



Chapter 5

Applications of Statics to Biomechanics

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The original version of this chapter was revised. An erratum to this chapter can be found at https://doi.org/10.1007/978-3-319-44738-4_16

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N. Özkaya et al., *Fundamentals of Biomechanics*, DOI 10.1007/978-3-319-44738-4_5

5.1 Skeletal Joints

The human body is rigid in the sense that it can maintain a posture, and flexible in the sense that it can change its posture and move. The flexibility of the human body is due primarily to the *joints*, or *articulations*, of the skeletal system. The primary function of joints is to provide mobility to the musculoskeletal system. In addition to providing mobility, a joint must also possess a degree of stability. Since different joints have different functions, they possess varying degrees of mobility and stability. Some joints are constructed so as to provide optimum mobility. For example, the construction of the shoulder joint (ball-and-socket) enables the arm to move in all three planes (triaxial motion). However, this high level of mobility is achieved at the expense of reduced stability, increasing the vulnerability of the joint to injuries, such as dislocations. On the other hand, the elbow joint provides movement primarily in one plane (uniaxial motion), but is more stable and less prone to injuries than the shoulder joint. The extreme case of increased stability is achieved at joints that permit no relative motion between the bones constituting the joint. The contacting surfaces of the bones in the skull are typical examples of such joints.

The joints of the human skeletal system may be classified based on their structure and/or function. *Synarthrodial joints*, such as those in the skull, are formed by two tightly fitting bones and do not allow any relative motion of the bones forming them. *Amphiarthrodial joints*, such as those between the vertebrae, allow slight relative motions, and feature an intervening substance (a cartilaginous or ligamentous tissue) whose presence eliminates direct bone-to-bone contact. The third and mechanically most significant type of articulations are called *diarthrodial joints* which permit varying degrees of relative motion and have articular cavities, ligamentous capsules, synovial membranes, and synovial fluid (Fig. 5.1). The *articular cavity* is the space between the articulating bones. The *ligamentous capsule* holds the articulating bones together. The *synovial membrane* is the internal lining of the ligamentous capsule enclosing the *synovial fluid* which serves as a lubricant. The synovial fluid is a viscous material which functions to reduce friction, reduce wear and tear of the articulating surfaces by limiting direct contact between them, and nourish the articular cartilage lining the surfaces. The *articular cartilage*, on the other hand, is a specialized tissue designed to increase load distribution on the joints and provide a wear-resistant surface that absorbs shock. Various diarthrodial joints can be further categorized as gliding (for example, vertebral facets), hinge (elbow and ankle), pivot (proximal radioulnar), condyloid (wrist), saddle (carpometacarpal of thumb), and ball-and-socket (shoulder and hip).

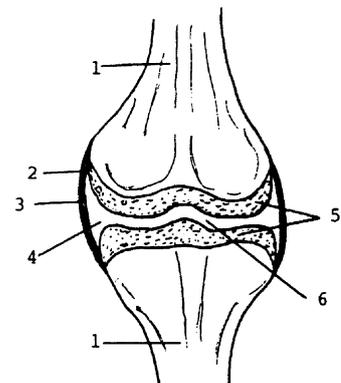


Fig. 5.1 A diarthrodial joint: (1) Bone, (2) ligamentous capsule, (3, 4) synovial membrane and fluid, (5, 6) articular cartilage and cavity

The nature of motion about a diarthrodial joint and the stability of the joint are dependent upon many factors, including the manner in which the articulating surfaces fit together, the properties of the joint capsule, the structure and length of the ligaments around the joint, and the number and orientation of the muscles crossing the joint.

5.2 Skeletal Muscles

In general, there are over 600 muscles in the human body, accounting for about 45° of the total body weight.

There are three types of muscles: cardiac, smooth, and skeletal. *Cardiac muscle* is the contractive tissue found in the heart that pumps the blood for circulation. *Smooth muscle* is found in the stomach, intestinal tracts, and the walls of blood vessels. *Skeletal muscle* is connected to the bones of the body and when contracted, causes body segments to move.

Movement of human body segments is achieved as a result of forces generated by skeletal muscles that convert chemical energy into mechanical work. The structural unit of skeletal muscle is the muscle fiber, which is composed of myofibrils. Myofibrils are made up of actin and myosin filaments. Muscles exhibit viscoelastic material behavior. That is, they have both solid and fluid-like material properties. Muscles are elastic in the sense that when a muscle is stretched and released it will resume its original (unstretched) size and shape. Muscles are *viscous* in the sense that there is an internal resistance to motion.

A skeletal muscle is attached, via soft tissues such as aponeuroses and/or tendons, to at least two different bones controlling the relative motion of one segment with respect to the other. When its fibers contract under the stimulation of a nerve, the muscle exerts a pulling effect on the bones to which it is attached. *Contraction* is a unique property of the muscle tissue. In engineering mechanics, contraction implies shortening under compressive forces. In muscle mechanics, contraction can occur as a result of muscle shortening or muscle lengthening, or it can occur without any change in the muscle length. Furthermore, the result of a muscle contraction is always tension: a muscle can only exert a pull. Muscles cannot exert a push.

There are various types of muscle contractions: a *concentric contraction* occurs simultaneously as the length of the muscle decreases (for example, the biceps during flexion of the forearm); a *static contraction* occurs while muscle length remains constant (the biceps when the forearm is flexed and held without any movement); and an *eccentric contraction* occurs as the length of the muscle increases (the biceps during the extension

of the forearm). A muscle can cause movement only while its length is shortening (concentric contraction). If the length of a muscle increases during a particular activity, then the tension generated by the muscle contraction is aimed at controlling the movement of the body segments associated with that muscle (eccentric contraction). If a muscle contracts but there is no segmental motion, then the tension in the muscle balances the effects of applied forces such as those due to gravity (isometric contraction).

The skeletal muscles can also be named according to the functions they serve during a particular activity. For example, a muscle is called *agonist* if it causes movement through the process of its own contraction. Agonist muscles are the primary muscles responsible for generating a specific movement. An *antagonist* muscle opposes the action of another muscle. *Synergic* muscle is that which assists the agonist muscle in performing the same joint motion.

5.3 Basic Considerations

In this chapter, we want to apply the principles of statics to investigate the forces involved in various muscle groups and joints for various postural positions of the human body and its segments. Our immediate purpose is to provide answers to questions such as: what tension must the neck extensor muscles exert on the head to support the head in a specified position? When a person bends, what would be the force exerted by the erector spinae on the fifth lumbar vertebra? How does the compression at the elbow, knee, and ankle joints vary with externally applied forces and with different segmental arrangements? How does the force on the femoral head vary with loads carried in the hand? What are the forces involved in various muscle groups and joints during different exercise conditions?

The forces involved in the human body can be grouped as internal and external. Internal forces are those associated with muscles, ligaments, and tendons, and at the joints. Externally applied forces include the effect of gravitational acceleration on the body or on its segments, manually and/or mechanically applied forces on the body during exercise and stretching, and forces applied to the body by prostheses and implements. In general, the unknowns in static problems involving the musculoskeletal system are the joint reaction forces and muscle tensions. Mechanical analysis of a joint requires that we know the vector characteristics of tension in the muscle including the proper locations of muscle attachments, the weights or masses of body segments, the centers of gravity of the body segments, and the anatomical axis of rotation of the joint.

5.4 Basic Assumptions and Limitations

The complete analysis of muscle forces required to sustain various postural positions is difficult because of the complex arrangement of muscles within the human body and because of limited information. In general, the relative motion of body segments about a given joint is controlled by more than one muscle group. To be able to reduce a specific problem of biomechanics to one that is statically determinate and apply the equations of equilibrium, only the muscle group that is the primary source of control over the joint can be taken into consideration. Possible contributions of other muscle groups to the load-bearing mechanism of the joint must be ignored. Note however that approximations of the effect of other muscles may be made by considering their cross-sectional areas and their relative positions in relation to the joint. Also, if the phasic activity of muscles is known via some experiments such as the electromyography (EMG) measurements of muscle signals, then the tension in different muscle groups may be estimated.

To apply the principles of statics to analyze the mechanics of human joints, we shall adopt the following assumptions and limitations:

- The anatomical axes of rotation of joints are known.
- The locations of muscle attachments are known.
- The line of action of muscle tension is known.
- Segmental weights and their centers of gravity are known.
- Frictional factors at the joints are negligible.
- Dynamic aspects of the problems will be ignored.
- Only two-dimensional problems will be considered.

These analyses require that the anthropometric data about the segment to be analyzed must be available. For this purpose, there are tables listing anthropometric information including average weights, lengths, and centers of gravity of body segments. See Chaffin, Andersson, and Martin (1999), and Winter (2004) for a review of the anthropometric data available.

It is clear from this discussion that we shall analyze certain idealized problems of biomechanics. Based on the results obtained and experience gained, these models may be expanded by taking additional factors into consideration. However, a given problem will become more complex as more factors are considered.

In the following sections, the principles of statics will be applied to analyze forces involved at and around the major joints of the human body. First, a brief functional anatomy of each joint and related muscles will be provided, and specific biomechanical problems will be constructed. For a more complete discussion about the functional anatomy of joints, see texts such as Nordin and Frankel (2011) and Thompson (1989). Next, an analogy will be formed between muscles, bones, and human joints, and certain mechanical elements such as cables, beams, and mechanical joints. This will enable us to construct a mechanical model of the biological system under consideration. Finally, the procedure outlined in Chap. 4.5 will be applied to analyze the mechanical model thus constructed. See LeVeau (2010) for additional examples of the application of the principles of statics to biomechanics.

5.5 Mechanics of the Elbow

The elbow joint is composed of three separate articulations (Fig. 5.2). The *humeroulnar joint* is a hinge (ginglymus) joint formed by the articulation between the spool-shaped trochlea of the distal humerus and the concave trochlear fossa of the proximal ulna. The structure of the humeroulnar joint is such that it allows only uniaxial rotations, confining the movements about the elbow joint to flexion (movement of the forearm toward the upper arm) and extension (movement of the forearm away from the upper arm). The *humeroradial joint* is also a hinge joint formed between the capitulum of the distal humerus and the head of the radius. The *proximal radioulnar joint* is a pivot joint formed by the head of the radius and the radial notch of the proximal ulna. This articulation allows the radius and ulna to undergo relative rotation about the longitudinal axis of one or the other bone, giving rise to pronation (the movement experienced while going from the palm-up to the palm-down) or supination (the movement experienced while going from the palm-down to the palm-up).

The muscles coordinating and controlling the movement of the elbow joint are illustrated in Fig. 5.3. The biceps brachii muscle is a powerful flexor of the elbow joint, particularly when the elbow joint is in a supinated position. It is the most powerful supinator of the forearm. On the distal side, the biceps is attached to the tuberosity of the radius, and on the proximal side, it has attachments at the top of the coracoids process and upper lip of the glenoid fossa. Another important flexor is the brachialis muscle which, regardless of forearm orientation, has the ability to produce elbow flexion. It is then the strongest flexor of the elbow. It has attachments at the lower half of the anterior portion of the humerus and the coronoid process of the

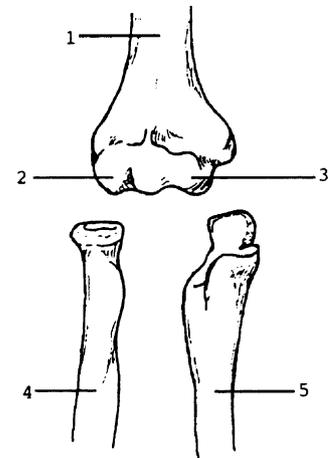


Fig. 5.2 Bones of the elbow: (1) humerus, (2) capitulum, (3) trochlea, (4) radius, (5) ulna

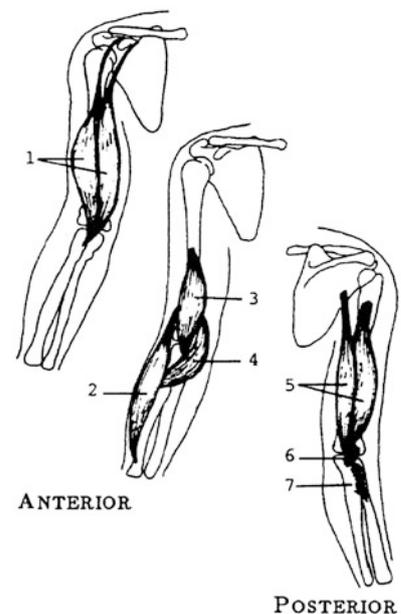


Fig. 5.3 Muscles of the elbow: (1) biceps, (2) brachioradialis, (3) brachialis, (4) pronator teres, (5) triceps brachii, (6) anconeus, (7) supinator

ulna. Since it does not inset on the radius, it cannot participate in pronation or supination. The most important muscle controlling the extension movement of the elbow is the triceps brachii muscle. It has attachments at the lower head of the glenoid cavity of the scapula, the upper half of the posterior surface of the humerus, the lower two-thirds of the posterior surface of the humerus, and the olecranon process of the ulna. Pronation and supination movements of the forearm are performed by the pronator teres and supinator muscles, respectively. The pronator teres is attached to the lower part of the inner condyloid ridge of the humerus, the medial side of the ulna, and the middle third of the outer surface of the radius. The supinator muscle has attachments at the outer condyloid ridge of the humerus, the neighboring part of the ulna, and the outer surface of the upper third of the radius.

