 Application Example: How to Design a 1-mH Inductor?

Again, after this theoretical excursion, let us design a practical inductor. Our goal is to construct a 1-mH inductance. According to Eq. (6.21) the required number of turns is

$$N = \sqrt{\frac{Ll}{\mu_0 A}} \quad (6.26)$$

We will select a coil length of $l = 5$ cm and an (average) coil radius of $r = 1$ cm. Equation (6.26) then yields

$$N = \sqrt{\frac{10^{-3} \times 5 \times 10^{-2}}{4\pi^2 \times 10^{-7} \times 10^{-4}}} = 356 \quad (6.27)$$

Such a coil can be wound on a former in the laboratory with a sufficiently thin wire, say AWG 28. A different approach to reducing the number of turns while maintaining, or even increasing, the inductance is to use a magnetic material within the coil, as shown in Fig. 6.11. Table 6.2 lists the magnetic permeability for a number of magnetic materials. The simplest magnetic core is an iron core. However, it is lossy since an alternating magnetic field creates so-called eddy currents in the conducting core, which are dissipated into heat. One solution to this problem is to use thin insulated sheets of iron, or *laminations*. Various *ferrites* (oxides of iron, or other metals) are an alternative to iron, which are ceramics and known as good electric insulators. Other types of losses may occur there, explanations of which go beyond the scope of our text. Once a soft ferrite with $\mu_r \sim 100$ is used in the design of the 1-mH inductor, the number of turns necessary to achieve the same inductance decreases by $\sqrt{\mu_r} = 10$. In our example, it becomes equal to 36 turns instead of over 300 turns. However, the magnetic core *cannot* be a short rod like that shown in Fig. 6.11c; it must form a closed loop shown in Fig. 6.11b.

Table 6.2. Relative permeability of some common materials.

Material	Relative static permeability
Air	1.0
Magnetic iron	200
Iron powder	2–75
Nickel	100
Permalloy (78.5 % nickel + 21.5 % iron)	8000
Soft ferrites with low losses at frequencies up to 100 MHz	20–800
Hard ferrites with low losses up to 1 MHz	1000–15,000

Inductor Marking

Leaded inductors have color codes, similar to resistors. A standardized *set of inductance base values* is defined in the industry. The inductance of any inductor can then be derived by multiplying one of the base numbers **1.0, 1.1, 1.3, 1.5, 1.9, 2.2, 2.7, 3.3, 3.9, 4.7, 5.8, 6.8, or 8.2** by powers of ten. Therefore, it is common to find inductances with values of 1.0, 2.7, 6.8 μH , and so on. Using this method, values ranging from 0.01 to 100 μH are customary in most applications.

Section 6.2 Dynamic Behavior of Capacitance and Inductance

6.2.1 Set of Passive Linear Circuit Elements

The three elements, resistance, capacitance, and inductance, constitute the fundamental set of passive circuit elements for any linear electric circuit. This is very similar to a mechanical system consisting of dashpot, spring, and mass, which form the basic set of any linear kinematic system. Having discussed the underlying DC concepts of capacitance and inductance, we now turn our attention to their dynamics. A simple example of dynamic behavior is given by a vacuum cleaner. If one manually unplugs the working vacuum cleaner from the wall outlet (please avoid doing so), a profound spark may appear. On the other hand, turning off the vacuum cleaner normally produces no spark. The reason for the spark is that breaking the current through a dynamic circuit element—an inductance—which models the coil of the motor, creates very large transient voltages. One reason for studying transients is the wish to avoid such sparks and to properly design the electric switch. The use of two dynamic circuit elements—the capacitance and inductance—is enormous, especially in power systems. Every electric motor is basically an inductance; most power motors need a power correction circuit that in turn requires a shunt capacitance. Some motors need starting capacitors or surge capacitors for large motors. On the other hand, the capacitance of logic gates is responsible for the so-called propagation delay. This delay determines a very important measure of the performance of a digital system, such as a computer, which is the maximum speed of operation. Thus, the capacitances and inductances are just everywhere, like mass and spring systems present everywhere in mechanical engineering. However, they are becoming most apparent when we consider a transient behavior, an alternating current, or high-frequency digital and communication circuits.

6.2.2 Dynamic Behavior of Capacitance

Both capacitance and inductance are *passive* circuit elements, which means that, like resistance, they do not deliver a net power increase to the circuit. Indeed, after charging, the capacitor is able to power a circuit, usually for a short period of time. At the end of the discharge cycle, it needs to be recharged. Therefore, we use the passive reference configuration for the capacitance in Fig. 6.13a. Lowercase letters denote time-varying voltage and current. The dynamic behavior of the capacitance is described by the well-known voltage-to-current relation (dynamic equation), which plays the role of “Ohm’s law” for the capacitance

$$i_C = C \frac{dv_C}{dt} \quad (6.28)$$

Equation (6.28) follows from the capacitance definition, $q_C = Cv_C$, given in the previous section. It is obtained after differentiation and using the equality

$$\frac{dq_C}{dt} = i_C \tag{6.29}$$

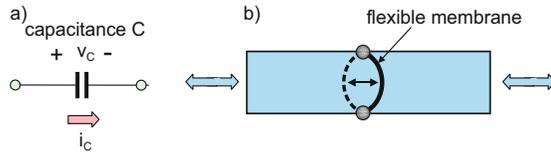


Fig. 6.13. Passive reference configuration for capacitance and its fluid mechanics analogy.

A fluid mechanics analogy of the dynamic capacitance effect corresponds to alternating fluid motion in Fig. 6.13b. A capacitance is represented by a *flexible membrane*. The capacitance value, C , corresponds to the *inverse stiffness*, $1/k$, of the membrane. When $k \rightarrow \infty$ and $C \rightarrow 0$ (a rigid membrane), the capacitance value tends to zero. The membrane becomes a solid wall, which blocks the alternating flow entirely. In another limiting case ($k \rightarrow 0$ or $C \rightarrow \infty$), the membrane moves with fluid and has no effect on the fluid flow. Intermediate cases correspond to a partial blocking. The electric current in Eq. (6.28) is not the DC conduction current, but the *displacement* current which was first introduced by Sir James Clerk Maxwell to complete Maxwell’s equations. This current can flow through empty (or free) space between two capacitor plates. It is not supported by directional motion of free charges, in contrast to the DC conduction current. The capacitor itself was invented yet in 1745 by Ewald Georg von Kleist (1700–1748), German lawyer and physicist, and by Dutch scientist Pieter Van Musschenbroek (1692–1761). Some also believe the Biblical Ark of the Covenant was protected by a first capacitor—the Leyden jar—capable of producing thousands of volts of static electricity.

Example 6.2: The voltage across a 100- μ F capacitor is shown in Fig. 6.14 that follows by a solid curve. At $t = 0$, the voltage is zero. Sketch the current through the capacitor to scale versus time.

Solution: We use Eq. (6.28) to find the current. In Fig. 6.14, $v_C(t) = 10^5(t - 10^{-6})$ V when t changes from 1 to 2 μ s, and $v_C(t) = 0$ at $t < 1$ μ s. At $t > 2$ μ s, the voltage is 100 mV. Therefore, the current is found in the form: $i_C(t) = 10^{-4} \times 10^5 = 10$ A when t changes from 1 to 2 μ s and $i_C(t) = 0$ otherwise. The result is shown in Fig. 6.14 by a dashed curve. We observe a strong current spike when the voltage across the capacitor changes rapidly, and we observe no current flow when the voltage across the capacitor remains constant.

Note that the relatively small voltage on the order of 100 mV in Fig. 6.14 leads to a very large current spike of 10 A(!) through the capacitance. The key point here is that the current

increase is due to the *rapid* change in voltage. Such a change can be created when the capacitor discharges through a small resistance. This is the reason why capacitors are routinely employed to deliver large currents, or high power levels, for a very short period of time. The high currents are common in motor starting circuits, in electronic flashes, in solenoids, and in various electromagnetic propulsion systems.

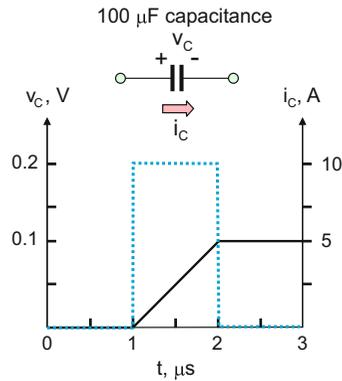


Fig. 6.14. Applied voltage across the capacitor (*solid curve*) and resulting current (*dashed curve*).

The capacitor is charged with an electric current. The voltage across the capacitor, from Eqs. (6.28) and (6.29), is given by

$$v_C = \frac{1}{C} \int_0^t i_C(t') dt' = \frac{q_C(t)}{C} \quad (6.30)$$

Equation (6.30) tells us that the capacitor voltage is equal to zero at the initial time, i.e., at $t = 0$. Once the current $i_C(t)$ is known, the voltage at any point in time is obtained by carrying out the integration in Eq. (6.30). At any time instant, the voltage is equal to the instantaneous stored charge $q_C(t)$ divided by capacitance. The current in Eq. (6.30) is either predefined or found from circuit considerations. The example that follows illustrates voltage calculations.

