Chapter 5
The Ontology Problem

The previous two chapters reviewed two important arguments against the idea that quantum mechanics provides *complete* descriptions of physical reality. Here we discuss a couple of other related concerns which might be summarized by the general question: even leaving aside the question of whether or not the description is complete, what kind of physical thing – what ontology, exactly – could the quantum wave function possibly represent, and how would that representation work?

5.1 Complexity and Reality

Every student of quantum mechanics learns that the wave function Ψ is *complex*: it has, in general, both a real part and an imaginary part. This is of course not surprising given the explicit appearance of the imaginary quantity $i = \sqrt{-1}$ in the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi. \tag{5.1}$$

Some textbooks make a big deal of the fact that Ψ is complex – arguing that (unlike complex numbers that are sometimes used as a matter of convenience in for example classical electrodynamics) the complexity of the quantum wave function is somehow a deep and fundamental requirement with profound implications. Indeed, one of these alleged implications is that the quantum mechanical wave function Ψ *cannot* represent a "physically real field" in the way, for example, that the classical electric and magnetic fields, $\vec{E}$ and $\vec{B}$ are supposed to. The classic *Quantum Physics* text by Eisberg and Resnick, for example, states:

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> The fact that wave functions are complex functions should not be considered a weak point in the quantum mechanical theory. Actually, it is a desirable feature because it makes it immediately apparent that we should not attempt to give to wave functions a physical existence in the same sense that water waves have a physical existence. The reason is that a complex quantity cannot be measured by any actual physical instrument. The 'real' world (using the term in its nonmathematical sense) is the world of 'real' quantities (using the term in its mathematical sense) [1, p. 134].

If such a view is correct, then evidently we would have to reject the idea that the quantum mechanical wave function provides a complete description of physical reality: if Ψ cannot represent something physically real at all, because it is not a mathematically real function, then certainly it cannot provide a faithful and full representation!

But I do not think this type of argument is convincing at all. It is simply false that one is somehow *required* to use complex numbers. For example, one could always break the quantum wave function apart into its real and imaginary parts:

$$\Psi(x,t) = f(x,t) + i\,g(x,t) \tag{5.2}$$

(where f and g are now *real* functions). Then, by plugging this ansatz into Schrödinger's equation and taking real and imaginary parts, we can break Schrödinger's (complex) equation into two coupled real equations:

$$-\hbar\frac{\partial g}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 f + Vf \tag{5.3}$$

and

$$\hbar\frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 g + Vg. \tag{5.4}$$

We could think of this set of equations as perhaps something like Maxwell's equations, which couple together the dynamics of the two (perfectly real!) fields $\vec{E}$ and $\vec{B}$.

It should be clear that one can always do this: a single complex field is mathematically equivalent to two coupled real fields. So the most the Eisberg/Resnick-type argument could establish is that, if one wants to regard the quantum wave function as representing something physically real, one would have to interpret it as representing these two coupled fields.

This may sound somewhat contrived and artificial, but actually there is an even closer parallel to all of this in Maxwellian electromagnetism that is worth pointing out. Take, for simplicity, the case of electromagnetic fields propagating in empty space (where the charge density ρ and the current density $\vec{j}$ both vanish). The four Maxwell equations are then

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \text{and} \qquad \vec{\nabla}\cdot\vec{B} = 0 \tag{5.5}$$

and then also

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{and} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}. \tag{5.6}$$

Now notice that we can rewrite these four equations – and indeed make them look a little simpler and nicer! – by re-writing them in terms of the *complex* quantity

$$\vec{F} = \vec{E} + ic\vec{B} \tag{5.7}$$

which is sometimes called the Riemann–Silberstein vector. (Note that the "c" is put into the definition so the units of the two terms on the right hand side are the same.) In terms of this quantity, it is easy to see that the first pair of Maxwell equations can be re-expressed as:

$$\vec{\nabla} \cdot \vec{F} = 0. \tag{5.8}$$

The real and imaginary parts, respectively, reproduce the two entries in Eq. (5.5). Now the cool thing is that the two entries in Eq. (5.6) can also be reproduced by writing

$$\vec{\nabla} \times \vec{F} = \frac{i}{c}\frac{\partial \vec{F}}{\partial t}. \tag{5.9}$$

You should take a second and check that, indeed, the real and imaginary parts of this equation correspond exactly to the two entries in Eq. (5.6).

One can even rearrange the last equation into the following form:

$$i\hbar\frac{\partial \vec{F}}{\partial t} = \hbar c \vec{\nabla} \times \vec{F} \tag{5.10}$$

whose structure is rather like that of Schrödinger's equation:

$$i\hbar\frac{\partial \vec{F}}{\partial t} = \hat{H}\vec{F} \tag{5.11}$$

where, evidently, the Hamiltonian operator here is

$$\hat{H} = \hbar c\, \vec{\nabla} \times \ . \tag{5.12}$$

Indeed, the complex vector $\vec{F}$ can (with some caveats) be understood as a kind of "quantum wave function for the photon." But we will not pursue this interesting connection any further here.

Our point is instead just that the dynamical equations – that is, Maxwell's equations – for the fields $\vec{E}$ and $\vec{B}$ can be re-expressed in an elegant form by combining $\vec{E}$ and $\vec{B}$ into a single, complex-valued quantity, $\vec{F}$. Yet nobody, I think, regards this as some kind of proof that $\vec{F}$ cannot correspond to anything physically real. It does! Its real part – $\vec{E}$ – corresponds to the physically real electric field, and its imaginary part

– $\vec{B}$ – corresponds to the physically real magnetic field. We should therefore allow this same flexibility of mathematical representation in the case of the quantum wave function Ψ and remain open to the possibility that, despite being complex-valued, it represents a physically real (or perhaps more than one coupled physically real) field.

5.2 Configuration Space

There is, however, a different mathematical fact about the quantum wave function Ψ that leads to a much more difficult and troubling question about the ontology it might conceivably describe: Ψ is a function on *configuration space*. So if Ψ represents one or more *fields* – things somehow like the electromagnetic fields $\vec{E}$ and $\vec{B}$ – the fields would be very, very unusual because they would live, not in ordinary three-dimensional physical space, but rather a higher-dimensional and seemingly purely abstract space.

This was, interestingly, a worry that arose almost immediately when Schrödinger first invented (and wrote down his dynamical equation for) the wave function. In 1926, for example, Schrödinger sent copies of his papers to a number of his colleagues and asked for their comments. Hendrik Lorentz, in his very first reply, praised Schrödinger's work for its physical/intuitive comprehensibility (compared to the more purely mathematical, and hence physically obscure, "matrix mechanics" which had previously been developed by Heisenberg and others). But Lorentz also raised a number of concerns about Schrödinger's wave mechanics, and the very first of these concerns had to do with the fact that the wave was not a wave in physical space but instead the abstract configuration space:

> Dear Colleague,
>
> I am finally getting around to answering your letter and to thanking you very much for kindly sending me the proof sheets of your three articles, all of which I have in fact received. Reading these has been a real pleasure to me. Of course the time for a final judgment has not come yet, and there are still many difficulties, it seems to me, about which I shall get to speak immediately. But even if it should turn out that a satisfactory solution cannot be reached in this way, one would still admire the sagacity that shows forth from your considerations, and one would still venture to hope that your efforts will contribute in a fundamental way to penetrating these mysterious matters.
>
> I was particularly pleased with the way in which you really construct the appropriate matrices and show that these satisfy the equations of motion. This dispels a misgiving that the works of Heisenberg, Born, and Jordan, as well as Pauli's, had inspired in me: namely, that I could not see clearly that in the case of the H-atom, for example, a solution of the equations of motion can really be specified. With your clever observation that the operators q and $\frac{\partial}{\partial q}$ commute or do not commute with each other in a similar way to the q and p in the matrix calculation, I began to see the point. In spite of everything it remains a marvel that equations in which the q's and p's originally signified coordinates and momenta, can be satisfied when one interprets these symbols as things that have quite another meaning, and only remotely recall those coordinates and momenta.
>
> If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only

> has to deal with the three coordinates x, y, z. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics. But your way of thinking has the advantage for this case too that it brings us closer to the real solution of the equations; the eigenvalue problem is the same in principle for a higher dimensional q-space as it is for a three dimensional space [2, p. 43–44].