Common injuries of the elbow include fractures and dislocations. Fractures usually occur at the epicondyles of the humerus and the olecranon process of the ulna. Another group of elbow injuries are associated with overuse, which causes an inflammatory process of the tendons of an elbow that has been damaged by repetitive motions. These include tennis elbow and golfer's elbow syndromes.

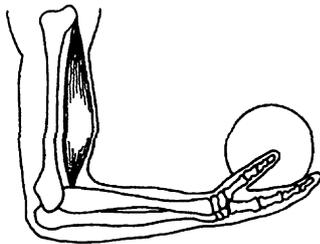


Fig. 5.4 Example 5.1

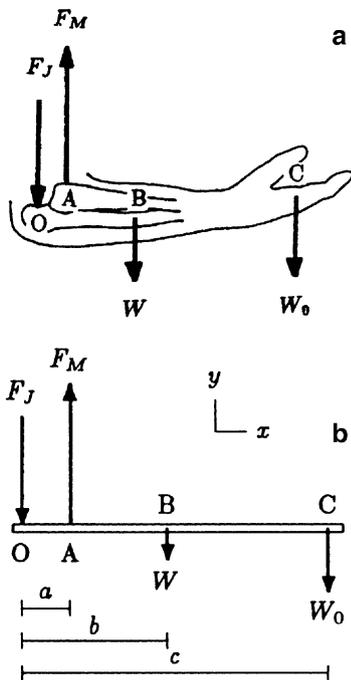


Fig. 5.5 Forces acting on the lower arm

Example 5.1 Consider the arm shown in Fig. 5.4. The elbow is flexed to a right angle and an object is held in the hand. The forces acting on the forearm are shown in Fig. 5.5a, and the free-body diagram of the forearm is shown on a mechanical model in Fig. 5.5b. This model assumes that the biceps is the major flexor and that the line of action of the tension (line of pull) in the biceps is vertical.

Point O designates the axis of rotation of the elbow joint, which is assumed to be fixed for practical purposes. Point A is the attachment of the biceps muscle on the radius, point B is the center of gravity of the forearm, and point C is a point on the forearm that lies along a vertical line passing through the center of gravity of the weight in the hand. The distances between point O and points A, B, and C are measured as a , b , and c , respectively. W_0 is the weight of the object held in the hand and W is the total weight of the forearm. F_M is the magnitude of the force exerted by the biceps on the radius, and F_J is the magnitude of the reaction force at the elbow joint. Notice that the line of action of the muscle force is assumed to be vertical. The gravitational forces are vertical as well. Therefore, for the equilibrium of the lower arm, the line of action of the joint reaction force must also be vertical (a parallel force system).

The task in this example is to determine the magnitudes of the muscle tension and the joint reaction force at the elbow.

Solution: We have a parallel force system, and the unknowns are the magnitudes F_M and F_J of the muscle and joint reaction forces. Considering the rotational equilibrium of the forearm about the elbow joint and assuming the (cw) direction is positive:

$$\sum M_O = 0$$

That is, $cW_O + bW - aF_M = 0$

$$\text{Then } F_M = \frac{1}{a} (bW + cW_O) \quad (\text{i})$$

For the translational equilibrium of the forearm in the y direction:

$$\sum F_y = 0$$

That is: $-F_J + F_M - W - W_O = 0$

$$\text{Then } F_J = F_M - W - W_O \quad (\text{ii})$$

For given values of geometric parameters a , b , and c , and weights W and W_O , Eqs. (i) and (ii) can be solved for the magnitudes of the muscle and joint reaction forces. For example, assume that these parameters are given as follows: $a = 4$ cm, $b = 15$ cm, $c = 35$ cm, $W = 20$ N, and $W_O = 80$ N. Then from Eqs. (i) and (ii):

$$F_M = \frac{1}{0.04} [(0.15)(20) + (0.35)(80)] = 775 \text{ N } (+y)$$

$$F_J = 775 - 20 - 80 = 675 \text{ N } (-y)$$

Remarks

- The numerical results indicate that the force exerted by the biceps muscle is about ten times larger than the weight of the object held in the position considered. Relative to the axis of the elbow joint, the length a of the lever arm of the muscle force is much smaller than the length c of the lever arm enjoyed by the load held in the hand. The smaller the lever arm, the greater the muscle tension required to balance the clockwise rotational effect of the load about the elbow joint. Therefore, during lifting, it is disadvantageous to have a muscle attachment close to the elbow joint. However, the closer the muscle is to the joint, the larger the range of motion of elbow flexion-extension, and the faster the distal end (hand) of the forearm can reach its goal of moving toward the upper arm or the shoulder.

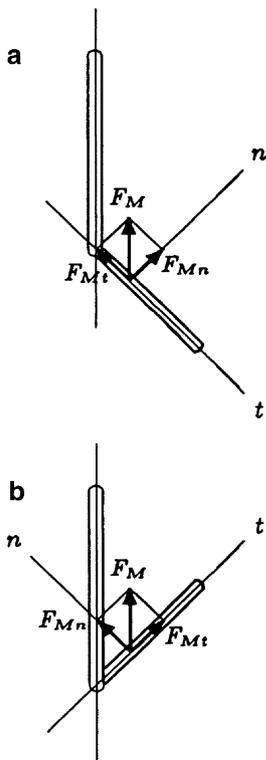


Fig. 5.6 Rotational (F_{Mn}) and stabilizing or sliding (F_{Mt}) components of the muscle force

- The angle between the line of action of the muscle force and the long axis of the bone upon which the muscle force is exerted is called the angle of pull and it is critical in determining the effectiveness of the muscle force. When the lower arm is flexed to a right angle, the muscle tension has only a rotational effect on the forearm about the elbow joint, because the line of action of the muscle force is at a right angle with the longitudinal axis of the forearm. For other flexed positions of the forearm, the muscle force can have a translational (stabilizing or sliding) component as well as a rotational component. Assume that the linkage system shown in Fig. 5.6a illustrates the position of the forearm relative to the upper arm. n designates a direction perpendicular (normal) to the long axis of the forearm and t is tangent to it. Assuming that the line of action of the muscle force remains parallel to the long axis of the humerus, F_M can be decomposed into its rectangular components F_{Mn} and F_{Mt} . In this case, F_{Mn} is the *rotational* (rotatory) component of the muscle force because its primary function is to rotate the forearm about the elbow joint. The tangential component F_{Mt} of the muscle force acts to compress the elbow joint and is called the *stabilizing* component of the muscle force. As the angle of pull approaches 90° , the magnitude of the rotational component of the muscle force increases while its stabilizing component decreases, and less and less energy is “wasted” to compress the elbow joint. As illustrated in Fig. 5.6b, the stabilizing role of F_{Mt} changes into a *sliding* or *dislocating* role when the angle between the long axes of the forearm and upper arm becomes less than 90° .

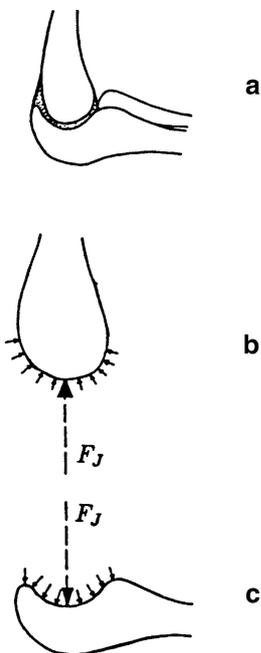


Fig. 5.7 Explaining the joint reaction force at the elbow

- The elbow is a diarthrodial (synovial) joint. A ligamentous capsule encloses an articular cavity which is filled with synovial fluid. Synovial fluid is a viscous material whose primary function is to lubricate the articulating surfaces, thereby reducing the frictional forces that may develop while one articulating surface slides over the other. The synovial fluid also nourishes the articulating cartilages. A common property of fluids is that they exert pressures (force per unit area) that are distributed over the surfaces they touch. The fluid pressure always acts in a direction toward and perpendicular to the surface it touches having a compressive effect on the surface. Note that in Fig. 5.7, the small vectors indicating the fluid pressure have components in the horizontal and vertical directions. We determined that the joint reaction force at the elbow acts vertically downward on the ulna. This implies that the horizontal components of these vectors cancel out (i.e., half pointing to the left and half pointing to the right), but their vertical components (on the ulna, almost all of them are pointing downward) add up to form the resultant force F_J (shown with a dashed arrow in Fig. 5.7c). Therefore, the joint reaction force F_J corresponds to the resultant of the distributed force system (pressure) applied through the synovial fluid.

• The most critical simplification made in this example is that the biceps was assumed to be the single muscle group responsible for maintaining the flexed configuration of the forearm. The reason for making such an assumption was to reduce the system under consideration to one that is statically determinate. In reality, in addition to the biceps, the brachialis and the brachioradialis are primary elbow flexor muscles.

Consider the flexed position of the arm shown in Fig. 5.8a. The free-body diagram of the forearm is shown in Fig. 5.8b. F_{M1} , F_{M2} , and F_{M3} are the magnitudes of the forces exerted on the forearm by the biceps, the brachialis, and the brachioradialis muscles with attachments at points A_1 , A_2 , and A_3 , respectively. Let θ_1 , θ_2 , and θ_3 be the angles that the biceps, the brachialis, and the brachioradialis muscles make with the long axis of the lower arm. As compared to the single-muscle system which consisted of two unknowns (F_M and F_J), the analysis of this three-muscle system is quite complex. First of all, this is not a simple parallel force system. Even if we assume that the locations of muscle attachments (A_1 , A_2 , and A_3), their angles of pull (θ_1 , θ_2 , and θ_3) and the lengths of their moment arms (a_1 , a_2 , and a_3) as measured from the elbow joint are known, there are still five unknowns in the problem (F_{M1} , F_{M2} , F_{M3} , F_J , and β , where the angle β is an angle between F_J and the long axes of the forearm). The total number of equations available from statics is three:

$$\sum M_O = 0: a_1 F_{M1} + a_2 F_{M2} + a_3 F_{M3} = bW + cW_O \quad (\text{iii})$$

$$\sum F_x = 0: F_{Jx} = F_{M1x} + F_{M2x} + F_{M3x} \quad (\text{iv})$$

$$\sum F_y = 0: F_{Jy} = F_{M1y} + F_{M2y} + F_{M3y} - W - W_O \quad (\text{v})$$

Note that once the muscle forces are determined, Eqs. (iv) and (v) will yield the components of the joint reaction force F_J . As far as the muscle forces are concerned, we have only Eq. (iii) with three unknowns. In other words, we have a statically indeterminate problem. To obtain a unique solution, we need additional information relating F_{M1} , F_{M2} , and F_{M3} .

There may be several approaches to the solution of this problem. The criteria for estimating the force distribution among different muscle groups may be established by: (1) using cross-sectional areas of muscles, (2) using electromyography (EMG) measurements of muscle signals, and (3) applying certain optimization techniques. It may be assumed that each muscle exerts a force proportional to its cross-sectional area. If S_1 , S_2 , and S_3 are the cross-sectional areas of the biceps, the brachialis, and the brachioradialis, then this criteria may be applied by expressing muscle forces in the following manner:

$$F_{M2} = k_{21} F_{M1} \quad \text{with} \quad k_{21} = \frac{S_2}{S_1} \quad (\text{vi})$$

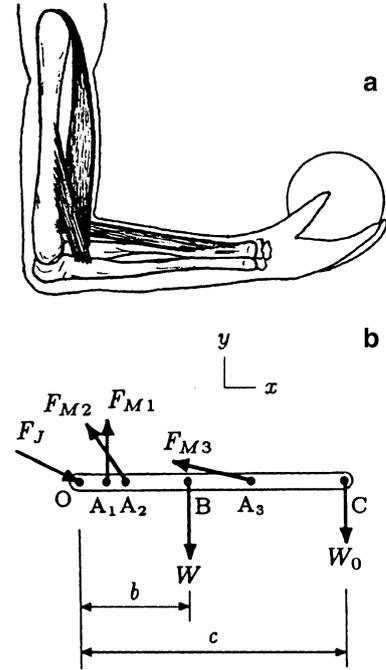


Fig. 5.8 Three-muscle system

$$F_{M3} = k_{31} F_{M1} \text{ with } k_{31} = \frac{S_3}{S_1} \quad (\text{vii})$$

If constants k_{21} and k_{31} are known, then Eqs. (vi) and (vii) can be substituted into Eq. (iii), which can then be solved for F_{M1} :

$$F_{M1} = \frac{bW + cW_O}{a_1 + a_2k_{21} + a_3k_{31}}$$

Substituting F_{M1} back into Eqs. (vi) and (vii) will then yield the magnitudes of the forces in the brachialis and the brachioradialis muscles. The values of k_{21} and k_{31} may also be estimated by using the amplitudes of muscle EMG signals.

This statically indeterminate problem may also be solved by considering some optimization techniques. If the purpose is to accomplish a certain task (static or dynamic) in the most efficient manner, then the muscles of the body must act to minimize the forces exerted, the moments about the joints (for dynamic situations), and/or the work done by the muscles. The question is, what force distribution among the various muscles facilitates the maximum efficiency? These concepts and relevant references will be discussed briefly in Sect. 5.11.

