Chapter 16: Electronic Diode and Diode Circuits

Overview

Prerequisites:
- Knowledge of basic circuit analysis

Objectives of Section 16.1:
- Understand basic principles of diode operation and its mechanical analogy
- Learn diode symbols
- Learn three operation regions of a diode
- Become familiar with common diode types and their functions

Objectives of Section 16.2:
- Solve diode circuits using the ideal-diode model
- Solve diode circuits using the constant-voltage-drop model
- Solve diode circuits using the exponential model and load-line method
- Learn about small-signal diode model and incremental resistance
- Establish applications of the superposition principle to diode circuits

Objectives of Section 16.3:
- Establish the concept of voltage reference/voltage regulator
- Analyze the voltage regulator with the Zener diode
- Become familiar with three major rectifier types: half-wave rectifier, full-wave rectifier, and bridge rectifier
- Study selected applications of diode rectifier circuits

Objectives of Section 16.4:
- Become familiar with clamper and voltage multiplier diode circuits
- Learn the function and topology of diode clipper circuits including hard and soft clippers

Application Examples:
- Automotive battery-charging system
- Envelope detector circuit for AM radio

S.N. Makarov et al., *Practical Electrical Engineering*, DOI 10.1007/978-3-319-21173-2_16

Keywords:
Small-signal diode, Forward-bias diode operation region, Reverse-bias diode operation region, Breakdown diode operation region, Zener breakdown voltage, Shockley's equation, Shockley's ideal-diode equation, Thermal voltage, Saturation current, Ideality factor of the diode, Built-in voltage of pn-junction, Switching diode, Maximum operating frequency, Varactor diode, Tuning diode, Zener diode, Schottky barrier diode, PIN diode, Photodiode, LED, Ideal diode, Ideal-diode model, Method of assumed states, Constant-voltage-drop model, Exponential diode model, Load line, Load-line method, DC operating point, Quiescent point, Bias point, Iterative method, Linearization procedure, Small-signal diode resistance, Incremental resistance, Small-signal diode model, Superposition principle for small-signal diode model, Method of Thévenin equivalent for diode circuits, Diode voltage reference, Diode voltage regulator, Forward-bias voltage regulator, Zener voltage regulator, Piecewise-linear diode model, Dynamic Zener resistance, Incremental Zener resistance, Line regulation, Half-wave diode rectifier, Conduction angle, Full-wave diode rectifier, Bridge diode rectifier, Envelope detector circuit, Peak detector circuit, Amplitude envelope, Modulating signal, Modulation depth, Demodulation, Linear region of operation of the envelope detector, Square-law region of operation of the envelope detector, Radio-frequency power meter, Diode clamper circuit, DC restorer circuit, Diode voltage multiplier, Diode voltage doubler, Diode voltage tripler, Diode voltage quadrupler, Positive clipper, Negative clipper, Double clipper, ESD protection clipper, Zener diode clipper, Hard limiter, Soft limiter

Section 16.1 Diode Operation and Classification

16.1.1 Circuit Symbol and Terminals

An electronic diode is the most basic *two-terminal* semiconductor device. The common diode is simply a sealed semiconductor silicon *pn-junction* shown in Fig. 16.1a. The diode symbol shown in Fig. 16.1b has a prominent arrow that indicates the proper direction of the current. The positive side (where the current enters and the voltage is more positive) is called *anode* and the negative side (where the current leaves) is called *cathode*. This terminology has been adopted from vacuum tubes, which were used as diodes in the past (circa 1910–1960). Thus, the diode, in contrast to the resistor, capacitor, and inductor, is a *polarized* device. The current i_D and the voltage across the diode υ_D follow the passive reference configuration, seen in Fig. 16.1b, since the diode is a passive device. The voltage υ_D is also called the *terminal voltage*. The general-purpose silicon and germanium diodes usually have one or two prominent black rings printed on their package terminations indicating the diode's cathode as depicted in Fig. 16.1c

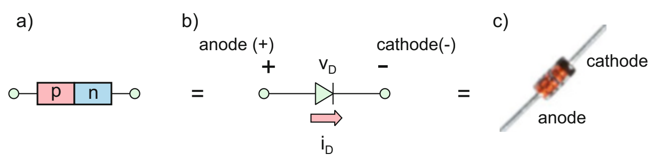


Fig. 16.1. (**a**) Internal composition of a Si diode, (**b**) circuit symbol, and (**c**) physical counterpart.

16.1.2 Three Regions of Operation

Figure 16.2 demonstrates an experimentally measured *volt–ampere* (or *v–i*) characteristic of a common *small-signal* (i.e., low-power) 1N4148 silicon *switching diode*. Such diodes are manufactured by many semiconductor companies. In Fig. 16.2, we plot the diode current i_D versus the voltage across the diode, υ_D. A closer look at Fig. 16.2 indicates that the diode has three distinct *operating regions*:

1. The *forward-bias region* characterized by the inequality $\upsilon_D > 0$
2. The *reverse-bias region* characterized by the inequality $-V_{Z0} < \upsilon_D < 0$
3. The *breakdown region* characterized by the inequality $\upsilon < -V_{Z0}$.

The two vertical asymptotes shown in Fig. 16.2 correspond to *Zener breakdown voltage* V_{Z0} and to a certain threshold voltage V_{D0}, which is close to the *built-in voltage of the pn-junction*, V_{BI}. The three regions of operation and the associated constants will be described in detail in the following sections.

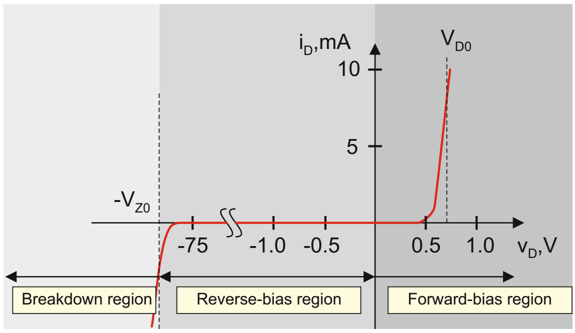


Fig. 16.2. Measured *v–i* characteristic of a 1N4148 Si switching diode to scale.

16.1.3 Mechanical Analogy of Diode Operation

The picture in Fig. 16.3 serves as a mechanical analogy for the diode, highlighting its major function: a one-way valve. When the mechanical pressure (equivalently the electric voltage) is sufficiently higher on the left side, the valve is open and a fluid (equivalently the electric current) flows from left to right at any speed. This corresponds to the nearly vertical *v–i* slope in the forward-bias region. However, a certain pressure drop (voltage of 0.7 V) is consumed by the spring attached to the valve. When the pressure gradient is opposite, the valve is closed and there is no fluid flow (no current). This situation corresponds to the reverse-bias region of operation.

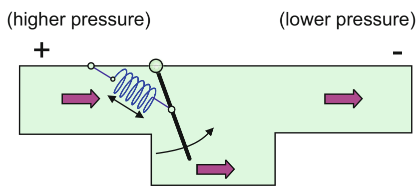


Fig. 16.3. Fluid flow analogy of the pn-junction behavior of an ideal diode—a flapper valve with a resistive force in the forward direction.

16.1.4 Forward-Bias Region: Switching Diode

The forward-bias region is characterized by positive terminal voltages. In this region, the diode current closely follows *Shockley's equation*:

$$i_D = I_S \left[\exp\left(\frac{v_D}{n V_T} \right) - 1 \right] \tag{16.1a}$$

This *Shockley's ideal-diode equation* (without the factor n) has been derived analytically based on the pn-junction equations. This equation is strongly temperature dependent. The reference is usually *room temperature* of 20 °C or 25 °C (*cabinet temperature*).

In Eq. (16.1a), the constant I_S[A] is the diode *saturation current*. For small-signal 1N4148-type diodes, a theoretical value of $I_S \approx 10^{-14}$ A or less at room temperature may be calculated. For real diodes, this value is much higher. The constant

$$V_T = kT/q \tag{16.1b}$$

with the unit of volts is called the *thermal voltage*. The dimensionless constant $1 \leq n$ is the *ideality factor of the diode*, which accounts for the deviation of the real diode from Shockley's ideal-diode equation. For small-signal, *discrete* silicon diodes, $n \approx 2$. For discrete Schottky diodes, as described below, $n \approx 1$. For diodes in integrated circuits, $n \approx 1$. Other parameters in Eqs. (16.1a) and (16.1b) are summarized in Table 16.1.

The goal of the vertical asymptote in the forward-bias region in Fig. 16.2 is to approximate the steep exponential function at practically relevant values of the diode current. As an approximation, the value of V_{D0} in Fig. 16.2 is close to the *built-in voltage of the pn-junction,* V_{BI}. For small-signal silicon diodes, $V_{BI} \approx 0.62 - 0.72\,\text{V}$. To be specific, we will assume $V_{D0} = 0.7\,\text{V}$, which is the commonly used value for small-signal silicon diodes.

Table 16.1. Useful constants for Shockley's equation.

Absolute temperature T (K)	$T = 273° + t°(\text{C})$
Electron charge q (C)	1.60218×10^{-19}
Boltzmann constant k (J/K)	1.38066×10^{-23}

Example 16.1: Figure 16.4a (and simultaneously Fig. 16.2) shows measured characteristics for the 1N4148 small-signal Si switching diode adopted from a Hitachi 1N4148 datasheet at 25 °C in the forward-bias region, i.e., the dotted curve. Compare the experimental curve with the Shockley's equation (16.1a) under the assumption that $n = 1.7, I_S = 1.1\,\text{nA}$.

Solution

We calculate the thermal voltage at 25 °C first and obtain $V_T = 26\,\text{mV}$. Next, we plot Shockley's equation in Fig. 16.4a based on the solid curve. The agreement between theory and experiment is quite satisfactory. At the same time, when we consider a wider electric current range and use a logarithmic scale to better resolve small and large currents, the deviation at larger currents becomes more visible. The corresponding graph is given in Fig. 16.4b. Figure 16.4 may be replicated in the laboratory for a 1N4148 diode from different manufacturers.

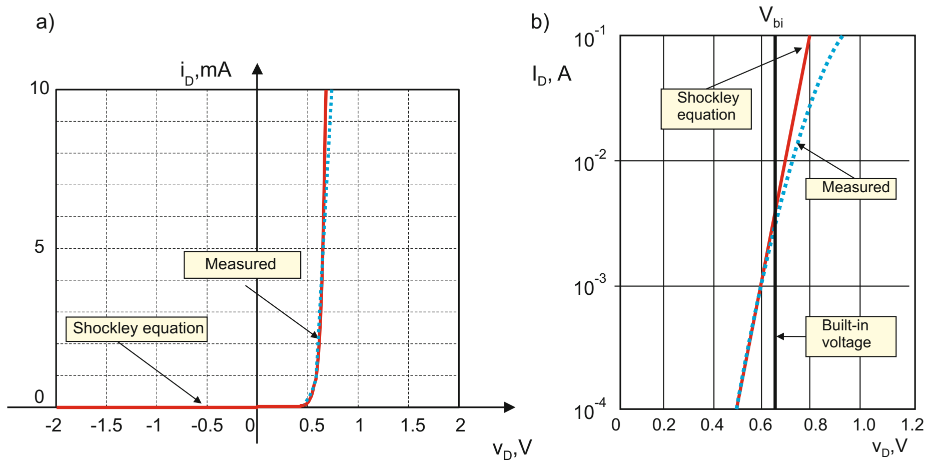


Fig. 16.4. (**a**) Shockley equation prediction (*solid curve*) versus measured data (*dotted curve*) for the 1N4148 small-signal Si switching diode at 25 °C and (**b**) enlarged scale.

Exercise 16.1: Determine the thermal voltage at room temperature (20 °C).

Answer: $V_T = 25\,\text{mV}$.

A diode operating in the forward-bias region is usually called a *switching diode*, meaning this is its major use. For many applications the diode *switching time* T plays an important role; it characterizes how fast the pn-junction current responds to reversing the diode voltage. For example, a gold-doped 1N4148 switching diode may have the switching times of about 2–4 ns. Even though this number might appear small, it is not suitable for radio frequencies. The *maximum operating frequency* of the diode is the inverse of its shortest switching time, $f_{max} \approx 1/T = 250\,\text{MHz}$ for the present switching diode.

16.1.5 Reverse-Bias Region: Varactor Diode

The reverse-bias region is characterized by negative terminal voltages. According to Shockley's equation (16.1a), in this region, $i_D = -I_S$ when $v_D \ll -V_T$. Thus, the reverse current should flow in the diode. While extremely small, the reverse current may even reach 1–20 nA due to various leakage effects. There is however a special diode which operates *only* in the reverse-bias region. It is called a *varactor* (from variable capacitor) *diode* or the *tuning diode*. The behavior of the varactor diode is based on the internal structure of the pn-junction, which effectively becomes a charge-free capacitor at negative (and even quite small positive) terminal voltages. The gap between the charged "capacitor plates" is determined by the terminal voltage, which controls the resulting capacitance. As a result, we obtain a *voltage-controlled capacitor* that has numerous applications in electronic communication circuits. The varactor diode is optimized to increase capacitance variations in response to the applied voltage. The circuit symbol for the varactor diode is seen in Fig. 16.5, where the schematic shows the built-in capacitor.

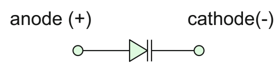


Fig. 16.5. Circuit symbol for a varactor (variable capacitor) diode.

16.1.6 Breakdown Region: Zener Diode

The steepness of the *v–i* diode curve in Fig. 16.2 in the breakdown region is utilized in the design of *voltage regulators*. The corresponding Zener breakdown voltage V_{Z0} is on the order of 75–100 V for switching diodes, which makes their use in the reverse-bias region impractical. A diode with a much smaller V_{Z0}, on the order of 5–20 V and specially designed to operate in the reverse-bias region, is called a *Zener diode*. The silicon-made Zener diode features a heavily doped pn-junction. The breakdown voltage is adjusted by a proper doping composition. Figure 16.6 compares circuit symbols for the switching diode and Zener diode, respectively. Under normal operating conditions in the reverse-bias region, the cathode of the Zener diode is more positive and the diode current flows from cathode to anode. Therefore, both the diode voltage and the diode current in Fig. 16.6b have positive values. The Shockley's equation is *not* used in the breakdown region. Instead, the behavior of the Zener diode is described by a piecewise-linear diode model. The diode breakdown is *not* destructive; the diode successfully functions in the breakdown region.

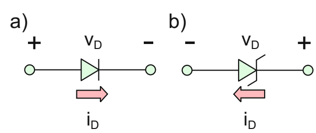


Fig. 16.6. Switching (**a**) versus Zener (**b**) diode. Note reversed voltage polarity/current direction.

16.1.7 Other Common Diode Types

Schottky Diode

The *Schottky* (barrier) *diode* does not use a pn-junction. Instead, a junction of a metal (anode) and an n-type semiconductor (cathode) are formed. Schottky diodes exhibit lower forward-bias voltages (0.15 to 0.5 V) and ultrafast switching speeds, since they are majority-carrier (conduction) devices, in contrast to the "slow" diffusion-current-based pn-junctions. Schottky diodes may employ different semiconductors including Si, GaAs, etc. in contact with metal (molybdenum, platinum, chromium). The corresponding circuit symbol is shown in Fig. 16.7.

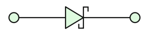


Fig. 16.7. Circuit symbol for a Schottky diode.

PIN Diode

A Si PIN diode is a *pin* semiconductor junction. In contrast to the standard junction, there exists a region of intrinsic (or lightly doped) silicon (*i*-layer) of low conductivity between doped p and n layers. When the diode voltage is high and positive, this region becomes filled with charge carriers. Consequently, the PIN diode becomes a *variable*, *voltage-controlled resistor*. This makes it useful as a switch or attenuator for radio-frequency signals. Another application is related to *photodetection*. In this case, the diode voltage is typically negative. When there is no ambient light, the region of intrinsic silicon has no charge carriers; hence, the diode is not conducting. When the ambient light is present, a photon collides with a single electron in the lattice. When the photon energy is sufficiently high, the electron leaves the crystal lattice and becomes a free carrier. Hence, the diode becomes conducting and operates as a photodetector.

Photodiode

The idea of the photodiode can be explained on the basis of the PIN diode. The light-induced carriers support a reverse current through the diode, the so-called photocurrent. Its intensity can be measured; it is proportional to the intensity of the incident light.

Light-Emitting Diode

A Light-Emitting Diode (LED) performs the opposite function of the photodiode. When a free conduction electron finds its place in the crystal lattice (becomes a valence electron), it loses energy, which is irradiated as a quantum of visible light. The LED junction is not a silicon junction, but is made of gallium arsenide (GaAs), another semiconductor material. Several different compounds may be involved and the junction becomes the *heterojunction*. The corresponding circuit symbol is shown in Fig. 16.8 (the symbol for the photodiode has the oppositely directed small arrows). Any general-purpose LED may operate as a solar cell—generate a nonzero voltage across its terminals when illuminated by light. A simple experiment is to connect an LED to the DMM and measure the voltage across it with and without the ambient light (cover it with your hand).

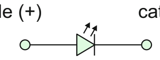


Fig. 16.8. Circuit symbol for an LED.

Historical: The history of diode discoveries is very rich. The term diode originates from the Greek roots *di*, meaning "two," and *ode*, meaning "path." It was suggested by William Eccles (1875–1966), British physicist and radio engineer, who worked with Guglielmo Marconi. The first solid-state diode was patented in 1899 by Karl Ferdinand Braun (1850–1918), German physicist and Nobel Prize laureate. Indian professor of physics, Jagadish Chandra Bose (1858–1937) was the first to use diodes to detect radio signals. The Zener diode is named in honor of American physicist Clarence Melvin Zener (1905–1993). The Schottky diode is named in honor of Walter Hermann Schottky (1886–1976), German professor and engineer at Siemens. The PIN diode was invented by Jun-ichi Nishizawa (1926–), a Japanese professor and engineer. British scientist Henry Joseph Round (1881–1966) was the first to report on the observation of light emission from crystals when subjected to an applied voltage. Russian scientist Oleg Losev (1903–1942) observed light emission from semiconductor junctions, the first LEDs. Nick Holonyak, Jr. (1928–), professor at the University of Illinois at Urbana-Champaign, invented and constructed the first practically useful LED.

