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chapter

Answers to Practice Problems in Chap. 4, Use of Statistics in Food Analysis

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1. What is the mean of the observations (mg/cup)?

$$\bar{x} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + \cdots + x_{n-1} + x_n}{n} = \frac{1967.1\,\text{mg / cup}}{6}$$
$$= 327.85\,\text{mg / cup, round to } 328\,\text{mg / cup}$$

What is the standard deviation of the observations (mg/cup)? Since we have < 30 observations, the correct formula for sample standard deviation is:

$$\text{SD}_{n-1} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum (x_i - 327.85\,\text{mg / cup})^2}{6-1}}$$
$$= \sqrt{\frac{460.215\,\text{mg}^2 / \text{cup}^2}{5}} = 9.593904\,\text{mg / cup}$$

Calculate the 96 % confidence interval for the true population mean (use t-score, not Z score, because n is small). We know that the formula for a CI is:

$$\text{CI}: \bar{x} \pm t_{\frac{\alpha}{2},\ \text{df}=n-1} \times \frac{\text{SD}}{\sqrt{n}}$$

We know the mean, SD, and n already, so we just need the t-score. First, calculate df:

$$\text{df} = n - 1 = 6 - 1 = 5$$

Next calculate C and $\alpha/2$:

$$\text{for 96\% CI, } C = 0.96 \rightarrow \frac{\propto}{2} = \frac{1-C}{2} = \frac{1-0.96}{2} = 0.02$$

Then, go to the t-table and find the t-score that corresponds to df = 5, $\alpha/2 = 0.02$:

$$t_{\frac{\alpha}{2},\ \text{df}=n-1} = t_{0.02,5} = 2.757$$

Putting it all together:

$$\text{CI}: \bar{x} \pm t_{\frac{\alpha}{2},\ \text{df}=n-1} \times \frac{\text{SD}}{\sqrt{n}}$$

$$327.85\ \text{mg / cup} \pm 2.757 \times \frac{9.593904\,\text{mg / cup}}{\sqrt{6}}$$
$$\rightarrow 327.85\,\text{mg / cup} \pm 10.7983\ \text{mg / cup}$$

What are the upper and lower limits of the 96 % confidence interval for the population mean?

The upper limit of the CI is:

$$\bar{x} + t_{\frac{\alpha}{2},\ \text{df}=n-1} \times \frac{\text{SD}}{\sqrt{n}} \rightarrow 327.85\frac{\text{mg}}{\text{cup}} + 10.7983\frac{\text{mg}}{\text{cup}}$$
$$= 338.648\,\text{mg / cup}$$

The lower limit of the CI is:

$$\bar{x} - t_{\frac{\alpha}{2},\ \text{df}=n-1} \times \frac{\text{SD}}{\sqrt{n}} \rightarrow 327.85\frac{\text{mg}}{\text{cup}} - 10.7983\frac{\text{mg}}{\text{cup}}$$
$$= 317.052\,\text{mg / cup}$$

Use a t-test to determine if the sample mean provides strong enough evidence that the population is "out of spec" (i.e., the actual population mean ≠ 343 mg cup). Since we have mean a "target" ($\mu = 343$ mg/cup) that we want to compare a sample against, we use a one sample t-test:

$$t_{\text{obs}} = \frac{|\bar{x} - \mu|}{\frac{\text{SD}}{\sqrt{n}}}$$

Plugging in the mean, SD, n, and μ values:

$$t_{\text{obs}} = \frac{|\bar{x} - \mu|}{\frac{\text{SD}}{\sqrt{n}}} = \frac{|327.85\,\text{mg / cup} - 343\,\text{mg / cup}|}{\frac{9.593904\,\text{mg / cup}}{\sqrt{6}}}$$
$$= \frac{|-15.15\,\text{mg / cup}|}{3.91669\,\text{mg / cup}} = 3.868$$

Based on t_{obs}, is there sufficient evidence that the population is "out of spec" (99 % confidence)?

The decision rule is that we need to compare $t_{\text{df},\,\alpha/2}$ vs. t_{obs}. Find $t_{\text{df},\,\alpha/2}$:

$$\text{for 99\% confidence,}$$
$$C = 0.99 \rightarrow \frac{\propto}{2} = \frac{1-C}{2} = \frac{1-0.99}{2} = 0.005$$
$$\text{df} = n - 1 = 6 - 1 = 5$$

From the t-score table shown, $t_{5,\,0.005} = 4.032$.
Next, compare $t_{\text{df},\,\alpha/2}$ vs. t_{obs}:

$$\text{if } t_{\text{obs}} > t_{\text{critical}} \rightarrow \text{there IS strong evidence that the true pop mean} \neq \mu$$

$$\text{if } t_{\text{obs}} < t_{\text{critical}} \rightarrow \text{there IS NOT strong evidence that the true pop mean} \neq \mu$$

In this case, since t_{obs} (3.868) < t_{critical} (4.032), there *IS NOT* sufficient evidence that the population is "out of spec" with 99 % confidence.

2. You should use a two-sample t-test to determine if the means are statistically different. We need the mean and the $\text{SD}_{n\text{-}1}$ for each population. Using your calculator or Excel, we get:

Line 1: n =5, $\bar{x}$=86.98, and SD=0.81670068
Line B: n = 5, $\bar{x}$=89.02, and SD=0.195 0.192353841

Next, calculate the pooled variance (s_p^2) from the two sample standard deviations:

$$\text{pooled variance} = s_p^2 = \frac{(n_1-1)\text{SD}_1^2 + (n_2-1)\text{SD}_2^2}{n_1+n_2-2}$$

$$= \frac{(5-1)(0.81670068)^2 + (5-1)(0.192353841)^2}{5+5-2} = 0.352$$

Now, calculate t_{obs}:

$$t_{\text{obs}} = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{|86.98-89.02|}{\sqrt{0.352\times\left(\frac{1}{5}+\frac{1}{5}\right)}}$$

$$= \frac{|-2.04|}{\sqrt{0.1408}} = 5.43662$$

Based on the t-test, is there strong evidence that the sample means are statistically different? For a two-sample t-test, the decision rule is:

$$t_{obs} > t_{\frac{\alpha}{2},\ \text{df}=n_1+n_2-2} \rightarrow \text{strong evidence that means are significantly different}$$

$$t_{obs} < t_{\frac{\alpha}{2},\ \text{df}=n_1+n_2-2} \rightarrow \text{insufficient evidence that means are significantly different}$$

$$95\%\ \text{confidence} \rightarrow C = 0.95,\ \frac{\alpha}{2} = \frac{1-C}{2} = \frac{1-0.95}{2} = \frac{0.05}{2} = 0.025$$

$$\text{df} = n_1 + n_2 - 2 = 5 = 5 - 2 = 8$$

Then, go to the t-table and find:

$$t_{\text{critical}}\left(t_{\frac{\alpha}{2},\ \text{df}=n_1+n_2-2}\right):\ t_{0.025,8} = 2.306$$

Since t_{obs} (5.44) > t_{critical} (2.31), there is strong evidence that the means differ (95 % confidence).