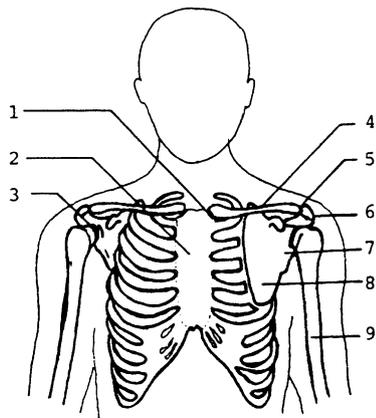


Fig. 5.9 The shoulder:
 (1) sternoclavicular joint,
 (2) sternum, (3) glenohumeral
 joint, (4) clavicle,
 (5) acromioclavicular joint,
 (6) acromion process, (7) glenoid
 fossa, (8) scapula, (9) humerus

5.6 Mechanics of the Shoulder

The bony structure and the muscles of the shoulder complex are illustrated in Figs. 5.9 and 5.10. The shoulder forms the base for all upper extremity movements. The complex structure of the shoulder can be divided into two: the shoulder joint and the shoulder girdle.

The shoulder joint, also known as the *glenohumeral articulation*, is a ball-and-socket joint between the nearly hemispherical humeral head (ball) and the shallowly concave glenoid fossa (socket) of the scapula. The shallowness of the glenoid fossa allows a significant freedom of movement of the humeral head on the articulating surface of the glenoid. The movements allowed are: in the sagittal plane, flexion (movement of the humerus to the front—a forward upward movement) and extension (return from flexion); in the coronal plane, abduction (horizontal upward movement of the humerus to the side) and adduction (return from abduction); and in the transverse plane, outward rotation (movement of the humerus around its long axis to the lateral side) and inward rotation (return from outward rotation). The configuration of the articulating surfaces of the shoulder joint also makes the joint more susceptible to instability and injury, such as dislocation. The stability of the joint is provided by the glenohumeral and coracohumeral ligaments, and by the muscles crossing the joint. The major

muscles of the shoulder joint are: deltoideus, supraspinatus, pectoralis major, coracobrachialis, latissimus dorsi, teres major, teres minor, infraspinatus, and subscapularis.

The bony structure of the shoulder girdle consists of the clavicle (collarbone) and the scapula (shoulder blade). The *acromioclavicular joint* is a small synovial articulation between the distal clavicle and the acromion process of the scapula. The stability of this joint is reinforced by the coracoclavicular ligaments. The *sternoclavicular joint* is the articulation between the manubrium of the sternum and the proximal clavicle. The stability of this joint is enhanced by the costoclavicular ligament. The acromioclavicular joint and the sternoclavicular joint both have layers of cartilage, called *menisci*, interposed between their bony surfaces.

There are four pairs of scapular movements: elevation (movement of the scapula in the frontal plane) and depression (return from elevation), upward rotation (turning the glenoid fossa upward and the lower medial border of the scapula away from the spinal column) and downward rotation (return from upward rotation), protraction (movement of the distal end of the clavicle forward) and retraction (return from protraction), and forward and backward rotation (rotation of the scapula about the shaft of the clavicle). Some of the main muscles that control and coordinate these movements are the trapezius, levator scapulae, rhomboid, pectoralis minor, serratus anterior, and subclavius.

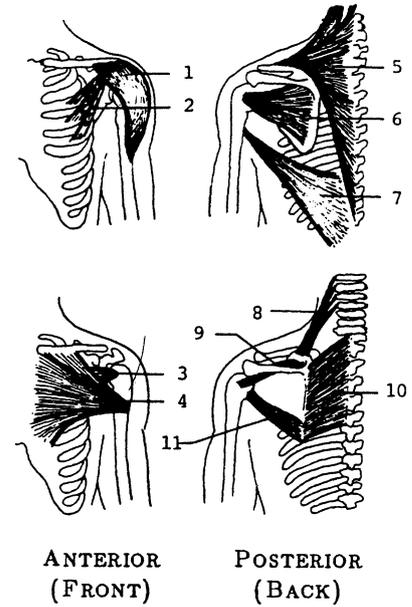


Fig. 5.10 Shoulder muscles: (1) deltoideus, (2) pectoralis minor, (3) subscapularis, (4) pectoralis major, (5) trapezius, (6) infraspinatus and teres minor, (7) latissimus dorsi, (8) levator scapulae, (9) supraspinatus, (10) rhomboideus, (11) teres major

Example 5.2 Consider a person strengthening the shoulder muscles by means of dumbbell exercises. Fig. 5.11 illustrates the position of the left arm when the arm is fully abducted to horizontal. The free-body diagram of the arm is shown in Fig. 5.12 along with a mechanical model of the arm. Also in Fig. 5.12, the forces acting on the arm are resolved into their rectangular components along the horizontal and vertical directions. Point O corresponds to the axis of rotation of the shoulder joint, point A is where the deltoid muscle is attached to the humerus, point B is the center of gravity of the entire arm, and point C is the center of gravity of the dumbbell. W is the weight of the arm, W_O is the weight of the dumbbell, F_M is the magnitude of the tension in the deltoid muscle, and F_j is the magnitude of the joint reaction force at the shoulder. The resultant of the deltoid muscle force makes an angle θ with the horizontal. The distances between point O and points A, B, and C are measured as a , b , and c , respectively.

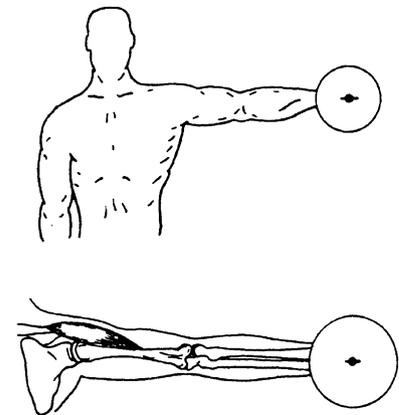


Fig. 5.11 The arm is abducted to horizontal

Determine the magnitude F_M of the force exerted by the deltoid muscle to hold the arm at the position shown. Also determine

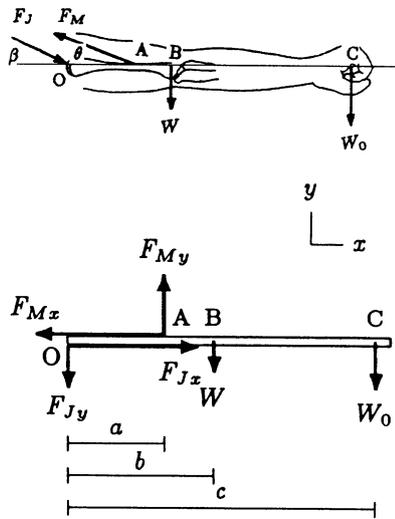


Fig. 5.12 Forces acting on the arm and a mechanical model representing the arm

the magnitude and direction of the reaction force at the shoulder joint in terms of specified parameters.

Solution: With respect to the xy coordinate frame, the muscle and joint reaction forces have two components while the weights of the arm and the dumbbell act in the negative y direction. The components of the muscle force are:

$$F_{Mx} = F_M \cos \theta \quad (-x) \tag{i}$$

$$F_{My} = F_M \sin \theta \quad (+y) \tag{ii}$$

Components of the joint reaction force are:

$$F_{Jx} = F_J \cos \beta \quad (+x) \tag{iii}$$

$$F_{Jy} = F_J \sin \beta \quad (-y) \tag{iv}$$

β is the angle that the joint reaction force makes with the horizontal. The line of action and direction (in terms of θ) of the force exerted by the muscle on the arm are known. However, the magnitude F_M of the muscle force, the magnitude F_J , and the direction (β) of the joint reaction force are unknowns. We have a total of three unknowns, F_M , F_J , and β (or F_M , F_{Jx} , and F_{Jy}). To be able to solve this two-dimensional problem, we have to utilize all three equilibrium equations.

First, consider the rotational equilibrium of the arm about the shoulder joint at point O. The joint reaction force produces no torque about point O because its line of action passes through point O. For practical purposes, we can neglect the possible contribution of the horizontal component of the muscle force to the moment generated about point O by assuming that its line of action also passes through point O. Note that this is not a critical or necessary assumption to solve this problem. If we knew the length of its moment arm (i.e., the vertical distance between O and A), we could easily incorporate the torque generated by F_{My} about point O into the analysis. Under these considerations, there are only three moment producing forces about point O. For the rotational equilibrium of the arm, the net moment about point O must be equal to zero. Taking counter-clockwise moments to be positive:

$$\begin{aligned} \sum M_O = 0 : \quad aF_{My} - bW - cW_O &= 0 \\ F_{My} &= \frac{1}{a} (bW + cW_O) \end{aligned} \tag{v}$$

For given a , b , c , W , and W_O , Eq. (v) can be used to determine the vertical component of the force exerted by the deltoid muscle. Equation (ii) can now be used to determine the total force exerted by the muscle:

$$F_M = \frac{F_{My}}{\sin \theta} \quad (\text{vi})$$

Knowing F_M , Eq. (i) will yield the horizontal component of the tension in the muscle:

$$F_{Mx} = F_M \cos \theta \quad (\text{vii})$$

The components of the joint reaction force can be determined by considering the translational equilibrium of the arm in the horizontal and vertical directions:

$$\sum F_x = 0 \quad \text{that is: } F_{Jx} - F_{Mx} = 0, \quad \text{then } F_{Jx} = F_{Mx} \quad (\text{viii})$$

$$\begin{aligned} \sum F_y = 0 \quad \text{that is: } & -F_{Jy} + F_{My} - W - W_O = 0 \\ \text{then } F_{Jy} = & F_{My} - W - W_O \end{aligned} \quad (\text{ix})$$

Knowing the rectangular components of the joint reaction force enables us to compute the magnitude of the force itself and the angle its line of action makes with the horizontal:

$$F_J = \sqrt{(F_{Jx})^2 + (F_{Jy})^2} \quad (\text{x})$$

$$\beta = \tan^{-1} \left(\frac{F_{Jy}}{F_{Jx}} \right) \quad (\text{xi})$$

Now consider that $a = 15 \text{ cm}$, $b = 30 \text{ cm}$, $c = 60 \text{ cm}$, $\theta = 15^\circ$, $W = 40 \text{ N}$, and $W_O = 60 \text{ N}$. Then:

$$\begin{aligned} F_{My} &= \frac{1}{0.15} [(0.30)(40) + (0.60)(60)] \\ &= 320 \text{ N} \quad (+y) \end{aligned}$$

$$F_M = \frac{320}{\sin 15^\circ} = 1236 \text{ N}$$

$$F_{Mx} = (1236)(\cos 15^\circ) = 1194 \text{ N} \quad (-x)$$

$$F_{Jx} = 1194 \text{ N} \quad (+x)$$

$$F_{Jy} = 320 - 40 - 60 = 220 \text{ N} \quad (-y)$$

$$F_J = \sqrt{(1194)^2 + (220)^2} = 1214 \text{ N}$$

$$\beta = \tan^{-1} \left(\frac{220}{1194} \right) = 10^\circ$$

Remarks

- F_{Mx} is the stabilizing component and F_{My} is the rotational component of the deltoid muscle. F_{Mx} is approximately four times larger than F_{My} . A large stabilizing component suggests that the horizontal position of the arm is not stable, and that the muscle needs to exert a high horizontal force to stabilize it.

- The human shoulder is very susceptible to injuries. The most common injuries are dislocations of the shoulder joint and the fracture of the humerus. Since the socket of the glenohumeral joint is shallow, the head of the humerus is relatively free to rotate about the articulating surface of the glenoid fossa. This freedom of movement is achieved, however, by reduced joint stability. The humeral head may be displaced in various ways, depending on the strength or weakness of the muscular and ligamentous structure of the shoulder, and depending on the physical activity. Humeral fractures are another common type of injuries. The humerus is particularly vulnerable to injuries because of its unprotected configuration.
- Average ranges of motion of the arm about the shoulder joint are 230° during flexion–extension, and 170° in both abduction–adduction and inward–outward rotation.

5.7 Mechanics of the Spinal Column

The human spinal column is the most complex part of the human musculoskeletal system. The principal functions of the spinal column are to protect the spinal cord; to support the head, neck, and upper extremities; to transfer loads from the head and trunk to the pelvis; and to permit a variety of movements. The spinal column consists of the cervical (neck), thoracic (chest), lumbar (lower back), sacral, and coccygeal regions. The thoracic and lumbar sections of the spinal column make up the trunk. The sacral and coccygeal regions are united with the pelvis and can be considered parts of the pelvic girdle.

