Chapter 5: Operational Amplifier and Amplifier Models

Overview

Prerequisites:

- Knowledge of major circuit elements (dependent sources) and their v-i characteristics (Chapter 2)
- Knowledge of basic circuit laws (Chapter 3) and Thévenin equivalent (Chapter 4)

Objectives of Section 5.1:

- Learn and apply the model of an operational amplifier including principle of operation, open-circuit gain, power rails, and input and output resistances
- Correlate the physical operational amplifier with the amplifier circuit model
- Establish the ideal-amplifier model
- Learn the first practical amplifier circuit—the comparator

Objectives of Section 5.2:

- Understand and apply the concept of negative feedback to an operational amplifier circuit
- Construct three canonic amplifier circuit configurations with negative feedback: the non-inverting amplifier, the inverting amplifier, and the voltage follower
- Understand the current flow in the amplifier circuit including the power transfer from the power supply to the load

Objectives of Section 5.3:

- Choose the proper resistance values for the feedback loop and learn how to cascade multiple amplifier stages
- Learn about input/output resistances of the amplifier circuit and establish load bridging and load matching conditions important in practice
- Find ways to eliminate the DC imperfections of the amplifier that become very apparent at high amplifier gains
- Use an amplifier IC with a single voltage supply (a battery)

Objectives of Section 5.4:

- Obtain the initial exposure to differential signals and difference amplifiers
- Build an instrumentation amplifier
- Connect an instrumentation amplifier to a resistive sensor

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Objectives of Section 5.5:

- Learn a general feedback system including closed-loop gain and error signal
- Apply the general feedback theory to voltage amplifier circuits
- Construct current, transresistance, and transconductance amplifiers with the negative feedback

Application Examples:

Operational amplifier comparator
Instrumentation amplifier in laboratory

Keywords:

Operational amplifier: (abbreviation op-amp, integrated circuit, dual in-line package, non-inverting input, inverting input, output terminal, power terminals, offset-null terminals, differential input voltage, open-circuit voltage gain, open-loop voltage gain, open-loop configuration, closed-loop configuration, power rails, voltage transfer characteristic, rail-to-rail, comparator, digital repeater, zero-level detector, circuit model, input resistance, output resistance, ideal amplifier, ideal-amplifier model, marking, summing point, common-mode input signal, differential input signal, summing-point constraints, first summing-point constraint, second summing-point constraint, sourcing current, sinking current, DC imperfections, input offset voltage, input bias current, input offset currents), Negative feedback, Feedback loop, Feedback as a dynamic process, Non-inverting amplifier, Inverting amplifier, Voltage follower (buffer) amplifier, Summing amplifier, Digital-to-analog converter, Binary counter, DC-coupled amplifier, AC-coupled amplifier, Capacitive coupling of an amplifier, Gain tolerance of an amplifier, Circuit model of a voltage amplifier, Input resistance of amplifier circuit, Output resistance of amplifier circuit, Load bridging (impedance bridging), Load matching (impedance matching), Cascading amplifier stages, Virtual-ground (integrated) circuit, Differential voltage of a sensor, Common-mode voltage of a sensor, Differential sensor, Single-ended sensor, Difference amplifier, Differential amplifier circuit gain, Common-mode amplifier circuit gain, Common-mode rejection ratio (CMRR), Unity common-mode gain stage, Instrumentation amplifier, Load cell, Current amplifier using op-amp, Transconductance amplifier using op-amp, Transresistance amplifier using op-amp, Howland current source (Howland current pump)
Linear feedback system: (forward gain? open-loop gain, feedback gain, feedback factor, summing node, difference node, closed-loop gain, error signal)

Section 5.1 Amplifier Operation and Circuit Models

The low-power amplifier *integrated circuit* (IC) is arguably the most widely employed discrete circuit component encountered in common electronic audio, control, and communication systems. Among amplifiers, the differential input, high-gain amplifier called the *operational amplifier* (or simply *op-amp*) has become a popular choice in many circuit applications. At this point, it is impossible for us to understand the internal operation of the amplifier IC without basic knowledge of semiconductor electronics, especially the junction transistor studied in the following chapters. Fortunately, the circuit model of an operational amplifier does not require knowledge of the IC fabrication steps, nor does it require an understanding of the internal transistor architecture. Conceptually, operational amplifiers can be introduced early in the book, which enables us to immediately proceed toward our goal of designing and building practical circuits.

5.1.1 Amplifier Operation

Symbol and Terminals

After the amplifier chip is fabricated as an integrated circuit and the bond wires are attached, it is permanently sealed in a plastic package. Often the encasing is done in a *dual in-line* (DIP-N) *package* with *N* denoting the number of IC pins. Figure 5.1 on the right shows an example of a DIP package. One IC chip may contain several independent individual amplifiers. We start analyzing the amplifier model by first labeling the terminals and introducing the amplifier circuit symbol (a triangle) as shown in Fig. 5.1 on the left. The amplifier is typically powered by a dual-polarity voltage power supply with three terminals: $\pm V_{CC}$ and common (ground) port of 0 V, see Section 3.2. The index *C* refers to the collector voltage of the internal transistors.

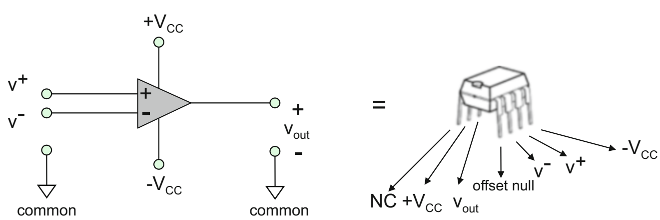


Fig. 5.1. Terminals of the operation amplifier (*left*); they also denote pins of the amplifier IC package (see a common LM 741 chip on the *right*). All voltages are referenced with respect to a common port of the dual-polarity voltage supply.

The amplifier has a total of *five* terminals, notably:

1. A *non-inverting input* with the input voltage υ^+ with respect to common
2. An *inverting input* with the input voltage υ^- with respect to common
3. An *output terminal* with the output voltage υ_{out} with respect to common

4. *Power terminal* $+V_{CC}$ with a positive voltage V_{CC} (+9 V) with respect to common
5. *Power terminal* $-V_{CC}$ with the negative voltage $-V_{CC}$ (-9 V) with respect to common

Each of the five terminals corresponds to a particular metallic pin of the IC package. *All* of the amplifier's terminals are used in an amplifier circuit and *none* of them should be left disconnected. However, the chip itself could have some *not connected* (NC) terminals that maintain symmetry and which are used as heat sinks, see Fig. 5.1 on the left. Also note that a number of amplifier ICs, including the LM74, may have extra terminals or pins, the so-called offset or offset-null terminals. These terminals are used to control the *input offset voltage* (an imperfection) of the amplifier.

Historical: The abbreviation for the operational amplifier is *op-amp*; this abbreviation is not quite official but is used by most practitioners. The term *operational amplifier* first appeared in a 1943 paper by John R. Ragazzini, an American electrical engineer and ECE professor. One of his students introduced the terms *inverting* and *non-inverting inputs*. One of his most notable students was Rudolf Kalman who became famous for the invention of the Kalman filters.

Open-Circuit or Open-Loop Voltage Gain

Once the amplifier chip is properly powered, its operation is quite simple: the output voltage is expressed through the two input voltages in the form

$$v_{\text{out}} = A(v^+ - v^-) \tag{5.1}$$

which is *identical* to the operation of the voltage-controlled voltage source introduced in Section 2.4. Here, $v^+ - v^-$ is the *differential input voltage* to the amplifier. The dimensionless constant A is called the *open-circuit voltage gain* of the amplifier. Quite frequently, the term *open-loop gain* is used and A is replaced by A_{OL}. Equation (5.1), which will be called the *amplifier equation*, is always valid. It does not matter if the amplifier is in the *open-loop configuration*, (i.e., no feedback loop is present) or in a *closed-loop configuration* (a feedback loop is present; see the next section). The amplifier IC is intentionally built in such a way as to provide the highest possible open-circuit gain; it is achieved using transistors connected in series such as the Darlington pair. Typically,

$$A \approx 10^5 - 10^8 \tag{5.2}$$

The exact gain value cannot be controlled precisely due to manufacturing tolerances. The open-circuit gain is often measured in V/mV. For example, the value of 160 V/mV corresponds to the gain value of 160,000. The open-circuit gain is difficult to measure.

Power Rails and Voltage Transfer Characteristic in the Open-Loop Configuration

Two power interconnects of the amplifier are often called *rails*. The term "rail" appears simply because the power interconnections are represented by two long horizontal wires in the circuit diagram connected to $+V_{CC}$ and $-V_{CC}$, respectively, which resemble long metal rails. The positive rail is $+V_{CC}$, and the negative rail is $-V_{CC}$. The power rails are interfaced to a laboratory dual-polarity voltage power supply that also provides a common (ground) port to be used later. The output amplifier voltage can never exceed the positive rail voltage or be less than the negative rail voltage. In other words,

$$-V_{CC} \le \upsilon_{out} \le V_{CC} \tag{5.3}$$

Should the output voltage found in Eq. (5.1) exceed V_{CC}, it will be forced to V_{CC}. Likewise, should the output voltage drop to less than $-V_{CC}$, it will be forced to $-V_{CC}$. In view of these physical constraints, Eq. (5.1) may be rewritten in the form

$$\begin{aligned} \upsilon_{out} &= A(\upsilon^+ - \upsilon^-), && |\upsilon_{out}| < V_{CC} \\ \upsilon_{out} &= +V_{CC}, && A(\upsilon^+ - \upsilon^-) > +V_{CC} \\ \upsilon_{out} &= -V_{CC}, && A(\upsilon^+ - \upsilon^-) < -V_{CC} \end{aligned} \tag{5.4}$$

Example 5.1: Plot to scale the output voltage of an operational amplifier with an open-circuit gain of $A = 10^5$ when the non-inverting input voltage υ^+ changes from -1 mV to $+1$ mV and the inverting input voltage υ^- is set to zero. The amplifier is powered by a ± 16-V dual voltage supply. This plot will give us the *voltage transfer characteristic* of the open-loop amplifier.

Solution: Amplifier Eq. (5.4) gives the result shown in Fig. 5.2 by a thick piecewise-linear curve. Due to the extremely high open-loop gain, the amplifier output is almost always *saturated*. This means that, except for a very narrow domain of input voltages on the order of ± 0.2 mV, the output simply follows the power rail voltage, either positive or negative. This is a very remarkable feature of the open-loop amplifier.

Power Rails in Practice

The power rail(s) of the amplifier or the supply voltage is specified in the datasheet. For example, the LM358 amplifier IC operates using a single supply 3 V to 32 V or dual supplies ± 1.5 V to ± 16 V. As we can see from this data, the amplifier does not necessarily operate using a dual voltage supply; a single supply ("single rail") can be used as well. The same amplifier chip (e.g., LM358) can be used either with the single voltage supply or with a dual supply. This question, although less important in theory, is very important in practice. Also note that, in practice, the output never exactly reaches the positive or negative rail voltages; there is always a voltage offset; it can vary from a minimum value of between 0.01 V and 0.05 V for certain special ICs (called the *rail-to-rail amplifiers*) all the way up to 1.8 V for common amplifiers (e.g., LM741).

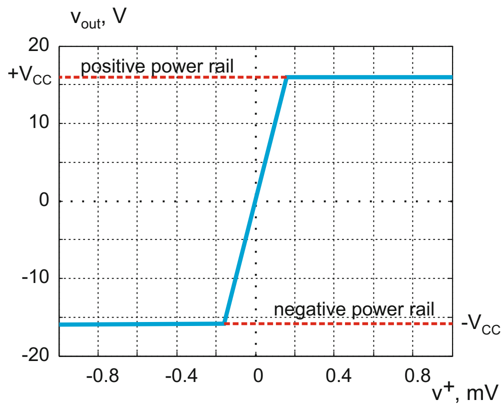


Fig. 5.2. Amplifier output voltage in the open-loop configuration. The open-loop gain is $A_{OL} = 10^5$ and the supply voltage is ± 16 V. Note that the scale for the input voltage is in mV.

5.1.2 Application Example: Operational Amplifier Comparator

A *comparator* is a circuit or a device that compares *two* input voltages and outputs a *digital voltage* (e.g., ± 10 V) as an indication of which input voltage is larger. Due to a very high gain, the operational amplifier in the open-loop configuration shown in Fig. 5.3a may operate as a basic comparator. Figure 5.3b shows one possible application of the comparator: *a digital repeater*. We assume that $v^- = V_{\text{threshold}} = 0$. The *input voltage to the comparator* $v^+(t)$ is a weak noisy digital signal shown in Fig. 5.3b. This signal is compared to a threshold level of zero volts (the threshold voltage in Fig. 5.3).

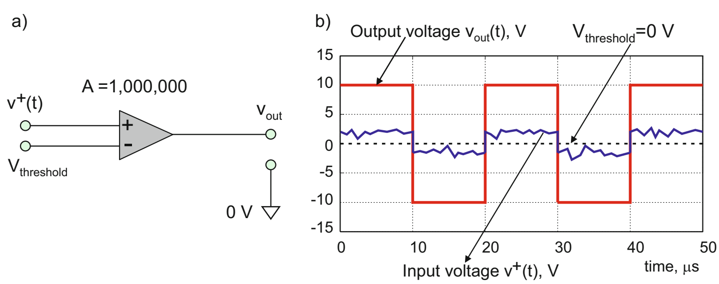


Fig. 5.3. A simple operational amplifier as a voltage comparator.

When the amplifier open-circuit gain tends to infinity (the transfer characteristic in Fig. 5.2 becomes a straight vertical line), Eq. (5.4) applied to the present case is reduced to

$$\begin{aligned} v_{\text{out}} &= +V_{\text{CC}}, & v^+(t) &> 0 \\ v_{\text{out}} &= -V_{\text{CC}}, & v^+(t) &< 0 \end{aligned} \tag{5.5}$$

Figure 5.3b shows the resulting output voltage for $V_{\text{CC}} = 10$ V. The weak input digital signal will thus be amplified and cleaned from noise, which is one major function of a *digital repeater*. In practice, dedicated comparators are used instead of this simple setup, which are much faster and have useful additional features. The comparator amplifier may also be employed for other purposes such as a *zero-level detector*.

Exercise 5.1: In Fig. 5.3, the threshold voltage of the comparator amplifier is changed to +5 V. What will be the output of the comparator circuit?
Answer: -10 V at any time instant.

5.1.3 Amplifier Circuit Model

Circuit Model

An equivalent circuit model of an amplifier is shown in Fig. 5.4. This circuit model is a two-port electric network. It includes three single circuit elements: an ideal voltage-controlled voltage source $A(v^+ - v^-)$ (Section 2.4), *input resistance* R_{in} *of the amplifier*, and *output resistance* R_{out} *of the amplifier*.

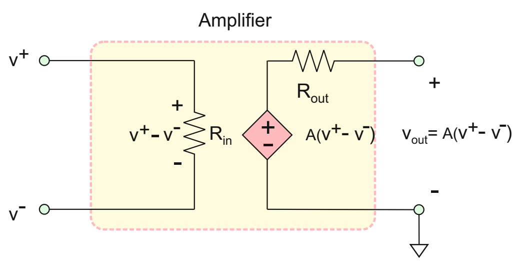


Fig. 5.4. Equivalent circuit model of an amplifier is in the shadow box as a two-port network. No load is connected. The ground of the output terminal is the common port.

Analysis of the Amplifier Circuit Model: Effect of Input/Output Resistances

The input/output resistances of an amplifier in Fig. 5.4 impose rather severe limitations on its desired operation. First, a large but finite input resistance always implies that some *input current*, $i_{\text{in}} = (v^+ - v^-)/R_{\text{in}}$, will flow into the amplifier as long as the input voltage signal is different from zero. Consequently, the amplifier would require not only the input voltage but also a certain amount of input power. As a result, the amplifier may appreciably *load* a sensor connected to its input, i.e., require more power than (a tiny) sensor can actually provide. Second, a finite output resistance limits the *output current* i_{out} to the amplifier; this resistance operates as a current limiting resistor which is studied in Chapter 3. Along with this, it also leads to the fact that the voltage across any load connected to the amplifier's output will *not* be equal to the desired output voltage given by Eq. (5.1), except for an open circuit. These limitations are quantified when we consider a circuit shown in Fig. 5.5. The circuit includes the amplifier model, an arbitrary source represented by its Thévenin equivalent v_S, R_S, and a load represented by its equivalent resistance R_L.

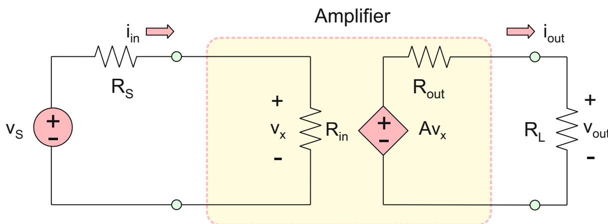


Fig. 5.5. Amplifier circuit model with connected source and load resistances.

Using the voltage division principle twice, the output voltage in Fig. 5.5 is expressed as

$$v_{out} = \underbrace{v_S \times \left(\frac{R_{in}}{R_{in} + R_S}\right)}_{v_X} \times A \times \left(\frac{R_L}{R_L + R_{out}}\right) \tag{5.6}$$

This result is quite different from the ideal behavior of the amplifier described by the perfect amplification of the source signal

$$v_{out} = Av_S \tag{5.7}$$

using the available open-circuit gain A of the amplifier.

Exercise 5.2: For the amplifier circuit in Fig. 5.5 with $A = 1000$, determine the output voltage given that $v_S = 1\ \text{mV}$, $R_S = 50\ \Omega$, and $R_L = 50\ \Omega$ for two cases:

A. $R_{in} = 1\ \text{M}\Omega$ and $R_{out} = 1\ \Omega$.
B. $R_{in} = 50\ \Omega$ and $R_{out} = 50\ \Omega$.

Answer:
Case A: $v_{out} = 0.98\ \text{V}$ (which is close to the ideal behavior, $v_{out} = 1.00\ \text{V}$).
Case A: $v_{out} = 0.25\ \text{V}$ (three quarters of the voltage gain are lost).

According to Eq. (5.6), $v_{out} < Av_S$ for any positive finite values of R_{in} and R_{out}. In order to make use of the full available open-circuit gain A of the amplifier, we should:

1. Design R_{in} as large as possible, ideally an open circuit, that is,
$$R_{in} = \infty \tag{5.8}$$
2. Design R_{out} as small as possible, ideally a short circuit, that is,
$$R_{out} = 0 \tag{5.9}$$

In this and only this case, the equality $v_{out} = Av_S$ will be satisfied exactly.

5.1.4 Ideal-Amplifier Model and First Summing-Point Constraint

The amplifier IC can then be described with a high degree of accuracy by using the so-called ideal-amplifier model. It is based on the best possible choices for input/output resistances as described by Eqs. (5.8) and (5.9), respectively. It is also based on the assumption that the open-loop gain in Eq. (5.1) is made as high as possible, i.e., equal to infinity. The ideal-amplifier model is an important theoretical and practical tool for the analysis of microelectronic amplifier circuits. This model will be used in the following sections of this chapter and in subsequent chapters. We can summarize the model of an ideal operational amplifier in concise form:

1. No current can flow into the amplifier (into either input terminal).
2. The open-loop gain A is infinitely high.
3. The input resistance R_{in} is infinitely high.
4. The output resistance R_{out} is zero.

Property 1 follows from property 3 and vice versa. One more condition of the ideal-amplifier model could be added, namely, that the power rails $\pm V_{CC}$ are exactly reached when operated in saturation. The ideal-amplifier model does not use the accurate internal amplifier circuit shown in Fig. 5.4 or in Fig. 5.5, respectively. Instead, a simple triangle symbol may be used for the ideal amplifier, which is shown in Figs. 5.1 and 5.3.

Exercise 5.3: Solve the previous exercise for the ideal-amplifier model.

Answer: Case A, B: $v_{out} = 1.00\ \text{V}$.

First Summing-Point Constraint

The *summing point* of an amplifier is the connection of the two inputs to the amplifier. The *common-mode input signal* is the half sum of the two input voltages, $(v^+ + v^-)/2$. The *differential input signal* is the input voltage difference, $v_x = v^+ - v^-$. Conditions applied to the amplifier's input are called *summing-point constraints*. The first *summing-point constraint* is applied to the ideal-amplifier model. It states that no current can flow into either of the amplifier terminals as shown in Fig. 5.6.

This is consistent with an infinitely high input resistance. The condition of no input current into the amplifier means that virtually no input power is necessary. For example,

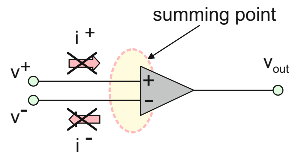


Fig. 5.6. The first summing-point constraint stipulates that no current flows into the ideal amplifier.

a tiny sensor, which does not deliver any appreciable power, could directly be connected to the input. The voltage from the sensor will still be accepted as the input of the amplifier. This condition is a convenient abstraction of the ideal-amplifier model. In reality, a very small input current does exist, typically on the order of nanoamperes (nA) for common amplifier ICs or picoamperes (pA) for ICs with an input JFET stage.

Realistic Values of Input/Output Resistances and Output Current

How far off from reality is the assumption of infinite input resistance? A review of the datasheets reveals that the input resistance of the common amplifier IC (e.g., LM741, LM1458) varies from 0.3 to 6 MΩ. The input resistance of JFET-input stage amplifiers (TL082) is on the order of 1 TΩ (10^{12} Ω). Now, how realistic is the assumption of zero output resistance? Note that if the output resistance were exactly zero, the amplifier would be able to source an infinite current (power) into a low-resistance load. Clearly, we cannot expect a large output power from a physically small amplifier IC. Therefore, we have to introduce a small internal output resistance, which appears to be on the order of 1–100 Ω. The corresponding *output short-circuit current* of the common amplifier ICs (LM741, LM1458, LM358) cannot exceed 40–60 mA; the current into a load is smaller. The output current of faster amplifier ICs (TL082) is even smaller. If the load requires more current than the chip can provide, then the output voltage will notably be clipped.