Note in particular the suddenness – the immediate finality – with which Lorentz simply dismisses the possibility that a function on configuration space could represent a physically real field: "I cannot interpret the waves and vibrations physically...."

Indeed, it is interesting that Lorentz does not just say that, in the case of two (or more) particles where the configuration space is 6- (or more) dimensional, he is confused about how to interpret Schrödinger's wave function. Instead, he says he would prefer in this case to go back to the physically obscure matrix mechanics. We can only speculate about exactly what he meant, and this was admittedly only a first impression, but one gets the feeling that he preferred to have no intuitive physical interpretation available at all, rather than one which was so obviously absurd as to suggest the existence of fields/waves in an unphysical, abstract space.

Einstein expressed a similar concern about Schrödinger's wave function in letters from this same period. Here are some excerpts, all quoted in Ref. [3]:

- "Schrödinger's conception of the quantum rules makes a great impression on me; it seems to me to be a bit of reality, however unclear the sense of waves in n-dimensional q-space remains". (May 1, 1926, to Lorentz)
- "Schrödinger's works are wonderful – but even so one nevertheless hardly comes closer to a real understanding. The field in a many-dimensional coordinate space does not smell like something real." (June 18, 1926, to Ehrenfest)
- "The method of Schrödinger seems indeed more correctly conceived than that of Heisenberg, and yet it is hard to place a function in coordinate space and view it as an equivalent for a motion. But if one could succeed in doing something similar in four-dimensional space, then it would be more satisfying." (June 22, 1926, to Lorentz)
- "Of the new attempts to obtain a deeper formulation of the quantum laws, that by Schrödinger pleases me most. If only the undulatory fields introduced there could be transplanted from the n-dimensional coordinate space to the 3 or 4 dimensional!" (August 21, 1926, to Sommerfeld)
- "Schrödinger is, in the beginning, very captivating. But the waves in n-dimensional coordinate space are indigestible..." (August 28, 1926, to Ehrenfest)
- "The quantum theory has been completely Schrödingerized and has much practical success from that. But this can nevertheless not be the description of a real process. It is a mystery." (February 16, 1927, to Lorentz)

Even Schrödinger himself admitted quite openly that, as a function on configuration space, the wave function can't really be understood as corresponding to some kind of physically real wave. In the abstract of "Wave Mechanics," his contribution to the 1927 Solvay conference, he wrote:

> Of course this use of the q-space is to be seen only as a mathematical tool, as it is often applied also in the old mechanics; ultimately ... the process to be described is one in space and time [4, p. 447].

In the body of the paper he elaborates on the crucial question:

> What does the ψ-function mean now, that is, *how does the system described by it really look like in three dimensions*? Many physicists today are of the opinion that it does not describe the occurrences in an individual system, but only the processes in an ensemble of very many like constituted systems that do not sensibly influence one another and are all under the very same conditions. I shall skip this point of view since others are presenting it. I myself have so far found useful the following perhaps somewhat naive but quite concrete idea. The classical system of material points does not really exist, instead there exists something that continuously fills the entire space and of which one would obtain a 'snapshot' if one dragged the classical system, with the camera shutter open, through *all* its configurations, the representative point in q-space spending in each volume element $d\tau$ a time that is proportional to the *instantaneous* value of $\psi\psi^*$. (The value of $\psi\psi^*$ for only *one* value of the argument t is thus in question.) Otherwise stated: the real system is a superposition of the classical one in all its possible states, using $\psi\psi^*$ as 'weight function' [4, p. 453].

The first view that Schrödinger mentions here – according to which the ψ function "does not describe ... an individual system" but instead characterizes "an ensemble of very many like constituted systems" – is the idea, argued for in the previous two chapters, that the wave function does *not* provide a complete description of physical reality.

But in contrast to these interpretations Schrödinger here suggests an alternative view in which physical reality really is faithfully described by the wavy, spread-out wave function. He speaks of "something that continuously fills the entire space" and then suggests that one could perhaps understand "the real system [as] a superposition of the classical one in all its possible states, using $\psi\psi^*$ as a 'weight function'."

For a single particle, whose wave function $\psi(\vec{x}, t)$ lives in ordinary, physical, three-dimensional space, one can understand this idea as saying that the "particle" is really a cloud whose density is given by the square of the wave function. For example, one could characterize the electron in terms of a mass density

$$\rho_m(\vec{x}, t) = m\,|\psi(\vec{x}, t)|^2 \tag{5.13}$$

or an electric charge density

$$\rho_e(\vec{x}, t) = e\,|\psi(\vec{x}, t)|^2 \tag{5.14}$$

where m and e are the total mass and charge of the electron. One often sees – in, for example, Chemistry textbooks – pictures of the "electron cloud" surrounding the nucleus for, say, different states of the Hydrogen atom. Such pictures invite you to think of the wave function in the way that Schrödinger was suggesting here.

The problem, of course, is that this interpretation doesn't make any sense as soon as one has a quantum system with more than one particle in it. Then the wave function Ψ is a function on configuration space, and so the "charge density"

$$\rho \sim |\Psi|^2 \tag{5.15}$$

would also be a charge density in this high-dimensional, abstract space. And that, to use Einstein's phrase, simply "does not smell like something real".

It is sometimes difficult for people to fully grasp the nature of the problem associated with the fact that the quantum mechanical wave function (for a system of more than 2 particles) is a function on configuration space. Basically the problem is that, mathematically, the wave function is – like the electric and magnetic fields of classical electrodynamics – a function of continuous spatial degrees of freedom which satisfies a dynamical evolution equation with the general structure of a *wave equation*, i.e., a partial differential equation relating spatial and temporal partial derivatives. Mathematically, in short, the wave function seems to look and act like a *field*. But unlike the familiar and unproblematic electric and magnetic fields, $\vec{E} = \vec{E}(\vec{x}, t)$ and $\vec{B} = \vec{B}(\vec{x}, t)$, we cannot ask for the value of the wave function at a point in three-dimensional space at a particular time: it is not $\Psi = \Psi(\vec{x}, t)$ but rather $\Psi = \Psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, t)$. So if the wave function describes a *field*, it would not appear to be a *physical* field in the sense that we are accustomed to thinking about $\vec{E}$ and $\vec{B}$ from electrodynamics. So if the wave function provides a description of some physically real things or stuff, the description must be in some sense indirect, abstract.

But whereas a single point in 3N-dimensional configuration space can be easily understood as an abstract representation of the configuration of N particles in 3-dimensional physical space (that was the kind of thing we discussed when "configuration space" came up in Chap. 1), it is hard to see what kind of thing a *field* in configuration space might be an abstract representation *of*. That is, when we are dealing with a multi-particle wave function in quantum mechanics, it is simply not clear, the way it is clear when we use configuration space in the context of the classical mechanics of particles, what we are talking about!

Of course, one apparent possibility (the one mentioned first by Schrödinger) would be that the wave function simply describes our incomplete *knowledge* of the state of a set of (literal, pointlike) particles. On this view, the wave function isn't a physical thing (like a field) at all – the wave function, that is, is epistemic, not ontological. As discussed already in Chap. 3, this interpretation is very difficult to reconcile with, for example, the existence of 2-slit interference. Still, it is helpful to have in mind as a possible way of avoiding the "ontology problem" which, it would seem, is going to afflict any theory that takes the quantum mechanical wave function seriously, as providing a direct and literal description of some kind of physically real thing. This problem would seem to be particularly worrisome for the orthodox viewpoint according to which the wave function provides a *complete* description of the physical state of the system being described. The *ontology* of orthodox quantum mechanics, that is, seems entirely mysterious: the one kind of thing that we can straightforwardly understand the wave function as describing (namely, a physical field living in the abstract, high-dimensional configuration space) seems unacceptably bizarre and absurd and, indeed, seems not really to be a legitimate physical "thing" at all.