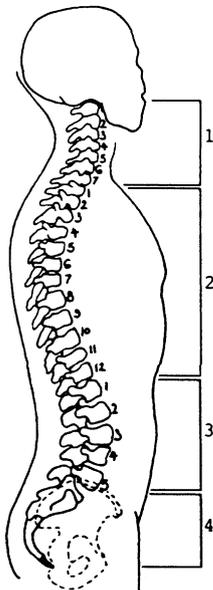


Fig. 5.13 The spinal column: (1) cervical vertebrae, (2) thoracic vertebrae, (3) lumbar vertebrae, (4) sacrum

The vertebral column consists of 24 intricate and complex vertebrae (Fig. 5.13). The articulations between the vertebrae are amphiarthrodial joints. A fibrocartilaginous disc is interposed between each pair of vertebrae. The primary functions of these intervertebral discs are to sustain loads transmitted from segments above, act as shock absorbers, eliminate bone-to-bone contact, and reduce the effects of impact forces by preventing direct contact between the bony structures of the vertebrae. The articulations of each vertebra with the adjacent vertebrae permit movement in three planes, and the entire spine functions like a single ball-and-socket joint. The structure of the spine allows a wide variety of movements including flexion–extension, lateral flexion, and rotation.

Two particularly important joints of the spinal column are those with the head (occiput bone of the skull) and the first cervical vertebrae, atlas, and the atlas and the second vertebrae, the axis. The *atlantooccipital joint* is the union between the first cervical vertebra (the atlas) and the occipital bone of the head. This is a double condyloid joint and permits movements of the head in

the sagittal and frontal planes. The *atlantoaxial joint* is the union between the atlas and the odontoid process of the head. It is a pivot joint, enabling the head to rotate in the transverse plane. The muscle groups providing, controlling, and coordinating the movement of the head and the neck are the prevertebrals (anterior), hyoids (anterior), sternocleidomastoid (anterior-lateral), scalene (lateral), levator scapulae (lateral), suboccipitals (posterior), and spleni (posterior).

The spine gains its stability from the intervertebral discs and from the surrounding ligaments and muscles (Fig. 5.14). The discs and ligaments provide intrinsic stability, and the muscles supply extrinsic support. The muscles of the spine exist in pairs. The anterior portion of the spine contains the abdominal muscles: the rectus abdominis, transverse abdominis, external obliques, and internal obliques. These muscles provide the necessary force for trunk flexion and maintain the internal organs in proper position. There are three layers of posterior trunk muscles: erector spinae, semispinalis, and the deep posterior spinal muscle groups. The primary function of the muscles located at the posterior portion of the spine is to provide trunk extension. These muscles also support the spine against the effects of gravity. The quadratus lumborum muscle is important in lateral trunk flexion. It also stabilizes the pelvis and lumbar spine. The lateral flexion of the trunk results from the actions of the abdominal and posterior muscles. The rotational movement of the trunk is controlled by the simultaneous action of anterior and posterior muscles.

The spinal column is vulnerable to various injuries. The most severe injury involves the spinal cord, which is immersed in fluid and protected by the bony structure. Other critical injuries include fractured vertebrae and herniated intervertebral discs. Lower back pain may also result from strains in the lower regions of the spine.

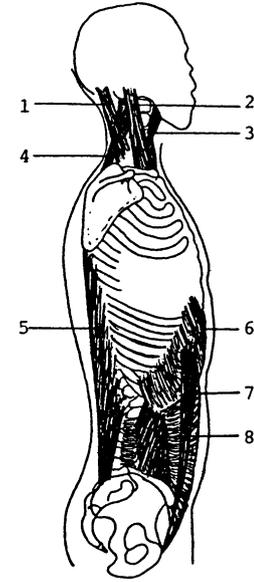


Fig. 5.14 Selected muscles of the neck and spine: (1) splenius, (2) sternocleidomastoid, (3) hyoid, (4) levator scapulae, (5) erector spinae, (6) obliques, (7) rectus abdominis, (8) transversus abdominis

Example 5.3 Consider the position of the head and the neck shown in Fig. 5.15. Also shown are the forces acting on the head. The head weighs $W = 50\text{ N}$ and its center of gravity is located at point C. F_M is the magnitude of the resultant force exerted by the neck extensor muscles, which is applied on the skull at point A. The atlantooccipital joint center is located at point B. For this flexed position of the head, it is estimated that the line of action of the neck muscle force makes an angle $\theta = 30^\circ$ and the line of action of the joint reaction force makes an angle $\beta = 60^\circ$ with the horizontal.

What tension must the neck extensor muscles exert to support the head? What is the compressive force applied on the first cervical vertebra at the atlantooccipital joint?

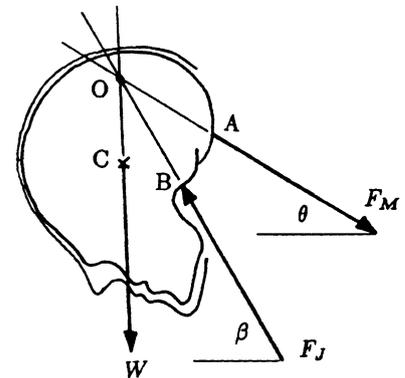


Fig. 5.15 Forces on the skull form a concurrent system

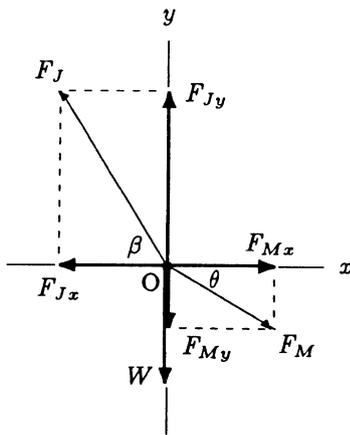


Fig. 5.16 Components of the forces acting on the head

Solution: We have a three-force system with two unknowns: magnitudes F_M and F_J of the muscle and joint reaction forces. Since the problem has a relatively complicated geometry, it is convenient to utilize the condition that for a body to be in equilibrium the force system acting on it must be either concurrent or parallel. In this case, it is clear that the forces involved do not form a parallel force system. Therefore, the system of forces under consideration must be concurrent. Recall that a system of forces is concurrent if the lines of action of all forces have a common point of intersection.

In Fig. 5.15, the lines of action of all three forces acting on the head are extended to meet at point O. In Fig. 5.16, the forces \underline{W} , \underline{F}_M , and \underline{F}_J acting on the skull are translated to point O, which is also chosen to be the origin of the xy coordinate frame. The rectangular components of the muscle and joint reaction forces in the x and y directions are:

$$F_{Mx} = F_M \cos \theta \tag{i}$$

$$F_{My} = F_M \sin \theta \tag{ii}$$

$$F_{Jx} = F_J \cos \beta \tag{iii}$$

$$F_{Jy} = F_J \sin \beta \tag{iv}$$

The translational equilibrium conditions in the x and y directions will yield:

$$\sum F_x = 0 \text{ that is : } -F_{Jx} + F_{Mx} = 0, \text{ then } F_{Jx} = F_{Mx} \tag{v}$$

$$\sum F_y = 0 \text{ that is : } -W - F_{My} + F_{Jy} = 0, \text{ then } F_{Jy} = W + F_{My} \tag{vi}$$

Substitute Eqs. (i) and (iii) into Eq. (v):

$$F_J \cos \beta = F_M \cos \theta \tag{vii}$$

Substitute Eqs. (ii) and (iv) into Eq. (vi):

$$F_J \sin \beta = W + F_M \sin \theta \tag{viii}$$

Substitute this equation into Eq. (viii), that is:

$$\frac{F_M \cdot \cos \theta}{\cos \beta} \cdot \sin \beta = W + F_M \sin \theta,$$

$$F_M \cdot \cos \theta \tan \beta = W + F_M \sin \theta, \text{ then}$$

$$\tan \beta = \frac{W + F_M \sin \theta}{F_M \cos \theta} \tag{ix}$$

Equation (ix) can now be solved for the unknown muscle force F_M :

$$F_M \cos \theta \tan \beta = W + F_M \sin \theta$$

$$F_M(\cos \theta \tan \beta - \sin \theta) = W$$

$$F_M = \frac{W}{\cos \theta \tan \beta - \sin \theta} \quad (\text{x})$$

Equation (x) gives the tension in the muscle as a function of the weight W of the head and the angles θ and β that the lines of action of the muscle and joint reaction forces make with the horizontal. Substituting the numerical values of W , θ , and β will yield:

$$F_M = \frac{50}{(\cos 30^\circ)(\tan 60^\circ) - (\sin 30^\circ)} = 50 \text{ N}$$

From Eqs. (i) and (ii):

$$F_{Mx} = (50)(\cos 30^\circ) = 43 \text{ N} \quad (+x)$$

$$F_{My} = (50)(\sin 30^\circ) = 25 \text{ N} \quad (-y)$$

From Eqs. (v) and (vi):

$$F_{Jx} = 43 \text{ N} \quad (-x)$$

$$F_{Jy} = 50 + 25 = 75 \text{ N} \quad (+y)$$

The resultant of the joint reaction force can be computed from either Eq. (iii) or Eq. (iv). Using Eq. (iii):

$$F_J = \frac{F_{Jx}}{\cos \beta} = \frac{43}{\cos 60^\circ} = 86 \text{ N}$$

Remarks

- The extensor muscles of the head must apply a force of 50 N to support the head in the position considered. The reaction force developed at the atlantooccipital joint is about 86 N.
- The joint reaction force can be resolved into two rectangular components, as shown in Fig. 5.17. F_{Jn} is the magnitude of the normal component of F_J compressing the articulating joint surface, and F_{Jt} is the magnitude of its tangential component having a shearing effect on the joint surfaces. Forces in the muscles and ligaments of the neck operate in a manner to counterbalance this shearing effect.

Example 5.4 Consider the weight lifter illustrated in Fig. 5.18, who is bent forward and lifting a weight W_0 . At the position shown, the athlete's trunk is flexed by an angle θ as measured from the upright (vertical) position.

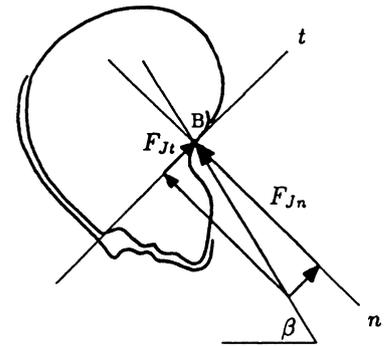


Fig. 5.17 Normal and shear components of the joint reaction force

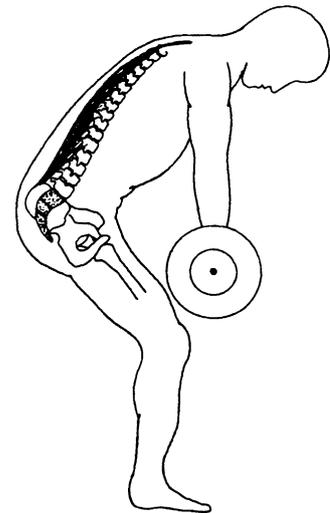


Fig. 5.18 A weight lifter

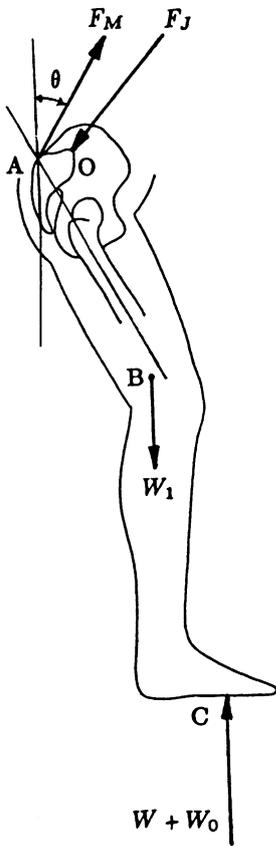


Fig. 5.19 Forces acting on the lower body of the athlete

The forces acting on the lower portion of the athlete’s body are shown in Fig. 5.19 by considering a section passing through the fifth lumbar vertebra. A mechanical model of the athlete’s lower body (the pelvis and legs) is illustrated in Fig. 5.20 along with the geometric parameters of the problem under consideration. W is the total weight of the athlete, W_1 is the weight of the legs including the pelvis, $(W + W_0)$ is the total ground reaction force applied to the athlete through the feet (at point C), F_M is the magnitude of the resultant force exerted by the erector spinae muscles supporting the trunk, and F_J is the magnitude of the compressive force generated at the union (point O) of the sacrum and the fifth lumbar vertebra. The center of gravity of the legs including the pelvis is located at point B. Relative to point O, the lengths of the lever arms of the muscle force, lower body weight, and ground reaction force are measured as a , b , and c , respectively.

Assuming that the line of pull of the resultant muscle force exerted by the erector spinae muscles is parallel to the trunk (i.e., making an angle θ with the vertical), determine F_M and F_J in terms of b , c , θ , W_0 , W_1 , and W .