Section 16.2 Diode Models

16.2.1 Ideal-Diode Model: Method of Assumed States

The *ideal-diode model* ignores all details of the *v–i* characteristic except the most fundamental one: the steep nonlinearity in the forward-bias region. Figure 16.9 shows the *v–i* characteristic of the *ideal diode*. We plot the current i_D versus the voltage v_D across the diode. The ideal diode is equivalent to an open circuit (no current) at negative or reverse-bias applied voltages ($v_D < 0$) and to a short circuit at any positive value of diode current ($i_D > 0 \Rightarrow v_D = 0$). For comparison, we also draw the *v–i* line for a resistor as a dashed line in Fig. 16.9.

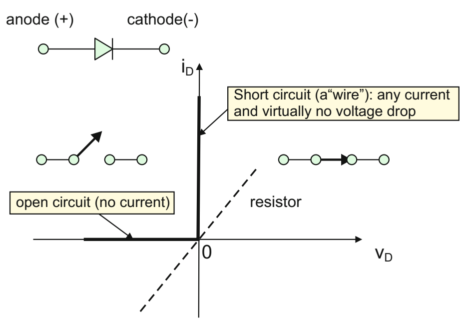


Fig. 16.9. The *v–i* characteristics of an ideal diode. The resistor *v–i* characteristic is a dashed line.

The ideal-diode model allows us to analyze an electric circuit with a diode using the *method of assumed states*. The ideal diode may only have two states: a short circuit (the diode is "ON") or an open circuit (the diode is "OFF"). During analysis, we make an intuitive guess (ON or OFF) and replace the diode either by a wire or by an open circuit, respectively. We then solve the rest of the circuit and check to see if our guess was right. For the ON-diode, we cannot check the voltage across the diode, since it is exactly zero for the ideal-diode model. However, we can check the current, which must flow in the direction of the diode arrow. If this is not the case, our guess is wrong. For the OFF-diode, we check the voltage across the diode. If this voltage is negative (or "reversed"), then it satisfies the condition of Fig. 16.9, and the diode is an open circuit as expected. Otherwise, the guess is wrong. Figure 16.10 shows the procedure.

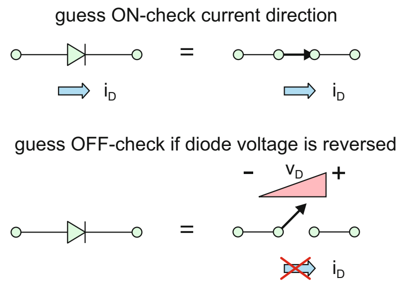


Fig. 16.10. Checking procedure for the method of assumed states.

Example 16.2: Figure 16.11a presents a DC diode circuit, a resistor network bridged with the ideal diode. Solve the circuit, i.e., find all voltages and currents assuming the ideal-diode model. Denote the solution for the diode voltage and diode current in the DC steady state by capital letters V_D, I_D, respectively.

Solution: We first assume the diode is OFF and replace it by an open circuit; see Fig. 16.11b. Thus, the diode current I_D is zero. The circuit becomes a combination of two independent voltage dividers connected to the same power supply. The absolute voltages of the anode and the cathode with respect to ground are obtained from the voltage division principle and are equal to 5 and 10 V, respectively. The voltage across the diode is obtained as $V_D = 5 - 10\,\text{V} = -5\,\text{V}$. This negative value shows that our guess is correct. After that, all circuit currents are found using Ohm's law. The total circuit current is 2 mA; it is equally divided between two independent voltage dividers. The circuit analysis in Fig. 16.11b is therefore complete; we have proved that the diode has a negative bias voltage. Consequently, it is *not necessary* to prove that the opposite guess will lead to a wrong conclusion, in this case, to a wrong direction of the diode current.

Exercise 16.2: For the purpose of completeness, present a solution for the diode state guess ON in Fig. 16.11a and show that there is a contradiction.

Answer: The assumed solution is given in Fig. 16.11c. The diode current of 0.75 mA flows though the diode in the opposite direction of the diode arrow. This is a contradiction; therefore, the guess is wrong.

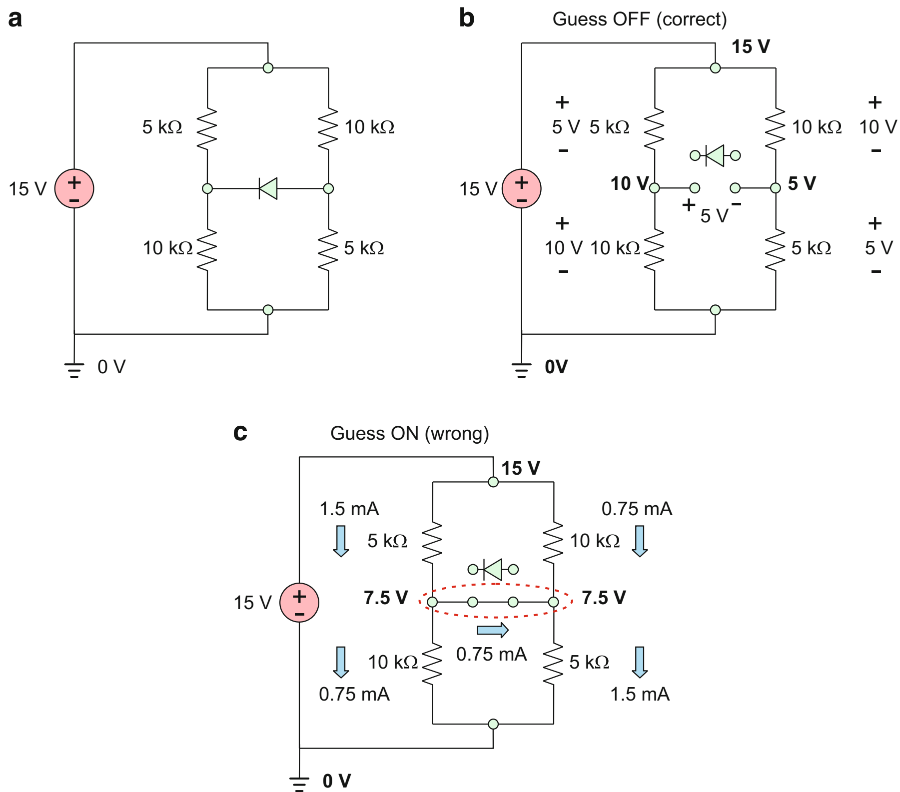


Fig. 16.11. (**a**) A resistor network bridged with an ideal diode. (**b**) Circuit analysis assuming the diode state is OFF (correct guess), leading to an open circuit. Absolute voltages versus ground are marked in *bold*. (**c**) Circuit analysis assuming the diode state ON (wrong guess), which is a short circuit. The absolute voltages with respect to ground are marked in *bold*.

We note that the equivalent circuit resistance in Fig. 16.11b is 7.5 kΩ, and in Fig. 16.11c it becomes 6.68 kΩ. It is interesting to note that the dissipated power V_S^2/R is greater in the second case. A correct diode state thus *minimizes* the circuit power.

Example 16.3: A diode circuit known as an OR logic gate is shown in Fig. 16.12. This circuit has two input voltages, V_1 and V_2, which may either be equal to 0 V or to 5 V. Solve the circuit by filling out Table 16.2 under the assumption that both diodes are ideal.

Example 16.3 (cont.):

Table 16.2. Output voltage of the diode circuit shown in Fig. 16.12 to be evaluated as a function of two input voltages.

V_1 (V)	V_2 (V)	V_{OUT}
0	0	
0	5	
5	0	
5	5	

Solution: For the first row of the table, all three voltages in Fig. 16.12 are zero. For the second row, diode D_2 is ON and diode D_1 is OFF. The last condition is confirmed by the reverse bias voltage, and the first condition is confirmed by the correct diode current. The output voltage is 5 V. For the third row of the table, the situation is opposite: diode D_1 is ON and diode D_2 is OFF. The output voltage is again 5 V. For the last row of the table, both diodes are ON and $V_{OUT} = 5\,\text{V}$.

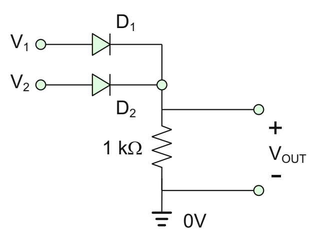


Fig. 16.12. An OR logic gate on the base of two ideal diodes.

If we assign a binary value of "1" to the voltage of 5 V and binary value of "0" to the voltage of 0 V, Table 16.2 from Example 16.3 can be rewritten in the form of a so-called truth table. Table 16.3 is a truth table for the OR logic gate constructed based on two diodes. An AND gate can be constructed in a similar fashion. The diode logic circuits (the so-called diode logic or DL) had once been popular, but they were quickly outperformed by the transistor logic circuits: the resistor-transistor logic (RTL) or the transistor-transistor logic (TTL).

Table 16.3. Output voltage of the diode circuit in Fig. 16.12 as a function of two input voltages in terms of binary numbers. This is known as a truth table.

V_1	V_2	V_{OUT}
0	0	0
0	1	1
1	0	1
1	1	1

16.2.2 Constant-Voltage-Drop Model

Figure 16.13 demonstrates the *constant-voltage-drop model* of a diode. This is the ideal-diode model, but with the inclusion of "turn-on" voltage V_{D0} from Fig. 16.2, which has been used to approximate Shockley's equation. The "voltage supply" V_{D0} and the diode have *the same polarity*. The constant-voltage-drop model is not a significant complication of the ideal-diode model: the *method of assumed states* is still applicable. However, it provides better accuracy and is therefore a popular and robust extension of the ideal-diode model. We will use $V_{D0} = 0.7\,V$ for silicon diodes. For the ON-diode, we additionally introduce the voltage drop across the diode of 0.7 V in the forward direction, calculate the diode current, and finally check the current direction.

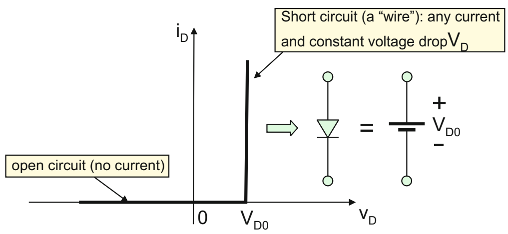


Fig. 16.13. The *v–i* characteristic of the constant-voltage-drop model. The diode in the forward-bias region is replaced by a DC voltage source having *the same polarity*.

Example 16.4: Determine diode current and diode voltage in a DC circuit shown in Fig. 16.14 using (A) ideal-diode model and (B) constant-voltage-drop model.

Solution (A): We assume the diode in Fig. 16.14 is ON and replace it by a short circuit. The correct current direction confirms this assumption. One has

$$V_D = 0, I_D = 3\,\text{mA} \tag{16.2a}$$

Solution (B): We again assume the diode in Fig. 16.14 is ON and replace it by a short circuit plus a 0.7-V voltage supply. The circuit current is found using KVL. The correct current direction confirms the initial assumption. One has

$$V_D = 0.7\,\text{V}, I_D = 2.3\,\text{mA} \tag{16.2b}$$

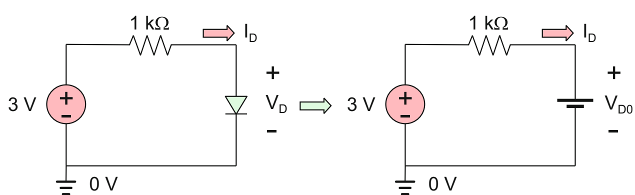


Fig. 16.14. A basic diode circuit solved with the constant-voltage-drop model.

It follows from Example 16.4 that the ideal-diode model less accurately predicts the circuit current (and other parameters) for relatively small supply voltages when compared with V_{D0}. However, its accuracy may be sufficient for voltages much greater than V_{D0}.

Exercise 16.3: Solve Example 16.4 when the supply voltages change to 120 V.

Answer: $V_D = 0, I_D = 120\,\text{mA}$ and $V_D = 0.7\,\text{V}, I_D = 119.3\,\text{mA}$.

16.2.3 Exponential Model in the Forward-Bias Region and Its Use

The most accurate *exponential diode model* makes use of Shockley's equation (16.1a) and is repeated here for convenience:

$$i_D = I_S \left[\exp\left(\frac{v_D}{nV_T}\right) - 1 \right] \tag{16.3a}$$

The saturation current I_S and the ideality constant n in Eq. (16.3a) can hardly be found in the diode datasheet. Instead, two pairs of measured values of diode voltage and current are usually used. Assume that we know V_{D1}, I_{D1} and V_{D2}, I_{D2} at a given temperature and that $V_{D1,2} \gg V_T$. By neglecting the factor 1 in Eq. (16.3a), one finds the ideality factor as

$$n = \frac{V_{D2} - V_{D1}}{V_T \ln(I_{D2}/I_{D1})} \tag{16.3b}$$

After that, the saturation current is evaluated in the form:

$$I_S = \frac{I_{D1}}{\exp\left(\frac{V_{D1}}{nV_T}\right) - 1} \tag{16.3c}$$

Exercise 16.4: A 1N4148 diode from Vishay Semiconductors has a current of 0.1 mA at 0.4 V and a current of 0.8 mA at 0.5 V at 25 °C. Determine the ideality factor and the saturation current at this temperature and in this range of diode currents.

Answer: $n = 1.9, I_S = 24.4\,\text{nA}$.

Exercise 16.5: Repeat Exercise 16.4 for a 1N4148 diode manufactured by Fairchild. This diode has a current of 0.1 mA at 0.5 V and a current of 5 mA at 0.7 V at 25°.

Answer: $n = 2.0, I_S = 5.7\,\text{nA}$.

16.2.4 Load-Line Analysis

With the exponential diode model, the circuit equation becomes a transcendental expression which complicates the analysis. A solution may be obtained *graphically*. Such a method is known as the *load-line method*. Consider the circuit in Fig. 16.14. On one hand, the diode current as a function of diode voltage is given by the Shockley's equation (16.3a). On the other hand, the *same* current is found with the help of KVL:

$$i_D = \frac{V_S - v_D}{R} \tag{16.4}$$

where $V_S = 3\,\text{V}, R = 1\,\text{k}\Omega$. In Fig. 16.15, we plot both dependencies on the same graph. The Shockley's equation is plotted to scale using $n = 1.7, I_S = 1.1\,\text{nA}$ at 25 °C. The linear relation of Eq. (16.4) or the solid line in Fig. 16.15 is the *load line*. The load line intersects the diode *v–i* curve at a point *Q*. The point *Q* is known as the DC *operating point* (or the *quiescent point*) of the circuit. Its coordinates V_D, I_D give us the required circuit solution. The load-line method is a general method for studying arbitrary linear circuits represented by its Thévenin equivalent and connected to a nonlinear load.

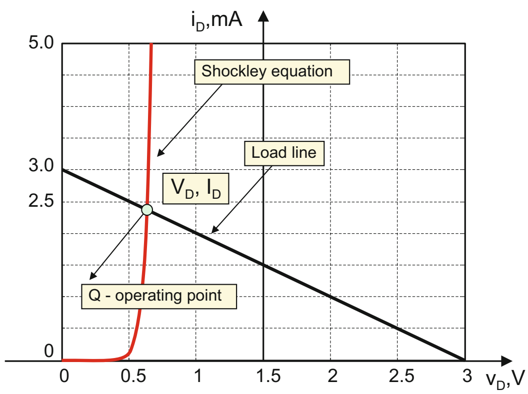


Fig. 16.15. Shockley's equation to scale and the load line for the circuit in Fig. 16.14.

Example 16.5: Determine diode (or circuit) current and diode voltage in the DC circuit shown in Fig. 16.14 using the diode *v–i* curve from Fig. 16.15.

Solution: A visual inspection of the operating point *Q* in Fig. 16.15 indicates that the diode voltage and the diode current may be approximately estimated as

$$V_D \approx 0.65\,\text{V}, I_D \approx 2.4\,\text{mA} \tag{16.5}$$

16.2.5 Iterative Solution

A higher accuracy can be achieved with a precise *iterative method*. Equating the right-hand sides of Eqs. (16.3a) and (16.4), we obtain

$$I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right]=\frac{V_S-v_D}{R}\Rightarrow v_D=nV_T\ln\left[\frac{V_S-v_D}{RI_S}+1\right] \tag{16.6a}$$

The transcendental equation (16.5a) for v_D is solved using a basic iterative scheme:

$$v_D^{n+1}=nV_T\ln\left[\frac{V_S-v_D^n}{RI_S}+1\right],\quad n=0,1,2,\ \ldots \tag{16.6b}$$

where the initial guess v_D^0 may be chosen arbitrarily. Table 16.4 shows the solution convergence for two choices of the initial diode voltage: 0.7 V and 0 V, respectively. In both cases, the convergence is excellent: an accurate solution is obtained after the second iteration already. The iterative algorithm works very well for lumped diode circuits.

Table 16.4. Convergence of the iterative algorithm for the diode circuit with a 1N4148 small-signal Si switching diode having $n=1.7, I_S=1.1\,\text{nA}$ at 25 °C.

v_D^0 (V) (init. guess)	v_D^1 (V)	v_D^2 (V)	v_D^3 (V)
0.7	0.6320	0.6333	0.6333
0.0	0.6436	0.6331	0.6333

Example 16.6: Compare performance of the four diode approximations using the DC circuit shown in Fig. 16.14.