Section 5.2 Negative Feedback

In most practical circuits, the amplifier IC is not used in open-loop configuration. Engineers have modified the open-loop condition into a negative feedback loop in order to set the gain to a desired value and ensure the amplifier's stability. This section provides you with all the essential knowledge needed to design an amplifier with negative feedback. The mathematical model introduced in this section is based on two conditions imposed at the amplifier's input; we term them the *summing-point constraints*:

a) No electric current flows into or out of the amplifier inputs.
b) The differential voltage at the amplifier's input is zero.

The first summing-point constraint has already been introduced in the previous section. The second summing-point constraint has yet to be derived. We will show that the two summing-point constraints, along with KCL and KVL, will enable us to solve *any* amplifier circuit that involves a negative feedback, no matter what the specific nature of the feedback loop is and regardless of whether it is DC, AC, or a transient circuit.

5.2.1 Idea of the Negative Feedback

The idea of the *negative feedback* goes way back—we may say almost to the Stone Age. Take a wooden rod of 1–2 feet in length. Hold the rod in the vertical position at the tip of your finger. You will probably succeed. Now, close your eyes and try to do the same. You will most likely fail. The reason for the failure is a breakdown of the feedback loop. This loop is created by visual control of the rod's position; you automatically apply a compensating acceleration to the bottom tip of the rod when it begins to fall. Another good example is driving a car and trying to stay in the center of the lane. The negative feedback for electronic amplifiers was first invented and realized by Harold S. Black (1898–1983), a 29-year-old American electrical engineer at Bell Labs. To many electrical engineers, this invention is considered perhaps the most important breakthrough of the twentieth century in the field of electronics because of its wide applicability. We will construct simple amplifier circuits of a given gain, using a resistive feedback loop. Being able to perform this task is already critical from the practical point of view.

5.2.2 Amplifier Feedback Loop: Second Summing-Point Constraint

We construct *the feedback loop*, as shown in Fig. 5.7, by connecting the output to the inverting input terminal. This was exactly the idea of Harold Black. The shadowed box in the feedback loop may represent one or more circuit elements. The feedback loop may be a simple wire, a resistance, a network of circuit elements (resistances, inductances, capacitances), etc. The *negative feedback* simply means that *the output voltage, or rather a portion of it, is returned back to the inverting input*.

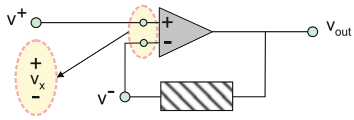


Fig. 5.7. A feedback loop around an amplifier.

Feedback as a Dynamic Process

According to Eq. (5.1) of the previous section, the output voltage is proportional to

$$v_x = v_{\text{in}}^+ - v_{\text{in}}^- \tag{5.10}$$

where v_x is the differential input voltage. Hence, v_x or v_x multiplied by a constant is returned to the input during a very short period of time. The feedback effect is inherently a very fast *dynamic process*, which leads to a static solution with quite remarkable properties. In the example that follows, we will attempt to model the effect of the feedback loop using several very simplifying assumptions.

Example 5.2: An amplifier with a feedback loop in Fig. 5.7 has v^+ fixed at +10 V. v^- is equal to 0 V at $t = 0$. We shall assume that 50 % of v^+ is returned back to the input in 1 µs. How does the differential voltage v_x change with time?

Solution:

1. At $t = 0$, $v_x = 10\,\text{V} - 0\,\text{V} = 10\,\text{V}$. Next, 50 % of 10 V is returned in 1 µs. The voltage v^- becomes equal to $0\,\text{V} + 5\,\text{V} = 5\,\text{V}$ after 1 µs.
2. At $t = 1$ µs, $v_x = 10\,\text{V} - 5\,\text{V} = 5\,\text{V}$. Next, 50 % of 5 V is returned in 1 µs. The voltage v^- becomes equal to $5\,\text{V} + 2.5\,\text{V} = 7.5\,\text{V}$ after 2 µs.
3. At $t = 2$ µs, $v_x = 10\,\text{V} - 7.5\text{V} = 2.5\,\text{V}$. Next, 50 % of 2.5 V is returned in 1 µs. The voltage v^- becomes equal to $7.5\,\text{V} + 1.25\,\text{V} = 8.75\,\text{V}$ after 3 µs.

The process further continues so that voltage v_x halves every microsecond. The process dynamic is shown in Table 5.1 and visualized in Fig. 5.8.

Table 5.1. Dynamics of the differential input voltage as a function of time for Example 5.2.

Time, µs	v^+	v^-	$v_x = v^+ - v^-$
0	10 V	0 V	10 V
1	10 V	5 V	5 V
2	10 V	7.5 V	2.5 V
3	10 V	8.75 V	1.25 V
4	10 V	9.375 V	0.625 V

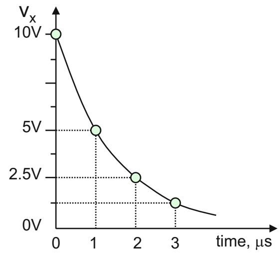


Fig. 5.8. Dynamics of the differential input voltage as a function of time.

Both Table 5.1 and Fig. 5.8 make clear that the differential voltage υ_x decays to zero very rapidly, once the feedback loop is introduced. Hence we arrive at the *second summing-point constraint*, which is valid only for the amplifiers with the negative feedback loop: *the differential input voltage to the amplifier is exactly equal to zero*. The second summing-point constraint is a close approximation to reality. Its accuracy depends on the value of the open-loop gain of the amplifier. If the open-loop gain were infinite, the second summing-point constraint would be exact.

5.2.3 Amplifier Circuit Analysis Using Two Summing-Point Constraints

Next, we will solve an amplifier circuit with negative feedback using the two summing-point constraints (SPC): (i) no current into or out of the input amplifier terminals and (ii) the differential input voltage is zero. The method of two summing-point constraints is an accurate solution method for a wide variety of amplifier circuits with the negative feedback. For amplifier circuits with a *single input*, we will denote the *input voltage to the amplifier circuit* by υ_{in}. Voltage υ_{in} may be equal to υ^+ or to υ^-, depending on amplifier type to be used.

Non-inverting Amplifier

The first amplifier configuration is the so-called non-inverting amplifier shown in Fig. 5.9. The feedback loop contains one resistance R_2. Another resistance R_1 shunts the inverting input to ground. The input voltage to the amplifier circuit is the voltage υ_{in} with respect to ground, or common in this case, which implies the use of the dual-polarity voltage power supply. The output voltage with respect to common is υ_{out}. We apply the first summing-point constraint and KCL to the node "*" in Fig. 5.9 and obtain

$$i_1 = i_2 \tag{5.11}$$

Equation (5.11) is further transformed using Ohm's law in the form

$$\frac{\upsilon^* - 0}{R_1} = \frac{\upsilon_{out} - \upsilon^*}{R_2} \tag{5.12}$$

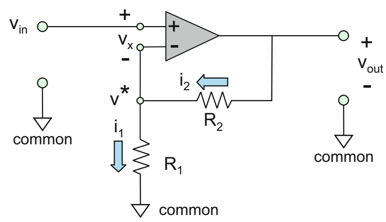


Fig. 5.9. Circuit diagram of the non-inverting amplifier. A dual power supply is not shown.

The second summing-point constraint yields

$$v^* = v_{\text{in}} \tag{5.13}$$

since $v_x = 0$. Equation (5.12) thus reads

$$\frac{v_{\text{in}}}{R_1} = \frac{v_{\text{out}} - v_{\text{in}}}{R_2} \Rightarrow \frac{v_{\text{out}}}{R_2} = \frac{v_{\text{in}}}{R_1} + \frac{v_{\text{in}}}{R_2} \tag{5.14}$$

As a result, we find that the voltage input-to-output relation becomes

$$v_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right) v_{\text{in}} \tag{5.15}$$

The amplifier circuit is solved: we have expressed the output voltage in terms of the input voltage and a resistor ratio. Equation (5.15) is the basic result in amplifier theory. It shows that the feedback loop allows us to precisely control the gain with two arbitrary resistances. One chooses the proper resistance combination to achieve any finite gain between one (setting $R_2 = 0$) and the open-loop (infinite) gain (setting $R_1 = 0$). In the last case, the negative input terminal becomes grounded; the feedback loop is irrelevant and can be replaced by an open circuit so that the amplifier again becomes the *comparator*. The gain expression

$$A_{\text{CL}} = \left(1 + \frac{R_2}{R_1}\right) \geq 1 \tag{5.16}$$

is called the *closed-loop gain* of the amplifier; it clearly relates the output voltage to the input voltage. Equation (5.16) is a dramatic illustration of the negative feedback. We started with an amplifier having a very large yet loosely predictable open-loop gain. Through applying the negative feedback, we arrived at a gain that is much smaller than the open-loop gain; however, it is controllable and stable. Equation (5.16) can be derived more simply using the voltage divider concept. Namely, resistors R_1, R_2 form a voltage divider between 0 V and the output voltage. Hence, the voltage at node (*) may be found. Equating this voltage to the input voltage gives us Eq. (5.16).

Exercise 5.4: Solve the circuit shown in Fig. 5.10, i.e., find the output voltage v_{out} with respect to common.

Answer: $v_{out} = \left(1 + \dfrac{R_2}{R_1}\right) v_{in} = \left(1 + \dfrac{1 \times 10^6}{5.1 \times 10^3}\right) \times 1 \ \text{mV} = 197 \ \text{mV}.$

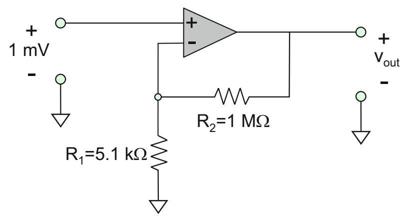


Fig. 5.10. A non-inverting amplifier circuit with an input voltage of 1 mV.

Inverting Amplifier

The next amplifier circuit configuration is the *inverting* amplifier shown in Fig. 5.11. Note that the input terminals are now flipped. The negative feedback loop is still present; it involves resistance R_2. Another resistance, R_1, shunts the non-inverting input to ground.

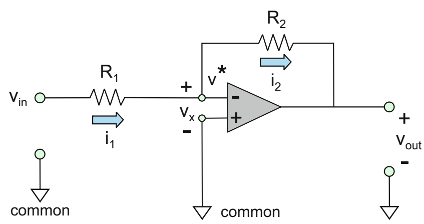


Fig. 5.11. Circuit diagram of the inverting amplifier; a dual power supply is used (not shown).

The input voltage to the amplifier circuit is the voltage v_{in} with respect to ground or common. The output voltage with respect to common is v_{out}. To solve the amplifier circuit, we use the same solution procedure as for the non-inverting amplifier. However, the final result will be quite different. We apply the first summing-point constraint and KCL to the node labeled "*" in Fig. 5.11 and again obtain

$$i_1 = i_2 \tag{5.17}$$

Equation (5.17) is transformed using Ohm's law,

$$\frac{v_{in} - v^*}{R_1} = \frac{v^* - v_{out}}{R_2} \tag{5.18}$$

The second summing-point constraint yields $v^* = 0$ since $v_x = 0$. Equation (5.18) then gives

$$v_{\text{out}} = -\frac{R_2}{R_1} v_{\text{in}} \tag{5.19}$$

The amplifier circuit is solved: we have expressed the output voltage in terms of the input voltage. Equation (5.19) is another key result in amplifier theory. The expression

$$A_{\text{CL}} = -\frac{R_2}{R_1} \tag{5.20}$$

is also called the *closed-loop gain* of the inverting amplifier; the gain again relates the output voltage to the input voltage. It is now negative, which means that the output voltage is inverted. This circumstance is hardly important for the AC signals where the voltage inversion is equivalent to a phase shift of π radians or 180 degrees. The feedback loop of the inverting amplifier also enables us to control the gain of the amplifier with two standard resistors. We can choose the proper resistance combination to achieve any finite gain between zero ($R_2 = 0$) and negative infinity ($R_1 = 0$). In Fig. 5.11 we clearly see how the amplifier gain is controlled by the voltage divider with resistors R_1 and R_2.

Exercise 5.5: Solve the inverting-amplifier circuit shown in Fig. 5.12, i.e., find the output voltage v_{out} with respect to common.

Answer: $v_{\text{out}} = -\frac{R_2}{R_1} v_{\text{in}} = -\frac{1 \times 10^4}{51} \times 1 \ \text{mV} = -196 \ \text{mV}.$

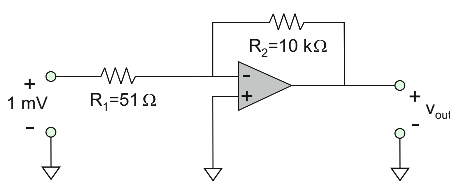


Fig. 5.12. An inverting amplifier circuit with an applied input voltage of 1 mV.

Voltage Follower or Buffer Amplifier

The third important member of the amplifier family is the *voltage follower* or *buffer* amplifier whose circuit is shown in Fig. 5.13. The negative feedback loop is just a wire.

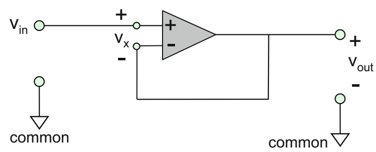


Fig. 5.13. Circuit diagram of the buffer amplifier; a dual power supply is used.

The use of the second summing-point constraint immediately leads to

$$v_{out} = v_{in}, \quad A_{CL} = 1 \tag{5.21}$$

so that the gain of this amplifier type is simply unity. Why do we need a unity-gain amplifier? The reason is that, while the buffer amplifier in Fig. 5.13 passes the voltage without change, it requires virtually no current at the input (virtually no input power) but, at the same time, could source a significant current (on the order of 20–40 mA) at the output, i.e., provide significant output power. In order words, it becomes in a certain sense a *power* amplifier. A simple example would be a capacitive sensor that cannot deliver currents on the order of 10 mA or even smaller currents; otherwise, the corresponding capacitor would immediately discharge. Such a sensor cannot directly be connected to an LED indicator that requires at least 10 mA. However, this sensor may deliver significant voltages, on the order of 1–5 V, which do not need to be amplified. The use of a buffer amplifier can nicely solve this connection problem. The above discussion directly leads us to the concept of *input resistance of the amplifier circuit* studied in the next section.

Exercise 5.6: Solve the voltage-follower circuit shown in Fig. 5.13, i.e., find the output voltage v_{out} with respect to common when the input voltage with respect to common is a) 1 V and b) 10 V. The amplifier is powered by a ±6-V dual supply.

Answer: a) $v_{out} = 1$ V; b) $v_{out} = 6$ V.

5.2.4 Mathematics Behind the Second Summing-Point Constraint

The second summing-point constraint might appear to be mysterious, at least at first sight. How does the amplifier accept the input signal if there is no current at the input and the differential input voltage is zero? Can we avoid using the second SPC, and at what cost? We will show that the second SPC is nothing but a handy tool to solve the amplifier circuit with the negative feedback, with a high degree of accuracy. Mathematically, the second SPC gives a leading (and usually very accurate) term of what is known as an *asymptotic expansion* with regard to a small parameter, here the inverse open-loop gain A^{-1}. Let us now ignore the second SPC and derive the gain equation for the buffer amplifier exactly. A similar derivation for the non-inverting amplifier is given as a homework problem. Looking at Fig. 5.13, we conclude that $v^{-} = v_{out}$ since the negative amplifier terminal is directly connected to the output. According to the amplifier equation, Eq. (5.1) of the previous section, we have

$$v_{out} = A(v^+ - v^-) = A(v_{in} - v_{out}) \tag{5.22}$$

Solving Eq. (5.22) for the output voltage yields

$$v_{out} = A\left(v_{in}^+ - v_{in}^-\right) = \frac{A}{1+A} v_{in} \tag{5.23}$$

Using a Maclaurin series expansion, we obtain with $A >> 1$ the result

$$\frac{A}{1+A} = \frac{1}{1+1/A} \approx 1 - 1/A \approx 1 \tag{5.24}$$

which is consistent with Eq. (5.21) and is very accurate since typically $A > 10^5$. A similar derivation holds for the non-inverting (or the inverting) amplifier configuration. With this in mind, the second SPC is clearly optional. Instead, the amplifier definition Eq. (5.1) may be used, along with the condition of the high open-loop gain. However, it is rather tedious to repeat the asymptotic analysis every time; so we prefer to use the accurate and simple summing-point constraint. The finite value of the open-circuit gain A becomes important for high-speed amplifiers with the feedback loop; see Chapter 10.

5.2.5 Current Flow in the Amplifier Circuit

The current flow in the complete amplifier circuit is illustrated in Fig. 5.14. The output current through the load resistance R_L of the amplifier circuit in Fig. 5.14 is provided by the dual-polarity power supply. In this sense, the amplifier is also a "valve" (similar to its building block, the transistor), which "opens" the power supply in response to the low-power (or virtually no-power) input voltage signal. In Fig. 5.14, you should note that standard resistor values (5 % or 1 % tolerance) may be slightly different from the values used in this figure for convenience. We consider the positive input voltage of 100 mV in Fig. 5.14a first. The non-inverting amplifier has a closed-loop gain A_{CL} equal to 50. The output voltage is thus +5 V, which is the push mode. The load current of 10 mA is found from Ohm's law. The feedback current of 0.1 mA is found using the second SPC and Ohm's law. The power supply current is the sum of both. The feedback current controls the gain of the amplifier, and the load current drives the load. The overall amplifier circuit efficiency (neglecting the loss in the IC itself) depends on the ratio of these two currents. Therefore, we should keep the feedback current small. The power current path is shown in Fig. 5.14a by a thick trace. The current at node A can only enter the upper power supply. Thus, it is the upper power supply being used. The amplifier is operating in the "push" mode, i.e., the *amplifier sources* the current. When the input voltage is negative as in Fig. 5.14b, the lower power supply is delivering power. Now, the *amplifier sinks* the current; it is operating in the "pull" mode.

However, the ratio of the load current and feedback loop current remains the same, at least for the ideal amplifier. Similar results are obtained for the inverting amplifier and the voltage follower, respectively.

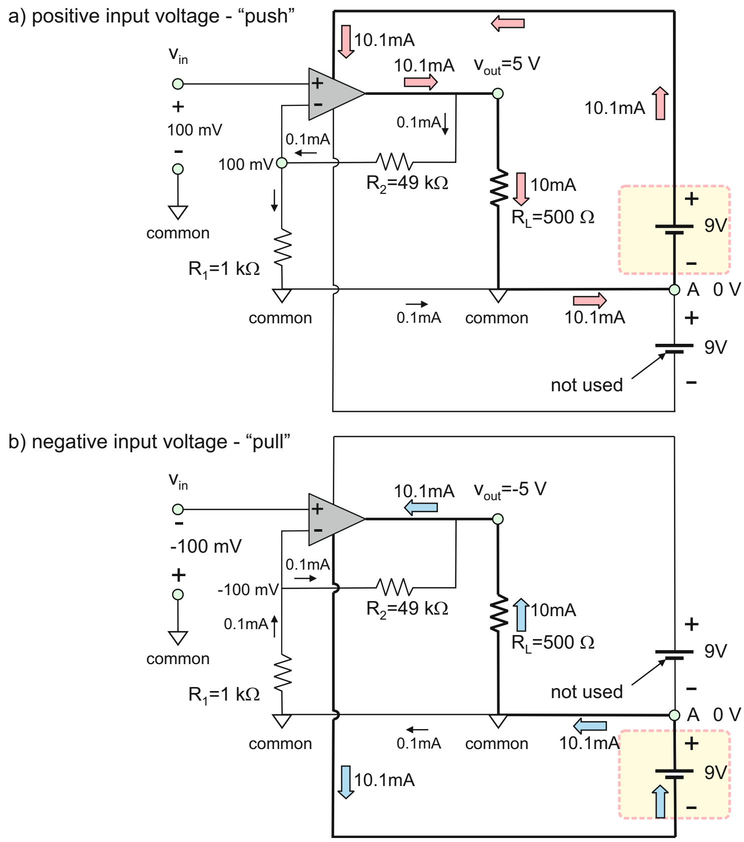


Fig. 5.14. Current flow in the non-inverting amplifier circuit operating in the (**a**) push mode and (**b**) pull mode. The path of the (relatively high) load current is marked in bold.

5.2.6 Multiple-Input Amplifier Circuit: Summing Amplifier

Figure 5.15 shows an important amplifier type on the basis of the inverting amplifier—the *summing amplifier*. A summing amplifier performs a simple mathematical operation: it sums several weighted input voltages. The summing amplifier is a prototype of the *binary-weighted-input digital-to-analog converte*r studied in Chapter 14. According to the KCL and to the first summing-point constraint, one has with reference to Fig. 5.15

$$i_F = i_1 + i_2 + i_3 \tag{5.25}$$

On the other hand, the second summing-point constraint (the differential voltage to the amplifier is zero and the inverting input is the common or virtual ground) yields

$$i_1 = \frac{v_1}{R_1}, \quad i_2 = \frac{v_2}{R_2}, \quad i_3 = \frac{v_3}{R_3} \tag{5.26}$$

in terms of input voltages v_1, v_2, v_3. Therefore, voltage v_{out} in Fig. 5.15 found from Eq. (5.25) is now written in the form

$$i_F = \frac{0 - v_{out}}{R_F} = i_1 + i_2 + i_3 = \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right) \Rightarrow v_{out} = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_2}v_2 - \frac{R_F}{R_3}v_3 \tag{5.27}$$

Example 5.3: An input to the amplifier circuit in Fig. 5.15a is a timing sequence shown in Fig. 5.15b. Such a sequence is known as a *binary counter*; it represents all three-bit binary numbers in an ascending order, with the time interval of 1 μs. The amplifier circuit is characterized by $R_F = 2\ \text{k}\Omega$, $R_1 = 40\ \text{k}\Omega$, $R_2 = 20\,\text{k}\Omega$, and $R_3 = 10\,\text{k}\Omega$. Plot the absolute output voltage to scale.

