# Chapter 4
The Locality Problem

Chapter 3 focused on Schrödinger's argument that quantum mechanical wave functions (evolving always in accordance with Schrödinger's own equation) cannot be understood as providing *complete* descriptions of the physical states of individual systems. In this chapter we focus on a second argument, due largely to Einstein, for the same conclusion – namely, the *incompleteness* of the quantum mechanical description of physical reality.

4.1 Einstein's Boxes

In the discussion period of an important international scientific conference in 1927, Einstein made what would turn out to be just the first of several important and related arguments purporting to prove that there is a contradiction between the idea that quantum mechanics already provides (with wave functions alone) complete descriptions of physical states, and the idea of "locality" that we reviewed in Chap. 1.

In this early argument, Einstein begins by asking us to consider a single particle (an electron, say) incident on a narrow slit, behind which there is a curved detection screen as indicated in Fig. 4.1. Behind the slit, the electron will diffract as we saw in Chap. 2, resulting in essentially spherical Schrödinger waves propagating toward the screen. Of course, each individual electron that is fired in will eventually be detected at some distinct point on the screen. Einstein's comments, which I quote here at length, focus on the apparent conflict between the spreading spherical wave and the distinct point of eventual detection.

Fig. 4.1 A single electron
approaches a narrow slit (O)
in a screen (S). Downstream
of the slit, the wave function
diffracts and spreads more or
less evenly over a curved
detection screen (P)
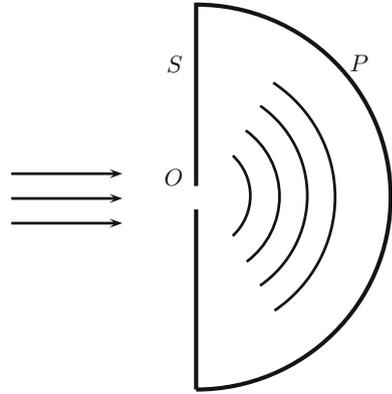
One can take two positions towards the theory with respect to its postulated domain of validity, which I wish to characterise with the aid of a simple example.

Let S be a screen provided with a small opening O [see Fig. 4.1] and P a hemispherical photographic film of large radius. Electrons impinge on S in the direction of the arrow.... Some of these go through O, and because of the smallness of O and the speed of the particles, are dispersed uniformly over the directions of the hemisphere, and act on the film.

Both ways of conceiving the theory now have the following in common. There are de Broglie waves, which impinge approximately normally on S and are diffracted at O. Behind S there are spherical waves, which reach the screen P and whose intensity at P is responsible for what happens at P.

We can now charaterise the two points of view as follows.

1. Conception I. – The de Broglie - Schrödinger waves do not correspond to a single electron, but to a cloud of electrons extended in space. The theory gives no information about individual processes, but only about the ensemble of an infinity of elementary processes.

2. Conception II. – The theory claims to be a complete theory of individual processes. Each particle directed towards the screen, as far as can be determined by its position and speed, is described by a packet of de Broglie - Schrödinger waves of short wavelength and small angular width. This wave packet is diffracted and, after diffraction, partly reaches the film P in a state of resolution.

According to the first, purely statistical, point of view $|\psi|^2$ expresses the probability that there exists at the point considered *a particular* particle of the cloud, for example at a given point on the screen.

According to the second, $|\psi|^2$ expresses the probability that at a given instant *the same* particle is present at a given point (for example on the screen). Here, the theory refers to an individual process and claims to describe everything that is governed by laws.

The second conception goes further than the first, in the sense that all the information resulting from I results also from the theory by virtue of II, but the converse is not true. It is only by virtue of II that the theory contains the consequence that the conservation laws are valid for the elementary process; it is only from II that the theory can derive the result of the experiment of Geiger and Bothe, and can explain the fact that in the Wilson [cloud] chamber the droplets stemming from an α-particle are situated very nearly on continuous lines.

But on the other hand, I have objections to make to conception II. The scattered wave directed towards P does not show any privileged direction. If $|\psi|^2$ were simply regarded as

the probability that at a certain point a given particle is found at a given time, it could happen that *the same* elementary process produces an action *in two or several* places on the screen. But the interpretation, according to which $|\psi|^2$ expresses the probability that *this* particle is found at a given point, assumes an entirely peculiar mechanism of action at a distance, which prevents the wave continuously distributed in space from producing an action in *two* places on the screen.

In my opinion, one can remove this objection only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time one localises the particle during the propagation. I think Mr de Broglie is right to search in this direction. If one works solely with the Schrödinger waves, interpretation II of $|\psi|^2$ implies to my mind a contradiction with the postulate of relativity [1].

Einstein's "Conception 2" is of course just the idea that quantum wave functions provide complete descriptions of the physical states of individual particles. If, on this view, the wave function is spread out across some (hemispherical) region of space, then the particle itself is literally smeared out across that region. (To use Schrödinger's phrase from the last Chapter, it is like a cloud or fog bank.)

The idea of "Conception 1", on the other hand, is that the wave function does *not* provide a complete description of an individual electron, but is instead a kind of collective description of a large ensemble of individual electrons with different individual properties. This is closely related to the "ignorance interpretation of superposition" we discussed in the last chapter. The simplest (and probably wrong) possibility along these lines is the idea that electrons really are like classical particles which follow definite trajectories through space. If, for example, we sent a million particles through, one at a time, they would each follow (say) some different path between the slit and the screen, with $|\psi(x)|^2$ representing the fraction of the million trajectories that go near a given point x. The wave function thus provides us with information about the *probability* for a given electron to be found somewhere, but this smeared-out probabilistic information is certainly not able to tell us the exact trajectory of any given particle.

Einstein acknowledges that certain empirical observations seem to support Conception 2. But then he argues that Conception 2 conflicts with the principle of locality – the point we want to focus on in this Chapter. The argument seems to be roughly as follows. Suppose Conception 2 is correct, i.e., suppose that each individual particle really is smeared out across the whole hemispherical region, with each part of the cloud evidently possessing the power to perhaps trigger a "flash" at the corresponding point on the screen. But there is always only and exactly one flash for a given electron that is sent through. So it must be, on this view, that when a certain bit of the cloud manages to trigger a "flash" at a certain point on the screen, the *rest* of the cloud instantaneously loses its potency. This is essentially just the idea that is described formally in the collapse postulate: when a position measurement is made, the wave function of the electron collapses to a position eigenstate (i.e., it goes to zero at all points where the successful detection did not occur). Einstein is really just pushing us to consider the implications of this if we take the wave function not just as some kind of incomplete information catalog, but as a faithful and full description of a kind of spread-out physical field or cloud. An interaction between one part of

the cloud and a measuring device at the location of that one part can dramatically change the structure – the "intensity" – of the cloud at distant locations.

But, Einstein argues, this conflicts with the principle of locality: as soon as the "flash" is triggered somewhere, the field/cloud must suddenly change ("collapse") at other locations in order to ensure that no *additional* flashes are produced elsewhere. And note that the effect must be *instantaneous*. If, for example, the message to other pieces of the cloud – saying something like "Urgent! A flash has already been produced elsewhere! So don't make a flash!" – propagated out at the speed of light, there would be a chance that the message would arrive too late: a single electron might thus sometimes produce two (or more) flashes. Since this is never in fact observed to occur, it must therefore be that – again assuming Interpretation 2 is correct – the signal propagates out infinitely fast. But this, of course, is supposed to be impossible according to the idea of "locality" that is especially strongly implied by Einstein's own relativity theory.

And so, Einstein suggests, if the only two possibilities are Conception 1 and Conception 2, we must adopt Conception 1 since Conception 2 contradicts the relativistic notion of local causality, i.e., no-faster-than-light-action-at-a-distance.

Let us review a couple of other formulations of the same basic argument to make sure its structure is clear.

Einstein gives a similar (but slightly simpler) example in a letter he wrote to Schrödinger in 1935. He asks Schrödinger to consider a ball that is placed inside a box into which a partition is then inserted, so that the ball is either on the left or on the right. But suppose that we cannot see inside the box and things are arranged so that it is impossible to tell which side the ball is in fact on. Suppose further that the two halves of the box are then separated (again without looking inside or determining in any other way which half contains the ball) and carried to distant locations, where they are finally opened and their contents examined. As in his discussion at the 1927 Solvey Conference, Einstein suggests that there are two possible ways to understand what is going on:

> Now I describe a state of affairs as follows: *the probability is* 1/2 *that the ball is in the first box*. Is this a complete description?
>
> NO: A complete description is: the ball *is* (or is not) in the first box. That is how the characterization of the state of affairs must appear in a complete description.
>
> YES: Before I open them, the ball is by no means in *one* of the two boxes. Being in a definite box only comes about when I lift the covers. This is what brings about the statistical character of the world of experience, or its empirical lawfulness. Before lifting the covers the state [of the distant box] is *completely* characterized by the number 1/2, whose significance as statistical findings, to be sure, is only attested to when carrying out observations [2, p. 69].