Example 6.3: A 1-μF capacitor is charged with an electric current, $i_C(t) = 1 \cdot t$ [mA]. The capacitance voltage is equal to zero at the initial time instance $t = 0$. When will the capacitor be charged to 10 V?

Solution: The integration yields

$$v_C = \frac{1}{C} \int_0^t i_C(t') dt' = \frac{0.001}{C} t^2 = 10 \Rightarrow t = 0.1 \text{ s} \quad (6.31)$$

6.2.3 Dynamic Behavior of Inductance

We use the passive reference configuration for the inductance in Fig. 6.15a. Lowercase letters denote time-varying voltage and current. The dynamic behavior of the inductance is described by the well-known voltage-to-current relation (*dynamic equation*), which plays the role of “Ohm’s law” for the inductance:

$$v_L = L \frac{di_L}{dt} \tag{6.32}$$

Equation (6.32) follows from the inductance definition, $\Phi = Li_L$, given in the previous section. We obtain Eq. (6.32) after differentiation and using the Faraday’s law of induction for the time derivative of the magnetic flux (the *plus* sign is used in Faraday’s law to be consistent with the passive reference configuration):

$$\frac{d\Phi(t)}{dt} = v_L \tag{6.33}$$

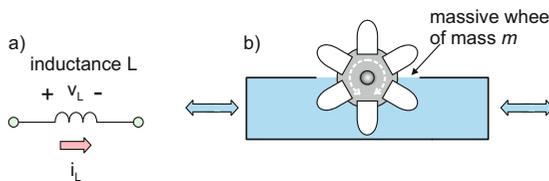


Fig. 6.15. Passive reference configuration for inductance and its fluid mechanics analogy.

A fluid mechanics analogy of the dynamic inductance effect is given here in terms of alternating current, which corresponds to alternating fluid motion in Fig. 6.15b. A massive wheel with rotational inertia in Fig. 6.15b represents inductance. The inductance value, L , corresponds to the *mechanical mass* m of the wheel. When $m \rightarrow \infty$ or $L \rightarrow 0$, the wheel does not responds to fluid oscillations and blocks the alternating fluid flow entirely. In the opposite case ($m \rightarrow 0$ or $L \rightarrow \infty$), the wheel has no effect on the fluid flow. Intermediate cases correspond to a partial blocking.

Example 6.4: The current through a 2-mH inductor is shown in Fig. 6.16 by a solid curve. At $t = 0$, the current is zero. Sketch the voltage across the inductance to scale versus time.

Solution: We use Eq. (6.32) to find the voltage across the inductance. In Fig. 6.16, $i_L(t) = 10^3(t - 10^{-6})$ A when t changes from 1 to 2 μ s and $i_L(t) = 0$ at $t < 1 \mu$ s. At $t > 2 \mu$ s, the current is 1 mA. Therefore, the voltage is found in the form: $v_L(t) = 2 \times 10^{-3} \times 10^3 = 2$ V when t changes from 1 to 2 μ s and $v_L(t) = 0$ otherwise. The result is shown in Fig. 6.16 by a dashed curve.

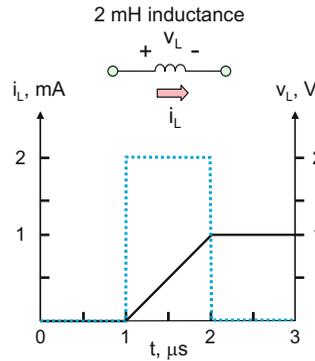


Fig. 6.16. Impressed current through an inductance (*solid curve*) and resulting voltage across the inductance (*dashed curve*).

Note that relatively small current, on the order of 1 mA, leads to a large voltage spike of 2 V across the inductance in Fig. 6.16. The key point here is again the *rapid* change in the current. If the current in the present example were on the order of 1 A, a voltage spike of 2000 V would be observed. This is the reason why inductors are routinely used to boost the voltage to a higher level. These high voltages are common in electric and electronic ignition systems including the most common car ignition plug. From Eqs. (6.32) and (6.33) the current through the inductance is given by

$$i_L = \frac{1}{L} \int_0^t v_L(t') dt' = \frac{\Phi(t)}{L} \quad (6.34)$$

Equation (6.34) implies that the current is equal to zero at the initial time, i.e., at $t = 0$. Once the voltage is known as a function of time, the current through the inductance at any time moment is obtained by the calculation of the integral in Eq. (6.34). At any time instant, the current is equal to the instantaneous magnetic flux $\Phi(t)$ divided by inductance.

Example 6.5: A 1-mH inductor is subject to applied voltage, $v_L(t) = 1 \cdot t$ [mV]. The inductance current is equal to zero at the initial time instance $t = 0$. When will the magnetic-field energy stored in the inductance reach 1 J?

Solution: The integration in Eq. (6.34) and using Eq. (6.18) for the energy stored in the inductor yield

$$i_L = \frac{1}{L} \int_0^t v_C(t') dt' = \frac{0.001}{L} t^2 = t^2 \text{ [A]} \Rightarrow \frac{1}{2} L (t^2)^2 = 1 \text{ J} \Rightarrow t = 6.7 \text{ s} \quad (6.35)$$

So, if the capacitance is associated with a spring, the inductance is associated with the mass, and the resistance is associated with a dash pot (damping element), then the entire electric circuit containing dynamic elements is nothing else but a mechanical system. Is this correct? Clearly the same analysis methods are applicable to both systems, electrical and mechanical! The model of an entire building in terms of lumped mechanical elements is in theory the same as the model of a complicated electric circuit. Both models can be analyzed by using the theory of linear systems, and both models follow the same control theory. A more difficult issue is related to nonlinear circuit elements.

6.2.4 Instantaneous Energy and Power of Dynamic Circuit Elements

An elegant derivation of the energy stored in a capacitance can also be obtained by integrating the power delivered (or taken) by the capacitance. The instantaneous electric power $p_C(t)$ can be written in the form

$$p_C(t) = v_C i_C = v_C C \frac{dv_C}{dt} = \frac{1}{2} C \frac{dv_C^2}{dt} \quad (6.36)$$

The stored energy is then the time integration of the power, i.e.,

$$E_C(t) = \int_0^t p_C(t') dt' = \int_0^t \frac{1}{2} C \frac{dv_C^2}{dt'} dt' = \frac{1}{2} C [v_C^2(t) - v_C^2(0)] \quad (6.37)$$

where the lower limit, $v_C(0)$, is the initial state of the capacitance. Suppose that $v_C(0) = 0$, i.e., the capacitance is initially uncharged and has zero stored energy. Then,

$$E_C(t) = \frac{1}{2} C v_C^2(t) \quad (6.38)$$

Equation (6.38) is the formal proof of the corresponding static result, Eq. (6.3), postulated in the previous section. We can derive the energy stored in the inductance using the same method—by integrating the power. The instantaneous power supplied to or obtained from the inductance has the form

$$p_L(t) = v_L i_L = i_L L \frac{di_L}{dt} = \frac{1}{2} L \frac{di_L^2}{dt} \quad (6.39)$$

The energy stored in the inductance is the integral of Eq. (6.39), i.e.,

$$E_L(t) = \int_0^t p_L(t') dt' = \int_0^t \frac{1}{2} L \frac{di_L^2}{dt'} dt' = \frac{1}{2} L [i_L^2(t) - i_L^2(0)] \quad (6.40)$$

Suppose that $i_L(0) = 0$, i.e., the inductance is initially “uncharged” or has no stored energy. Then,

$$E_L(t) = \frac{1}{2}Li_L^2(t) \quad (6.41)$$

Equation (6.41) is the formal proof of the corresponding static result, Eq. (6.18), postulated in the previous section. The series and parallel combinations of inductances and capacitances may also be analyzed using the dynamic element equations; the laws obtained in the previous section will be confirmed.

Exercise 6.8: Determine instantaneous power supplied to the capacitance in Fig. 6.14 at A. $t = 2 \mu\text{s}$ and B. $t = 1 \mu\text{s}$.

Answer: 1 W and 0 W, respectively.

Exercise 6.9: Repeat the previous exercise for the inductance shown in Fig. 6.16.

Answer: 0.002 W and 0 W, respectively.