Solution: After plugging in the numbers, Eq. (5.27) is transformed to

$$|v_{out}| = 0.05v_1 + 0.1v_2 + 0.2v_3 \tag{5.28}$$

Figure 5.15c shows the result. This is a staircase approximation of the straight line.

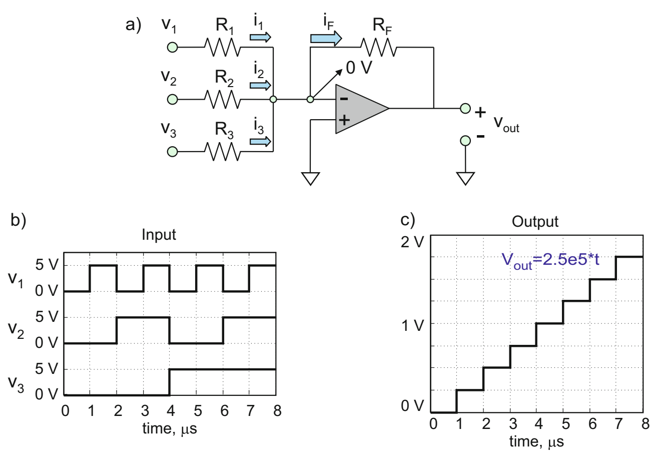


Fig. 5.15. (**a**) Circuit diagram of a summing amplifier. (**b**) and (**c**) Typical input and output voltages.

A large collection of practical amplifier circuits with the negative feedback exists. Some of them are *DC-coupled amplifiers* (considered here), some are intended for amplification of AC voltage signals with zero mean (the so-called AC-coupled amplifiers).

Section 5.3 Amplifier Circuit Design

Now that the theory of the negative feedback loop has been established, we can turn our attention to the laboratory. Our hope is to be immediately successful with our designs. However, a number of questions will arise almost instantly. They raise issues such as how to choose the resistor values, how to connect the sensor as part of the input load, and how to use an amplifier chip with a single power supply (a battery).

5.3.1 Choosing Proper Resistance Values

There are several rules regarding how to choose resistances R_1, R_2 controlling the feedback loop in both non-inverting and inverting configurations. They are:

1. Resistances R_1, R_2 cannot be too small. Imagine that in Fig. 5.14 of Section 5.2, the resistor values are changed to $R_1 = 1\ \ \Omega,\ \ R_2 = 49\ \ \Omega$. The same non-inverting gain will be achieved and the same output voltage will be obtained. However, the feedback loop current now becomes 100 mA instead of 0.1 mA. The general-purpose op-amp chips are not capable of delivering such large currents. Furthermore, the ohmic losses in the feedback loop become high. Therefore, one should generally use

$$R_1, R_2 \geq 50 - 100\ \ \Omega \tag{5.29a}$$

2. Resistances R_1, R_2 cannot be too large. Let us assume that resistance R_2 equals 100 MΩ. This means that this physical resistor and the feedback loop represent almost an "open circuit." Unwanted electromagnetic signals may couple into such a circuit through the related electric field difference across its terminals. This effect is known as *capacitive coupling*. Furthermore, the very large resistances increase the parasitic effect of the input offset current. Plus, very large resistances are unstable—their values depend on moisture, temperature, etc. Therefore, one should generally use

$$R_1, R_2 \leq 1\,\mathrm{M}\Omega \tag{5.29b}$$

3. When a precision design is not warranted, inexpensive 5 % tolerance resistors may be used. Otherwise, 1 % or even 0.1 % tolerance resistors are employed. Moreover, in lieu of fixed resistors, we may use one or two potentiometers to make the gain adjustable.
4. The load resistance should be sufficiently large in order not to overdrive the amplifier. A good choice is

$$R_L \geq 100\,\Omega \tag{5.29c}$$

This requires an output current of 20 mA at $v_{\mathrm{out}} = 2\,\mathrm{V}$ when R_L is exactly 100 Ω. If $R_\mathrm{L} < 100\ \ \Omega$, the amplifier output voltage may decrease compared to the expected value due to amplifier's inability to source/sink sufficient current.

Example 5.4: A non-inverting amplifier circuit with a gain of 11 is needed in the configuration depicted in Fig. 5.16. Identify one set of proper resistance values.

Solution:
To satisfy Eq. (5.29), we simply choose the round numbers

$$R_1 = 1\ \text{k}\Omega,\ \ R_2 = 10\ \text{k}\Omega,\ \ R_\text{L} = 100\ \ \Omega \tag{5.30}$$

However, other choices are indeed possible. For example, the set

$$R_1 = 100\ \text{k}\Omega,\ \ R_2 = 1\ \text{M}\Omega,\ \ R_\text{L} = 100\ \ \Omega \tag{5.31}$$

will solve the problem too.

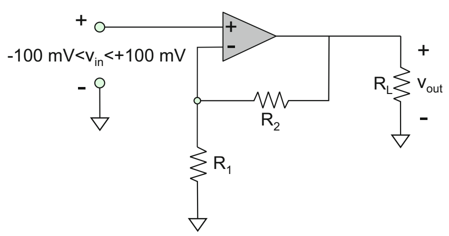


Fig. 5.16. A non-inverting amplifier with unknown resistance values.

Discrete Resistance Values and Potentiometers

To achieve a proper gain, we sometimes have to use "strange" resistor values like 49 kΩ (Fig. 5.14 of Section 5.2). Do such resistors really exist? For 5 % tolerance resistors, they do not. However, for 1 % tolerance resistors, you can find the standard value of 48.7 kΩ, which is close to the above value. When the exact resistor values are not available, a potentiometer can be used as a variable resistor. Furthermore, an externally controlled potentiometer in the feedback loop is also important when a *variable-gain amplifier* is needed, for example, for applications that require *automatic gain control*.

Gain Tolerance

What about the *gain tolerance*? The feedback resistor tolerances indeed determine the gain tolerance. If the resistor tolerance is X, then the gain tolerance is $2X$. This result is valid for both the inverting and the non-inverting amplifier. The corresponding proof uses an asymptotic expansion for the gain about its unperturbed value. We consider the worst-case scenario for the inverting amplifier and obtain

$$A_{CL} = -\frac{R_2(1+X)}{R_1(1-X)} \approx -\frac{R_2}{R_1}(1+X)(1+X) \approx -\frac{R_2}{R_1}(1+2X) \quad \text{for } X << 1 \tag{5.32}$$

The non-inverting gain is treated similarly. For example, if two resistors of an inverting amplifier circuit are $R_1 = 1$ kΩ, $R_2 = 100$ kΩ and both resistors have 5 % tolerances, then the amplifier gain is equal to -100 with tolerance of 10 %. Similarly the gain of the non-inverting amplifier circuit becomes 101 with tolerance of slightly less than 10 %.

5.3.2 Model of a Whole Voltage Amplifier Circuit

Any *voltage amplifier circuit with the negative feedback* loop may be modeled in a form similar to Fig. 5.5. The corresponding model is shown in Fig. 5.17.

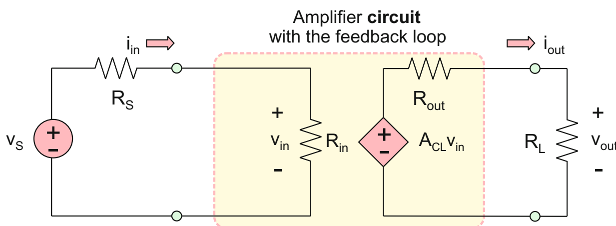


Fig. 5.17. Equivalent model of an amplifier circuit with a negative feedback loop.

First, the open-circuit gain A of the amplifier is replaced by the closed-loop gain $A_{CL} << A$. Second, the differential input voltage v_x is replaced by v_{in}. Third, resistances R_{in} and R_{out} in Fig. 5.17 now become *input and output resistances of the amplifier circuit*, not the amplifier itself. This difference may be quite important in practice.

Input/Output Resistances of Basic Amplifier Circuits Using Ideal-Amplifier Model

The solution simplifies for the ideal-amplifier model; it is shown in Fig. 5.18. We assume ideal operational amplifiers in all three cases. If there were no feedback loop, R_{in} would be exactly equal to the input resistance of the amplifier itself; this is for an ideal operational amplifier $R_{in} = \infty$. When the feedback loop is present, a more general definition should be used, namely,

$$R_{in} \equiv \frac{v_{in}}{i_{in}}, \tag{5.33}$$

where v_{in} is the amplifier's circuit input voltage and i_{in} is now the current into the amplifier's circuit *with the feedback loop*. Figure 5.18 illustrates the corresponding calculation for the three basic amplifier types. For both the non-inverting amplifier and the voltage follower circuits, we have $R_{in} = \infty$, $R_{out} = 0$. However, for the inverting amplifier circuit, $R_{in} = R_1$, $R_{out} = 0$, since an input current can still flow into the feedback loop but not into the amplifier itself. Thus, the inverting amplifier circuit potentially provides greater flexibility in the input resistance simply by varying R_1. To ensure the necessary gain, R_2 has to be chosen accordingly.

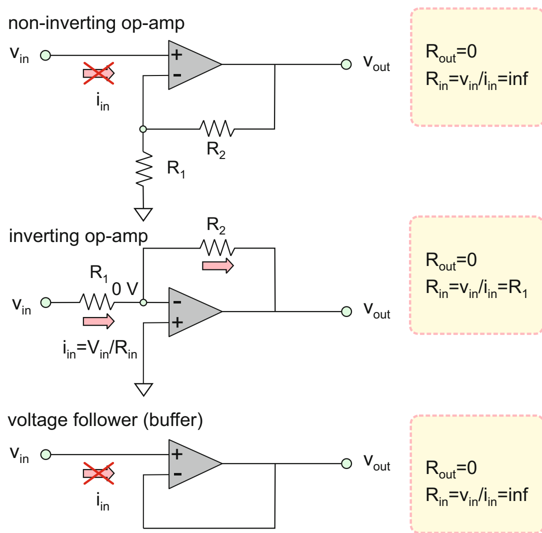


Fig. 5.18. Input and output resistances of amplifier circuits with the ideal operational amplifier.

5.3.3 Voltage Amplifier Versus Matched Amplifier

A sensor (input load to an amplifier circuit) "sees" an amplifier circuit as a simple resistance R_{in} as depicted in Fig. 5.19. The sensor is represented by its Thévenin equivalent circuit.

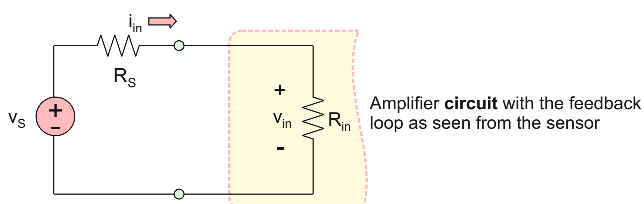


Fig. 5.19. Sensor's equivalent circuit and amplifier's equivalent circuit as seen from the sensor.

Input Load "Bridging"

The general-purpose operational amplifier is a *voltage amplifier* (also called a *signal amplifier*) and not a *power amplifier*. The input voltage matters, not the input power. For the voltage divider circuit in Fig. 5.19, the input voltage v_{in} to the amplifier circuit is *maximized* when R_{in} is maximized. Therefore, R_{in} must satisfy the inequality

$$R_{in} >> R_S \tag{5.34}$$

An appropriate value would be $R_{in} = 100R_S$, for example. Equation (5.34) is sometimes called *load bridging* (or *impedance bridging*) *condition*, where the "load" resistance R_{in} seen by the Thévenin source is much larger than the source resistance R_S. The load bridging is automatically satisfied for the non-inverting amplifier or for the voltage follower. For the inverting amplifier, one should use large values of R_1, for instance,

$$R_1 = 100R_S \tag{5.35}$$

Bridging connections are used to maximize the voltage transfer from a sensor to an amplifier. Even more importantly, the amplifier does not appreciably load the sensor.

Example 5.5: A sensor is given by its Thévenin equivalent circuit in Fig. 5.19 where the sensor voltage v_S is small. The sensor's equivalent resistance R_S is 100 Ω. An inverting amplifier circuit is needed to generate an amplified version of the sensor's voltage. The output voltage should be $\approx -100v_S$.

Solution: The corresponding circuit is shown in Fig. 5.20. The input voltage to the amplifier circuit is computed by voltage division:

$$v_{in} = \frac{R_1}{R_1 + R_S} v_S \tag{5.36}$$

If $R_S = 100\ \ \Omega,\ \ R_1 = 10\ \ \mathrm{k}\Omega$, then $v_{in} \approx v_T$ and there is almost no loss of voltage signal strength across resistance R_S. Therefore, a pair of resistors with $R_1 = 10\ \ \mathrm{k}\Omega,\ \ R_2 = 1\ \ \mathrm{M}\ \Omega$ will solve the problem, with the amplifier voltage gain of -100. If, however, we choose $R_1 = 0.25R_S = 25\,\Omega$, then $v_{in} = 0.2v_S$ and 80 % of available voltage signal strength will be lost! Even if the remaining voltage signal is still appreciable (above the *sensitivity threshold* of the amplifier), the necessary amplifier gain becomes not -100, but -500. An increase in gain leads to an increase in additive voltage noise at the output. Therefore, in the best case, the amplified signal will be a noisier version of the corresponding signal in the previous design.

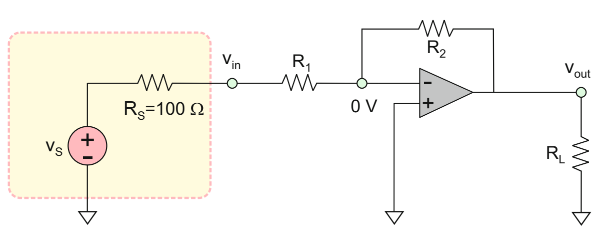


Fig. 5.20. A 100 Ω sensor connected to the inverting amplifier. Note the common connections.

Example 5.6: A sensor is given by its Thévenin equivalent in Fig. 5.19 where the sensor voltage υ_S is small. The sensor's equivalent resistance R_S may vary in time but is always less than 100 Ω. An inverting amplifier is needed that generates an amplified version of the sensor's voltage, which is $\approx -100\upsilon_S$.

Solution: With reference to Fig. 5.20, the input voltage to the amplifier is again given by Eq. (5.36). This equation is further transformed to

$$\upsilon_{in} = \frac{R_1}{R_1 + R_S(t)}\upsilon_S \approx \upsilon_S \tag{5.37}$$

if we choose $R_1 = 100\max(R_S(t)) = 10\,\mathrm{k}\Omega$. In other words, not only have we provided amplification but we also *eliminated* the effect of the sensor's resistance variation by proper load bridging. Therefore, a pair of resistors with $R_1 = 10\ \mathrm{k}\Omega,\ \ R_2 = 1\ \ \mathrm{M}\Omega$ will solve the problem, with the amplifier output of $\approx -100\upsilon_S$, irrespective of the specific value of R_S.

Input Load Matching

And yet, in many modern applications related to radio-frequency (RF) circuits, the *load matching* (but not the load bridging) may be a critical condition. RF amplifiers are internally designed for matching to a precise 50 Ω load at *both* the input and the output—see Fig. 5.21. The reason is that voltage and current signals in conductors behave like propagating electromagnetic waves at high frequencies. If there is no matching, then multiple wave reflections between the amplifier and its input and/or output loads can occur, resulting in superimposing the previous signal onto the next signal.

Fig. 5.21. A RF amplifier to be matched to a 50 Ω at both the input and output.

Therefore, the amplifier circuit optimized for proper load matching (which also achieves maximum power transfer from the input load to the amplifier circuit, see the generator theorems) may still be critical in many high-frequency applications.

Example 5.7: Construct an amplifier circuit matched to an input source with $R_S = 50\ \Omega$. The amplifier's voltage gain is $|A_{CL}| = 20$. The sign of A_{CL} (either positive or negative) is not important since an AC input signal is assumed.

Example 5.7 (cont.):

Solution: Two possible solutions are shown in Fig. 5.22. In the first case, we use the inverting amplifier; in the second case, a smart trick is employed: a non-inverting amplifier with a 50-Ω shunt resistor. You should note that the maximum power transfer will be achieved for the *entire* amplifier circuit, including the shunt resistor.

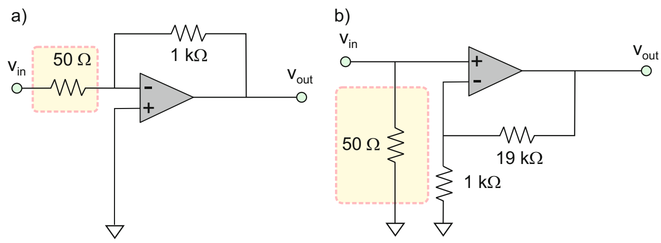


Fig. 5.22. Two possible amplifier configurations matched to a 50-Ω input resistance.

5.3.4 Cascading Amplifier Stages

Assume that we need an inverting amplifier with an overall gain of −1000. The input to the amplifier is a sensor as in Fig. 5.19 with the equivalent resistance given by $R_S = 1\,\text{k}\Omega$. If we require load bridging according to Eq. (5.35), we arrive at

$$R_1 = 100R_S = 100\,\text{k}\Omega \tag{5.38}$$

This yields

$$R_1 = 100\ \ \text{k}\Omega \Rightarrow R_2 = 100\ \ \text{M}\Omega \tag{5.39}$$

Such a resistance value is too large to satisfy Eq. (5.29b); it perhaps will not even be included in your laboratory kit (although the ECE shop may still have such resistors). What should we do? The answer lies in *cascading the amplifier stages* as shown in Fig. 5.23. We use the non-inverting amplifier with a gain of 10 as the first stage; this simultaneously provides the load bridging condition. We use the inverting amplifier with a gain of −100 and with the reasonable resistor values as the second stage. The key point of cascading is to realize that the overall gain of the cascade amplifier is given by the *product* (not the sum!) of the individual stage gains, i.e.,

$$A_{CL} = A_{CL1} \times A_{CL2} = 10 \times (-100) = -1000 \tag{5.40}$$

The same result is valid for more than two stages. The proof for two stages is simple:

$$A_{CL} \equiv \frac{v_{out}}{v_{in}} = \frac{A_{CL2} v_{out}^1}{v_{in}} = \frac{A_{CL1} A_{CL2} v_{in}}{v_{in}} = A_{CL1} A_{CL2} \tag{5.41}$$

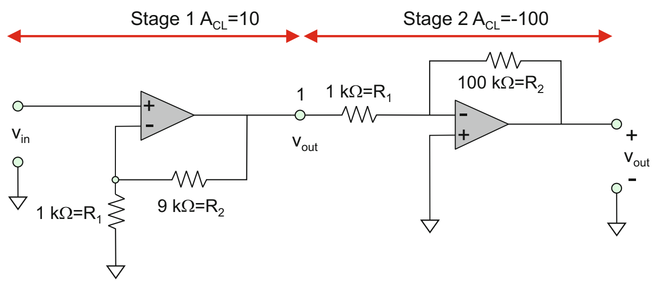


Fig. 5.23. Cascading two amplifier stages into a high-gain circuit. The first stage is a non-inverting amplifier with a gain of 10; the second stage is an inverting amplifier with a gain of -100. The overall gain is therefore -1000.

The cascading of individual amplifier stages is a simple and powerful tool to build high-gain amplifier circuits. Cascading has a number of remarkable features, some of which are studied here:

1. The gains of the individual stages multiply.
2. The gain per stage generally should not exceed 100 (absolute value) in order to avoid instability.
3. The first stage in Fig. 5.23 sees R_1 of the second stage as its output load resistance. Therefore, R_1 should be large enough.
4. The amplifier ICs usually include two (*dual* op-amp) or even four (*quad* op-amp) individual amplifiers in one package. Therefore, they are ideally suited for building multistage amplifiers.
5. The effect of an input offset voltage (an amplifier imperfection studied next) is primarily important for the first stage, but it then loses its significance with every subsequent stage.

Note that cascading is equivalent to a series combination of individual amplifiers. Parallel configurations also exist, particularly in *analog-to-digital converters*.

Example 5.8: The input to the amplifier is a sensor in Fig. 5.19 with an equivalent resistance given by $R_S = 100\ \Omega$ and an equivalent sensor voltage, v_S. An amplifier circuit is needed that generates ~10,000 v_S at its output. The load bridging condition must be satisfied.

Example 5.8 (cont.):

Solution:

A. A single-stage, non-inverting amplifier with $R_1 = 100\ \Omega$, $R_2 = 1\ \mathrm{M}\Omega$ might do the job, including load bridging. However, the gain per stage (10,000) is far too high for stable operation.

B. A series combination of two inverting stages will do a much better job: the first inverting op-amp assures load bridging, and it consists of $R_1 = 10\ \mathrm{k}\Omega$, $R_2 = 1\ \mathrm{M}\Omega$. The second inverting amplifier has exactly the same resistor values: $R_1 = 10\ \mathrm{k}\Omega$, $R_2 = 1\ \mathrm{M}\Omega$. And the overall gain is given by $A_{\mathrm{CL}} = (-100) \times (-100) = 10,000$.