Solution: In this case, there are three unknowns: F_M , F_{Jx} , and F_{Jy} . The lengths of the lever arms of the muscle force, ground reaction force, and the gravitational force of the legs including the pelvis are given as measured from point O. Therefore, we can apply the rotational equilibrium condition about point O to determine the magnitude F_M of the resultant force exerted by the erector spinae muscles. Considering clockwise moments to be positive:

$$\sum M_O = 0 : aF_M + bW_1 - c(W + W_0) = 0$$

Solving this equation for F_M will yield:

$$F_M = \frac{c(W + W_0) - bW_1}{a} \tag{i}$$

For given numerical values of b , c , θ , W_0 , W_1 , and W , Eq. (i) can be used to determine the magnitude of the resultant muscle force. Once F_M is calculated, its components in the x and y directions can be determined using:

$$F_{Mx} = F_M \sin \theta \tag{ii}$$

$$F_{My} = F_M \cos \theta \tag{iii}$$

The horizontal and vertical components of the reaction force developed at the sacrum can now be determined by utilizing the translational equilibrium conditions of the lower body of the athlete in the x and y directions:

$$\sum F_x = 0 \text{ That is, } F_{Mx} - F_{Jx} = 0, \text{ then } F_{Jx} = F_{Mx} \tag{iv}$$

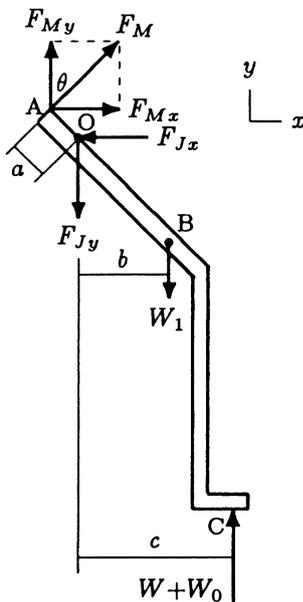


Fig. 5.20 Free-body diagram

$$\sum F_y = 0 \text{ That is, } F_{My} - F_{Jy} - W_1 + (W + W_0) = 0, \quad (\text{v})$$

$$\text{then } F_{Jy} = F_{My} + W + W_0 - W_1$$

Assume that at an instant the athlete is bent so that his trunk makes an angle $\theta = 45^\circ$ with the vertical, and that the lengths of the lever arms are measured in terms of the height h of the athlete and the weights are given in terms of the weight W of the athlete as: $a = 0.02h$, $b = 0.08h$, $c = 0.12h$, $W_0 = W$, and $W_1 = 0.4W$. Using Eq. (i):

$$F_M = \frac{(0.12h)(W + W) - (0.08h)(0.4W)}{0.02h} = 10.4 W$$

From Eqs. (ii) and (iii):

$$F_{Mx} = (10.4 W)(\sin 45^\circ) = 7.4 W$$

$$F_{My} = (10.4 W)(\cos 45^\circ) = 7.4 W$$

From Eqs. (iv) and (v):

$$F_{Jx} = 7.4 W$$

$$F_{Jy} = 7.4 W + W + W - 0.4 W = 9.0 W$$

Therefore, the magnitude of the resultant force on the sacrum is:

$$F_J = \sqrt{(F_{Jx})^2 + (F_{Jy})^2} = 11.7 W$$

Remarks

- The results obtained are quite significant. While the athlete is bent forward by 45° and lifting a weight with magnitude equal to his own body weight, the erector spinae muscles exert a force more than 10 times the weight of the athlete and the force applied to the union of the sacrum and the fifth lumbar vertebra is about 12 times that of the body weight.

5.8 Mechanics of the Hip

The articulation between the head of the femur and the acetabulum of the pelvis (Fig. 5.21) forms a diarthrodial joint. The stability of the hip joint is provided by its relatively rigid ball-and-socket type of configuration, its ligaments, and by the large and strong muscles crossing it. The femoral head fits well into the deep socket of the acetabulum. The ligaments of the hip joint, as well as the labrum (a flat rim of fibrocartilage), support and hold the femoral head in the acetabulum as the femoral

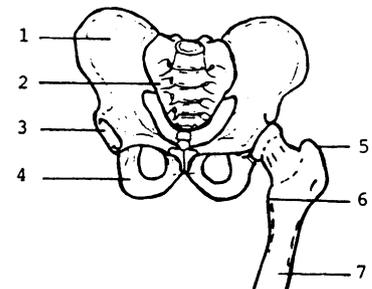


Fig. 5.21 Pelvis and the hip: (1) ilium, (2) sacrum, (3) acetabulum, (4) ischium, (5) greater trochanter, (6) lesser trochanter, (7) femur

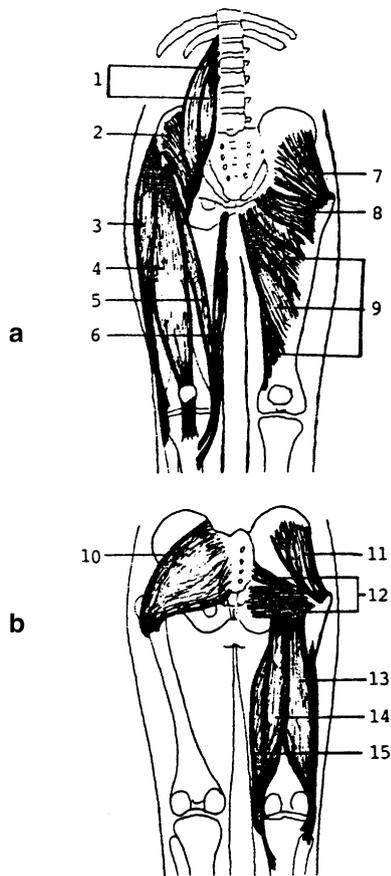


Fig. 5.22 Muscles of the hip (a) anterior and (b) posterior views: (1) *psaos*, (2) *iliacus*, (3) *tensor fascia latae*, (4) *rectus femoris*, (5) *sartorius*, (6) *gracilis*, (7) *gluteus minimus*, (8) *pectineus*, (9) *adductors*, (10, 11) *gluteus maximus and medius*, (12) *lateral rotators*, (13) *biceps femoris*, (14) *semitendinosus*, (15) *semimembranosus*

head moves. The construction of the hip joint is such that it is very stable and has a great deal of mobility, thereby allowing a wide range of motion required for activities such as walking, sitting, and squatting. Movements of the femur about the hip joint include flexion and extension, abduction and adduction, and inward and outward rotation. In some instances, the extent of these movements is constrained by ligaments, muscles, and/or the bony structure of the hip. The articulating surfaces of the femoral head and the acetabulum are lined with hyaline cartilage. Derangements of the hip can produce altered force distributions in the joint cartilage, leading to degenerative arthritis.

The pelvis consists of the ilium, ischium, and pubis bones, and the sacrum. At birth and during growth the bones of the pelvis are distinct. In adults the bones of the pelvis are fused and form synarthrodial joints which allow no movement. The pelvis is located between the spine and the two femurs. The position of the pelvis makes it relatively less stable. Movements of the pelvis occur primarily for the purpose of facilitating the movements of the spine or the femurs. There are no muscles whose primary purpose is to move the pelvis. Movements of the pelvis are caused by the muscles of the trunk and the hip.

Based on their primary actions, the muscles of the hip joint can be divided into several groups (Fig. 5.22). The *psaos*, *iliacus*, *rectus femoris*, *pectineus*, and *tensor fascia latae* are the primary hip flexors. They are also used to carry out activities such as running or kicking. The *gluteus maximus* and the hamstring muscles (the *biceps femoris*, *semitendinosus*, and *semimembranosus*) are hip extensors. The hamstring muscles also function as knee flexors. The *gluteus medius* and *gluteus minimus* are hip abductor muscles providing for the inward rotation of the femur. The *gluteus medius* is also the primary muscle group stabilizing the pelvis in the frontal plane. The *adductor longus*, *adductor brevis*, *adductor magnus*, and *gracilis* muscles are the hip adductors. There are also small, deeply placed muscles (outward rotators) which provide for the outward rotation of the femur.

The hip muscles predominantly suffer contusions and strains occurring in the pelvis region.

Example 5.5 During walking and running, we momentarily put all of our body weight on one leg (the right leg in Fig. 5.23). The forces acting on the leg carrying the total body weight are shown in Fig. 5.24 during such a single-leg stance. F_M is the magnitude of the resultant force exerted by the hip abductor muscles, F_J is the magnitude of the joint reaction force applied

by the pelvis on the femur, W_1 is the weight of the leg, W is the total weight of the body applied as a normal force by the ground on the leg. The angle between the line of action of the resultant muscle force and the horizontal is designated by θ .

A mechanical model of the leg, rectangular components of the forces acting on it, and the parameters necessary to define the geometry of the problem are shown in Fig. 5.25. O is a point along the instantaneous axis of rotation of the hip joint, point A is where the hip abductor muscles are attached to the femur, point B is the center of gravity of the leg, and point C is where the ground reaction force is applied on the foot. The distances between point A and points O, B, and C are specified as a , b , and c , respectively. α is the angle of inclination of the femoral neck to the horizontal, and β is the angle that the long axis of the femoral shaft makes with the horizontal. Therefore, $\alpha + \beta$ is approximately equal to the total neck-to-shaft angle of the femur.

Determine the force exerted by the hip abductor muscles and the joint reaction force at the hip to support the leg and the hip in the position shown.

Solution 1: Utilizing the Free-Body Diagram of the Leg For the solution of the problem, we can utilize the free-body diagram of the right leg supporting the entire weight of the person. In Fig. 5.25a, the muscle and joint reaction forces are shown in terms of their components in the x and y directions. The resultant muscle force has a line of action that makes an angle θ with the horizontal. Therefore:

$$F_{Mx} = F_M \cos \theta \quad (\text{i})$$

$$F_{My} = F_M \sin \theta \quad (\text{ii})$$

Since angle θ is specified (given as a measured quantity), the only unknown for the muscle force is its magnitude F_M . For the joint reaction force, neither the magnitude nor the direction is known. With respect to the axis of the hip joint located at point O, a_x in Fig. 5.25b is the moment arm of the vertical component F_{My} of the muscle force, and a_y is the moment arm of the horizontal component of the muscle force F_{Mx} . Similarly, $(b_x - a_x)$ is the moment arm for W_1 and $(c_x - a_x)$ is the moment arm for the force W applied by the ground on the leg.

From the geometry of the problem:

$$a_x = a \cos \alpha \quad (\text{iii})$$

$$a_y = a \sin \alpha \quad (\text{iv})$$

$$b_x = b \cos \beta \quad (\text{v})$$

$$c_x = c \cos \beta \quad (\text{vi})$$

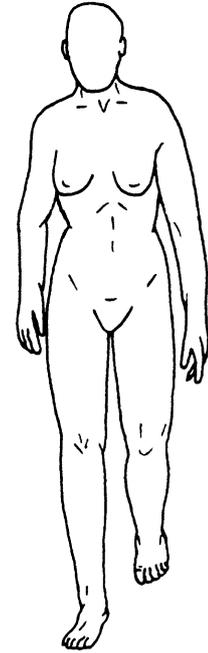


Fig. 5.23 Single-leg stance

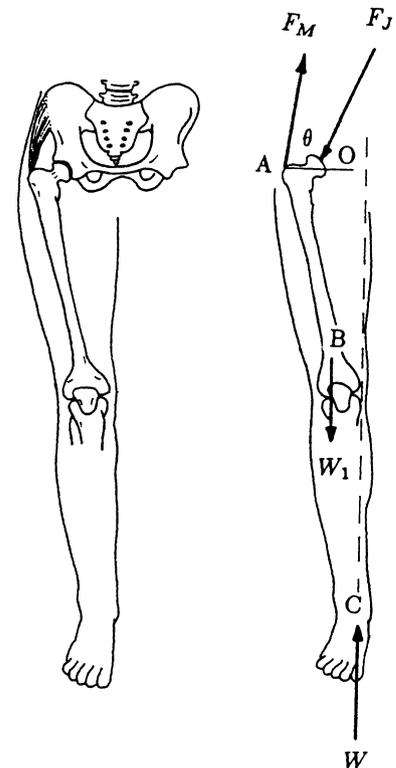


Fig. 5.24 Forces acting on the right leg carrying the entire weight of the body

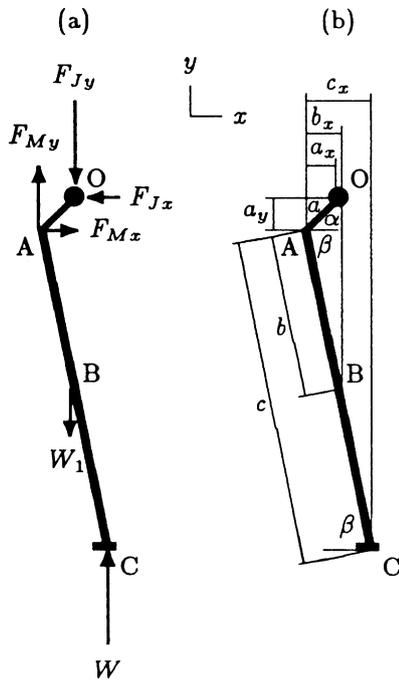


Fig. 5.25 Free-body diagram of the leg (a) and the geometric parameters (b)

Now that the horizontal and vertical components of all forces involved, and their moment arms with respect to point O are established, the condition for the rotational equilibrium of the leg about point O can be utilized to determine the magnitude of the resultant muscle force applied at point A. Assuming that the clockwise moments are positive:

$$\sum M_O = 0: a_x F_{My} - a_y F_{Mx} - (c_x - a_x)W + (b_x - a_x)W_1 = 0$$

Substituting Eqs. (i) through (vi) into the above equation:

$$(a \cos \alpha)(F_M \sin \theta) - (a \sin \alpha)(F_M \cos \theta) - (c \cos \beta - a \cos \alpha)W + (b \cos \beta - a \cos \alpha)W_1 = 0$$

Solving this equation for the muscle force:

$$F_M = \frac{(cW - bW_1) \cos \beta - a(W - W_1) \cos \alpha}{a(\cos \alpha \sin \theta - \sin \alpha \cos \theta)} \quad (\text{vii})$$

Notice that the denominator of Eq. (vii) can be simplified as $a \sin(\theta - \alpha)$. To determine the components of the joint reaction force, we can utilize the horizontal and vertical equilibrium conditions of the leg:

$$\sum F_x = 0: F_{Jx} = F_{Mx} = F_M \cos \theta \quad (\text{viii})$$

$$\sum F_y = 0: F_{Jy} = F_{My} + W - W_1$$

$$F_{Jy} = F_M \sin \theta + W - W_1 \quad (\text{ix})$$

Therefore, the resultant force acting at the hip joint is:

$$F_J = \sqrt{(F_{Jx})^2 + (F_{Jy})^2} \quad (\text{x})$$

Assume that the geometric parameters of the problem and the weight of the leg are measured in terms of the person's height h and total weight W as follows: $a = 0.05h$, $b = 0.20h$, $c = 0.52h$, $\alpha = 45^\circ$, $\beta = 80^\circ$, $\theta = 70^\circ$, and $W_1 = 0.17W$. The solution of the above equations for the muscle and joint reaction forces will yield $F_M = 2.6W$ and $F_J = 3.4W$, the joint reaction force making an angle $\varphi = \tan^{-1}(F_{Jy}/F_{Jx}) = 74.8^\circ$ with the horizontal.