6.2.5 DC Steady State

According to Eq. (6.28) when voltage across the capacitance does not change with time, the capacitance becomes an *open circuit* (no current) under DC steady-state condition, i.e.,

$$\frac{dv_C}{dt} = 0 \Rightarrow i_C = C \frac{dv_C}{dt} = 0 \quad (6.42)$$

This is to be expected since a DC current cannot flow through empty space between two capacitor plates. Similarly, according to Eq. (6.32) the inductance becomes a *short circuit* for the DC steady state when current across the inductance does not change in time, i.e.,

$$\frac{di_L}{dt} = 0 \Rightarrow v_L = L \frac{di_L}{dt} = 0 \quad (6.43)$$

In other words, there is no voltage drop across a (long) bent piece of wire, which is the inductor, for DC currents. Equations (6.36) and (6.37) allow us to establish the behavior of *any transient electric circuit* in the long run, after the circuit behavior has been stabilized. The *transient circuit* is a circuit with dynamic elements and a switch. Figure 6.17 shows one such circuit that consists of a number of dynamic (and static) elements and a switch. The switch connects the voltage source to the rest of the circuit as the switch closes at $t \rightarrow 0$.

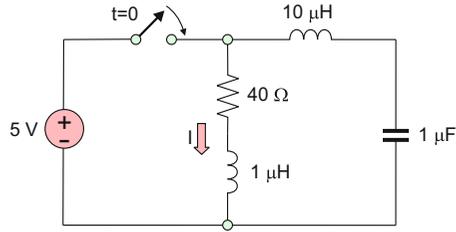


Fig. 6.17. A circuit is used to study the DC steady state. The switch closes at $t = 0$. The behavior of the circuit at $t \rightarrow \infty$ is sought after the circuit has stabilized.

Example 6.6: Find current I in Fig. 6.17 at $t \rightarrow \infty$, i.e., under DC steady state.

Solution: Immediately after the switch in Fig. 6.17 closes, the voltages and currents in the circuit may be subject to a complicated response. In particular the voltage across certain dynamic elements may be higher than the voltage of the power supply of 5 V. However, in the long run as $t \rightarrow \infty$, the circuit behavior stabilizes and we reach the DC steady state. The capacitance in Fig. 6.17 becomes an open circuit and may be ignored. Both inductances can be replaced by a wire (short circuit). The resulting DC circuit is shown in Fig. 6.18. Thus, we obtain

$$I = \frac{5 \text{ V}}{40 \text{ } \Omega} = 125 \text{ mA} \quad (6.44)$$

Other more complicated circuits can be analyzed in exactly the same way.

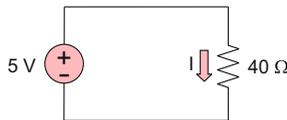


Fig. 6.18. A DC equivalent of the circuit in Fig. 6.17 under steady-state conditions.

6.2.6 Behavior at Very High Frequencies

At very high frequencies, the behavior of the two dynamic circuit elements is exactly the *opposite*: the capacitance becomes a *short* circuit, whereas the inductance becomes an *open* circuit. To establish this fact, we can either use the fluid mechanics analogies or the dynamic equations themselves. For example, the inertia of a massive wheel (inductance) will prevent any very fast movements of it so that the oscillating fluid flow will be entirely blocked when the oscillation frequency tends to infinity. On the other hand, the forces on a flexible membrane of zero mass will be so high for a rapidly oscillating fluid flow that its finite stiffness no longer matters. The membrane will simply be moving along with the fluid, which means the full transmission through its counterpart—the capacitance.

The behavior of dynamic elements at very high frequencies is exactly as important as the behavior at DC; it will be studied quantitatively in Chapter 9.

Example 6.7: Illustrate how is the capacitance becoming a short circuit at very high frequencies using the capacitor's dynamic equation as a starting point.

Solution: Assume that there is a periodic current with the amplitude of 1 A, $i_C(t) = 1 \text{ A} \cos \omega t$ through a 10- μF capacitance. The resulting capacitor voltage is given by Eq. (6.30), $v_C(t) = 1/(\omega C) \sin \omega t$. When $\omega = 10^8 \text{ rad/s}$, the capacitor voltage has the amplitude of 1 mV. This small voltage approximately corresponds to a short circuit. When ω increases, the voltage amplitude is reduced even further.

Section 6.3 Application Circuits Highlighting Dynamic Behavior

6.3.1 Bypass Capacitor

Let us consider the circuit shown in Fig. 6.19a. It includes a voltage source represented by its Thévenin equivalent and a load represented by its equivalent resistance R_L . The source generates a voltage in the form of a (large) DC component V_S and superimposed (small) AC signal $v_s(t)$. This setup could model a nonideal DC voltage power supply, which does not create the exactly DC voltage. In fact, a weak AC component may be present. This AC component (also called the *noise component*) has a frequency of either $60 \times n$ Hz, where n is an integer (USA, Canada, parts of South America, Saudi Arabia, etc.), or $50 \times n$ Hz (the rest of the world) and appears due to a not quite perfect rectification of the primary AC power. As an aside, switching power supplies create noise spikes at much higher frequencies. The weak AC component may lead to circuit oscillations, especially when dealing with high-gain amplifiers. It therefore should be removed from the load, or “filtered out” as engineers often say. The idea is to use a capacitor C in *parallel* with the (imperfect) power supply and in parallel with the load, the so-called bypass capacitor. This capacitor ideally becomes a short circuit for the high-frequency noise component of the source and shorts it out (or bypasses). The corresponding circuit diagram is shown in Fig. 6.19b. An electrolytic capacitor is typically used as the bypass capacitor.

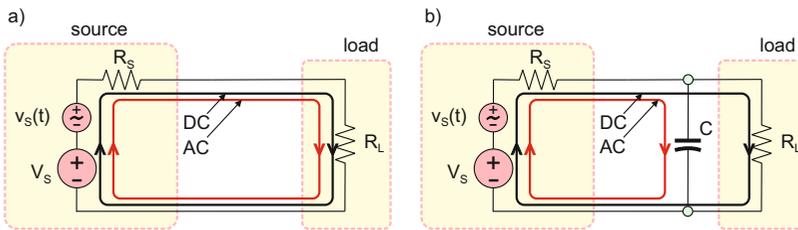


Fig. 6.19. Model of a voltage source connected to a load with a bypass capacitor.

In many cases, the undesired noise source in a circuit is not a low-frequency noise source of a nonideal power supply, but rather a high-frequency noise generator. Examples include high-speed DC motors, analog-to-digital converters, and other digital circuits. Radio-frequency (RF) high-speed amplifiers are also very sensitive to RF noise that is created by connectors and wires which can act like antennas. By placing a bypass capacitor as closely as possible to the power supply pins of every chip, such RF noise sources may be eliminated. Bypass capacitors are so prevalent that they are encountered in virtually *every* working piece of electronic equipment.

Example 6.8: Explain the operation of bypassing a DC motor.

Solution: A DC motor with a *bypass capacitor* is also described by the model shown in Fig. 6.19b. In this case, the source voltage $V_S + v_S(t)$ becomes the induced electromotive force (*emf*), E , of the motor. The induced emf is still a DC voltage but with quite a significant high-frequency noise component created by the spinning rotor comprised of a finite number of individual switched coils. The source resistance becomes the armature and brush resistance R_M , that is, $R_S = R_M$. The load resistance R_L may, for example, be the oscilloscope resistance. We consider a small DC fan motor directly connected to a 5-V power supply shown in Fig. 6.20. The motor creates a substantial high-frequency noise seen on the oscilloscope in Fig. 6.20, left (with 100 mV per division resolution). The oscilloscope measures the voltage across the motor, which is the 5-V DC component plus the noise component. Once a 1000- μF capacitor is connected in parallel with the motor (one may call it a *shunt capacitor*), the resulting voltage becomes a highly stable 5-V DC (see Fig. 6.20, right).

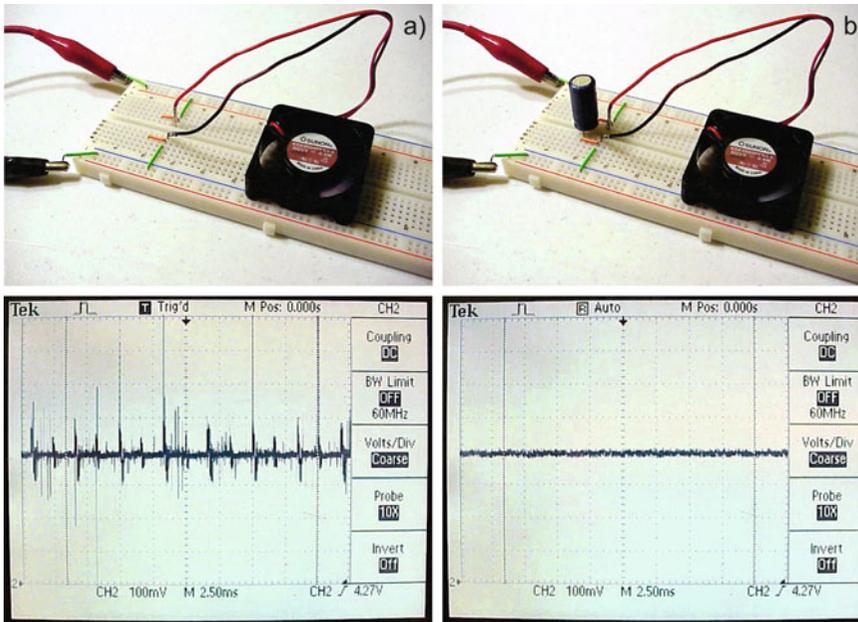


Fig. 6.20. Effect of bypass capacitor on the high-frequency noise created by a DC motor.