C. A series combination of two non-inverting stages will perform equally well: we choose $R_1 = 10\ \mathrm{k}\Omega$, $R_2 = 1\ \mathrm{M}\Omega$ for the first stage and $R_1 = 10\ \mathrm{k}\Omega$, $R_2 = 1\ \mathrm{M}\Omega$ for the second stage. The overall gain then yields $A_{\mathrm{CL}} = 101 \times 101 = 10,201 \approx 10,000$.

5.3.5 Amplifier DC Imperfections and Their Cancellation

In general, *DC imperfections of the operational amplifier* can have a severe influence on its performance for high-gain amplifiers. Below, we study two types of imperfections, the *input offset voltage* V_{OS} and the *input (bias and offset) currents*, and provide a simple way of how to cancel the corresponding output offset voltage.

Input Offset Voltage

The input offset voltage results in a nonzero output voltage when the two input terminals of the amplifier are shorted out. It arises due to a small asymmetry in the input differential transistor stage inside the amplifier chip. It is fixed for a certain chip but varies from chip to chip. General-purpose amplifiers have the input offset voltage V_{OS} in the range of $1\ \mathrm{mV} - 6\ \mathrm{mV}$. The offset can be modeled as a small DC source of strength, V_{OS}, in series with one of the input terminals to the amplifier as shown in Fig. 5.24. The input offset voltage produces a similar effect for any amplifier configuration, including the comparator where its effect becomes most dramatic. A large triangle in Fig. 5.24 indicates the actual amplifier chip, whereas the small triangle is the ideal amplifier without DC imperfections. The circuit in Fig. 5.24 is analyzed using the two summing-point constraints. Since the negative feedback is present, v_x must be zero, which yields

$$v_{\mathrm{in}} + V_{\mathrm{OS}} = v^* = \frac{R_1}{R_1 + R_2} v_{\mathrm{out}} \Rightarrow v_{\mathrm{out}} = A_{\mathrm{CL}}(v_{\mathrm{in}} + V_{\mathrm{OS}}), \quad A_{\mathrm{CL}} = 1 + \frac{R_2}{R_1} \tag{5.42}$$

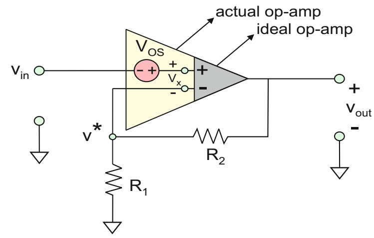


Fig. 5.24. Circuit for the non-inverting amplifier with an input offset voltage.

Exercise 5.7: The amplifier circuit in Fig. 5.24 has the closed-loop gain of $A_{CL} = 100$ and an input offset voltage of $V_{OS} = 5$ mV. What is the general expression for the output voltage?

Answer:

$$v_{out} = 100 v_{in} + 0.5 \text{ V} \quad (5.43)$$

where 0.5 V is the resulting output offset voltage to the amplifier.

Canceling the Output Offset Voltage

In some amplifiers like the LM741, special *offset-null terminals* are available for trimming the output DC voltage to zero. Figure 5.25a shows the concept. The wiper of the potentiometer is to be connected to the negative supply rail. Both inputs to the amplifier should be connected to the common port during the adjusting procedure, which implies that the output voltage is trimmed to zero. If the offset-null terminals are not available, the voltage at the common port, which controls the feedback loop, might be subject to a small offset. Figure 5.25b shows a circuit that can be used to eliminate the DC imperfections for a particular non-inverting amplifier in the laboratory. A compensating voltage offset is introduced by means of an adjustable voltage divider with a potentiometer. To achieve a good degree of accuracy, one should set

$$R_P << R \quad (5.44)$$

Furthermore, R_1 should be considerably larger than R. This condition can be avoided by a further modification of the present voltage divider. The output offset voltage is trimmed to zero when the input to the amplifier in Fig 5.26b is connected to the common port (circuit ground). This ensures that the effect of the input offset voltage will be eliminated entirely, for *any* value of the input voltage. Note that such an adjustment has to be done for every discrete temperature point, since V_{OS} depends on temperature.

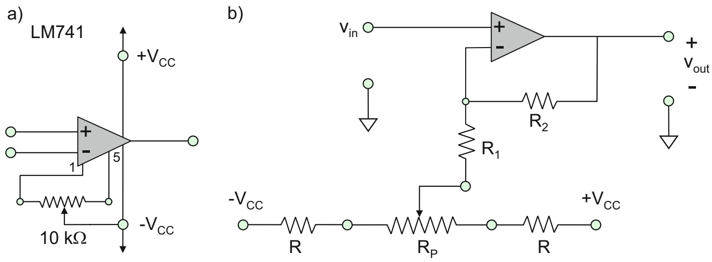


Fig. 5.25. (**a**) Output DC offset voltage for the LM741 is reduced to zero by adjusting the potentiometer placed between its offset-null pins and (**b**) a similar operation performed with the virtual ground of the feedback loop.

Input Bias and Offset Currents

In reality, for the op-amp to operate, there will be very small currents into the input terminals (into transistor bases), typically on the order of 100 nA. When those currents flow through the feedback resistances, they create corresponding voltages which appear as an output offset voltage as well. Let a current of 100 nA flow *into* the negative terminal of the amplifier in Fig. 5.24. If the input to the amplifier is grounded, this current must flow through resistance R_2. Therefore, it will create the extra output DC voltage of

$$v_{out} = +100 \ \text{nA} \times R_2 \tag{5.45}$$

when the input voltage to the amplifier v_{in} is exactly zero. To appreciate its value, we can use a resistance $R_2 = 1 \ \text{M}\Omega$ as an example. This yields

$$v_{out} = 0.1 \ \text{V} \tag{5.46}$$

at the output. Fortunately, the currents flowing into the amplifier are nearly the *same* for either terminal. Therefore, their average (the *input bias current*) considerably exceeds their difference (the *input offset current*). There is a way to eliminate the larger effect of the input bias current. It consists of modifying the circuits for the non-inverting and inverting amplifier by adding one extra resistance $R = R_1||R_2$ as shown in Fig. 5.26.

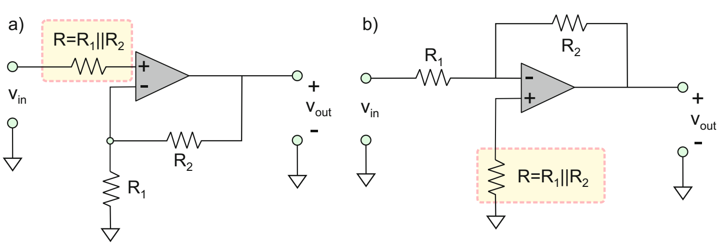


Fig. 5.26. Cancellation of the effect of input bias currents.

From the amplifier's gain point of view, the resistance R has a negligible, if any, effect. The proof of the cancellation effect for the non-inverting amplifier circuit is as follows. When both the input and output to the amplifier in Fig. 5.26a are grounded (connected to the common port), the input current source at the non-inverting input sees resistance R and the input current source at the inverting input sees the parallel combination of R_1, R_2, respectively. Making R equal to $R_1||R_2$ yields an offset differential voltage that is zero.

5.3.6 DC-Coupled Single-Supply Amplifier: Virtual-Ground Circuit

A single voltage supply is a battery. We consider the non-inverting amplifier circuit driven by a 9-V battery. To handle the problem of not being able to generate negative voltages, a *virtual-ground circuit* may be used as shown in Fig. 5.27. We simply divide the voltage of the battery by two, with two large, equal resistances R, and assign this voltage of 4.5 V to the common port. The battery terminal voltages then formally become ±4.5 V versus the common port. The power supply so constructed is unfortunately not exactly the dual-polarity voltage power supply. Namely, both resistances R have to be large to avoid ohmic losses in the virtual-ground circuit. If they are, we cannot source/sink an appreciable output current into the common terminal since these resistances simultaneously operate as current limiters. An alternative solution is to reference the output to ground (which is the negative terminal of the battery), but not to the *virtual ground* of 4.5 V. When referenced to ground, the circuit in Fig. 5.27 has one remarkable property. Namely, if the input vs. ground is the (small) sensor voltage v_{in} plus 4.5 V, then the output vs. ground is the amplified sensor voltage, $A_{CL}v_{in}$, plus the *same* 4.5 V offset. In other words, the offset voltage of the virtual ground is not amplified! This statement is proved in the following example.

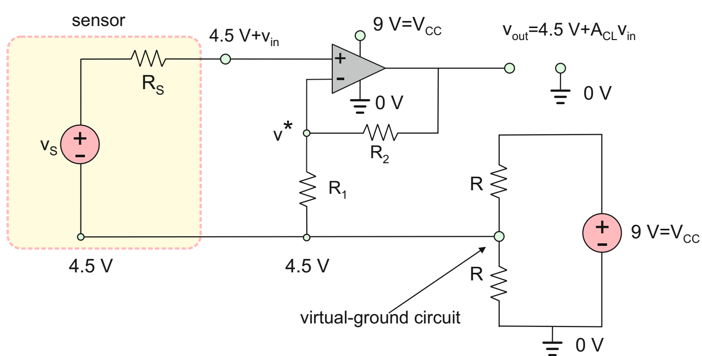


Fig. 5.27. A non-inverting amplifier driven by a single voltage supply, a battery. Absolute voltages versus ground (negative terminal of the battery) are shown.

Example 5.9: Solve the circuit in Fig. 5.27.

Solution: The solution is based on the two summing-point constraints. There is no current into the input terminals so that resistances R_1 and R_2 again form a voltage divider but now between v_{out} and 4.5 V. By voltage division, the voltage at node (*) becomes

$$v^* = 4.5 \ \text{V} + \frac{R_1}{R_1 + R_2}(v_{out} - 4.5 \ \text{V}) \tag{5.47}$$

At the same time, using the second summing-point constraint for the amplifier with negative feedback, we obtain:

$$v^* = 4.5 \ \text{V} + v_{in} \tag{5.48}$$

Equating Eqs. (5.47) and (5.48), we arrive at the expected result:

$$v_{out} = \ 4.5 \ \text{V} + A_{CL} v_{in}, \quad A_{CL} = 1 + \frac{R_2}{R_1} \tag{5.49}$$

Feedback resistances R_1, R_2 should be much larger than resistance R in Fig. 5.27, in order to assure a flawless circuit operation. Yet another solution is to use (Zener) diodes in the bias circuit. Special *virtual-ground integrated circuits* exist that support single-supply amplifier operation. They generate an output precisely midway between the two supply rails.

Section 5.4 Difference and Instrumentation Amplifiers

5.4.1 Differential Input Signal to an Amplifier

Consider a sensing element that is a variable resistance. The sensor configuration is a Wheatstone bridge (Section 3.3) as seen in Fig. 5.28. Since the amplifier is powered by a dual supply with three terminals, $\pm V_{CC}$, and common (ground) port, the same power supply is connected to the bridge. We will use the positive rail and the ground rail in Fig. 5.28 since the sensor (e.g., the strain gauge) may require lower voltages than the amplifier chip itself.

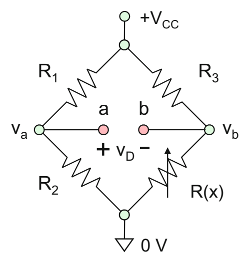


Fig. 5.28. A Wheatstone bridge sensor to be connected to an amplifier circuit.

Both voltages v_a, v_b have to be used when reading sensor information. They may be written in terms of the other two voltages v_D, v_{CM}:

$$v_a = v_{CM} + \frac{1}{2}v_D, \quad v_b = v_{CM} - \frac{1}{2}v_D \tag{5.50a}$$

$$v_D = v_a - v_b, \;\; v_{CM} = 0.5(v_a + v_b) \tag{5.50b}$$

Here, v_D is the differential component of the combined input signal or the *differential voltage* and v_{CM} is the sum component of the combined input signal or the *common-mode voltage*. Only the differential voltage is usually important for sensor reading; the common-mode voltage does not carry any information. When the bridge is exactly balanced, i.e., when

$$\frac{R_1}{R_2} = \frac{R_3}{R(x)}, \tag{5.51a}$$

the differential voltage is exactly zero:

$$v_D = 0 \tag{5.51b}$$

Still, the common-mode DC voltage given by

$$v_{CM} = v_{CC}\left(\frac{R_2}{R_2 + R_1}\right) \tag{5.51c}$$

can have any large positive value. For example, it is $v_{CC}/2$ when all resistances are equal. Even if the bridge in Fig. 5.28 uses $\pm V_{CC}$ power rails, it is hardly possible that the common-mode voltage is set to zero since absolutely identical resistors do not exist.

Exercise 5.8: In the Wheatstone bridge in Fig. 5.28, we assume $R_2 = 1.1R_1$, $R(x) = 1.1R_3$, and $V_{CC} = 6$ V. What are the differential and common-mode voltages?

Answer: $v_D = 0$, $v_{CM} = 3.14\,\text{V}$.

5.4.2 Difference Amplifier: Differential Gain and Common-Mode Gain

The sensor in Fig. 5.28 is the *differential sensor* with *three* terminals: *a*, *b*, and ground. How do we amplify the differential voltage? Reviewing the inverting and non-inverting amplifier types reveals that they are not appropriate for this purpose: we simply do not have two input terminals to be connected to nodes *a* and *b* in Fig. 5.28. Only *one* input terminal referenced to common (ground) is available. Note that *single-ended sensors* with *two* terminals (plus and ground) indeed exist and may be used. In that case, inverting or non-inverting amplifiers will function well. However, the overall design accuracy may deteriorate compared to the differential design. Therefore, a new amplifier type with two input terminals should be introduced. It is the *difference amplifier* shown in Fig. 5.29.

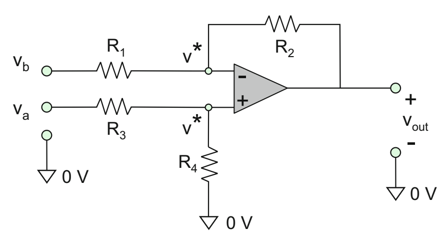


Fig. 5.29. A difference amplifier.

Both inputs to this amplifier are referenced to common (ground) port. First, we see that the difference amplifier is an inverting amplifier with the negative feedback loop. However, the second input signal is now added to its positive terminal through a voltage divider. The analysis of this amplifier type is done using two summing-point constraints. A shortcut is to recognize, with the help of the first SPC, that in Fig. 5.29 we have two voltage dividers: one between resistances R_1, R_2 and another between resistances R_3, R_4. Therefore, for the voltage at node (*), one has, using the first voltage divider,

$$v^* = v_b + \frac{R_1}{R_1 + R_2}(v_{out} - v_b) \tag{5.52a}$$

and, according to the second voltage divider,

$$v^* = v_a + \frac{R_3}{R_3 + R_4}(0\ \text{V} - v_a) \tag{5.52b}$$

Both expressions must be equal to each other due to the second SPC (the differential input voltage in a negative feedback amplifier is zero). Therefore,

$$\begin{gathered} v_b + \frac{R_1}{R_1 + R_2}(v_{out} - v_b) = v_a - \frac{R_3}{R_3 + R_4}v_a \Rightarrow \\ \frac{R_1}{R_1 + R_2}v_{out} = \left(\frac{R_1}{R_1 + R_2} - 1\right)v_b - \left(\frac{R_3}{R_3 + R_4} - 1\right)v_a \end{gathered} \tag{5.52c}$$

To create a voltage difference, i.e., $v_a - v_b$, between the input voltages on the right-hand side of Eq. (5.52c), we select

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \tag{5.52d}$$

as the necessary condition. Then, both factors in parentheses on the right-hand side of Eq. (5.52c) become equal. This yields the basic equation of the difference amplifier,

$$v_{out} = \frac{R_2}{R_1}(v_a - v_b) = \frac{R_2}{R_1}v_D \tag{5.53}$$

Equation (5.53) is a simple, yet highly useful result for amplifier circuit design. Namely, once the amplifier in Fig. 5.29 is connected to the sensor in Fig. 5.28, the differential voltage $v_D = v_a - v_b$ is amplified with the gain of R_2/R_1 (the *differential amplifier circuit gain*). At the same time, the undesired common-mode voltage $v_{CM} = 0.5(v_a + v_b)$ is completely *rejected*, i.e., amplified with a gain of 0, no matter what specific values the input voltages have versus ground. In other words, the *common-mode amplifier circuit gain* is zero. Note that the ratio of two gains (*differential gain* versus *common-mode gain*) is an important characteristic of the difference-amplifier circuit. It is called the *common-mode rejection ratio* (CMRR). In our case, this ratio is clearly infinity. Unfortunately, in reality, this value is finite though quite large. One obvious reason is a possible mismatch in resistance ratios in Eq. (5.52d), which will not allow us to obtain Eq. (5.53) exactly. A certain portion of $v_{CM} = 0.5(v_a + v_b)$ will be present at the output.

Example 5.10: Find the output voltage of the amplifier circuits shown in Fig. 5.30 below. Assume the ideal amplifier and exact resistance values.

Solution: We check Eq. (5.52d) first and conclude that the circuit in Fig. 5.30 is a true difference amplifier: it rejects the common-mode voltage. The differential voltage to the amplifier is -0.01 V. Using Eq. (5.53) gives us an output voltage of $v_{out} = -0.1$ V. If the resistance ratios were not equal to each other, a common-mode signal would be present at the output. In that case, the complete amplifier equation (5.52c) should to be used.

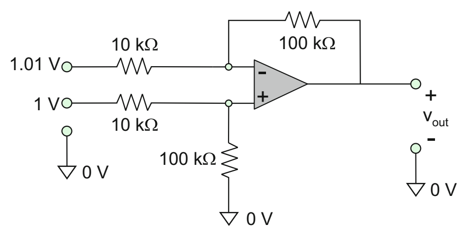


Fig. 5.30. Difference amplifier for Example 5.10.

To minimize the effect of bias currents (Section 5.3), we should choose

$$R_1 = R_3, \quad R_2 = R_4 \tag{5.54}$$

5.4.3 Application Example: Instrumentation Amplifier

Motivation for an Instrumentation Amplifier

Figure 5.31 shows a 700-Ω uniaxial strain gauge. The strain gauge is attached to an aluminum slab. Figure 5.31 also shows a Wheatstone bridge intended for strain measurement with the present device. The dual-polarity supply voltage is ±7.5 V. You may build the Wheatstone bridge according to Fig. 5.31a and connect it to a DMM, with the DMM leads attached to terminals *a* and *b*, respectively. In this configuration, the DMM measures the differential voltage v_D. Using the potentiometer, you may balance the bridge, i.e., reduce voltage v_D at no strain to a minimum, which should be within the range $0\ \text{V} \pm 3\ \text{mV}$. The highest DMM resolution should be used. Now, by applying a strong bending force to the slab with two hands, you probably could obtain a maximum voltage change of $\pm 2\ \text{mV}$.

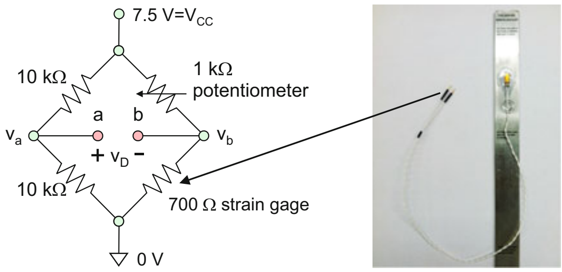


Fig. 5.31. A Wheatstone bridge sensor where the strain gauge forms one of the four resistors.

The positive voltage change corresponds to one bending direction, the negative change, to the opposite direction. Next, you may connect an oscilloscope instead of the DMM and use the DC-coupled settings and the highest voltage resolution of 20 mV per division. You will find that the noisy signal line on the screen hardly changes when you try your best. So are you not strong enough? Well, no. These are the realistic differential voltages for strain gauges which correspond to gauge resistance changes on the order of 1 Ω. Even *smaller* voltage changes are often encountered in practice. Therefore, an accurate amplification of an extremely small differential voltage should be done while rejecting the large common-mode voltage (3.75 V in the present case). This nontrivial task is accomplished by an *instrumentation amplifier*.

How to Build an Instrumentation Amplifier?

The initial guess is probably to use the difference amplifier from Fig. 5.30. For this amplifier, $R_1 = R_3 = 10\ \text{k}\Omega$, $R_2 = R_4 = 100\ \text{k}\Omega$. However, we encounter two problems. The first one is that the amplifier should not perturb the sensor operation. In other words, the amplifier's input resistance must be large compared to any of the resistances in the Wheatstone bridge. The *differential-mode input resistance* to the amplifier in Fig. 5.30 is $R_{\text{in}} = 2R_1 = 2R_3$ (check problem 5.82 at the end of this chapter). Therefore, we may wish to increase $R_1 = R_3$ by a factor of 10, i.e., choose $R_1 = R_3 = 100\ \text{k}\Omega$. The second problem is the amplifier gain. An overall gain of 1000 is required in Eq. (5.53) in order to obtain appreciable output voltages on the order of $\pm$ 2 V. This gain is too high; prohibitively large resistor values $R_2 = R_4 = 100\ \text{M}\Omega$ would be needed in Fig. 5.30.

Concept of an Instrumentation Amplifier

Thus, the solution for at least one problem is clear: we need to add an extra amplifier stage. One way of doing so is shown in Fig. 5.32. Two non-inverting amplifiers are added to both inputs of the difference amplifier. This design achieves two goals simultaneously. First, it isolates the amplifier circuit from the Wheatstone bridge since the non-inverting amplifiers have an infinite input resistance and do not sink any current from the bridge. Second, it adds the extra gain; in other words, it eases the burden on the difference amplifier in the second stage. The difference amplifier becomes mainly responsible for rejecting the common-mode signal and the amplification of the differential signal.

