Chapter 2: Major Circuit Elements

Overview

Prerequisites:

- Knowledge of university physics: electricity and magnetism (optional)
- Knowledge of vector calculus (optional)

Objectives of Section 2.1:

- Realize the difference between circuit elements and circuit components
- Review (derive) Ohm's law
- Become familiar with the v-i characteristic of the resistance including limiting cases
- Realize the importance of ohmic losses in long cables
- Become familiar with discrete fixed resistors and with resistive sensing elements

Objectives of Section 2.2:

- Realize the meaning of a passive nonlinear circuit element and its v-i characteristic
- Define two resistance types (static and dynamic) for a nonlinear passive circuit element
- Present two examples of nonlinear elements: ideal diode and a threshold switch

Objectives of Section 2.3:

- Introduce the concept of independent voltage and current sources and become familiar with their v-i characteristics
- Introduce the concept of practical voltage/current sources including their v-i characteristics
- Obtain initial exposure to the operation principles of voltage sources including specific examples

Objectives of Section 2.4:

- Become familiar with the concept of a dependent source
- Become familiar with four major types of dependent sources
- Obtain initial exposure to transfer characteristics of dependent sources
- Become familiar with ideal time-varying and AC sources

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Objectives of Section 2.5:

- Formalize the meaning of voltmeter and ammeter from the viewpoint of open and short circuits
- Obtain a clear understanding of the circuit ground and its role in the circuit
- Review different ground types

Application Examples:

- Power loss in transmission lines and cables
- Resistive sensing elements
- DC voltage generator with permanent magnets
- Chemical battery

Keywords:
Circuit elements, Circuit components, *v-i* characteristic, Resistance, Polarity, Voltage difference, Voltage drop, Voltage polarity, Passive reference configuration, Ohm's law, Linear passive circuit element, Conductance, Siemens, mho, Short circuit, Open circuit, Ohmic conductor, Mobility of charge carriers, Material conductivity, Material resistivity, Electric circuit, American Wire Gauge (AWG), Resistor, Fixed resistors, Surface Mound Devices (SMD) (resistance value, geometry size), Variable resistor, Potentiometer, Resistive sensors, Photoresistor, Photocell, Negative temperature coefficient (NTC), Thermistor equation, Thermistor constant, Thermocouple, Peltier-Seebeck effect, Strain gauge, Strain sensitivity, Gauge factor, Strain gauge equation, Potentiometric position sensor, Nonlinear passive circuit elements, Non-ohmic circuit elements, Radiation resistance, Ideal diode, Shockley equation, Static resistance, Dynamic resistance, Small-signal resistance, Differential resistance, Incremental resistance, (DC) Operating point, Quiescent point, Electronic switch, Solid-state switch, Switch threshold voltage, Two-terminal switch, Three-terminal switch, Unidirectional switch, Bidirectional switch, Independent ideal voltage source, Active reference configuration, Nonlinear passive circuit elements, Non-ohmic circuit elements, Radiation resistance, Static resistance, Ideal diode, Shockley equation, Dynamic resistance, Small-signal resistance, Differential resistance, Incremental resistance, (DC) Operating point, Quiescent point, Electronic switch, Solid-state switch, Switch threshold voltage, Two-terminal switch, Three-terminal switch, Unidirectional switch, Bidirectional switch, Independent ideal voltage source, Active reference configuration, Practical voltage source, Maximum available source current, Maximum available source power, Open-circuit source voltage, Short-circuit circuit current, Internal source resistance, Independent ideal current source, Practical current source, Charge separation principle, Faraday's law of induction, Lorentz force, Instantaneous generator voltage, Average generator voltage, Battery voltage, Battery capacity, Battery energy storage, Dependent sources, Voltage-controlled voltage source, Current-controlled voltage source, Voltage-controlled current source, Current-controlled current source, Open-circuit voltage gain, Transresistance, Transconductance, Short-circuit current gain, Voltage amplifier, Current amplifier, Transresistance amplifier, Transconductance amplifier, Transfer characteristic, AC voltage source, Ideal voltmeter, Ideal ammeter, Earth ground, Chassis ground, Common (neutral) terminal (ground), Forward current, Return current, Absolute voltages in a circuit

Section 2.1 Resistance: Linear Passive Circuit Element

2.1.1 Circuit Elements Versus Circuit Components

Circuit Elements

Similar to mechanical mass, spring, and damper used in analytical dynamics, *circuit elements* are simple hypothetic ideal models. Every circuit element is characterized by its *unique voltage/current dependence* called the *υ-i characteristic*. Most of the *υ-i* characteristics reflect general physical laws. A list of the circuit elements includes:

1. Resistance
2. Capacitance
3. Inductance
4. Ideal electric transformer
5. Voltage source (independent and dependent)
6. Current source (independent and dependent)
7. Ideal switch
8. Ideal (Shockley) diode
9. Logic gates (NOT, AND, OR).

Circuit elements may be *linear* (resistance) or *nonlinear* (ideal diode), *passive* (resistance) or *active* (voltage source), *static* (resistance) or *dynamic* (capacitance/inductance), or both. Although all circuit elements studied here are static ones, the extension to the case of time-varying voltage and current is often trivial.

Circuit Components

Circuit components are numerous hardware counterparts of the circuit elements. Examples of the circuit components include resistor, capacitor, inductor, battery, etc. The circuit components may be modeled as combinations of the ideal circuit elements with one dominant desired element (e.g., resistance) and several parasitic ones (e.g., parasitic inductance and capacitance of a physical resistor). Another example is a battery, which is modeled as an ideal voltage source in series with a (small) resistance. In practice, we attempt to model any existing or newly discovered circuit component as a *combination* of the well-known circuit elements. The same is valid for more complicated structures targeted by electrical, mechanical, and biomedical engineers. An example is a human body, the response of which is modeled as a combination of resistance and capacitance.

2.1.2 Resistance

Symbols and Terminals

Figure 2.1 shows the circuit symbol for *resistance* with current direction and voltage polarity: positive voltage applied to the left terminal and a negative voltage applied to the right terminal cause a current to flow from left to right, as depicted in Fig. 2.1b. As a circuit element, the resistance is fully symmetric: terminals 1 and 2 in Fig. 2.1 may be interchanged without affecting its operation. Thus, the resistance does not have *a polarity*.

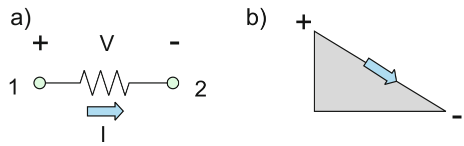


Fig. 2.1. Resistance symbol along with the voltage and current.

Voltage Across the Resistance

The *voltage difference* (or *voltage drop* or simply *voltage*) across the resistance, *V*, in Fig. 2.1 is a *signed quantity*. The voltage is measured in volts (V), named in honor of Italian physicist Alessandro Volta (1745–1827), who invented the first battery. Plus and minus signs across the resistance indicate the *voltage polarity*. Specifically, a plus sign denotes a (presumably) higher *absolute voltage level* versus *ground* than the minus sign; see Fig. 2.1b. For example, let us assume that the value *V* in Fig. 2.1a is positive and equal to 1 V. This means that the electric field spends positive work equal to

$$1\text{V} \times 1\text{C} = \;\; 1\text{J} \tag{2.1}$$

when moving the charge of 1 C through the resistance from left to right in Fig. 2.1a. Similarly, the positive work of one joule is to be spent by an external force to move one coulomb of charge across a potential difference of one volt *against* the electric field. In power electronics, quantities of 1 kV (1000 V), even 1 MV (10^6 V) for voltage, are customary. In sensors and cellular phone circuits, for example, voltages are usually much lower. Values of 1 mV (10^{-3} V) or even 1 μV (10^{-6}V) may be recorded. Voltage applied to the resistance causes electric current flow.

Current Through Resistance: Passive Reference Configuration

The net current *I* flowing through the resistance is shown in Fig. 2.1a by an arrow. The current is measured in amperes (A), named in honor of French physicist and mathematician André-Marie Ampère (1775–1836). For example, the value of *I* in Fig. 2.1a is 1 A. This means that one coulomb of charges passes through the resistance in one second:

$$1\text{A} \times 1\text{s} = 1\text{C} \tag{2.2}$$

The electric current flow *through* the resistance (and any other circuit element) is a *directed quantity*; the arrow shows its direction. A useful fluid mechanics analogy for the resistance is water (electric current) that flows down the “voltage” hill in Fig. 2.1b. The relation between voltage polarity and current direction depicted in Fig. 2.1b is known as the *passive reference configuration*. It is commonly used for all passive circuit elements such as resistances, diodes, capacitances, and inductances. Physically, the passive reference configuration means that the resistance consumes electric power, but does not create it. In power electronics, currents of several A, even kA (1000 A), are customary.

In digital and communication circuits, however, currents are usually low; therefore, units of 1 μA (10^{-6}A) or 1 mA (10^{-3}A) are commonly used.

Ohm's Law: Resistance and Conductance

According to *Ohm's law*, the voltage V across the resistance and the current I through the resistance are related by a simple linear expression

$$V = RI \tag{2.3}$$

with the proportionality constant R known as the *resistance*. This expression was first established by German mathematician and physicist Georg Simon Ohm (1789–1854) in 1827 but was coldly received by the scientific community at that time. It took nearly 14 years before the Royal Society of London finally recognized his work and his discovery is now known as Ohm's law. The unit of resistance R carries his name ohm and the Greek symbol Ω. The unit follows from Eq. (2.3) as volt over ampere:

$$1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}} \tag{2.4}$$

The resistance is the *linear passive circuit element*. Resistance values vary typically between 1 Ω and 100 MΩ. The reciprocal of the resistance is the conductance, G:

$$G = \frac{1}{R} \tag{2.5}$$

The unit of conductance, Ω^{-1}, is called *siemens* (S) in honor of Ernst Werner von Siemens (1816–1892), a German inventor and the founder of what is today Europe's largest electrical engineering company (Siemens AG). An older American equivalent of that unit is *mho* (℧) or ohm spelled backwards! Conductance is useful in the circuit analysis.

Exercise 2.1: A voltage of 20 V is applied to a 1-MΩ resistance. Determine the current through the resistance.

Answer: 20 μA.

Exercise 2.2: A voltage of 20 V is applied to a 1-mS conductance. Determine current through the conductance.

Answer: 20 mA.

2.1.3 *υ-i* Characteristic of the Resistance: Open and Short Circuits

Figure 2.2 plots the linear dependence given by Eq. (2.3) for two distinct resistances. The corresponding plot is known as the *υ-i characteristic* (or the *υ-i dependence*). We use small letters *υ-i* to maintain consistency with the following study of time-varying circuits. The *υ-i* characteristic is the "business card" of the circuit element—every circuit element has its own *υ-i* characteristic. Once the *υ-i* characteristic is known, the circuit element is characterized completely. The following is true with reference to Fig. 2.2a:

1. The slope of the *υ-i* dependence for the resistance is equal to $1/R$ or G.
2. Smaller resistance leads to a stepper *υ-i* dependence (large currents).
3. Larger resistance leads to a flatter *υ-i* dependence (small currents).
4. The negative part of the *υ-i* dependence simply means simultaneous switching voltage polarity and current direction, respectively, in Fig. 2.1.

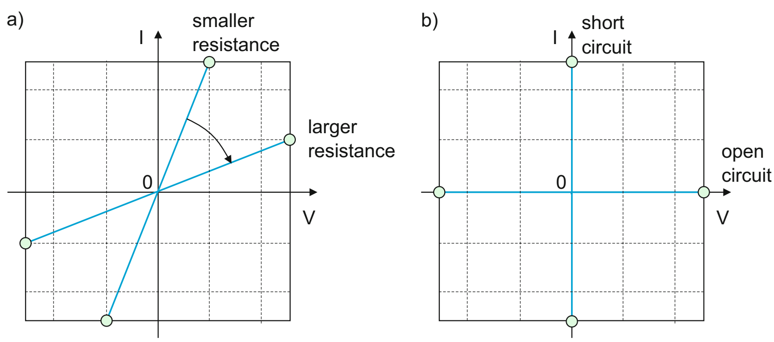


Fig. 2.2. *υ-i* Characteristics for resistances and for the short and open circuits, respectively.

Open and Short Circuits

Two limiting cases of the resistance *υ-i* characteristics are the *short circuit* and the *open circuit*, as seen in Fig. 2.3.

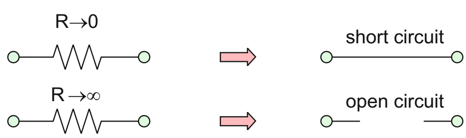


Fig. 2.3. Transformation of resistance to a short and an open circuit, respectively.

When $R \to 0$, the resistance becomes a short circuit, or an ideal wire. There is no voltage drop V across the wire, but any current I can flow through it. Therefore, the *υ-i* characteristic of the short circuit is the straight vertical line in Fig. 2.2b. When $R \to \infty$, the resistance becomes an open circuit, or an ideal vacuum gap. There is no current I through the gap at any value of the applied voltage V. Therefore, the *υ-i* characteristic of the open circuit is the straight horizontal line in Fig. 2.2b.

Exercise 2.3: Every vertical division in Fig. 2.2a is 0.1 A; every horizontal division is 1 V. Find resistances for two *υ-i* dependencies in the figure.

Answer: $R = 4\ \Omega$ and $R = 25\ \Omega$, respectively.

Exercise 2.4: An *ideal switch* is open when $V < 0$ and is closed when $V \geq 0$. Plot the *υ-i* characteristic given that only a positive current $I > 0$ can flow.

Answer: Horizontal line $I = 0$ when $V < 0$ and vertical line $V = 0$ when $I > 0$.

2.1.4 Power Delivered to the Resistance

Voltage V across the resistance is work in joules necessary to pass 1 C of charge through the resistance. Since there are exactly I coulombs passing through the resistance in one second, the power P delivered to the resistance must be the product of work per unit charge and the number of charges passing through the element in one second: $P = VI$. The power P has indeed the units of watts ($1\,\mathrm{V} \times 1\,\mathrm{A} = 1\,\mathrm{J}/1\,\mathrm{s} = 1\,\mathrm{W}$). When the *υ-i* characteristic of the resistance is examined, the power is equal to the area of the shaded rectangles in Fig. 2.4. Using Ohm's law, Eq. (2.3) gives us three equivalent definitions of the absorbed power by a resistance:

$$P = VI \quad \text{Basic definition, valid for any passive circuit element} \tag{2.6}$$

$$P = \frac{V^2}{R} \quad \text{Power for resistance in terms of voltage} \tag{2.7}$$

$$P = RI^2 \quad \text{Power for resistance in terms of current} \tag{2.8}$$

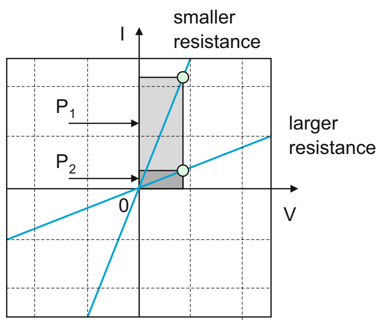


Fig. 2.4. *υ-i* Characteristics for the resistances and power rectangles. $P_{1,2}$ are absorbed powers.

Despite their obvious nature, all three equations are useful in practice. In particular, Eq. (2.7) indicates that a small resistance absorbs more power than the large resistance *at the same applied voltage*; this is seen in Fig. 2.4. Imagine for a moment that we know the voltage across the resistance, but do not know the current. This happens if a number of circuit elements are connected in parallel to a known voltage source. Then Eq. (2.7) is used to find the power. However, if the current is known, but the voltage is not (a number of elements connected in series to a current source), then Eq. (2.8) is employed.

Example 2.1: A voltage of 10 V is applied to a 2.5-Ω resistance. Determine the absorbed electric power in three possible ways as stated by Eqs. (2.6) through (2.8).

Solution: To apply two of the three equations, current through the resistance is needed. From Ohm's law, $I = V/R = 4$ A. The electric power delivered to the resistance can thus be determined in three ways:

$$P = VI = 40\,\text{W} \quad \text{Basic power definition, passive reference configuration}$$

$$P = \frac{V^2}{R} = 40\,\text{W} \quad \text{Power for resistance in terms of voltage}$$

$$P = RI^2 = 40\,\text{W} \quad \text{Power for resistance in terms of current}$$

2.1.5 Finding Resistance of Ohmic Conductors

An *ohmic conductor* satisfies Ohm's law given by Eq. (2.3). Finding its resistance is equivalent to the derivation of Ohm's law under certain assumptions. Let us consider a conducting circular cylinder subject to an applied voltage V in Fig. 2.5. The cylinder has length l and a cross-sectional area A.

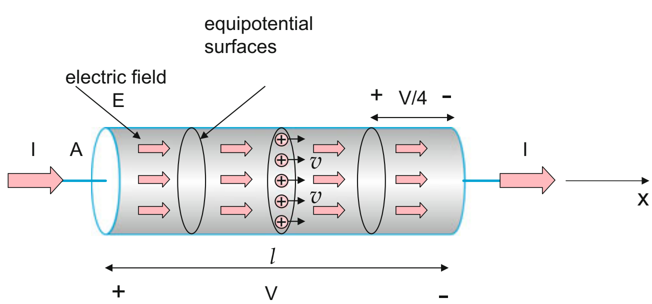


Fig. 2.5. Finding the resistance of a conducting cylinder.

Finding Total Current

The net electric current in a metal or other conductor is defined as the net flux of *positive* charge carriers directed along the conductor axis x:

$$I = Aqn\,\upsilon \tag{2.9}$$

Here, nq is the *volumetric charge density* of *free charges* q with concentration n in coulombs per cubic meter, C/m^3, and υ is the magnitude of the average charge velocity in m/s. In the one-dimensional model of the current flow, the average velocity vector is directed along the x-axis seen in Fig. 2.5. Since the electrons have been historically assigned a negative charge, the electric current direction is *opposite* to the direction of electron motion in a conductor. The electron carries an elemental charge of $-q = -1.60218 \times 10^{-19}$ C. Because A and nq are constants for a given conductor, the electric current is simply associated with the charge's mean velocity υ.