Perhaps we can thus summarize "the ontology problem" as follows: in quantum mechanics, there simply is nothing in the theory other than the wave function Ψ with which to describe the physical state of a microscopic system; but it simply is not clear how the wave function Ψ might be understood as describing some material structures in three-dimensional physical space. Put simply, it is just not at all clear, from the mathematical formulation of the theory, what sort of physical things quantum mechanics might be *about*.

In Sect. 5.4 we will consider an old idea of Schrödinger's for trying to address the question: what kind of (field-like) physical reality might the wave function be an abstract representation *of*? But first, let us develop a bit further our thinking about the nature of the ontology problem in general.

5.3 Ontology, Measurement, and Locality

To help flesh out the problem and to emphasize its fundamentality, let's explore the connections between the ontology problem and the other two problems we discussed in the previous two chapters.

Recall first the measurement problem. Here is a quick summary of the way we presented the problem in Chap. 3, using the concrete example of the particle-in-a-box whose energy is to be measured by a device which will indicate the result with the position of a pointer:

> If the particle-in-a-box starts out in an energy eigenstate (such that it *has* a definite pre-measurement energy E_n), the Schrödinger equation evolution of the particle-pointer system proceeds according to
>
> $$\psi_n(x)\,\phi(y) \quad \rightarrow \quad \psi_n(x)\,\phi(y - \lambda E_n T). \tag{5.16}$$
>
> This is unproblematic since the final state $\Psi(x, y, T) = \psi_n(x)\,\phi(y - \lambda E_n T)$ describes the particle (still) being in a state of definite energy E_n and describes the pointer as having moved a definite distance $(\lambda E_n T)$ past its initial position, i.e., correctly and unambiguously indicating that the energy of the particle was E_n.
>
> However, if the particle-in-a-box instead starts out in a superposition of energy eigenstates, the Schrödinger evolution of the particle-pointer wave function looks like this:
>
> $$\left[\sum_i c_i \psi_i(x)\right] \phi(y) \quad \rightarrow \quad \sum_i c_i\,\psi_i(x)\,\phi(y - \lambda E_i T). \tag{5.17}$$
>
> This is highly problematic since the final state does not seem to attribute any particular position to the pointer; instead, the pointer is in an (entangled) superposition of many distinct locations, and this simply doesn't seem to correspond to the observed behavior of real pointers.

Our main goal in Chap. 3, following Schrödinger, was to emphasize the difficulty of understanding macroscopic superpositions. It was from this point of view that we stressed the problematic character of the final state in Eq. (5.17), as against the comparatively unproblematic final state in Eq. (5.16).

But now we'd like to cycle back and ask: is the final state in Eq. (5.16) really so unproblematic? When we said so before, we simply took for granted that we could understand a product state like

$$\Psi(x, y, T) = \psi_n(x)\,\phi(y - \lambda E_n T) \tag{5.18}$$

as saying "the particle-in-a-box is in the nth energy eigenstate" *and* "the pointer is a distance $\lambda E_n T$ to the right of its ready position". But what is really going on when we interpret this state this way?

First of all, we are assuming that when the overall wave function is a product, we can simply "peel apart" the factors and take them individually as describing the states of the individual sub-systems in question. As straightforward as this is in the case of a product state, though, it is simply not possible in general: as soon as the overall state fails to be a product, nothing like this straightforward mathematical "peeling apart" procedure is possible. So we should probably be suspicious of the propriety of this procedure in general, i.e., even in the special kind of situation where it is technically possible.

Second, our descriptions of the physical meanings of the individual factors were a little bit abstract. For example, to say that the particle-in-a-box has a certain energy is really just to say that an appropriate energy measuring device will respond in a certain way if it is allowed to interact with the particle. A more direct and literal description of the state of the particle (in terms of constitutive rather than dispositional properties, one might say) would instead just say: its wave function has a certain spatial structure, namely, that of $\psi_n(x)$.

The overall point is that if we simply take the quantum mechanical description literally, then the state of the particle-pointer system is just given by its wave function Ψ. And this, taken at face value, means that there are some regions of the abstract (here, two-dimensional) configuration space where Ψ has "high intensity". This is depicted, in Fig. 5.1, for the final states $\Psi(x, y, T)$ associated with the two cases from Eqs. (5.16) and (5.17). And then the point is that, even in the simpler case depicted in the left panel, which we previously regarded as relatively unproblematic, there is a question about how this wave function corresponds to the kinds of physical objects we thought we were describing. This high intensity region of configuration space – the "blob" colored grey in the left panel of Fig. 5.1 – doesn't exactly look like a particle-in-a-box (in some particular state) and a pointer (with some reasonably sharp location), each moving in a one-dimensional physical space.

To be clear, the claim here is not that this problem is insoluble. The obvious response would be to insist that the "blob" in configuration space *does* represent the state of our two particles, but the representation is somehow indirect or abstract. After all, the single dot in the right-hand panel of Fig. 1.12, back in Chap. 1, didn't really "look like" the two particles depicted, in ordinary physical space, in the left-hand panel of that same Figure. And yet we have no trouble understanding how the one thing can perfectly well represent the other. Isn't it just the same here with our quantum mechanical wave functions?

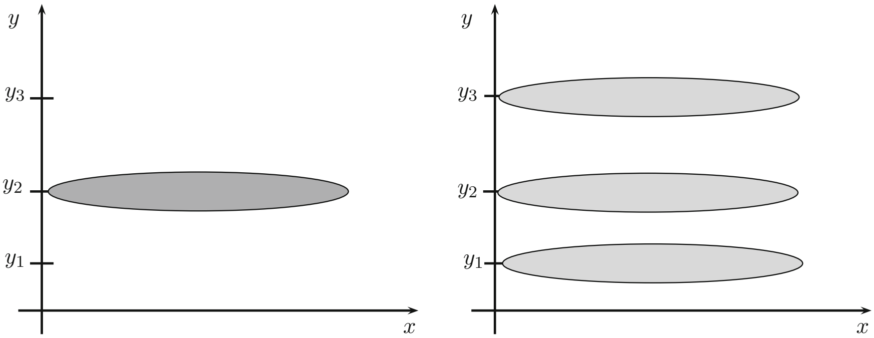

Fig. 5.1 "Configuration space cartoons" showing the intensity of $\Psi(x, y, T)$ for the two cases discussed in the text. The *left panel* corresponds to Eq. (5.16), in which the particle-in-a-box (whose energy is being measured) begins in the nth energy eigenstate (here the case $n = 2$ is depicted) and the pointer's final position is y_n (so, here, y_2). The right panel corresponds to Eq. (5.17), in which the particle-in-a-box begins in a superposition of several energy eigenstates, and so the pointer ends up in an entangled superposition involving several different positions (here y_1, y_2, and y_3). The novel point being developed in the present chapter is that, in addition to the difficulties associated with the wave function depicted in the *right panel*, there is a deeper kind of problem: even the single blob in the *left panel* does not, on its face, "look like" two particles (the particle-in-the-box and the pointer) moving in one spatial dimension. The relationship between the quantum mechanical wave function, and some ontology of objects in three-dimensional space, remains obscure

The point is: it might well be. But unlike the case of classical mechanics, where we *started out* with a clear ontology of particles (moving and interacting in three-dimensional space) and then constructed abstract representations, like configuration space, to describe these particles in a new way; here, in the quantum mechanical case, we *only* have, as yet, the abstract representation. We don't yet know what kind of reality, what sort of physically real objects or stuff, in three-dimensional space, the wave functions might be abstract representations *of*.