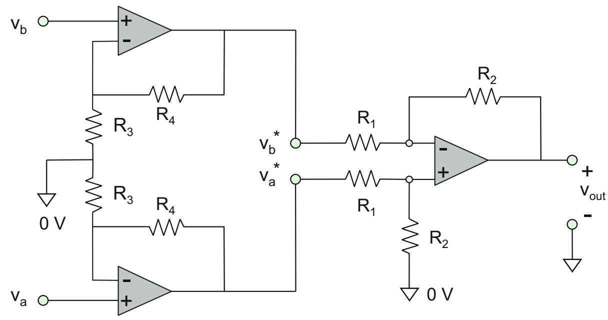


Fig. 5.32. An important step toward the instrumentation amplifier: we add non-inverting amplification stages at the input.

However, another problem arises there. The two non-inverting amplifiers amplify voltages υ_a, υ_b (close to 3.75 V in the present case). Therefore, at any appreciable gain (say, $A_{CL} = 1 + R_4/R_3 = 10$), they simply saturate and will not function! To avoid this issue, we use a simple yet critical change shown in Fig. 5.33 where we remove the common-port connection from the non-inverting stage. The circuit in Fig. 5.33 behaves completely differently compared to the original circuit in Fig. 5.32. We no longer have the output voltages $\upsilon_a^*, \upsilon_b^*$ given by

$$\upsilon_a{}^* = A_{CL}\upsilon_a, \quad \upsilon_a{}^* = A_{CL}\upsilon_a, \quad A_{CL} = 1 + \frac{R_4}{R_3} \tag{5.55}$$

Instead, those voltages now become

$$\upsilon_a{}^* = \upsilon_a + \frac{R_4}{2R_3}(\upsilon_a - \upsilon_b), \quad \upsilon_b{}^* = \upsilon_b - \frac{R_4}{2R_3}(\upsilon_a - \upsilon_b) \tag{5.56}$$

The details of the derivation are seen in Fig. 5.33. The currents and voltages labeled in this figure are obtained using two summing-point constraints. The key observation is that the absolute voltages υ_a, υ_b are no longer amplified but are simply passed through. Only the differential voltage $\upsilon_D = \upsilon_a - \upsilon_b$ is amplified. The circuit in Fig. 5.33 is also a "difference amplifier," and it may be called the *unity common-mode gain stage*. The final step in the construction of the *instrumentation amplifier* is to connect both stages together. Figure 5.34 gives the final circuit that can be employed in conjunction with the Wheatstone bridge for the strain gauge shown in Fig. 5.31. Here, a quad op-amp chip (LM148 series) is used; it has four individual amplifiers inside the chip. For our circuit, we need three of them. The circuit is powered by a $\pm 7.5 - \text{V}$ dual supply. According to Eqs. (5.53) and (5.56), the overall (differential) gain in Fig. 5.34 becomes

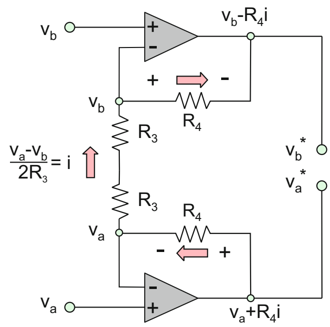


Fig. 5.33. Next step toward the instrumentation amplifier: we convert the non-inverting stage to a unity common-mode gain amplifier.

$$v_{\text{out}} = \frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}\right)(v_{\text{a}} - v_{\text{b}}) = \frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}\right)v_{\text{D}}, \quad A_{\text{CL}} = \frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}\right) \tag{5.57}$$

The overall common-mode gain is exactly zero (ideal resistances). Choosing resistance values from Fig. 5.34 gives us an overall differential gain of 1010. We retain a certain gain (10) of the differential stage in order to have the gain of no more than 100 per stage.

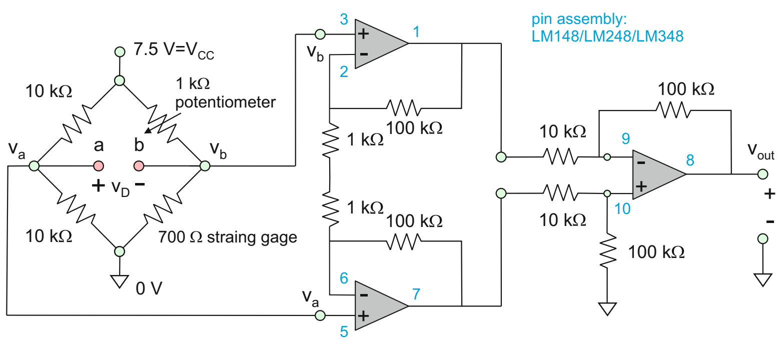


Fig. 5.34. The complete instrumentation amplifier for the strain gauge testing.

5.4.4 Instrumentation Amplifier in Laboratory

The operation of the circuit in Fig. 5.34 is shown in Fig. 5.35. The oscilloscope resolution is 1 V per division. At no applied strain, the oscilloscope connected to the amplifier output (the ground terminal of the oscilloscope is connected to circuit ground) shows a relatively small output voltage signal. When a bending moment is applied, as shown in Fig. 5.35a, the output voltage rises to approximately 2 V. This voltage is sufficient to light up a yellow LED (light-emitting diode) indicator connected between the output port and common port. When the opposite bending moment is applied, as in Fig. 5.35b, the output voltage drops to approximately –2 V. This voltage, taken with a negative sign, is again sufficient to light up a red LED connected from the common port to the output port.

The states in Fig. 5.35 have been achieved by a proper tuning of the potentiometer in the Wheatstone bridge. Thus, we have built a simple, yet useful, uncalibrated, uniaxial, stress-monitoring system. Frequently, the output of an instrumentation amplifier is connected to an analog-to-digital converter (ADC) and then to a computer system.

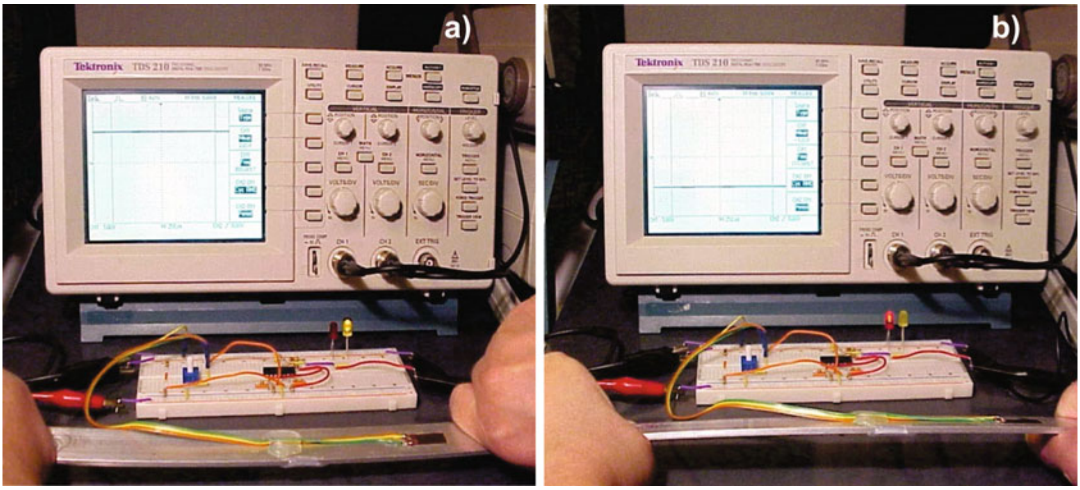


Fig. 5.35. Operation of the instrumentation amplifier with the strain gauge attached to a metal slab. (**a**) A "positive" bending moment is applied and (**b**) a "negative" bending moment is applied. The oscilloscope resolution is 1 V per division in every case.

Load Cell and Other Uses

The circuit in Fig. 5.34 gives us the idea of a commercial *load cell*. Strain gauges are commercially available in prefabricated modules such as load cells that measure force, tension, compression, and torque. All four resistors of the Wheatstone bridge may be strain gauges. Load cells typically use a full-bridge configuration and contain four leads for bridge excitation and measurement. The manufacturers provide calibration and accuracy information. However, the load cells do *not* normally include the instrumentation amplifier itself. Another mechanical engineering example where the differential amplifier is quite useful is a thermocouple. When measuring a thermocouple in a noisy environment, the noise from the environment appears as an offset on both input leads, making it a common-mode voltage signal. Many other examples indeed exist, particularly in biomedical engineering. The instrumentation amplifier is used to amplify an output signal from virtually any analog differential sensor instrument. Also note that instrumentation amplifiers with precision resistors are available as separate integrated circuits. Those ICs have a much better performance than an instrumentation amplifier wired on the protoboard. Other instrumentation amplifier types exist, which are different from the topology of the instrumentation amplifier circuit in Fig. 5.34. In principle, it is possible to design an instrumentation amplifier circuit with only *two* amplifier gain stages. The summary to this chapter provides an example used in practice.

Section 5.5 General Feedback Systems

5.5.1 Signal-Flow Diagram of a Feedback System

Although the negative feedback was first quantified by electrical engineers, it is extensively observed, studied, and employed in many mechanical, biomedical, chemical, and other systems. Figure 5.36 shows a generic structure of a *linear feedback system*.

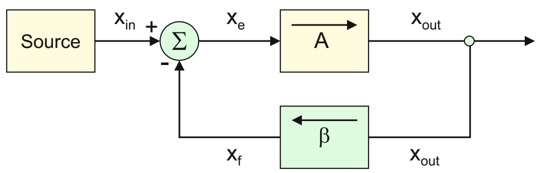


Fig. 5.36. A negative feedback loop for an arbitrary system (signal-flow diagram).

Figure 5.36 is a simplified *signal-flow diagram* or *controls block diagram*. The variable x is a *signal*; it may be voltage, current (electrical engineering) or displacement, velocity (mechanical engineering), etc. These three blocks have special names in control theory:

1. The first block (basic voltage amplifier in terms of ECE) is called the *forward* or *open-loop gain* A (often denoted by A_{OL}). It operates according to the ideal-amplifier rule

$$x_o = Ax_{\mathrm{e}}, \quad A = \mathrm{const} > 0 \tag{5.58}$$

2. The second block is called the *feedback gain*. It operates according to a linear rule

$$x_f = \beta x_{\mathrm{out}}, \quad \beta = \mathrm{const} > 0 \tag{5.59}$$

where β is called the *feedback factor*.

3. The block where the input signal and feedback signal are compared and subtracted from one another is called the *summing* (or *difference*) *node*.

5.5.2 Closed-Loop Gain and Error Signal

Mathematically, from Eq.(5.59), one has

$$x_{\mathrm{e}} = x_{\mathrm{in}} - x_f = x_{\mathrm{in}} - \beta x_{\mathrm{out}} \tag{5.60}$$

According to Eqs. (5.58) and (5.60), we obtain

$$x_{\mathrm{out}} = Ax_{\mathrm{e}} = A(x_{\mathrm{in}} - \beta x_{\mathrm{out}}) \tag{5.61}$$

Solving for x_{out} gives us the *closed-loop gain* A_{CL} (sometimes denoted by G or A_f)

$$x_{\text{out}} = \frac{A}{1 + A\beta} x_{\text{in}} \Rightarrow A_{\text{CL}} = \frac{A}{1 + A\beta} \tag{5.62}$$

If the *open-loop* gain A is made arbitrarily large (ideally approaching ∞), then the *closed-loop* gain approaches

$$A_{\text{CL}} \approx \frac{1}{\beta} \tag{5.63}$$

The significance of Eq. (5.63) cannot be overstated. It means that as long as the open-loop gain A is large enough, the closed-loop gain A_{CL} will approach the constant value $1/\beta$, which is precisely controlled by an external passive feedback network. In other words, manufacturing uncertainties in A and the potential nonlinear behavior of A are eliminated since A itself is eliminated. The price for this operation is a significant reduction of the overall system gain. Equation (5.63) implies that

$$A\beta >> 1 \Rightarrow A_{CL} = \frac{1}{\beta} << A \tag{5.64}$$

Clearly, the feedback loop reduces the initial gain substantially. And yet, despite this drawback, the closed-loop gain A_{CL} may still be large enough and sufficient for amplification as long as A is made very large. Thus, the feedback loop in Fig. 5.36 is a simple and powerful means to control the operation of an arbitrary high-gain system.

Exercise 5.9: The open-loop gain A in Fig. 5.36 varies between two extreme values of $A = 1000 \pm 200$ ($\pm 20\,\%$ gain variation) depending on the system parameters. The forward gain block is used in the closed-loop configuration with the feedback factor β of 0.1. Approximate the two extreme values of the closed-loop gain, A_{CL}.

Answer: $A_{\text{CL}} = A/(1 + A\beta) = 9.90 \pm 0.02$ or $\pm 0.2\%$ closed-loop gain variation.

Error Signal

The second question of interest is finding the *error signal*, x_e, in Fig. 5.36, which corresponds to the differential input voltage for an amplifier circuit with the negative feedback. Substitution of Eq. (5.62) into Eq. (5.60) yields

$$x_e = x_{\text{in}} - \beta \frac{A}{1 + A\beta} x_{\text{in}} = \frac{1}{1 + A\beta} x_{\text{in}} \tag{5.65}$$

When the open-loop gain A is large and furthermore $A\beta >> 1$, one obtains

$$x_e \approx 0 \tag{5.66}$$

As applied to the amplifier circuits, Eq. (5.66) is exactly the second summing-point constraint or the condition of the zero differential input amplifier voltage under presence of the negative feedback.

Exercise 5.10: The open-loop gain A in Fig. 5.36 is 10,000. The forward gain block is used in the closed-loop configuration with the feedback factor β of 0.1. Determine the error signal x_e if the input voltage signal is 1 mV.

Answer: $x_e = 0.999\ \mu V \approx 1\ \mu V$.

5.5.3 Application of General Theory to Voltage Amplifiers with Negative Feedback

Two circuits shown in Fig. 5.37 are the buffer amplifier circuit and the non-inverting amplifier circuit, respectively. The goal is to find the closed-loop gain A_{CL} of the amplifier circuit when the given open-circuit voltage gain A is large but finite. In this case, the second summing-point constraint cannot be applied. Therefore, simple Eqs. (5.16) and (5.21) obtained previously need to be modified.

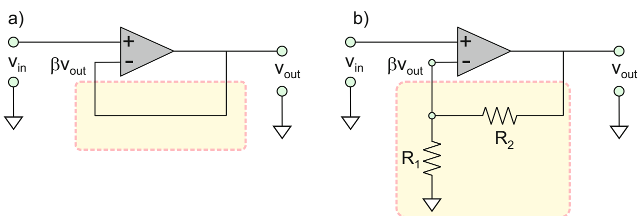


Fig. 5.37. Two amplifier circuits with negative feedback networks indicated by a shaded rectangle.

Both circuits from Fig. 5.37 have the form of the feedback system as in Fig. 5.36. The feedback network is indicated by a shaded rectangle. The signal x is now the voltage. Since A is given, the only problem is to find the feedback factor, β. For the buffer amplifier, the feedback factor is clearly one. For the non-inverting amplifier circuit, the feedback loop is the voltage divider, where β is determined by the resistance ratio. Note that the voltage divider model implies no current into amplifier's input terminals. Specifically, βv_{out} is equal to $R_1/(R_1 + R_2)v_{out}$. Therefore,

$$\beta = 1 \ - \text{ buffer ampl. circuit;} \quad \beta = \frac{R_1}{R_1 + R_2} - \text{ non-inv. ampl. circuit} \tag{5.67}$$

Substitution into Eq. (5.62) gives us two expressions for the closed-loop gain:

$$A_{CL} = \frac{A}{1+A} \ - \text{ buffer ampl. circuit;} \quad A_{CL} = \frac{A}{1+A\frac{R_1}{R_1+R_2}} - \text{ non-inv. ampl. circuit} \tag{5.68}$$

The first equation (5.68) coincides with Eq. (5.23) obtained in Section 5.2 using the accurate circuit analysis. So does second equation (5.68) when we repeat the same analysis for the non-inverting amplifier configuration. If $A \to \infty$, then the simple gain expressions—Eqs. (5.16) and (5.21)—derived with the help of the second summing-point constraint are obtained from Eqs. (5.68). The analysis of the inverting amplifier requires more efforts since this amplifier type is not exactly the voltage amplifier but rather a transresistance amplifier considered next.

Exercise 5.11: The open-loop (open-circuit) gain A of a non-inverting amplifier circuit with $R_1 = 1\,\mathrm{k}\Omega, R_2 = 9\,\mathrm{k}\Omega$ is 10,000. Determine the closed-loop gain.

Answer: $A_{\mathrm{CL}} = 9.99$, which is by 0.1 % different from $A_{\mathrm{CL}} = 1 + \frac{R_2}{R_1} = 10$.

Last but not least, we emphasize another significant advantage of the negative feedback. When the input and output resistances of the amplifier model in Fig. 5.5 have finite values (which occurs in practice), the negative feedback loop effectively *increases* the input resistance and *decreases* the output resistance, i.e., makes the entire amplifier circuit look closer to the ideal-amplifier model.

5.5.4 Voltage, Current, Transresistance, and Transconductance Amplifiers with the Negative Feedback

At the end of this short section, we consider four basic amplifier circuits with negative feedback, which correspond to the four basic dependent sources studied in Chapter 2: *voltage amplifier*, *transconductance amplifier*, *transresistance amplifier*, and *current amplifier*. Figure 5.38 shows the corresponding circuit diagrams. Load resistance R_{L} is introduced for the transconductance amplifier and the current amplifier, respectively, where the output is the load current. Although every amplifier circuit may be represented in the form similar to the feedback diagram in Fig. 5.36 and analyzed accordingly, only a simplified treatment will be given here. It utilizes the condition $A \to \infty$ and the resulting second summing-point constraint. Using the both summing-point constraints, we may obtain, with reference to Fig. 5.38,

$$v_{\mathrm{out}} = \left(1 + \frac{R_2}{R_1}\right) v_{\mathrm{in}} \qquad \text{voltage amplifier} \tag{5.69a}$$

$$i_{\mathrm{out}} = G_{\mathrm{F}} v_{\mathrm{in}}, \quad G_{\mathrm{F}} = 1/R_{\mathrm{F}} \qquad \text{transconductance amplifier} \tag{5.69b}$$

$$v_{\mathrm{out}} = -R_{\mathrm{F}} i_{\mathrm{in}} \qquad \text{transresistance amplifier} \tag{5.69c}$$

$$i_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right) i_{\text{in}} \qquad \text{current amplifier} \tag{5.69d}$$

Note that other more elaborate circuits may be considered; some of them are analyzed in the corresponding homework problems.

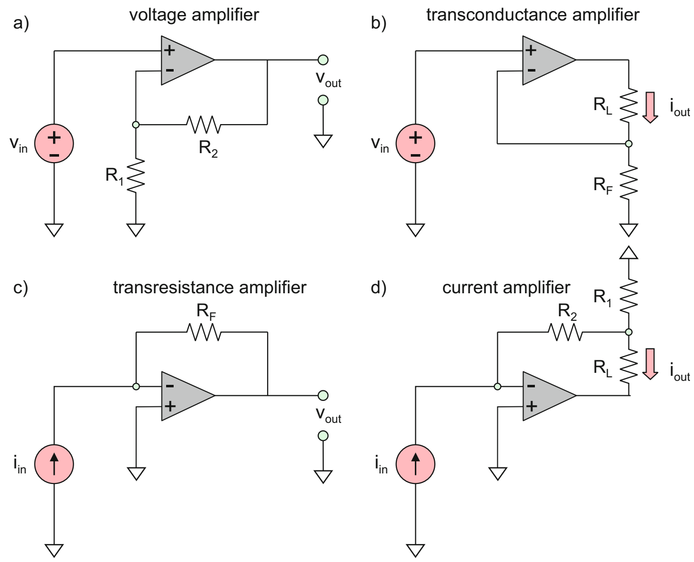
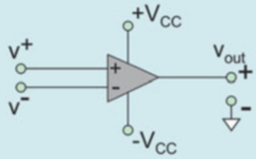
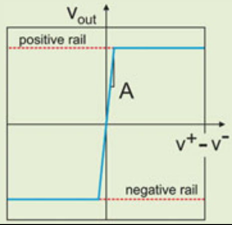
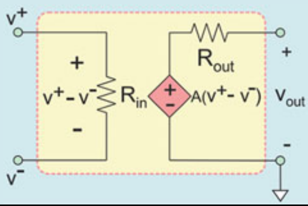
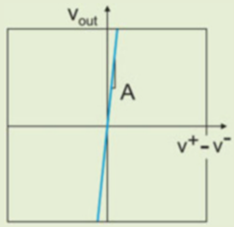
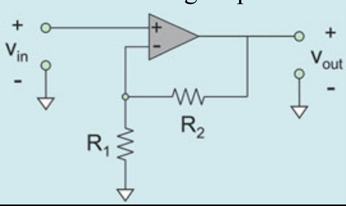
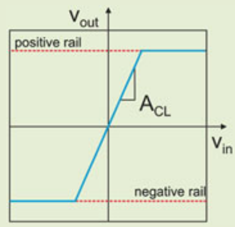
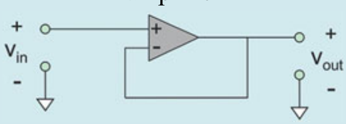
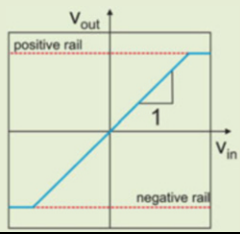
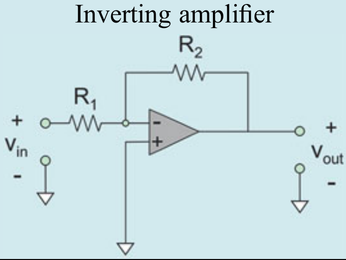
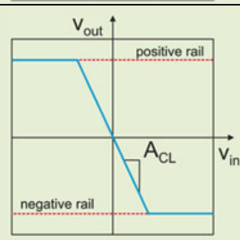
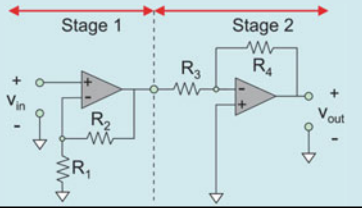
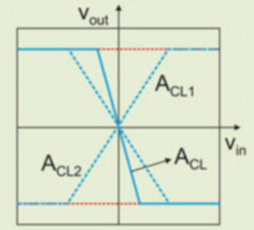
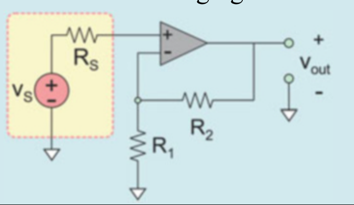
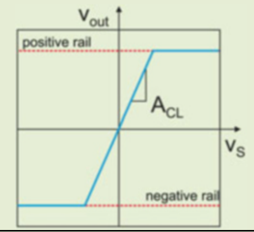
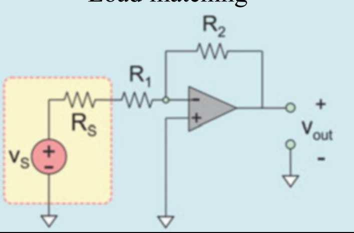
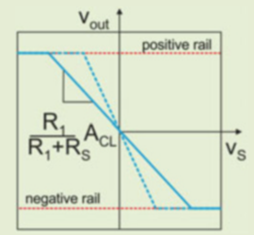
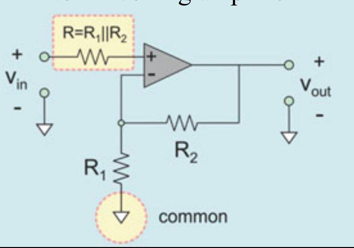
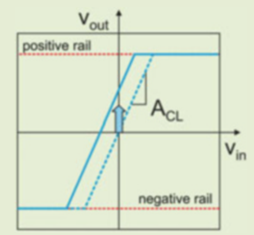
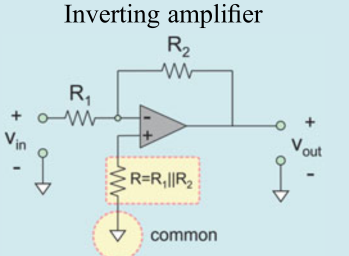
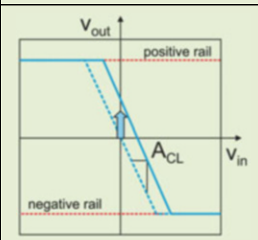


Fig. 5.38. Four basic amplifier types with negative feedback.