Solution: We collect the results of the two previous examples and Table 16.4:

Iterative method (most accurate):	$V_D=0.6333\,\text{V}, I_D=2.367\,\text{mA}$
Load-line method (visual inspection):	$V_D=0.65\,\text{V}, I_D=2.4\,\text{mA}$
Constant-voltage-drop model:	$V_D=0.7\,\text{V}, I_D=2.3\,\text{mA}$
Ideal-diode mode (least accurate):	$V_D=0, I_D=3\,\text{mA}$

The conclusion of this example is that the simple constant-voltage-drop model and the load-line analysis perform quite well compared to the most-accurate solution.

16.2.6 Linearization About a Bias Point: Small-Signal Diode Model

The *linearization procedure* for the diode (or any other nonlinear circuit element) is illustrated in Fig. 16.16. When compared to Fig. 16.15, this figure uses the same data, but employs a finer voltage scale. Quite often, a signal-processing radio-frequency diode is set in a DC circuit, which provides a certain DC operating point V_D, I_D shown in Fig. 16.16. The DC circuit so constructed is called the *bias circuit*.

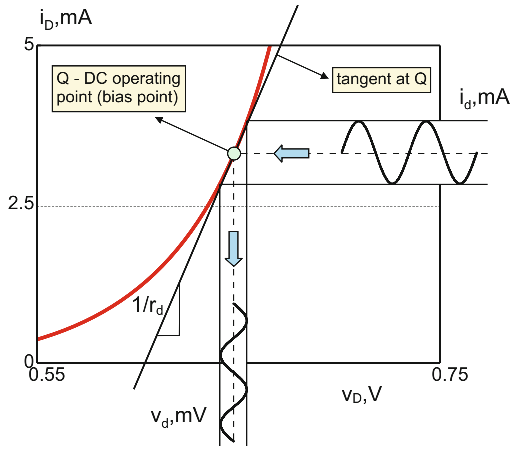


Fig. 16.16. Linearization procedure for a switching diode with $n = 1.7, I_S = 1.1\,\mathrm{nA}$ at 25 °C.

Consequently, the DC operating point of the diode is called the *bias point* (*quiescent point*). Further, a very weak AC signal $v_d(t), i_d(t)$ is added. Though weak, this signal contains information to be processed. The diode voltage and diode current become

$$\begin{aligned} v_D(t) &= V_D + v_d(t) \\ i_D(t) &= I_D + i_d(t) \end{aligned} \qquad |v_d(t)| < V_T \tag{16.7a}$$

The inequality in Eq. (16.7a) means that the AC signal amplitude should be much less than 25 mV. The *linearization* concept states that this weak AC signal will satisfy not the complicated nonlinear diode expression, but the familiar *linear* Ohm's law in the form:

$$v_d(t) = r_d i_d(t) \tag{16.7b}$$

where r_d is the *small-signal diode resistance* or *incremental resistance* determined by the slope of the *v*–*i* dependence in Fig. 16.16. Thus, the diode becomes a resistor for small signals, which greatly simplifies the AC analysis. Our goal is to find this small-signal resistance as a function of V_D and I_D. To do so, we use the asymptotic expansion (Taylor series) with regard to the small parameter v_d/V_T:

$$\exp\left(\frac{v_D}{nV_T}\right) = \exp\left(\frac{V_D}{nV_T}\right)\exp\left(\frac{v_d}{nV_T}\right) = \exp\left(\frac{V_D}{nV_T}\right)\left(1 + \frac{v_d}{nV_T} + O\left(\left(\frac{v_d}{nV_T}\right)^2\right)\right) \tag{16.7c}$$

and neglect all terms on the order of $(v_d/nV_T)^2$ or less denoted by the symbol O. Substitution into Shockley's equation yields

$$i_d = I_S \exp\left(\frac{V_D}{nV_T}\right)\frac{v_d}{nV_T} \tag{16.8}$$

Since V_D/V_T is usually much greater than one, we can use $I_S \exp(V_D/nV_T)$ instead of $I_D = I_S(\exp(V_D/nV_T) - 1)$. This gives the small-signal diode resistance r_d in the form:

$$i_d = \frac{I_D}{nV_T} v_d \Rightarrow r_d = \frac{nV_T}{I_D} \tag{16.9}$$

Mathematically, r_d is the inverse slope of the tangent line at the bias point in Fig. 16.16. The small-signal resistance is high at small bias currents and is low otherwise.

16.2.7 Superposition Principle for Small-Signal Diode Model

A circuit with the DC bias and a small AC signal is solved using the *superposition principle* shown in Fig. 16.17. We solve the nonlinear DC diode circuit in Fig. 16.17a first. The DC solution is used to find the small-signal diode resistance in Fig. 16.17b. The linear AC circuit in Fig. 16.17b is solved next. The complete solution is the sum of both.

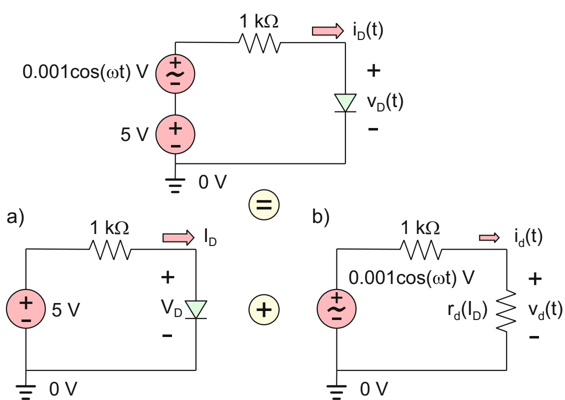


Fig. 16.17. Illustration of the superposition principle for small-signal diode circuits.

Example 16.7: Determine the diode voltage for the circuit in Fig. 16.17 which superimposes a weak AC voltage signal and the DC bias. Use the constant-voltage-drop-diode model. Assume temperature of 25 °C and $n = 1.7$.

Solution: We apply the method of assumed states to the DC circuit in Fig. 16.17a and obtain $I_D = 4.3\,\text{mA}$. The small-signal diode resistance is $r_d = \frac{nV_T}{I_D} \approx 10\,\Omega$. The linear small-signal circuit in Fig. 16.17b is a voltage divider; the small-signal diode voltage is therefore given by $v_d(t) = \frac{10}{1010} \times 0.001 \cos \omega t \approx 10 \cos \omega t\,\mu\text{V}$. The total diode voltage is finally obtained in the form $v_d(t) = 0.7\,\text{V} + 10 \cos \omega t\,\mu\text{V}$.

Section 16.3 Diode Voltage Regulators and Rectifiers

This section studies major diode circuits such as rectifiers and regulators. These circuits have applications both in power electronics and communication. When the primary application area is power electronics, we attempt to designate the input AC voltage as the source voltage $\upsilon_S(t)$ and the output voltage as the load voltage $\upsilon_L(t)$. The same is done for voltage regulator circuits. Otherwise, we keep the notation $\upsilon_{in}(t)$ for the input voltage to the circuit and $\upsilon_{out}(t)$ for the output voltage, respectively. As to the phase, we pick the initial phase in the form that is either most convenient for the graphical representation of the problem or is of general convention. Specifically, we choose $\sin \omega t$ for basic rectifier circuits and $\cos \omega t$ for a signal-processing envelope (peak) detector circuit. We will label each individual diode in a circuit as D_1, D_2, etc.

16.3.1 Voltage Reference and Voltage Regulator

The first useful diode circuit is the *voltage reference circuit* shown in Fig. 16.18. In amplifier and actuator circuits, it is often desired to have a *fixed* voltage reference with respect to ground, which is not affected by variations of the source voltage V_S and/or by particular values of the load resistance R_L. A resistive voltage divider is unable to do this. However, a diode circuit shown in Fig. 16.18 solves this problem. Indeed, the voltage reference can only be a multiple of the diode voltage drop $V_{D0} = 0.7\,\text{V}$.

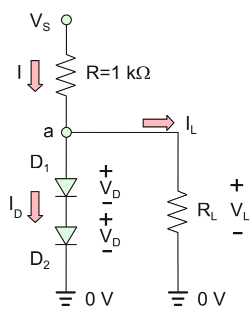


Fig. 16.18. Diode voltage reference for the load resistor R_L.

The circuit shown in Fig. 16.18 simultaneously serves as a basic *forward-bias voltage regulator*. The voltage regulator is a circuit that provides a constant DC voltage between its terminals, no matter how much the voltage supply changes. The circuit in Fig. 16.18 is the *shunt voltage regulator*.

Example 16.8: Determine the load voltage and diode current for the regulator circuit in Fig. 16.18 given that $V_S = 9\,\text{V} \pm 0.5\,\text{V}$ for two different values of the load resistance, $R_L = 1\,\text{M}\Omega$ and $R_L = 1\,\text{k}\Omega$. Use the constant-voltage-drop-diode model to evaluate the circuit.

Example 16.8 (cont.):

Solution: We apply the method of assumed states to the DC circuit in Fig. 16.18. We assume that both diodes in Fig. 16.18 are ON and replace them by short circuits plus two 0.7 V voltage supplies in series; this gives us

$$V_L = V_a = 1.4\,\text{V} \tag{16.10}$$

in Fig. 16.18. The load current is therefore $I_L = 1.4/R_L$. The current through the top resistor in Fig. 16.18 is $I = (V_S - 1.4)/R$. The guess ON implies the correct direction of the diode current, that is,

$$I_D = I - I_L = (V_S - 1.4)/R - 1.4/R_L > 0 \tag{16.11}$$

The substitutions show that for any set of values $V_S = 9\,\text{V} \pm 0.5\,\text{V}$ and $R_L = 1\,\text{M}\Omega$ or $R_L = 1\,\text{k}\Omega$, the inequality (16.11) is satisfied. Thus, the initial guess ON is always true. This means that the diode combination in Fig. 16.18 provides a constant voltage reference of 1.4 V given by Eq. (16.10) irrespective of the power supply voltage variations and/or load resistance variations.

The constant-voltage-drop-diode model employed in Example 16.18 may be improved using the small-signal diode approximation from the previous section. The key observation is that the small-signal diode resistance, specifically developed to study weak AC signals, is equally applicable for the study of arbitrary small variations of diode voltages such as the variations encountered in the present problem.

Exercise 16.6: Does the voltage regulator in Example 16.8 still operate properly when:

A. The supply voltage $V_S = 5\,\text{V} \pm 1\,\text{V}$ is used?
B. The load resistance of $R_L = 100\,\Omega$ is used?

Answer: A. Yes. B. No.

16.3.2 Voltage Regulator with Zener Diode

Shunt Voltage Regulator with Zener Diode

If a voltage regulator with an output voltage of 5–20 V is required, the use of forward-biased diodes becomes impractical since many of them have to be used. The *standard voltage regulator* makes use of the Zener diode operating in the breakdown region as shown in Fig. 16.19a where R_L is the load resistance. Interestingly, the circuit in this figure and the circuit in Fig. 16.18 have in fact the same topology if we replace two forward-biased diodes by one reverse-biased Zener diode.

Piecewise-Linear Diode Model

The Zener diode in the breakdown region is described by a *piecewise-linear diode model*. As the name implies, this model approximates the reverse-bias region by a linear dependence shown in Fig. 16.19b. A datasheet for the Zener diode typically specifies:

1. The *dynamic* or *incremental Zener diode resistance* r_Z, which is the inverse slope of the straight-line asymptote in Fig. 16.19b
2. At least one *diode test point* V_{ZT}, I_{ZT} on the diode breakdown *v–i* curve that is also shown in Fig. 16.19b

This information is sufficient to construct the linear model in the breakdown region as

$$V_D = r_Z I_D + V_{Z0} \tag{16.12a}$$

where the *Zener breakdown voltage* V_{Z0} is found from the datasheet parameters as

$$V_{Z0} = V_{ZT} - r_Z I_{ZT} \tag{16.12b}$$

Example 16.9: A 1N5231B Zener diode with $V_{ZT} = 5.1\,\text{V}, I_{ZT} = 20\,\text{mA}$ and $r_Z = 17\,\Omega$ is used in the voltage regulator circuit in Fig. 16.19a. Determine the load voltage given that $V_S = 9\,\text{V} \pm 1\,\text{V}$ and the load has a very high (infinite) resistance.

Solution: We apply the method of assumed states to the DC circuit in Fig. 16.19 and assume that the Zener diode operates in the breakdown region. Substituting the diode data into Eq. (16.12b) yields the breakdown voltage of $V_{Z0} = 4.76\,\text{V}$. The resulting circuit is shown in Fig. 16.19c where the infinite load resistance is replaced by an open circuit. The circuit (diode) current is given by $I = I_D - (V_S - 4.76)/(R + r_z)$. Therefore, $V_L = r_z I + 4.76$. When the supply voltage is $V_S = 9\,\text{V} \pm 1\,\text{V}$, the load voltage becomes $V_L = 4.831\,\text{V} \pm 17\,\text{mV}$. Thus, the voltage regulation function of the circuit is quantified. The load voltage response to ± 1 V supply change is known as *line regulation*. In the present example, the line regulation is 17 mV per 1 V or 17 mV/V. Finally, we must justify the initial guess of diode operation. This is done by checking the diode current: $I_D = 4.2\,\text{mA} \pm 1\,\text{mA}$. Since the current flows in the reverse direction, the initial assumption is also justified.

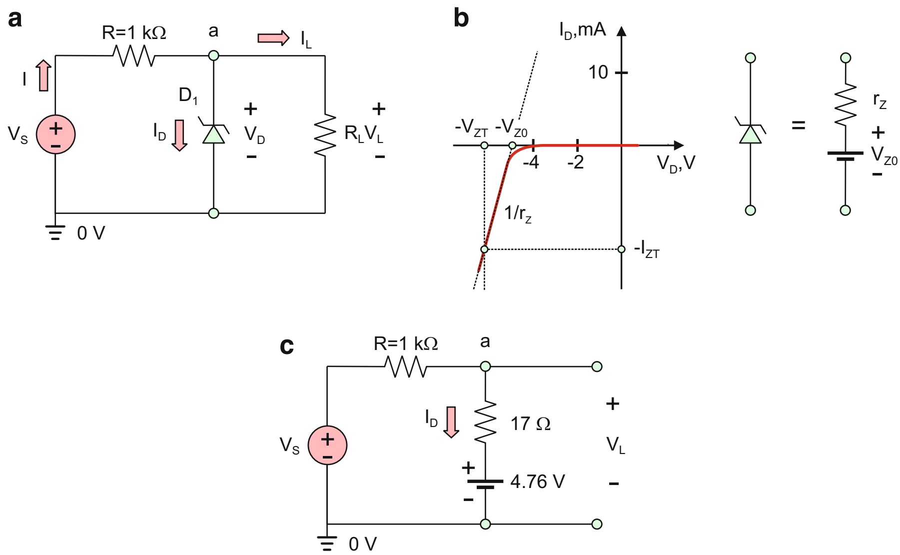


Fig. 16.19. (**a**) Shunt voltage regulator on the base of a Zener diode. (**b**). Piecewise-linear diode model for the Zener diode in the breakdown region. (**c**) The circuit from (**a**) with the Zener diode replaced by its equivalent circuit and with the open-circuited load.

16.3.3 Half-Wave Rectifier

A diode rectifier is perhaps the most important application of the diode. Mostly large AC currents are rectified, i.e., converted to DC currents. The diode rectifier forms an input stage for electronic DC power supplies including switching-mode power supplies. Furthermore, diode rectifiers form a basis for battery-charging circuits including automotive applications. The diode rectifier is also an important part of wireless energy-harvesting and communication devices. In particular, any passive RFID tag uses a diode rectifier to convert the (very weak) AC power of a received electromagnetic signal to DC electric power. The concept of a simple (half-wave) diode rectifier is shown in Fig. 16.20. An AC power supply with sinusoidal voltage (either positive or negative with respect to ground) is connected to a load via a diode in series. We assume that $v_S(t) = V_m \sin \omega t$. Let us use the ideal-diode model first. When the voltage versus ground is positive, the diode is in the ON state and can be replaced by a wire. All electric power is transferred to the load and the current through the load flows from top to bottom. When the voltage is negative, the diode is OFF, and the load acquires no current. Hence, the current through the load always flows in one direction (or does not flow at all).

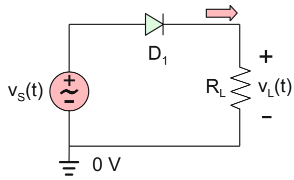


Fig. 16.20. Half-wave diode rectifier with a resistive load. The source in this figure is usually a secondary winding of a power transformer.

Mathematically, the load voltage for the ideal-diode rectifier is expressed by

$$v_L(t) = v_S(t) \quad \text{if} \quad v_S(t) > 0 \tag{16.13a}$$

$$v_L(t) = 0 \quad \text{if} \quad v_S(t) \le 0 \tag{16.13b}$$

Note that a battery charger is constructed in a similar way, with the load resistor in Fig. 16.20 replaced by a battery. A current-limiting resistor must also be added. Figure 16.21a shows the load voltage (rectified voltage) for the AC source $v_S(t) = V_m \sin \omega t$ with $V_m = 90\,\text{V}$ and $f = \omega/(2\pi) = 0.5\,\text{Hz}$. Other signals that are not necessarily sinusoidal, potentially not even periodic, may be rectified in a similar way.