Finding Average Carrier Velocity

In order to find υ, we use the following method. The total voltage drop V applied to a sufficiently long, conducting cylinder is *uniformly* distributed along its length following the equally spaced equipotential surfaces; this is schematically shown in Fig. 2.5. This fact has been proved in Chapter 1. Such a voltage distribution corresponds to a *constant uniform* electric field within the cylinder, which is also directed along the cylinder axis. The magnitude of the field, E, with the units of V/m, is given by

$$E = V/l \tag{2.10}$$

The electric field creates a Coulomb force acting on an individual positive charge q. The Coulomb force is directed along the field; its magnitude F is given by

$$F = qE \tag{2.11}$$

The key is a *linear* relation between the charge velocity υ and force F or, which is the same, a linear relation between the charge velocity υ and the applied electric field E, i.e.,

$$\upsilon = \mu E \tag{2.12}$$

where μ is the so-called mobility of charge carriers, with the units of $m^2/(V{\cdot}s)$. Carrier mobility plays an important role in semiconductor physics. With the help of Eqs. (2.10) and (2.12), the expression for the total current Eq. (2.9) is transformed to

$$V = \left(\frac{l}{Aqn\mu}\right)I = \left(\frac{l}{A\sigma}\right)I = RI, \quad \sigma = qn\mu, \quad R = \frac{l}{A\sigma} \tag{2.13}$$

This is the expression for the resistance of a cylindrical conductor. *Material conductivity* σ is measured in S/m. Its reciprocal is the *material resistivity* $\rho = 1/\sigma$ measured in ℧·m.

Example 2.2: Estimate resistance R of a small doped Si disk with the length l of 5 μm, cross section of $A = 10^{-4}$ cm^2, uniform electron, concentration (carrier concentration) of $n = 10^{17}$ cm^{-3}, and carrier mobility of $\mu_n = 1450$ cm^2/(V·s).

Solution: Resistance calculations are usually simple when the one-dimensional model of a conducting cylinder or a disk is used. However, one must be careful with the units. Units of cm are customary in semiconductor physics. Therefore, one should first convert all different units of length to meters (or to centimeters). After that, we use the definition of the resistance given by Eq. (2.13) and obtain (units of meters are used):

$$R = \frac{l}{Aqn\mu_n} = \frac{5 \times 10^{-6}}{10^{-8} \times 1.602 \times 10^{-19} \times 10^{23} \times 0.145} = 0.215\ \Omega \tag{2.14}$$

Table 2.1 lists conductivities of common materials. What is the major factor that determines the conductivity of a particular conducting material? According to Eq. (2.13), there are two such parameters: charge concentration and charge mobility.

Table 2.1. DC conductivities of conductors, semiconductors, and insulators (25 °C, multiple sources).

Material	Class	σ (S/m)	Material	Class	σ (S/m)
Silver	Conductor	6.1×10^7	Seawater	Semiconductor	4
Copper	Conductor	5.8×10^7	Human/animal tissues	Semiconductor	0.1–2.0
Gold	Conductor	4.1×10^7	Germanium	Semiconductor	2
Aluminum	Conductor	4.0×10^7	Fresh water	Semiconductor	0.01
Brass	Conductor	2.6×10^7	Wet soil	Semiconductor	0.01–0.001
Tungsten	Conductor	1.8×10^7	Dry soil	Semiconductor	0.001–0.0001
Zinc	Conductor	1.7×10^7	Intrinsic silicon (Si)	Semiconductor	4.4×10^{-4}
Nickel	Conductor	1.5×10^7	Gallium arsenide (GaAs)	Semiconductor	10^{-6}
Iron	Conductor	1.0×10^7	Glass	Insulator	10^{-12}
Tin	Conductor	0.9×10^7	Porcelain	Insulator	10^{-14}
Lead	Conductor	0.5×10^7	Hard rubber	Insulator	10^{-15}
Graphite	Conductor	0.003×10^7	Fused quartz	Insulator	10^{-17}
Carbon	Conductor	0.003×10^7	Teflon	Insulator	10^{-23}
Magnetite	Conductor	0.002×10^7			

It is mostly the different concentration of free charge carriers n that makes the resistance of two materials quite different. For example, $n = 8.46 \times 10^{28}\text{m}^{-3}$ in copper (a good conductor), whereas it may be $n = 10^{16}\text{m}^{-3}$ in a moderately doped silicon crystal (doped semiconductor). However, it is also the difference in mobility μ that represents the "friction" experienced by the "gas" of free charges with density n that is moving through the solid or liquid conductor under the applied voltage (electric field).

Exercise 2.5: Using Table 2.1 in chapter's summary, determine the total resistance of an aluminum wire having a length of 100 m and a cross-sectional area of 1 mm^2.

Answer: 2.5 Ω.

2.1.6 Application Example: Power Loss in Transmission Wires and Cables

All metal wires and cables are ohmic conductors. Electric power absorbed by an ohmic conductor is transformed into heat. This is known as electric power loss. We can apply Eqs. (2.6)–(2.8) and Eq. (2.13) in order to determine the loss of electric power in transmission lines and/or cables. This question has significant practical importance. Figure 2.6 outlines the corresponding *electric circuit*. The electric circuit is a closed path for electric current. Resistance R_L characterizes the load. Only Ohm's law is used to analyze this circuit, along with the current continuity. No other circuit laws are necessary. We also consider a voltage source in Fig. 2.6. The voltage sources will be studied next.

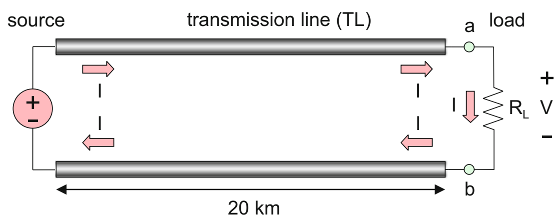


Fig. 2.6. A long transmission power line carrying a steady-state current I to the load resistance.

According to Eq. (2.13), the wire resistance is inversely proportional to its diameter. In the USA, the *American Wire Gauge* (AWG) system was developed to classify the wire diameters of conductors. You probably have heard an electrician refer to a gauge 12 household wiring. This implies a wire diameter of about 2 mm, or 0.08″. Table 2.2 reports common AWG numbers and maximum current strengths.

Table 2.2. American Wire Gauge (AWG) wire parameters. The maximum current is given for solid copper (Source: Handbook of Electronic Tables and Formulas for American Wire Gauge).

AWG #	Diameter (inches)	Diameter (mm)	Resistance per 1000 ft or 304.8 m (Ω)	Maximum current in (A) for power transmission
24	0.0201	0.51054	25.67	0.577
22	0.0254	0.64516	16.14	0.92
20	0.0320	0.81280	10.15	1.50
18	0.0403	1.02362	6.385	2.30
16	0.0508	1.29032	4.016	3.70
14	0.0640	1.62814	2.525	5.90
12	0.0808	2.05232	1.588	9.30
10	0.1019	2.58826	0.999	15.0
Gauges 10 through 1 are not shown				
0 (1 aught)	0.3249	8.252	0.09827	150
00 (2 aught)	0.3648	9.266	0.07793	190
000 (3 aught)	0.4096	10.404	0.06180	239
0000 (4 aught)	0.4600	11.684	0.04901	302

Example 2.3: An AWG 0 aluminum transmission grid cable schematically shown in Fig. 2.6 has a wire diameter of 8.25 mm and a cross-sectional area of 53.5 mm^2. The conductivity of aluminum is 4.0×10^7 S/m. The total cable length (two cables must run to a load) is 40 km. The system delivers 1 MW of DC power to the load. Determine the power loss in the cable when *load* voltage V and *load* current I are given by:

1. $V = 40$ kV and $I = 25$ A
2. $V = 20$ kV and $I = 50$ A
3. $V = 10$ kV and $I = 100$ A

Why is high-voltage power transmission important in power electronics?

Solution: We find the total cable resistance from Eq. (2.13):

$$R = \rho\frac{L}{A} = \frac{L}{\sigma A} = \frac{40 \times 10^3}{4.0 \times 10^7 \times 53.5 \times 10^{-6}} = 18.7\,\Omega \qquad (2.15)$$

The same load current I flows through the load modeled by a resistor R_L and through the cables in Fig. 2.6. Therefore, power loss in the cables may be found using Eq. (2.8). Knowing the load voltage (or the voltage across the cable) is not necessary. The power loss in the cables is thus given by $P = RI^2$. For the three different cases corresponding to the same load power, we obtain

Example 2.3 (cont.):

$$\begin{aligned}&1.\ P = RI^2 = 11.7\,\text{kW or } 1.17\ \%\ \text{of the load power}\\&2.\ P = RI^2 = 46.8\,\text{kW or } 4.68\ \%\ \text{of the load power}\\&3.\ P = RI^2 = 187\,\text{kW or } 18.7\ \%\ \text{of the load power}\end{aligned} \tag{2.16}$$

Clearly, the high-voltage power transmission allows us to reduce the power loss in long cables very significantly while transmitting the same power to the load. Therefore, the high-voltage transmission lines passing through the country have typical voltages between 100 kV and 800 kV.

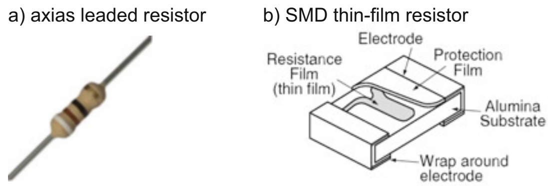
In circuit analysis in the laboratory, we usually consider *ideal* or perfectly conducting wires whose resistance is zero. This is justified since the wire lengths for most practical circuit applications are so short that the voltage drop is negligibly small.

2.1.7 Physical Component: Resistor

Fixed Resistors

Resistance is constructed intentionally, as a separate circuit component. This component is called the *resistor*. A common axial leaded carbon film 0.25-W resistor deployed on a solderless protoboard is seen in Fig. 2.7a. Those carbon film resistors are typically manufactured by coating a homogeneous layer of pure carbon on high-grade ceramic rods. After a helical groove is cut into the resistive layer, tinned connecting leads of electrolytic copper are welded to the end-caps. The resistors are then coated with layers of tan-colored lacquer. The common *Surface Mount Device* (SMD) thin-film resistor is shown in Fig. 2.7b. Manufacturing process variations result in deviations from the normal resistor values; they are known as tolerances and reported to the end user through an *extra color ring* (for leaded axial resistors) or an *extra digit* (for SMD resistors). Typical power ratings for the axial resistors are 1/6 W, ¼ W, ½ W, 1 W, 2 W, and 3 W. When the power delivered to the resistor considerably exceeds the particular rating, the resistor may burn out, releasing a prominent "carbon" smell. The axial resistors have color codes shown in Fig. 2.7c. To find the value of the resistor depicted in Fig. 2.7a, we first encounter the tolerance code, which will typically be gold, implying a 5 % tolerance value. Starting from the *opposite* end, we identify the first band, and write down the number associated with that color; in Fig. 2.7a it is 9 (white). Then, we read the next band (brown) and record that number; it is 1. After this we read the multiplier black, which is 0. The resistor value in Fig. 2.7a is consequently $R = 91 \times 10^0 = 91\ \Omega$.

The surface mount resistors, also known as SMD resistors, do not have color codes. The SMD resistors are labeled numerically as **102** $= 10 \times 10^2 = 1\ \text{k}\Omega$, **271** $= 27 \times 10^1 = 270\ \Omega$, etc. Along with this, the SMD resistors, similar to other SMD components, do have codes corresponding to their geometry size. Each size is described as a four-digit number. The first two digits indicate length, and the last two digits indicate



c) Color codes for axial resistors

Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Gray	White
0	1	2	3	4	5	6	7	8	9

Fig. 2.7. (**a**) A leaded axial resistor, (**b**) a thin-film resistor, and (**c**) color codes for leaded resistors.

width (in 0.01″, or 10 mils units). Some popular SMD resistor sizes are 0603 (0.06″ × 0.03″, or 60 × 30 mils, or 1.6 × 0.8 mm), 0805 (0.08″ × 0.05″), and 1206 (0.12″ × 0.06″).

Variable Resistors (Potentiometers)

The simplest *variable resistor* is a *potentiometer*. A picture is shown in Fig. 2.8a along with its equivalent electric schematic in Fig. 2.8b. The potentiometer is used either as a voltage divider, discussed later in the text, or as a variable resistor. By rotating the potentiometer shaft, it is possible to obtain any resistance value up to the maximum potentiometer value. The adjustable resistance is obtained between terminals 1 and 2 or 2 and 3 of the potentiometer, respectively. You should remember that the potentiometer is a nonpolar device. This means that it can be placed into the circuit in any orientation. With this knowledge the joking engineer telling you that "all resistors in your circuit are backwards" should not cause any fear.

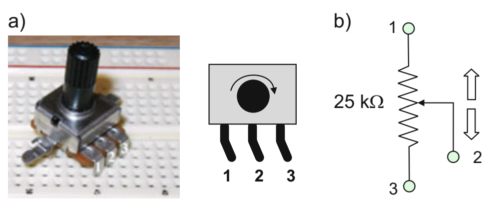


Fig. 2.8. A rotary 25-kΩ potentiometer rated at 0.25 W and its equivalent circuit schematic.

2.1.8 Application Example: Resistive Sensors

There are a variety of sensor types—*resistive sensors*—which use electric resistance variation to measure a mechanical or a thermal quantity. Some of them are shown in Fig. 2.9. As a first example, we consider a temperature sensor based on a *thermistor* (a resistor), with a resistance that varies when ambient temperature changes; see Fig. 2.9a.

The word thermistor is a contraction of the words "thermal" and "resistor." As a second example of a resistance subject to ambient conditions, we will consider a *photoresistor* (*photocell*) shown in Fig. 2.9b. The final example is a *strain gauge* shown in Fig. 2.9c.

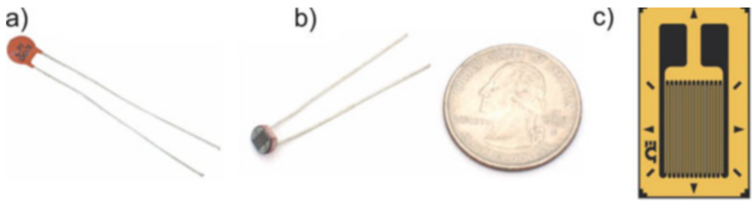


Fig. 2.9. Sensing elements which change their resistances when ambient conditions change.

Thermistor

The thermistor changes its resistance as temperature increases or decreases. General-purpose thermistors are made out of *metal oxides* or other *semiconductors*. Successful semiconductor thermistors were developed almost simultaneously with the first transistors (1950s). For a metal-oxide thermistor, its resistance *decreases* with increasing temperature. Increasing the temperature increases the number of free carriers (electrons) and thus increases the sample conductivity (decreases its resistance). Shown in Fig. 2.9a is a very inexpensive NTC—*negative temperature coefficient*—leaded thermistor. According to the manufacturer's datasheet, it reduces its resistance from approximately 50 kΩ at room temperature (about 25 °C) by 4.7 % for every degree Celsius (or Kelvin) and reaches about 30 kΩ at body temperature according to the *thermistor equation*:

$$R_1 = R_2 \exp\left[B\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \tag{2.17}$$

where $T_1,\ T_2$ are two *absolute* temperatures always given in degrees K. Temperature T_2 corresponds to a room temperature of 25 °C so that $R_2{=}R_{25^\circ\mathrm{C}}$, temperature T_1 is the observation temperature, and B is the *thermistor constant*, which is equal to 4200 K in the present case. Equation (2.17) is a nontrivial result of the solid-state physics theory. We emphasize that Eq. (2.17) is more accurate than the temperature coefficient of the thermistor—the above referenced value of −4.7 %. Typical applications include temperature measurement, control, compensation, power supply fan control, and printed circuit board (PCB) temperature monitoring. Inexpensive thermistors operate from −30 °C to approximately +130 °C. At higher temperatures, thermocouples should be used.

Thermocouple

Figure 2.9 does not show one more important temperature sensor—the *thermocouple*—which is used to measure large temperatures and large temperature differences. It operates based on a completely different principle. The thermocouple does not significantly change its resistance when temperature changes. Instead, it generates an electric current and the associated voltage, when the junction of the two metals is heated or cooled, known as the *Peltier-Seebeck effect*; this voltage can be correlated to temperature. Therefore, the thermocouple, strictly speaking, is not a resistive sensor.

Photoresistor (Photocell)

An idea similar to the thermistor design applies. Quanta of light incident upon the photocell body create new free charge carriers—new electron-hole pairs in a semiconductor. If the concentration of free charges increases, the resistance of the sample decreases according to Eq. (2.13). The resistance is inversely proportional to the concentration. The photocell in Fig. 2.9b is characterized by very large nonlinear variations of the resistance in response to ambient light.

Strain Gauge

The *strain gauge* measures mechanical strain. The operation is based on Eq. (2.13), which defines the resistance through material conductivity σ, the length of the resistor l, and its cross section A. When the resistor, which may be a trace on the base of a metal alloy, is stretched, its length l increases and its cross section A decreases. Hence, the resistance R increases due to *both* of these effects simultaneously; changes in the resistance may be made visible for small strains. Shown in Fig. 2.9c is an inexpensive uniaxial strain gauge with a nominal resistance of 350 Ω; typical resistances are 120, 350, 600, 700, and 1000 Ω. The gauge changes its resistance R in proportion to the *strain sensitivity* S_G of the wire's resistance, also called the *gauge factor* (GF). For a strain gauge, the relative resistance variation, $\Delta R/R$, is estimated based on known values of the strain sensitivity, S_G, and strain, ε. The *strain gauge equation* has the form

$$\Delta R/R = S_G\varepsilon \tag{2.18}$$

The dimensionless strain sensitivity S_G varies around 2. The strain (a relative elongation) is a dimensionless quantity. It is measured in micro-strains, $\mu\varepsilon$, where one $\mu\varepsilon$ is 10^{-6}. Typical strain values under study are on the order of 1000 $\mu\varepsilon$. Using Eq. (2.18) this yields a relative resistance variation as small as 0.2 %. Because of this, the circuits for the strain measurements should be designed and built with great care. Temperature compensation efforts are also required. Since the relative resistance changes are very small, the strain gauge is a *linear* device: the strain is directly proportional to resistance variations.