Summary

Amplifier circuit	Operation	Formulas
Open-loop operational amplifier (comparator) 		Operation with $\pm V_{CC}$ power rails: $v_{out} = A(v^+ - v^-), \quad \lvert v_{out}\rvert < V_{CC}$ Open-circuit (open-loop) gain A is very high
Amplifier circuit model 		– Valid for *any* voltage amplifier or voltage amplifier circuit, but R_{in}, R_{out}, A are different in every case; – Ideal amplifier model (useful simplification): $R_{in} = \infty$, $R_{out} = 0$, $A = \infty$
Negative feedback for the ideal-amplifier model: differential input voltage is zero (2nd SPC)		
Non-inverting amplifier 		For ideal-amplifier model: $v_{out} = A_{CL} v_{in}, \quad A_{CL} = 1 + \dfrac{R_2}{R_1}$ $R_{in} = \infty$, $R_{out} = 0$ Exact: $A_{CL} = A\left(1 + A\dfrac{R_1}{R_1 + R_2}\right)^{-1}$
Voltage follower (buffer) amplifier 		For ideal-amplifier model: $v_{out} = A_{CL} v_{in}, \quad A_{CL} = 1$ $R_{in} = \infty$, $R_{out} = 0$ Exact: $A_{CL} = \dfrac{A}{1 + A}$
Inverting amplifier 		For ideal-amplifier model: $v_{out} = A_{CL} v_{in}, \quad A_{CL} = -\dfrac{R_2}{R_1}$ $R_{in} = R_1$, $R_{out} = 0$ Exact: $A_{CL} = -\dfrac{R_2}{R_1} A \left(A + 1 + \dfrac{R_2}{R_1}\right)^{-1}$

(continued)

Cascaded amplifier 		– Gains of individual stages multiply; – Input resistance of the amplifier circuit is the input resistance of stage 1; – Individual stage gain should not exceed 100
Input load bridging versus input load matching		
Load bridging 		– Source (sensor) sees the amplifier as an open circuit: $$v_{\text{out}} = v_S \times A_{CL}$$ – No current from the source can flow into amplifier circuit
Load matching 		– Source (sensor) sees the amplifier as resistance $R_{\text{in}} = R_1$: $$v_{\text{out}} = \frac{R_{\text{in}}}{R_S + R_{\text{in}}} v_S \times A_{CL}$$ – Matching condition $R_S = R_{\text{in}}$ is important for high-freq. circuits
DC imperfections and their cancellation		
Non-inverting amplifier 		– Short-circuited output voltage may be trimmed to zero using the offset null terminal – Short-circuited output voltage may be trimmed to zero by adjusting common-terminal voltage; – Extra resistance R eliminates the effect of the input bias current
Inverting amplifier 		The same as above

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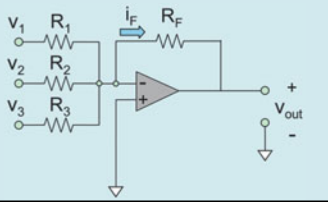
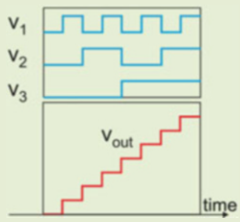
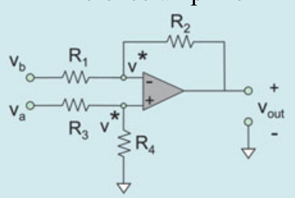
<table>
<tr><th colspan="3">Multiple-input amplifier circuits</th></tr>
<tr>
<td>Summing amplifier

</td>
<td>
</td>
<td>– The summing amplifier sums several weighted input voltages:
$$v_{\text{out}} = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_2}v_2 - \frac{R_F}{R_3}v_3$$
– Used as a prototype of the digital to analog converter</td>
</tr>
<tr>
<td>Difference amplifier

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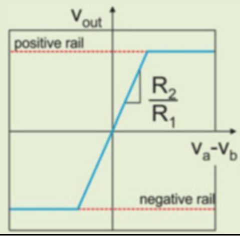
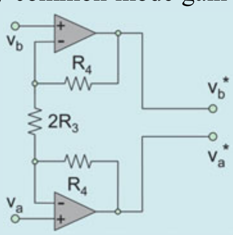
<td>
</td>
<td>– True difference amplifier:
$$\frac{R_2}{R_1} = \frac{R_4}{R_3},\ v_{\text{out}} = \frac{R_2}{R_1}(v_a - v_b)$$
– Rejects common-mode voltage
– For the general difference amplifier circuit see Eq. (5.52c)</td>
</tr>
<tr>
<td>Unity common-mode gain stage

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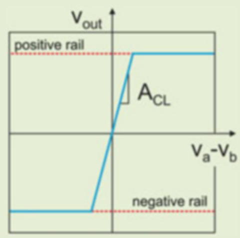
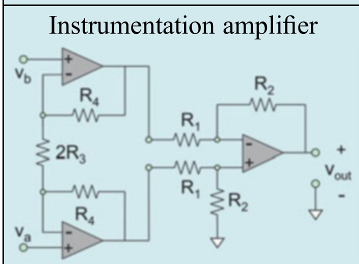
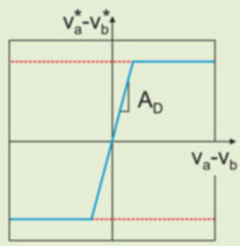
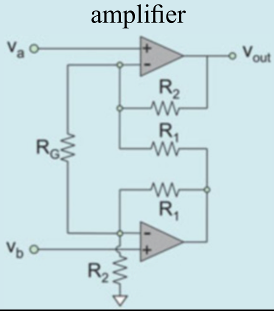
<td>
</td>
<td>– Differential gain:
$$v_a{}^* - v_b{}^* = A_D(v_a - v_b)$$
$$A_D = 1 + \frac{R_4}{R_3}$$
– Common-mode gain:
$$v_a{}^* + v_b{}^* = A_{CM}(v_a + v_b)$$
$$A_{CM} = 1$$</td>
</tr>
<tr>
<td>Instrumentation amplifier

</td>
<td>
</td>
<td>Output voltage:
$$v_{\text{out}} = \frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3}\right)(v_a - v_b)$$
Closed-loop differential gain:
$$A_{CL} = \frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3}\right)$$
– Rejects common-mode voltage</td>
</tr>
<tr>
<td>Simpler instrumentation amplifier

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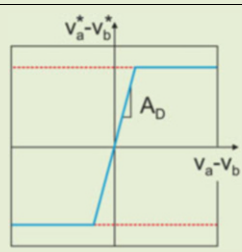
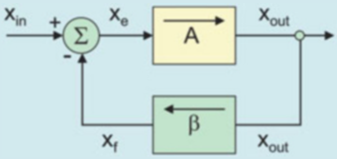
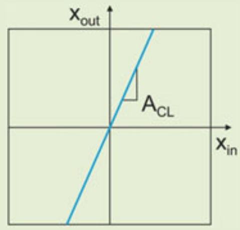
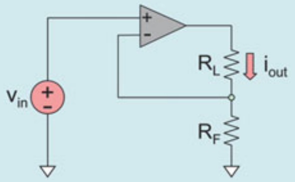
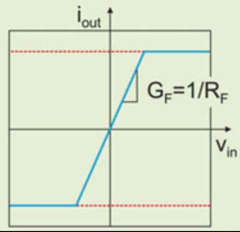
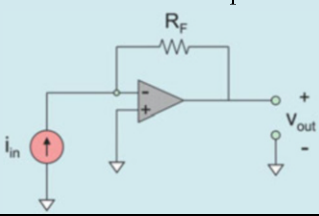
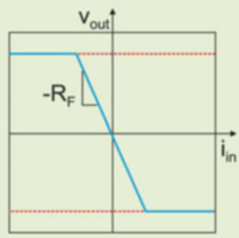
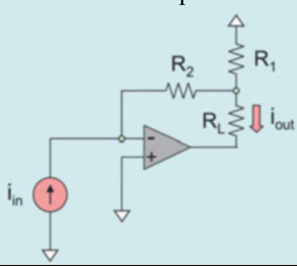
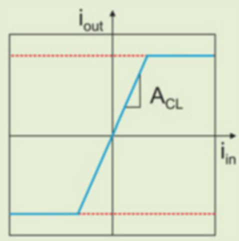
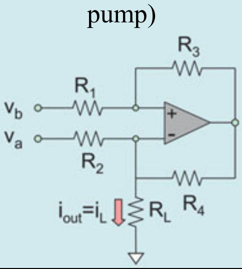
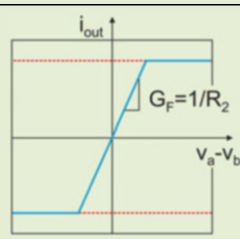
<td>
</td>
<td>Output voltage:
$$v_{\text{out}} = \left(2\frac{R_2}{R_G} + \left(1 + \frac{R_2}{R_1}\right)\right)(v_a - v_b)$$
– Closed-loop differential gain:
$$A_{CL} = 2\frac{R_2}{R_G} + \left(1 + \frac{R_2}{R_1}\right)$$
– Rejects common-mode voltage</td>
</tr>
</table>

(continued)

General feedback systems and amplifiers with negative feedback		
Signal—flow diagram 		Closed-loop gain: $x_{\text{out}} = A_{\text{CL}} x_{\text{in}},\ A_{\text{CL}} = \frac{A}{1 + A\beta} \approx \frac{1}{\beta}$ Error signal: $x_e = \frac{1}{1 + A\beta} x_{\text{in}} \approx 0$
Transconductance amplifier 		Closed-loop operation: $i_{\text{out}} = G_{\text{F}} v_{\text{in}},\ G_{\text{F}} = 1/R_{\text{F}}$ Variations of this simple circuit are possible
Transresistance amplifier 		Closed-loop operation: $v_{\text{out}} = -R_{\text{F}} i_{\text{in}}$ Variations of this simple circuit are possible
Current amplifier 		Closed-loop operation: $i_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right) i_{\text{in}}$ Variations of this simple circuit are possible
Howland amplifier (current pump) 		Closed-loop operation: $(R_1/R_3 = R_2/R_4)$ $i_{\text{out}} = G_{\text{F}}(v_a - v_b),\ G_{\text{F}} = 1/R_2$ Variations of this clever circuit are possible

Problems

5.1 Amplifier operation and circuit models

5.1.1 Amplifier Operation

Problem 5.1. An operational amplifier has five terminals.

A. Sketch the amplifier symbol.
B. Name each of the op-amp terminals and describe its function in one sentence per terminal.
C. Can the amplifier IC have more than five terminals? Explain.

Problem 5.2. You may wonder about the meaning of the two letters preceding amplifier marking, e.g., LM741. Each of the semiconductor companies has its own abbreviation, e.g., LM for an amplifier designed and manufactured by the National Semiconductor Corporation (acquired by Texas Instruments in 2011), AD for an amplifier manufactured by Analog Devices, MC for STMicroelectronics, TL for Texas Instruments, etc. The same chip, e.g., LM741, may be manufactured by several semiconductor chip makers. The part number is given by a numerical code that is imprinted on the top of the package. An MC1458 amplifier IC chip is shown in the figure below. This IC is a *dual operational amplifier*. In other words, one such IC package contains two separate operational amplifiers.

N
DIP8
(Plastic Package)

A. Download the amplifier's datasheet from http://www.datasheetcatalog.com
B. Redraw the figure to this problem in your notes and label the pins for the non-inverting input, the inverting input, and the output of the operational amplifier #1.
C. Label pins for $+V_{CC}$ and $-V_{CC}$.

Problem 5.3. What is the minimum number of pins required for:

A. The dual operational amplifier (the corresponding IC package contains two separate operational amplifiers)?
B. The *quad operational amplifier* (the corresponding IC package contains four separate operational amplifiers)?

Problem 5.4. An operational amplifier has an open-circuit gain of $A = 2 \times 10^5$ and is powered by a dual source of ± 10 V. It is operated in the open-circuit configuration. What is the amplifier's open-circuit output voltage υ_{out} if

A. $\upsilon^+ = 0\text{V},\ \ \upsilon^- = 0\text{V}$
B. $\upsilon^+ = +1\text{V},\ \ \upsilon^- = +1\text{V}$
C. $\upsilon^+ = +1\text{V},\ \ \upsilon^- = 0\text{V}$
D. $\upsilon^+ = 0\text{V},\ \ \upsilon^- = -1\text{V}$
E. $\upsilon^+ = +1\text{mV},\ \ \upsilon^- = 0\text{V}$
F. $\upsilon^+ = -1\text{mV},\ \ \upsilon^- = 0\text{V}$
G. $\upsilon^+ = 10\mu\text{V},\ \ \upsilon^- = 0\text{V}$
H. $\upsilon^+ = 0\text{V},\ \ \upsilon^- = 10\mu\text{V}$

Problem 5.5. Based on the solution to Problem 5.4, why do you think the operational amplifier is seldom used in the open-loop configuration, at least in analog electronics?

Problem 5.6. Using the website of the National Semiconductor Corporation, determine the maximum and minimum supply voltages (operating with the dual-polarity power supply) for the following amplifier's ICs:

A. LM358
B. LM1458
C. LM741

Which amplifier IC from the list may be powered by two AAA batteries?

Problem 5.7. Plot to scale the output voltage of the operation amplifier with an open-circuit gain $A = 5 \times 10^4$ when the non-inverting input voltage υ^+ changes from -2 mV to +2 mV and the inverting input voltage υ^- is equal to -1 mV. The amplifier is powered by a ± 12-V dual voltage supply. Label the axes.

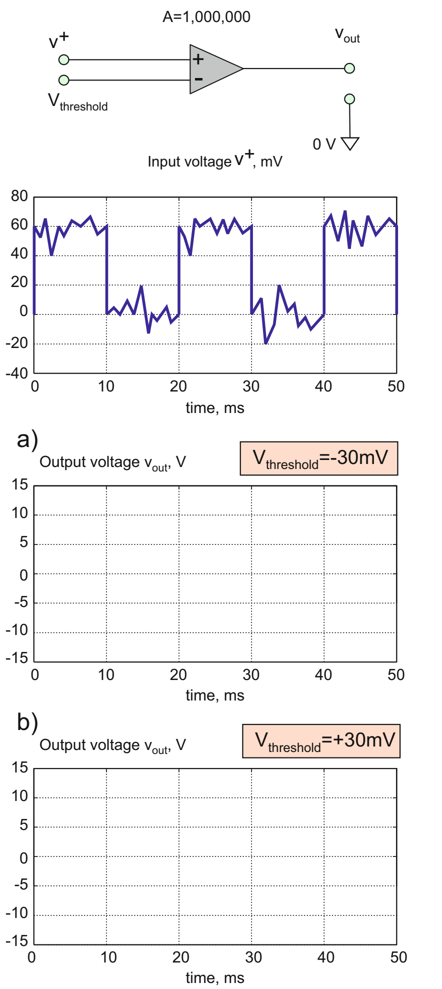
Problem 5.8. Repeat the previous problem for $A = 5 \times 10^5$.

5.1.2 Operational Amplifier Comparator

Problem 5.9. In a circuit shown in the figure below, an operational amplifier is driven by a ±10-V dual power supply (not shown). The open-circuit DC gain of the amplifier is $A = 1,000,000$. Sketch to scale the output voltage to the amplifier when

a) $V_{\text{threshold}} = -30$ mV
b) $V_{\text{threshold}} = +30$ mV

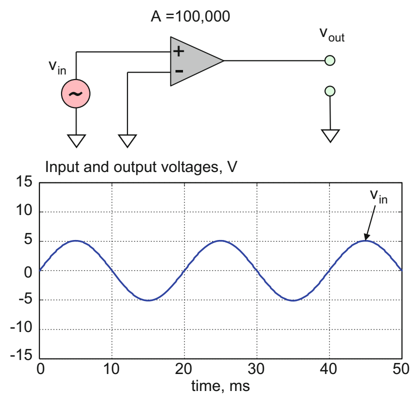
Assume that the amplifier hits the power rails in saturation.



Problem 5.10. Based on the solution to the previous problem, why do you think the operational amplifier in the open-loop configuration may be useful for digital circuits?

Problem 5.11. Solve Problem 5.9 when the input voltage is applied to the inverting input and the threshold voltage is applied to the non-inverting input.

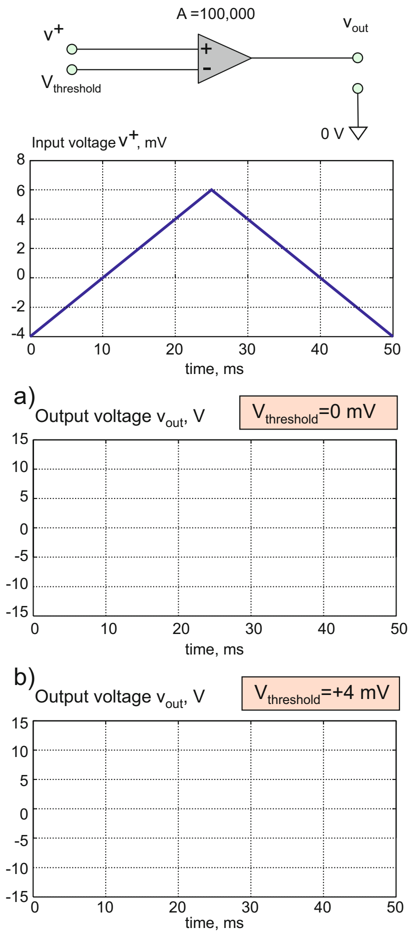
Problem 5.12. The circuit shown in the figure is a *zero-level detector*. An operational amplifier in the open-loop configuration is driven by a ±10-V dual power supply (not shown). The open-circuit amplifier gain is 100,000. Sketch the output voltage to scale. Assume that the amplifier hits the power rails in saturation.



Problem 5.13. In a circuit shown in the figure below, an operational amplifier is driven by a ±15-V dual power supply (not shown). The open-circuit gain of the amplifier is $A = 100,000$. Sketch to scale the output voltage to the amplifier when

a) $V_{\text{threshold}} = 0$ mV
b) $V_{\text{threshold}} = +4$ mV

Assume that the amplifier hits the power rails in saturation.



5.1.3 Amplifier Circuit Model
5.1.4 Ideal-Amplifier Model

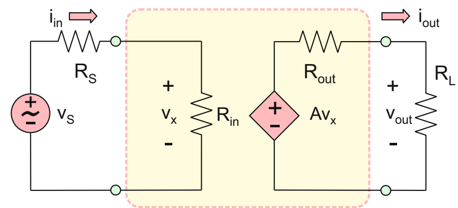
Problem 5.14

A. Draw the circuit model of an operational amplifier.

B. Describe the meaning of the amplifier as the voltage-controlled voltage source in your own words.

Problem 5.15. For an equivalent amplifier circuit with $A = 1500$ shown in the figure below, determine the output voltage given that $v_S(t) = 1\cos\omega t$ [mV], $R_S = 50\ \Omega$, $R_L = 50\ \Omega$ for three cases:

A. $R_{in} = 100\ \text{k}\Omega$ and $R_{out} = 2\ \Omega$.
B. $R_{in} = 50\ \Omega$ and $R_{out} = 25\ \Omega$.
C. $R_{in} = \infty$ and $R_{out} = 0$.