Pointing out that quantum mechanics suffers from an "ontology problem" is thus largely a plea for help: anyone who says that quantum mechanical wave functions should be taken seriously, as corresponding in some sense to physical reality (as opposed, for example, to our incomplete knowledge), should be asked to explain in concrete, mundane detail how that alleged correspondence works. They should tell us what sorts of things (particles?) or stuff (fields??) quantum mechanics is *about*, and clarify in precise mathematical detail the relationship between those things (and/or that stuff) and quantum mechanical wave functions. Until or unless this is done, I think we have to admit that the connection with the three-dimensional reality of direct experience remains puzzling, not only for the final state described in Eq. (5.17), but also – already – for the state described in Eq. (5.16).

Let us then turn to exploring how the "ontology problem" relates to the other of the two big worries we explored previously: the "locality problem".

The locality problem, recall, was that – if one regards quantum mechanical wave functions as providing complete descriptions of physical states – then quantum mechanics evidently violates the relativistic notion of local causality. We developed Bell's careful formulation of local causality in Chap. 1 and then showed explicitly, in Chap. 3, how local causality is violated by ordinary quantum mechanics (assumed complete) in the "Einstein's Boxes" scenario. But you might have noticed that we never applied our explicit formulation of local causality to the two-entangled-particle EPR scenario in the same way.

The reason for this has to do with the ontology problem. Recall that, in Bell's formulation of locality, we need to compare the probabilities assigned to some event, conditioned on a *complete specification of the physical state on a slice across the past light cone of that event*, when some distant event is, and is not, also specified. In the EPR-Bohm situation, in which the two particles are jointly in the spin singlet state

$$\Psi = \frac{1}{\sqrt{2}} \left[\psi^1_{+z}\, \psi^2_{-z} \; - \; \psi^1_{-z}\, \psi^2_{+z} \right], \tag{5.19}$$

for example, we might ask whether

$$P\,[\,A\,|\,\mathcal{C}_\Sigma\,] \; = \; P\,[\,A\,|\,\mathcal{C}_\Sigma,\; B\,] \tag{5.20}$$

where A is, say, the event "particle 1 comes out spin-up along the z-direction", B is the event "particle 2 comes out spin-up along the z-direction", and $\mathcal{C}_\Sigma$ is a complete specification of the physical state in region Σ indicated in Fig. 5.2.

Intuitively, one wants to say that locality is violated, because, according to quantum mechanics (assumed complete) there is just an irreducible 50/50 probability for a z-spin measurement on particle 1 to come out up/down. So the probability on the left hand side of Eq. (5.20) is 50%. Whereas if we specify also B – that a z-spin measurement on particle 2 comes out spin-up – then the probability assigned to particle 1 being spin-up along the z-direction, i.e., the right hand side of Eq. (5.20) is instead zero. So locality is violated, just as in the one-particle Einstein's Boxes scenario.

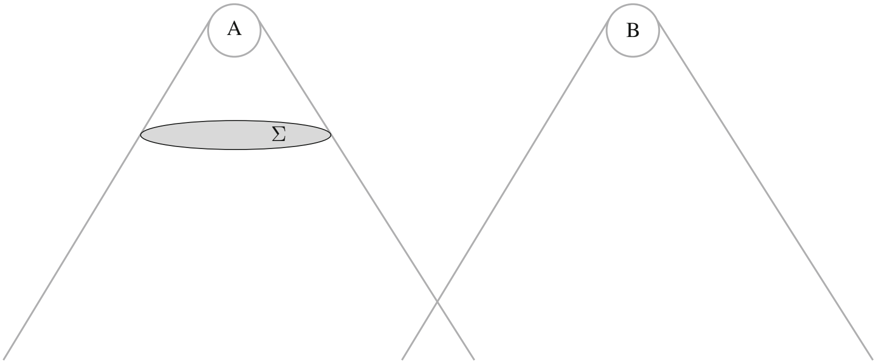

Fig. 5.2 Space-time diagram for the attempted application of Bell's formulation of local causality to the Einstein–Podolsky–Rosen scenario

But is it really so clear that, for example, $P\,[A|\mathcal{C}_\Sigma] = 50\%$? What, exactly, is $\mathcal{C}_\Sigma$ here? Again, intuitively, one wants to say: it is the complete quantum mechanical description of the state of particle 1, i.e., the wave function of particle 1. But when particles 1 and 2 are in an entangled state, *there is simply no such thing as* "the wave function of particle 1". One cannot "peel apart" the two-particle state into two one-particle states when the two-particle state is entangled.

This may be slightly obscure since, in the EPR-Bohm scenario, we are largely suppressing the spatial degrees of freedom of the two particles and focusing on the (more abstract) spin degrees of freedom. But (as we saw already in Chap. 2) it is perfectly possible for the spatial degrees of freedom of two particles to be entangled. Indeed, you will recall that the original EPR argument was framed in terms of an example involving the entanglement of the spatial degrees of freedom of two particles. Here is another example that is a little better suited to our immediate needs here: suppose that particle 1 is definitely in a room a million miles to the West, either on the left side of the room (ψ_L^W) or on the right side of the room (ψ_R^W). Similarly, particle 2 is in a different room, a million miles to the East, either on the left side of that room (ψ_L^E) or the right side of that room (ψ_R^E). Then suppose in particular that the two particles are in the following entangled state

$$\Psi(x_1, x_2) = \frac{1}{\sqrt{2}} \left[\psi_L^W(x_1)\, \psi_L^E(x_2) + \psi_R^W(x_1)\, \psi_R^E(x_2) \right] \tag{5.21}$$

which can be understood as a superposition of "both particles are on the left sides of their respective rooms" and "both particles are on the right sides of their respective rooms".

It should be clear that, if the quantum mechanical state of the two particles jointly is given by Eq. (5.21), it is impossible to assign a one-particle wave function to either particle alone. If you have even the slightest doubt about this, take a minute and try to work out what you think "the wave function of particle 1" is, for example.[1]

The problem here is of course just the problem we've been focusing on in this chapter: quantum mechanical wave functions for multiple-particle systems are something like fields, but in an abstract configuration space rather than ordinary three-dimensional physical space. So it is simply not meaningful to, for example, consider "the part" of such a wave function that describes goings-on in a particular region of space such as Σ in Fig. 5.2. There is no such "part" because the wave function doesn't live in (ordinary, three-dimensional, physical) space to begin with.

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[1] Experts may object here that, although one cannot assign a one-particle *wave function* to either particle in this kind of situation, one may nevertheless describe the state of each particle separately using something called a reduced density matrix. This is in some sense true but is nevertheless ultimately unhelpful. The reduced density matrix for particle 1 is a formal way of expressing that particle 1 is either on the left, or the right, side of its room, with 50/50 probability. And then similarly for particle 2. But then, crucially, the two reduced density matrices together fail to capture everything that is implied by Eq. (5.21); in particular, the *correlation* between the positions of the two particles (namely the fact that either both are on the left or both are on the right), is lost. So it is simply not true that the two reduced density matrices jointly capture, in the form of state descriptions for the two particles separately, the full state of the two-particle system as given in Eq. (5.21).

So what does this mean, with respect to the question of whether quantum mechanics (assumed complete) is a local theory? One might think that, by showing that we cannot cleanly apply Bell's formulation of locality to diagnose the theory as non-local in the EPR scenario, we leave the door open to the claim that perhaps the theory is, after all, consistent with relativistic local causality. But that is wrong, for several reasons.

First, we shouldn't forget that we were already able to diagnose quantum mechanics (assumed complete) as non-local in the simpler, one-particle "Einstein's Boxes" scenario, where the weirdness associated with multi-particle wave functions never came up. So if the question is just "Is ordinary quantum mechanics (assumed complete) a local theory?" that question has already been decisively and conclusively answered in the negative before we even get around to considering the EPR scenario.