Potentiometric Position Sensor

Another general resistive sensor is a *potentiometric* (or *potentiometer*) *position sensor*. Its operation becomes apparent when we rotate the potentiometer dial in Fig. 2.8a. A change in the resistance is directly proportional to the rotation angle. The resistance variation can be converted to voltage variation and then measured. Similar potentiometer sensors for measuring linear motion also exist.

Sensitivity of Resistive Sensors

One major difference between different resistive sensing elements is a very different degree of the relative resistance variations. For the photocell, the resistance variation is up to 100 times. For the thermistor, the resistance variation can be as much as 50 %. For the strain gauge, the resistance variations do not exceed 0.5 %.

Circuit Symbols

There are several similar but not identical standards for *circuit symbols* related to resistance: International standard IEC 60617, American ANSI standard Y32 (IEEE Std 315), etc. Figure 2.10 shows popular circuit symbols for variable resistances.

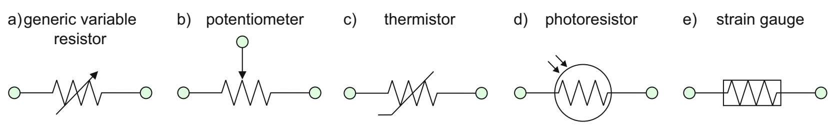


Fig. 2.10. Circuit symbols for variable resistances: (**a**) generic variable resistance, (**b**) potentiometer, (**c**) thermistor, (**d**) photoresistor, and (**e**) strain gauge.

Section 2.2 Nonlinear Passive Circuit Elements

2.2.1 Resistance as a Model for the Load

It might appear at first glance that the resistance causes mainly power loss and heating. However, the concept of heating elements in household appliances or power losses in long cables covers only a small subset of applications. Important resistance applications are related to the resistive sensors studied previously. Resistances are widely used in the circuits to provide different voltage values, i.e., *bias* circuit components such as diodes and transistors. Last but not least, resistances model an arbitrary power-absorbing device, *the load*, which can be mechanical, acoustical, microwave, optical, etc. From a circuit point of view it *does not matter* how the electric power supplied to a load is eventually transformed. The circuit delivers certain power to the load, but cares little about whether the power is converted into heat to warm a heating pan, light to illuminate our house, mechanical power to drive a motor, or electromagnetic radiation generated by a cell phone. Circuit analysis is concerned with the power delivered to a power-absorbing device, *the load*, leaving its utilization and conversion to other engineering disciplines. Therefore, many practical loads can be replaced by a simple *load resistance* R_L. Such a resistance is often called the *equivalent resistance*, $R_L = R_{eq}$. It is also valid for AC circuits. For AC circuits it is convenient to use *rms* (root mean square) voltages, which are equivalent to DC voltages and thus provide us with the same power value delivered to the load. Figure 2.11 shows two examples of the load replacement with the equivalent resistance: a light source radiating visible light and an antenna radiating microwaves.

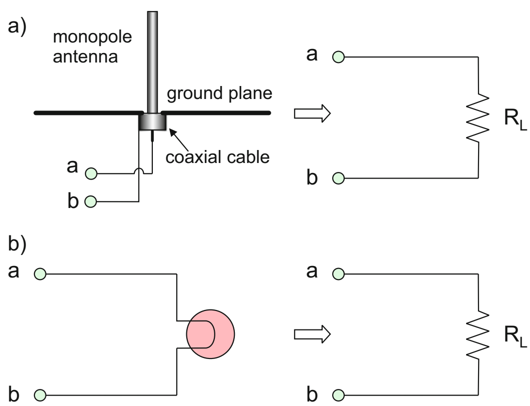


Fig. 2.11. (**a**) Radiating monopole antenna is modeled as a resistance. (**b**) Light source is approximately modeled as a resistance.

Example 2.4: A small commercial monopole antenna shown in Fig. 2.11a is rated at R_{eq} = 50 Ω in the ISM band of 902–928 MHz. When an rms voltage of 10 V is applied to the antenna, what is the total amount of power radiated by the antenna?

Example 2.4 (cont.):

Solution: We use Eq. (2.7) and obtain $P = V_{\text{rms}}^2/R_{\text{eq}} = 2$ W. All of this power is radiated in the form of an outgoing electromagnetic wave. There is no heat loss. The resistance R_{eq} is called the *radiation resistance* in such a case. The above analysis is valid only in a certain frequency range.

A load that exactly follows Ohm's law Eq. (2.3) is called the *linear load*. While the transmitting antenna in Fig. 2.11a is a linear load, an incandescent light bulb in Fig. 2.11 is not. Most of the loads deviate from the linear Ohm's law.

2.2.2 Nonlinear Passive Circuit Elements

Nonlinear passive circuit elements do not satisfy Ohm's law with the constant resistance R over a wide range of voltages. Therefore, they are also called *non-ohmic circuit elements*. The non-ohmic elements may be described by a similar expression:

$$V = R(V)I \Leftrightarrow R(V) \equiv \frac{V}{I(V)} \tag{2.19}$$

but with a *variable resistance* $R(V)$. For passive elements, $R(V) > 0$. The resistance $R(V)$ is known as the *static* or *DC resistance*. Figure 2.12 depicts the v-i characteristics for three distinct passive circuit elements.

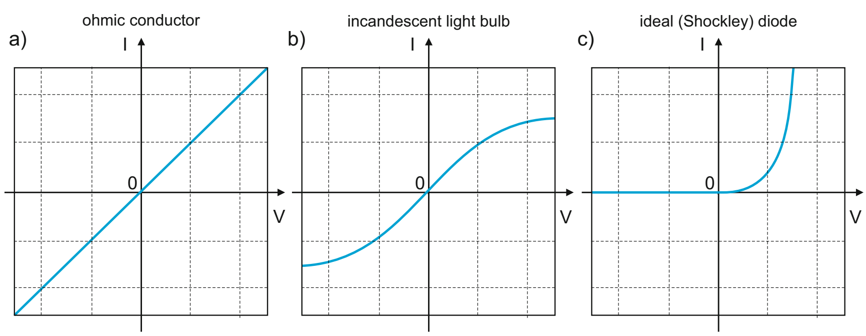


Fig. 2.12. Three v-i characteristics: (**a**) linear—resistance; (**b**) nonlinear—incandescent light bulb; and (**c**) nonlinear—ideal or Shockley diode.

The first element is an ohmic element (*ohmic conductor*) with a constant resistance R. The corresponding v-i characteristics is a straight line—the circuit element is linear. The second element corresponds to an incandescent light bulb. Its resistance R increases when the applied voltage V increases (the conductivity of the radiating filament of wire decreases with increasing absorbed power and wire temperature). Hence, the v-i characteristic bends down and deviates from the straight line—see Fig. 2.12b. This element only approximately follows Ohm's law. It is therefore the nonlinear circuit element. The third element in Fig. 2.12 corresponds to an *ideal (Shockley) diode*. The diode does not conduct at negative applied voltages. At positive voltages, its *v-i* characteristic is very sharp (exponential). The diode is also the nonlinear circuit element. Strictly speaking, the *v-i* characteristic of the incandescent light bulb does not belong to the list of circuit elements due to its limited applicability. However, the ideal diode is an important nonlinear circuit element. The nonlinear elements are generally *polar* (non-symmetric) as Fig. 2.12c shows.

2.2.3 Static Resistance of a Nonlinear Element

Once the *v-i* characteristic is known, we can find the static resistance $R(V)$ of the nonlinear circuit element at any given voltage V_0. For example, the *v-i* characteristic of the ideal diode shown in Fig. 2.12c is described by the exponential *Shockley equation*:

$$I = I_S\left[\exp\left(\frac{V}{V_T}\right) - 1\right] \tag{2.20}$$

In Eq. (2.20), the constant I_S [A] is the diode *saturation current*. The saturation current is very small. The constant V_T [V] is called the *thermal voltage*.

Example 2.5: Give a general expression for the diode resistance $R(V)$ using Eq. (2.20) and find its terminal values at $V \to 0$ and $V \to \infty$, respectively. Then, calculate static diode resistance R_0 and diode current I_0 when the voltage across the diode is $V_0 = 0.55$ V. Assume that $I_S = 1 \times 10^{-12}$ A and $V_T = 25.7\,\text{mV}$.

Solution: Using Eq. (2.20) we obtain

$$R(V) = \frac{V}{I_S\left[\exp\left(\frac{V}{V_T}\right) - 1\right]} \tag{2.21}$$

When $V \to 0$, we can use a Taylor series expansion for the exponent. Keeping only the first nontrivial term, one has $\exp(V/V_T) \approx 1 + V/V_T$. Therefore,

$$R(V) \to \frac{V_T}{I_S} \text{ when } V \to 0 \;\; (\text{or } V/V_T << 1) \tag{2.22}$$

This value is very large, in excess of 1 GΩ. The diode is thus the open circuit with a good degree of accuracy.

On the other hand, at large V, the exponential factor in Eq. (2.21) greatly increases. Therefore,

$$R(V) \to 0 \text{ when } V \to \infty \tag{2.23}$$

Example 2.5 (cont.):

The diode becomes virtually a short circuit as indicated by the almost vertical slope in Fig. 2.12c. Finally, we obtain the particular values for $V_0 = 0.55\,\text{V}$:

$$R_0 = 275\ \Omega, \quad I_0 = 2.00\ \text{mA} \tag{2.24}$$

2.2.4 Dynamic (Small-Signal) Resistance of a Nonlinear Element

Equally, and perhaps even more important, is the concept of a *dynamic* (or *small-signal*) resistance, r, of the nonlinear circuit element. Other equivalent names include *differential resistance* or *incremental resistance*. Quite often the voltage across the element and the current through it are given by

$$V = V_0 + \upsilon, \quad I = I_0 + i \tag{2.25}$$

where V_0 and I_0 are the DC (constant-value) voltage and current related to each other through the static resistance, $V_0 = R_0 I_0$. These values are set with the help of an external DC circuit. On the other hand, quite small components υ and i describe a very weak time-varying (AC or pulse) signal. Though weak, this signal contains the major information to be processed. A receiver circuit in your cell phone is an example. Now, how are υ and i related to each other? The answer is still given by Ohm's law but written in terms of the *dynamic resistance*, i.e.,

$$\upsilon = ri, \quad r \equiv \left.\frac{dV}{dI}\right|_{V=V_0,\ \ I=I_0} \tag{2.26}$$

This derivative is to be evaluated at the *operating point* $V_0,\ I_0$ (also called the *quiescent point* or *Q-point*). Figure 2.13 provides a graphical proof of Eq. (2.26) using the example of an ideal diode. The zoomed-in version of Fig. 2.12c has been used. The *dynamic (small-signal) resistance* is thus the *inverse slope* of the υ-i characteristic at the operating point. The exact mathematical proof is performed using a Taylor series expansion.

Exercise 2.6: Determine the small-signal resistance r for the ohmic circuit element with $V = RI,\ \ R = \text{const.}$

Answer: $r = R$ for any operating point.

The dynamic diode resistance plays a decisive role in the design of amplifiers based on junction transistors. The ideal-diode circuit element becomes a part of the transistor circuit model. The dynamic resistance is also critical for amplifiers which use other transistor types. From the mathematical point of view, finding the static and dynamic resistance is simply finding the function and its first derivative at the operating point.

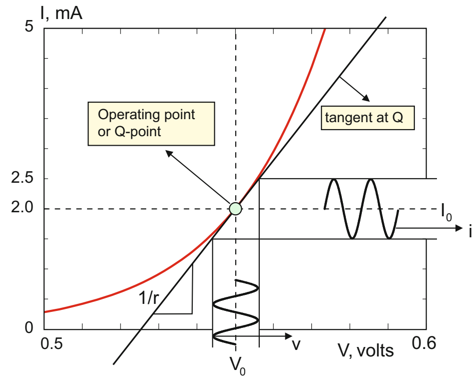


Fig. 2.13. Finding dynamic resistance for an ideal diode.

Example 2.6: Give a general expression for the dynamic diode resistance r using Eq. (2.20) at an arbitrary operating point with current I_0. Then, calculate the dynamic diode resistance r when the voltage across the diode is V_0 =0.55 V. Assume that $I_S = 1 \times 10^{-12}$ A and $V_T = 25.7$ mV.

Solution: Using Eq. (2.20) we obtain

$$V = V_T \ln\left[1 + \frac{I}{I_S}\right] \Rightarrow r(I) = \frac{V_T}{I + I_S} \Rightarrow r \approx \frac{V_T}{I_0} \tag{2.27}$$

since the saturation current I_S may be neglected. At $V_0 = 0.55$ V we obtain $I_0 = 2.00$ mA—see Eq. (2.24). Therefore,

$$r = 12.8\ \ \Omega\ \ \text{when}\ \ V_0 = 0.55\ \text{V}. \tag{2.28}$$

2.2.5 Electronic Switch

In an *electronic switch* (or a *solid-state switch*), an electric quantity (voltage or current) acts as a stimulus—it opens and closes the switch. The electronic switch is inherently a nonlinear device. Figure 2.14 shows a *two-terminal switch* and its (idealized) v-i characteristic. The voltage across the switch V is generated by the main circuit. When this voltage reaches a certain *switch threshold voltage* V_{Th}, the switch becomes closed, it can conduct any current. Further, the voltage across the switch does not change. Switches of this type usually involve pn-junction diodes. For example, the diode v-i characteristic from Fig. 2.12c may approximate the step function in Fig. 2.14b. The *unidirectional switch* shown in Fig. 2.14 can pass the current only in one direction. *Bidirectional switches* can pass current in both directions.

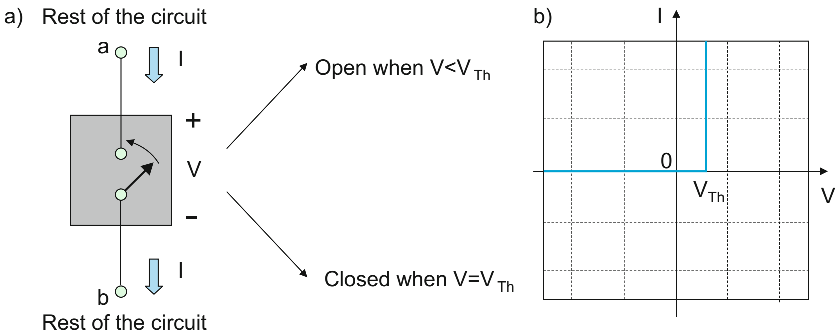


Fig. 2.14. (**a**) Two-terminal unidirectional threshold switch and (**b**) its ideal *υ-i* characteristic.

Figure 2.15 shows another, *three-terminal electronic switch*. The voltage V controlling the switch operation is now generated by a separate (control) circuit. It is still referenced to the common circuit ground. When the control voltage reaches a certain *switch threshold voltage* V_{Th} *or exceeds* it, the switch closes. The switches of this type involve transistors, either junction or field effect. A distinct feature of the switch in Fig. 2.15 is that the control voltage may have arbitrary values, including $V > V_{\mathrm{Th}}$. Therefore, its *υ-i* characteristic involves all states to the right of the vertical line in Fig. 2.14.

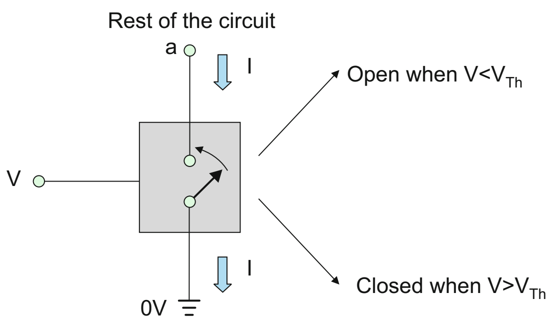


Fig. 2.15. Three-terminal unidirectional threshold switch (pull-down switch).

The switch shown in Fig. 2.15 and its pull-up counterpart are the "heart" of any digital circuit, which is essentially a nonlinear switching circuit. Chapter 15 provides an introduction to digital switching circuits.

Exercise 2.7: Based on conditions of example 2.5, determine when a diode switch closes. This condition approximately corresponds to the diode current of 10 mA.

Answer: The diode voltage should be equal to 0.594 V or approximately 0.6 V.

Section 2.3 Independent Sources

2.3.1 Independent Ideal Voltage Source

An *independent ideal voltage source* is an important circuit element. Figure 2.16a shows the corresponding circuit symbol for the DC (steady-state) source. As a circuit element, the voltage source is not symmetric: terminals 1 and 2 (commonly labeled as plus and minus or red and black) may not be interchanged without affecting its operation. In other words, the voltage source is a *polar device*. The voltage source generates a positive *voltage difference* (or *voltage drop* or simply *voltage*) across its terminals, $V_S > 0$, the polarity of which is shown in Fig. 2.16. The term *independent* means that voltage V_S does not vary because of different parameters of an electric circuit (not shown in the figure), which is *implied to be connected* to the voltage source.

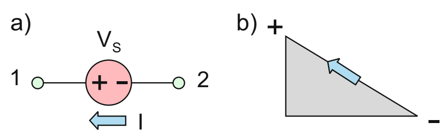


Fig. 2.16. Symbol for an ideal voltage source along with the voltage and current behavior.

Current Through the Voltage Source: Active Reference Configuration

The current I flowing through the voltage source is shown in Fig. 2.16 by an arrow. The relation between voltage polarity and current direction depicted in Fig. 2.16 is known as the *active reference configuration*. It is commonly used for all active circuit elements such as voltage and current sources, either dependent or independent. A useful fluid mechanics analogy for the voltage source is water (electric current) that is pushed up the “voltage” hill in Fig. 2.16b by external means. Alternatively, one may think of a water pump that is characterized by a constant pressure drop. The active reference configuration means that the voltage source supplies electric power to the circuit. This configuration is the opposite of the passive reference configuration for the resistance.

v-i Characteristic of the Voltage Source

Figure 2.17a plots the *v-i characteristic* of the ideal voltage source. The term *ideal* literally means that the *v-i* characteristic is a straight vertical line: the ideal voltage source is capable of supplying *any* current to *any* circuit while keeping the *same* voltage V_S across its terminals. In reality, it is not the case since the high currents mean high powers. Therefore, a laboratory power supply—the physical counterpart of the ideal voltage source—will be eventually overloaded as shown in Fig. 2.17b. Figure 2.17c shows a common way of drawing the *v*-i characteristic for the voltage source with the axes *interchanged*. For the purposes of consistency, the *x*-axis will always be used as the voltage axis throughout the text.