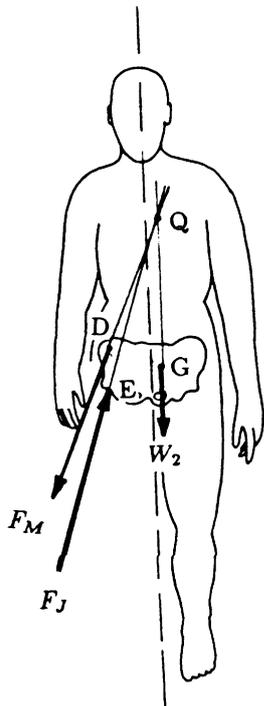


Fig. 5.26 Forces acting on the pelvis during a single-leg (right leg) stance

Solution 2: Utilizing the Free-Body Diagram of the Upper Body Here we have an alternative approach to the solution of the same problem. In this case, instead of the free-body diagram of the right leg, the free-body diagram of the upper body (including the left leg) is utilized. The forces acting on the upper body are shown in Figs. 5.26 and 5.27. F_M is the magnitude of the resultant force exerted by the hip abductor muscles applied on the pelvis at point D. θ is again the angle between the line of action of the resultant muscle force and the horizontal. F_J is the magnitude of the reaction force applied by the head of the femur on the hip joint at point E. $W_2 = W - W_1$ (total body

weight minus the weight of the right leg) is the weight of the upper body and the left leg acting as a concentrated force at point G. Note that point G is not the center of gravity of the entire body. Since the right leg is not included in the free-body, the left-hand side of the body is “heavier” than the right-hand side, and point G is located to the left of the original center of gravity (a point along the vertical dashed line in Fig. 5.27) of the person. The location of point G can be determined utilizing the method provided in Sect. 4.12.

By combining the individual weights of the segments constituting the body under consideration, the problem is reduced to a three-force system. It is clear from the geometry of the problem that the forces involved do not form a parallel system. Therefore, for the equilibrium of the body, they have to form a concurrent system of forces. This implies that the lines of action of the forces must have a common point of intersection (point Q in Fig. 5.27), which can be obtained by extending the lines of action of \underline{W}_2 and \underline{F}_M . A line passing through points Q and E designates the line of action of the joint reaction force \underline{F}_J . The angle φ that \underline{F}_J makes with the horizontal can now be measured from the geometry of the problem. Since the direction of \underline{F}_J is determined through certain geometric considerations, the number of unknowns is reduced by one. As illustrated in Fig. 5.28, the unknown magnitudes F_M and F_J of the muscle and joint reaction forces can now be determined simply by translating \underline{W}_2 , \underline{F}_M , and \underline{F}_J to point Q, and decomposing them into their components along the horizontal (x) and vertical (y) directions:

$$\begin{aligned} F_{Mx} &= F_M \cos \theta \\ F_{My} &= F_M \sin \theta \\ F_{Jx} &= F_J \cos \varphi \\ F_{Jy} &= F_J \sin \varphi \end{aligned} \quad (\text{xi})$$

For the translational equilibrium in the x and y directions:

$$\begin{aligned} \sum F_x &= 0 \quad \text{That is, } -F_{Mx} + F_{Jx} = 0, \text{ then } F_{Jx} = F_{Mx} \\ \sum F_y &= 0 \quad \text{That is, } F_{Jy} - W_2 - F_{My} = 0, \text{ then } F_{Jy} = F_{My} + W_2 \end{aligned}$$

Considering Eq. (xi):

$$F_J \cos \varphi = F_M \cos \theta, \text{ and} \quad (\text{xii})$$

$$F_J \sin \varphi = F_M \sin \theta + W_2 \quad (\text{xiii})$$

From Eq. (xii):

$$F_J = \frac{F_M \cos \theta}{\cos \varphi}$$

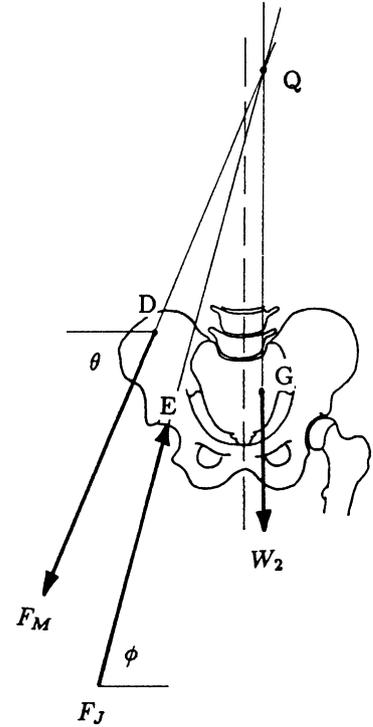


Fig. 5.27 Forces involved form a concurrent system

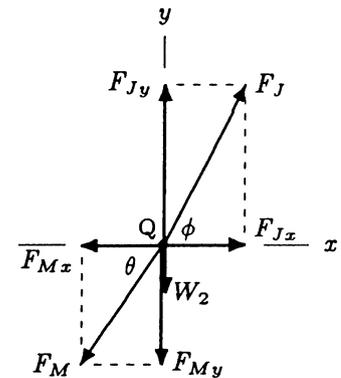


Fig. 5.28 Resolution of the forces into their components

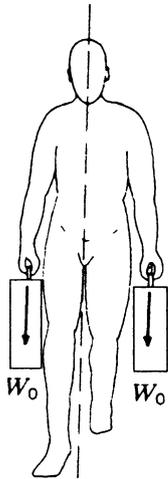


Fig. 5.29 Carrying a load in each hand

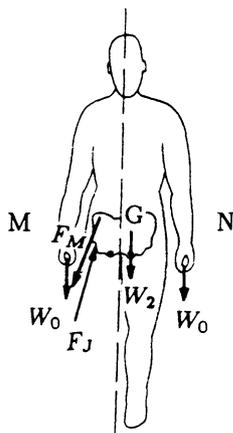


Fig. 5.30 Forces acting on the upper body

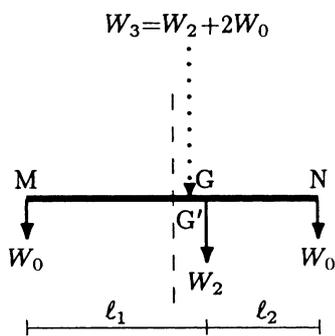


Fig. 5.31 W_3 is the resultant of the three-force system

Substituting this equation into Eq. (xiii) will yield:

$$\frac{F_M \cos \theta}{\cos \varphi} \sin \varphi = F_M \sin \theta + W_2, \text{ that is}$$

$$\frac{F_M \cos \theta}{\cos \varphi} \sin \varphi - F_M \sin \theta = W_2$$

$$F_M \left(\frac{\cos \theta \sin \varphi - \sin \theta \cos \varphi}{\cos \varphi} \right) = W_2, \text{ then}$$

$$F_M = \frac{\cos \varphi W_2}{\sin (\varphi - \theta)}, \text{ and}$$

$$F_J = \frac{\cos \theta W_2}{\sin (\varphi - \theta)}$$

For example, if $\theta = 70^\circ$, $\varphi = 74.8^\circ$, and $W_2 = 0.83 W$ (W is the total weight of the person), then the last two equations will yield $F_M = 2.6 W$ and $F_J = 3.4 W$.

How would the muscle and hip joint reaction forces vary if the person is carrying a load of W_0 in each hand during single-leg stance (Fig. 5.29)?

The free-body diagram of the upper body while the person is carrying a load of W_0 in each hand is shown in Fig. 5.30. The system to be analyzed consists of the upper body of the person (including the left leg) and the loads carried in each hand. To counterbalance both the rotational and translational (downward) effects of the extra loads, the hip abductor muscles will exert additional forces, and there will be larger compressive forces generated at the hip joint.

In this case, the number of forces is five. The gravitational pull on the upper body (W_2) and on the masses carried in the hands (W_0) form a parallel force system. If these parallel forces can be replaced by a single resultant force, then the number of forces can be reduced to three, and the problem can be solved by applying the same technique explained above (Solution 2). For this purpose, consider the force system shown in Fig. 5.31. Points M and N correspond to the right and left hands of the person where external forces of equal magnitude (W_0) are applied. Point G is the center of gravity of the upper body including the left leg. The vertical dashed line shows the symmetry axis (midline) of the person in the frontal plane, and point G is located to the left of this axis. Note that the distance l_1 between points M and G is greater than the distance l_2 between points N and G. If l_1, l_2, W_2 , and W_0 are given, then a new center of gravity (point G') can be determined by applying the technique of finding the center of gravity of a system composed of a number of parts whose centers of gravity are known (see Sect. 5.14). By intuition, point G' is located somewhere between the symmetry axis and point G. In other words, G' is closer to the right hip joint, and therefore, the length of the moment arm

of the total weight as measured from the right hip joint is shorter as compared to the case when there is no load carried in the hands. On the other hand, the magnitude of the resultant gravitational force is $W_3 = W_2 + 2W_0$, which over compensates for the advantage gained by the reduction of the moment arm.

Once the new center of gravity of the upper body is determined, including the left leg and the loads carried in each hand, Eqs. (xi) and (xii) can be utilized to calculate the resultant force exerted by the hip abductor muscles and the reaction force generated at the hip joint:

$$F_M = \frac{\cos \phi' (W_2 + 2W_0)}{\cos \theta \sin \phi' - \sin \theta \cos \phi'}$$

$$F_J = \frac{\cos \theta (W_2 + 2W_0)}{\cos \theta \sin \phi' - \sin \theta \cos \phi'}$$

Here, Eqs. (xi) and (xii) are modified by replacing the weight W_2 of the upper body with the new total weight $W_3 = W_2 + 2W_0$, and by replacing the angle ϕ that the line of action of the joint reaction force makes with the horizontal with the new angle ϕ' (Fig. 5.32). ϕ' is slightly larger than ϕ because of the shift of the center of gravity from point G to point G' toward the right of the person. Also, it is assumed that the angle θ between the line of action of the muscle force and the horizontal remains unchanged.

What happens if the person is carrying a load of W_0 in the left hand during a right-leg stance (Fig. 5.33)?

Assuming that the system we are analyzing consists of the upper body, left leg, and the load in hand, the extra load W_0 carried in the left hand will shift the center of gravity of the system from point G to point G'' toward the left of the person. Consequently the length of the lever arm of the total gravitational force $W_4 = W_2 + W_0$ as measured from the right hip joint (Fig. 5.34) will increase. This will require larger hip abductor muscle forces to counterbalance the clockwise rotational effect of W_4 and also increase the compressive forces at the right hip joint.