Second, "not cleanly diagnosable as non-local" is not the same as "local". Indeed, in the spirit of Pauli's memorable phrase "not even wrong", it would probably be most accurate to summarize the situation by describing quantum mechanics here as "not *even* non-local". Remember what "locality" means: the causal influences that objects, moving and interacting in three-dimensional space, exert on one another, always propagate at the speed of light or slower. A theory which fails to provide a clear ontology of objects moving and interacting in three-dimensional space – for example, a theory which posits something like a physical field that lives in a more abstract space – doesn't even rise to the level of making the question, of whether causal influences always propagate at the speed of light or slower, or not, meaningful. In this sense I think it would be appropriate to say that ordinary quantum mechanics (assumed complete) is even less local than, for example, our paradigmatic example of a non-local theory: Newtonian mechanics with instantaneous gravitational forces. Whereas Newtonian mechanics provides a perfectly coherent "local ontology" (of objects, in this case particles, that move and interact in three-dimensional space) but posits "dynamical non-locality" (in the form of the gravitational interactions), quantum mechanics does not even appear to provide a coherent local ontology.

And third, it *is* possible to cleanly diagnose quantum mechanics as violating a modified version of Bell's formulation, as follows. Suppose we accept Bell's formulation as reasonable, at least for the kinds of theories to which it can be cleanly applied. This means we accept, if the two probabilities in Eq. (5.20) are different, when $\mathcal{C}_\Sigma$ is a complete specification of the physical state of region Σ in Fig. 5.2 according to some candidate theory, that the theory is non-local. But then surely if, in place of $\mathcal{C}_\Sigma$, we conditionalize both probabilities on *even more information* – say, a complete specification of the physical state of region Σ and some other stuff besides – we must still conclude that the theory is non-local if the two probabilities are different. That is, we may take

$$P\,[A\,|\,\mathcal{C}\,] \;=\; P\,[\,A\,|\,\mathcal{C},\; B\,], \tag{5.22}$$

where A and B are situated as in Fig. 5.3 and $\mathcal{C}$ *includes* (but is not necessarily restricted to providing merely) a complete specification of events in Σ, as a *necessary condition* for locality.

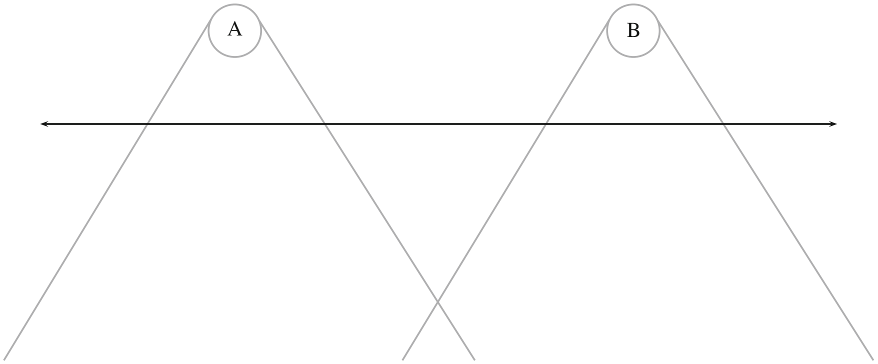

Fig. 5.3 Space-time diagram for the attempted application of Bell's formulation of local causality to the Einstein–Podolsky–Rosen scenario

In particular, we may let $\mathcal{C}$ denote a complete specification of events *throughout the entire universe*, at some moment in time prior to the events A and B, i.e., on the unbounded "slice" indicated by the horizontal black line in Fig. 5.3. If Eq. (5.22) is violated, for some theory, even with this hugely expanded $\mathcal{C}$, then surely the theory should be considered non-local.

The nice thing about this modified version of Bell's formulation is that multi-particle quantum mechanical wave functions are at least well-defined at particular moments in time. That is, we needn't any longer worry about the impossibility of extracting, from an entangled two-particle wave function, the "part" that pertains to a certain region of space; instead, we can just take the theory at face value and accept that, somehow, the ontologically mysterious two-particle wave function $\Psi(x_1, x_2, t)$ provides a complete description of the state of the two particles in question at time t. And so Ψ (supplemented, as appropriate, by any relevant macroscopic goings-on, but these play no important role here and will be suppressed for simplicity) can play the role of $\mathcal{C}$ in Eq. (5.22), which thus reduces to

$$P\,[\,A\,|\,\Psi\,] \;=\; P\,[\,A\,|\,\Psi,\;B\,]\,. \tag{5.23}$$

But these two probabilities are simply not equal, in just exactly the intuitive way we sketched at the beginning of this discussion: if A refers to particle 1 emerging as "spin-up" along the z-direction, and B refers to particle 2 emerging as "spin-up" along the z-direction, then the probability on the left hand side is 50% whereas the one on the right hand side is zero. So our necessary condition for locality is violated, and we must conclude that the theory is non-local.

Hopefully this long digression about the status of quantum mechanics, with respect to our concept of relativistic local causality, has illuminated an important loose end from Chap. 4. But of course the real point of this discussion, in the context of the present chapter, is to stress the problematical character of multi-particle quantum mechanical wave functions, taken as somehow providing complete descriptions of the physical states of those multi-particle systems.

5.4 Schrödinger's Suggestion for a Density in 3-Space

So far in this chapter we have been exploring the nature and fundamentality of the ontology problem. The (ultimately untenable) idea that the wave function is purely epistemic, with the ontology of the theory just being ordinary literal particles (like in classical mechanics), was mentioned as at least one possible way of eluding the problem. In this section, we turn to the one other remotely plausible proposed solution that I know of. I think, at the end of the day, this proposed solution is also unsatisfactory, for reasons that we will discuss. Nevertheless, it is worth exploring, because having some relatively clear and concrete ideas in mind, for the sort of three-dimensional physical reality that quantum mechanical wave functions might be (complete) descriptions of, will help clarify the nature of the difficulty and will put us in a better position to appreciate some more sophisticated proposals that we'll explore in subsequent chapters.

In the letter he wrote back to Lorentz (in response to Lorentz's letter that was quoted back in Sect. 3.2) Schrödinger proposed an answer to the puzzle about the wave function Ψ (and hence any density functions proportional to $|\Psi|^2$) being a function on the $3N$-dimensional configuration space:

> My dear Professor Lorentz,
>
> You have rendered me the extraordinary honor of subjecting the train of thought in my latest papers to a profound analysis and criticism on eleven closely written pages. I cannot find words with which to thank you sufficiently for this precious gift that you have thereby made to me; I am deeply distressed that I have made such excessive demands on your time in this way.....
>
> 1. You mention the difficulty of projecting the waves in q-space, when there are more than three coordinates, into ordinary three dimensional space and of interpreting them physically there. I have been very sensitive to this difficulty for a long time but believe that I have now overcome it. I believe, (and I have worked it out at the end of the third article), that the physical meaning belongs not to the quantity itself but rather to a *quadratic* function of it. *There* [i.e., in the article] I chose [a somewhat more complicated quadratic function of ψ]. *Now* I want to choose more simply $\psi\bar{\psi}$ [$= |\psi|^2$], that is, the square of the absolute value of the quantity ψ. If we now have to deal with N particles, then $\psi\bar{\psi}$ (just as ψ itself) is a function of $3N$ variables or, as I want to say, of N three dimensional spaces, R_1, R_2,..., R_N. Now first let R_1 be identified with the real space and integrate $\psi\bar{\psi}$ over R_2, R_3, ..., R_N; second, identify R_2 with the real space and integrate over R_1, R_3, ..., R_N; and so on. The N individual results are to be added after they have been multiplied by certain constants which characterize the particles (their charges, according to the former theory). I consider the result to be the electric charge density in real space [2, pp. 55–56].

That is, Schrödinger's original idea for physically interpreting the meaning of the wave function – even in cases of $N \geq 2$ particles where the wave function is a function on $3N$-dimensional configuration space – is to use the wave function to construct a (mass or) charge density for each particle separately, and to then add these together to get the total (say) charge density.