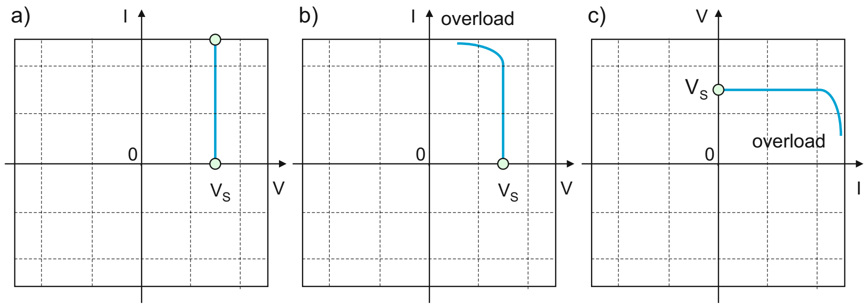


Fig. 2.17. υ-i Characteristics for (**a**) ideal voltage source used in the circuit analysis and (**b**) its physical counterpart—a regulated laboratory power supply. (**c**) Typical way of drawing the υ-i characteristic for the voltage source with the axes interchanged.

Symbols for Independent Voltage Source

Multiple symbols may be used in a circuit diagram to designate the independent ideal voltage source—see Fig. 2.18. All these symbols are equivalent from the circuit point of view, as long as we imply the ideal source. However, their physical counterparts are quite different. The general symbol in Fig. 2.18a implies either an AC to DC converter (the laboratory power source) or a battery. The symbol in Fig. 2.18b relates to a battery and Fig. 2.18c–d depicts battery banks.

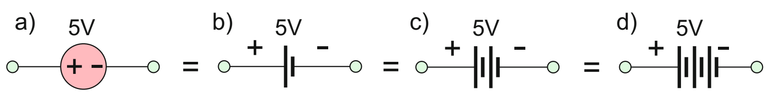


Fig. 2.18. (**a**) Generic DC voltage source, (**b**) single battery, and (**c**) and (**d**) battery banks. All symbols in the circuit diagram are equivalent.

Example 2.7: Solve an electric circuit shown in Fig. 2.19—determine circuit current I and voltage across the resistance V.

Solution: We use the graphical solution—plot the υ-i characteristic of the 2 kΩ resistance and the υ-i characteristic of the voltage source on the same graph to scale—see Fig. 2.19b. The intersection point is the desired solution: $V = 3$ V, $I = 1.5$ mA. Indeed, this simple solution implicitly uses circuit laws (KVL and KCL) studied next.

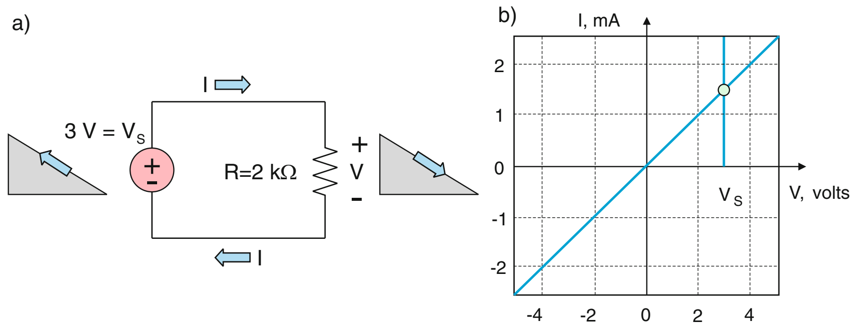


Fig. 2.19. Electric circuit solution in graphical form.

2.3.2 Circuit Model of a Practical Voltage Source

Any *practical voltage source* is modeled as a *combination* of an ideal voltage source V_S and an ideal resistance R *in series*—see Fig. 2.20a. The resistance R reflects the non-ideality of the practical source: it limits the *maximum available source current* and the *maximum available source power* by (similar to the current-limiting resistor):

$$I_{max} = V_S/R, \quad P_{max} = V_S I_{max} = V_S^2/R \tag{2.29}$$

Voltage V_S is called the *open-circuit voltage* of the source for an obvious reason. Similarly, current I_{max} is called the *short-circuit current* of the source. Once both quantities are measured in laboratory, resistance R (called the *internal source resistance*) may be found using Eq. (2.29).

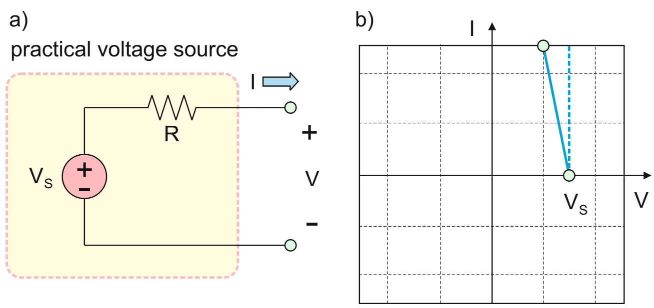


Fig. 2.20. Circuit model of a practical voltage source and its *v*-i characteristic.

Exercise 2.8: The open-circuit voltage of a voltage source is 9 V; the short-circuit current is 2 A. Determine the internal source resistance.

Answer: 4.5 Ω.

The v-i characteristic of the practical voltage source is the plot of source current I versus voltage V available from the source in Fig. 2.20a. This voltage is generally less than V_S since any nonzero current I causes a voltage drop of RI across resistance R. One has

$$V = V_S - RI \Rightarrow I = \frac{V_S - V}{R} \tag{2.30}$$

This v-i characteristic is plotted in Fig. 2.20b by a solid line. The deviation from the straight vertical line characterizes the degree of non-ideality. Emphasize that any laboratory power supply is indeed a practical voltage source. However, using a special circuit, its input is *regulated* so that the output voltage does depend on the output current, at least over a reasonable range of currents. Therefore, instead of Fig. 2.20b we arrive at a more reliable voltage source from Fig. 2.17b.

Exercise 2.9: Determine internal source resistance for the source illustrated in Fig. 2.20b given that every horizontal division is 3 V and every vertical division is 1 A.

Answer: 0.6 Ω.

2.3.3 Independent Ideal Current Source

An *independent ideal current source* is dual of the ideal voltage source. Figure 2.21a shows the corresponding circuit symbol for the DC (constant-current) source. As a circuit element, the constant-current source is *directional*: terminals 1 and 2 indicate the direction of the current flow. The current source generates a positive constant current, $I_S > 0$, which flows from terminal 2 to terminal 1 in Fig. 2.21. The term *independent* means that current I_S does not vary because of different parameters of an electric circuit (not shown in the figure), which is *implied to be connected* to the current source.

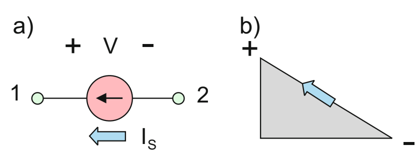


Fig. 2.21. Symbol for the ideal current source along with voltage and current designations.

Voltage Across the Current Source: Active Reference Configuration

Once the current source is connected to a circuit, a voltage V will be created across it. The voltage polarity is indicated in Fig. 2.21a. The relation between voltage polarity and current direction shown in Fig. 2.21 is again the *active reference configuration*, similar to the voltage source. A useful fluid mechanics analogy for the electric current source is a water pump that creates a constant water supply (e.g., 0.5 ft^3/s). Indeed, this water pump will be characterized by a certain pressure difference across its terminals, which is the analogy of voltage V in Fig. 2.21.

υ-i Characteristic of the Current Source

Figure 2.22a plots the *υ-i characteristic* of an ideal current source. Compared to the ideal voltage source, the graph is rotated by 90 degrees. The term *ideal* again means that the *υ-i* characteristic is the straight horizontal line: the ideal current source is capable of creating *any* voltage across its terminals while keeping the *same* current I_S flowing into the circuit. In reality, it is not the case since high voltages mean high powers. Therefore, a laboratory current power supply—the physical counterpart of the ideal current source—will eventually be overloaded as shown in Fig. 2.17b. The current laboratory supplies are rarely used (one common use relates to transistor testing); they are less common than the voltages supplies. However, the current sources are widely used in transistor circuits, both integrated and discrete. There, the current sources are created using dedicated transistors. Furthermore, photovoltaic sources are essentially current sources.

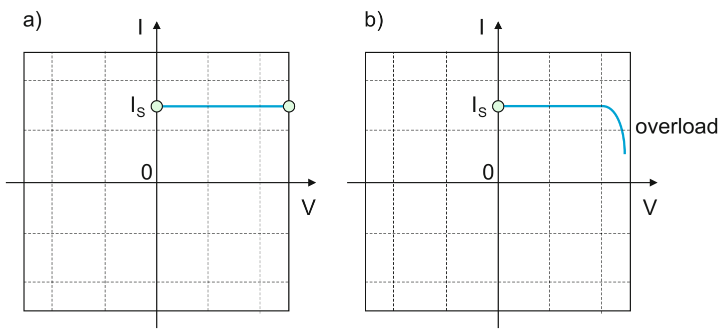


Fig. 2.22. *υ*-i Characteristics of (**a**) an ideal current source and (**b**) its physical counterpart.

Symbols for Independent Current Source

A few equivalent symbols may be used in a circuit diagram to designate the independent ideal current source; see Fig. 2.23. All these symbols are equivalent as long as we imply the ideal source. The symbol in Fig. 2.23a is used in North America, the symbol in Fig. 2.23b is European, and the symbol in Fig. 2.23 may be also found in older texts.

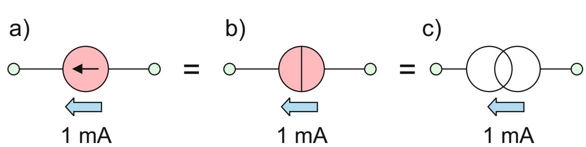


Fig. 2.23. Equivalent symbols of the current source in the circuit diagram.

Example 2.8: Solve an electric circuit shown in Fig. 2.24—determine voltage V across the source and the resistance.

Solution: Similar to the voltage source, we use a graphical solution—plot the v-i characteristic of the 2-kΩ resistance and the v-i characteristic of the current source on the same graph to scale; see Fig. 2.24b. The intersection point gives us the desired solution: $V = 3$ V. Note that the solutions for this example and the solution for Example 2.7 coincide. This means that, under certain conditions, we can interchange both sources without affecting the circuit performance. Indeed, the graphical solution implicitly uses the circuit laws (KVL and KCL) studied in detail next.

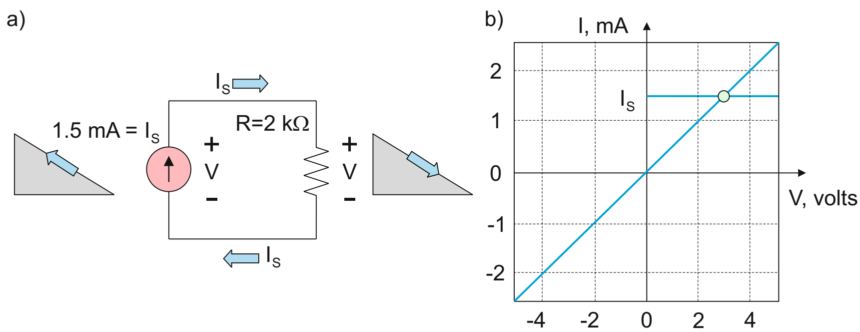


Fig. 2.24. Electric circuit solution in the graphical form.

2.3.4 Circuit Model of a Practical Current Source

Any *practical current source* is modeled as a *combination* of the ideal current source I_S and the ideal resistance R *in parallel*—see Fig. 2.25a. The resistance R reflects the non-ideality of the practical source: it limits the *maximum available source voltage* and the *maximum available source power* by

$$V_{max} = RI_S, \quad P_{max} = V_{max} I_S = RI_S^2 \tag{2.31}$$

Voltage V_{max} is again called the *open-circuit voltage* of the source. Similarly, current I_S is called the *short-circuit current* of the source. Once both the quantities are measured, resistance R (called the *internal source resistance*) may be found using Eq. (2.31).

Exercise 2.10: The open-circuit voltage of a current source is 9 V; the short-circuit current is 2 A. Determine the internal source resistance.

Answer: 4.5 Ω.

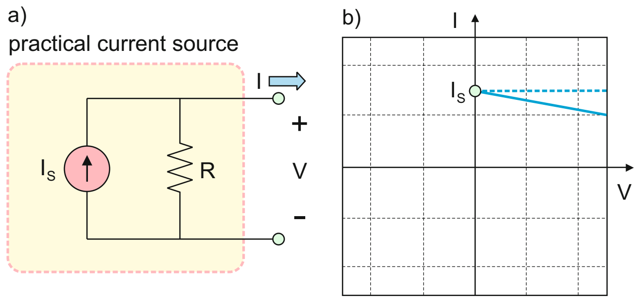


Fig. 2.25. Circuit model of a practical current source and its υ-i characteristic.

The υ-i characteristic of the practical current source is the plot of current I available from the source versus voltage V across the source in Fig. 2.25b. This current is generally less than I_S since a portion of I_S flows through the internal resistance R, i.e.,

$$I = I_S - \frac{V}{R} \tag{2.32}$$

This υ-i characteristic is plotted in Fig. 2.25b by a solid line. The deviation from the straight horizontal line characterizes the degree of non-ideality.

Exercise 2.11: Determine the internal source resistance for the source illustrated in Fig. 2.25b given that every horizontal division is 3 V and every vertical division is 1 A.

Answer: 15 Ω.

2.3.5 Operation of the Voltage Source

Operation of a voltage power supply of any kind (an electric generator, a chemical battery, a photovoltaic cell, etc.) might be illustrated based on the *charge separation principle* very schematically depicted in Fig. 2.26. We need to deliver electric power to a load modeled by an equivalent resistance R_L. First, we consider in Fig. 2.26 a charged capacitor with a charge Q connected to a load resistor R_L at an initial time moment. The capacitor voltage V is related to charge by $V = Q/C$ where C is the (constant) capacitance. The capacitor starts to discharge and generates a certain load current I_L. At small observation times, the change in Q is small, so is the change in V. Therefore, the capacitor initially operates as a voltage power supply with voltage V. However, when time progresses, the capacitor discharges and the voltage V eventually decreases.

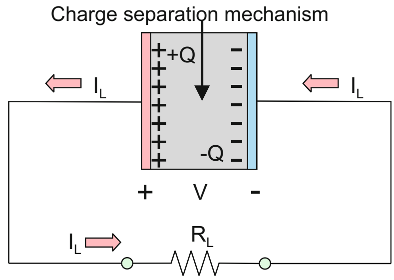


Fig. 2.26. Power source schematically represented as a capacitor continuously charged by a charge separation mechanism—the *charge pump*.

How could we keep V constant, i.e., continuously charge the capacitor? A charge separation mechanism should be introduced between the hypothetic capacitor plates to continuously compensate for the charge leakage. That mechanism may have the forms:

1. For an electromechanical generator, this is the *Lorentz force* that acts on individual electrons in a conductor and pushes them to one conductor terminal while creating the opposite charge density on the opposite conductor terminal. The macroscopic effect of the Lorentz force is the *Faraday's law of induction.*
2. For a battery, these are chemical reactions at the electrodes which cause a charge separation, i.e., positive metal ions dissolve in the electrolyte and leave excess electrons in the metal electrode on the left in Fig. 2.26.
3. For the photovoltaic cell, this is a built-in potential of the semiconductor pn-junction that separates light-generated negative carriers (electrons) and positive carriers (holes) as shown in Fig. 2.26.

Indeed, the capacitor analogy in Fig. 2.26 is only an illustrative approach, especially for electromechanical power generation. Below, we will consider a few specific examples.

2.3.6 Application Example: DC Voltage Generator with Permanent Magnets

A realistic electromechanical voltage source—a basic *DC generator* with permanent magnets—is shown in Fig. 2.27. This generator setup makes use of the *Lorentz force*:

$$\vec{f} \equiv q\left(\vec{v} \times \vec{B}\right) \tag{2.33}$$

The Lorentz force acts on charge q moving with a velocity $\vec{v}$ in an external magnetic field with the vector flux $\vec{B}$ measured in tesla (T). The force itself is measured in newtons. The cross symbol in Eq. (2.33) denotes the vector product of two vectors evaluated according to the right-hand rule. Shown in Fig. 2.27a are two permanent magnets (stator of the generator) responsible for creating the magnetic flux $\vec{B}$ emanating from the north pole

(N) and terminating at the south pole (S). The armature (rotor) rotates clockwise in Fig. 2.27b with the armature velocity $\vec{v}$.

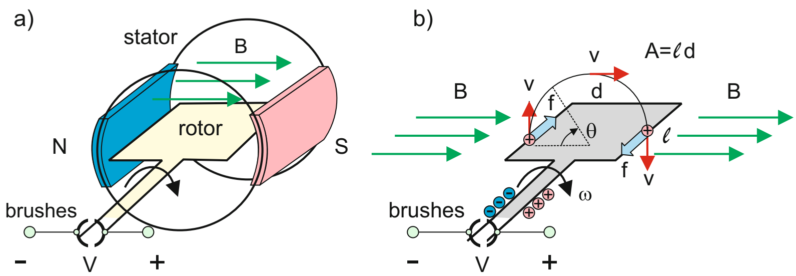


Fig. 2.27. Charge separation in a DC electromechanical generator.

When the flux density $\vec{B}$ is applied, every positive charge $+q$ in the armature segment l will experience a Lorentz force with the magnitude $f = +qvB$ which will move this charge toward the right terminal of the armature in Fig. 2.27b. Similarly, every negative charge $-q$ in the armature would experience the equal but oppositely directed Lorentz force $f = -qvB$ which will move this charge toward the left terminal. Hence, a charge separation occurs along the armature which will give rise to an induced voltage V. Total work W of the Lorentz force on a charge q along the entire armature path in Fig. 2.27b is given by $W = 2lf$. This work divided by the amount of charge determines the equivalent voltage that will be developed on the generator terminals, i.e., the *instantaneous generator voltage* $V = W/q = 2lvB$. If the armature rotates at an angular speed ω (rad/s), the charge velocity perpendicular to the field is given by $v = d/2\omega \cos\theta$ (m/s). Plugging in this expression and averaging over angles θ from 0 to $\pi/2$, we obtain the *average generator voltage* in the form

$$V = ldB\langle\cos\theta\rangle = (2/\pi)AB\omega \quad [\mathrm{V}] \tag{2.34}$$

where A is the armature area. If the rotor has N turns, the result is multiplied by N. The same expression for the voltage is obtained using the *Faraday's law of induction*. A *regulator circuit* is necessary to obtain a flat DC voltage without ripples. Any brushed DC motor operates as a generator when its shaft is rotated with a certain speed. The generated open-circuit voltage may be observed in laboratory with the oscilloscope.