Note that the NO and YES alternatives map exactly onto Conception 1 and Conception 2 from the 1927 discussion. According to the NO view (and Conception 1), the description of the state of the system in terms of probabilities is incomplete, there being, in reality, an actual fact of the matter about the location of the particle. According to the YES view (and Conception 2), the description in terms of probabilities is complete because the actual fact of the matter regarding the location of the ball (particle) only comes into existence with the act of measurement – the particle

is, prior to observation, a kind of cloud that is in fact spread 50/50 between the two half-boxes.

For the case of a literal (classical, macroscopic) ball, the YES view is not very plausible, and Einstein asserts that "the man on the street would only take the [NO] interpretation seriously." But for a single electron, the YES view is essentially the standard claim that quantum mechanics already provides a complete description of physical states. Einstein didn't accept this view and wanted to argue that the NO view was the correct one not only for the classical particle but for the electron as well. He constructs that argument, just as in 1927, by bringing in the idea of locality:

> *My* way of thinking is now this: properly considered, one cannot [refute the completeness doctrine, i.e., Conception 2, i.e., the YES view] if one does not make use of a supplementary principle: the 'separation principle.' That is to say: 'the second box, along with everything having to do with its contents, is independent of what happens with regard to the first box (separated partial systems).' If one adheres to the separation principle, then one thereby excludes the [YES] point of view, and only the [NO] point of view remains, according to which the above state description is an *incomplete* description of reality, or of the real states [3].

The principle of locality, that is, seems to imply that we can *find out* the contents of the distant box, without physically affecting it (or its contents) at all, merely by examining the contents of the nearby box. This seems to force on us the following dilemma: either (i) the distant box already did or didn't contain the particle (in which case the earlier statement that there was a 50% probability of its being found there is revealed as decidedly incomplete) – or (ii) the act of examining the contents of the *nearby* box instantaneously affects the physical contents of the *distant box* (changing it from a half-cloud with 50% potency to create a full-fledged particle, to either *nothing* or a full-fledged particle). But this latter option indeed seems to imply a violation of the relativistic notion of locality, i.e., seems to imply a kind of instantaneous action-at-a-distance.

Heisenberg, interestingly, presented a nice version of Einstein's argument in which he points out that it could be re-formulated in terms of a single photon that impinges on a half-silvered mirror (see Fig. 4.2):

> ...one other idealized experiment (due to Einstein) may be considered. We imagine a photon which is represented by a wave packet built up out of Maxwell waves. It will thus have a certain spatial extension and also a certain range of frequency. By reflection at a semi-transparent mirror, it is possible to decompose it into two parts, a reflected and a transmitted packet. There is then a definite probability for finding the photon either in one part or in the other part of the divided wave packet. After a sufficient time the two parts will be separated by any distance desired; now if an experiment yields the result that the photon is, say, in the reflected part of the packet, then the probability of finding the photon in the other part of the packet immediately becomes zero. The experiment at the position of the reflected packet thus exerts a kind of action (reduction of the wave packet) at the distant point occupied by the transmitted packet, and one sees that this action is propagated with a velocity greater than that of light [4, p. 39].

This is particularly interesting because Heisenberg – one of the creators and advocates of the "orthodox completeness doctrine" – seems here to concede that Einstein's argument really does establish the nonlocality of quantum theory, at least if
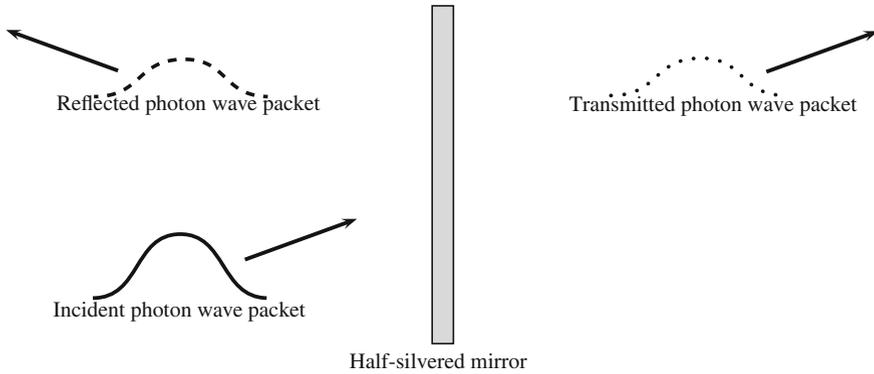
Fig. 4.2 Heisenberg's suggested setup for Einstein's Boxes argument: a single photon (*black*) is incident on a half-silvered mirror (i.e., a beam-splitter); a transmitted wave packet (*dotted*) contains 50% of the total probability for the particle to be detected, and a reflected packet (*dashed*) contains the other 50% of the total probability. As Heisenberg writes, the detection of the particle "at the position of the reflected packet thus exerts a kind of action (reduction of the wave packet) at the distant point occupied by the transmitted packet, and one sees that this action is propagated with a velocity greater than that of light [4]"

one assumes that it indeed provides *complete* descriptions of physical systems. But, as he goes on to state, Heisenberg doesn't believe that this implies any conflict with relativity theory: "However, it is also obvious that this kind of action can never be utilized for the transmission of signals so that it is not in conflict with the postulates of the theory of relativity." [4] It is surely correct that the nonlocality here cannot be used for superluminal communication. But the idea that this makes it compatible with relativity is quite dubious. Einstein, for example, obviously disagreed: he, apparently, thought that relativity prohibited instantaneous-action-at-a-distance as such, not merely that which can be somehow used by humans to build a telephone. (The question of compatibility with relativity will arise in several later Chapters as well. We set it aside here so as to focus on Einstein's argument that the completeness of the quantum mechanical description implies nonlocality... which Einstein, at least, regarded as implying "a contradiction with the postulate of relativity" [1].)

Here, finally, is one last statement of the "Einstein's Boxes" argument, this time as formulated in 1964 in a book by Louis de Broglie:

> Suppose a particle is enclosed in a box B with impermeable walls. The associated wave Ψ is confined to the box and cannot leave it. The usual interpretation asserts that the particle is 'potentially' present in the whole of the box B, with a probability $|\Psi|^2$ at each point. Let us suppose that by some process or other, for example, by inserting a partition into the box, the box B is divided into two separate parts B_1 and B_2 and that B_1 and B_2 are then transported to two very distant places, for example to Paris and Tokyo. The particle, which has not yet appeared, thus remains potentially present in the assembly of the two boxes and its wave function Ψ consists of two parts, one of which, Ψ_1, is located in B_1 and the other, Ψ_2, in B_2. The wave function is thus of the form $\Psi = c_1\Psi_1 + c_2\Psi_2$, where $|c_1|^2 + |c_2|^2 = 1$.

> The probability laws of wave mechanics now tell us that if an experiment is carried out in box B_1 in Paris, which will enable the presence of the particle to be revealed in this box, the probability of this experiment giving a positive result is $|c_1|^2$, whilst the probability of it giving a negative result is $|c_2|^2$. According to the usual interpretation, this would have the following significance: because the particle is present in the assembly of the two boxes prior to the observable localization, it would be immediately localized in box B_1 in the case of a positive result in Paris. This does not seem to me to be acceptable. The only reasonable interpretation appears to me to be that prior to the observable localization in B_1, we know that the particle was in one of the two boxes B_1 and B_2, but we do not know in which one, and the probabilities considered in the usual wave mechanics are the consequence of this partial ignorance. If we show that the particle is in box B_1, it implies simply that it was already there prior to localization. Thus, we now return to the clear classical concept of probability, which springs from our partial ignorance of the true situation. But, if this point of view is accepted, the description of the particle given by the customary wave function Ψ, though leading to a perfectly *exact* description of probabilities, does not give us a *complete* description of the physical reality, because the particle must have been localized prior to the observation which revealed it, and the wave function Ψ gives no information about this.
>
> We might note here how the usual interpretation leads to a paradox in the case of experiments with a negative result. Suppose that the particle is charged, and that in the box B_2 in Tokyo a device has been installed which enables the whole of the charged particle located in the box to be drained off and in so doing to establish an observable localization. Now, if nothing is observed, this negative result will signify that the particles is not in box B_2 and it is thus in box B_1 in Paris. But this can reasonably signify only one thing: the particle was already in Paris in box B_1 prior to the drainage experiment made in Tokyo in box B_2. Every other interpretation is absurd. How can we imagine that the simple fact of having observed *nothing* in Tokyo has been able to promote the localization of the particle at a distance many thousands of miles away? [5]

That is a very nice summary of the argument.

Now that we have (hopefully) made the "Einstein's Boxes" argument fairly clear in a qualitative way, let us try to make it a little more formally rigorous using Bell's formulation of "locality" from Chap. 1. This is of course somewhat anachronistic in the sense that Bell did not propose this definition of "locality" until 1976 (or, in its final version, 1990). So Einstein, for example, certainly never presented his argument in exactly this form. Still, it will help us to understand exactly the structure of the argument and the role of locality in particular.