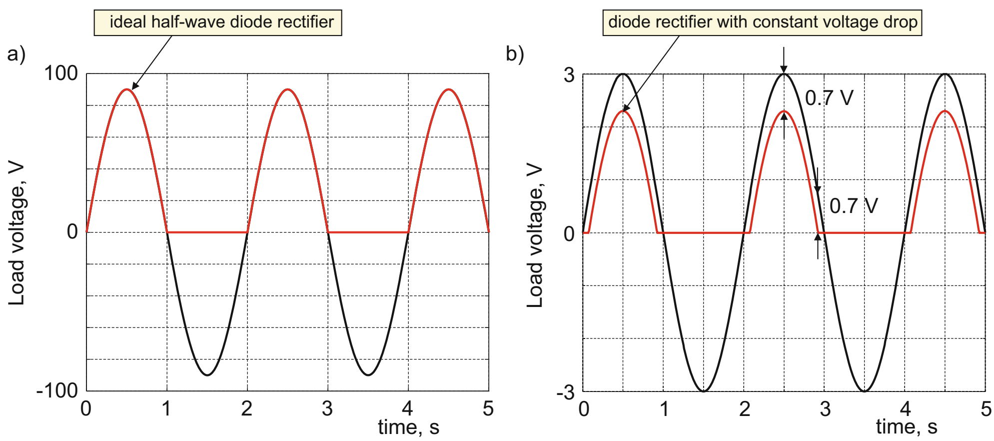


Fig. 16.21. Rectified (or load) voltage versus source voltage. (**a**) The result for the ideal half-wave diode rectifier or rectifier with a high input signal. (**b**) The result using the constant-voltage-drop-diode model for a moderate input signal.

Exercise 16.7: Plot source voltage and load voltage for the half-wave rectifier with $V_m = 3\,\text{V}$ and $f = 0.5\,\text{Hz}$ using the constant-voltage-drop-diode model.

Exercise 16.7 (cont.):

Answer: The plot is given in Fig. 16.21b. The non-ideal diode in Fig. 16.21b conducts not over the entire positive half cycle characterized by the dimensionless angle of π (in terms of ωt), but over a smaller *conduction angle* given by $\pi - 2\sin^{-1}(V_{D0}/V_m)$ where $V_{D0} = 0.7\,\text{V}$.

16.3.4 Full-Wave Rectifier with a Dual Supply

A drawback of the half-wave diode rectifier is that half of the AC signal (and of the AC power) is being lost. One solution to the problem is a *full-wave diode rectifier* shown in Fig. 16.22. This particular rectifier circuit uses a *dual AC voltage supply*, which is realized in practice by a secondary winding of a power transformer with a center tap at node *b*. We denote every *identical* individual supply in Fig. 16.22 by $\upsilon_S(t)$.

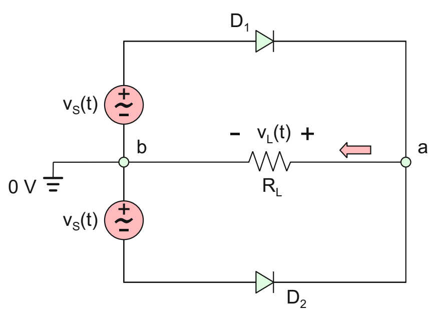


Fig. 16.22. The full-wave diode rectifier with a dual AC supply. The dual source in this figure is usually a center-tapped secondary winding of a power transformer.

We assume that $\upsilon_S(t) = V_m \sin\omega t$ in Fig. 16.22 and analyze the circuit using the ideal-diode model and the method of assumed states. When the top supply in Fig. 16.22 is positive versus ground, diode D_1 will be ON and diode D_2 will be OFF. The positive half-cycle will be rectified. However, when the top supply is negative versus ground, diode D_1 will be OFF and diode D_2 will be ON. The negative half-cycle will be rectified. However, the load current will *always* be directed from right to left in Fig. 16.22. Mathematically, the load voltage for the ideal full-wave diode rectifier is expressed by

$$\upsilon_L(t) = |\upsilon_S(t)| \quad \text{for any} \;\; \upsilon_S(t) \tag{16.14}$$

Figure 16.23a shows the load voltage (rectified voltage) for the AC source $\upsilon_S(t) = V_m \sin\omega t$ with $V_m = 90\,\text{V}$ and $f = \omega/(2\pi) = 0.5\,\text{Hz}$.

Exercise 16.8: Plot the source voltage and the load voltage for the full-wave rectifier with $V_m = 3\,\text{V}$ and $f = 0.5\,\text{Hz}$ using the constant-voltage-drop-diode model and give the general mathematical expression for the load voltage corresponding to this model.

Answer: The plot is given in Fig. 16.23b. The load voltage becomes

$$\begin{aligned} &v_L(t) = v_S(t) - 0.7 && \text{if } v_S(t) > 0.7 \\ &v_L(t) = |v_S(t) + 0.7| && \text{if } v_S(t) < -0.7 \\ &v_L(t) = 0 && \text{if } |v_S(t)| \le 0.7 \end{aligned} \tag{16.15}$$

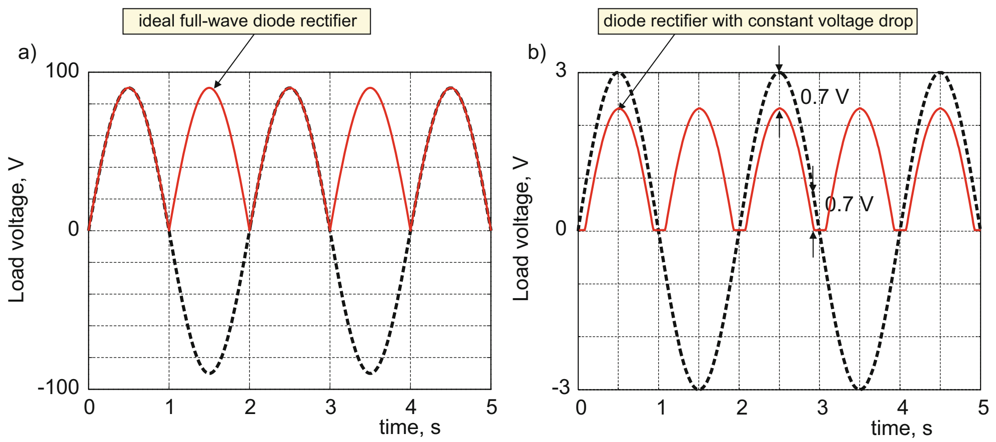


Fig. 16.23. Rectified (or load) voltage versus source voltage. (**a**) The result for the ideal full-wave diode rectifier or full-wave rectifier with a high input signal. (**b**) The result using the constant-voltage-drop-diode model for a moderate input signal.

16.3.5 Diode Bridge Rectifier

The *diode bridge rectifier* also does the full-wave rectification. However, it is operating using a single AC supply, similar to the half-wave rectifier. Four diodes are used instead of two. Nowadays this is not a serious drawback since diodes are inexpensive. The corresponding circuit is shown in Fig. 16.24. The circuit is again analyzed using the ideal-diode model and the method of assumed states. Figure 16.25 shows the result of this analysis: the current flow at positive and negative voltages of the AC power supply, respectively. When the voltage is positive, diodes D_2 and D_4 are ON and diodes D_1 and D_3 are OFF. When it is negative, the opposite is true: diodes D_1 and D_3 are ON, but diodes D_2 and D_4 are OFF. We can see from Fig. 16.25 that the current through the load resistor flows in the same direction at both positive and negative voltages. Thus, the rectification is achieved and the negative phase of the AC signal is not lost. The ideal-diode bridge rectifier is characterized by Eq. (16.14). In sum, the full-wave rectifier delivers twice as much power to the load as the half-wave rectifier does.

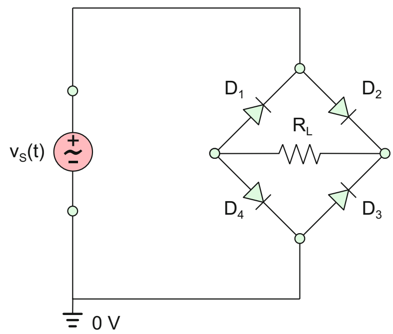


Fig. 16.24. The full-wave diode bridge rectifier: four diodes are bridged by the load resistor R_L.

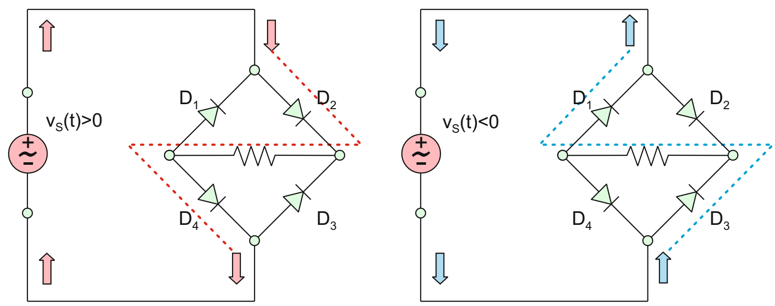


Fig. 16.25. Current flow in the full-wave diode bridge rectifier assuming ideal diodes. Circuit on the left is for positive applied voltage; circuit on the right is for negative applied voltage.

Exercise 16.9: Will the diode bridge rectifier follow Fig. 16.23a, b obtained for the full-wave bridge rectifier with the dual supply?

Answer: Fig. 16.23a will be identical for both rectifier types. However, Fig. 16.23b will be different.

16.3.6 Application Example: Automotive Battery-Charging System

When an automotive generator starts to work, it recharges the battery and supplies electric power for all electronic systems in the vehicle. The automotive generators have a long history. Until the early 1960s, DC generators have been driven by a belt on the crankshaft. After that, they have been replaced by three-phase AC generators, the alternators. Three coils of an alternator stator may be *delta* or *wye* connected; see Chapter 11. Since the mid-1980s, the *delta* connection has been used far more frequently. In this application example, we apply our prior knowledge of the diode rectifier circuits to understand the operation of an automotive alternator with a rectifier circuit.

Diode Circuit Transformation

Figure 16.26a shows part of a *three-phase* diode rectifying circuit that rectifies a voltage from one of the *three* stator coils of an alternator (three-phase voltage generator studied in Chapter 11) and charges the battery. Every stator coil is an *independent* voltage source that is offset 120° with regard to the others. The complete rectifying circuit includes *six* diodes; every voltage power supply uses *four* of them. Figure 16.26a shows an equivalent circuit for *one* such power supply (one phase); the charging battery is replaced by a resistor load. Two other circuits are identical. The circuit uses chassis ground; this is typical in automotive applications. A node-by node analysis (the nodes are marked as * and ** in Fig. 16.26a) shows that the circuit in Fig. 16.26a may be converted to the circuit shown in Fig. 16.26b. This circuit is "almost" the familiar full-wave rectifier from Fig. 16.24, except that the load is now connected to the chassis ground instead of the opposite end of the bridge. The opposite bridge end is also grounded. We remember, however, that the chassis ground is just a metal case, and it conducts the electric current exactly as an ordinary metal wire does. Thus, we can restore the missing connection and put the load in the middle of the bridge to obtain *exactly* the rectifier circuit of Fig. 16.24.

Three-Phase Diode Rectifier with Delta-Connected Alternator

Figure 16.26c shows the complete rectifying system for a delta-connected automotive alternator. Every independent power supply $v_{ab}(t)$, $v_{bc}(t)$, and $v_{ca}(t)$ is connected to its own bridge rectifier circuit shown in Fig. 16.26a, b. Those circuits share common diodes so that the total number of diodes is six. We reduce each circuit to the standard bridge rectifier model in Fig. 16.25. We can apply the method of assumed states to each individual circuit based on the ideal-diode model. The individual power supply voltages are given by $v_{ab}(t) = V_m \cos(\omega t)$, $v_{bc}(t) = V_m \cos(\omega t - 120°)$, and $v_{ca}(t) = V_m \cos(\omega t + 120°)$. The rectified load voltages have the form shown in Fig. 16.23b to within a phase shift.

Although every individual diode circuit is nonlinear, the currents add up at the load so that their linear superposition applies for the *net* load voltage and current. Figure 16.26d shows the rectified load voltage found as the sum of the three voltage contributions; the hardware prototype of the rectifier is shown below. We emphasize that modeling and simulation of automotive electrical power systems facilitate efficient design of the next generation of vehicles.

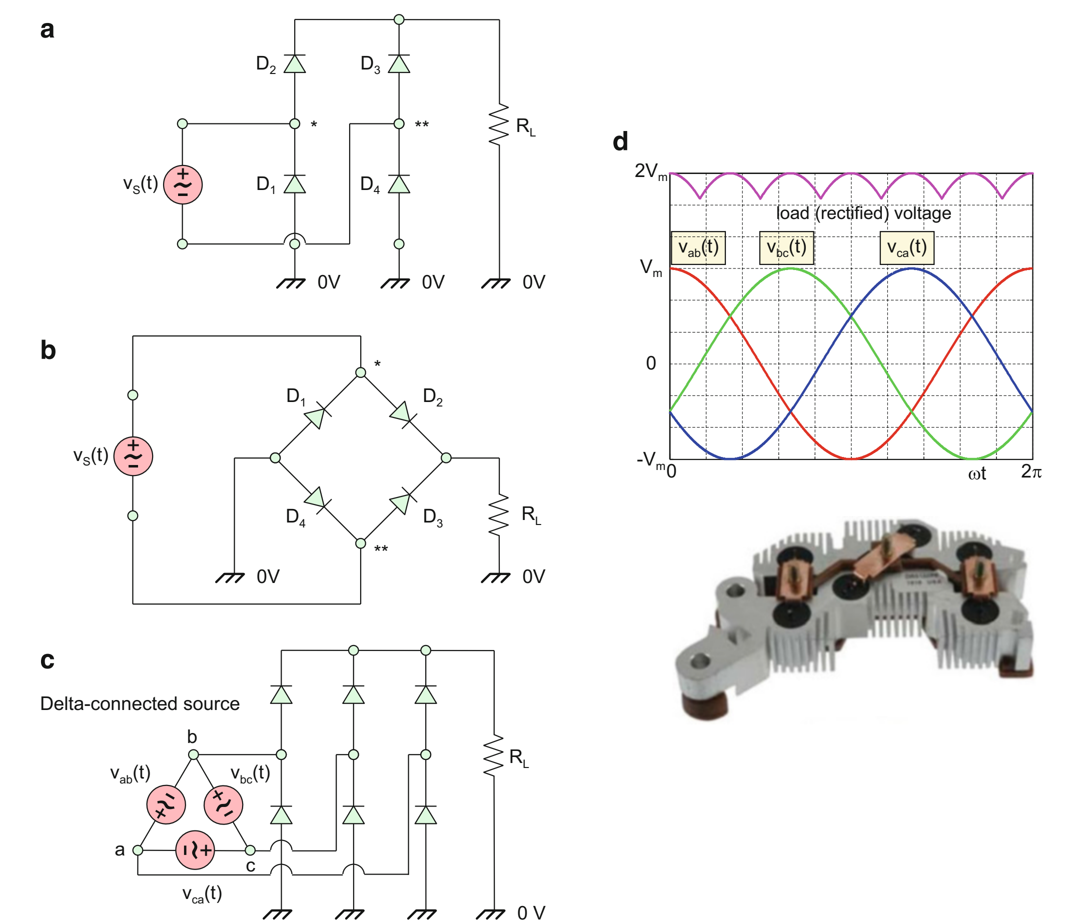


Fig. 16.26. (**a**) Model of one phase of an automotive battery-charging system. The voltage source is a winding of the alternator (three-phase voltage generator). (**b**) Conversion of the circuit depicted in (**a**) to the full-wave diode rectifier. (**c**) Delta-connected alternator and the diode rectifier bridge. (**d**) *Top*: rectified generator voltage; *bottom*: alternator rectifier circuit. Six power diodes with heat sinks are shown (with permission from General Motors).

16.3.7 Application Example: Envelope (or Peak) Detector Circuit

We now discuss a very different, low-power, application of the diode rectifier concept called the *envelope detector circuit* or the *peak detector circuit*. Such a rectifier circuit is primarily used for the *demodulation* of radio-frequency signals up to the very high frequencies of 60 GHz. It is also used for radio-frequency power measurements and RF power harvesting. A simple envelope detector circuit is shown in Fig. 16.27a. This circuit is identical to the half-wave rectifier circuit in Fig. 16.20, except for the addition of a capacitance C. The source voltage $v_S(t)$ is now the input voltage $v_{in}(t)$ and the load voltage $v_L(t)$ is now the output voltage $v_{out}(t)$.

Amplitude-Modulated Signal

The received and sufficiently amplified radio-frequency signal at the input to the envelope detector has the form:

$$v_{\text{in}}(t) = V_{\text{m}}[1 + m(t)] \cos \omega t \tag{16.16}$$

It is shown in Fig. 16.27b where $V_{\text{m}} \cos \omega t$ is the *radio-frequency carrier*. The carrier supports the radio transmission, but does not carry any useful information itself. The information is hidden in a low-frequency *amplitude envelope* seen in Fig. 16.27b. Mathematically, the envelope is described by a function $m(t)$, $|m(t)| < 1$ in Eq. (16.16). The parameter $m(t)$ is a *modulating signal*, a slowly varying function of time when compared to the carrier frequency. For example, the modulating signal may be a voice signal or a digital binary code (ON/OFF). In the simplest case of a pure modulating tone,

$$m(t) = A_{\text{m}} \cos \Omega t, \quad \Omega \ll \omega, \quad 0 \le A_{\text{m}} \le 1 \tag{16.17}$$

where Ω is the frequency of the modulating signal and A_{m} is the dimensionless *modulation depth*. The modulation depth is the *amplitude* of the modulating signal. The purpose of the envelope detector circuit is to recover the envelope of an amplitude-modulated signal.

Exercise 16.10: Determine the carrier frequency and the frequency of the modulating pure-tone signal in Fig. 16.27b.

Answer: 600 kHz and 100 kHz, respectively.