Exercise 2.12: Determine average open-circuit generator voltage in Fig. 2.27 given $A = 0.1\ \mathrm{m}^2$, $B = 0.2\ \mathrm{T}$, $\omega = 20$ rad/s (191 rpm), and the armature with 20 turns.

Answer: 5.1 V.

2.3.7 Application Example: Chemical Battery

A chemical reaction in a battery induces a continuous charge separation. The "charge pump" so constructed, once connected to a load, is able to create a continuous electric current into a load and a voltage difference across it. You are probably aware of the quest to improve the venerable battery. Extensive coverage in the media and in technical journals frequently reports on new chemical compounds and control circuits. They target smaller, more powerful rechargeable batteries for such diverse devices as portable computers, cell phones, sensors, and automobiles. Specifically, it is the automotive sector which implements hybrid vehicle technology where powerful electro motors in conjunction with high-performance batteries are supplementing, even completely replacing, conventional combustion engines. For a standard *chemical battery*, the two important parameters are *battery voltage* and *battery capacity*. The capacity, Q, is a new quantity that is needed because of a battery's inability to provide constant current and power for an infinite time duration. How a battery behaves over time is illustrated in Fig. 2.28 where we monitor the power and current as a function of time. A key time constant is the so-called discharge time, which is critically dependent on the attached load.

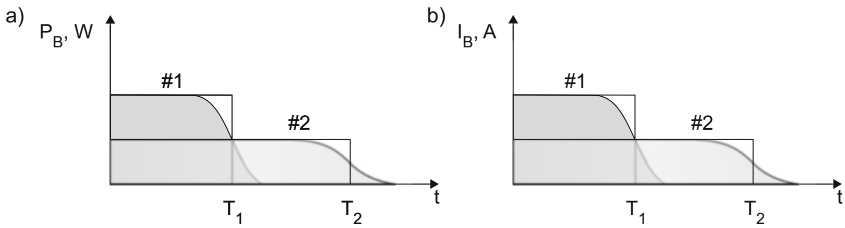


Fig. 2.28. Generic plots of delivered power, P_B, and electric current, I_B, for two different loads labeled #1 and #2. The discharge times T_1 and T_2 correspond to the loads #1 and #2, respectively.

When a load of resistance R_L is connected to the battery, and the battery's internal resistance R is negligibly small compared to that resistance, the circuit current, I_B, and the power delivered by the battery, P_B, are determined based on Ohm's law:

$$I_B = \frac{V_B}{R}, \quad P_B = V_B I_B \tag{2.35}$$

The *total energy*, E_B, stored in the battery and then delivered to the circuit is a *fixed constant*. Its value depends on the battery type and size. The total energy in joules is given by the time integral of delivered power over time, i.e.,

$$E_B = \int_0^\infty P_B(t')dt' \tag{2.36}$$

We can assume that the total energy is a finite constant; it follows from Eq. (2.36) that the delivered power must drop to zero at a finite time T. This is schematically shown in

Fig. 2.28 for two different load resistances, #1 and #2, which require two different circuit currents. Even though the two power curves in Fig. 2.28a are different, the area under those curves, denoting the total energy stored in the battery, remains *the same* to a sufficient degree of accuracy. The battery's terminal voltage V_B also remains *approximately constant* over the entire operation cycle and even afterwards. It is the battery's current I_B that finally sharply decreases with time and causes a drop in power, as seen in Fig. 2.28b. Let us consider the simplest case where the current is a constant for $t < T$ and at $t = T$ drops to zero and stays zero for $t > T$. From Eq. (2.36), it follows that

$$E_B = \int_0^\infty P_B(t')dt' = \int_0^T V_B I_B dt' = [T I_B] V_B \tag{2.37}$$

The expression in the square brackets is the definition of the *battery capacity*, Q:

$$Q \equiv T I_B = \frac{E_B}{V_B} \tag{2.38}$$

Since the battery terminal voltage is always known, its capacity determines the total energy stored in the battery. The capacity is measured in A·h (Ah) or for small batteries in mA·h (mAh). The capacity rating that manufacturers print on a battery is based on the product of 20 h multiplied by the maximum constant current that a fresh battery can supply for 20 h at 20°C while keeping the required terminal voltage. The physical size of batteries in the USA is regulated by the American National Standards Institute (ANSI) and the International Electrotechnical Commission (IEC). Table 2.3 lists the corresponding parameters of some common batteries.

Example 2.9: A 12-V battery rated at a capacity of $Q = 100$ A·h may deliver 5 A over a 20-h period, 2.5 A over a 40-h period, or 10 A over a 10-h period. Find the total energy delivered by the battery provided that its internal resistance is negligibly small.

Solution: The total energy delivered by the battery is equal to

$$E_B = V_B Q = 12 \cdot 100 \ \ \mathrm{V \cdot A \cdot h} = 1200 \ \ \mathrm{W \cdot h} = 4.32\,\mathrm{MJ} \tag{2.39}$$

It remains constant for each case. This example shows that the electric energy can be measured either in joules or in Wh or more often in kWh. Clearly, 1 Wh = 3600 J.

Circuit Model of a Battery

As a practical voltage source, a battery always has a small, but finite *internal* resistance, R. Battery's equivalent circuit therefore includes the ideal voltage source and the internal resistance in series—see Fig. 2.29. Even though the values of R are small, the internal resistance has critical implications affecting both the battery's efficiency and its ability to provide a high instantaneous power output. In general, it is difficult to directly measure

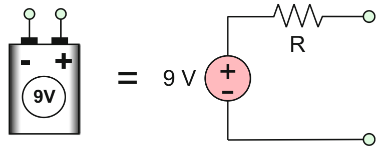


Fig. 2.29. Circuit model of a battery: the ideal voltage source in series with an internal resistor.

the internal resistance of batteries, you would need a calibrated load resistor and sophisticated measurement equipment to precisely measure voltages and currents.

Exercise 2.13: A 12-V battery has an internal resistance of 10 Ω. What are the maximum current and the maximum power that the battery can output?

Answer: $I_{\max} = 1.2\ \text{A}$, $P_{\max} = 14.4\ \text{W}$

Many battery types have been developed for a wide range of applications. They differ both in *battery energy storage* per kg of weight, or unit volume, and in power delivery per kg of weight, or per unit volume. In particular, modern heavy-duty, deep-cycle batteries may sport the following properties:

$$\text{Energy storage}: \quad 150\,\text{W}\cdot\text{h}/\text{l}, \tag{2.40}$$

$$\text{Power density}: \quad 2\ \text{kW}/\text{l}. \tag{2.41}$$

You can compare Eqs. (2.40) and (2.41) with the last row of Table 2.3 and establish the approximate density of the battery device.

Table 2.3. Characteristics of batteries (from multiple datasheets).

Battery size/type	Rechargeable	Voltage (cell)	Capacity (A·h)	Resistance (R)
AAA	No	1.5	1.3	100–300mΩ for alkaline battery per cell
AA	No	1.5	2.9	
C	No	1.5	8.4	
D	No	1.5	20.5	
9 V	No	9.0	0.6	~400 mΩ
Lithium batteries	Yes	3.6–3.7	0.7–1.5	~300 mΩ
Lead acid starter battery (automotive, deep cycle)	Yes	12.6	~600A for 30 s at 32 °F before voltage drops to 7.20 V	$<$100 mΩ
Deep-cycle marine, electric vehicles	Yes	Variable: ~ 30 W·h per kg of weight, or ~ 108 kJ per kg of weight		~200 mΩ

Section 2.4 Dependent Sources and Time-Varying Sources

2.4.1 Dependent Versus Independent Sources

If the strength of the source (voltage or current) does not vary because of variation of the circuit parameters, the source is an *independent source*. The voltage and current sources considered previously are the independent sources. However, if the strength of the source is controlled by some dedicated circuit parameters, the sources are called *dependent sources*. Figure 2.30 shows circuit symbols (diamonds) for the dependent voltage and current sources. The ideal dependent sources are the important circuit elements, along with the independent sources. We explain the notations in Fig. 2.30 as follows.

1. The ideal independent and dependent sources may generate not only the steady-state (DC) voltages and currents, but also *arbitrary time-varying* voltages and currents. To underscore this fact, we will use the *lowercase* notations for voltages and currents, respectively.
2. The dependent sources generate (or *output*) voltage or current in response to some *input* voltage or current—the stimulus. To underscore this fact, we will use the subscript $_{\mathrm{OUT}}$ for the generated voltage or current strengths. This is in contrast to the subscript $_{\mathrm{S}}$, which always denotes the independent sources.
3. The stimulus voltage and the stimulus current (not yet shown in Fig. 2.30) will be denoted by v_{in} and i_{in}, respectively.

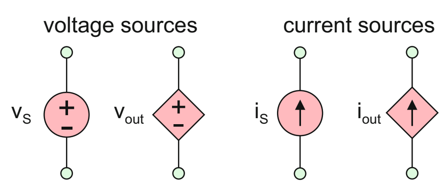


Fig. 2.30. Circuit symbols for ideal independent and dependent sources, respectively. Lowercase notations for voltages and currents are used to underscore possible time variations.

2.4.2 Definition of Dependent Sources

The stimulus voltage v_{in} is the voltage across a certain resistance. Likewise, the stimulus current i_{in} is the current through a certain resistance. Figure 2.31 shows *four major types of dependent sources* where the stimulus and the response are combined into one block—the shaded rectangle. Such a combination reflects the physical reality since this block usually corresponds to a single circuit component—a transistor or an amplifier. Emphasize that the resistances in Fig. 2.31 may be reduced to an *open circuit* for dependent voltage sources or to a *short circuit* for dependent current sources, respectively, if required. Also note that another circuit element may be present in place of the resistance, for example, the ideal diode. The bottom (ground) nodes in every circuit in Fig. 2.31 may be interconnected to emphasize the same voltage reference.

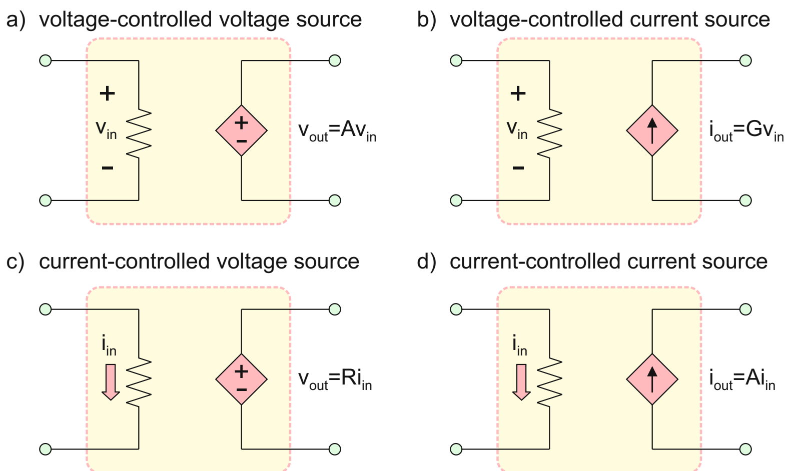


Fig. 2.31. Four major types of dependent sources.

Voltage-Controlled Voltage Source

This dependent source is shown in Fig. 2.31a. The source voltage or the output voltage follows the input voltage according to a linear law

$$v_{\text{out}} = Av_{\text{in}} \tag{2.42}$$

where the dimensionless constant A is called the *open-circuit voltage gain* of the dependent source. However, units of V/V or V/mV are often used. For example, the expressions $A = 5\ \text{V/mV}$ and $A = 5000$ are equivalent. Equation (2.42) is valid *irrespective* of the circuits connected to the dependent source to the right and to the left in Fig. 2.31a. In this sense, the voltage-controlled voltage source is the ideal circuit element. Such a source is a *voltage amplifier.*

Voltage-Controlled Current Source

This dependent source is shown in Fig. 2.31b. The source current or the output current follows the input voltage according to a linear law:

$$i_{\text{out}} = Gv_{\text{in}} \tag{2.43}$$

where the constant G with units of $\text{A/V} = \Omega^{-1} = \text{S}$ is called the *transconductance* of the dependent source, similar to the name conductance. For example, the expressions

$G = 0.5 \ \ \mathrm{A/mV}$ and $G = 500 \ \ \mathrm{S}$ are equivalent. Emphasize that the transconductance has nothing in common with the conductance (inverse resistance) of a passive resistor. Equation (2.43) is also valid *irrespective* of the circuits connected to the dependent source to the right and to the left in Fig. 2.31b. In this sense, the voltage-controlled current source is again the ideal circuit element. Such a source is a *transconductance amplifier*.

Current-Controlled Voltage Source

This dependent source is shown in Fig. 2.31c. The source voltage or the output voltage follows the input current through the resistance in Fig. 2.31c according to a linear law:

$$v_{\mathrm{out}} = R\, i_{\mathrm{in}} \tag{2.44}$$

where the constant R with units of $\mathrm{V/A} = \Omega$ is called the *transresistance* of the dependent source, similar to the name resistance. For example, the expressions $R = 5\mathrm{V/mA}$ and $R = 5000 \ \ \Omega$ are equivalent. Emphasize that the transresistance has nothing in common with the resistance of a passive resistor. Equation (2.44) is again valid *irrespective* of the circuits connected to the dependent source to the right and to the left in Fig. 2.31c. In this sense, the current-controlled voltage source is also an ideal circuit element. Such a source is a *transresistance amplifier*.

Current-Controlled Current Source

The last dependent source is shown in Fig. 2.31d. The source current or the output current follows the input current according to a linear law:

$$i_{\mathrm{out}} = A\, i_{\mathrm{in}} \tag{2.45}$$

where the dimensionless constant A is called the *short-circuit current gain* of the dependent source. However, units of A/A or A/mA are often used. For example, the expressions $A = 0.5 \ \ \mathrm{A/mA}$ and $A = 500$ are equivalent. We repeat that Eq. (2.45) is valid *irrespective* of the circuits connected to the dependent source to the right and to the left in Fig. 2.31d—the voltage-controlled voltage source is the ideal circuit element. Such a source is a *current amplifier*.

2.4.3 Transfer Characteristics

The dependent sources do not possess the v-i characteristic. Instead, a *transfer characteristic* of the source is used, which relates the output voltage or current to the input voltage or current. For example, the transfer characteristic of the voltage-controlled voltage source follows Eq. (2.42). It is a straight line in the v_{in}, v_{out} plane (the *xy*-plane), with the slope equal to A. Other linear transfer characteristics are obtained similarly.

Example 2.10: Solve a circuit shown in Fig. 2.32a—determine current i through the 1-kΩ resistance. The independent voltage source is given by $v_S = 0.5 + 2\cos 2t \ \ [\mathrm{V}]$; the open-circuit voltage gain of the dependent voltage source is 5 V/V.

Solution: The input voltage is simply the independent-source voltage, $v_{in} = v_S$. The output voltage is $v_{out} = 5v_{in} = 5v_S$. The output current follows Ohm's law:

$$i = \frac{v_{OUT}}{1 \ \mathrm{k\Omega}} = 2.5 + 10\cos 2t \quad [\mathrm{mA}] \tag{2.46}$$

Note that all circuit parameters now become time dependent. However, this does not change the solution compared to the steady-state case.

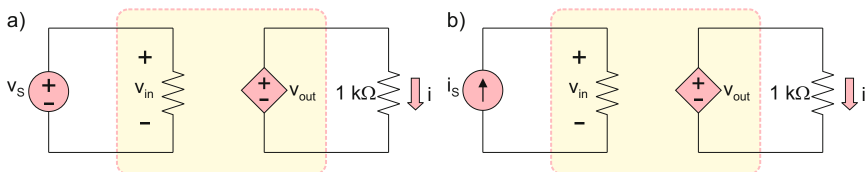


Fig. 2.32. Two circuits with the dependent voltage-controlled voltage source.

Exercise 2.14: Solve an electric circuit shown in Fig. 2.32b—determine current i through the 1-kΩ resistance. The independent current source is given by $i_S = 0.5 + 2\cos 2t \ \ [\mathrm{mA}]$; the open-circuit voltage gain of the dependent source is 5 V/V. The leftmost resistance in Fig. 2.32b (often called the *input resistance*) is 1 kΩ.

Answer: $i = 2.5 + 10\cos 2t \ [\mathrm{mA}]$ (the same answer as in Example 2.10).

2.4.4 Time-Varying Sources

Figure 2.33 shows a number of commonly used symbols for the voltage source—the ideal circuit element—which differentiate its time-related behavior. Figure 2.33a shows the steady-state ideal DC voltage source. Figure 2.33b indicates an arbitrary (either steady-state or variable) ideal voltage source. Figure 2.33c–d indicates a time-harmonic ideal *AC* (alternating current) *voltage source* described by a cosine function in the form

$$v_S(t) = V_m \cos(\omega t + \varphi) \ \ [\mathrm{V}] \tag{2.47}$$

where V_m is the AC *source amplitude* with the units of volts, ω is the AC *source angular frequency*, and φ is the phase in degrees or radians. The AC current sources do not have special symbols—the symbols from Fig. 2.30 are used. The same is valid for the dependent AC sources, both voltage and current.

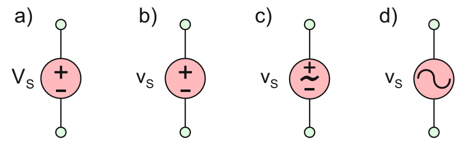


Fig. 2.33. Symbols for independent voltage source which imply (**a**) the DC source (capital V_S), (**b**) an arbitrary source (lowercase v_S), and (**c**) and (**d**) AC time-harmonic sources (lowercase v_S).