The bypass capacitor in Fig. 6.20 may be considered as a part of the *snubber RC circuit*, which includes a capacitance and a small series resistance. The snubber circuits are used to suppress high-voltage spikes in inductive switching systems like electric motors.

6.3.2 Blocking Capacitor

Quite often, an opposite scenario is desired—we want to block a DC component at the load—see Fig. 6.21. An example is an audio amplifier (the source) connected to a speaker (the load). The audio amplifier may generate an unwanted DC component, which may overheat the speaker coil made of a very thin wire. The idea is to use a capacitor C in series with the (imperfect) amplifier and in series with the load. This *blocking or decoupling capacitor* will block the DC current at the load as shown in Fig. 6.21b.

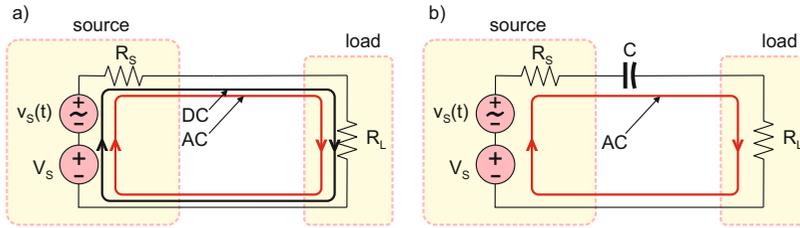


Fig. 6.21. Model of a voltage source connected to a load with a blocking capacitor.

6.3.3 Decoupling Inductor

A *decoupling inductor* is the complement of the bypass capacitor. Consider the circuit shown in Fig. 6.22a. It includes a source represented by its Norton equivalent $I_S + i_s(t)$, R_S and a load with an equivalent resistance R_L . The source generates an electric current in the form of a DC component and a superimposed AC signal. Assume that we would like to have only the direct current at the load. The idea is to use an inductor L in series with the load, the so-called decoupling inductor as shown in Fig. 6.22b. At a sufficiently high frequency, this inductor will block the AC component at the load.

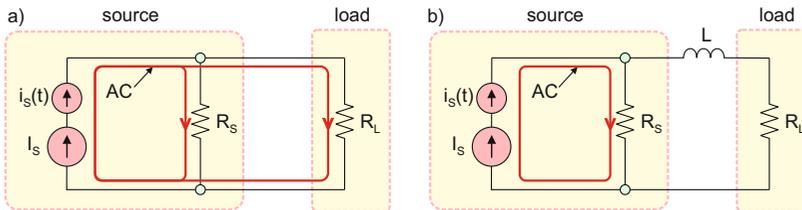


Fig. 6.22. Model of a current source connected to a load with a decoupling inductor.

A common application of the decoupling inductor is the so-called radio-frequency (RF) *inductor choke*. Here, we'd rather intend to redirect the alternating current. The inductor choke prevents the very weak alternating current received by an antenna to be lost in the low-resistance DC power supply, which powers an amplifier. Instead, it forces the current to flow directly into the input port of the amplifier. In order to model the choke effect in Fig. 6.22, we should in fact interchange the role of two resistances: we consider the load resistance as the DC supply resistance and the source resistance as the desired input resistance of the amplifier.

6.3.4 Amplifier Circuits With Dynamic Elements: Miller Integrator

Amplifier circuits with the *dynamic elements* in the *negative feedback loop* can serve different purposes. In particular, they operate as *active filters*. Here, we will introduce the operation concept and present simple examples. The *Miller integrator circuit* is an inverting-amplifier circuit considered in Chapter 5, but with the feedback resistance R_2 replaced by a capacitance C —see Fig. 6.23. Given a time-varying input voltage signal, the capacitor will conduct a current. Therefore, the negative feedback is still present, even though we now have a capacitance instead of the resistance in the feedback loop.

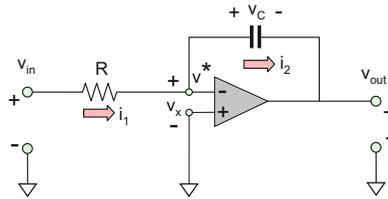


Fig. 6.23. Miller integrator circuit.

The circuit analysis uses two summing-point constraints: no current into the amplifier and zero differential input voltage. Therefore, the node voltage v^* in Fig. 6.23 is also zero. The currents $i_1 = i_R$ and $i_2 = i_C$ in Fig. 6.23 are equal to each other. This yields

$$\begin{aligned} \overbrace{C \frac{dv_C}{dt}}^{i_C} &= C \frac{d(v^* - v_{\text{out}})}{dt} = -C \frac{dv_{\text{out}}}{dt} = \overbrace{\frac{v_R}{R}}^{i_R} = \frac{v_{\text{in}} - v^*}{R} = \frac{v_{\text{in}}}{R} \Rightarrow \\ -C \frac{dv_{\text{out}}}{dt} &= \frac{v_{\text{in}}}{R} \Rightarrow v_{\text{out}} = -\frac{1}{RC} \int_0^t v_{\text{in}}(t') dt' - V_C \end{aligned} \quad (6.45)$$

where V_C is a constant (the initial voltage across the capacitor at $t = 0$). Thus, an integral of the input voltage (weighted by $-1/(RC)$) is provided at the output. Interestingly, the *time constant* τ of the integrator, $\tau = RC$, has the unit of seconds.

Example 6.9: The analog *pulse counter* is an integrator circuit shown in Fig. 6.23 that counts monopolar voltage pulses simply by integrating the input voltage as time progresses. Assume that the input to the amplifier is the voltage shown in Fig. 6.24, where every rectangular voltage pulse of 10-ms duration corresponds to a car passing through a gate. Given that $R = 10 \text{ k}\Omega$, $C = 0.1 \text{ }\mu\text{F}$, and that the initial value of the output voltage is reset to zero, how many cars should pass the gate in order to reach the output voltage threshold of -6 V ?

Solution: The time constant τ of the integrator, $\tau = RC$, is equal to 1 ms.

Example 6.9 (cont.):

According to Eq. (6.45),

$$v_{out} = -1000 \text{ s}^{-1} \sum_{n=1}^N (0.06 \text{ V} \times 0.01 \text{ s}) = -N \times 0.6 \text{ V} \tag{6.46}$$

where N is the number of pulses (cars). Equating the above expression to -6 V gives $N = 10$. The time interval between passing cars is not important.

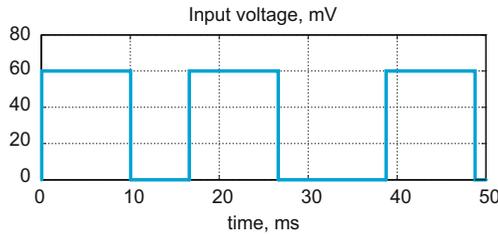


Fig. 6.24. Input signal to the amplifier in Example 6.9.

Along with Example 6.9, other applications of the Miller integrator include various wave-shaping circuits.

6.3.5 Compensated Miller Integrator

The circuit in Fig. 6.23 will not function in the laboratory, when a realistic amplifier chip is used that is different from the ideal-amplifier model. The reason is that the capacitance is equivalent to an open circuit at DC. Therefore, the feedback loop is simply missing in the Miller integrator at DC, and the entire amplifier circuit becomes a comparator with a very high open-loop gain. A small random *input offset voltage*, V_{OS} , which is present for

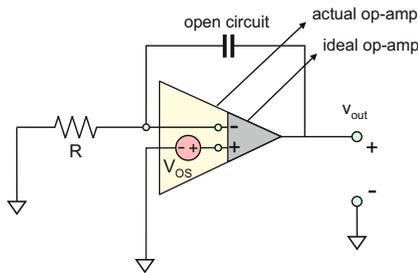


Fig. 6.25. Effect of the input offset voltage on the integrator circuit at DC.

any realistic amplifier IC, will saturate the amplifier toward one of the power rails (depending on the sign of V_{OS}) even if its input is at zero volts (grounded)—see Fig. 6.25.