The overall setup is illustrated in Fig. 4.3. The event "A" refers to the examination of the contents of the left half-box. If the particle is indeed found in the left half-box, we will denote this $A = +1$, whereas if that box is found to be empty we will denote this $A = 0$. Similarly, the events $B = +1$ and $B = 0$ will refer, respectively, to the finding and not-finding of a particle in the right half-box when it is opened and its contents are examined. In the Figure, the quantum mechanical wave function $\psi = \psi_L + \psi_R$ describing the state of the particle-in-the-boxes is depicted as the dashed and dotted packets, each containing 50% of the total probability associated with eventually finding the particle. (Note that here ψ_L and ψ_R are not separately normalized. For example, $\int |\psi_L|^2 dx = 1/2$.)

Now recall that Bell's definition of locality requires us to compare the probabilities assigned to an event like (for example) $A = +1$, when events from region B are, and aren't, conditioned upon. The definition also involves a complete specification
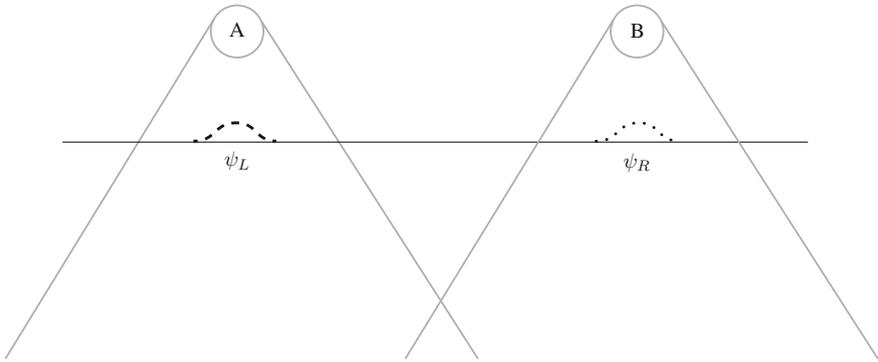
Fig. 4.3 Space-time diagram of the "Einstein's Boxes" setup. The events "A" and "B" represent observations in which the half-boxes are opened and their contents examined. The *dashed* (ψ_L) and *dotted* (ψ_R) curves represent the parts of the particle's quantum mechanical wave function ($\psi = \psi_L + \psi_R$) contained in the separated half-boxes. At the time corresponding to the horizontal *black line*, this wave function ψ is supposed, according to orthodox quantum mechanics, to provide a complete description of the state of the particle

of events $\mathcal{C}_\Sigma$ in a "slice" Σ across the backwards light cone of A. Here, Σ can just be the intersection of the horizontal black line in the figure with the backwards light cone of A. $\mathcal{C}_\Sigma$ then evidently includes (for the particle-in-the-box), the wave packet, ψ_L, contained in the left half-box, as well as all of the physical details about the left half-box itself, the transportation method, the observer and particle-detection apparatus, etc. It is clear, though, that according to quantum theory, we can say, for example, that even if all of these complicated details were specified, there would still just be an irreducible 50% probability assigned to the event $A = +1$. That is:

$$P\left[A = +1 \mid \mathcal{C}_\Sigma \right] = \frac{1}{2}. \tag{4.1}$$

However, consider now the event B – the examination of the contents of the right half-box. The particle is either found there ($B = +1$) or not ($B = 0$). But in either case, specifying the outcome of this other observation changes the probability assigned to the event $A = +1$. For example:

$$P\left[A = +1 \mid \mathcal{C}_\Sigma, B = +1 \right] = 0. \tag{4.2}$$

That is, the probability of finding the particle in the left half-box *given that it is found in the right half-box* is *zero*. And, similarly, the probability of finding the particle in the left half-box given that it is *not* found in the right half-box is *one*:

$$P\left[A = +1 \mid \mathcal{C}_\Sigma, B = 0 \right] = 1. \tag{4.3}$$

So we have a clear *violation* of Bell's locality condition,

$$P\big[\,A\,|\,\mathcal{C}_\Sigma\,\big] \;\neq\; P\big[\,A\,|\,\mathcal{C}_\Sigma,\,B\,\big], \tag{4.4}$$

as well as a violation of the modified condition,

$$P\big[\,A\,|\,\mathcal{C}_\Sigma,\,B\,\big] \;\neq\; P\big[\,A\,|\,\mathcal{C}_\Sigma,\,B'\,\big]. \tag{4.5}$$

Events from region B affect the probability assigned to A, even when the physical state of a (properly situated) slice across the past light cone of A has been specified (by assumption) completely.

What do we make of this? Basically it is just bringing out the fact that the collapse of the wave function violates locality, if we take the wave function seriously as representing a kind of physically-real field. The collapse provides a mechanism whereby one measurement (here, the examination of the contents of the right half-box) influences the state of a distant system (here, the contents of the left half-box) and thereby influences the probabilities assigned to events that are influenced by that system (here, the examination of the contents of the left half-box).

We must be absolutely clear, though, that this in no way establishes the real existence of nonlocal causal influences in nature! Instead, it merely establishes the existence of nonlocal causal influences in a certain *theory*, namely, the version of quantum theory according to which wave functions provide complete descriptions of the physical states of microscopic systems. A crucial assumption in our analysis, that is, was the assumption that the quantum mechanical wave function ψ_L provided a complete specification of the contents of the left half-box! We could – rather obviously – avoid the violation of locality by considering instead a different theory, according to which a complete specification of the contents of the half-boxes – at the time corresponding to the horizontal black line in the Figure – attributes the particle definitely to one, or the other, of the half-boxes. This is precisely what Einstein was suggesting when he said, at the end of his remarks in 1927, that one should "not describe the process solely by the Schrödinger wave, but [should in addition localise] the particle during the propagation." [1]

What the Einstein's boxes argument shows, then, is that we face a dilemma between "locality" and "completeness". If quantum mechanical wave functions provide complete descriptions of microscopic systems, then the theory must violate locality in order to make correct statistical predictions. In short, completeness implies that the collapse of the wave function is a physical, dynamical process, which conflicts with relativistic locality. On the other hand, we could preserve locality by denying the completeness doctrine and considering instead a hidden variable theory in which, even when the wave function involves a superposition between the particle being in the left and the right half-boxes, the particle is in fact (although unbeknownst to us) already in one place or the other. (The pilot-wave theory of de Broglie and Bohm, the subject of our Chap. 7, is a hidden variable theory of just this sort.)

4.2   EPR

In 1935, Einstein co-authored with Boris Podolsky and Nathan Rosen the most
famous criticism of the idea that quantum mechanical wave functions provide com-
plete descriptions of physical states. The argument of the EPR paper has the same
basic structure as that of the earlier and simpler "boxes" type argument: if quantum
mechanics is complete this would imply a violation of locality. Or equivalently: if we
believe in the principle of relativistic local causality, we must reject the completeness
doctrine.

It was discovered only rather recently that the entire text of the EPR paper – "Can
Quantum-Mechanical Description of Physical Reality be Considered Complete?"
[6] – was written by Podolsky (after conversations with Einstein and Rosen) and sent
in for publication before Einstein had even seen the manuscript. Einstein wrote, in a
private letter to Schrödinger, that the main point of the argument had not been made
very clear: "the essential thing is, so to speak, smothered by the formalism [2]." So
we should be a bit cautious about treating the EPR paper as providing an accurate
presentation of Einstein's views. But the paper is so famous and so important that
we will review it rather carefully. The subsequent section then discusses some of
Einstein's own later expressions of the same basic argument.

Here is the abstract of the EPR paper, which lays out the argument to be presented:

> In a complete theory there is an element corresponding to each element of reality. A sufficient
> condition for the reality of a physical quantity is the possibility of predicting it with certainty,
> without disturbing the system. In quantum mechanics in the case of two physical quantities
> described by non-commuting operators, the knowledge of one precludes the knowledge of
> the other. Then either (1) the description of reality given by the wave function in quantum
> mechanics is not complete or (2) these two quantities cannot have simultaneous reality.
> Consideration of the problem of making predictions concerning a system on the basis of
> measurements made on another system that had previously interacted with it leads to the
> result that if (1) is false then (2) is also false. One is thus led to conclude that the description
> of reality as given by a wave function is not complete [6].

The structure here is a bit convoluted, so let us delve in and try to understand it better.

The explanation of what it means for a theory to be "complete" seems clear
and uncontroversial. In the paper, EPR elaborate what they describe as a necessary
condition for calling a theory "complete": "*every element of the physical reality must
have a counterpart in the physical theory*". The overall goal of the paper will thus
be to establish the existence of *more* elements of reality than have counterparts in
quantum wave functions. In particular, the argument can be understood as an attempt
to establish that a single particle can have *both* a definite momentum *and* a definite
position, something that is forbidden in quantum mechanics since the position and
momentum operators do not commute. (This implies that there is no wave function
that is simultaneously an eigenstate of both position and momentum.)

In order to try to establish the existence of these properties, EPR require a "suffi-
cient condition for the reality of a physical quantity". As they elaborate in the main
text, this criterion is as follows:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity [6].