Problem 5.16. Name one reason why we should attempt to:

A. Make the input resistance (impedance) to the amplifier as high as possible.

B. Make the output resistance (impedance) to the amplifier as low as possible.

Problem 5.17

A. List all conditions of the ideal-amplifier model.

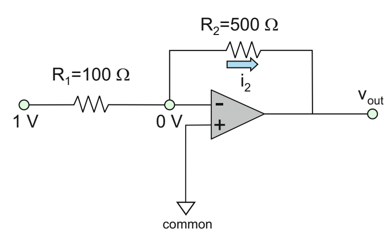
B. What is the short-circuit output current of the ideal amplifier?

Problem 5.18

A. What is the first summing-point constraint?

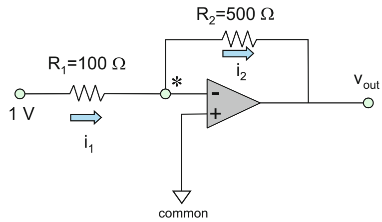
B. What is the equivalent formulation of the first summing-point constraint in terms of the input resistance (impedance) to the amplifier?

Problem 5.19. An amplifier circuit is shown in the figure below. The first summing-point constraint applies. Determine current i_2.



Problem 5.20. An amplifier circuit is shown in the figure below. The first summing-point constraint applies. An ideal operational amplifier has an open-circuit gain of $A = 2\times10^5$.

Determine the output voltage, V_{out}. You are not allowed to use any of the materials of the next section!
Hint: Denote the unknown voltage at node * by v*, express v* in terms of v_{out}, and then solve for v_{out}.



Problem 5.21. An ECE laboratory project uses the LM358 amplifier IC.

A. What semiconductor company has developed this chip?
B. Is the chip from the lab project necessarily manufactured by this company? (See http://www.datasheetcatalog.com/ for manufacturers' datasheets related to this product.)
C. Use the Digi-Key distributor's website and estimate average cost for this amplifier chip (DIP-8 package) in today's market.

Problem 5.22. An ECE laboratory project uses the TL082 amplifier IC.

A. What semiconductor company has developed this chip?
B. Is the chip from the lab project necessarily manufactured by this company? (See http://www.datasheetcatalog.com/ for manufacturers' datasheets related to this product.)
C. Use the Digi-Key distributor's website to estimate the average cost for this amplifier chip (DIP-8 package) in today's market.

5.2 Negative Feedback

5.2.2 Amplifier Feedback Loop. Second Summing-Point Constraint

5.2.3 Amplifier Circuit Analysis Using Two Summing-point Constraints

Problem 5.23

A. Name the two summing-point constraints used to solve an amplifier circuit.
B. Which summing-point constraint remains valid without the negative feedback?

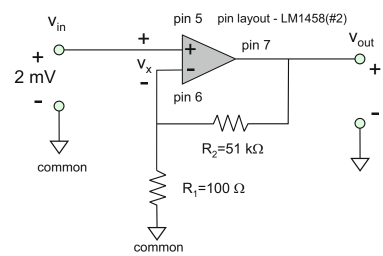
Non-inverting Amplifier

Problem 5.24

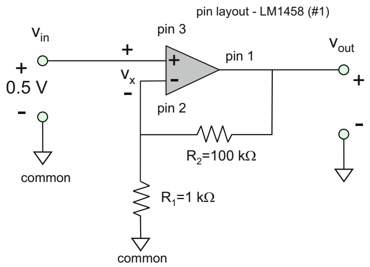
A. Draw the circuit diagram of the basic non-inverting amplifier configuration.
B. Accurately derive the expression for the amplifier gain in terms of the resistances, assuming an ideal operational amplifier.

Problem 5.25. Using the two summing-point constraints, solve the ideal-amplifier circuit shown in the figure if the input voltage has the value of 2 mV.

A. Label and determine the currents in the feedback loop.
B. Determine the output voltage of the amplifier versus the common port.



Problem 5.26. Determine the output voltage of the ideal operational amplifier shown in the figure. The amplifier is driven by a ±10-V dual power supply.



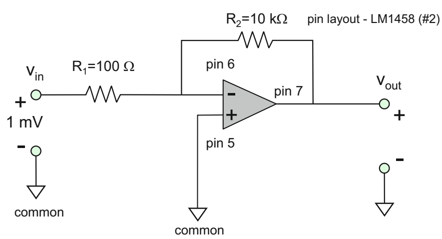
Inverting Amplifier

Problem 5.27

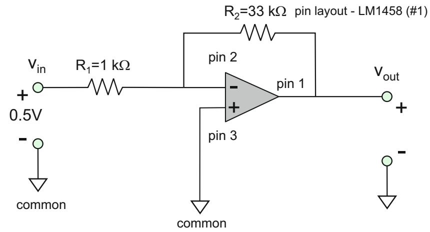
A. Draw the circuit diagram of the basic inverting amplifier configuration.
B. Give the expression for the amplifier gain in terms of the resistances, assuming an ideal operational amplifier.

Problem 5.28. Using the two summing-point constraints, solve the ideal-amplifier circuit shown in the figure that follows if the input voltage is 1 mV.

A. Label and determine the currents in the feedback loop.
B. Determine the output voltage of the amplifier versus the common port.



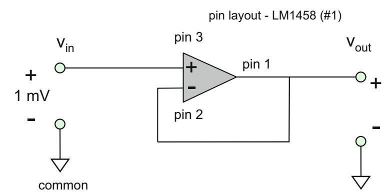
Problem 5.29. Determine the output voltage of the ideal operational amplifier shown in the figure. The amplifier is driven by a ±10-V dual power supply.



Voltage Follower

Problem 5.30

A. Using only the first summing-point constraint (SPC), solve the circuit shown in the figure, i.e., determine the output voltage of the amplifier versus the common port.
B. What function does this amplifier have? Why is it important?



Exercises on the Use of the Negative Feedback

Problem 5.31. (A review problem) For three basic ideal-amplifier circuits:

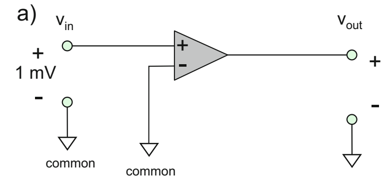
Inverting amplifier
Non-inverting amplifier
Voltage follower

(each includes negative feedback) present

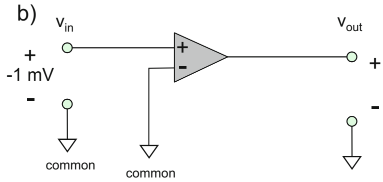
1. A circuit diagram
2. Expression for the amplifier gain

Problem 5.32. Determine the output voltage of amplifier configurations shown in the figure that follows. The amplifier is powered by a ±9-V dual-polarity voltage power supply. Assume an ideal operational amplifier.

a)

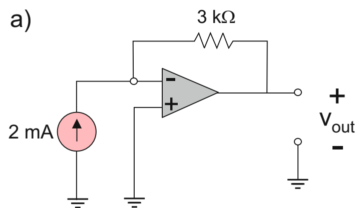


b)

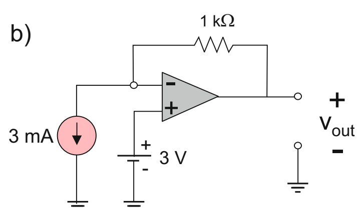


Problem 5.33. Each of the circuits shown in the figures below employs negative feedback. Find the output voltage v_{out} vs. ground (or common). *Hint:* The ground symbol in an amplifier circuit usually has the *same* meaning as the common port.

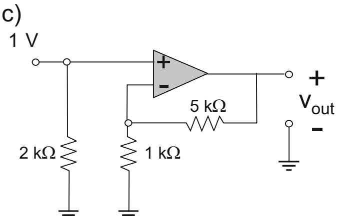
a)



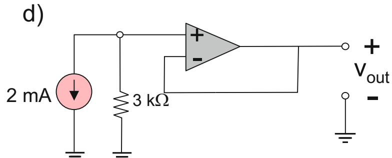
b)



c)

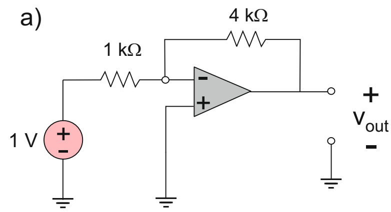


d)

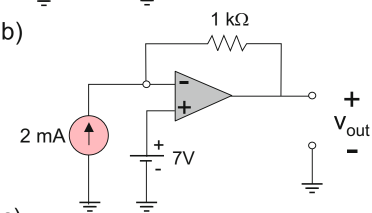


Problem 5.34. Each of the circuits shown in the figures below employs negative feedback. Find the output voltage V_{out} vs. ground (or common). *Hint:* The ground symbol in the amplifier circuit usually has the same meaning as the common port.

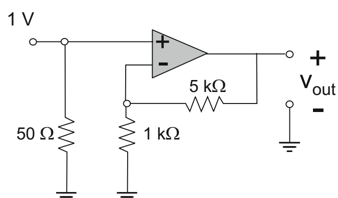
a)



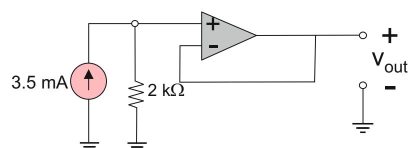
b)



c)

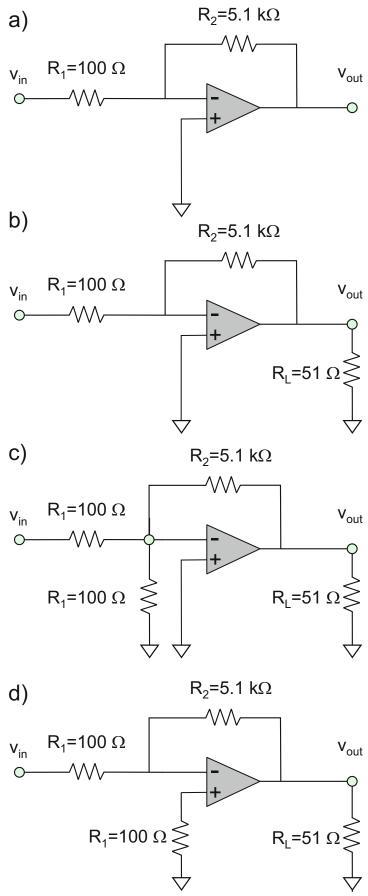


d)

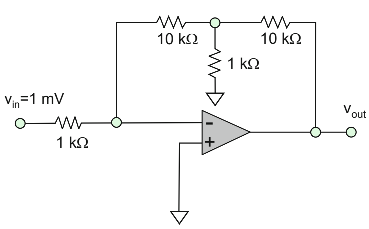


Problem 5.35. Each of the circuits shown in the figures below employs an inverting amplifier.

1. Solve each circuit (find v_{out}) with an input voltage of 1 mV.
2. Based on this solution, find the closed-loop voltage gain A_{CL} of the corresponding amplifier circuit.

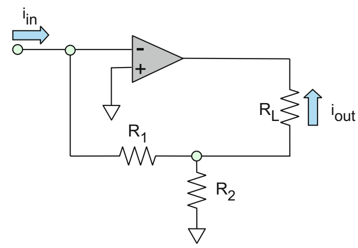


Problem 5.36. An inverting amplifier that achieves high-gain magnitude with a smaller range of resistance values is shown in the figure below. Find its output voltage υ_{out} vs. ground (or common port) and the resulting amplifier gain.



Problem 5.37. The amplifier circuit shown in the figure employs negative feedback.

A. Find the value of the output current i_{out} if the input current is 1 mA, $R_1 = 9\ \text{k}\Omega$, $R_2 = 1\ \text{k}\Omega$.
B. Why do you think this amplifier type is known as the *current amplifier*? To answer this question quantitatively, analytically express the output current i_{out} (current through the load) in terms of the unknown input current i_{out} and two arbitrary resistor values, $R_{1,2}$.



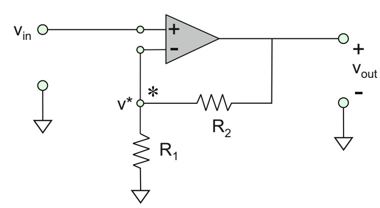
5.2.4 Mathematics Behind the Second Summing-Point Constraint

Problem 5.38

A. Derive an expression for the closed-loop gain of the non-inverting amplifier based only on the definition of the output voltage $\upsilon_{out} = A\left(\upsilon_{in}^{+} - \upsilon_{in}^{-}\right)$, without using the second summing-point constraint.
B. Determine the exact gain value when

$$A = 2 \times 10^5$$

$$R_1 = 1\ \text{k}\Omega, R_2 = 9\ \text{k}\Omega$$



Problem 5.39

A. Derive an expression for the closed-loop gain of the inverting amplifier based only on the definition of the output voltage υ_{out}

$= A\left(v_{in}^{+} - v_{in}^{-}\right)$, without using the second summing-point constraint.

B. Determine the exact gain value when

$$A = 2 \times 10^5$$

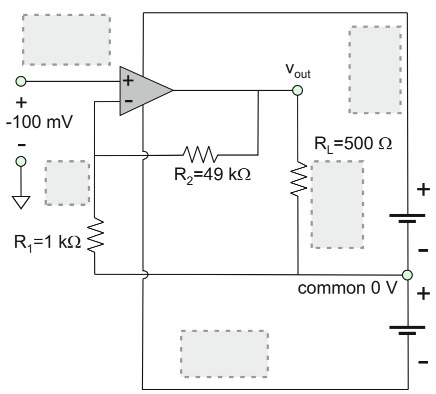
$$R_1 = 1\ \text{k}\Omega, R_2 = 10\ \text{k}\Omega$$

5.2.5 Current Flow in the Amplifier Circuit

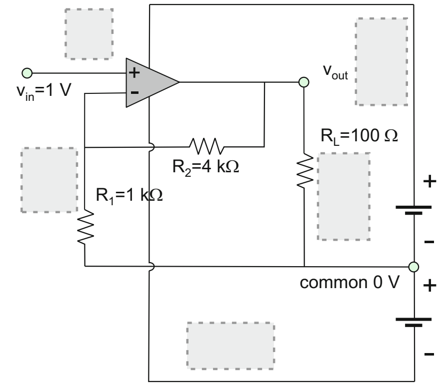
Problem 5.40. The amplifier circuit shown in the figure below is powered by a ±9-V dual-polarity voltage power supply.

A. Redraw the amplifier schematic in your notes.
B. Show the current direction in every wire of the circuit by an arrow and write the corresponding current value close to each arrow.

Hint: Change the polarity of the input voltage and the voltage sign if you have trouble operating with negative values.

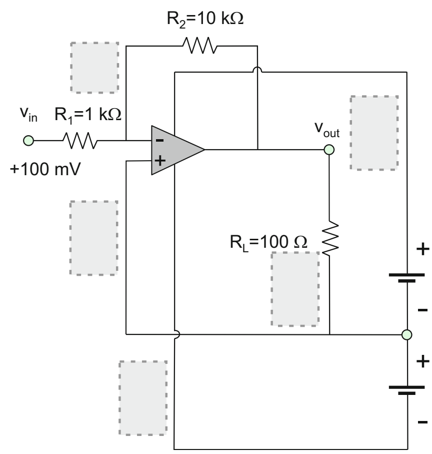


Problem 5.41. Repeat the previous problem for the circuit shown in the figure below.



Problem 5.42. The amplifier shown in the figure below is powered by a ±9-V dual-polarity voltage power supply.

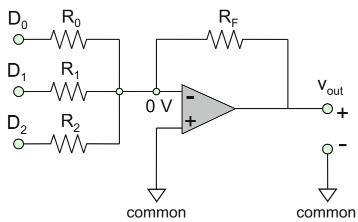
A. Redraw the amplifier schematic in your notes.
B. Show the current direction in every wire of the circuit by an arrow and write the corresponding current value close to each arrow.



Problem 5.43. Repeat the previous problem when the input voltage to the amplifier is -100 mV.

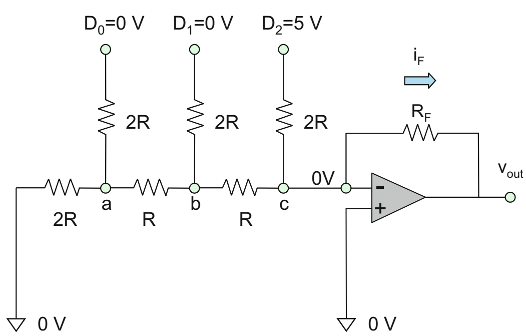
5.2.6 Multiple-Input Amplifier Circuit: Summing Amplifier

Problem 5.44. By solving the amplifier circuit shown in the figure, fill out the table that follows. Assume that $R_0 = R_F$, $R_1 = R_F/2$, and $R_2 = R_F/4$.

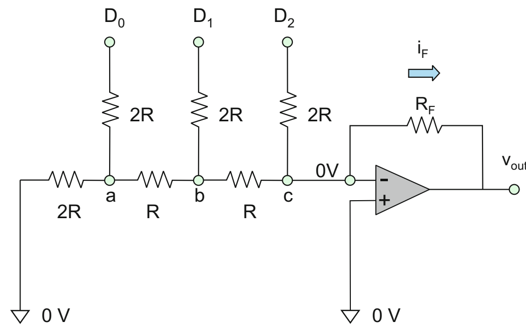


D_2, V	D_1, V	D_0, V	v_{out}, V
0	0	0	
0	0	5	
0	5	0	
0	5	5	
5	0	0	
5	0	5	
5	5	0	
5	5	5	

Problem 5.45. The amplifier circuit shown in the figure below employs negative feedback. This configuration is known as a three-bit *digital-to-analog converter* (DAC) on the base of an R/2R ladder. By solving the amplifier circuit, determine its output voltage in terms of resistances R, R_F, given the input voltages $D_0 = 0\ \text{V}, D_1 = 0\ \text{V},\ D_2 = 5\ \text{V}$.



Problem 5.46. By solving the amplifier circuit shown in the following figure, determine its output voltage in terms of resistances R, R_F, given the input voltages $D_0 = 0\ \text{V}, D_1 = 5\ \text{V}, D_2 = 0\ \text{V}$.



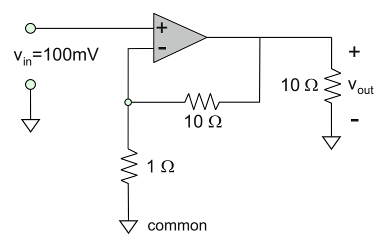
5.3 Amplifier Circuit Design

5.3.1 Choosing Proper Resistance Values

Problem 5.47. State the limitations on the feedback resistances and the output load resistance of an amplifier circuit.

Problem 5.48. The non-inverting amplifier shown in the figure below has been wired in laboratory.

A. Do you have any concerns with regard to this circuit?
B. If you do, draw the corrected circuit diagram.



5.3.2 Model of a Whole Amplifier Circuit

5.3.3 Input Load Bridging or Matching

Problem 5.49. For three basic amplifier circuits

Inverting amplifier
Non-inverting amplifier
Voltage follower

(each includes negative feedback), present

1. A circuit diagram
2. An expression for the closed-loop amplifier circuit gain
3. An expression for the input resistance (impedance)
4. An expression for output resistance (impedance)

Problem 5.50

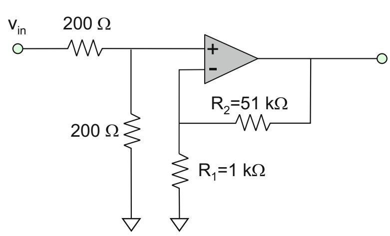
A. Explain in your own words the concept of load bridging (impedance bridging).

B. Which amplifier, the non-inverting or inverting, should be subject to load bridging?

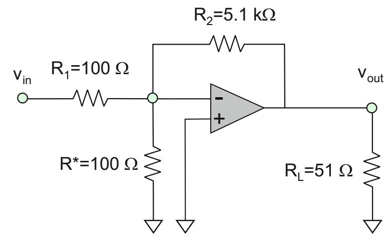
Problem 5.51. An electromechanical sensor is given by its Thévenin equivalent wherein the sensor voltage v_S is small. The sensor's equivalent resistance R_S may vary in time; but it is always less than 1 kΩ. An inverting amplifier is needed that generates an amplified version of the sensor's voltage. The output voltage should be $\approx -100 v_S$. Draw the corresponding circuit diagram and specify one possible set of resistor values.

Problem 5.52. Construct an amplifier circuit matched to a 100-Ω load. The amplifier's gain is $|A_{CL}| = 100$. The sign of the gain (positive or negative) is not important and the input AC signal.
Hint: Multiple solutions many exist. Present at least two solutions.

Problem 5.53. Find the input resistance (impedance) to the amplifier circuit shown in the figure below.

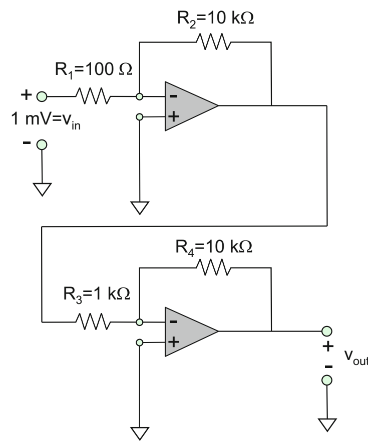


Problem 5.54. Find the input resistance (impedance) to the amplifier circuit shown in the figure below.