It can be observed from the geometry of the system analyzed that a shift in the center of gravity from point G to point G'' toward the left of the person will decrease the angle between the line of action of the joint reaction force and the horizontal from ϕ to ϕ'' . For the new configuration of the free-body shown in Fig. 5.34, Eqs. (xi) and (xii) can again be utilized to calculate the required hip abductor muscle force and joint reaction force produced at the right hip (opposite to the side where the load is carried):

$$F_M = \frac{\cos \phi'' (W_2 + W_0)}{\cos \theta \sin \phi'' - \sin \theta \cos \phi''}$$

$$F_J = \frac{\cos \theta (W_2 + W_0)}{\cos \theta \sin \phi'' - \sin \theta \cos \phi''}$$

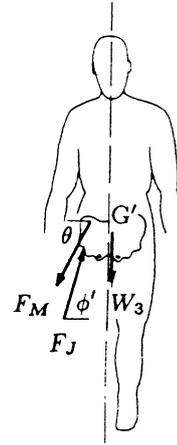


Fig. 5.32 The problem is reduced to a three-force concurrent system

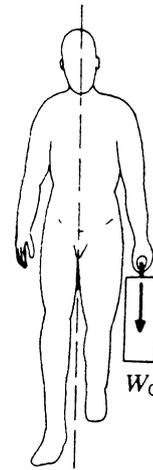


Fig. 5.33 Carrying a load in one hand

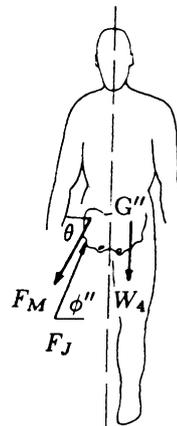


Fig. 5.34 Forces acting on the upper body

Remarks

- When the body weight is supported equally on both feet, half of the supra-femoral weight falls on each hip joint. During walking and running, the entire mass of the body is momentarily supported by one joint, and we have analyzed some of these cases.
- The above analyses indicate that the supporting forces required at the hip joint are greater when a load is carried on the opposite side of the body as compared to the forces required to carry the load when it is distributed on either side. Carrying loads by using both hands and by bringing the loads closer to the midline of the body is effective in reducing required musculoskeletal forces.
- While carrying a load on one side, people tend to lean toward the other side. This brings the center of gravity of the upper body and the load being carried in the hand closer to the midline of the body, thereby reducing the length of the moment arm of the resultant gravitational force as measured from the hip joint distal to the load.
- People with weak hip abductor muscles and/or painful hip joints usually lean toward the weaker side and walk with a so-called abductor gait. Leaning the trunk sideways toward the affected hip shifts the center of gravity of the body closer to that hip joint, and consequently reduces the rotational action of the moment of the body weight about the hip joint by reducing its moment arm. This in return reduces the magnitude of the forces exerted by the hip abductor muscles required to stabilize the pelvis.

- Abductor gait can be corrected more effectively with a cane held in the hand opposite to the weak hip, as compared to the cane held in the hand on the same side as the weak hip.

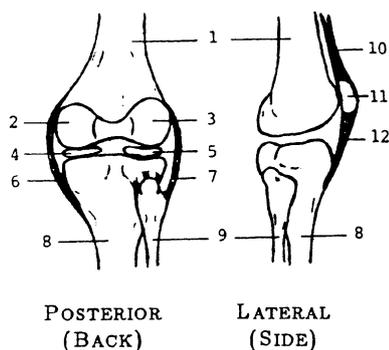


Fig. 5.35 The knee: (1) femur, (2) medial condyle, (3) lateral condyle, (4) medial meniscus, (5) lateral meniscus, (6) tibial collateral ligament, (7) fibular collateral ligament, (8) tibia, (9) fibula, (10) quadriceps tendon, (11) patella, (12) patellar ligament

5.9 Mechanics of the Knee

The knee is the largest joint in the body. It is a modified hinge joint. In addition to flexion and extension action of the leg in the sagittal plane, the knee joint permits some automatic inward and outward rotation. The knee joint is designed to sustain large loads. It is an essential component of the linkage system responsible for human locomotion. The knee is extremely vulnerable to injuries.

The knee is a two-joint structure composed of the tibiofemoral joint and the patellofemoral joint (Fig. 5.35). The *tibiofemoral joint* has two distinct articulations between the medial and

lateral condyles of the femur and the tibia. These articulations are separated by layers of cartilage, called *menisci*. The lateral and medial menisci eliminate bone-to-bone contact between the femur and the tibia, and function as shock absorbers. The *patellofemoral joint* is the articulation between the patella and the anterior end of the femoral condyles. The patella is a “floating” bone kept in position by the quadriceps tendon and the patellar ligament. It increases the mechanical advantage of the quadriceps muscle, improving its pulling effect on the tibia via the patellar tendon. The stability of the knee is provided by an intricate ligamentous structure, the menisci and the muscles crossing the joint. Most knee injuries are characterized by ligament and cartilage damage occurring on the medial side.

The muscles crossing the knee protect it, provide internal forces for movement, and/or control its movement. The muscular control of the knee is produced primarily by the quadriceps muscles and the hamstring muscle group (Fig. 5.36). The quadriceps muscle group is composed of the rectus femoris, vastus lateralis, vastus medialis, and vastus intermedius muscles. The rectus femoris muscle has attachments at the anterior-inferior iliac spine and the patella, and its primary actions are the flexion of the hip and the extension of the knee. The vastus lateralis, medialis, and intermedius muscles connect the femur and tibia through the patella, and they are all knee extensors. The biceps femoris, semitendinosus, and semimembranosus muscles make up the hamstring muscle group, which help control the extension of the hip, flexion of the knee, and some inward–outward rotation of the tibia. Semitendinosus and semimembranosus muscles have proximal attachments on the pelvic bone and distal attachments on the tibia. The biceps femoris has proximal attachments on the pelvic bone and the femur, and distal attachments on the fibula. There is also the popliteus muscle that has attachments on the femur and tibia. The primary function of this muscle is knee flexion. The other muscles of the knee are sartorius, gracilis, gastrocnemius, and plantaris.

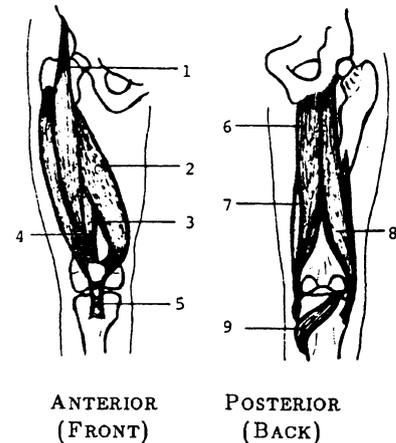


Fig. 5.36 Muscles of the knee: (1) rectus femoris, (2) vastus medialis, (3) vastus intermedius, (4) vastus lateralis, (5) patellar ligament, (6) semitendinosus, (7) semimembranosus, (8) biceps femoris, (9) gastrocnemius

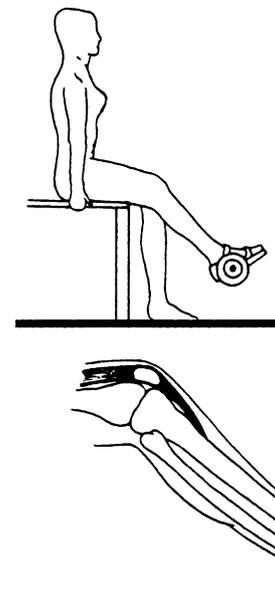


Fig. 5.37 Exercising the muscles around the knee joint

Example 5.6 Consider a person wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen the quadriceps muscles (Fig. 5.37).

Forces acting on the lower leg and a simple mechanical model of the leg are illustrated in Fig. 5.38. W_1 is the weight of the lower leg, W_0 is the weight of the boot, F_M is the magnitude of the tensile force exerted by the quadriceps muscle on the tibia through the patellar tendon, and F_J is the magnitude of the tibiofemoral joint reaction force applied by the femur on the tibial plateau. The tibiofemoral joint center is located at point O, the patellar tendon is attached to the tibia at point A, the center

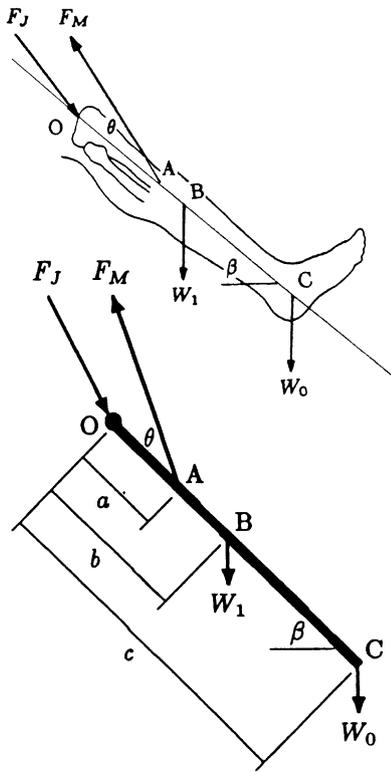


Fig. 5.38 Forces acting on the lower leg

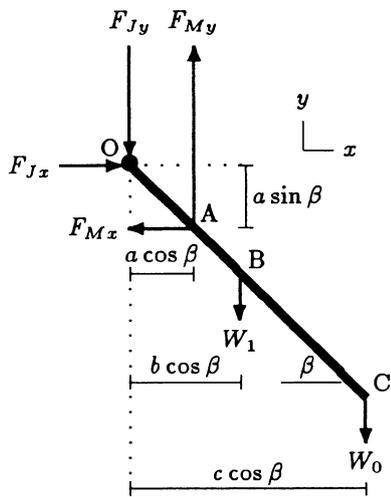


Fig. 5.39 Force components, and their lever arms

of gravity of the lower leg is located at point B, and the center of gravity of the weight boot is located at point C. The distances between point O and points A, B, and C are measured as a , b , and c , respectively. For the position of the lower leg shown, the long axis of the tibia makes an angle β with the horizontal, and the line of action of the quadriceps muscle force makes an angle θ with the long axis of the tibia.

Assuming that points O, A, B, and C all lie along a straight line, determine F_M and F_J in terms of a , $b \cdot c$, θ , β , W_1 , and W_0 .

Solution: Horizontal (x) and vertical (y) components of the forces acting on the leg and their lever arms as measured from the knee joint located at point O are shown in Fig. 5.39. The components of the muscle force are:

$$F_{Mx} = F_M \cos(\theta + \beta) \tag{i}$$

$$F_{My} = F_M \sin(\theta + \beta) \tag{ii}$$

There are three unknowns, namely F_M , F_{Jx} , and F_{Jy} . For the solution of this two-dimensional (plane) problem, all three equilibrium conditions must be utilized. Assuming that the counterclockwise moments are positive, consider the rotational equilibrium of the lower leg about point O:

$$\sum M_O = 0: \quad (a \cos \beta)F_{My} - (a \sin \beta)F_{Mx} - (b \cos \beta)W_1 - (c \cos \beta)W_0 = 0$$

Substituting Eqs. (i) and (ii) into the above equation, and solving it for F_M will yield:

$$F_M = \frac{(bW_1 + cW_0) \cos \beta}{a[\cos \beta \sin(\theta + \beta) - \sin \beta \cos(\theta + \beta)]} \tag{iii}$$

Note that this equation can be simplified by considering that $[\cos \beta \sin(\theta + \beta) - \sin \beta \cos(\theta + \beta)] = \sin \theta$, that is:

$$F_M = \frac{(bW_1 + cW_0) \cos \beta}{a \sin \theta}$$

Equation (iii) yields the magnitude of the force that must be exerted by the quadriceps muscles to support the leg when it is extended forward making an angle β with the horizontal. Once F_M is determined, the components of the reaction force developed at the knee joint along the horizontal and vertical directions can also be evaluated by considering the translational equilibrium of the lower leg in the x and y directions:

$$\begin{aligned} \sum F_x = 0: \quad & F_{Jx} = F_{Mx} = F_M \cos(\theta + \beta) \\ \sum F_y = 0: \quad & F_{Jy} = F_{My} - W_0 - W_1 \\ & F_{Jy} = F_M \sin(\theta + \beta) - W_0 - W_1 \end{aligned}$$

The magnitude of the resultant compressive force applied on the tibial plateau at the knee joint is:

$$F_J = \sqrt{(F_{Jx})^2 + (F_{Jy})^2} \quad (\text{iv})$$

$$\varphi = \arctan\left(\frac{F_{Jy}}{F_{Jx}}\right)$$

Assume that the geometric parameters and the weights involved are given as: $a = 12\text{ cm}$, $b = 22\text{ cm}$, $c = 50\text{ cm}$, $W_1 = 150\text{ N}$, $W_0 = 100\text{ N}$, $\theta = 15^\circ$, and $\beta = 45^\circ$, then by using Eqs. (iii) and (iv):

$$F_M = 1956\text{ N}, \quad F_J = 1744\text{ N}, \quad \varphi \cong 56^\circ$$

Remarks

- The force \underline{F}_M exerted by the quadriceps muscle on the tibia through the patellar tendon can be expressed in terms of two components normal and tangential to the long axis of the tibia (Fig. 5.40). The primary function of the normal component \underline{F}_{Mn} of the muscle force is to rotate the tibia about the knee joint, while its tangential component \underline{F}_{Mt} tends to translate the lower leg in a direction collinear with the long axis of the tibia and applies a compressive force on the articulating surfaces of the tibiofemoral joint. Since the normal component of \underline{F}_M is a sine function of angle θ , a larger angle between the patellar tendon and the long axis of the tibia indicates a larger rotational effect of the muscle exertion. This implies that for large θ , less muscle force is wasted to compress the knee joint, and a larger portion of the muscle tension is utilized to rotate the lower leg about the knee joint.

- One of the most important biomechanical functions of the patella is to provide anterior displacement of the quadriceps and patellar tendons, thus lengthening the lever arm of the knee extensor muscle forces with respect to the center of rotation of the knee by increasing angle θ (Fig. 5.41a). Surgical removal of the patella brings the patellar tendon closer to the center of rotation of the knee joint (Fig. 5.41b), which causes the length of the lever arm of the muscle force to decrease ($d_2 < d_1$). Losing the advantage of having a relatively long lever arm, the quadriceps muscle has to exert more force than normal to rotate the lower leg about the knee joint.