AC Source Polarity

Since the voltage in Eq. (2.47) is alternating, the polarity of the AC voltage source is also variable. This circumstance is reflected in Fig. 2.33d where the source polarity is not shown at all. However, for reference purposes, and when the multiple sources of the same frequency are present in the circuit, it is always useful to designate the source polarity. Reversing the AC source polarity means changing the phase in Eq. (2.47) by ±180°.

Section 2.5 Ideal Voltmeter and Ammeter: Circuit Ground

2.5.1 Ideal Voltmeter and Ammeter

The ubiquitous voltmeter and ammeter are devices designed to measure voltages and currents. Both devices are usually assembled in one unit known as a digital multimeter (DMM). From the circuit point of view, the *ideal voltmeter* is an *open circuit* which conducts zero current as shown in Fig. 2.34. An *ideal ammeter* is a *short circuit* which conducts any current with zero resistance—see the same figure. In reality, the voltmeter will conduct a small leakage current, and the ammeter will exhibit a small resistance.

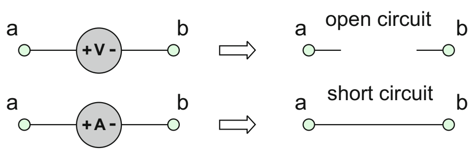


Fig. 2.34. Circuit equivalencies for ideal voltmeter and ammeter.

These features guarantee that the connection of the measurement device will not change the circuit operation. Figure 2.35 shows the proper connection of the voltmeter and ammeter to measure current through circuit element *A* and voltage across this element.

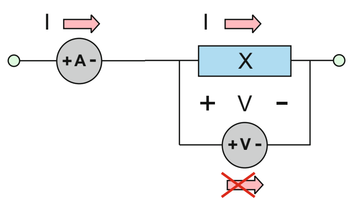


Fig. 2.35. Correct connection of voltmeter and ammeter for voltage and current measurements.

The ammeter is always connected *in series* with element *X*. In other words, to connect the ammeter we must *break* the circuit either before or after element *X*. Since the ammeter has no resistance, it acts just like an ideal wire and thus does not perturb the electric circuit. On the other hand, the voltmeter is always connected *in parallel* with element *X*. The circuit current *I* in Fig. 2.35 cannot flow through the voltmeter, which acts as an open circuit. As required, it will flow through element *X*. We conclude that an ideal voltmeter does not perturb the circuit either. Generally, voltage measurements are simpler to perform than current measurements.

Wrong Connections of Ammeter and Voltmeter

The ammeter connected in parallel will *short out* the element *A*: the current will flow through the ammeter. If the element *A* were a load, there would no longer be a load resistance in the circuit. And with no attached load, the power supply will deliver the largest possible current, which will likely burn out the ammeter fuses or destroy other circuit elements. The voltmeter is an open circuit. Connecting it in series is equivalent to physically breaking the circuit. The circuit will no longer properly function.

2.5.2 Circuit Ground: Fluid Mechanics Analogy

Consider first a fluid mechanics analogy of an ungrounded electric circuit shown in Fig. 2.36a.

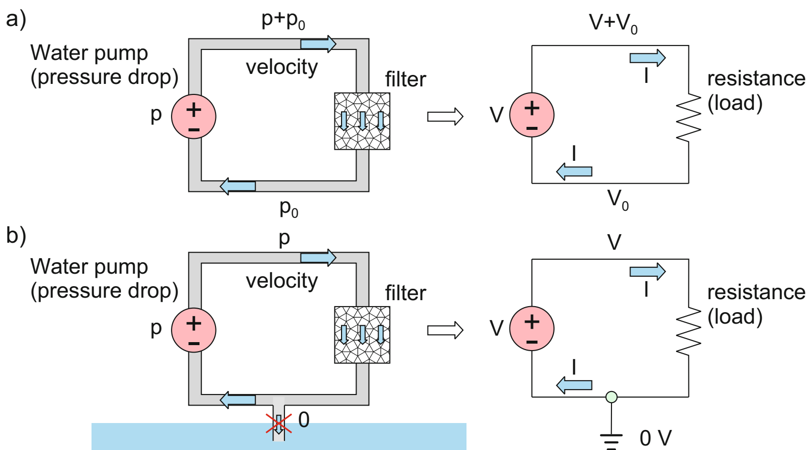


Fig. 2.36. A large reservoir at atmospheric pressure attached to a pumping system serves as an analogy to the ground connection in an electric circuit.

A water pump creates a constant pressure difference p between its terminals, which forces water to move through the filter. The pressure water pump is less common than a water pump of a constant flux; however, it exactly corresponds to the voltage power supply of the electric circuit. For entirely closed (*isolated*) pumping systems, such as those shown in Fig. 2.36a, the water pressure inside the system can in principle have an arbitrary pressure deviation p_0 from the ambient atmospheric pressure. A large p_0 is in practice undesirable since if the system breaks, then a large pressure difference with regard to atmospheric pressure will cause high-speed water leakage. Similarly, an isolated electric circuit may have an arbitrary voltage V_0 versus ground voltage, due to static charge accumulation. We could make the reference level equal to atmospheric pressure (make p_0 equal to zero) if we connect tubing to a large water reservoir at atmospheric pressure as shown in Fig. 2.36b. There is indeed no water flow through such a connection; but the pressure level is normalized. A similar situation takes place for the electric ground shown in Fig. 2.36b. By connecting a point in the circuit to a ground, we normalize the circuit voltage to the earth's voltage level, which we define to be 0 V, and eliminate any static charges. There is *no current flow* through the ground connection, except, maybe, for the first time moment. Therefore, this connection is only a *voltage reference point*. A similar analogy holds for a current source (pump of a constant flux).

2.5.3 Types of Electric Ground

Figure 2.37 shows three different types of electric ground connections. The first one is the *earth ground*. A true earth ground, as defined by the National Electrical Code (USA), physically consists of a conductive pipe or rod driven into the earth to a minimum depth of 8 feet. Obviously, it is not always possible to physically connect the circuit directly to the earth. Some examples include a cell phone, an automobile, or an airplane.

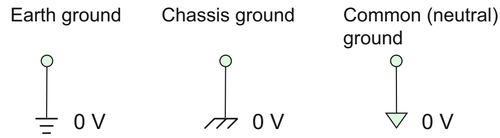


Fig. 2.37. Different ground types: earth ground, chassis ground, and common (neutral) ground.

The second ground type is the *chassis ground*. It is the physical *metal* frame or structure of an automobile, an airplane, a desktop computer, a cell phone, or other electrical devices; the term *case* is very similar in meaning. The chassis ground primarily involves a connection to the metal case. It is implied that the case should eventually discharge due to contact with other objects or with earth. The term *ground plane* for planar printed circuits, which is usually the copper bottom of a printed circuit board, is equivalent to chassis ground. The third ground type in Fig. 2.37 is the *common terminal* or *common ground*. The word *common* is typical for many circuits including the amplifier circuits considered next, when a dual-polarity power supply is used. Here two identical batteries are connected in series, plus to minus. The common terminal of the dual power supply so designed serves as the reference ground; even a metal case is not necessarily required. The AC analog of the common ground is the *neutral terminal* of your wall plug. Frequently, different ground types may be interconnected. For example, the neutral terminal of the wall plug should be connected to earth ground at a certain location. The chassis ground of a large truck may be connected to the physical ground by a little flexible strip nearly touching the asphalt.

2.5.4 Ground and Return Current

We have already seen that electric current in a circuit always flows in closed loops. This is a simple and yet a very critical property of an electric circuit. The steady-state current that flows to a load is sometimes called *forward current*, whereas the current that returns to the power supply is the *return current*. Can the chassis ground itself be used as a part of this loop for the return current? The answer is yes, and Fig. 2.38a depicts this situation as an example. Here a 9 V battery is powering an incandescent light bulb. For the chassis ground in Fig. 2.38a, the circuit is correctly drawn, but putting two wires into the soil, as shown in Fig. 2.38b, will fail owing to the high resistance of the earth. The use of the ground to establish a path for the return current is quite common for the chassis ground (automotive electronics) and also for the common ground. However, it should not be attempted for the earth ground connection. Emphasize that, in many circuit diagrams, the difference between the chassis ground, the common ground, and the true earth ground is often *ignored*. Namely, the symbol of the earth ground used in the circuit often implies either the chassis ground or the common ground, i.e., the (physically grounded or not) current return path.

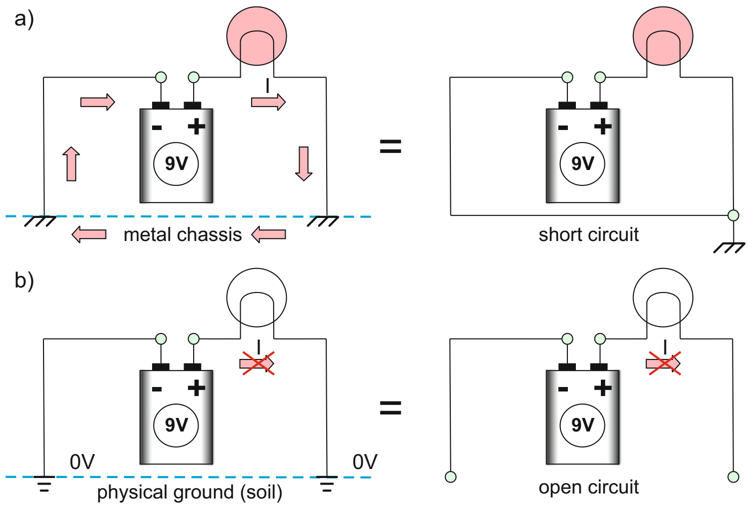


Fig. 2.38. (**a**) The return current path for the chassis ground is metal; it can be replaced by a wire. (**b**) There is no current return path, since soil (dry or wet) is a very poor conductor. The circuit is therefore open and not functioning.

2.5.5 Absolute Voltage and Voltage Drop Across a Circuit Element

The electric ground serves as a voltage reference point in a circuit. It allows us to use *two* types of voltages in the circuit:

1. The *absolute voltage* at a certain circuit node
2. The *voltage drop* or simply the *voltage across a circuit element*

Figure 2.39 shows the concept. Voltages $V_{a,b,c,d}$ are absolute voltages measured versus ground at nodes *a, b, c, d* in Fig. 2.39. Voltages $V_{A,B,C}$ give the voltage drop across the circuit elements A, B, and C. Indeed, the ideal wires remain the equipotential surfaces (have the same absolute voltage). Taking into account the polarity of the voltages $V_{A,B,C}$ shown in Fig. 2.39, one has for the node voltages

$$V_a = 0\ \text{V},\ V_b = V_a + V_A = 10\ \text{V},\ V_c = V_b - V_B = 5\ \text{V},\ V_d = V_c - V_C = 0\ \text{V} \quad (2.48)$$

Note that both voltage types—absolute voltage and voltage across a circuit element—are often denoted by the same letter V (in the DC case) and may be easily misplaced. Both of them are widely used in electric circuit analyses. The hint is that the voltage across a circuit element always has the polarity labeled with ± sign, whereas the absolute voltage often has not.

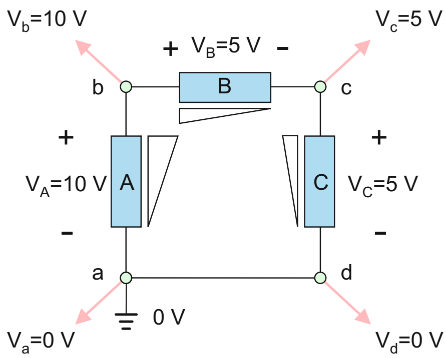


Fig. 2.39. Absolute voltages measured versus ground in a grounded electric circuit and voltages across individual circuit elements. Note that there is no voltage drop across *ideal* wires.

Exercise 2.14: Determine the absolute voltages at nodes 1 through 6 in the circuit shown in Fig. 2.40.

Answer: Clearly, $V_1 = 0\text{V}$ since node 1 is directly connected to ground. Then,

$$V_2 = V_1 + V_A = 6 \text{ V}, \quad V_3 = V_2 + V_B = 12 \text{ V}, \quad V_4 = V_3 - V_C = 9 \text{ V},$$
$$V_5 = V_4 - V_D = 3 \text{ V}, V_6 = V_5 - V_E = 0 \text{ V}$$

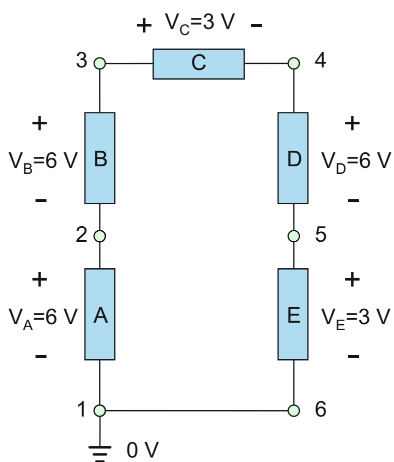
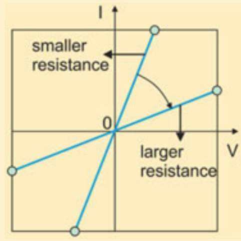
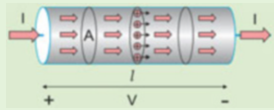
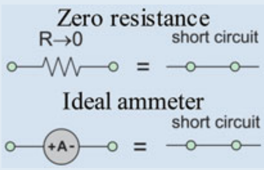
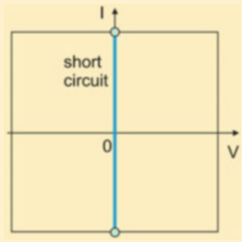
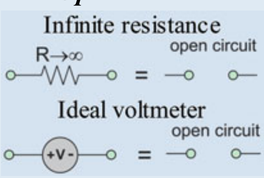
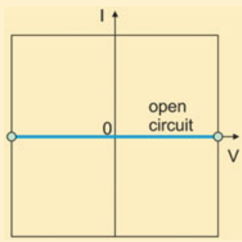
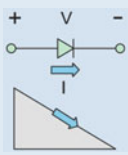
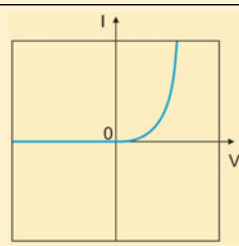
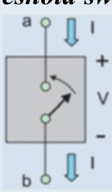
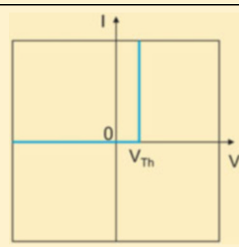
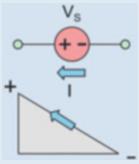
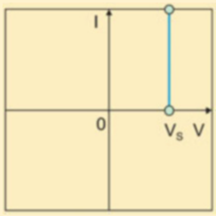
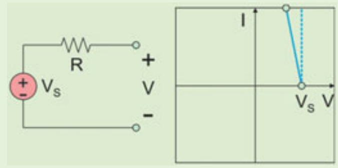
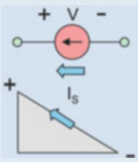
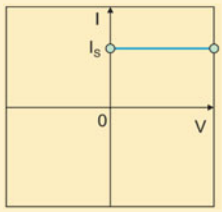
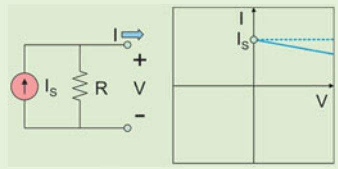
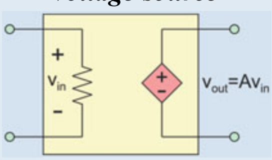
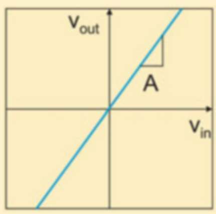
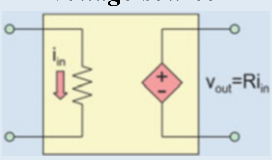
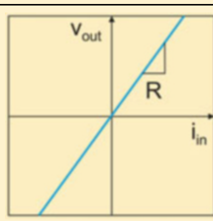
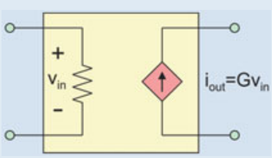
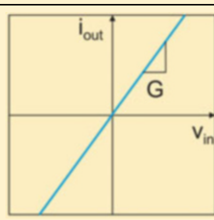
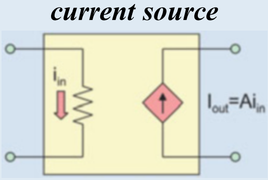
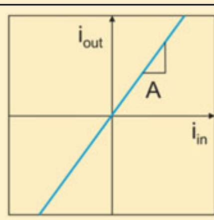


Fig. 2.40. Voltages across circuit elements in a grounded electric circuit.