5.3.4 Cascading Amplifier Stages

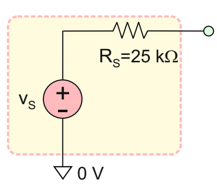
Problem 5.55. For the amplifier circuit shown in the figure, find the output voltage and the input resistance (show units).



Problem 5.56. A sensor with Thévenin voltage (source voltage) $v_S = 2.5$ mV shown in the figure is to be connected to an amplifier circuit. An amplified replica of the sensor's voltage, $v_{out} \approx 1000\, v_S$, is needed at the circuit's output.

A. Present one possible circuit diagram and specify all necessary resistor values.

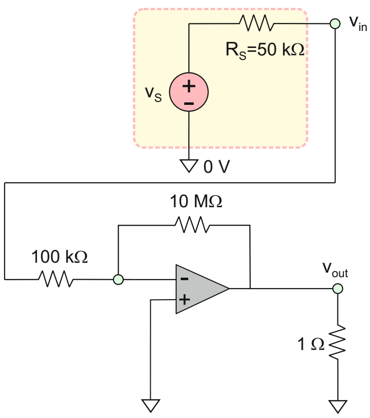
B. Present another (distinct) circuit diagram and specify all necessary resistor values.



Problem 5.57. An amplifier in the configuration shown in the figure below is connected to a

sensor with Thévenin (source) voltage $v_S = 25$ mV. An amplified replica of the sensor's voltage, $v_{out} \approx -100\, v_S$, is needed at the output.

A. Do you have any concerns with regard to this circuit?
B. If you do, draw an appropriate circuit diagram.



Problem 5.58. An amplifier circuit is needed with the closed-loop gain $A_{CL} = +1000$. The input resistance (impedance) to the circuit should be 5 kΩ. Present two alternative circuit diagrams and specify the necessary resistor values. The first circuit must use inverting amplifiers and the second circuit-non-inverting amplifiers.

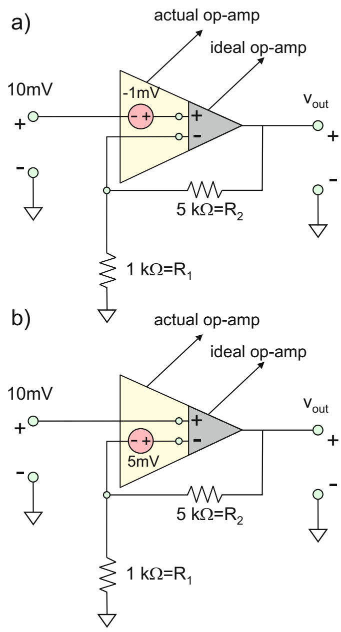
Problem 5.59. An amplifier circuit is needed with the closed-loop gain $A_{CL} = +10,000$. The input resistance (impedance) to the circuit should be as high as possible. Present the corresponding circuit diagram and specify the necessary resistor values.

Problem 5.60. An amplifier circuit is needed with a positive gain of 1000 ± 20 %. The input resistance (impedance) to the circuit should be 1 kΩ. Present the circuit diagram and specify the necessary resistor values including tolerance.

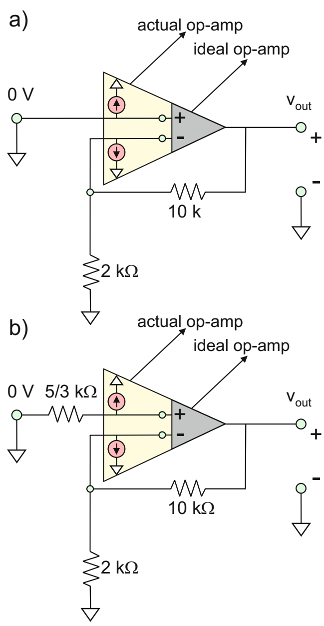
Problem 5.61. An amplifier circuit is needed with a positive gain of 5000 ± 5 %. The input resistance (impedance) should be as high as possible. Present one possible circuit diagram and specify the necessary resistor values including tolerance.

5.3.5 Amplifier DC Imperfections and Their Cancellation

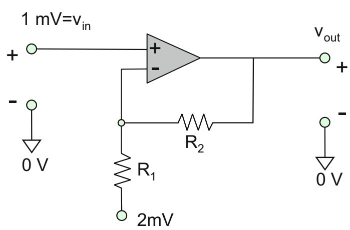
Problem 5.62. Determine the output voltage to nonideal operational amplifier circuits (with the nonzero input offset voltage) shown in the figures below.



Problem 5.63. Determine the output voltage to nonideal operational amplifier circuits (with nonzero input currents) shown in the figures below. The input terminal is connected directly to the common terminal (grounded). The strength of every bias current source is 100 nA. *Hint:* The upper bias current source does not contribute to the solution.

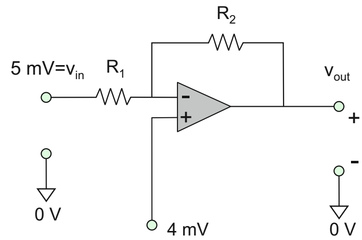


Problem 5.64. For the amplifier circuit shown in the figure below, determine the output voltage. Use $R_1 = R_2 = 1\ \text{k}\Omega$.



Problem 5.65. In the previous problem, denote the terminal voltage of 2 mV by V_{off}, the input voltage of 1 mV by v_{in}, the output voltage by v_{out}, and the amplifier gain by A_{CL}. Derive an analytical formula that determines V_{off} in terms of V_{in} given that the output voltage v_{out} to the amplifier is exactly zero.

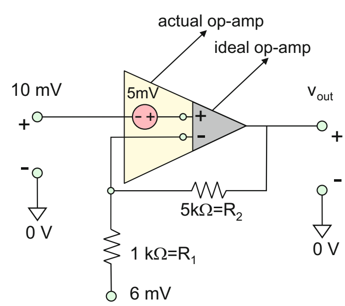
Problem 5.66. For the amplifier circuit shown in the figure below, determine the output voltage. Use $R_1 = 1\ \text{k}\Omega$ and $R_2 = 4\ \text{k}\Omega$.



Problem 5.67. In the previous problem, denote the terminal voltage of 4 mV by V_{off}, the input voltage of 5 mV by v_{in}, the output voltage by v_{out}, and the amplifier gain by A_{CL}. Derive an analytical formula that determines V_{off} in terms of v_{in} given that the output voltage to the amplifier is exactly zero.

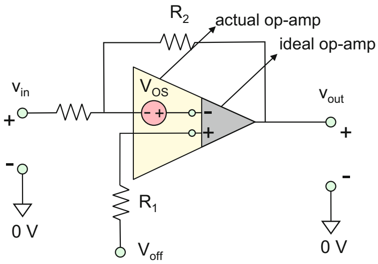
Problem 5.68. For the circuit shown in the figure below:

A. Determine the output voltage of the nonideal operational amplifier circuit (with the nonzero input offset voltage).
B. Does the amplifier circuit really follow the ideal-amplifier circuit law: $v_{\text{out}} = A_{\text{CL}} v_{\text{in}}$?
C. What happens if the input voltage changes from 10 mV to 20 mV?

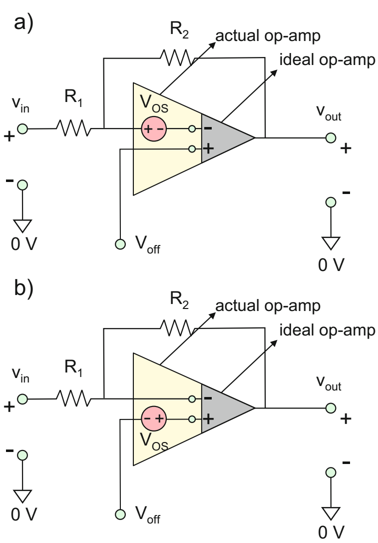


Problem 5.69. Solve the previous problem with the offset voltage in the feedback loop changed from 6 mV to 5 mV.

Problem 5.70. In problem 5.68, denote the terminal voltage of 6 mV by V_{off}, the input voltage of 10 mV by v_{in}, the input offset voltage by V_{OS}, the output voltage by v_{out}, and the amplifier gain by A_{CL}. Derive an analytical formula that determines V_{off} in terms of V_{OS} given that the output voltage v_{out} to the amplifier must exactly follow the ideal-amplifier gain law: $v_{out} = A_{CL} v_{in}$.



Problem 5.71. For two nonideal operational amplifier circuits (with the nonzero input offset voltage) shown in the figures below, determine the necessary offset voltage, V_{off}, which ensures that the output voltage, v_{out}, to the amplifier exactly follows the ideal-amplifier gain law: $v_{out} = A_{CL} v_{in}$. You need to express this voltage in terms of other circuit parameters that are given in figures a) and b).

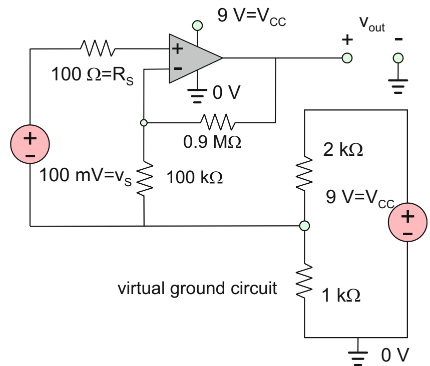


5.3.6 DC-Coupled Single-Supply Amplifier

Problem 5.72. For the single-supply amplifier circuit shown in the figure:

A. Determine the output voltage versus circuit ground (the negative terminal of the voltage power supply).

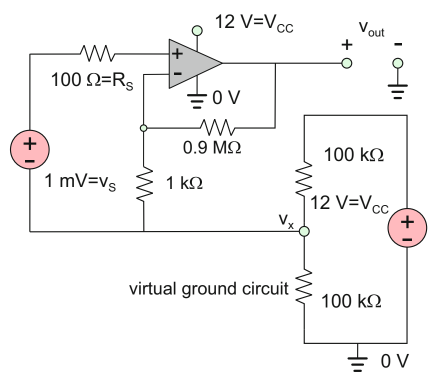
B. What potential problem do you see with this circuit? How could you fix it?



Problem 5.73. For the single-supply amplifier circuit shown in the figure:

A. Determine the output voltage versus circuit ground (the negative terminal of the voltage power supply).

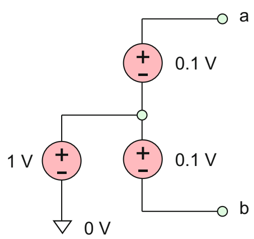
B. Do you see any problem with this circuit?



5.4 Difference and Instrumentation Amplifiers

5.4.1 Differential Input Signal to an Amplifier

Problem 5.74. The model of an input signal from a three-terminal sensing device is shown in the figure below. What are the differential and common-mode voltages at terminals *a* and *b*?



Problem 5.75. The Wheatstone bridge in Fig. 5.27 is connected to $\pm V_{CC}$ rails instead of $+V_{CC}$ and ground. Furthermore, $R_2 = 1.1R_1$, $R(x) = 1.1R_3$, and $V_{CC} = 6\,\mathrm{V}$. What are the differential and common-mode voltages?

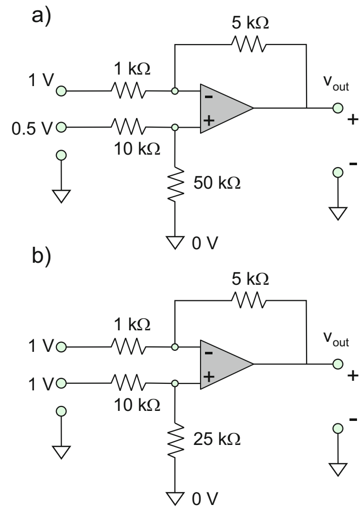
Problem 5.76. The Wheatstone bridge in Fig. 5.27 is connected to ground and $-V_{CC}$ rails instead of $+V_{CC}$ and ground. Given that $R_2 = 1.05R_1$, $R(x) = 1.05R_3$, and $V_{CC} = 6$ V, determine the differential and common-mode voltages.

5.4.2 Difference Amplifier

Problem 5.77. Design a difference amplifier with a differential gain of 20. Present the circuit diagram and specify one possible set of resistor values. In the circuit diagram, label the input voltages as v_a, v_b and express the output voltage in terms of v_a, v_b.

Problem 5.78. Repeat the previous problem for a differential gain of 100.

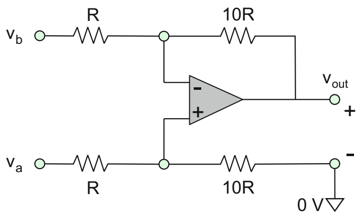
Problem 5.79. Find the output voltage to the difference-amplifier circuits shown in the figures below. Assume the ideal-amplifier model and exact resistance values.



Problem 5.80. Your technician needs to control a process using two sensors with output voltages v_1 and v_2, respectively. The weighted difference in sensor reading, $v = 1v_1 - 0.5v_2$, is critical for the product quality. The technician reads voltage v_1 and then voltage v_2 and then uses a calculator to find v. Help the technician, i.e., sketch for him a difference-amplifier circuit that will directly output v to the DMM. The negative terminal of the DMM is always grounded. Specify one possible set of resistor values.

Problem 5.81. Repeat the previous problem when the weighted difference in sensor reading, $v = 10v_1 - 5v_2$, needs to be processed.

Problem 5.82. For the circuit shown in the figure, find the output voltage if the input voltages are $v_b = 1\,\mathrm{V}$ and $v_a = 1.01\,\mathrm{V}$, respectively. Assume the ideal-amplifier model and exact resistance values.

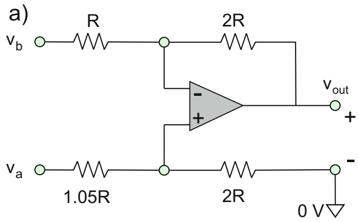


Problem 5.83

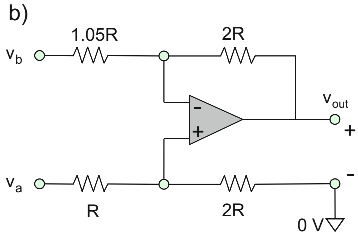
A. For the circuits shown in the figures below, find the output voltage if the input voltages are $v_a = 1\,\mathrm{V}$ and $v_b = 1\,\mathrm{V}$, respectively Assume the ideal amplifier and exact resistance values.

B. What is the value of the common-mode gain in every case?

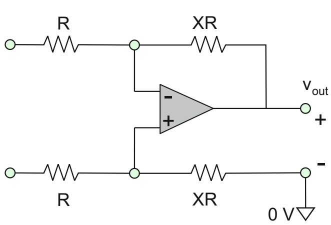
a)



b)



Problem 5.84. For the difference-amplifier circuit shown in the figure below, find the *differential-mode resistance (impedance)* to the amplifier. The differential-mode resistance is defined as the ratio of a voltage of a power supply placed between terminals *a* and *b* to the current that flows through this power supply.



5.4.3 Instrumentation Amplifier

Problem 5.85

A. Why is the original difference amplifier not used as an instrumentation amplifier?

B. Why is the circuit in Fig. 5.31 not used as the instrumentation amplifier?

Problem 5.86

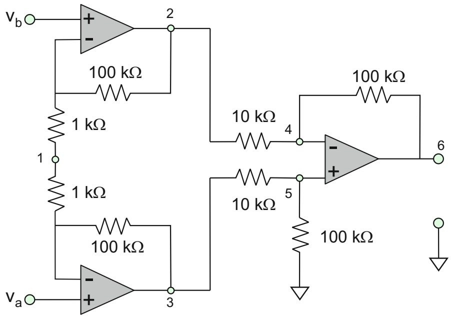
A. Find the differential gain and the common-mode gain for the amplifier circuit shown in Fig. 5.32. The differential output voltage is $v_a* - v_b*$, and the common-mode output voltage is $0.5(v_a* + v_b*)$.

B. Find the differential gain and the common-mode gain for the amplifier circuit shown in Fig. 5.31.

Problem 5.87. Design an instrumentation amplifier with a differential gain of 210. Present the corresponding circuit diagram and specify one possible set of resistance values. In the circuit diagram, label the input voltages as v_a, v_b and express the output voltage in terms of v_a, v_b.

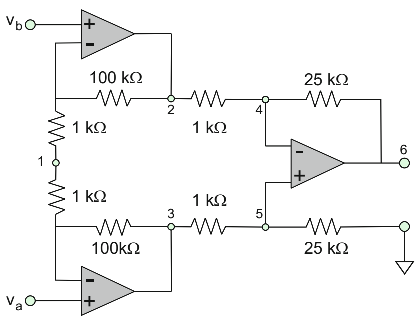
Problem 5.88. Design an instrumentation amplifier with a differential gain of 1010. Present the corresponding circuit diagram and specify one possible set of resistor values. In the circuit diagram, label the input voltages as v_a, v_b and express the output voltage in terms of v_a, v_b.

Problem 5.89. The following voltages are measured: $v_a = 3.750$ V and $v_b = 3.748$ V. Find voltages versus circuit ground (common port of the dual supply) for every labeled node in the circuit shown in the figure below. The amplifier circuit is powered by a $\pm 10 - \mathrm{V}$ dual supply. Assume exact resistance values and the ideal-amplifier model.



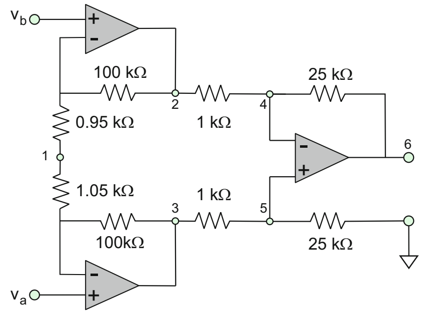
Problem 5.90. Repeat the previous problem when node 1 is grounded.

Problem 5.91. The following voltages are measured: $\upsilon_a = 5.000$ V and $\upsilon_b = 5.001$ V. Find voltages versus circuit ground (common port of the dual supply) for every labeled node in the circuit shown in the figure below. The amplifier circuit is powered by a $\pm 10 - \text{V}$ dual supply. Assume exact resistance values and the ideal-amplifier model.



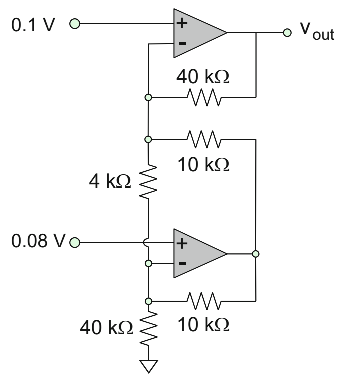
Problem 5.92. Repeat the previous problem when node 1 is grounded.

Problem 5.93. The following voltages are measured: $\upsilon_a = 5.000$ V and $\upsilon_b = 5.001$ V. Find voltages versus circuit ground (common port of the dual supply) for every labeled node in the circuit shown in the figure below. The amplifier circuit is powered by a $\pm 10 - \text{V}$ dual supply. Assume exact resistance values and the ideal-amplifier model.



Problem 5.94

A. Find the output voltage for the amplifier circuit shown in the figure below.
B. Denote the input voltage of 0.1 V by υ_a, the input voltage of 0.08 V by υ_b, the 10-kΩ resistor by R_1, the 40-kΩ resistor by R_2, and the 10-kΩ resistor by R_G. Express the output voltage in the general form, in terms of two input voltages and the resistances.



5.5 General Feedback Systems

5.5.1 Signal-flow Diagram of a Feedback System

5.5.2 Closed-Loop Gain and Error Signal

Problem 5.95. The block diagram of Fig. 5.35 is applied to a voltage amplifier.

A. Given the input signal $x_{in} = 10$ mV, the error signal $x_e = 1$ μV, and the output signal $x_{out} = 1$ V, determine the open-loop gain and the feedback factor.
B. Given the ratio of input to error signal $x_{in}/x_e = 100$ and the feedback factor of 0.1, determine the open-loop gain.

Problem 5.96. The open-loop gain A in Fig. 5.35 varies between two extreme values of $A = 10,000 \pm 2,000$ (± 20 % gain variation) depending on the system parameters. The forward gain block is used in the closed-loop configuration with the feedback factor β of 0.1. Determine the two extreme values of the closed-loop gain, A_{CL}.

Problem 5.97. The open-loop gain A in Fig. 5.35 is 100,000. The forward gain block is used in the closed-loop configuration with the feedback factor β of 1. Determine the error signal, x_e, if the input voltage signal is 1 mV.

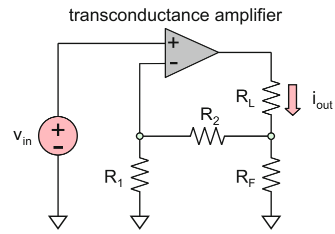
Problem 5.98. The closed-loop gain of a non-inverting amplifier circuit with $R_1 = 1\,\text{k}\Omega$, $R_2 = 100\ \text{k}\Omega$ is 99. Determine the open-circuit gain A of the amplifier chip.

5.5.3 Application of General Theory to Voltage Amplifiers with Negative Feedback

5.5.4 Voltage, Current, Transresistance, and Transconductance Amplifiers with the Negative Feedback

Problem 5.99. Derive the gain Eq. (5.69) for the amplifier circuits shown in Fig. 5.37.

Problem 5.100. The circuit shown in the figure that follows is a feedback transconductance amplifier. Express i_{out} in terms of v_{in}.



Problem 5.101. The amplifier circuit shown in the figure that follows is the *Howland current source* widely used in biomedical instrumentation; its output is the current through the load resistance.

A. Classify the amplifier circuit in terms of four basic amplifier topologies and mention the most important circuit features.
B. Derive its gain equation $i_{out} = (v_a - v_b)/R_2$ given that $R_1/R_3 = R_2/R_4$.