Let us express formally this idea that Schrödinger wrote in words. For an N-particle system with wave function

$$\Psi = \Psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, t) \tag{5.24}$$

we may construct the electric charge density of the ith particle this way:

$$\rho_i(\vec{x}, t) = q_i \int |\Psi|^2 \, \delta^3(\vec{x}_i - \vec{x}) \, d^3x_1 \, d^3x_2 \cdots d^3x_N \tag{5.25}$$

where q_i is the charge of the ith particle. Note that we integrate over the coordinates of all N particles here, but the δ-function makes the integration over the coordinates of the ith particle trivial. So the expression here is equivalent to saying, as Schrödinger expressed it in the letter: for the ith particle, *only* integrate over the coordinates associated with all of the *other* particles; this gives a function of $\vec{x}_i$ (and of course t); now regard this as a function on physical space, i.e., just identify these three remaining coordinates with the three coordinates in physical space, i.e., $x_i = x$, $y_i = y$, and $z_i = z$.

The total charge density for all N particles is then defined by simply summing the charge densities for the individual particles:

$$\rho_{\text{total}}(\vec{x}, t) = \sum_{i=1}^{N} \rho_i(\vec{x}, t). \tag{5.26}$$

And it is this total charge density which Schrödinger proposes as the physical reality described by the wave function Ψ.

Schrödinger eventually gave up on this picture, for reasons which have to do with another of the worries raised by Lorentz in his original letter: Schrödinger's equation implies that wave packets *spread*, as we saw in Chap. 2, and so it turns out that, as time evolves, these nice little "clouds" – lumps of nonzero charge density – would diffuse into an increasingly blurry haze that doesn't seem to correspond to the relatively sharp macroscopic world that, as we saw in Chap. 3, Schrödinger was at pains to make sure to capture in the fundamental theory. And the situation appears to be even worse when we consider the possibility of (entangled) superpositions of macroscopically distinct states, as illustrated for example by Schrödinger's cat. (Appearances, however, might in this case turn out to be misleading, as we will see in Chap. 10 when we examine the "many-worlds" interpretation of Hugh Everett.)

For now, though, let us set aside these sorts of concerns about whether Schrödinger's suggestion – for interpreting the wave function Ψ as describing a "density of stuff" in three-dimensional, physical space – is ultimately viable. If we are going to understand the wave function Ψ as providing a complete description of some continuous, field-like ontology, *something* like Schrödinger's suggestion will be necessary, so we should try to understand the suggestion as carefully as possible.

A concrete example should help. Let's therefore consider a one-dimensional "box" in which particles can be confined – but suppose, like the box that appeared in the last chapter, this box has been split in half and the two halves have been carried to distant locations. For definiteness, suppose in particular that each half box has a width L

and the two boxes are separated by a distance d:

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < L \\ 0 & \text{for } d < x < d + L \\ \infty & \text{otherwise} \end{cases} \tag{5.27}$$

Now, for example, a particle that is confined to the (half-) box on the left might have wave function

$$\psi_L(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) & \text{for } 0 < x < L \\ 0 & \text{otherwise} \end{cases} \tag{5.28}$$

while a particle that is instead confined to the (half-) box on the right might have wave function

$$\psi_R(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{\pi(x-d)}{L}\right) & \text{for } d < x < L + d \\ 0 & \text{otherwise} \end{cases} . \tag{5.29}$$

And then of course it is possible that a particle might find itself *split* between the two (half-) boxes, i.e., in a superposition of being on the left and being on the right:

$$\psi_{L+R} = \frac{1}{\sqrt{2}} \left[\psi_L(x) + \psi_R(x)\right]. \tag{5.30}$$

Note that, for a single particle in these various states, Schrödinger's electric charge density acts the way one would naively expect: if the particle's quantum state is ψ_L then that particle's charge is smeared throughout (but is exclusively contained in) the left box; if the particle's quantum state is ψ_R then the particle's charge is smeared throughout (but exclusively contained in) the right box; and if the particle's quantum state is ψ_{L+R} the particle's charge is half in the left box and half in the right box.

But of course our goal here is to consider situations in which there are now *two* (or more, but two will suffice) particles involved. Suppose, for simplicity, that we have two particles with identical electric charges, q, but that the particles are distinguishable. (Then we don't need to worry about the Pauli Exclusion Principle, the symmetry/anti-symmetry properties of the two-particle wave function under exchange, etc.) For example, perhaps particle 1 (whose coordinate we call x_1) is an electron and particle 2 (whose coordinate we call x_2) is a muon.

Then let us consider three possible quantum states – call them A, B, and C – that these two particles might be in:

- In state A

  $$\psi_A = \psi_L(x_1)\, \psi_R(x_2) \tag{5.31}$$

  particle 1 is definitely in the (half-) box on the left, and particle 2 is definitely in the (half-) box on the right.
- In state B

$$\psi_B = \psi_{L+R}(x_1)\, \psi_{L+R}(x_2) \tag{5.32}$$

particle 1 is in a superposition of being on the left and being on the right (it is "smeared out" evenly between the two half-boxes), and so is particle 2.

- Finally, state C

$$\psi_C = \frac{1}{\sqrt{2}} \left[\, \psi_L(x_1)\, \psi_R(x_2) \;+\; \psi_R(x_1)\, \psi_L(x_2) \,\right] \tag{5.33}$$

is an entangled superposition of (on the one hand) a state in which particle 1 is on the left and particle 2 is on the right, and (on the other hand) a state in which particle 1 is on the right and particle 2 is on the left.

These three different quantum states are sketched, in the two-dimensional configuration space, in Fig. 5.4. And it should be clear – both from the mathematical definitions of the three states and from the pictures in the Figure – that the states are indeed not all the same: there are real, measurable things that would be different in the three different cases.

For example, if we prepare a large ensemble of particle pairs in state A, and then measure their positions, we will always find particle 1 on the left and particle 2 on the right. In particular, note that we will never find the two particles in the same place! Each particle has a distinct position (the same for all elements in the ensemble) and so the positions of the two particles are perfectly correlated.

If we instead prepare a large ensemble of particle pairs in state B and then measure their positions, we will find that particle 1 is found on the left half the time and on the right half the time, and the same for particle 2, and the positions of the two particles are totally uncorrelated: 25% of the time we will find both particles on the left, 25% of the time we will find particle 1 on the left and particle 2 on the right, 25% of the

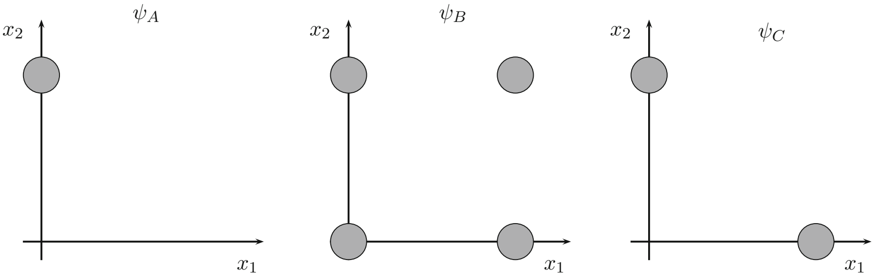

Fig. 5.4 Three different quantum states that a pair of particles might be in, schematically represented in the two-dimensional configuration space. (The *grey circles* represent regions in configuration space where the wave function is nonzero.) In the state ψ_A, particle 1 is on the left ($x_1 \approx 0$) and particle 2 is on the right ($x_2 \approx d$). In the state ψ_B, both particles are "smeared" between being on the *left* and on the *right*. State ψ_C, finally, is a superposition of "particle 1 is on the *left* and particle 2 is on the *right*" and "particle 1 is on the *right* and particle 2 is on the *left*"

time we will find particle 1 on the right and particle 2 on the left, and 25% of the time we will find both particles on the right.

And things are different yet again if we now prepare a large ensemble of particle pairs in state C and measure their positions. It is again the case (as with state B) that particle 1 is found half of the time on the left and half of the time on the right, and the same is again true for particle 2 as well, but now the positions are perfectly correlated: whenever particle 1 is found on the right, particle 2 is found on the left, and vice versa. The particles are somehow definitely in different places, even though neither particle has a definite location!