Note that the input offset voltage source, V_{OS} , may be added to either amplifier terminal. Hence, the voltage across the capacitance will approach the rail voltage and the capacitance itself will become permanently charged. The *dynamics* of this process can be analyzed explicitly, starting with some initial voltage value, say $v_C = 0$ V. In this case, we are allowed to use the negative feedback. For example, given that $R = 10\text{ k}\Omega$, $C = 0.1\text{ }\mu\text{F}$, $V_{OS} = 5\text{ mV}$, it takes exactly 1 s to reach the output voltage of 5 V! A similar effect is created by input bias currents to the amplifier. To overcome this issue, a large resistance, R_F , is introduced in parallel with C in order to maintain the negative feedback at DC and discharge the capacitance as needed—see Fig. 6.26. If, for example, $R_F = 10\text{ M}\Omega$, then the capacitance will discharge over time on the order of $R_F C = 1\text{ s}$. This estimate is comparable with the estimate for the charging time. As a result, a balance will be established that results in a certain nonzero v_{out} with the output of the amplifier grounded. A further quantitative discussion may be carried out.

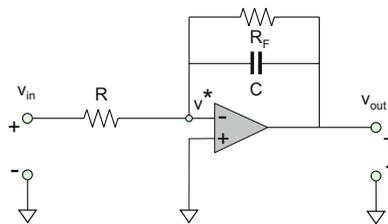


Fig. 6.26. Miller integrator improved with a large resistance, R_F , in the feedback loop.

6.3.6 Differentiator and Other Circuits

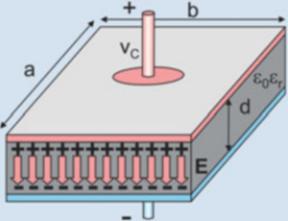
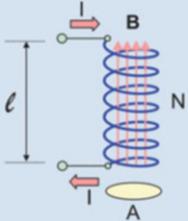
When the resistance and the capacitance in Fig. 6.23 are interchanged, a *differentiator amplifier circuit* (or *active differentiator*) is obtained; its output signal is a derivative of the input signal. The corresponding solution is studied in one of the homework problems. The differentiators are rarely used in practice since they attempt to amplify any input noise (they become “noise magnifiers”). The reason for this is an *infinitely high gain* of the amplifier circuit at *high* frequencies, when the capacitance becomes a short circuit. A small resistance added *in series* with the capacitance reduces this effect and assures the finite gain similar to the standard inverting amplifier.

Exercise 6.10: Draw an integrator circuit with an inductance instead of the capacitance.

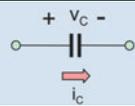
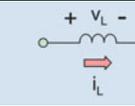
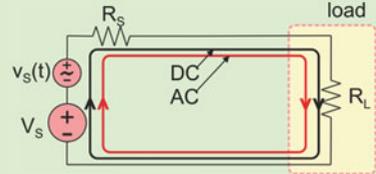
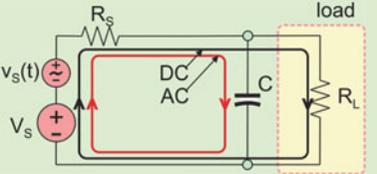
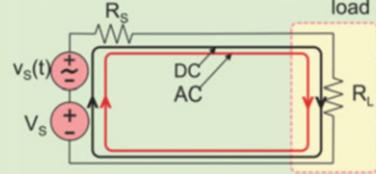
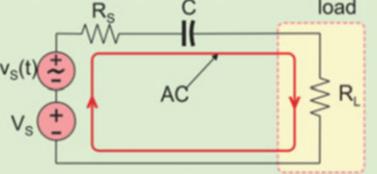
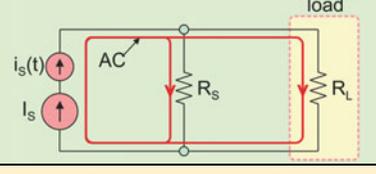
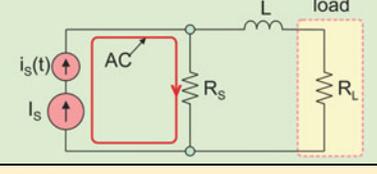
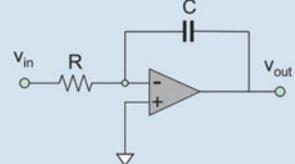
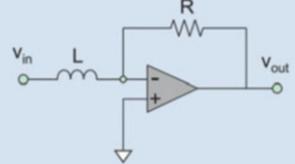
Answer: The circuit in Fig. 6.23 but with the resistance replaced by an inductance and with the capacitance replaced by the resistance.

Inductances might be used instead of capacitances; the amplifier circuits so constructed will be either a differentiator or an integrator. We again pass the corresponding analysis to the homework exercises. However, the physical inductors tend to have a significant series resistance and are more bulky. Last but not least, we may ask ourselves a question: as long as the amplifier circuits can perform multiplication, addition (or subtraction), and integration (or differentiation), can we now build an *analog computer*, which operates with analog voltages and replaces its digital counterpart at least for simple computational tasks? The answer is yes, we can. In fact, this was done a long time ago, in the mid-1960s.

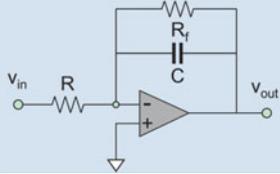
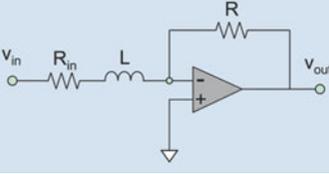
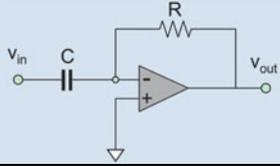
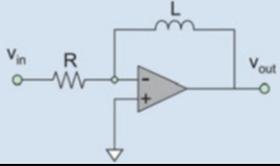
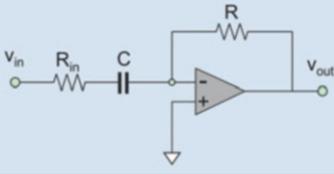
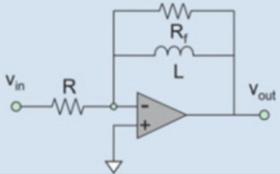
Summary

Static capacitance and inductance																														
Property	Capacitance 	Inductance 																												
Definition	$C = \frac{Q}{V} > 0$ Q —charge of either conductor; V —volt. between two conductors Units: F = C/V	$L = \frac{\Phi}{I} > 0$ Φ —magnetic flux through the circuit; I —circuit current Units: H = (V · s)/A																												
Physical meaning	Charge on either conductor produced by 1 V voltage difference between the two conductors	Magnetic flux through the circuit produced by 1 A of current in the same circuit																												
Stored energy, J (static or dyn.)	$E = \frac{1}{2} CV^2$ or $E(t) = \frac{1}{2} Cv(t)^2$	$E = \frac{1}{2} LI^2$ or $E(t) = \frac{1}{2} Li(t)^2$																												
Series/parallel combinations	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ in series $C_{eq} = C_1 + C_2 + C_3$ in parallel	$L_{eq} = L_1 + L_2 + L_3$ in series $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$ in parallel																												
Basic models (no fringing fields)	 $C = \frac{\epsilon_0 A}{d}$ or $C = \frac{\epsilon_r \epsilon_0 A}{d}$ (diel. material inside) $A = ab, \epsilon_0 = 8.854187 \times 10^{-12}$ F/m	 $L = \frac{\mu_0 AN^2}{l}$ or $L = \frac{\mu_0 \mu_r AN^2}{l}$ (closed magnetic core) $\mu_0 = 4\pi \times 10^{-7}$ H/m																												
Models with fringing																														
Capacitance of a parallel-plate square capacitor (Table 6.1)	<table border="1"> <tr> <td>d/a</td> <td>0.01</td> <td>0.1</td> <td>0.2</td> <td>0.3</td> <td>0.4</td> <td>0.5</td> </tr> <tr> <td>C_{exact}/C</td> <td>1.043</td> <td>1.307</td> <td>1.554</td> <td>1.787</td> <td>2.013</td> <td>2.237</td> </tr> <tr> <td>d/a</td> <td>0.6</td> <td>0.7</td> <td>0.8</td> <td>0.9</td> <td>1.0</td> <td></td> </tr> <tr> <td>C_{exact}/C</td> <td>2.459</td> <td>2.680</td> <td>2.901</td> <td>3.122</td> <td>3.343</td> <td></td> </tr> </table>		d/a	0.01	0.1	0.2	0.3	0.4	0.5	C_{exact}/C	1.043	1.307	1.554	1.787	2.013	2.237	d/a	0.6	0.7	0.8	0.9	1.0		C_{exact}/C	2.459	2.680	2.901	3.122	3.343	
d/a	0.01	0.1	0.2	0.3	0.4	0.5																								
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d/a	0.6	0.7	0.8	0.9	1.0																									
C_{exact}/C	2.459	2.680	2.901	3.122	3.343																									
Inductance of a finite-radius solenoid	$L_{exact} = L \left(1 - \frac{8w}{3\pi} + \frac{w^2}{2} - \frac{w^4}{4} \right), \quad w = \frac{r}{l} < 1; r$ is the radius of the coil																													
Inductance of a solenoid with a finite core	$L \approx \frac{0.5\pi\mu_0 l^* N^2}{\ln\left[\frac{l^*}{r}\right]} \left(1 - \frac{l}{2l^*} \right), \mu_r \geq 100$, see Fig. 6.12c																													