As we will discuss in Chap. 6, this criterion became the focal point of Bohr's attempt to rebut the EPR argument. But it has always seemed perfectly valid to me. In any case, it is a little hard to understand the idea, so let's think it through with a simple concrete example.

Suppose someone hands you a shoebox with a wine glass in it and you want to determine whether the wine glass is shattered, or intact. One way of doing this might be to shake the box vigorously and listen for the tinkling sound of shattered pieces of glass hitting one another. But if the question is whether the glass in the box was shattered *originally*, when the box was first handed to you, this method does not really work: the act of shaking the box may very well result in an originally intact glass shattering! So hearing tinkling glass pieces inside would *not* effectively establish that the glass had already been broken prior to your shaking. The shaking itself may have brought that shattered state about.

By contrast, suppose (to complicate the scenario slightly) that the glass came to be in the box by the following procedure. There were two glasses on the shelf; one of them was perfectly intact, and one of them was already broken. Then your trusted friend flipped a coin and thereby randomly selected one of the glasses to seal up in the box; suppose we are certain that he was extremely careful so that, if the intact glass was the one selected for inclusion in the box, the glass was not broken during the act of putting it in the box. The second glass is then left on the shelf and the cupboard door is closed. Now in this situation, another method of determining the state of the glass in the box presents itself: simply open the cupboard and see which glass is there! If the intact glass is there in the cupboard, it must be the already-shattered glass that is in the box, and vice versa. This way of determining the contents of the box – in which we never interact directly with the box or its contents at all – ensures that the determined state of the glass in the box faithfully represents the true original state of the glass. We preclude the possibility that our act of determining the state has somehow affected and changed the state. This is the basic scheme that EPR will use to try to show that (in a certain special situation) a particle can be said to possess simultaneously definite values of both position and momentum.

EPR thus consider the following situation. Suppose two particles have interacted and gotten into an entangled state but then spatially separated so they are now far apart from one another. See Fig. 4.4. Assuming the particles are well-separated in regard to their y coordinates, we then focus our attention on the degrees of freedom x_1 and x_2. Suppose in particular that the particles are in the following entangled state:

$$\Psi(x_1, x_2) = \delta(x_1 - x_2) = \int \delta(x_1 - x)\delta(x_2 - x)\, dx \qquad (4.6)$$

Pictured in the two-dimensional configuration space, this state is a "ridge" along the diagonal line $x_1 = x_2$. One can think of it as a superposition of states, over all possible values of x, in which both particles are definitely located at position x. That is, the
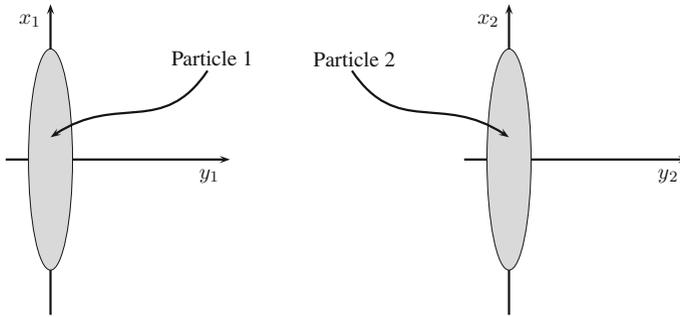
Fig. 4.4 Two particles which have previously interacted are spatially separated but remain in an entangled state. In particular, the spatial degrees of freedom x_1 and x_2 are entangled. We assume that, say, the part of the quantum state associated with the y coordinates is a simple product of two well-separated wave packets, centered (say) at $y_1 = 0$ and $y_2 = 0$ for the coordinate systems shown here. (Note that we use distinct coordinate systems for the two particles such that, for example, $y_1 = 0$ and $y_2 = 0$ are perhaps a million miles apart from one another!) So the particles are entangled (in so far as their positions along x are concerned) but they are unambiguously well-separated in space in regard to their y positions

state does not attribute a definite position to particle 1 or to particle 2 – both particles are maximally smeared out. But they are smeared out in a perfectly correlated way: measurement of the position x_1 of particle 1 immediately tells us the position x_2 of particle 2 because (even though neither x_1 nor x_2 has a well-defined value prior to such measurements) x_1 and x_2 are definitely equal to one another.

But this means it is possible to determine the position of particle 2 *indirectly* – without disturbing the physical state of particle 2 at all – by measuring the position x_1 of its distant entangled partner. And so, by the reality criterion, it follows that the distant particle must already *have* a definite position even when no such position is attributed to it by the pre-measurement wave function, Eq. (4.6). And note that this is already sufficient to show that that pre-measurement wave function did not provide a complete description of the state of the two particles: particle 2 *has* a definite position, but the wave function doesn't tell us about this at all.

EPR, however, go farther. The two-particle wave function can be re-written in this alternative (but mathematically equivalent) form

$$\Psi(x_1, x_2) = \delta(x_1 - x_2) = \frac{1}{2\pi} \int e^{ik(x_1 - x_2)} \, dk = \frac{1}{2\pi} \int e^{ikx_1} e^{-ikx_2} \, dk. \quad (4.7)$$

The last expression can be understood as saying that the state is a superposition – over all possible values of k – of states in which particle 1 has momentum $p_1 = \hbar k$ and particle 2 has momentum $p_2 = -\hbar k$. So the state can also, alternatively, be understood as a state in which neither particle has any definite momentum value, but the momenta of the two particles are perfectly (anti-) correlated: $p_1 = -p_2$.

But this means it is possible to determine the momentum of particle 2 *indirectly* – without disturbing it at all – by measuring the momentum p_1 of its distant entangled partner. And so, by the reality criterion, it follows that the distant particle must already *have* a definite momentum even when no such momentum is attributed to it by the pre-measurement wave function. It is, in short, the same story again with momentum as it was before with position. So in addition to possessing a definite position (about which the pre-measurement wave function was silent), particle 2 apparently *also* possesses a definite *momentum* (about which the pre-measurement wave function was also silent). So that wave function provides, at best, a decidedly – a doubly – incomplete description of the state of the particle. There are at least these two physical properties, position and momentum, which in reality have sharp well-defined values, for which there is no corresponding element in the theoretical description. And in a way it's even worse than that, for by establishing the real existence of both position and momentum (for the one distant particle) EPR show not only that the particular wave function in Eq. (4.6) fails to provide a complete description, but that no wave function possibly could provide a complete description. For there is, simply, no such thing as a wave function that is simultaneously a position and momentum eigenstate.

That is the essential argument. But I have explained it here in my own words, and my version doesn't appear to correspond perfectly to the logical structure of the EPR paper's abstract. Let us try to understand that. First of all, what should we make of this disjunction (from the paper's abstract), "that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality"? This sounds very complicated but is actually quite trivial. Suppose two operators (for example position and momentum) fail to commute. Clearly, either the corresponding physical properties (1) *can* have simultaneous reality, or (2) *cannot* have simultaneous reality. If they *can*, then quantum mechanics is necessarily incomplete, because there is no wave function that is simultaneously an eigenstate for (i.e., there is no wave function that simultaneously attributes definite real values to) the two properties in question. So the trivial disjunction I wrote two sentences back is equivalent to the one from the EPR text.

Now, in the paper, EPR continue the argument as follows. Having established the disjunction between (1) and (2) just discussed, they write:

> Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

Where and what, exactly, is the argument described in the first sentence here? It is again somewhat obscure and confusing, but actually this is just the essential argument we reviewed before. The way it is presented in the text is along the following lines. Thinking of the state as $\Psi = \int \delta(x_1 - x)\delta(x_2 - x)\, dx$ it is obvious that, if we measure the position of particle 1 and find it, say, at $x_1 = X$, the state of the two-particle system *collapses* to $\delta(x_1 - X)\delta(x_2 - X)$ which (being a product state) implies that we can

attribute the following wave function to particle 2:

$$\psi_2 = \delta(x_2 - X). \tag{4.8}$$

On the other hand, thinking of the (2-particle pre-measurement) state as $\Psi = \frac{1}{2\pi} \int e^{ikx_1} e^{-ikx_2} \, dk$ it is obvious that, if we measure the momentum of particle 1 and find, say, $p_1 = P$, the state of the two-particle system *collapses* to $\frac{1}{2\pi} e^{iPx_1/\hbar} e^{-iPx_2/\hbar}$ which (being a product state) implies that we can attribute the following wave function to particle 2:

$$\psi_2 = e^{i(-P/\hbar)x_2}. \tag{4.9}$$

EPR write: "Thus, *it is possible to assign two different wave functions ... to the same reality* (the second system after the interaction with the first)."

But then, *assuming that wave functions provide a complete description of the state of the particle*, i.e., assuming the negation of statement (1) from before, we have – from Eq. (4.8) – that particle 2 has a definite position, and – from Eq. (4.9) – that particle 2 has a definite momentum. Which indeed contradicts statement (2) from before.