Operation of an Envelope Detector

The envelope carries information. Therefore, it should be extracted at the receiver, i.e., the received signal should be *demodulated* or *downconverted*. This is the underlying principle of wireless communications. The goal of the envelope detector is to perform this task. We assume the ideal-diode model for simplicity. Without the capacitor, the circuit performs identical to the half-wave diode rectifier as shown in Fig. 16.27c. However, when the capacitor in Fig. 16.27a is present, the situation changes.

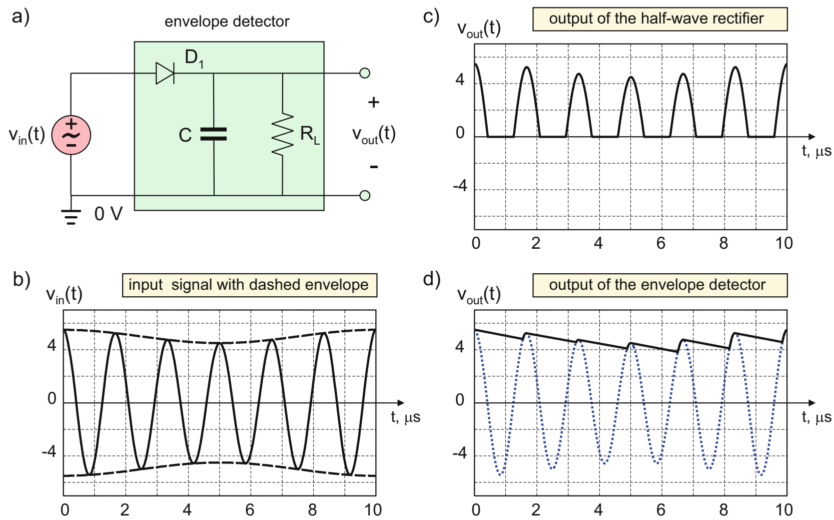


Fig. 16.27. Envelope detector circuit and its operation; (**a**) circuit, (**b**) modulated input signal, (**c**) output signal without capacitor *C*, and (**d**) output signal with capacitor *C*.

The capacitor is charged when the diode conducts, i.e., when the input voltage $v_{in}(t)$ is positive. This process continues until the capacitor voltage $v_C(t)$ reaches the *peak* input voltage of the positive phase. Beyond this point, the diode becomes reverse biased and the capacitor voltage remains the same since there is no discharge path through the diode. However, there is another discharge path through resistor R_L. If R_L is large, the capacitor still discharges slowly between two time periods. Its voltage shown in Fig. 16.27c by a solid curve approximately follows the signal envelope. So does the output voltage of the circuit, which is indeed equal to the capacitor voltage, $v_{out}(t) = v_C(t)$. The appropriate envelope extraction implies a proper choice for the time constant τ of the corresponding RC circuit, $\tau = R_L C$. This constant should satisfy two inequalities:

$$\tau \gg T_{carrier} = \frac{2\pi}{\omega}, \quad \tau \ll T_{modulation} = \frac{2\pi}{\Omega} \tag{16.18}$$

where $T_{carrier}$, $T_{modulation}$ are the period of the carrier signal and the (minimum) period of the modulation signal, respectively. The first inequality in Eq. (16.18) ensures that there are no significant ripples between two consecutive short periods of the carrier. When this inequality is satisfied, the small ripple voltage is equal to $(T_{carrier}/\tau)V_m$ given no modulation. The second inequality in Eq. (16.18) ensures that the output to the envelope detector does follow the modulation signal, but does not stay the same all the time. Figure 16.27 corresponds to the so-called linear region of operation of the envelope detector where its output voltage is proportional to the modulating signal. Figure 16.28 shows a basic AM radio receiver circuit constructed in an undergraduate laboratory. The envelope detector marked by a circle block follows the front-end amplifier block.

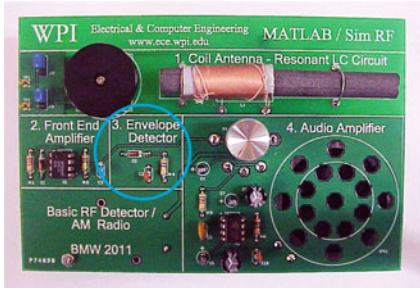


Fig. 16.28. AM radio receiver circuit with the envelope detector.

Example 16.10: Design an envelope detector given the carrier frequency of $f = 1\,\text{MHz}$. The modulation is a human voice, with the maximum passing modulation frequency of $5\,\text{kHz}$.

Solution: First, we find the period of the carrier and the period of the modulating signal: $T_{\text{carrier}} = 1\,\mu\text{s}, T_{\text{modulation}} = 200\,\mu\text{s}$. A number of choices may satisfy Eq. (16.18). One possible choice is given by $R_{\text{d}} = 50\,\text{k}\Omega, C_{\text{d}} = 0.5\,\text{nF}$. Thus, $\tau = R_{\text{L}}C = 25\,\mu\text{s}$.

Modeling Envelope Detector and Square-Law Region of Operation

The circuit shown in Fig. 16.27 (and other similar diode circuits) can be analyzed using SPICE. In some cases, it is also useful to have an analytical description. KCL gives

$$C\frac{dv_{\text{out}}}{dt} = i_{\text{D}} - \frac{v_{\text{out}}}{R_{\text{L}}} \tag{16.19a}$$

The corresponding nonlinear ODE for the envelope detector has the form:

$$\frac{dv_{\text{out}}}{dt} + \frac{v_{\text{out}}}{\tau} = \frac{R_{\text{L}}I_{\text{S}}}{\tau}\exp[(v_{\text{in}} + V_{\text{bias}} - v_{\text{out}})/V_{\text{T}}] \tag{16.19b}$$

where V_{bias} is an extra DC bias voltage applied to the diode. Equation (16.19b) is then solved numerically as done in a number of software packages including MATLAB. The model of the envelope detector studied thus far implies large input signals. For small input signals, an appropriate DC bias voltage can be applied to the diode. For very small input signals, which are less than thermal voltage $V_{\text{T}} \approx 0.026\,\text{V}$, the (modified) small-signal diode model developed in the previous section can be used. Its key modification is that the last square term on the right-hand side of Eq. (16.7c) must be retained. As a result, the circuit behavior becomes quite different. The output voltage now follows the *square* of the input voltage. Thus, the envelope detector operates not in the *linear region* (as for large input signals), but in the *square-law region*, where its output voltage is proportional to the power of the received signal, and not to its linear envelope. In the square-law region, the envelope detector is a simple and versatile *radio-frequency (RF) power meter*.

Section 16.4 Diode Wave-Shaping Circuits

This section studies common wave-shaping (signal-processing) diode circuits such as clamper circuits, voltage doublers and multipliers, and clipper circuits. These circuits find applications in power electronics. We keep the notation $v_{in}(t)$ for the input voltage of the circuit and $v_{in}(t)$ for the output voltage of the circuit, respectively. As to the phase of an AC source, we choose the input voltage in the form $v_{in}(t) = -V_m \sin \omega t = V_m \cos(\omega t + \pi/2)$, which is perhaps most convenient for the graphical representation of the clamper and multiplier circuit operation.

16.4.1 Diode Clamper Circuit (DC Restorer)

The *diode clamper circuit* or *DC restorer circuit* without a load resistor is shown in Fig. 16.29a. It is identical to the envelope (or peak) detector circuit seen in Fig. 16.27a with the load resistor removed. However, the output voltage is now the (reverse) voltage across the diode or the voltage across the voltage supply and capacitance C_1 *in series*. This change leads to different applications. A similar situation occurs for filter circuits, where voltages across different elements lead to different filter types.

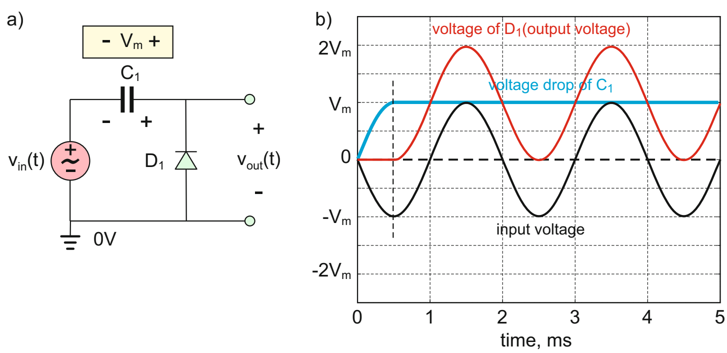


Fig. 16.29. Diode clamper circuit and the corresponding voltage waveforms. The steady-state value of the capacitor voltage is shown in the *box*.

The idea behind the circuit in Fig. 16.29a is explained using the ideal-diode model. We assume that the input voltage has the period of 2 ms and starts with a negative phase, i.e., $v_{in}(t) = -V_m \sin \omega t$ in Fig. 16.29b. The capacitor starts to charge when diode D_1 conducts, i.e., when the input voltage $v_{in}(t)$ is negative. The charge polarity is shown in Fig. 16.29a. This process continues until the capacitor voltage $v_C(t)$ reaches the *peak* input voltage V_m. Afterwards, the diode becomes reverse biased and the capacitor voltage remains the same over an infinitely large number of periods as in Fig. 16.29b since there is no discharge path through the diode. The output voltage with respect to circuit ground is simply the sum of the capacitor voltage and the supply voltage; it is shown in

Fig. 16.29b as the top curve. We conclude that, in the clamper circuit, the lowest peak of a waveform is *clamped* to zero with respect to ground, hence its name. This is important for square digital waveforms in *pulse-width modulation*, which are generated as entirely positive pulse trains, but transmitted with zero mean values. The clamper circuit lifts up the entire signal and thus makes it possible to measure and utilize the variable *duty cycle* of the pulse train, which carries information. This process effectively restores the lost DC reference voltage with respect to the receiver ground; here is another name of the *DC restorer circuit*.

Exercise 16.11: How do voltage waveforms in Fig. 16.29b change if the diode polarity in Fig. 16.29a is reversed and the input voltage becomes $v_S(t) = V_m \sin \omega t$?

Answer: All three curves in Fig. 16.29b will be mirror-reflected with respect to the x-axis. Thus, the output (diode) voltage will always be negative.

Exercise 16.12: How will the diode voltage in Fig. 16.29b change if the input voltage has an extra DC component of $-V_m$?

Answer: The diode voltage with respect to circuit ground will not change. However, the steady-state capacitor voltage will double.

When a finite load resistance R_L is connected in shunt with the diode D_1 in Fig. 16.29a, the ideal output waveform in Fig. 16.29b will be distorted and it will no longer have only positive output voltages.

16.4.2 Diode Voltage Doubler and Multiplier

Diode voltage multiplier circuits are employed to generate a high-voltage DC signal from an AC source. Such circuits generally avoid using an electric transformer in applications where its use is impractical due to size, safety, loss, and other issues. For example, voltage multipliers can boost low-voltage radio-frequency signals received by an antenna. They can also be used to generate high static voltages for special power supplies. Figure 16.30a shows a circuit for the *diode voltage doubler* invented in 1914 by Heinrich Greinacher, a German-Swiss experimenter. Typically, the output capacitance C_2 is larger than the series coupling capacitance C_1. The idea behind the circuit in Fig. 16.29a will be explained using the ideal-diode model and a combination of already studied *cascaded* circuit blocks. Here again, we note an obvious analogy with the AC filter circuits, which may also combine multiple filter stages with different features in one.

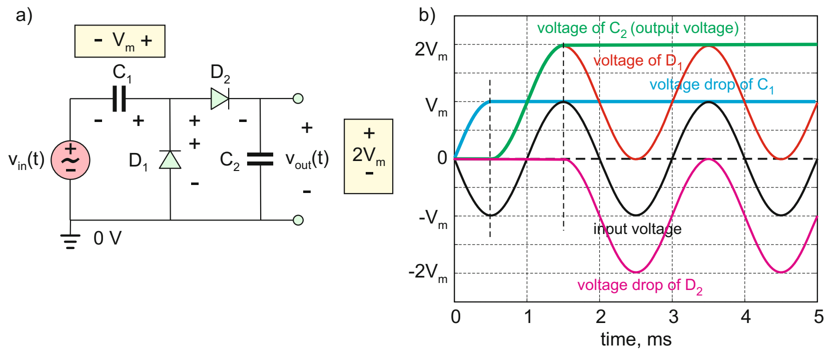


Fig. 16.30. Diode voltage doubler circuit (Greinacher circuit) and the corresponding voltage waveforms. The steady-state values of the capacitor voltages are shown in the *boxes*.

We note that the *first stage* of the doubler is the *clamper circuit* from Fig. 16.29. The voltage across diode D_1 is exactly the curve depicted in Fig. 16.29b; see also Fig. 16.30b. This voltage is now an input voltage to the *second stage* of the circuit which is the envelope detector circuit in Fig. 16.27a without the load resistor. Therefore, the circuit in Fig. 16.30 generates a positive DC voltage of $2V_{\mathrm{m}}$ when excited by the input signal $v_{\mathrm{in}}(t) = -V_{\mathrm{m}} \sin \omega t$; see Fig. 16.30b. It is therefore named a *doubler circuit*.

Exercise 16.13: How will the output voltage of the doubler circuit in Fig. 16.30b change if the input voltage has an extra DC component of $-V_{\mathrm{m}}$?

Answer: The output voltage versus ground will not change.

A natural extension of the doubler circuit is the *diode voltage quadrupler circuit* shown in Fig. 16.31. We will describe the circuit operation and determine the output voltage in the AC steady state. The key point is again the stage-by-stage analysis. The quadrupler circuit in Fig. 16.31 includes two *cascaded* diode voltage doublers. However, the output of the first doubler is not the DC voltage of $2V_{\mathrm{m}}$ across capacitor C_2, but the voltage drop v_{D2} across diode D_2. According to KVL, $v_{\mathrm{D2}} = v_{\mathrm{in}} + v_{\mathrm{C1}} - v_{\mathrm{C2}}$. If we employ the already existing curves $v_{\mathrm{C1}}, v_{\mathrm{C2}}$ from Fig. 16.30b, we obtain the voltage drop v_{D2} in the form shown in Fig. 16.30b and repeated in Fig. 16.31b. Now, we employ this voltage, which is $v_{\mathrm{in}}(t) - V_{\mathrm{m}}$ for the AC steady state, as the *input* voltage to the second voltage doubler. The second voltage doubler is also using the new reference ground point of $2V_{\mathrm{m}}$—the virtual ground. The lowest peak of v_{D2} is clamped to zero volts versus the new ground point and then rectified; see Fig. 16.31b. Hence, we obtain the DC voltage of $4V_{\mathrm{m}}$ at the output of the quadrupler circuit versus the original circuit ground as shown in Fig. 16.31a. Multiplier circuits of a different order are constructed in a similar way.

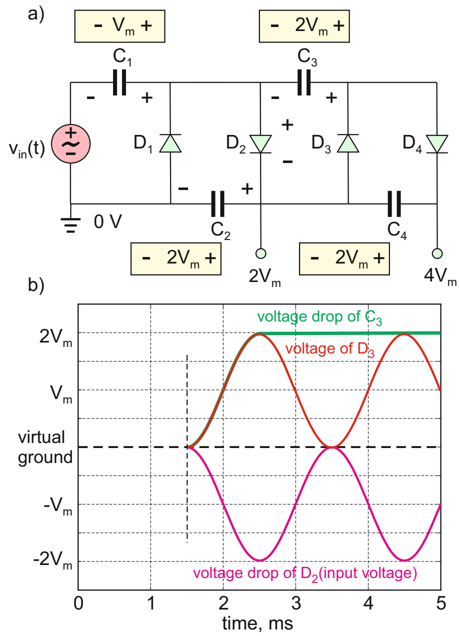


Fig. 16.31. Voltage quadrupler diode circuit and the corresponding voltage waveforms. The steady-state values of the capacitor voltages are shown in the *boxes*.

16.4.3 Positive, Negative, and Double Clipper

Diode clipper circuits derive their name by operating in such a way that a part of the input signal *is clipped off* at the output. Another name is the *diode limiter circuits*, which often implies a protection function of such circuits against overload, electrostatic discharge, etc. Figure 16.32a shows the *positive diode clipper circuit*. The circuit topology is exactly that of the half-wave diode rectifier in Fig. 16.20. However, the output voltage is now the diode voltage, not the resistor voltage.

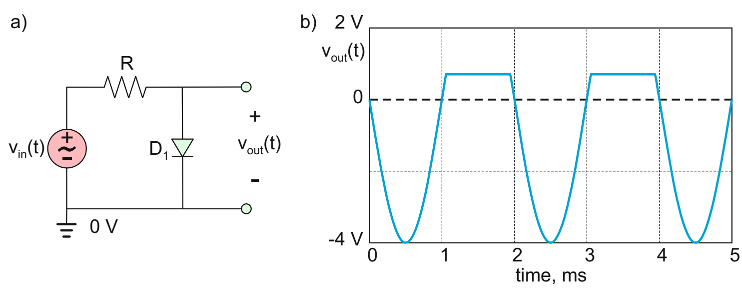


Fig. 16.32. Positive diode clipper circuit and the corresponding voltage waveform.

When the constant-voltage-drop-diode model (with the voltage drop of 0.7 V) is used, the circuit output has the form shown in Fig. 16.32b given that the input voltage is $v_{\mathrm{in}}(t) = -V_{\mathrm{m}} \sin \omega t, \;\; V_{\mathrm{m}} = 4\,\mathrm{V}$. The positive signal voltages above +0.7 V are thus clipped off. By analogy with the positive clipper, we can also introduce the *negative diode clipper circuit* and the *double diode clipper circuit*, see Fig. 16.33. The negative clipper circuit clips negative signal voltages below −0.7 V, whereas the double clipper circuit clips both halves of the voltage waveform if they exceed ±0.7 V. Similar clipper circuits can be constructed with the Zener diodes. A single Zener diode may in fact clip voltages of both polarities, in the forward-bias region and in the breakdown region, respectively.