(continued)

Dynamic behavior		
Property	Capacitance	Inductance
Dynamic model- Passive ref. conf.		
Dynamic model- $v-i$ characteristic	$i_C = C \frac{dv_C}{dt}$	$v_L = L \frac{di_L}{dt}$
Dynamic model- charge and flux	$\frac{dq_C}{dt} = i_C$	$\frac{d\Phi(t)}{dt} = v_L$ (passive ref. conf.)
Behavior at DC		
Behavior at very high frequencies		
Bypassing/Decoupling		
Bypass capacitor		
Blocking capacitor		
Decoupling inductor		
Amplifier circuits with capacitor/inductor		
Property	Capacitance	Inductance
Miller Integrator (open-loop amplifier at DC)		
Time-domain operation	$\frac{dv_{out}}{dt} = -\frac{v_{in}}{\tau}, \tau = RC$	$\frac{dv_{out}}{dt} = -\frac{v_{in}}{\tau}, \tau = \frac{L}{R}$

(continued)

<p>Compensated integrator with a finite gain at DC</p>		
<p>Time-domain operation</p>	$\frac{dv_{out}}{dt} + \frac{v_{out}}{R_f C} = -\frac{v_{in}}{\tau}, \tau = RC$	$\frac{dv_{out}}{dt} + \frac{R_{in}}{L} v_{out} = -\frac{v_{in}}{\tau}, \tau = \frac{L}{R}$
<p>Differentiator circuit (infinite gain at very high frequencies)</p>		
<p>Time-domain operation</p>	$v_{out} = -\tau \frac{dv_{in}}{dt}, \tau = RC$	$v_{out} = -\tau \frac{dv_{in}}{dt}, \tau = \frac{L}{R}$
<p>Differentiator with a finite gain at very high frequencies</p>		
<p>Time-domain operation</p>	$R_{in} C \frac{dv_{out}}{dt} + v_{out} = -\tau \frac{dv_{in}}{dt}, \tau = RC$	$\frac{L}{R_f} \frac{dv_{out}}{dt} + v_{out} = -\tau \frac{dv_{in}}{dt}, \tau = L/R$

Problems

6.1 Static Capacitance and Inductance

6.1.1 Capacitance, Self-capacitance, and Capacitance to Ground

6.1.2 Application Example: ESD

Problem 6.1.

- Describe in your own words the physical meaning of capacitance.
- Suggest a way to memorize the expression for the capacitance of two conductors.
- What is approximately the self-capacitance of a human body?
- What is approximately the capacitance of a human body (to ground)?
- How does the human-body self-capacitance change in embryo pose (yoga)?

Problem 6.2. A metal square plate with the side of 10 mm has the self-capacitance $C_{\text{self}} = 0.41$ pF.

- Estimate the capacitance of a capacitor formed by two such parallel plates separated by 30 mm. Compare this value to the exact result of 0.23 pF.
- Estimate the capacitance of the plate to ground when the separation distance is 30 mm. Compare this value to the exact result of 0.43 pF.

Problem 6.3. Draw the basic electric-circuit model of a human body and specify the generic element values.

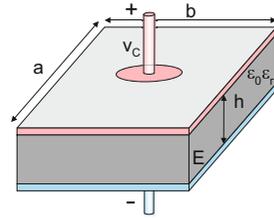
6.1.3 Parallel-Plate Capacitor

6.1.4 Capacitances in Parallel and in Series

Problem 6.4. For the parallel-plate capacitor schematically shown in the figure,

$$a = 10 \text{ cm}, b = 20 \text{ cm},$$

$$h = 1 \text{ mm}, \epsilon_r = 12$$



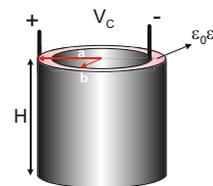
- Determine capacitance of the capacitor, C .
- Determine the electric field strength, E (in V/m), within the capacitor volume and total charge, Q , on either capacitor plate if the applied voltage is 25 V.
- Determine the electric field energy stored in the capacitor if the applied voltage is 25 V.

Problem 6.5. Solve the previous problem when the separation distance, h , between the plates is reduced to 100 μm .

Problem 6.6. For the enclosed-cylinder capacitor shown in the figure, $a = 10$ cm, $b = 9.99$ cm, $H = 5$ cm, $\epsilon_r = 16$, the electrodes are the inner and outer cylinder surfaces, respectively.

- Determine the capacitance of the capacitor, C .
- Determine the electric field strength, E (in V/m), within the capacitor volume and total charge, Q , on either capacitor plate if the applied voltage is 50 V.
- Determine the electric field energy stored in the capacitor if the applied voltage is 25 V.

Hint: The capacitance per unit area of the device is that of the parallel-plate capacitor.

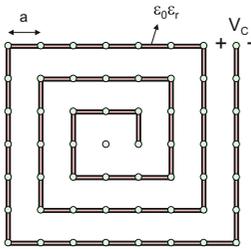


Problem 6.7. Solve the previous problem when the separation distance between the two electrodes is reduced to $25\ \mu\text{m}$.

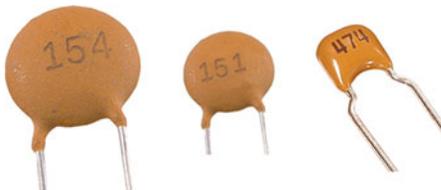
Problem 6.8. A cross section of the rolled capacitor is approximated by a rational spiral shown in the figure. Here, $a = 0.25\ \text{cm}$, $\epsilon_r = 10$, the separation distance between the two conductors is $20\ \mu\text{m}$, and the height of the entire roll is $1.5\ \text{cm}$.

- Determine capacitance of the capacitor, C .
- Determine the electric field strength, E (in V/m), within the capacitor and total charge, Q , on either capacitor plate if the applied voltage is $12\ \text{V}$.
- Determine the electric field energy stored in the capacitor if the applied voltage is $12\ \text{V}$.

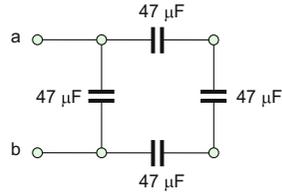
Hint: The capacitance per unit area of the device is that of the parallel-plate capacitor.



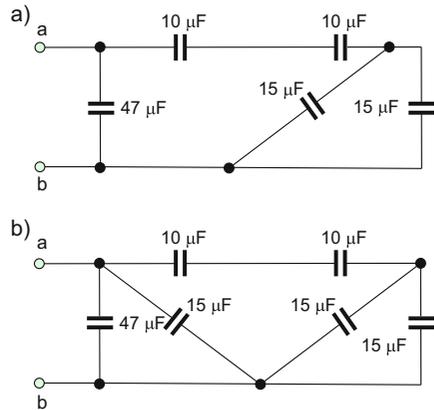
Problem 6.9. Determine the capacitance of the three leaded capacitors shown in the figure (from left to right).



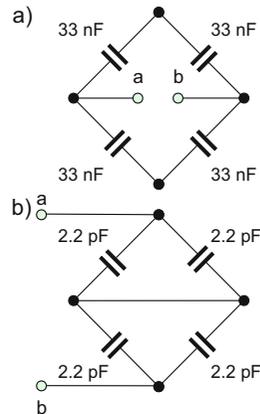
Problem 6.10. Find the equivalent capacitance for the circuit shown in the following figure.



Problem 6.11. Find the equivalent capacitance for each circuit shown in the figure below.

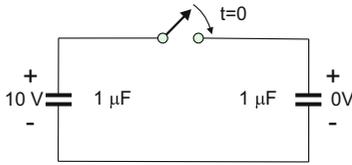


Problem 6.12. Find the equivalent capacitance for each circuit shown in the following figure.



Problem 6.13. Two $1\text{-}\mu\text{F}$ capacitances have an initial voltage of $10\ \text{V}$ and $0\ \text{V}$, respectively (before the switch is closed), as shown in the figure. Find the total electric energy stored in the system before the switch is closed. Find the

voltage across each capacitance and the total stored energy after the switch is closed. What could have happened to the missing energy?



6.1.7 Self-inductance (Inductance) and Mutual Inductance

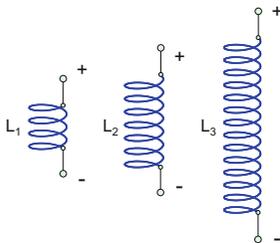
6.1.8 Inductance of a Solenoid With and Without Magnetic Core

6.1.9 Circuit Symbol. Inductances in Series and in Parallel

Problem 6.14.

- A. Describe in your own words the physical meaning of inductance.
- B. Do you think a straight wire has a certain inductance per unit length? You might want to ask the TA and/or browse the Web and present the corresponding expression (if any).