Thus, the way it is presented in the actual EPR paper, the argument has the following extremely convoluted structure: either (1) or (2), but denying (1) requires one to also deny (2), and so one cannot consistently deny (1); that is, one must accept (1). That is, to be sure, logically valid. But it is also needlessly convoluted. The heart of the argument is simply the idea that, for spatially-separated but appropriately-entangled pairs of particles, we can determine, with certainty, the value of some property of one of the particles without actually *messing with it* at all, but by instead messing with its entangled partner and using the correlations built into the entangled state to infer something about the undisturbed particle. It is just like the example of the wine glass in the box. No wonder Einstein thought Podolsky's version of the argument was unnecessarily confusing!

4.3   Einstein's Discussions of EPR

I mentioned in the last section that although the paper grew out of discussions between Einstein, Podolsky, and Rosen, Podolsky actually wrote the EPR paper and submitted it for publication before Einstein had had a chance to see it or comment. And Einstein was somewhat frustrated and disappointed with how it came out. We began to see in the last section how Podolsky's version of the argument seemed needlessly convoluted, and undoubtedly that is part of what frustrated Einstein. But Einstein also specifically remarked that the main point had been "smothered". What was this main point that got buried in Podolsky's write-up?

Almost certainly it was the concept of "locality" which, as we have already seen in the discussion of the simpler "boxes" type arguments, was quite central to Einstein's

thinking about this kind of situation, but which hardly appears explicitly in the EPR paper itself. It is, though, implied in the application of the "reality criterion". Why do we think that, if we only make an actual measurement on particle 1, particle 2 is not disturbed at all? Well, evidently, because the two particles are spatially separated – they are distant from one another – and surely nothing we do *here* can have an immediate effect *over there*. That, presumably, is the idea – that is, it is only if we make the locality assumption that we are entitled to actually *apply* the reality criterion in the case at hand. But this is not made very clear. The closest we come, in the actual EPR paper, to an explicit mention of "locality" is in the penultimate paragraph of the paper:

> One could object to this conclusion [that quantum mechanics is incomplete] on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. On this point of view, since either one or the other, but not both simultaneously, of the quantities [momentum and position of particle 2] can be predicted, they are not simultaneously real. This makes the reality of [particle 2's momentum and position] depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this [6].

Here EPR seem to be anticipating the objection that, as we will see, Bohr makes against their argument: you cannot measure *both* the position *and* the momentum of the nearby particle, so you cannot determine both of these properties for the distant particle. You can only do one or the other. I think EPR are right to point out that this objection would make the real state of particle 2 "depend [nonlocally!] upon the process of measurement carried out on the first system." It doesn't matter whether you *do* in fact determine the position or momentum of particle 2 by measuring the corresponding property of particle 1; the mere fact that you *could* do so implies that those distant properties exist. Locality implies that the state of the distant particle is unaffected by what happens here – even the choice of whether or not to in fact go ahead with a certain kind of measurement.

So, I think, EPR here make a good and valid point, but it remains unfortunate that this is practically the only place they stress the notion of locality. As mentioned earlier, establishing the incompleteness of quantum mechanical descriptions doesn't even require establishing that the distant particle has *both* a definite position *and* a definite momentum. Its merely possessing, say, a definite position – when its state is described by the entangled wave function, Eq. (4.6) – is completely sufficient. And the point is, locality plays a crucial role already in the argument that a single such property, for the distant particle, can be established. Podolsky should have made all of this clearer.

Anyway, to round out our understanding of the EPR argument, we consider finally some of Einstein's own commentaries on the argument and related issues.

To begin with, in the same 1935 letter to Schrödinger from which I earlier quoted his remarks about the "boxes" example, Einstein writes:

> The preceding [boxes] analogy corresponds only very imperfectly to the quantum mechanical example in the [EPR] paper. It is, however, designed to make clear the point of view that

is essential to me. In quantum mechanics one describes a real state of affairs of a system by means of a normed function ψ of the coordinates (of configuration space). The temporal evolution is uniquely determined by the Schrödinger equation. One would now very much like to say the following: ψ stands in a one-to-one correspondence with the real state of the real system. The statistical character of measurement outcomes is exclusively due to the measuring apparatus, or the process of measurement. If this works, I talk about a complete description of reality by the theory. However, if such an interpretation doesn't work out, then I call the theoretical description 'incomplete'.... [2, p. 71]

Einstein continues:

Now what is essential is exclusively that [the wave functions, relating to the distant particle, that arise from different kinds of measurements on the nearby particle] are in general different from one another. I assert that this difference is incompatible with the hypothesis that the ψ description is correlated one-to-one with the physical reality (the real state). After the [particles separate], the real state of [the two particle system] consists precisely of the real state of [particle 1] and the real state of [particle 2], which two states have nothing to do with one another. *The real state of [particle 2] thus cannot depend upon the kind of measurement I carry out on [particle 1].* ('Separation hypothesis' from above.) But then for the same state of [particle 2] there are two (in general arbitrarily many) equally justified $\psi_{[2]}$, which contradicts the hypothesis of a one-to-one or complete description of the real states [3].

This passage has the virtue of making clearer the exact sense in which Einstein understood the "incompleteness" of the quantum mechanical description: a failure of the one-to-one correspondence between real states and theoretical descriptions.

In his auto-biographical contribution to the collection *Albert Einstein: Philosopher Scientist* from 1949, Einstein also gave an extensive discussion of the EPR-type argument for quantum incompleteness. We quote it at length here:

Physics is an attempt conceptually to grasp reality as it is thought independently of its being observed. In this sense one speaks of 'physical reality'. In pre-quantum physics there was no doubt as to how this was to be understood. In Newton's theory reality was determined by a material point in space and time; in Maxwell's theory, by the field in space and time. In quantum mechanics it is not so easily seen. If one asks: does a ψ-function of the quantum theory represent a real factual situation in the same sense in which this is the case of a material system of points or of an electromagnetic field, one hesitates to reply with a simple 'yes' or 'no'; why? What the ψ-function (at a definite time) asserts, is this: What is the probability for finding a definite physical magnitude q (or p) in a definitely given interval, if I measure it at time t? The probability is here to be viewed as an empirically determinable, and therefore certainly as a 'real' quantity which I may determine if I create the same ψ-function very often and perform a q-measurement each time. But what about the single measured value of q? Did the respective individual system have this q-value even before the measurement? To this question there is no definite answer within the framework of the [existing] theory, since the measurement is a process which implies a finite disturbance of the system from the outside; it would therefore be thinkable that the system obtains a definite numerical value for q (or p), the measured numerical value, only through the measurement itself. For the further discussion I shall assume two physicists, A and B, who represent a different conception with reference to the real situation as described by the ψ-function.

A. The individual system (before the measurement) has a definite value of q (i.e., p) for all variables of the system, and more specifically, *that* value which is determined by a measurement of this variable. Proceeding from this conception, he will state: The ψ-function is no exhaustive description of the real situation of the system but an incomplete description; it expresses only what we know on the basis of former measurements concerning the system.

B. The individual system (before the measurement) has no definite value of q (i.e., p). The value of the measurement only arises in cooperation with the unique probability which is given to it in view of the ψ-function only through the act of measurement itself. Proceeding from this conception, he will (or, at least, he may) state: the ψ-function is an exhaustive description of the real situation of the system.

We now present to these two physicists the following instance: There is to be a system which at the time t of our observation consists of two partial systems S_1 and S_2, which at this time are spatially separated and (in the sense of the classical physics) are without significant reciprocity. The total system is to be completely described through a known ψ-function ψ_{12} in the sense of quantum mechanics. All quantum theoreticians now agree upon the following: If I make a complete measurement of S_1, I get from the results of the measurement and from ψ_{12} an entirely definite ψ function ψ_2 of the system S_2. The character of ψ_2 then depends upon *what kind* of measurement I undertake on S_1.

Now it appears to me that one may speak of the real factual situation of the partial system S_2. Of this real factual situation, we know to begin with, before the measurement of S_1, even less than we know of a system described by the ψ-function. But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former. According to the type of measurement which I make of S_1, I get, however, a very different ψ_2 for the second partial system.... Now, however, the real situation of S_2 must be independent of what happens to S_1. For the same real situation of S_2 it is possible therefore to find, according to one's choice, different types of ψ-function. (One can escape from this conclusion only by either assuming that the measurement of S_1 (telepathically) changes the real situation of S_2 or by denying independent real situations as such to things which are spatially separated from each other. Both alternatives appear to me entirely unacceptable.)

If now the physicists, A and B, accept this consideration as valid, then B will have to give up his position that the ψ-function constitutes a complete description of a real factual situation. For in this case it would be impossible that two different types of ψ-functions could be co-ordinated with the identical factual situation of S_2.

The statistical character of the present theory would then have to be a necessary consequence of the incompleteness of the description of the systems in quantum mechanics, and there would no longer exist any ground for the supposition that a future basis of physics must be based upon statistics [7].

This passage, I think, makes very clear: (i) Einstein's belief that the *randomness* of quantum mechanics is in fact not inherent to nature, but is instead a result of the incompleteness of the theory's descriptions of nature; (ii) his hope that a future theory might *complete* the quantum descriptions and thereby restore the principle of *determinism* to physical theory; and (iii) his *reasons*, based in particular on the locality principle to which he stresses we must "absolutely hold fast", for this belief and this hope.