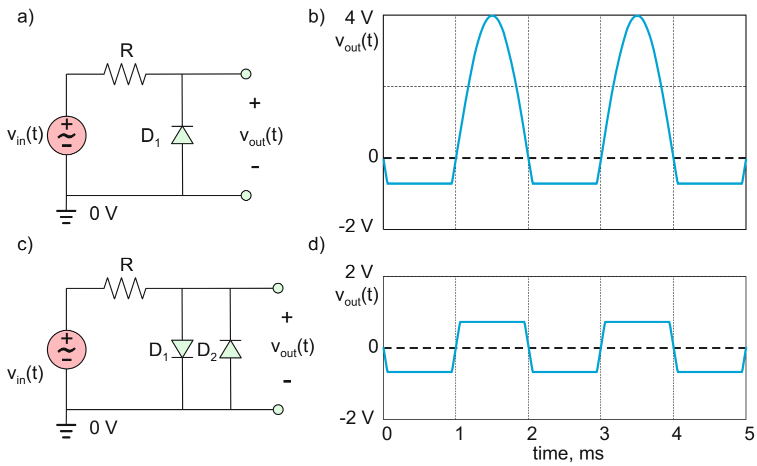


Fig. 16.33. Negative clipper circuit, double clipper circuit, and the corresponding voltage waveforms.

Example 16.11: Figure 16.34a shows a diode clipper circuit commonly used for electrostatic discharge (ESD) protection. Describe its operation and plot to scale the output voltage over three periods given the input voltage of $v_{\text{in}}(t) = -V_{\text{m}} \sin \omega t$ with $V_{\text{m}} = 10$ V and $V_{\text{CC}} = 5\,\text{V}$.

Solution: We will use the method of assumed states and the constant-voltage-drop model to solve the diode circuit. When $v_{\text{in}}(t) < -0.7\,\text{V}$, diode D_2 conducts. The output voltage stays at −0.7 V. When $v_{\text{in}}(t) > -0.7\,\text{V}$, diode D_2 is an open circuit with no influence on the solution. However, diode D_1 starts to conduct when $v_{\text{in}}(t) > V_{\text{CC}} + 0.7\,\text{V}$. In this case, the output voltage is the voltage across D_1 plus +0.7 V. Thus, the output voltage has the form shown in Fig. 16.34b. In other words, the diode circuit in Fig. 16.34a prevents the signal from exceeding the power supply rails by more than 0.7 V.

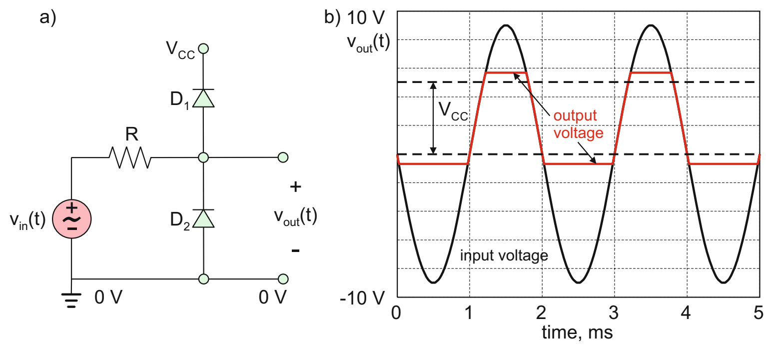


Fig. 16.34. ESD discharge protection clipper circuit and the corresponding voltage waveforms.

16.4.4 Transfer Characteristic of a Diode Circuit

Since the diode circuits are inherently nonlinear, the meaning of the amplitude/phase *transfer function*, which is common for linear circuits, cannot be applied. Instead, the diode circuits are described by their *transfer characteristic*. The transfer characteristic of the diode circuit is simply the *ratio of the instantaneous output voltage to the instantaneous input (source) voltage*. Thus, in order to find the transfer characteristic, we consider the circuit behavior in the DC steady state. The transfer characteristic is well defined for the diode circuits containing *only* diodes and resistances. Those are simple rectifiers and clippers/limiter circuits. On the other hand, the transfer characteristic of the diode circuits with dynamic circuit elements (clamper circuits, voltage doublers and multipliers) is not well defined since it is time dependent. Figure 16.35 shows the transfer characteristics of three clipper circuits from Figs. 16.32 and 16.33, respectively.

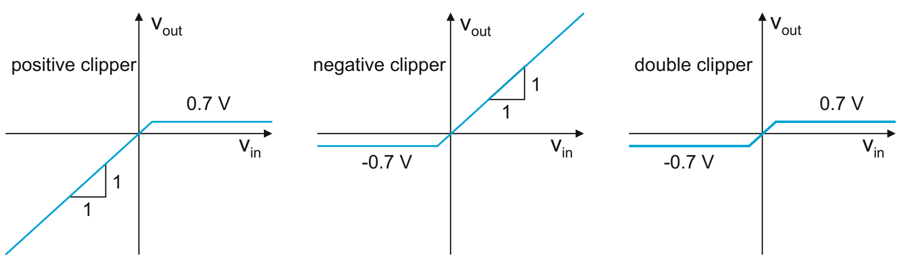


Fig. 16.35. Transfer characteristics of diode clipper circuits.

Example 16.12: The clipper (limiter) circuits studied in Figs. 16.32 and 16.33 are called the *hard limiters*. *Soft limiters* also exist: they are characterized by a *smoothe*r transfer characteristic. The circuit shown in Fig. 16.36a is the *positive soft limiter*. Plot its transfer characteristic given that $R_2 = 0.5R_1$.

Example 16.12 (cont.):

Solution: We use the method of assumed states and the constant-voltage-drop model to solve the diode circuit.

When $\upsilon_{\text{in}} < 0.7\,\text{V}$, diode D_1 does not conduct. The output voltage is exactly the input voltage which corresponds to the straight line of slope 1 in Fig. 16.36b. When $\upsilon_{\text{in}} > 0.7\,\text{V}$, the output voltage is the voltage across resistance R_2 and the diode combined, that is, $\upsilon_{\text{out}} = R_2/(R_1 + R_2)(\upsilon_{\text{in}} - 0.7) + 0.7\,\text{V}$. Given $R_2 = 0.5R_1$ this voltage corresponds to the straight line of slope 1/3 in Fig. 16.36b. Other variations are possible.

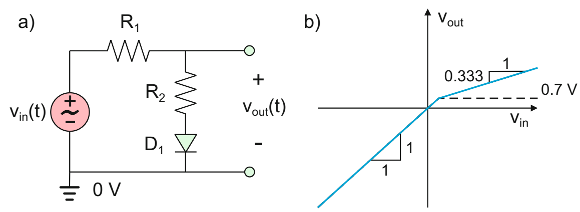
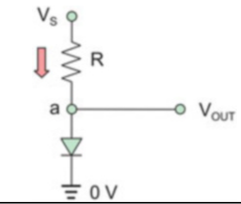
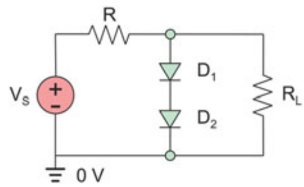
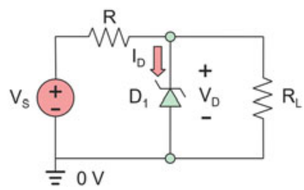
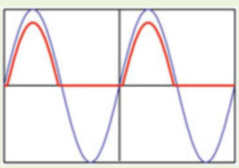
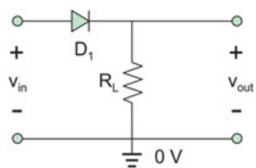
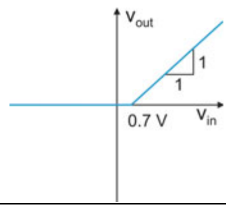
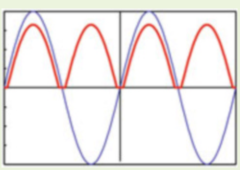
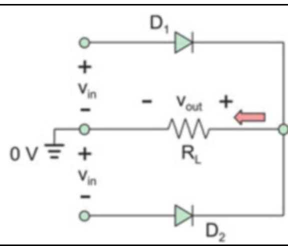
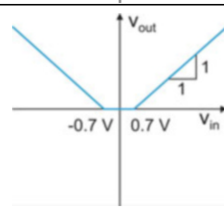
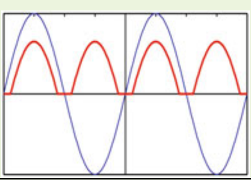
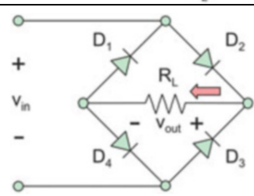
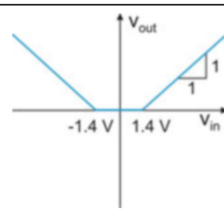


Fig. 16.36. Transfer characteristic of the soft limiter circuit.

Summary

Common diode circuits—circuits with diodes and resistors		
Sensors, voltage references/regulators		
1. Diode temperature sensor		1. Uses temperature dependence of diode pn-junction parameters 2. Resistor R determines necessary diode current 3. Typical sensitivity is minus 2 mV to minus 4 mV per 1 °C
2. Forward-bias voltage reference/voltage regulator		1. Provides a fixed reference voltage in a circuit 2. Provides a constant DC voltage to the load 3. Solved using the constant-voltage-drop model or the small-signal diode model
3. Zener voltage regulator		For high-resistance load: $V_L = r_Z \frac{V_S - V_{Z0}}{R + r_Z} + V_{Z0}$, $V_{Z0} = V_{ZT} - r_Z I_{ZT}$
Rectifiers and clippers		
4. Half-wave rectifier 		
5. Full-wave rectifier 		
6. Full-wave bridge rectifier 		

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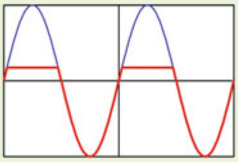
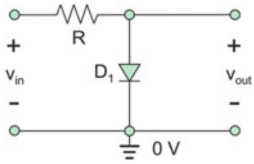
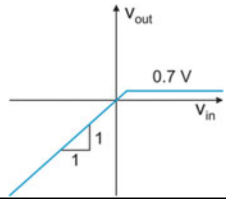
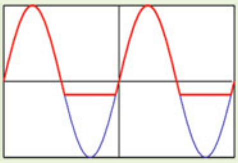
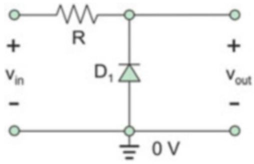
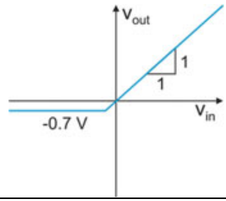
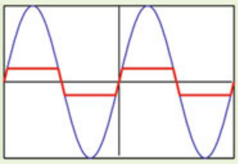
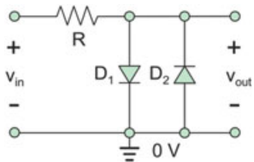
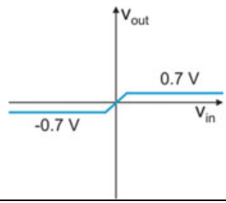
</td>
<td>

Clips positive voltages</td>
<td>
</td>
</tr>
<tr>
<td>8. Negative clipper/limiter
</td>
<td>

Clips negative voltages</td>
<td>
</td>
</tr>
<tr>
<td>9. Double clipper/limiter
</td>
<td>

Clips pos./neg. voltages</td>
<td>
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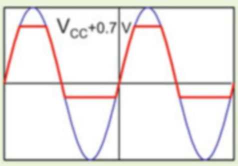
</tr>
<tr>
<td>10. ESD protection circuit

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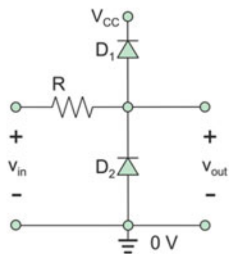
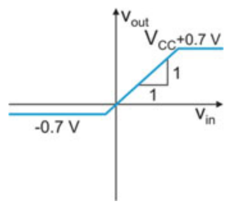
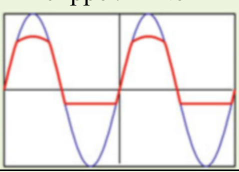
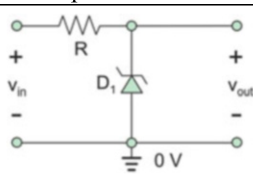
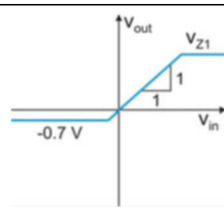
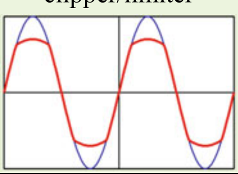
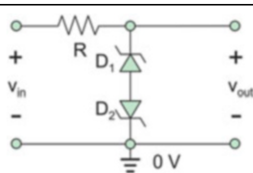
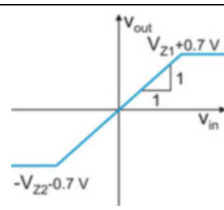
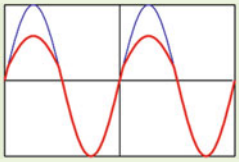
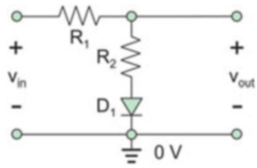
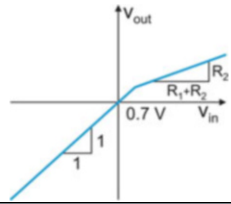
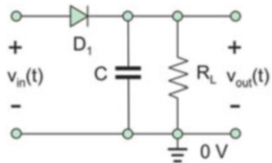
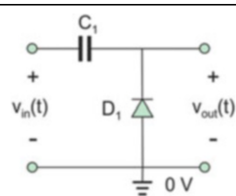
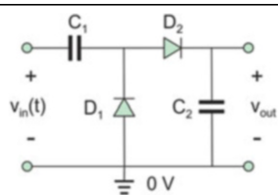
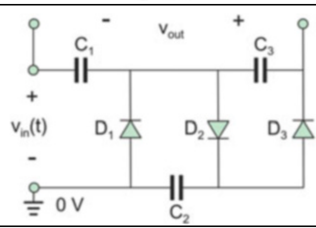
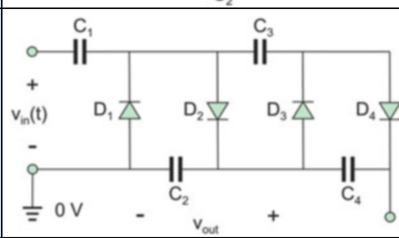
<td>

Clips signals outside power rails</td>
<td>
</td>
</tr>
<tr>
<td>11. Zener diode clipper/limiter
</td>
<td>

Clips positive/negative voltages</td>
<td>
</td>
</tr>
<tr>
<td>12. Zener diode double clipper/limiter
</td>
<td>

Clips positive/negative voltages</td>
<td>
</td>
</tr>
</table>

(continued)

13. Soft clipper/limiter 	 Smoothly clips positive voltages	
Common diode circuits—circuits with diodes, capacitors, and resistors		
14. Envelope detector or peak detector		Outputs signal envelope $\tau = R_L C$ $\tau >> T_{\text{carrier}},\ \tau << T_{\text{modulation}}$ $\frac{dv_{out}}{dt} + \frac{v_{out}}{\tau} = \frac{R_L I_S}{\tau} \times$ $\exp[(v_{in} + V_{bias} - v_{out})/V_T]$
15. Clamper circuit or DC restorer		1. Lowest peak of a signal is *clamped* to zero volts versus ground 2. Has no effect on already clamped signals 3. Used in clock recovery circuits and in PWM
16. Diode voltage doubler		1. Outputs the DC voltage of $2V_m$ if the input signal has the amplitude of V_m 2. Constructed as a combination of the clamper and the envelope detector
17. Diode voltage tripler		1. Outputs the DC voltage of $3V_m$ if the input signal has the amplitude of V_m 2. Constructed as a combination of the voltage doubler and the envelope detector
18. Diode voltage quadrupler		1. Outputs the DC voltage of $4V_m$ if the input signal has the amplitude of V_m 2. Constructed as a combination of two voltage doublers

Problems

16.1 Diode Operation and Classification

16.1.1 Circuit Symbol and Terminals

16.1.2 Three Regions of Operation

16.1.3 Mechanical Analogy of Diode Operation

16.1.4 Forward-Bias Region: Switching Diode

16.1.5 Reverse-Bias Region: Varactor Diode

16.1.6 Breakdown Region: Zener Diode

16.1.7 Other Diode Types

Problem 16.1. Draw the circuit symbol for the diode, labeling the anode and the cathode. In what direction does the electric current flow?

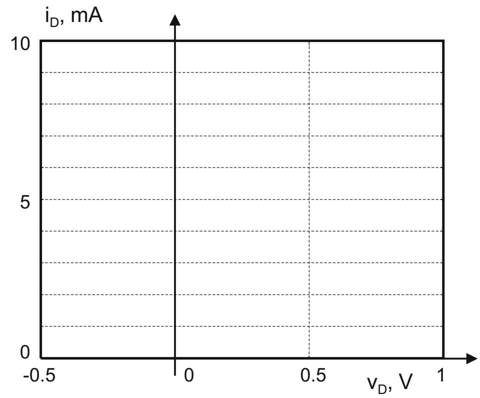
Problem 16.2. A package for a small-signal 1N4148 Si switching diode (yellow package) is shown in the figure. Where is its anode, on the left or on the right?

Problem 16.3.
A. Sketch the typical *v–i* diode curve
B. Indicate three regions of diode operation and write the name of each region on the figure.