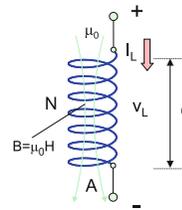
Problem 6.15. Three air-core inductors of the same cross section are shown in the following figure. The inductor length is proportional to the number of turns. Find the ratios of inductances: $L_2/L_1, L_3/L_2, L_3/L_1$.



Problem 6.16. The solenoid shown in the figure has a diameter $d = 1$ cm and a length $l = 10$ cm.

- A. Find the solenoid's inductance, L , using the common assumption $d/l \ll 1$.
- B. Determine the magnetic field energy stored within the inductance and in the

surrounding space if the applied current, I_L , is 0.5 A.



Problem 6.17.

- A. Obtain more accurate answers to the previous problem using the precise expression for the inductance of a solenoid, which is Eq. (6.22).
- B. Estimate the relative error of the common expression for solenoid's inductance, Eq. (6.21).

Problem 6.18.

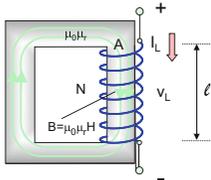
- A. Design a 100- μ H air-core inductor (determine the necessary number of turns) with a radius of 1 cm and a length of 10 cm.
- B. Determine the magnetic field energy stored in the inductor if the applied current, I_L , is 1.0 A.

Problem 6.19.

- A. Design a 0.5-mH air-core inductor (determine the necessary number of turns) having a radius of 0.5 cm and a length of 10 cm.
- B. Determine the magnetic field energy stored in the inductor if the applied current, I_L , is 1.0 A.

Problem 6.20. The solenoid shown in the figure has diameter, $d = 1$ cm, length, $l = 10$ cm, and a closed magnetic core of the same circular cross section with $\mu_r = 1000$.

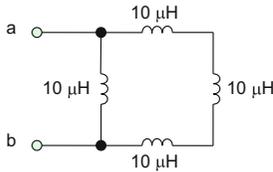
- A. Find the solenoid's inductance, L , using the common assumption $d/l \ll 1$.
- B. Determine the magnetic field energy stored in the inductor if the applied current, I_L , is 0.5 A.
- C. Compare two of your answers to the answers in Problem 6.16.
- D. Determine voltage across the inductor assuming the ideal (zero-resistance) wire.



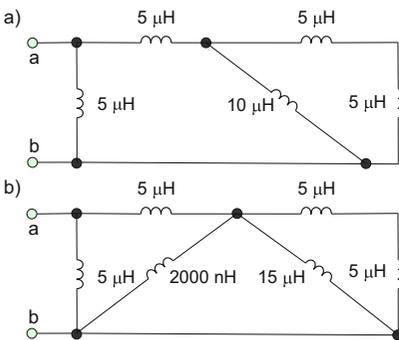
Problem 6.21.

- A. Obtain a more accurate solution to the previous problem using the precise expression for the inductance of a solenoid, Eq. (6.22). To do so, propose your own modification of this expression to include the effect of the magnetic core.
- B. Estimate the relative error of the simplified expression for the inductance, Eq. (6.21).

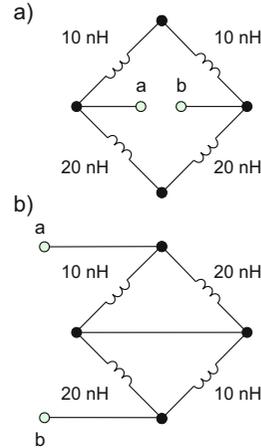
Problem 6.22. Find the equivalent inductance for the circuit shown in the following figure.



Problem 6.23. Find the equivalent inductance for circuits shown in the figure that follows.



Problem 6.24. Find the equivalent inductance for circuits shown in the figure.



Problem 6.25. For the inductor shown in Fig. 6.11, $l^* = 2l = 10$ cm and $r = 0.75$ cm. How do the inductances with the magnetic core (with $\mu_r \geq 100$) and without the core compare to each other?

Problem 6.26. Repeat the previous problem with $l^* = 15$ cm, but still $2l = 10$ cm. The other parameters are the same.

6.2 Dynamic Behavior of Capacitance and Inductance

- 6.2.2 Dynamic Behavior of Capacitance
- 6.2.3 Dynamic Behavior of Inductance
- 6.2.4 Instantaneous Energy and Power of Dynamic Circuit Elements

Problem 6.27.

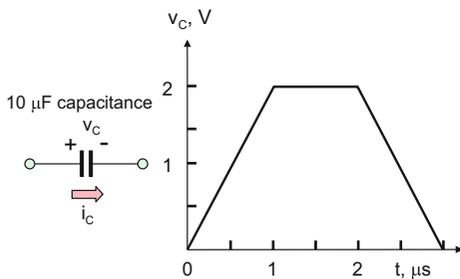
- A. Sketch the circuit symbol for the capacitance and the inductance; label the corresponding voltages and currents.
- B. Write two dynamic equations for the capacitance and the inductance which relate the voltages and currents.
- C. Express the units for the inductance and capacitance through volts and amperes.

D. Which capacitance and inductance values are typical in electronic circuits?

Problem 6.28.

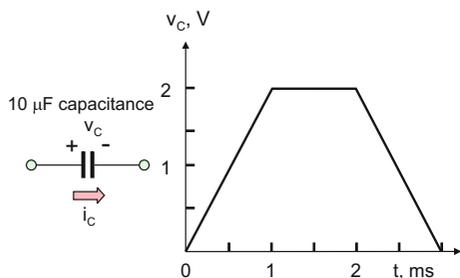
- A. Sketch the hydrodynamic analogies for the capacitance and the inductance.
- B. Which mechanical quantities are associated with the capacitance and inductance?

Problem 6.29. The voltage across a 10- μF capacitance is shown in the following figure. At $t = 0$, the voltage is zero. Sketch the current through the capacitance to scale versus time.



Problem 6.30. Repeat the previous problem for the voltage shown in the following figure.

- A. How is the solution different from the previous problem?
- B. For creating large currents, should the voltage across the capacitance change slowly or quickly?



Problem 6.31. A 10- μF capacitance is charged by the current $i_C = 1\text{mA} \cdot \cos^2(1000t)$. At $t = 0$, the capacitance voltage is zero.

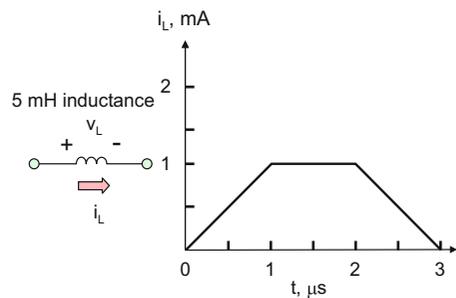
- A. Using software of your choice (MATLAB is recommended), sketch

the capacitance voltage to scale versus time over the interval from 0 to 0.05sec.

- B. How much time is approximately necessary to charge the capacitance to 1.5 V?
- C. Solve the same problem analytically.

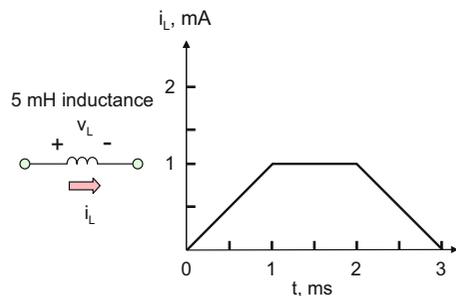
Problem 6.32. Repeat the previous problem with the capacitance changed to 1 μF . Does the 10-V charging time increase or decrease?

Problem 6.33. The current through a 5-mH inductance is shown in the following figure. At $t = 0$, the current is zero. Sketch the voltage across the inductance to scale versus time.



Problem 6.34. Repeat the previous problem for the current shown in the figure.

- A. How is the solution different from the previous problem?
- B. For creating large voltages, should the current through the inductance change slowly or quickly?



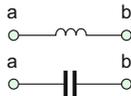
Problem 6.35. The voltage across a 3-mH inductance is given by $v_L(t) = 10\text{mV} \cdot \cos^2(1000t)$. The current through the

inductance is equal to zero at the initial time $t = 0$. Using software of your choice (MATLAB is recommended), plot the current through the inductance for $t \leq 50$ ms.

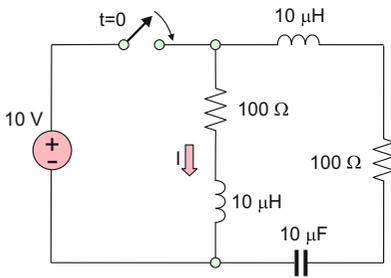
6.2.5 DC Steady State

Problem 6.36.

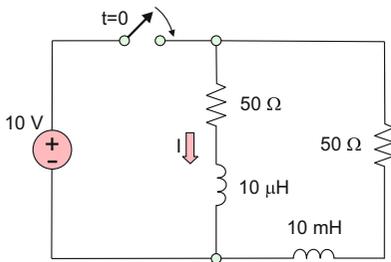
- A. Draw the equivalent circuit for the inductance shown in the figure at DC.
- B. Repeat for the capacitance.