I want to stress, in particular, that Einstein's belief that a proper theory would restore determinism – captured in his oft-quoted remark "God does not play dice" – was in no way based on a philosophical unwillingness to contemplate fundamental, irreducible randomness. Instead it was based on his argument that consistency with relativistic locality required one to reject the claim that quantum mechanical wave functions provide complete descriptions of the physical states of the systems they describe.

Einstein gave another, much briefer, summary of his position in his other contribution (called "Reply to Criticisms") to the same 1949 book:

> By this way of looking at the matter it becomes evident that the paradox forces us to relinquish one of the following two assertions:
>
> 1. the description by means of the ψ-function is complete.
>
> 2. the real states of spatially separated objects are independent of each other.
>
> On the other hand, it is possible to adhere to (2) if one regards the ψ-function as the description of a (statistical) ensemble of systems (and therefore relinquishes (1)). However, this view blasts the framework of the 'orthodox quantum theory' [8].

In that same essay, Einstein states: "I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems." [8]

In this section, I have tried to stress that, for Einstein, establishing the real existence of (for example) *both* the position *and* the momentum of a distant particle (by making one or the other measurement on the nearby particle) was not really necessary to establish the incompleteness of the quantum mechanical descriptions of states. (He wrote to Schrödinger that whether the EPR argument establishes the reality of, for example, both position and momentum "ist mir *wurst*" – roughly, "I couldn't care less!" [2]) Instead, his preferred argument merely pointed out that one's choice of measurements to make on the nearby particle produced (via wave function collapse) different wave functions for the distant particle; and so, if the actual state of that distant particle remains unaffected by such measurements on the nearby particle, we have a failure of completeness in the sense of one-to-one correspondence between wave functions and physical states.

But I also don't want to make it appear that Einstein's views were totally different from those presented in our summary of the EPR paper itself. Indeed, it seems that – although he did not think this was necessary to establish the conflict between locality and completeness – Einstein did accept that the EPR-type argument does establish that, if locality is true, the distant system must already possess (for example) both position and momentum. For example, in a 1938 letter to Tanya Ehrenfest, Einstein wrote "Here, however, I [cannot] reconcile myself to the following, that a manipulation undertaken on A has an influence on B; thus I see myself required to suppose, as actually or physically realized at B, everything relating to measurement outcomes on B that can be predicted with certainty, on the basis of some measurement or other undertaken on A." [2, p. 63]

4.4 Bohm's Reformulation

In 1951, the young physicist David Bohm published a textbook on quantum theory. The book is somewhat unusual in emphasizing conceptual questions and containing lengthy prose discussions (in addition to the more standard mathematical presenta-

tion). The book is also of interest because it treats the subject in a very orthodox, Copenhagen way; but shortly after its publication, Bohm met with Einstein, who evidently convinced Bohm of the inadequacy of this approach (and in particular convinced him of the "incompleteness" of the orthodox quantum state descriptions). And in the following year, 1952, Bohm would produce (or really, because similar ideas had been proposed, but then prematurely abandoned, 25 years earlier by de Broglie, reproduce) a fully-worked-out "hidden variable theory" that we will discuss in depth in Chap. 7.

For our present purposes, though, we want to focus on Bohm's 1951 presentation of "The Paradox of Einstein, Podolsky, and Rosen" which marks an important (if also seemingly minor and merely technical) advance that would play an important role in Bell's Theorem, the subject of our Chap. 8.

Here is Bohm's lead-in to the discussion:

> What the authors [of EPR] wished to do with their criteria for reality was to show that the above interpretation of the present quantum theory is untenable and that the wave function cannot possibly contain a complete description of all physically significant factors (or 'elements of reality') existing within a system. If their contention could be proved, then one would be led to search for a more complete theory, perhaps containing something like hidden variables, in terms of which the present quantum theory would be a limiting case [9, p. 612].

And then here is Bohm's presentation of the EPR argument, re-framed in terms of the *spins* of a pair of spin 1/2 particles:

> We have modified the experiment somewhat [compared to the way it was presented in the actual EPR paper], but the form is conceptually equivalent to that suggested by them, and considerably easier to treat mathematically.

> Suppose that we have a molecule containing two atoms in a state in which the total spin is zero and that the spin of each atom is $\hbar/2$. Roughly speaking, this means that the spin of each particle points in a direction exactly opposite to that of the other, insofar as the spin may be said to have any definite direction at all. Now suppose that the molecule is disintegrated by some process that does not change the total angular momentum. The two atoms will begin to separate and will soon cease to interact appreciably. Their combined spin angular momentum, however, remains equal to zero, because by hypothesis, no torques have acted on the system.

> Now, if the spin were a classical angular momentum variable, the interpretation of this process would be as follows: While the two atoms were together in the form of a molecule, each component of the angular momentum of each atom would have a definite value that was always opposite to that of the other, thus making the total angular momentum equal to zero. When the atoms separated, each atom would continue to have every component of its spin angular momentum opposite to that of the other. The two spin-angular-momentum vectors would therefore be correlated. These correlations were originally produced when the atoms interacted in such a way as to form a molecule of zero total spin, but after the atoms separate, the correlations are maintained by the deterministic equations of motion of each spin vector separately, which bring about conservation of each component of the separate spin-angular-momentum vectors.

> Suppose now that one measures the spin angular momentum of any one of the particles, say No. 1. Because of the existence of correlations, one can immediately conclude that the angular-momentum vector of the other particle (No. 2) is equal and opposite to that of No. 1. In this way, one can measure the angular momentum of particle No. 2 indirectly by measuring the corresponding vector of particle No. 1.

Let us now consider how this experiment is to be described in the quantum theory. Here, the investigator can measure either the x, y, or z component of the spin of particle No. 1, but not more than one of these components, in any one experiment. Nevertheless, it still turns out as we shall see that whichever component is measured, the results are correlated, so that if the same component of the spin of atom No. 2 is measured, it will always turn out to have the opposite value. This means that a measurement of any component of the spin of atom No. 1 provides, as in classical theory, an indirect measurement of the same component of the spin of atom No. 2. Since, by hypothesis, the two particles no longer interact, we have obtained a way of measuring an arbitrary component of the spin of particle No. 2 without in any way disturbing that particle. If we accept the definition of an element of reality ... suggested by ERP, it is clear that after we have measured σ_z for particle 1, then σ_z for particle 2 must be regarded as an element of reality, existing separately in particle No. 2 alone. If this is true, however, this element of reality must have existed in particle No. 2 even before the measurement of σ_z for particle No. 1 took place. For since there is no interaction with particle No. 2, the process of measurement cannot have affected this particle in any way. But now let us remember that, in each case, the observer is always free to reorient the apparatus in an arbitrary direction while the atoms are still in flight, and thus to obtain a definite (but unpredictable) value of the spin component in any direction that he chooses. Since this can be accomplished without in any way disturbing the second atom, we conclude that ... precisely defined elements of reality must exist in the second atom, corresponding to the simultaneous definition of all three components of its spin. Because the wave function can specify, at most, only one of these components at a time with complete precision, we are then led to the conclusion that the wave function does not provide a complete description of all elements of reality existing in the second atom.

If this conclusion were valid, then we should have to look for a new theory in terms of which a more nearly complete description was possible [9].

As mentioned, Bohm himself became convinced of the need for such a "new theory" – and indeed produced one! – in the following year. But at the time of this writing he continued to accept the Copenhagen philosophy, according to which the EPR argument is not valid. We will review the Copenhagen philosophy (and in particular Bohr's reply to the EPR argument) in Chap. 6.

For now, let's just focus on the technical aspects of Bohm's reformulation of the EPR scenario in terms of the spins of two spin 1/2 particles. Using the notation of Chap. 2, in which for example "ψ^1_{+z}" denotes a state in which particle 1 is "spin up" along the z-direction, the state of total spin zero described by Bohm is the following:

$$\Psi = \frac{1}{\sqrt{2}} \left[\psi^1_{+z} \psi^2_{-z} - \psi^1_{-z} \psi^2_{+z} \right]. \tag{4.10}$$

This is a superposition of (one the one hand) a state in which particle 1 is "spin up" along z and particle 2 is "spin down" along z and (on the other hand) a state in which particle 1 is "spin down" and particle 2 is "spin up". The minus sign (i.e., the relative phase between the two terms in the superposition) turns out to be important. The qualitatively similar superposition with a "+" sign is also a state in which the z-component of the total spin is zero, but (unlike the state with the minus sign) the *magnitude* (squared) of the total spin is not zero. Indeed, this other state – with the "+" sign – naturally goes together with the "both particles are spin up" and "both particles are spin down" states to form a so-called *triplet* of states (with one unit of

total spin angular momentum, but with the z-component being -1, 0, and $+1$ in the three states).

By contrast, the state in Eq. (4.10) is sometimes called the *singlet* state because it alone has zero units of total spin angular momentum – and therefore zero for its z-component and indeed also all other components. I will leave the exploration of some of the mathematical aspects of this state for you to work through in the Projects.