Problem 16.4. Determine thermal voltage, which is present in Shockley equation at:
A. 0 °C
B. 10 °C
C. 20 °C
D. 40 °C

Problem 16.5. Plot the *v–i* characteristic of a diode with:
A. $n = 1.0, I_S = 1.1\,\text{nA}$
B. $n = 2.0, I_S = 1.1\,\text{nA}$

at room temperature of 25 °C on the same figure. Use the figure that follows as a template. Label each curve. Use the value of 1.60218×10^{-19} C for the electron charge and the value of 1.38066×10^{-23} J/K for the Boltzmann constant.



Problem 16.6. Plot the *v–i* characteristic of a diode with:
A. $n = 1.0, I_S = 1\,\text{nA}$
B. $n = 1.0, I_S = 0.01\,\text{nA}$

at room temperature of 25 °C on the same figure. Use the figure to the previous problem as a template. Label each curve. Use the value of 1.60218×10^{-19} C for the electron charge and the value of 1.38066×10^{-23} J/K for the Boltzmann constant.

Problem 16.7. For a diode with $n = 2.0$, the following measurement is taken: $v_D = 0.65\,\text{V}$ and $i_D = 1\,\text{mA}$. Given thermal voltage of 26 mV, determine diode's saturation current I_S.

Problem 16.8. For a diode with $I_S = 1\,\text{pA}$, the following measurement is taken: $v_D = 0.62\,\text{V}$ and $i_D = 1\,\text{mA}$. Given thermal voltage of 26 mV, determine diode's ideality constant, n.

Problem 16.9. At which forward-bias voltage in terms of V_T does the diode conduct a current of $10^4 I_S$ given the ideality factor of two?

Problem 16.10. A diode with $n = 2.0$ is to be used as a temperature sensor in the forward-bias region (it is a common diode application). Determine:
A. The corresponding change in the diode voltage (initial value, final value, and the difference) when the temperature rises from 20 to 60 °C

B. Sensitivity of the device in mV/°C

The diode current is fixed at 1 mA. You are given that $I_S = 1\,\text{nA}$ at 20 °C and that I_S doubles in value for every 5 °C. Use the value of 1.60218×10^{-19} C for the electron charge and the value of 1.38066×10^{-23} J/K for the Boltzmann constant.

Problem 16.11. Answer the following questions:

A. Which diode is used as a variable capacitor? Draw its symbol.
B. Which diode operates in the breakdown region? Draw its symbol.
C. Draw the circuit symbol for the Schottky barrier diode.
D. Draw the circuit symbol for the photodiode.

16.2 Diode Models

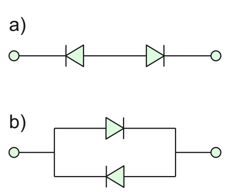
16.2.1 Ideal-Diode Model: Method of Assumed States

Problem 16.12.

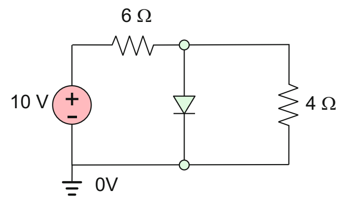
A. What is an ideal-diode model?
B. Draw its volt-ampere characteristic using the voltage axis from −5 V to 5 V and the current axis from −10 mA to +10 mA as shown in the figure that follows. On the same graph draw the *v–i* characteristic for a 250 Ω resistor to scale.

Problem 16.13. After solving a circuit with ideal diodes, what check is necessary for diodes initially assumed to be ON? OFF?

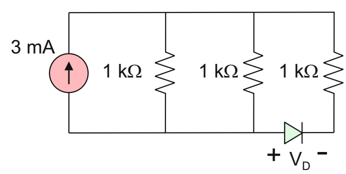
Problem 16.14. Present equivalent circuits for the two-diode configurations shown in the figure, assuming ideal diodes.



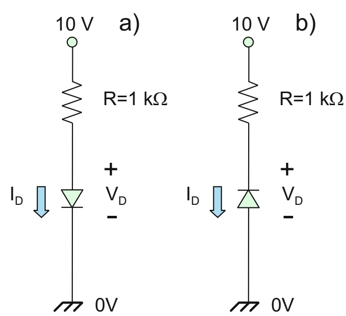
Problem 16.15. Determine the electric current through the 1 kΩ resistor for the circuit shown in the figure below, assuming the ideal diode.



Problem 16.16. Assuming the ideal-diode model, find the voltage across the diode and the diode current for the circuit shown in the following figure. Denote the solution for the diode voltage and diode current in the DC steady state by capital letters V_D and I_D, respectively.



Problem 16.17. For the circuits shown in the figure that follows, find values of the diode current and voltage across the diode assuming that the diodes are ideal. Use capital letters V_D and I_D to denote the solution for the diode voltage and diode current in the DC steady state.

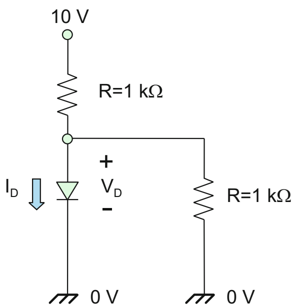


Problem 16.18. Using the ideal-diode model, you need to design a circuit for the diode temperature sensor described in Problem 16.10.

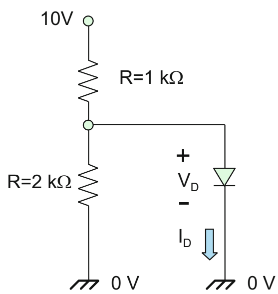
The diode current must be fixed at 1 mA. The power supply voltage is fixed at 9 V.

A. Present the corresponding circuit diagram and specify the component (s) values.
B. Label the sensor output voltage.

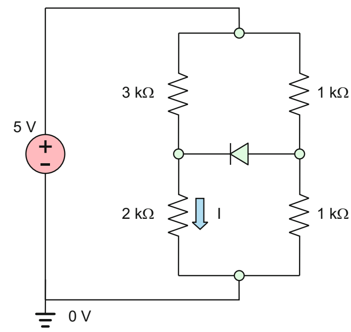
Problem 16.19. A diode circuit is shown in the figure that follows. Find the values of the diode current and the voltage across the diode, assuming that the diode is ideal.



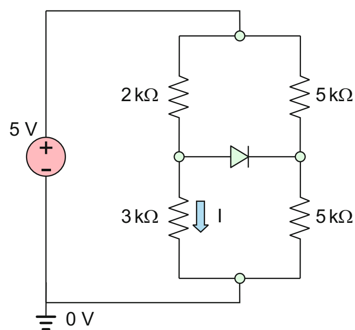
Problem 16.20. For the diode circuit shown in the following figure, determine the values of the diode current and the voltage across the diode, assuming that the diode is ideal.



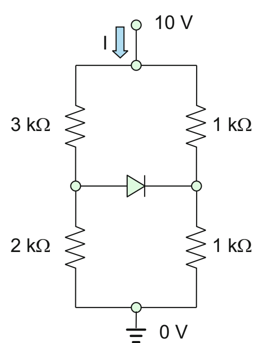
Problem 16.21. Assuming the ideal-diode model, find current *I* for the circuit shown in the figure below.



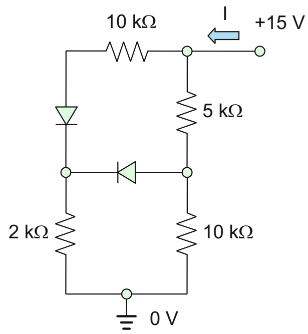
Problem 16.22. Assuming the ideal-diode model, find current *I* for the circuit shown in the figure below.



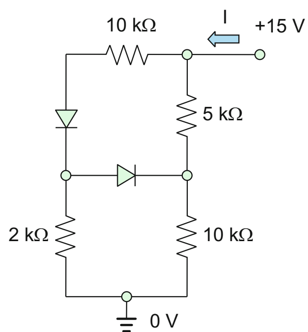
Problem 16.23. For the circuit shown in the figure below, determine circuit current *I*, assuming that the diode is ideal. The ground path is simultaneously the current return path.



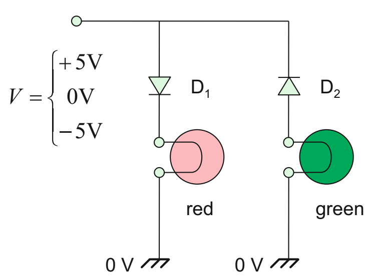
Problem 16.24. For the circuit below find circuit current *I*, assuming that both diodes are ideal. The ground path is simultaneously the current return path.



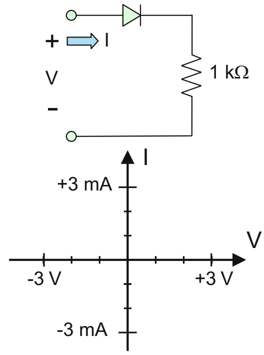
Problem 16.25. For the circuit shown in the figure below, find circuit current I, assuming that the diodes are ideal. The ground path is simultaneously the current return path.



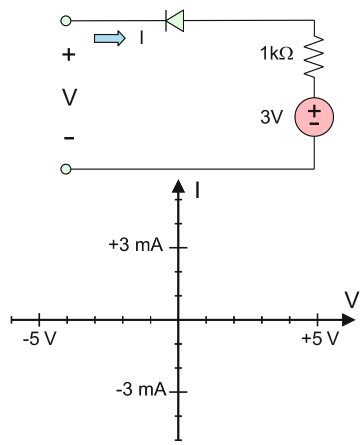
Problem 16.26. The circuit shown in the figure below can be used as a signaling system using one wire plus a common ground return. At any moment, the input has one of three voltage values shown in the figure. What is the status of the lamps for each input value?



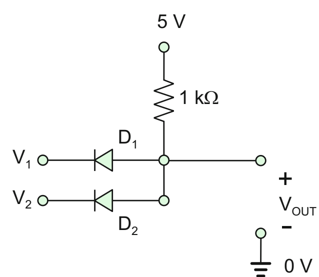
Problem 16.27. Sketch I versus V to scale for the circuit shown in the following figure. Assume the ideal-diode model and allow V to range from -3 V to 3 V.



Problem 16.28. Sketch I versus V to scale for the circuit shown in the figure. Assume the ideal-diode model and allow V to range from -5 V to 5 V.



Problem 16.29. For the circuit shown in the figure below, fill out Table 16.5.



What type of logic gate is it?

Table 16.5. Output voltage of the diode circuit as a function of two input voltages.

V_1 (V)	V_2 (V)	V_{OUT}
0	0	
0	5	
5	0	
5	5	

Problem 16.30. A freshman ECE student attends class if all of the following conditions are satisfied:

A. He/she feels that this lecture might be useful.
B. There are no other more important things to do.
C. The way to the Department is cleaned up from snow.

Every morning he/she "votes" by simultaneously pushing any appropriate combination of three 5-V buttons (A, B, C) placed in parallel. A simple diode circuit is needed that lights a green LED when there is time to go to the lecture.

Problem 16.31. A small county board is composed of three commissioners. Each commissioner votes on measures presented to the board by pressing a 5-V button indicating whether the commissioner votes for or against a measure. If two or more commissioners vote for a measure, it passes. You are asked to help with a an ideal-diode circuit that takes the three votes as inputs and lights a green LED to indicate that a measure passed. You can use as many diodes/resistors as you need.

1. Explain your reasoning for building the diode circuit.
2. Present the appropriate circuit diagram.

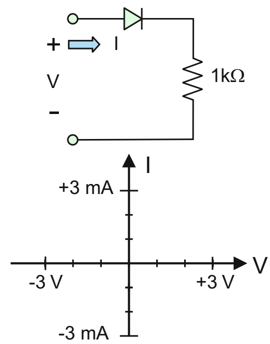
16.2.2. Constant-Voltage-Drop Model

Problem 16.32. What is the constant-voltage-drop-diode model? Draw the corresponding v–i diagram.

Problem 16.33. Sketch I versus V to scale for the circuit shown in the following figure using:

A. Ideal-diode model
B. Constant-voltage-drop-diode model with the turn-on voltage of 1 V

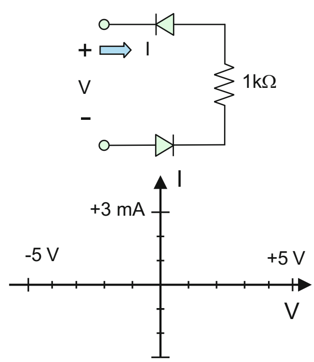
Allow V to range from −3 V to 3 V.



Problem 16.34. Sketch I versus V to scale for the circuit shown in the figure using:

A. Ideal-diode model
B. Constant-voltage-drop-diode model with the turn-on voltage of 1 V

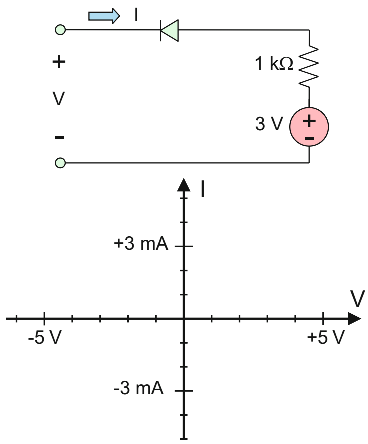
Allow V to range from −5 V to 5 V.



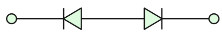
Problem 16.35. Sketch I versus V to scale for the circuit shown in the figure using:

A. Ideal-diode model
B. Constant-voltage-drop-diode model with the turn-on voltage of 1 V

Allow V to range from −5 V to 5 V.



Problem 16.36. Present equivalent circuit for the two-diode configuration shown in the figure, assuming the constant-voltage-drop-diode model with the turn-on voltage of 1 V.



Problem 16.37. Using the constant-voltage-drop-diode model, you need to design a circuit for the diode temperature sensor described in Problem 16.10. The diode current must be fixed at 1 mA. The power supply voltage is fixed at 9 V.

A. Present the corresponding circuit diagram and specify the component (s) values.
B. Label the sensor output voltage.

16.2.3 Exponential Model in the Forward-Bias Region and Its Use

16.2.4 Load-Line Analysis

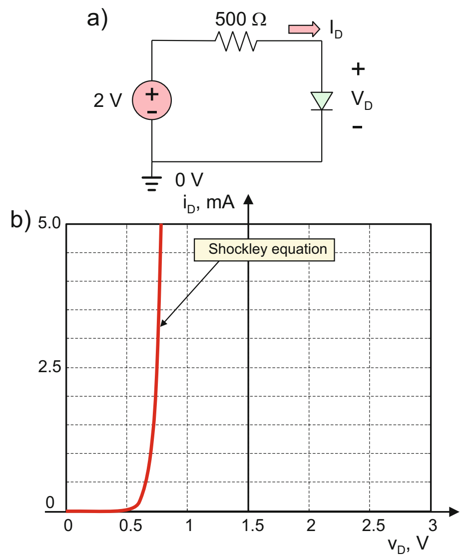
16.2.5 Iterative Solution

Problem 16.38. A 1N4148 diode manufactured by Fairchild has a current of 0.7 mA at 0.6 V and a current of 8 mA at 0.725 V, all at 25°. Determine the ideality factor and the saturation current of Shockley equation at this temperature.

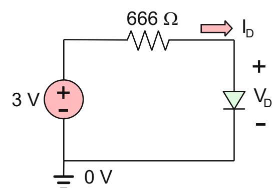
Problem 16.39. A 1N4148 diode manufactured by Hitachi has a current of 0.15 mA at 0.6 V and a current of 1.5 mA at 0.7 V, all at *minus* 25°. Determine the ideality factor and the saturation current of Shockley equation at this temperature.

Problem 16.40. In the circuit shown in the figure below, the diode is described in terms of the exponential forward-bias model with the Shockley equation plotted in the figure.

A. Graphically determine the solution—the DC operating point V_D, I_D using the load-line method.
B. Compare the obtained diode current with that found in the constant-voltage-drop model.

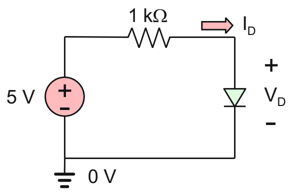


Problem 16.41. Repeat the previous problem for the circuit shown in the figure that follows.



Problem 16.42. In the circuit shown in the figure that follows, the diode is described in terms of the exponential forward-bias model where the ideality factor and the saturation

current of Shockley equation are $n = 1.5, I_S = 1\,\mathrm{nA}$. Given thermal voltage of 0.026 V, determine the exact DC operating point (diode voltage and diode current V_D, I_D) with the help of the iterative solution.



Problem 16.43. Repeat the previous problem when the diode saturation current changes to 3 nA. All other parameters remain the same.

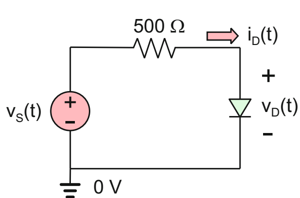
16.2.6 Linearization About a Bias Point: Small-Signal Diode Model

16.2.7 Superposition Principle for Small-Signal Diode Model

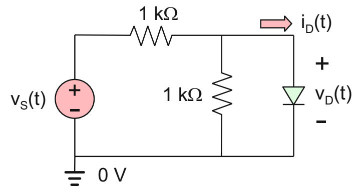
Problem 16.44. Determine the small-signal diode resistance for two limiting cases:

A. When diode bias voltage (DC operating voltage) V_D tends to zero
B. When diode bias current (DC operating current) I_D tends to infinity

Problem 16.45. In the circuit shown in the following figure, $v_S(t) = 10 + 0.005\cos\omega t[\mathrm{V}]$. Determine diode voltage. Use the constant-voltage-drop-diode model for the diode with the turn-on voltage of 0.7 V. Assume the operating temperature of 25 °C and $n = 2.0$.