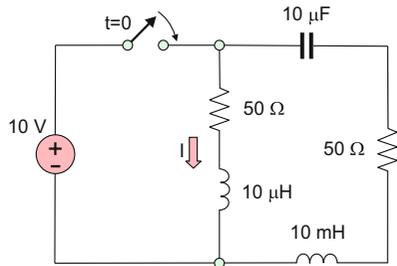
Problem 6.37. Find current I in the circuit shown in the figure that follows at $t \rightarrow \infty$, in the DC steady state. The switch closes at $t = 0$.



Problem 6.38. Find current I in the circuit shown in the figure at $t \rightarrow \infty$, in the DC steady state. The switch closes at $t = 0$.

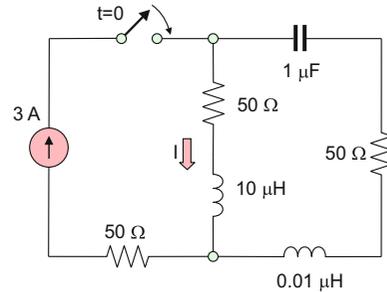


Problem 6.39. Find current I in the circuit shown in the following figure at $t \rightarrow \infty$, in the DC steady state. The switch closes at $t = 0$.

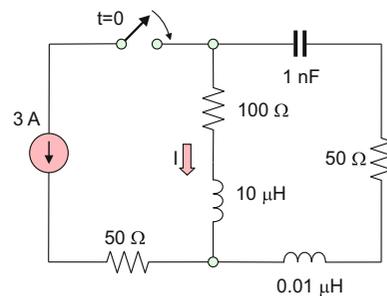


Problem 6.40.

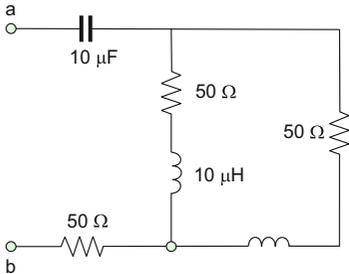
- A. Find current I in the circuit shown in the figure at $t \rightarrow \infty$, in the DC steady state. The switch closes at $t = 0$.
- B. Find voltage across the capacitance in the DC steady state and label its polarity in the figure.



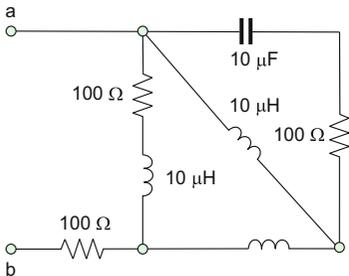
Problem 6.41. Find the voltage across the capacitance in the DC steady state and label its polarity in the figure at $t \rightarrow \infty$, in the DC steady state. The switch opens at $t = 0$.



Problem 6.42. Determine the equivalent resistance between terminals a and b for the circuit shown in the following figure in the DC steady state.



Problem 6.43. Determine the equivalent resistance between terminals a and b for the circuit shown in the following figure in the DC steady state.



6.3 Application Circuits Highlighting Dynamic Behavior

6.3.1 Bypass Capacitor

6.3.2 Blocking Capacitor

6.3.3 Decoupling Inductor

Problem 6.44. Describe the purpose of a

- A. Bypass capacitor
- B. Blocking capacitor
- C. Decoupling inductor

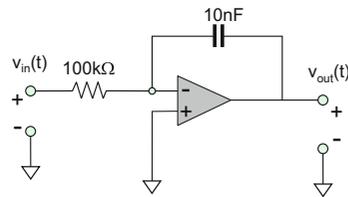
in your own words. Specify the placement of each component: in series or in parallel with the source.

6.3.4 Amplifier Circuits with Dynamic Elements: Miller Integrator

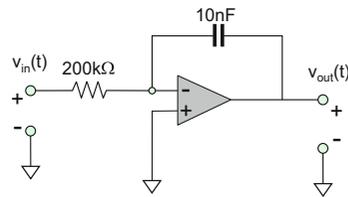
6.3.5 Compensated Miller Integrator

6.3.6 Differentiator and Other Circuits

Problem 6.45. The input voltage to the Miller integrator circuit with the ideal amplifier shown in the figure is a series of rectangular voltage pulses. Each is 50 mV tall and 8 ms wide. Given that the initial value of the output voltage is zero, how many voltage pulses are necessary to reach the negative output voltage threshold of -8 V?

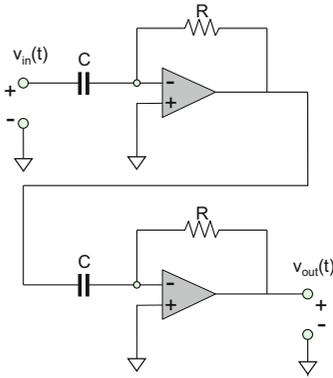


Problem 6.46. The input voltage to the Miller integrator circuit with the ideal amplifier shown in the figure is a series of rectangular voltage pulses. Each is 50 mV tall and 16 ms wide. Given that the initial value of the output voltage is zero, how many voltage pulses are necessary to reach the negative output voltage threshold of -9.2 V?

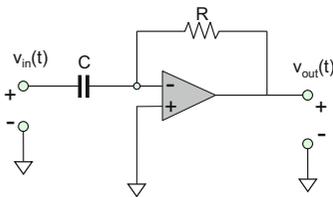


Problem 6.47. How would you modify the circuit to the previous problem when the positive threshold voltage of $+9.2$ V should be reached at the output?

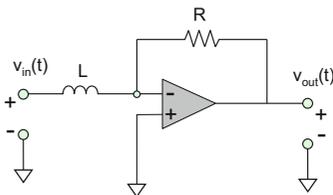
Problem 6.48. For the circuit shown in the following figure, express the output voltage, $v_{out}(t)$, as a function of time in terms of the input voltage, $v_{in}(t)$, and circuit parameters R , C . Assume the ideal amplifier.



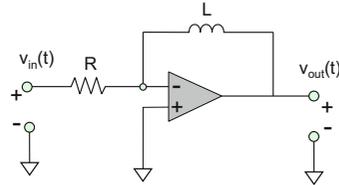
Problem 6.49. For the circuit shown in the following figure, express the output voltage, $v_{out}(t)$, as a function of time in terms of the input voltage, $v_{in}(t)$, and circuit parameters R , C . Assume ideal amplifiers.



Problem 6.50. For the circuit shown in the following figure, express the output voltage, $v_{out}(t)$, as a function of time in terms of the input voltage, $v_{in}(t)$, and circuit parameters R , L . Assume ideal amplifier.

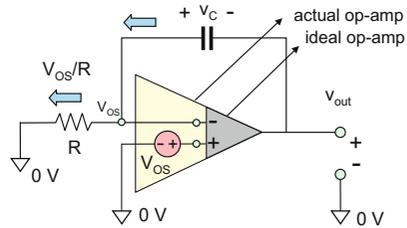


Problem 6.51. For the circuit shown in the following figure, express the output voltage, $v_{out}(t)$, as a function of time in terms of the input voltage, $v_{in}(t)$, and circuit parameters R , L . Assume ideal amplifier.



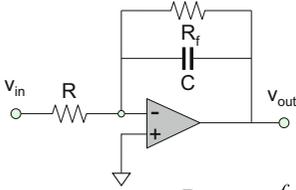
Problem 6.52. Explain why is the Miller integrator typically used with a shunt resistance, R_F .

Problem 6.53. For the circuit shown in the figure, assume that the voltage across the capacitance at $t = 0$ is zero. Also assume that the negative feedback is present. Derive the dynamic expression for $v_C(t)$ at any positive time instance.

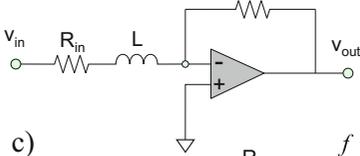


Problem 6.54. Establish the gain of amplifier circuits shown in the figure that follows.

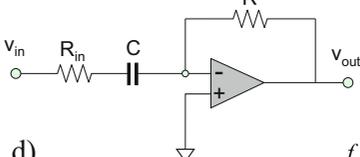
a) $f \rightarrow 0, \text{DC}$



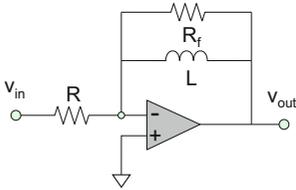
b) $f \rightarrow 0, \text{DC}$



c) $f \rightarrow \infty$



d) $f \rightarrow \infty$



Problem 6.55. For two circuits shown in the figure that follows, obtain an analytical expression for the output voltage as a function of time and circuit parameters when the input voltage has the form $v_{in}(t) = 1 \exp(-\alpha t)$ [mV].