But it should be clear, at least, that this EPR-Bohm state allows a simplified version of an EPR-type argument in the following way. The state in Eq. (4.10) is not an eigenstate of σ_z for particle 2. It is instead an entangled superposition of two states in which σ_z for particle 2 has two distinct values ($+1$ and -1). So according to the usual completeness doctrine, particle 2 has no definite value of σ_z when the state of the two particles is given by Eq. (4.10). However, by measuring σ_z of particle 1, we can determine – seemingly without disturbing particle 2 in any way – σ_z for particle 2: if particle 1 turns out to be spin-up, then particle 2 is spin-down, and vice versa. *After* such a measurement on particle 1, it thus seems clear that particle 2 *has* a definite z-spin. Then, either it had that definite z-spin value all along – in which case the quantum mechanical state description of Eq. (4.10) is revealed as having been incomplete – or its z-spin value only crystallized, from some earlier "blurry" state, as a result of the measurement on particle 1. But this latter possibility involves a kind of "spooky action-at-a-distance", i.e., a violation of local causality. We thus have to either accept the non-locality (and face the seemingly daunting task of trying to reconcile it with relativity) or abandon the completeness doctrine.

4.5 Bell's Re-Telling

John Bell – whose seminal 1964 theorem will be the subject of Chap. 8 – wrote extensively on the foundations of quantum theory and the EPR argument in particular. One of his presentations in particular is so amusing and beautiful and clear that I cannot help but include it here. It is from a paper with the intriguing title "Bertlmann's socks and the nature of reality". The paper begins:

> The philosopher on the street, who has not suffered a course in quantum mechanics, is quite unimpressed by Einstein–Podolsky–Rosen correlations. He can point to many examples of similar correlations in everyday life. The case of Bertlmann's socks is often cited. Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable. But when you see that the first sock is pink you can be already sure that the second sock will not be pink. Observation of the first, and experience of Bertlmann, gives immediate information about the second. There is no accounting for tastes, but apart from that there is no mystery here. And is not the EPR business just the same? [10]

Bell then reviews Bohm's reformulation of the EPR setup, with the two entangled spin-1/2 particles, including also a nice discussion of the difficulty of understanding the results of individual Stern–Gerlach spin measurements in terms of classical

magnetic dipoles one of whose pre-existing components is simply revealed by the measurement. Bell continues:

> Phenomena of this kind made physicists despair of finding any consistent space-time picture of what goes on on the atomic and subatomic scale. Making a virtue of necessity, and influenced by positivistic and instrumentalist philosophies, many came to hold not only that it is difficult to find a coherent picture but that it is wrong to look for one – if not actually immoral then certainly unprofessional. Going further still, some asserted that atomic and subatomic particles do not *have* any definite properties in advance of observation. There is nothing, that is to say, in the particles approaching the [Stern–Gerlach] magnet, to distinguish those subsequently deflected up from those subsequently deflected down. Indeed even the particles are not really there. [Note: to help prevent the reader from getting lost in quotes within quotes, passages that Bell quotes from other authors are *italicized* in the remainder of this block quote as well as the following one. In particular, the following italicized passages are quotations from Peterson, Heisenberg, Zilsel, Pauli, and Born.]

> For example, [Bohr's colleague Peterson recalled that] *Bohr once declared when asked whether the quantum mechanical algorithm could be considered as somehow mirroring an underlying quantum reality: 'There is no quantum world. There is only an abstract quantum mechanical description. It is wrong to think that the task of physics is to find out how Nature* is*. Physics concerns what we can say about Nature'.*

> And for Heisenberg *...in the experiments about atomic events we have to do with things and facts, with phenomena that are just as real as any phenomena of daily life. But the atoms or the elementary particles are not as real; they form a world of potentialities or possibilities rather than one of things or facts.*

> And [Zilsel recollects] *Jordan declared, with emphasis, that observations not only* **disturb** *what has to be measured, they* **produce** *it. In a measurement of position, for example, as performed with the gamma ray microscope, 'the electron is forced to a decision. We compel it* **to assume a definite position***; previously it was, in general, neither here nor there; it had not yet made its decision for a definite position... If by another experiment the* **velocity** *of the electron is being measured, this means: the electron is compelled to decide itself for some exactly defined value of the velocity... we ourselves produce the results of measurement'.*

> It is in the context of ideas like these that one must envisage the discussion of the Einstein–Podolsky–Rosen correlations. Then it is a little less unintelligible that the EPR paper caused such a fuss, and that the dust has not settled even now. It is as if we had come to deny the reality of Bertlmann's socks, or at least of their colours, when not looked at. And as if a child had asked: How come they always choose different colours when they *are* looked at? How does the second sock know what the first has done?

> Paradox indeed! But for the others, not for EPR. EPR did not use the word 'paradox'. They were with the man in the street in this business. For them these correlations simply showed that the quantum theorists had been hasty in dismissing the reality of the microscopic world. In particular Jordan had been wrong in supposing that nothing was real or fixed in that world before observation. For after observing only one particle the result of subsequently observing the other (possibly at a very remote place) is immediately predictable. Could it be that the first observation somehow fixes what was unfixed, or makes real what was unreal, not only for the near particle but also for the remote one? For EPR that would be an unthinkable 'spooky action at a distance'. To avoid such action at a distance they have to attribute, to the space-time regions in question, *real* properties in advance of observation, correlated properties, which *predetermine* the outcomes of these particular observations. Since these real properties, fixed in advance of observation, are not contained in quantum formalism, that formalism for EPR is *incomplete*. It may be correct, as far as it goes, but the usual quantum formalism cannot be the whole story [10].

That, I submit, is as clear as anyone will ever make the EPR argument, and it would seem appropriate to end the Chapter on that note.

But in the paper Bell goes on to discuss another aspect of Einstein's worries about the quantum theory, and I think it will be very illuminating to include this here as well:

> It is important to note that to the limited degree to which *determinism* plays a role in the EPR argument, it is not assumed but *inferred*. What is held sacred is the principle of 'local causality' – or 'no action at a distance'. Of course, mere *correlation* between distant events does not by itself imply action at a distance, but only correlation between the signals reaching the two places. These signals, in the idealized example of Bohm, must be sufficient to *determine* whether the particles go up or down. For any residual undeterminism could only spoil the perfect correlation.

> It is remarkably difficult to get this point across, that determinism is not a *presupposition* of the analysis. There is a widespread and erroneous conviction that for Einstein determinism was always *the* sacred principle. The quotability of his famous 'God does not play dice' has not helped in this respect. Among those who had great difficult in seeing Einstein's position was Born. Pauli tried to help him in a letter of 1954:

> ...*I was unable to recognize Einstein whenever you talked about him in either your letter or your manuscript. It seemed to me as if you had erected some dummy Einstein for yourself, which you then knocked down with great pomp. In particular, Einstein does not consider the concept of 'determinism' to be as fundamental as it is frequently held to be (as he told me emphatically many times)... he **disputes** that he uses as a criterion for the admissibility of a theory the question: 'Is it rigorously deterministic?' ... he was not at all annoyed with you, but only said you were a person who will not listen.*

> Born had particular difficulty with the Einstein–Podolsky–Rosen argument. Here is his summing up, long afterwards, when he edited the Born-Einstein correspondence:

> *The root of the difference between Einstein and me was the axiom that events which happen in different places A and B are independent of one another, in the sense that an observation on the state of affairs at B cannot teach us anything about the state of affairs at A.*

> Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could *influence*, immediately, affairs at the other.

> These references to Born are not meant to diminish one of the towering figures of modern physics. They are meant to illustrate the difficulty of putting aside preconceptions and listening to what is actually being said. They are meant to encourage *you*, dear listener, to listen a little harder [10].

Bell then closes this section of his paper by quoting the following "summing-up by Einstein himself", which is from Einstein's 1948 *Dialectica* essay:

> If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas in physics, one is first of all struck by the following: the concepts of physics relate to a real outside world.... It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space.'

> The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B...

> There seems to me no doubt that those physicists who regard the descriptive methods of quantum mechanics as definitive in principle would react to this line of thought in the

following way: they would drop the requirement ... for the independent existence of the physical reality present in different parts of space; they would be justified in pointing out that the quantum theory nowhere makes explicit use of this requirement.

I admit this, but would point out: when I consider the physical phenomena known to me, and especially those which are being so successfully encompassed by quantum mechanics, I still cannot find any fact anywhere which would make it appear likely that (that) requirement will have to be abandoned.

I am therefore inclined to believe that the description of quantum mechanics ... has to be regarded as an incomplete and indirect description of reality, to be replaced at some later date by a more complete and direct one [11].

And that seems like an entirely fitting way to close this chapter.

Projects:

4.1 What is the Bothe-Geiger experiment that Einstein mentions in his 1927 Solvay remarks? Do a little research and report back. (Hint: it relates to Compton scattering and something called the Bohr–Kramers–Slater or "BKS" theory, which was a kind of pre-cursor to the formal quantum theory that eventually developed.)