Now, for our purposes here, what is interesting about all this is the following. These three genuinely different, empirically distinguishable quantum states *all produce the exact same charge distribution* according to Eq. (5.26). That is, the "physical reality" of these three states – if Schrödinger's early idea about how to interpret the physical meaning of the wave function is correct – is the same. But this, I submit, simply cannot be correct. The three states are different – *physically* different – as proved by the fact that the outcome statistics for various kinds of measurements are different for the three states.

And so, it seems, Schrödinger's idea cannot be correct, or at least cannot be the whole story. It provides a nice way of understanding how the wave function (on configuration space) might describe some kind of material stuff in three-dimensional physical space. But, in mathematical terms, one loses a lot of information by "projecting down" the wave function Ψ into the charge density field ρ. The correspondence, that is, is many-to-one in the sense that there are very different – and meaningfully different, physically different – wave functions that correspond to the same charge density. So Schrödinger's suggestion is a nice try, and as we will see in later chapters it has a role to play in some more sophisticated candidate theories, but it apparently cannot just be the case that Schrödinger's charge density ρ *is*, all alone, the ontology of the world described by quantum mechanics.

5.5 So Then What?

The obvious follow-up question to the previous sentence would be: "OK, so if quantum mechanical wave functions aren't really, or aren't just, descriptions of something like Schrödinger's charge density ρ, what *are* they descriptions of?" You would probably therefore expect this next section to start introducing some other possible ideas, for the ontology of the quantum world, one of which maybe turns out to be right, or at least viable. But unfortunately I don't know of any other possible ideas. If there is some other way of taking quantum mechanical wave functions seriously, as somehow more or less directly describing some kind of field or fields in regular physical space, I don't know about it.

Well, actually, there is another idea on this issue that is sometimes suggested. I think it is based on a confusion, and is not viable at all, but perhaps it is worth mentioning here if only to pre-empt ongoing confusion. The idea is that the ontology problem somehow magically goes away when we remember that our best current theory is not non-relativistic quantum mechanics but rather quantum field theory (QFT). The idea is that, perhaps unlike non-relativistic quantum mechanics (NRQM), QFT provides a straightforward and obvious and unproblematic ontology, namely, one of *fields* (in physical space).

But this is simply not true. Fields with definite configurations play exactly the same role in QFT that particles with (for example) definite positions play in non-relativistic quantum mechanics. That is, it is possible, in QFT, to write down a quantum state which can be interpreted as describing a field with a definite configuration, just as it is possible, in NRQM, to write down a quantum state (namely, a δ function) which can be interpreted as describing a particle with a definite position. But just as a generic quantum state for a single particle in NRQM will *not* attribute any particular position to the particle, so a generic quantum state for a field in QFT will *not* attribute any particular configuration to the field. That is, just as a typical quantum state for a single particle in NRQM can be understood as a superposition over a continuous infinity of different positions (think $\psi(x,t) = \int \psi(a,t)\,\delta(x-a)\,da$), so a typical quantum state for a field in QFT can be understood as a superposition over a continuous infinity of different field configurations. So unless one openly rejects the idea that quantum states provide complete state descriptions and adopts a naive (and, of course, ultimately untenable) "ignorance interpretation" of quantum states one simply cannot claim that QFT describes fields which always possess definite configurations.

There are several reasons for confusion about this. One is that, in the way that QFT is traditionally presented, one does not typically deal with generic quantum states, but instead focuses almost exclusively on (asymptotic) initial and final states corresponding to various incoming or outgoing particles in a scattering experiment, and, in a certain sense, these states can be understood in terms of fields with definite configurations. The analog in NRQM would be focusing on calculating the probability that, if a particle starts at a certain position x_0 at $t = 0$, it will be detected at some other position x_f at $t = T$. One can see why focusing exclusively on this kind of case might reinforce the belief that it is perfectly viable to think that, according to quantum mechanics, particles always have definite positions. A similar thing is happening when people get the idea that QFT is just unproblematically about fields (in physical space) with evolving, but always unproblematically definite, configurations.

A second and deeper reason for the confusion, though, is just that most people have not really thought carefully about these kinds of issues, even in the context of NRQM. Perhaps they tend to think exclusively about one-particle examples, and so have in mind an ontology of single-particle waves running around through physical space. Or perhaps they do hold some naive version of the "ignorance interpretation", according to which the ontology is something like classical (i.e., literal) particles, with wave functions providing only some kind of very incomplete description of their states. Or perhaps they don't have any particular ontological picture in mind, but are instead happy to just play games with mathematical symbols without thinking about (and without even acknowledging that *someone* should think about) what the symbols correspond to in physical reality. In any case, and whatever the ultimate reasons, most physicists have simply not appreciated or accepted that there is some problem associated with understanding what NRQM wave functions might describe exactly – and so they are open to the (in fact rather ridiculous) suggestion that there is definitely no such problem in quantum field theory.

So where does that leave us? If, as "realist" and literal-minded physicists like Einstein and Schrödinger seem to have assumed, the idea of quantum mechanics providing *complete* descriptions of physical states means that in some sense physical reality "looks exactly like" wave functions, that view seems very difficult to maintain. The measurement problem shows that wave functions (obeying Schrödinger's equation all the time) seem unable to capture the definite outcomes that we always observe in measurements; Einstein et al. pointed out that the assumption of completeness appears to generate a direct conflict with the idea of relativistic local causality; and now we have seen that even leaving these other worries aside it is simply not clear how one might regard wave functions as directly and faithfully describing a three-dimensional physical reality at all, since wave functions are (in general) functions on an abstract, multi-dimensional configuration space.

As mentioned before, we will return to some of the ideas of this chapter when we study Everett's many-worlds version of quantum theory in Chap. 10, and also when we study the so-called "spontaneous collapse" theories in Chap. 9. We will also encounter, in Chap. 7, another candidate version of quantum theory according to which wave functions alone are *not* the whole story and which attempt to give perfectly clear descriptions of physical processes in three-dimensional space in terms of the thing that is postulated to exist *in addition to* the wave function. But before turning to those alternative theories, we will explore, in the next chapter, the so-called "Copenhagen interpretation" and try to understand better the point of view that people like Einstein and Schrödinger were reacting against when they complained that a coherent description of micro-physical reality was needed, but not provided by ordinary quantum mechanics.

Projects

5.1 Decompose the (complex) Schrödinger equation into two (real) equations by substituting in $\Psi = Re^{iS/\hbar}$ and then separating the real and imaginary parts of the resulting equation. (This will hardly be obvious, but it turns out these two resulting equations are rather interesting. One of them can be understood as expressing local conservation of probability, and the other turns out to be exactly the Hamilton–Jacobi equation from classical mechanics, but with an extra – somehow purely quantum – term in the potential energy. You might be interested to google "Hamilton–Jacobi equation" if you haven't encountered it before, and see what that is all about. But really the point of this question is just to let you practice decomposing a single equation involving a complex quantity into two equations involving two real quantities.)

5.2 Show that there exist plane-wave solutions, of Eq. (5.10), of the form $\vec{F}(\vec{x}, t) = \vec{F}_0 \, e^{i\vec{k}\cdot\vec{x}-\omega t}$. What, exactly, can $\vec{F}_0$ be? (Hint: don't forget about the additional constraint imposed by Eq. (5.8).) Identify the type of polarization (linear? circular?) associated with your solution.

5.3 Take the dot product of $\vec{F}^*$ (the complex conjugate of $\vec{F}$) with Eq. (5.10). Then write down a second equation which is the dot product of $\vec{F}$ with the complex conjugate of Eq. (5.10). Now subtract your two equations and show that you get

$$\frac{\partial}{\partial t}\left(\vec{F}^* \cdot \vec{F}\right) = -ic\vec{\nabla} \cdot \left(\vec{F} \times \vec{F}^*\right). \tag{5.34}$$

This is an important result in classical electromagnetism which you may (or may not!) have seen before. Put it in more familiar terms by using $\vec{F} = \vec{E} + ic\vec{B}$ to re-express it in terms of $\vec{E}$ and $\vec{B}$, and interpret the result. (Hint: it has something to do with the Poynting vector, $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$. and the electromagnetic field energy density, $\rho = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$.) For extra credit, what does this correspond to in regular non-relativistic quantum mechanics? (If you're not sure, you could follow the same procedure and see what happens: multiply Schrödinger's equation by Ψ^*, then multiply the complex conjugate of Schrödinger's equation by Ψ, then subtract and simplify...)