Problem 16.46. Repeat the previous problem for the circuit shown in the following figure.



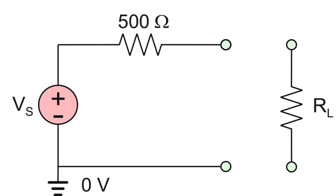
Problem 16.47.

A. Obtain the next term of the asymptotic expansion in Eq. (16.7c) so that the error will be on the order of $(v_d/nV_T)^3$.
B. Derive the expression of the *nonlinear* small-signal diode resistance as a constant term plus a term that depends on the small-signal diode voltage.

16.3 Diode Voltage Regulators and Rectifiers

16.3.1 Voltage reference and voltage regulator

Problem 16.48. You are given a variable voltage source $V_S = 5\,\mathrm{V} \pm 0.5\,\mathrm{V}$ represented by its Thévenin equivalent shown in the figure below. You are also given a load represented by its equivalent resistance of $R_L = 1000\,\Omega$. Construct a diode voltage regulator circuit which outputs the constant voltage of 2.1 V to the load.



A. Present the corresponding circuit diagram.
B. Determine load voltage and diode current for the regulator circuit for two extreme cases $V_S = 5\,\mathrm{V} \pm 0.5\,\mathrm{V}$ of the supply voltage variation. Use the constant-voltage-drop-diode model with the turn-on voltage of 0.7 V.

Problem 16.49. Repeat the previous problem when Thévenin resistance of the source changes to 1 kΩ.

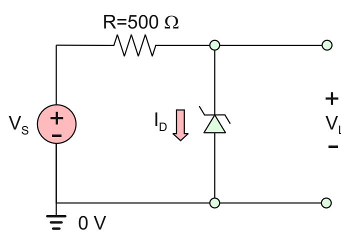
Problem 16.50. You are given a (variable) voltage source V_S represented by its Thévenin equivalent with resistance R_T and a load represented by its equivalent resistance of R_L. A forward-bias diode voltage regulator is used to keep the load voltage constant. Using the constant-voltage-drop-diode model, answer two questions:

A. What is the maximum possible regulated load voltage if $R_L = R_T$?
B. What is the maximum possible regulated load voltage if $R_L = 10R_T$?

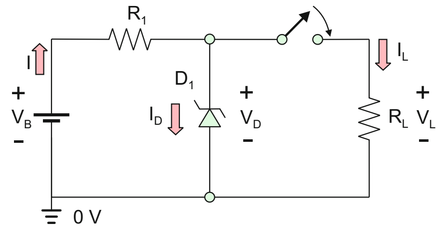
16.3.2 Voltage regulator with Zener diode

Problem 16.51. A 1N5231B Zener diode with the test point $V_{ZT} = 5.1\,\text{V}, I_{ZT} = 20\,\text{mA}$ and with the dynamic resistance $r_Z = 17\,\Omega$ is used in the voltage regulator circuit shown in the figure below.

A. Determine load voltage for the regulator circuit given that $V_S = 9\,\text{V} \pm 1\,\text{V}$ and that the load has a very high (infinite) resistance.
B. Determine line regulation



Problem 16.52. In a typical 12-V automotive application, battery voltage may vary between 10.5 and 14.1 V. The ECM (engine control module) determines fuel delivery and spark advance to control emissions based on several sensors connected to the engine. Many of these sensors require a stable 5-V reference that can be achieved through the use of a Zener diode. The figure that follows shows the corresponding circuit using a 1N4733A Zener diode to provide a stable 5-V reference.



The Zener diode has a reference (test) voltage of 5.1 V at a reference (test) current of 49 mA.

A. Choose a value for resistor R_1 to limit the current through the Zener diode to approximately 50 mA with the sensor disconnected (switch OPEN).
B. Given that the battery voltage may vary between 10.5 and 14.1 V, determine how much the Zener voltage (voltage across the Zener diode) fluctuates with the sensor disconnected (switch OPEN). *Note:* This Zener diode has a dynamic resistance of 7 Ω at the test current of 49 mA.
C. What minimum load resistance can be connected to the circuit without the voltage drop more than 0.5 V from 5 V?
D. Plot load voltage as function of the load resistance in the range 0–1000 Ω for two extreme battery voltages.
E. If the switch is closed and the load resistance is $100\,\Omega$, what is the *power efficiency* of this circuit for two extreme values of the battery voltage? *Note:* Efficiency percentage = $P_{LOAD}/P_{BAT} \times 100\,\%$.

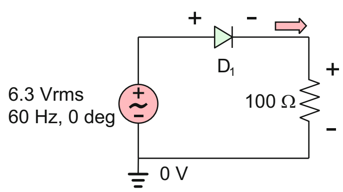
Problem 16.55. Using software of your choice (MATLAB is recommended), plot the output of a half-wave diode rectifier to scale over a time period from 0 to 8 s when the input voltage is given by

$$v_S(t) = V_m \sin \omega t + 0.5 V_m \sin 2\omega t$$

with
Signal frequency—0.5 Hz
Signal amplitude—$V_m = 9\,\text{V}$.
Assume the ideal diode.

Problem 16.56. Using the constant-voltage-drop model for a diode with the turn-on voltage of 0.7 V, determine the following parameters for the circuit shown in the figure:

A. The peak positive voltage across the load
B. The average voltage across the load
C. The peak diode current
D. The average diode current
E. The peak negative voltage across the diode



Note: The voltage source shown represents the output of a typical step-down transform and is given in *rms*.

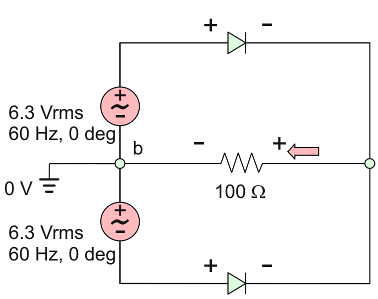
16.3.4 Full-wave rectifier with a dual supply

16.3.5 Diode bridge rectifier

16.3.6 Application example: Automotive battery-charging system

Problem 16.57. Using the constant-voltage-drop model for the diode with the turn-on voltage of 0.7 V, determine the following parameters for the circuit shown in the figure:

A. The peak positive voltage across the load
B. The average voltage across the load
C. The peak diode current
D. The average diode current
E. The peak negative voltage across each diode

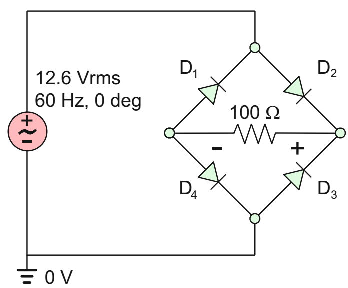


Problem 16.58.

A. Draw a schematic of the full-wave diode bridge rectifier.
B. Indicate current flow in the full-wave diode rectifier at positive and negative applied voltages.
C. If all diodes in the rectifier are changed to the opposite direction, will the rectifier function or not?

Problem 16.59. Using the constant-voltage-drop model for the diode with the turn-on voltage of 0.7 V, determine the following parameters for the circuit shown in the figure:

A. The peak positive voltage across the load
B. The average voltage across the load
C. The peak diode current
D. The average diode current
E. The peak negative voltage across each diode



Note: The voltage source shown represents the output of a typical step-down transform and is given in *rms*.

Problem 16.60. For an automotive battery-charging system schematically shown in Fig. 16.26c, plot the individual rectified voltages and the output voltage (voltage across the load) as a function of time over the interval 0–0.01 s. Every power supply in the figure is a sinusoidal AC voltage source with $V_m = 15\,\text{V}$, $f = 100\,\text{Hz}$. All three voltage power supplies are 120° out of phase with regard to each other—have the phase angles of 0 and $\mp 120°$. Any software can be used (MATLAB is recommended).

16.3.7 Application example: Envelope (or peak) detector circuit

Problem 16.61.

A. Explain the function of the envelope detector in your own words.
B. What is the difference between linear and square-law regions of operation for the envelope detector?
C. When does the envelope detector operate in the linear region? In the square-law region?

Problem 16.62. Design an envelope detector given the carrier frequency of $f = 1.7\,\text{MHz}$. The modulation is a human voice, with the maximum passing modulation frequency of 20 kHz.

A. Draw the circuit diagram of the envelope detector.
B. Specify one possible set of values for R_L, C.

Problem 16.63. Design an envelope detector (specify one possible set of values for R_L, C) given the carrier frequency of $f = 10\,\text{MHz}$. The modulation is a digital signal, with the bit rate of 100 kbps. *Hint:* The equivalent frequency of the digital signal is the bit rate in Hz.

Problem 16.64. A MATLAB script that follows models an envelope detector circuit in Fig. 16.27 by solving the exact circuit ODE given by Eq. (16.19b)

$$\frac{dv_{out}}{dt} = \frac{1}{\tau}\left(R_L I_S \exp[(v_{in} + V_{bias} - v_{out})/V_T] - v_{out}\right)$$

```
%   Input signal
%   carrier freq., Hz
f       = 1e6;
%   modulation freq., Hz
fm      = 2e4;   Am = 0.5;
%   time array
t       = linspace(0, 4/fm, 1e6);
%   envelope
E       = 1 + Am*cos(2*pi*fm*t);
%   input signal ampl., V
Vm      = 0.10;
%   input signal, V
vin     = Vm*E.*cos(2*pi*f*t);
%   Envelope detector
 %   capacitance, F
C    = 10e-9;
%   resistance, Ohm
R    = 5e4;
%   time constant, sec
tau = R*C;
%   Boltzmann constant [J/K]
k    = 1.38066e-23;
%   electron charge [C]
q    = 1.60218e-19;
%   temperature [K]
T    = 298;
%   thermal voltage [V]
VT   = k*T/q;
%   saturation current, A
Is   = 1e-9;
%   bias voltage, V
Vbias  = 0.0;
%   Numerical solution
%    (first-order Euler)
vout    = zeros(size(t));
dt      = t(2)- t(1);
iD      = zeros(size(t));
%   starting voltage
vout(1)    = Vbias + vin(1) -0.60;

for n = 1:length(t)-1
    iD(n)  = Is*(exp((vin(n) ...
    +Vbias-vout(n))/VT )-1);
    vout(n+1)   = vout(n)  +...
    dt/tau*(R*iD(n)  - vout(n));
end
%   Graphics
t       = t(end/2:end);
vout    = vout(end/2:end);
vin     = vin(end/2:end);
subplot(1,2,1); plot(t, vin+Vbias);
grid on; axis square
subplot(1,2,2); plot(t, vout);
grid on; axis square
```

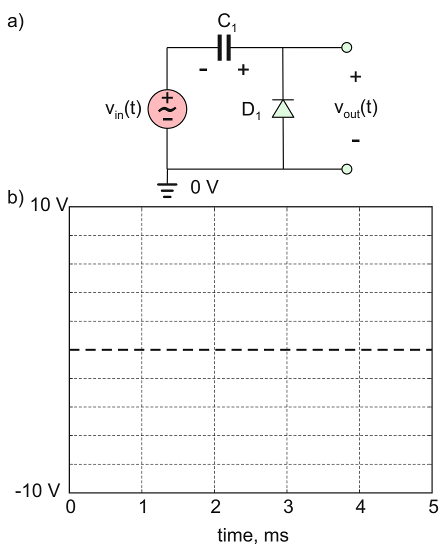
with a particular set of design parameters of your choice. Which bias voltage (0, 1, 4, or 8 V) is most beneficial for the performance of your circuit? Justify your answer.

16.4 Diode Wave-Shaping Circuits

16.4.1 Diode clamper circuit (DC restorer)

16.4.2 Diode voltage doubler and multiplier

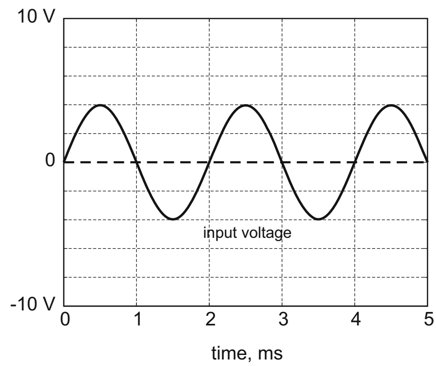
Problem 16.65. In the clamper circuit shown in the figure below, $v_{\text{in}}(t) = V_{\text{m}} \sin \omega t$ and $V_{\text{m}} = 4\,\text{V}$. The wave period is 2 ms. Given the ideal-diode model, plot the input voltage, voltage across capacitor C_1, and voltage across diode D_1 (the output voltage of the circuit) to scale versus time. Clearly label each curve.



Problem 16.66. In the clamper circuit shown in the figure for Problem 16.65, $v_{\text{in}}(t) = V_{\text{m}} - V_{\text{m}} \sin \omega t$ and $V_{\text{m}} = 4\,\text{V}$. The wave period is 2 ms.

A. Given the ideal-diode model, plot the input voltage, voltage across capacitor C_1, and voltage across diode D_1 to scale versus time. Clearly label each curve.
B. Based on your solution, what conclusion could you make about the operation of a clamper circuit subject to strictly positive versus the ground point (already clamped) AC signals?

Problem 16.67. The input voltage for the voltage doubler circuit in Fig. 16.30a is shown in the figure below. Plot the voltage across capacitor C_1 and the voltage across capacitor C_2 (the output voltage) to scale versus time. Clearly label each curve.



Problem 16.68.

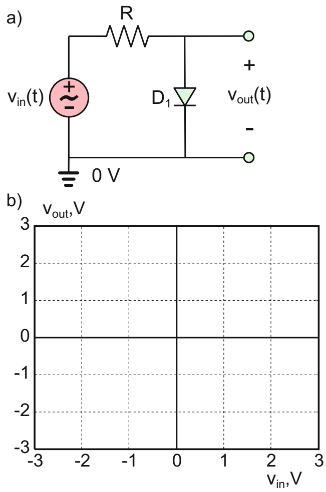
A. Construct a *voltage tripler diode circuit*, which outputs the DC voltage of $3V_{\text{m}}$ for the input AC signal of amplitude V_{m} and zero mean. Present the corresponding circuit diagram.
B. How many capacitors and diodes are you using?
C. Could you extrapolate your answer to a voltage multiplier diode circuit, which outputs the DC voltage of $5V_{\text{m}}$?

16.4.3 Positive, negative, and double clipper

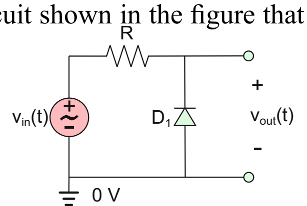
16.4.4 Transfer characteristic of a diode circuit

Problem 16.69. For the positive clipper diode circuit, sketch the voltage transfer characteristic to scale assuming

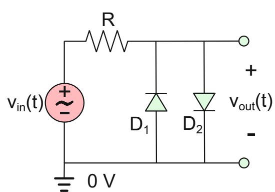
A. Ideal-diode model
B. Constant-voltage drop model



Problem 16.70. Repeat Problem 16.69 for the diode circuit shown in the figure that follows.



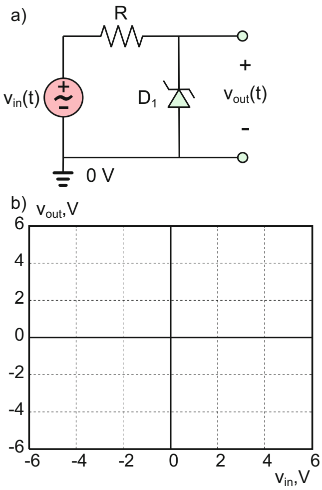
Problem 16.71. Repeat Problem 16.69 for the diode circuit shown in the figure that follows.



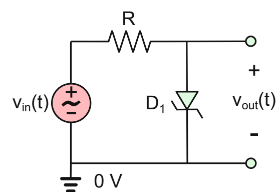
Problem 16.72. Given the Zener breakdown voltage of 4 V for D_1, for the circuit shown in the figure below, sketch the voltage transfer characteristic to scale assuming:

A. Ideal-diode model in the forward-bias region
B. Constant-voltage drop model in the forward-bias region

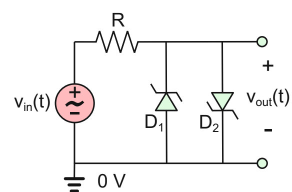
Always use the constant-voltage-drop model in the breakdown region.



Problem 16.73. Repeat Problem 16.72 for the diode circuit shown in the following figure.



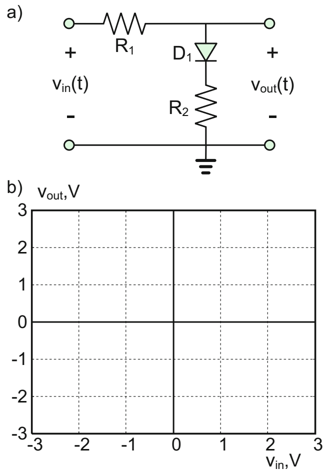
Problem 16.74. Repeat Problem 16.72 for the diode circuit shown in the figure below assuming the Zener breakdown voltage of 4 V for D_1 and 5 V for D_2.



Problem 16.75. Repeat Problem 16.72 for the diode circuit shown in the figure below assuming the Zener breakdown voltage of 2 V for D_1 and 4 V for D_2.

Problem 16.76. For the following diode circuit, sketch the voltage transfer characteristic to scale given that $R_1 = 1\,\text{k}\Omega$ and $R_2 = 1\,\text{k}\Omega$, and assuming:

A. Ideal-diode model
B. Constant-voltage drop model
Label the endpoint voltages.



Problem 16.77. Repeat Problem 16.76 for the diode circuit shown in the figure below. Assume $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 1\,\mathrm{k}\Omega$, and $R_3 = 1\,\mathrm{k}\Omega$. Label the endpoint voltages.