4.2 In the text, our application of Bell's formulation of "locality" to the Einstein's Boxes argument consisted of showing that quantum theory (with the completeness assumption and with the collapse postulate) violates locality. One could also, however, put the same pieces together in a slightly different way – showing that quantum theory (with the completeness assumption but *not* the collapse postulate) implies, if you assume locality, that there should be a nonzero probability for detecting the same one particle *twice*, once in the left half-box and once again in the right half-box. Explain carefully how this argument would go.

4.3 Show that the commutator $[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x}$ of $\hat{x}$ and $\hat{p}$ is the constant $i\hbar$. Hint: let $[\hat{x}, \hat{p}]$ act on an arbitrary function $f(x)$, using $\hat{x} = x$ and $\hat{p} = -i\hbar\frac{d}{dx}$, and show that you get $i\hbar f(x)$.

4.4 Prove that, if the commutator of two operators $\hat{A}$ and $\hat{B}$ is the (nonzero) constant c, then there cannot exist a state ψ which is a simultaneous eigenstate of both $\hat{A}$ and $\hat{B}$. Use this, along with the results of Project [4.3], to argue that in quantum mechanics there cannot be a state which attributes a sharp value to position and momentum simultaneously – a point that was crucial in the EPR argument.

4.5 Give a careful summary of the argument for incompleteness that Einstein gives in his "Autobiographical Notes" (quoted in Sect. 4.3).

4.6 One assumption of all these EPR-type arguments, that is sometimes taken for granted and not given the attention it maybe deserves, is the assumption that the statistical predictions of quantum mechanics in the relevant situations are actually *correct*. For example, in the "boxes" type argument, it is assumed that, indeed, each particle will only be found at one place later. An early experiment by Ádám, Jánossy, and Varga (ÁJV) attempted to test this prediction, but their results were not very conclusive. Here is a bit of description, though, from John Clauser, who re-did a more convincing version of the experiment in the 1970s:

> As an original heretic to the standard religion, [Schrödinger] persuaded *Ádám, Jánnosy and Varga* (ÁJV) to actually perform [this] experiment[:] two independent photodetectors are placed respectively in the transmitted and reflected beams of a half-silvered mirror. If photons have a particle-like character, i.e., if their detectable components are always spatially bounded and well localized, then photons impinging on the half-silvered mirror will not be split in two at this mirror. On the other hand, if they are purely wave-like in nature, ... then they can and will be split into two independent classical wave packets at this mirror. This fact then implies that if they are purely wave-like (in this classical sense), then the two detectors will show coincidences when a single temporally localized photon is directed at said mirror. One of these independent wave packets will be transmitted to illuminate the first detector, and the other will be reflected to illuminate the second detector, and both detectors will then have a finite probability of detecting the same photon (classical wave packet). This latter possibility, however, violates the predictions [of quantum theory] which prohibits such coincidences. ÁJV thus searched for anomalous coincidences between photomultiplier tubes that viewed the reflected and transmitted beams behind a half -silvered mirror [12].

Take a look at Clauser's experimental paper, Ref. [13], reporting the results of his later version of this kind of experiment. Summarize his experimental setup and findings.

4.7 Read the actual EPR paper [6] and report back, sharing any insights, confusions, and/or questions.

4.8 Recall the quantum mechanical collapse postulate: when a measurement of some observable is made, the quantum state changes suddenly and discontinuously into the particular eigenstate (of that observable) corresponding to the actually-realized outcome of the measurement. The theme of Chap. 3 could perhaps be summarized by saying that there are two different ways one could interpret this collapse rule, and each seems problematic: first, if you think of the collapse as describing a real physical change in the state of the system, i.e., as a dynamical process, this seems impossible to reconcile with the *normal* system dynamics (namely Schrödinger's equation); whereas, second, if you think of the collapse as describing a mere updating of information, i.e., as describing a change in our *knowledge* not implying any change in the physical state of the thing described, then the quantum state descriptions are revealed as obviously incomplete. Explain how the EPR dilemma between locality and completeness can be understood in these same terms, with the two horns of the dilemma corresponding exactly to the two views (dynamic vs. epistemic) one might take to wave function collapse. What, exactly, does the EPR argument then *add* to the arguments from Chap. 3? How is it different or better?

4.9 Illustrate the EPR scenario on a space-time diagram showing (i) the preparation of the two-particles at some central source, (ii) the separating of the two entangled particles, and (iii) the measuring equipment that will perhaps be used to measure some property of one or both properties. Can Bell's formulation of "locality" be used to rehearse a more formally rigorous version of the argument, along the lines of what we did in the Chapter with the Einstein's Boxes argument? If so, explain how; or if not, explain why not. (Note: this is a bit of a

trick question, for reasons that will be discussed in the following Chapter. But it is still well worth thinking about here.)

4.10 Re-write the "EPR-Bohm state" (that is, the spin-singlet state of the two spatially-separated spin 1/2 particles), Eq. (4.10), in terms of the states ψ_{+x} and ψ_{-x}, whose relation to the states ψ_{+z} and ψ_{-z} are explained in Chap. 2. Use your re-writing to argue that not only, as explained in the text, a measurement of σ_z on particle 1 provides an indirect determination of the value of σ_z of particle 2, but that also a measurement of σ_x on particle 1 provides an indirect determination of the value of σ_x of particle 2. Discuss how and whether and under what assumptions this all implies that the distant particle must possess (contrary to what you'd say if you believe the quantum state provides a complete description) definite values of *both* σ_x *and* σ_z.

4.11 Re-write the EPR-Bohm spin state in terms of the states ψ_{+n} and ψ_{-n} from Chap. 2. Thus show that the singlet state takes the same mathematical form for spin components along *any* arbitrary axis, and that therefore the argument from the text, establishing the real existence of σ_z for the distant particle (which argument was already generalized to σ_x in Project 4.10) applies to *all* possible axes.

4.12 Suppose two spin 1/2 particles are prepared in the EPR-Bohm spin state, Eq. (4.10). Now suppose that the spin of one of the particles (say, particle 1) is measured along the z-direction, and the spin of the other particle (2) is measured along the direction $\hat{n}$ (in the $x - z$-plane and making an angle θ with respect to the z-axis). What are the probabilities for the four possible joint outcomes (i.e., "particle 1 is spin-up/spin-down along z and particle 2 is spin-up/spin-down along n)? To answer this, write the EPR-Bohm state as a linear combination of four terms of the form $\psi^1_{\pm z}\psi^2_{\pm n}$ and read off the probabilities as the absolute squares of the coefficients. Finally, compute the "correlation coefficient", defined here as the expected value of the *product* of the two outcomes, taking spin-up/spin-down as $+1/-1$:

$$C = (+1)(+1)P_{++} + (+1)(-1)P_{+-} + (-1)(+1)P_{-+} + (-1)(-1)P_{--}. \quad (4.11)$$

Does the correlation coefficient make sense in various limiting cases like $\theta = 0$?

4.13 Sometimes the EPR argument (say, in the Bohm version in terms of spin) is explained as follows: "you can determine *both* σ_x and σ_z for the same one particle at the same time, by measuring one of these quantities *directly* (i.e., by actually measuring it on that particle) and then by also measuring the other quantity *indirectly* (i.e., by actually measuring that same quantity on the *other* particle and then attributing the opposite value to the particle in question)." The conclusion is then something like: "...so both of these properties must be real (or, at least, you can learn more about them than is supposed to be allowed by the uncertainty principle) and QM is incomplete." Is this a good argument? Discuss its merits and its relation to the actual EPR argument.

4.14 Here is a paragraph from the Wikipedia page on the "EPR Paradox" (grabbed on Jan 7, 2016):

> While EPR felt that the paradox showed that quantum theory was incomplete and should be extended with hidden variables, the usual modern resolution is to say that due to the common preparation of the two particles (for example the creation of an electron-positron pair from a photon) the property we want to measure has a well defined meaning only when analyzed for the whole system while the same property for the parts individually remains undefined. Therefore if similar measurements are being performed on the two entangled subsystems, there will always be a correlation between the outcomes resulting in a well defined global outcome, i.e., for both subsystems together. However, the outcomes for each subsystem separately at each repetition of the experiment will not be well defined or predictable. This correlation does not imply any action of the measurement of one particle on the measurement of the other, therefore it doesn't imply any form of action at a distance. This modern resolution eliminates the need for hidden variables, action at a distance or other structures introduced over time in order to explain the phenomenon.

Pretend that you are Einstein, magically transported to the present day, with both internet access and too much free time. Write a few paragraphs that you would post on the Wikipedia discussion page, explaining to the other contributors how and why the "modern resolution" described here is inadequate, and clarifying your original argument.

4.15 In Bell's re-telling of the EPR argument, he stresses that determinism is "not assumed, but *inferred*." This may be somewhat confusing since the notion of "determinism" did not play much of a role in our earlier presentations of the EPR argument. What, exactly, is the role of "determinism" in the argument? Is Bell correct?

4.16 Provide a detailed explanation of the sort of theoretical model which could explain the quantum mechanical predictions for the Einstein's Boxes scenario in a perfectly local way. Then do the same for the EPR-Bohm scenario.

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