5.4 Consider the two-particle entangled state given in Eq. (5.21). Draw a configuration space cartoon showing the regions of the two-dimensional configuration space where this state has nonzero intensity.

5.5 Consider the "Einstein's Boxes" scenario from the point of view of Schrödinger's suggested interpretation of ψ in terms of charge density. In particular, suppose an electron is "split" between two well-separated half-boxes. What is its electric charge density? What is the electric charge density after the half-boxes are opened and the electron is found (complete!) in one half-box or the other? Explain, finally, how one can apply Bell's formulation of "locality" to diagnose this theoretical description as non-local.

5.6 Consider a case of measurement (described quantum mechanically) from the point of view of Schrödinger's suggested interpretation of ψ. Take the example from Chap. 3: a one-dimensional particle-in-a-box in a superposition of energy eigenstates has its energy measured and recorded in the final position of a "pointer" so that the final quantum state is the right hand side of Eq. (5.17). Calculate and sketch a picture of the electric charge density after the measurement interaction.

5.7 Maybe, instead of adding up all the one-particle charge densities from Eq. (5.25) to get the total charge density of Eq.(5.26), we should instead regard each particle's individual charge density ρ_i as physically real. (Physical reality, on this view, would not consist of just one undifferentiated total charge density, but would instead involve distinct charge densities for each particle.) Does this view survive the argument made in Sect. 5.4? In particular, are the "physical realities" implied by the three states (ψ_A, ψ_B, and ψ_C) all the same on this view?

5.8 Interview one of your other physics teachers and ask her whether she agrees with Bohr that the wave function provides a complete description of quantum systems. (She will probably say yes, but you never know.) Then ask her to elaborate by explaining, for the case of the electron in a Hydrogen atom in the ground state, how she pictures the electron. (Presumably she will describe something like the "cloud" picture implied by taking $|\psi|^2$ as a density-of-stuff, but, again, you never know.) Finally ask her how to understand what physical reality is described by an entangled two-particle state like the one in Eq. (5.21). Summarize her viewpoint.

5.9 In a nice historical paper on "Schrödinger's route to wave mechanics" [5] Linda Wessels explains that "In the case of a single classical particle ψ could be interpreted as a wave function describing a matter wave. For a system of n classical particles, however, ψ was a function of $3n$ spatial coordinates and therefore described a wave in a $3n$-dimensional space that could not be identified with ordinary physical space. To give his theory a wave interpretation Schrödinger would either have to show how the ψ in $3n$-dimensional space determined n waves in 3-dimensional space, or reformulate the theory so that it would yield directly the required n wave functions." She then adds, citing a 1962 interview with Carl Eckart conducted by John Heilbron: "The obvious solution would be to rewrite the equations of wave mechanics so that even for a system of several 'particles', only three-dimensional wave functions would be determined. C. Eckart has reported that at one time he attempted this and remarked that *it was something that initially 'everybody' was trying to do.*" (Emphasis added.) It might be a little intimidating to know that "everybody" tried something already, and nobody succeeded... but how hard can it be, really? See if you can come up with some other way or ways to convert Schrödinger's $\Psi(x_1, x_2, \ldots, x_N)$ into N "single particle" waves. Test your ideas out on examples like the one from Sect. 5.4.

5.10 In one version of the EPR-type argument we discussed in the previous chapter, Einstein pointed out that if "completeness" means a one-to-one correspondence between physically real states and quantum wave functions, then (assuming locality!) QM is not complete. One could also summarize the discussion of Sect. 5.4 by saying that there is a failure of one-to-one correspondence between physical states (as understood according to Schrödinger's suggestion) and wave functions. (Namely: the three different wave functions discussed there would all correspond to the same one physical state.) Compare and contrast these two different applications of the "one-to-one correspondence" idea. What exactly is being argued for in the two cases and how do the arguments relate?

5.11 Consider the two electrons in a diatomic Hydrogen *molecule*. What exactly do chemists and physicists think the two electrons are doing, and how does this relate to their joint wave function? For example, find a "picture" of a Hydrogen molecule in a chemistry or physics book (or online). (Or interview a physicist or chemist, and ask them to draw a Hydrogen molecule.) What exactly is pictured? Is it Schrödinger's early suggestion? Something else? Summarize and explain.

5.12 One weird idea that has been suggested, as a way of dealing with "the ontology problem", is to take the $3N$-dimensional configuration space seriously, as the fundamental physical space. (Evidently N here should be what we would ordinarily think of as the total number of particles in the universe.) Then there is no longer any problem understanding how the wave function could directly and faithfully and completely describe what's real. On this view, physical reality would consist of something like a (complex valued) field (or maybe two coupled real-valued fields!) in this 3N-dimensional space. The three-dimensional world would then be somehow "emergent" from this more basic reality. What do you think of this idea?

5.13 It was mentioned at the end of Sect. 5.4 that one of Schrödinger's reasons for abandoning his early suggestion (about the physical meaning of Ψ) had to do with the inevitable spreading-out of wave packets we first encountered in Chap. 2. But, you might wonder, isn't (for example) the electron in a Hydrogen atom *bound*? Won't its wave function remain forever localized around the proton without spreading? So... is there really a problem with spreading, or not? (Hint: consider both the electron and the proton in the Hydrogen atom. A complete answer will involve constructing an explicit one-dimensional toy model of a Hydrogen atom and depicting the time-evolution of its quantum state in the two-dimensional configuration space.) What if the Hydrogen atom is bound to some other atoms in a molecule?

5.14 Show that, as claimed in the text, the "three genuinely different, empirically distinguishable quantum states [ψ_A, ψ_B, and ψ_C] *all produce the exact same charge distribution* according to Eq. (5.26)."

5.15 The main conclusion of Sect. 5.4 was that Schrödinger's charge density ontology cannot be correct, or at least cannot be the whole story. Maybe supplementing the charge density with some additional properties would do the trick? What sort of thing would you need to *add* to the ontology to give appropriate *physical* differences between the states described by ψ_A, ψ_B, and ψ_C?

5.16 Two particles of identical charge q are in the same length-L box, in the entangled state

$$\psi(x_1, x_2) = \frac{1}{\sqrt{2}} \left[\psi_1(x_1)\, \psi_2(x_2) + i\, \psi_3(x_1)\, \psi_1(x_2)\right] \tag{5.35}$$

where the ψ_ns are the usual particle-in-a-box energy eigenstates. According to Schrödinger's early suggestion about the physical meaning of the wave function, Eq. (5.25), what is the charge density $\rho_1(x)$ associated with particle 1? Calculate it explicitly. Will $\rho_1(x)$ change in time?

5.17 Consider the following version of (something like) the EPR argument:

> A pair of spatially-separated particles is in the entangled spin state
>
> $$\Psi = \frac{1}{\sqrt{2}} \left[\psi^1_{+z}\psi^2_{-z} - \psi^1_{-z}\psi^2_{+z}\right]. \tag{5.36}$$
>
> If one measures the spin (say, along the z-direction) of particle 1, the two particle state will, according to QM, collapse – either to $\psi^1_{+z}\psi^2_{-z}$ or to $\psi^1_{-z}\psi^2_{+z}$. That is, after and as a result of the measurement on particle 1, the spin state of particle 2 will become either ψ^2_{+z} or ψ^2_{-z}. But prior to the measurement the spin state of particle 2 was neither

of these. Therefore, the spin state of particle 2 has been (non-locally) affected by the measurement on (the distant) particle 1.

What, from the point of view of the issue raised in this chapter, is not quite right about this argument? Do you think that the not-quite-right-ness of this argument suggests that, contra EPR, QM is actually a local theory? Explain.

5.18 What is "the ontology problem"? Summarize in your own words.

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