Summary

<table>
<tr><th colspan="3">Passive circuit elements</th></tr>
<tr><th>Name and symbol</th><th>v–i Characteristic</th><th>Physical counterpart (component)</th></tr>
<tr><td>Resistance
</td><td></td><td>Resistor

$V = RI, \quad \sigma = qn\mu, \quad R = \frac{l}{A\sigma}$
$P = VI = RI^2 = V^2/R$</td></tr>
<tr><td>Short circuit:
</td><td></td><td>Short wire of almost zero resistance
Ammeter</td></tr>
<tr><td>Open circuit:
</td><td></td><td>Air gap of almost infinite resistance
Voltmeter</td></tr>
<tr><td>Ideal diode
</td><td></td><td>Electronic diode
Transistor junctions
Solar cell
$I = I_S\left[\exp\left(\frac{V}{V_T}\right) - 1\right]$
Static resistance: $R_0 \equiv V_0/I(V_0)$
Dynamic resist.: $r \equiv dV/dI|_{V_0,\, I_0}$</td></tr>
<tr><td>Threshold switch
</td><td></td><td>Diode
Transistor
Open circuit when $V < V_{Th}$
Short circuit when $V = V_{Th}$</td></tr>
</table>

(continued)

Active circuit elements		
Name and symbol	v–i Characteristic	Physical counterpart (component)
Independent voltage source 		***Practical voltage source*** 
Independent current source 		***Practical current source*** 
Voltage-controlled voltage source 		***Transistor Amplifier*** $v_{out} = Av_{in}$ A—open-circuit *voltage* gain [V/V, V/mV] (dimensionless)
Current-controlled voltage source 		***Transistor Amplifier*** $v_{out} = Ri_{in}$ R—transresistance [V/A, V/mA] (units of resistance, Ω)
Voltage-controlled current source 		***Transistor Amplifier*** $i_{out} = Gv_{in}$ G—transconductance [A/V] (units of conductance, Ω^{-1})
Current-controlled current source 		***Transistor Amplifier*** $i_{out} = Ai_{in}$ A—short-circuit *current* gain [A/A, A/mA] (dimensionless)

Problems

2.1 Resistance: Linear Passive Circuit Element

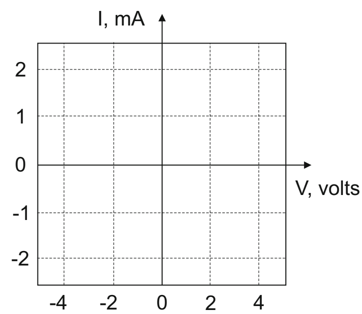
2.1.2 Resistance

2.1.3 *υ-i* Characteristic of the Resistance: Open and Short Circuits

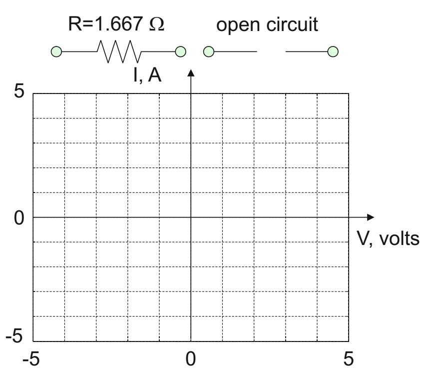
2.1.4 Power Delivered to the Resistance

2.1.5 Finding Resistance of Ohmic Conductors

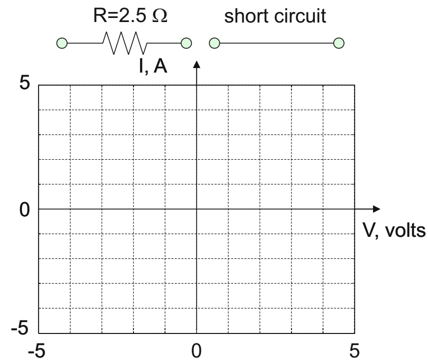
Problem 2.1. Plot *υ-i* characteristics of the following resistances: (A) 8 kΩ, (B) 2 kΩ, (C) 1 kΩ, and (D) 500 Ω. Clearly label each characteristic.



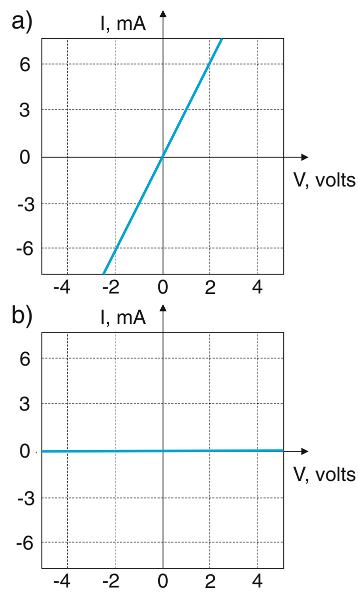
Problem 2.2. Plot *υ-i* characteristics of the following resistances: (A) 1.667 Ω. (B) Open circuit on the same graph.



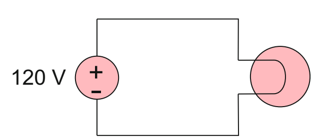
Problem 2.3. Plot *υ-i* characteristics of the following resistances: (A) 2.5 Ω. (B) Short circuit on the same graph.



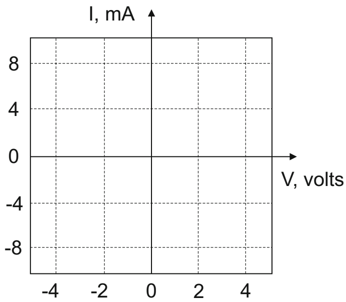
Problem 2.4. Given *υ-i* characteristics of a resistance determine the corresponding conductance. Show units.



Problem 2.5. An incandescent energy-saving light bulb ("soft white") from General Electric is rated to have the wattage of 57 W when the applied AC voltage is 120 V *rms* (root mean square). This means that the corresponding DC voltage providing the *same* power to the load is exactly 120 V. When the bulb is modeled as a resistance, what is the equivalent resistance value?



Problem 2.6. The power absorbed by a resistor from the ECE laboratory kit is 0.2 W. Plot the *v-i* characteristics of the corresponding resistance to scale given that the DC voltage across the resistor was 10 V.



Problem 2.7. The number of free electrons in copper per unit volume is $n = 8.46 \times 10^{28} \frac{1}{m^3}$. The charge of the electron is -1.60218×10^{-19} C. A copper wire of cross section 0.25 mm^2 is used to conduct 1A of electric current.

A. Sketch the wire, the current direction, and the direction of electron motion.
B. How many coulombs per one second is transported through the conductor?
C. How fast do the electrons really move? In other words, what is the average electron velocity?

Problem 2.8. Repeat the above problem when the conductor's cross section is increased to 5 mm^2.

Problem 2.9. A copper wire having a length of 1000 ft and a diameter of 2.58826 mm is used to conduct an electric current of 5 A.

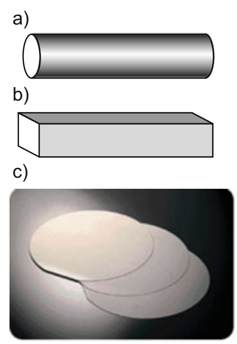
A. What is wire's total resistance? Compare your answer to the corresponding result of Table 2.2.
B. What is the power loss in the wire?

Problem 2.10

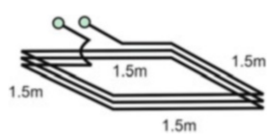
A. A copper wire having a length of 100 m and a cross section of 0.5 mm^2 is used to conduct an electric current of 5 A. What is the power loss in the wire? Into what is this power loss transformed?
B. Solve task A when the wire cross section is increased to 2.5 mm^2.

Problem 2.11. Determine the total resistance of the following conductors:

A. A cylindrical silver rod of radius 0.1 mm, length 100 mm, and conductivity 6.1×10^7 S/m.
B. A square graphite bar with the side of 1 mm, length 100 mm, and conductivity 3.0×10^4 S/m.
C. A semiconductor doped Si wafer with the thickness of 525 μm. Carrier mobility is $\mu = 0.15$ m^2/(V·s). Carrier concentration is $n = 10^{23}$ m^{-3}. Carrier charge is 1.6×10^{-19} C. The resistance is measured between two circular electrodes with the radius of 1 mm each, which are attached on the opposite sides of the wafer. Assume uniform current flow between the electrodes.



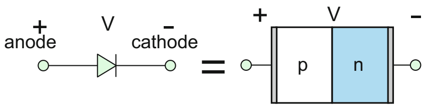
Problem 2.12. A setup prepared for a basic wireless power-transfer experiment utilizes a square multi-turn loop schematically shown in the figure, but *with 40 full turns*. A #22 gauge copper wire with the diameter of 0.645 mm is used. Total loop resistance, *R*, is needed. Please assist in finding the loop resistance (show units).



Problem 2.13. Estimate resistance, R_n (show units), of the n-side of a Si pn-junction diode in

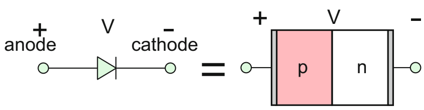
the figure that follows. We model the n-side by a Si bar having the following parameters:

1. Length of $L = 0.0005$ cm $= 5$ μm.
2. Cross section of $A = 0.01$ cm $\times$ 0.01 cm $= 1 \times 10^{-4}$ cm^2.
3. *Uniform* electron concentration (carrier concentration) of $n = 10^{17}$ cm^{-3}. This value is typical for a Si diode pn-junction. Carrier mobility is $\mu_n = 1450$ cm^2/(V·s).

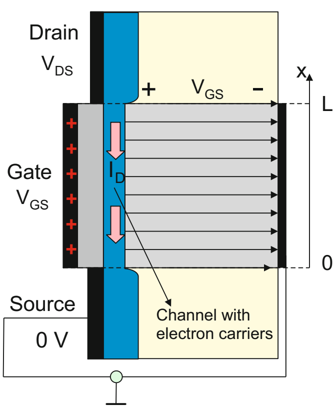


Problem 2.14. Estimate resistance, R_p (show units), of the p-side of a Si pn-junction diode in the figure that follows. We model the p-side by a Si bar having the following parameters:

1. Length of $L = 0.0005$ cm $= 5$ μm.
2. Cross section of $A = 0.01$ cm $\times$ 0.01 cm $= 1 \times 10^{-4}$ cm^2.
3. *Uniform* hole concentration (carrier concentration) of $n = 10^{17}$ cm^{-3}. This value is typical for a Si diode pn-junction. Carrier mobility is $\mu_n = 500$ cm^2/(V·s).



Problem 2.15. A cross section of the most popular NMOS transistor is shown in the following figure.



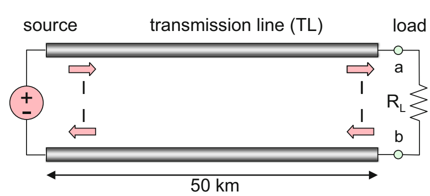
The transistor has three terminals (metal contacts): drain (with voltage $V_{DS} > 0$ vs. source), gate (with voltage $V_{GS} > 0$ vs. source), and source itself (grounded). The source is also connected to a metal conductor on the other side of the semiconductor body. Accordingly, there are *two* types of the electric field within the semiconductor body: the *horizontal field* created by V_{GS}, and the *vertical field* created by V_{DS}. The horizontal field fills a *conducting channel* between the drain and the source with charge carriers, but has *no effect* on the vertical charge motion. The resulting carrier concentration in the channel is given by $n = N(V_{GS} - V_{Th}) > 0$, $N =$ const, $V_{Th} =$ const. The individual carrier charge is q. Given the channel cross section A, the carrier mobility μ, and the channel length L, determine transistor current I_D and transistor resistance (drain-to-source resistance) R_{DS}. Express both results in terms of quantities listed above *including* V_{GS} and V_{Th}.

2.1.6 Application Example: Power Loss in Transmission Wires and Cables

Problem 2.16. An AWG 0000 aluminum transmission grid cable has the wire diameter of 11.68 mm and the area of 107 mm^2. The conductivity of aluminum is 4.0×10^7 S/m. The total cable length (two cables must run to a load) is 100 km. The system delivers 10 MW of DC power to a load. Determine the power loss in the cable (show units) when load voltage and current are given by:

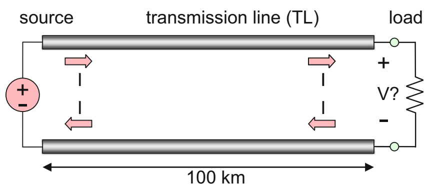
1. $V = 200$ kV and $I = 50$ A
2. $V = 100$ kV and $I = 100$ A
3. $V = 50$ kV and $I = 200$ A

Why do you think the high-voltage power transmission is important in power electronics?

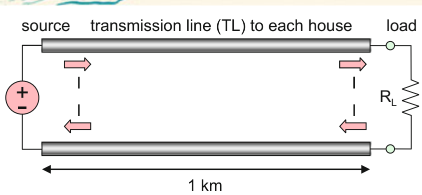


Problem 2.17. Solve the previous problem when the total cable length (two cables must run to a load) is increased to 200 km.

Problem 2.18. An AWG 00 aluminum transmission grid cable has the wire diameter of 9.266 mm. The conductivity of aluminum is 4.0×10^7 S/m. A power transmission system that uses this cable is shown in the figure that follows. The load power is 1 MW. Determine the minimum necessary load voltage V that guarantees us a 1 % relative power loss in the cables.



Problem 2.19. An AC-direct micro-hydropower system is illustrated in the figure that follows.



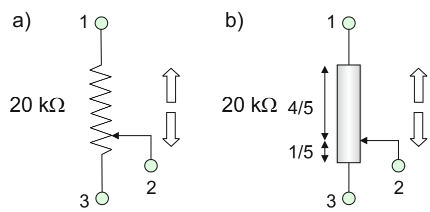
Reprinted from *Micro-Hydropower Systems* Canada 2004, ISBN 0-662-35880-5.

The system uses a single phase induction generator with the rms voltage (equivalent DC voltage) of 240 V. The system serves four small houses, each connected to the generator via a *separate* transmission line with the *same* length of 1000 m. Each line uses AWG#10 aluminum wire with the diameter of 2.59 mm. The conductivity of aluminum is 4.0×10^7 S/m. The house load is an electric range with the resistance of 20 Ω. Determine total power delivered by the generator, P_{total}, total power loss in the transmission lines, P_{loss}, and total useful power, P_{useful} (show units).

2.1.7 Physical Component: Resistor

Problem 2.20. A leaded resistor has color bands in the following sequence: brown, black, red, gold. What is the resistor value?

Problem 2.21. Potentiometer operation may be schematically explained as moving sliding contact #2 in the following figure along a uniform conducting rod with the total resistance of 20 kΩ. Determine resistance between terminals 1 and 2 as well as between terminals 2 and 3 of the potentiometer, when the sliding contact is at one fifth of the rod length.



2.2 Nonlinear Passive Circuit Elements

2.2.2 Nonlinear Passive Circuit Elements

2.2.3 Static Resistance

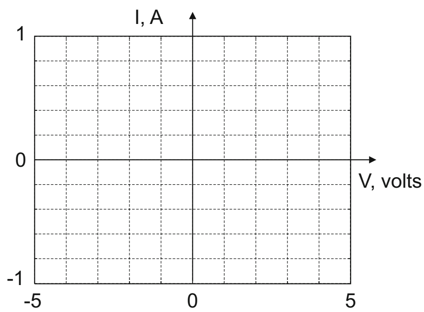
2.2.4 Dynamic (Small-Signal) Resistance

2.2.5 Electronic Switch

Problem 2.22. A nonlinear passive circuit element—the ideal diode—is characterized by the *υ-i* characteristic in the form $I = I_S\left[\exp\left(\frac{V}{V_T}\right) - 1\right]$ with $I_S = 1\times 10^{-13}$ A and $V_T = 25.7$ mV. Find the static diode resistance R_0 and the diode current I_0 when (A) $V_0 = 0.40$ V, (B) $V_0 = 0.50$ V, (C) $V_0 = 0.55$ V, and (D) $V_0 = 0.60$ V.

Problem 2.23. Find the dynamic (small-signal) resistance r of a nonlinear passive circuit element—the ideal diode—when the operating DC point V_0, I_0 is given by the solutions to the previous problem. Consider all four cases.

Problem 2.24. A nonlinear passive circuit element is characterized by the υ-i characteristic in the form $I = I_S \frac{V/V_S}{\sqrt{1+(V/V_S)^2}}$ with $I_S = 1$ A and $V_S = 1$ V. Plot the υ-i characteristic to scale. Next, find the static element resistance R_0, the element current I_0, and the corresponding dynamic element resistance r when (A) $V_0 = 0.1$ V, (B) $V_0 = 1.0$ V, and (C) $V_0 = 5.0$ V.

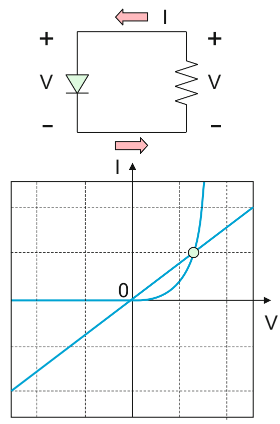


Problem 2.25. Repeat the previous problem when $I_S = 0.5$ A and $V_S = 0.5$ V. All other parameters remain the same. Consider the following DC operating points: (A) $V_0 = 0.05$ V, (B) $V_0 = 0.50$ V, and (C) $V_0 = 2.50$ V.

Problem 2.26. A DC circuit shown in the following figure includes two interconnected passive elements: an ideal diode and a resistance. One possible circuit solution is given by an intersection of two υ-i characteristics marked by a circle in the same figure. This solution predicts a non-zero circuit current and a positive voltage across both circuit elements.

A. Is this solution an artifact (a mistake has been made somewhere)?
B. Is this solution true (the circuit so constructed might function)?

Justify your answer.

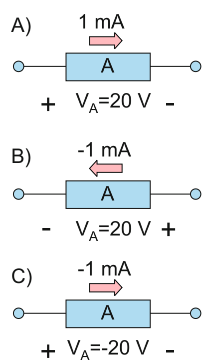


2.3 Independent Sources

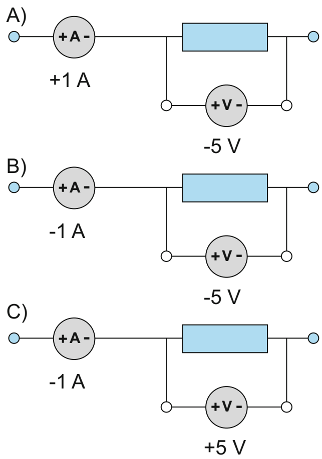
2.3.1 Independent Ideal Voltage Source

2.3.2 Circuit Model of a Practical Voltage Source

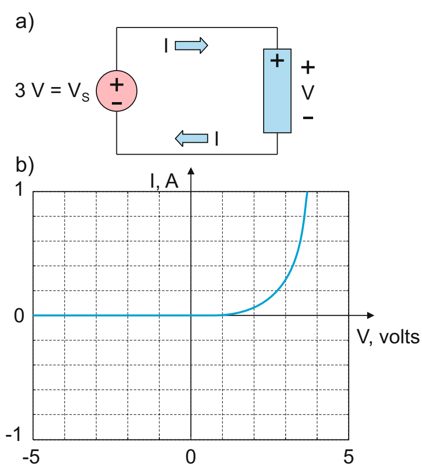
Problem 2.27. In the following figure, determine if the element is a resistance or a voltage source. Find the power delivered to element A or taken from element A in every case.



Problem 2.28. Based on voltage and current measurements, determine if the circuit element is a resistance or a voltage source. Readings of the ammeter and voltmeter are shown in the following figure.

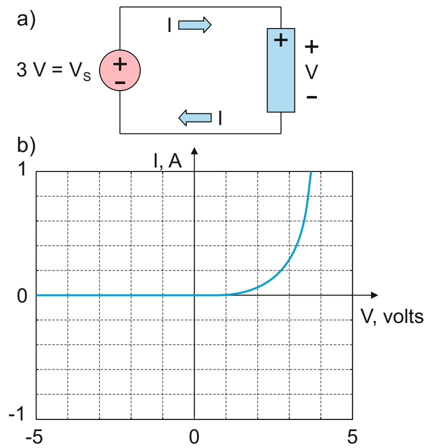


Problem 2.29. The figure that follows shows a circuit with a passive nonlinear circuit element shown by a rectangle.

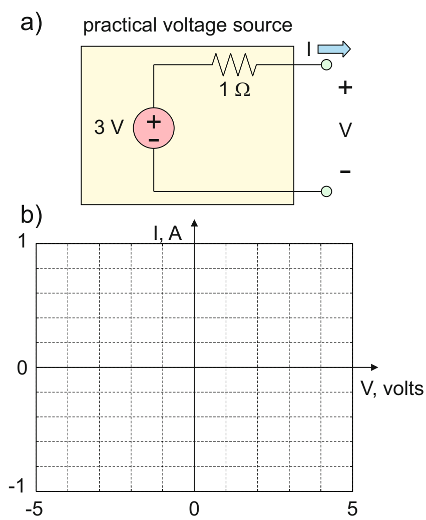


The polarity (direction of current inflow for passive reference configuration) of the element is labeled by a sign plus. The *v*-*i* characteristic of the element is also shown in the figure. Determine current *I* and voltage *V*.

Problem 2.30. Repeat the previous problem for the circuit shown in the figure that follows.

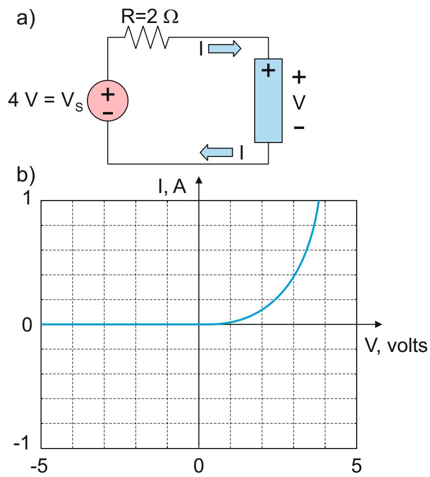


Problem 2.31. Plot to scale the *v*-*i* characteristic of the practical voltage source shown in the following figure.



Problem 2.32. The following figure shows a circuit with a passive nonlinear circuit element labeled by a rectangle. Element's polarity (direction of current inflow for passive reference configuration) of the element is indicated by a sign plus. The *v*-*i* characteristic of the

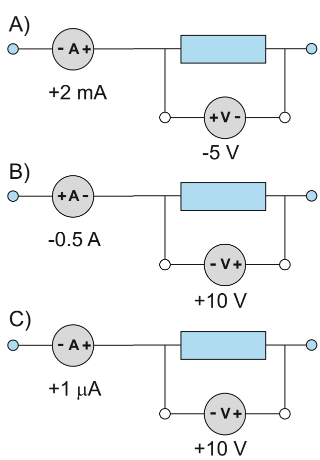
element is also shown in the figure. Determine circuit current *I*.



2.3.3 Independent Ideal Current Source

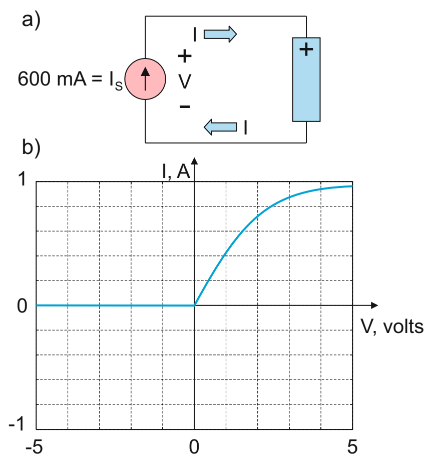
2.3.4 Circuit Model of a Practical Current Source

Problem 2.33. Readings of the ammeter and voltmeter are shown in the following figure.

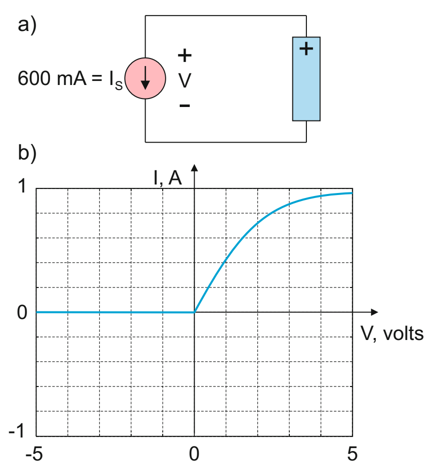


Based on voltage and current measurements, determine if the element is a resistance or a current source. Then, find the power delivered to the circuit element or taken from it in every case.

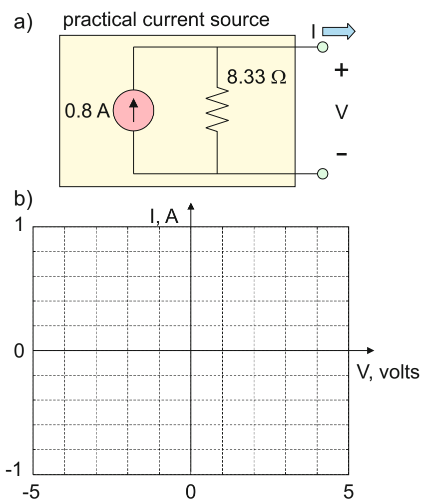
Problem 2.34. The following figure shows a circuit with a passive nonlinear circuit element shown by a rectangle. Element's polarity (direction of current inflow for passive reference configuration) of the element is labeled by a sign plus. The *υ-i* characteristic of the element is also shown in the figure. Determine current *I* and voltage *V*.



Problem 2.35. Repeat the previous problem for the circuit shown below.

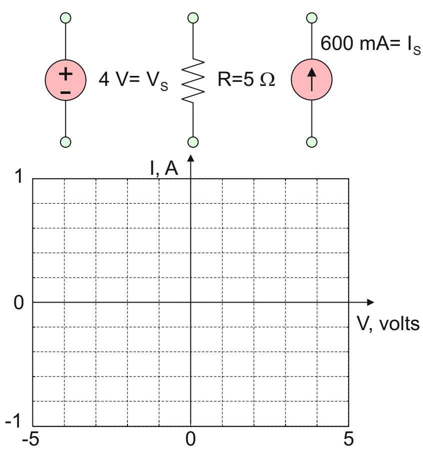


Problem 2.36. Plot to scale the v-i characteristic of the practical current source shown in the following figure.

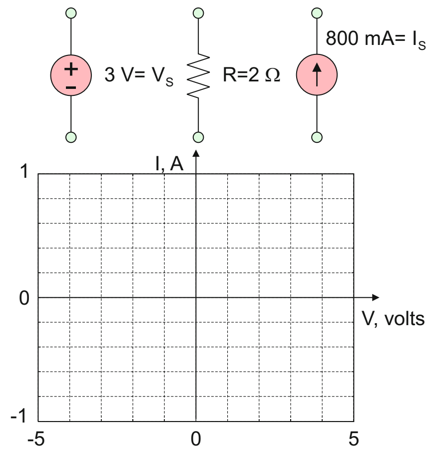


Review Problems

Problem 2.37. For every circuit element shown in the following figure, plot its v-i characteristic on the same graph.



Problem 2.38. Repeat the previous problem: for every circuit element shown in the figure below, plot its v-i characteristic on the same graph.



2.3.7 Application Example: Chemical Battery

Problem 2.39. The electronics aboard a certain sailboat consume 96 W when operated from a 24 V source.

A. If a certain fully charged deep-cycle marine battery is rated for 24 V and 100 A h, for how many hours can the electronics be operated from the battery without recharging? (The ampere-hour rating of the battery is the battery capacity—the operating time to discharge the battery multiplied by the current).

B. How much energy in kilowatt hours is initially stored in the battery?

Problem 2.40. A motor of a small, unmanned electric vehicle consumes 120 W and operates from a 24-V battery source. The source is rated for 200 Ah.

A. For how many hours can the motor be operated from the source (a battery bank) without recharging?

B. How much energy in kilowatt hours is initially stored in the battery source?

Problem 2.41. A certain sensing device operates from a 6-V source and consumes 0.375 W of power over a 20-h time period. The source is a combination of four fully charged AAA batteries, 1.5 V each, assembled in series. The batteries discharge by the end of the 20-h period.

A. What is the expected capacity of a typical AAA battery used, in mAh?
B. How much energy in Joules was stored in each AAA battery?

Problem 2.42. How many Joules are in 1 kWh and how many N·m does this correspond to?

2.4 Dependent Sources and Time-Varying Sources

2.4.1 Dependent Versus Independent Sources

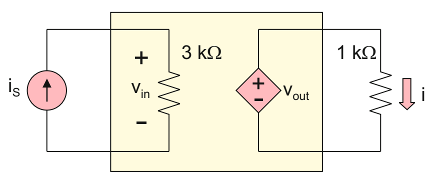
2.4.2 Definition of Dependent sources

2.4.3 Transfer Characteristics

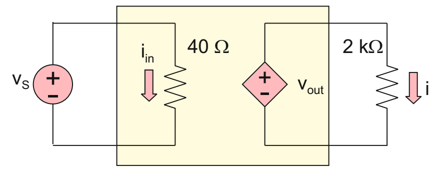
2.4.4 Time-Varying Sources

Problem 2.43. Draw circuit diagrams for four major types of dependent sources, label stimulus voltage/current and output voltage/current. Describe operation of each dependent source.

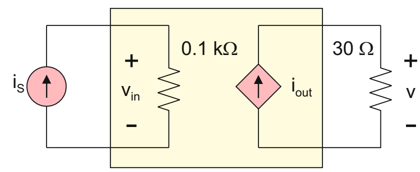
Problem 2.44. Solve an electric circuit shown in the following figure—determine current i through the 2-kΩ resistance. The independent current source is given by $i_S = 0.2 - 1.5\cos 5t$ [mA] ; the open-circuit voltage gain of the dependent source is 12 V/V.



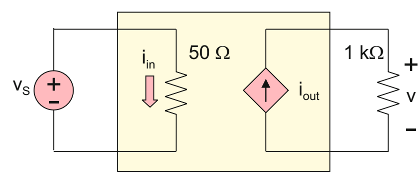
Problem 2.45. Solve an electric circuit shown in the following figure—determine current i through the 2-kΩ resistance. The independent voltage source is given by $v_S = -0.3 + 0.7\cos 6t$ [V] ; the transresistance of the dependent source is 250 V/A.



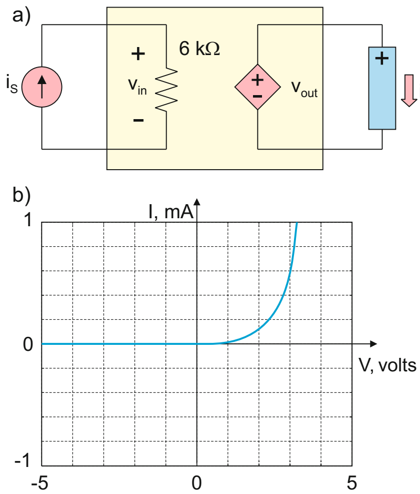
Problem 2.46. Solve an electric circuit shown in the following figure—determine voltage v through the 30-Ω resistance. The independent current source is given by $i_S = -0.05 - 0.2\cos 2t$ [mA] ; the transconductance of the dependent source is 10 A/V.



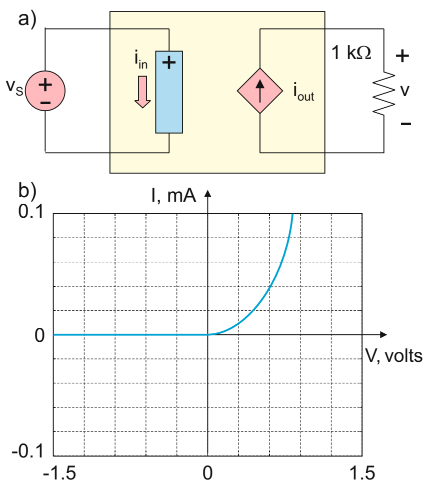
Problem 2.47. Solve an electric circuit shown in the following figure—determine voltage v through the 1-kΩ resistance. The independent voltage source is given by $v_S = 0.05 + 0.1\cos 4t$ [V] ; the short-circuit current gain of the dependent source is 10 A/A.



Problem 2.48. Solve an electric circuit shown in the following figure—determine current i through a nonlinear passive circuit element shown by a rectangle. Element's polarity (direction of current inflow for passive reference configuration) is labeled by a sign plus. The v-i characteristic of the element is also shown in the figure. The independent current source is given by $i_S = 0.1$ A ; the open-circuit voltage gain of the dependent source is 5 V/V.



Problem 2.49. Solve the electric circuit shown in the following figure—determine voltage v across the 1-kΩ resistance.



A nonlinear passive circuit element is shown by a rectangle. Element's polarity (direction of current inflow for passive reference configuration) of the element is labeled by a sign plus. The v-i characteristic of the element is also shown in the figure. The independent voltage source is given by $v_S = 0.6$ V; the short-circuit current gain of the dependent source is 100 A/A.

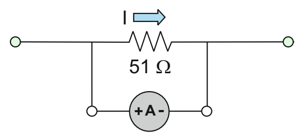
2.5 Ideal Voltmeter and Ammeter: Circuit Ground

2.5.1 Ideal Voltmeter and Ammeter

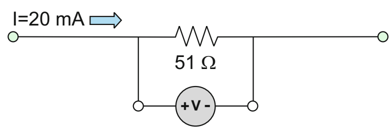
2.5.3 Types of Electric Ground

2.5.4 Ground and Return Current

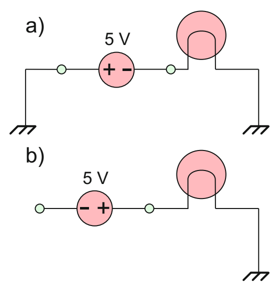
Problem 2.50. You attempt to measure electric current through a resistance as part of a circuit. Is the following figure appropriate? What is the current across the 51-Ω resistor?



Problem 2.51. You attempt to measure voltage across a resistance in the circuit. Is the following figure correct? What is the voltmeter's reading, assuming an ideal instrument?

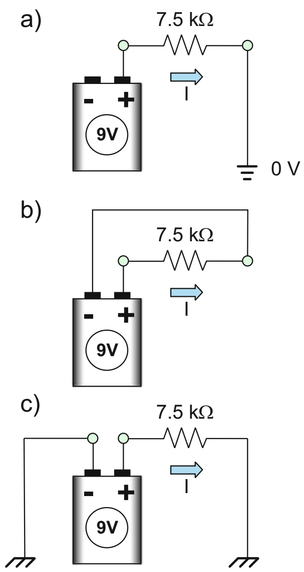


Problem 2.52. Two circuits with an incandescent light bulb are shown in the following figure. Will they function? Explain.

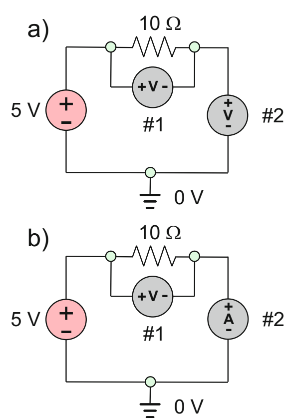


Problem 2.53. A 9-V battery is connected to a 7.5-kΩ resistor shown in the following figure.

Find current I in every case. (a) The negative terminal is left disconnected; (b) the negative terminal is connected to the positive terminal through the resistor; (c) both terminals are connected to chassis ground.

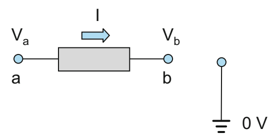


Problem 2.54. What is the voltmeters' (ammeter's) reading in the figure below?



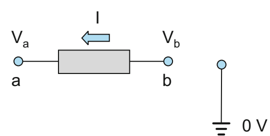
2.5.5 Absolute Voltage and Voltage Drop Across a Circuit Element

Problem 2.55. Determine if the circuit element shown in the following figure is a resistance, a voltage source, or a wire (short circuit). Absolute voltages at points a and b are measured versus ground.



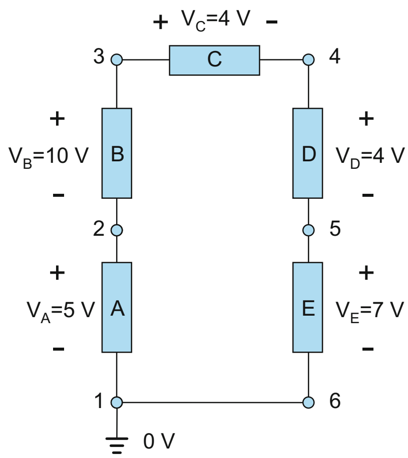
1. $V_a = 3\,\text{V}, \quad V_b = 3\text{V}, \quad I = 1\,\text{A}$
2. $V_a = 3\,\text{V}, \quad V_b = 1\text{V}, \quad I = -1\,\text{A}$
3. $V_a = -2\,\text{V}, \quad V_b = -5\,\text{V}, \quad I = 2\,\text{A}.$

Problem 2.56. Determine if the circuit element shown in the following figure is a resistance, a voltage source, or a wire (short circuit). Absolute voltages at points a and b are measured versus ground.



1. $V_a = 6\,\text{V}, \quad V_b = 3\,\text{V}, \quad I = 1\,\text{A}$
2. $V_a = 1\,\text{V}, \quad V_b = 1\,\text{V}, \quad I = -1\,\text{A}$
3. $V_a = -7\text{V}, \quad V_b = -5\text{V}, \quad I = -2\,\text{A}.$

Problem 2.57. Determine absolute voltages at nodes 1 through 6 in the circuit shown in the following figure.



Problem 2.58. Determine voltages across circuit elements A, B, C, D, and E in the circuit shown in the following figure.