- The human knee has a two-joint structure composed of the tibiofemoral and patellofemoral joints. Notice that the quadriceps muscle goes over the patella, and the patella and the muscle form a pulley-rope arrangement. The higher the tension in the muscle, the larger the compressive force (pressure) the patella exerts on the patellofemoral joint.

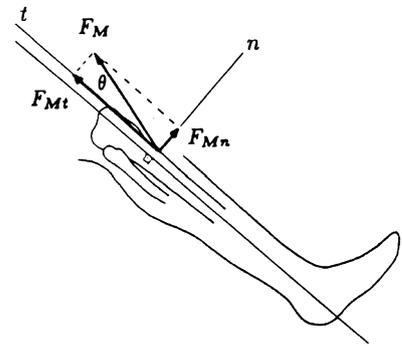


Fig. 5.40 Rotational and translatory components of \underline{F}_M

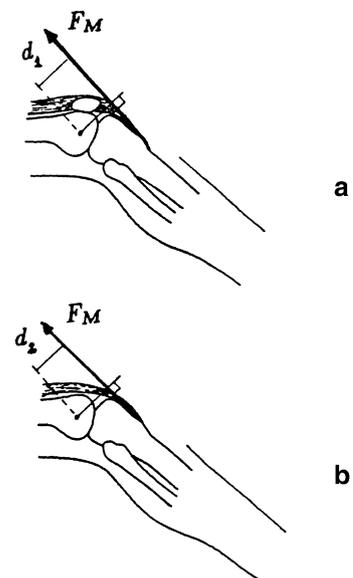


Fig. 5.41 Patella increases the length of the lever arm

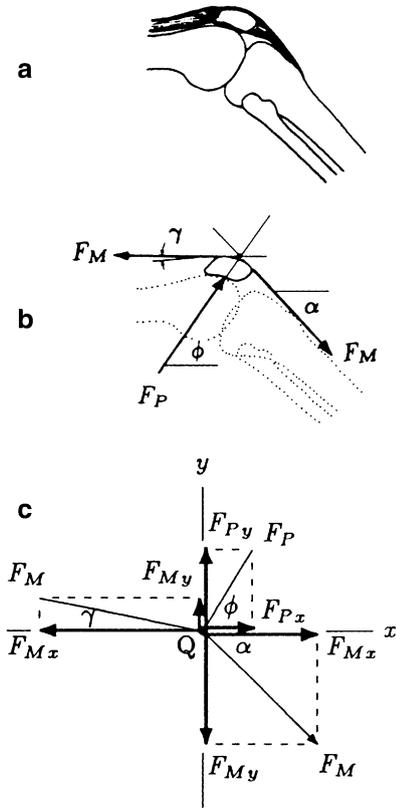


Fig. 5.42 Static analysis of the forces acting on the patella

We have analyzed the forces involved around the tibiofemoral joint by considering the free-body diagram of the lower leg. Having determined the tension in the patellar tendon, and assuming that the tension is uniform throughout the quadriceps, we can calculate the compressive force applied on the patellofemoral joint by considering the free-body diagram of the patella (Fig. 5.42). Let F_M be the uniform magnitude of the tensile force in the patellar and quadriceps tendons, F_P be the magnitude of the force exerted on the patellofemoral joint, α be the angle between the patellar tendon and the horizontal, γ be the angle between the quadriceps tendon and the horizontal, and ϕ be the unknown angle between the line of action of the compressive reaction force at the joint (Fig. 5.42b) and the horizontal. We have a three-force system and for the equilibrium of the patella it has to be concurrent.

We can first determine the common point of intersection Q by extending the lines of action of patellar and quadriceps tendon forces. A line connecting point Q and the point of application of \underline{F}_P will correspond to the line of action of \underline{F}_P . The forces can then be translated to point Q (Fig. 5.42c), and the equilibrium equations can be applied. For the equilibrium of the patella in the x and y directions:

$$\sum F_x = 0: F_P \cos \phi = F_M(\cos \gamma - \cos \alpha) \tag{v}$$

$$\sum F_y = 0: F_P \sin \phi = F_M(\sin \alpha - \sin \gamma) \tag{vi}$$

These equations can be solved simultaneously for angle ϕ and the magnitude F_P of the compressive force applied by the femur on the patella at the patellofemoral joint:

From Eq. (v):

$$F_P = \frac{F_M(\cos \gamma - \cos \alpha)}{\cos \phi}$$

From Eq. (vi):

$$F_P = \frac{F_M(\sin \alpha - \sin \gamma)}{\sin \phi}, \text{ that is}$$

$$\frac{F_M(\cos \gamma - \cos \alpha)}{\cos \phi} = \frac{F_M(\sin \alpha - \sin \gamma)}{\sin \phi}, \text{ then}$$

$$\sin \phi(\cos \gamma - \cos \alpha) = \cos \phi(\sin \alpha - \sin \gamma),$$

$$\tan \phi = \frac{\sin \alpha - \sin \gamma}{\cos \gamma - \cos \alpha} \text{ and}$$

$$\phi = \tan^{-1} \left(\frac{\sin \alpha - \sin \gamma}{\cos \gamma - \cos \alpha} \right)$$

Once angle φ is determined, then the magnitude of force exerted on the patellofemoral joint F_P can also be determined:

$$F_P = F_M \left(\frac{\cos \gamma - \cos \gamma}{\cos \varphi} \right)$$

5.10 Mechanics of the Ankle

The ankle is the union of three bones: the tibia, fibula, and the talus of the foot (Fig. 5.43). Like other major joints in the lower extremity, the ankle is responsible for load-bearing and kinematic functions. The ankle joint is inherently more stable than the knee joint which requires ligamentous and muscular restraints for its stability.

The ankle joint complex consists of the tibiotalar, fibulotalar, and distal tibiofibular articulations. The *ankle (tibiotalar) joint* is a hinge or ginglymus-type articulation between the spool-like convex surface of the trochlea of the talus and the concave distal end of the tibia. Being a hinge joint, the ankle permits only flexion–extension (dorsiflexion–plantar flexion) movement of the foot in the sagittal plane. Other foot movements include inversion and eversion, inward and outward rotation, and pronation and supination. These movements occur about the foot joints such as the subtalar joint between the talus and calcaneus and the transverse tarsal joints, talonavicular and calcaneocuboid.

The ankle mortise is maintained by the shape of the three articulations, and the ligaments and muscles crossing the joint. The integrity of the ankle joint is improved by the medial (deltoid) and lateral collateral ligament systems, and the interosseous ligaments. There are numerous muscle groups crossing the ankle. The most important ankle plantar flexors are the gastrocnemius and soleus muscles (Fig. 5.44). Both the gastrocnemius and soleus muscles are located in the posterior compartment of the leg and have attachments to the posterior surface of the calcaneus via the Achilles tendon. The gastrocnemius crosses the knee and ankle joints and has functions in both. In the knee, it collaborates with knee flexion and in the ankle is the main plantar flexor. The plantar extensors or dorsiflexors are anterior muscles. They are the tibialis anterior, extensor digitorum longus, extensor hallucis longus, and peroneus tertius muscles. The primary function of the lateral muscles (the peroneus longus and peroneus brevis) is to exert and plantarflex the ankle.

The ankle joint responds poorly to small changes in its anatomical configuration. Loss of kinematic and structural restraints due to severe sprains can seriously affect ankle

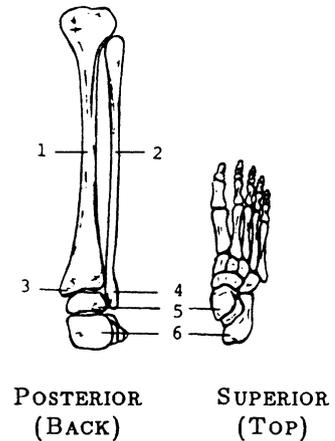


Fig. 5.43 The ankle and the foot: (1) tibia, (2) fibula, (3) medial malleolus, (4) lateral malleolus, (5) talus, (6) calcaneus

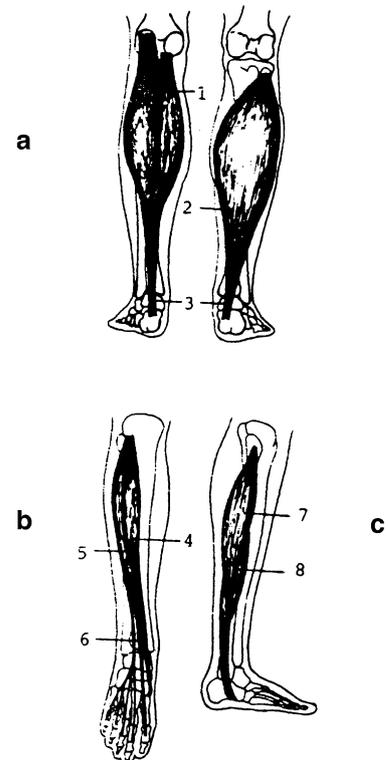


Fig. 5.44 Ankle muscles (a) posterior, (b) anterior, and (c) lateral views: (1) gastrocnemius, (2) soleus, (3) Achilles tendon, (4) tibialis anterior, (5) extensor digitorum longus, (6) extensor hallucis longus, (7) peroneus longus, (8) peroneus brevis

stability and can produce malalignment of the ankle joint surfaces. The most common ankle injury, inversion sprain, occurs when the body weight is forcefully transmitted to the ankle while the foot is inverted (the sole of the foot facing inward).

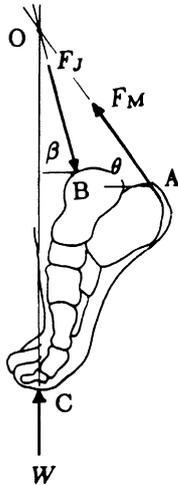


Fig. 5.45 Forces acting on the foot form a concurrent system of forces

Example 5.7 Consider a person standing on tiptoe on one foot (a strenuous position illustrated). The forces acting on the foot during this instant are shown in Fig. 5.45. W is the person’s weight applied on the foot as the ground reaction force, F_M is the magnitude of the tensile force exerted by the gastrocnemius and soleus muscles on the calcaneus through the Achilles tendon, and F_J is the magnitude of the ankle joint reaction force applied by the tibia on the dome of the talus. The weight of the foot is small compared to the weight of the body and is therefore ignored. The Achilles tendon is attached to the calcaneus at point A, the ankle joint center is located at point B, and the ground reaction force is applied on the foot at point C. For this position of the foot, it is estimated that the line of action of the tensile force in the Achilles tendon makes an angle θ with the horizontal, and the line of action of the ankle joint reaction force makes an angle β with the horizontal.

Assuming that the relative positions of points A, B, and C are known, determine expressions for the tension in the Achilles tendon and the magnitude of the reaction force at the ankle joint.

Solution: We have a three-force system composed of muscle force \underline{F}_M , joint reaction force \underline{F}_J , and the ground reaction force \underline{W} . From the geometry of the problem, it is obvious that for the position of the foot shown, the forces acting on the foot do not form a parallel force system. Therefore, the force system must be a concurrent one. The common point of intersection (point O in Fig. 5.45) of these forces can be determined by extending the lines of action of \underline{W} and \underline{F}_M . A straight line passing through both points O and B represents the line of action of the joint reaction force. Assuming that the relative positions of points A, B, and C are known (as stated in the problem), the angle (say β) of the line of action of the joint reaction force can be measured.

Once the line of action of the joint reaction force is determined by graphical means, the magnitudes of the joint reaction and muscle forces can be calculated by translating all three forces involved to the common point of intersection at O (Fig. 5.46). The two unknowns F_M and F_J can now be determined by applying the translational equilibrium conditions in the horizontal (x) and vertical (y) directions. For this purpose, the joint reaction

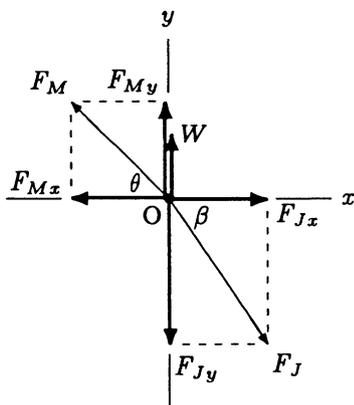


Fig. 5.46 Components of the forces acting on the foot

and muscle forces must be decomposed into their rectangular components first:

$$F_{Mx} = F_M \cos \theta$$

$$F_{My} = F_M \sin \theta$$

$$F_{Jx} = F_J \cos \beta$$

$$F_{Jy} = F_J \sin \beta$$

For the translational equilibrium of the foot in the horizontal and vertical directions:

$$\sum F_x = 0: F_{Jx} = F_{Mx}, \text{ that is } F_J \cos \beta = F_M \cos \theta$$

$$\sum F_y = 0: F_{Jy} = F_{My} + W, \text{ that is } F_J \sin \beta = F_M \sin \theta + W$$

Simultaneous solutions of these equations will yield:

$$F_M = \frac{W \cos \beta}{\cos \theta \sin \beta - \sin \theta \cos \beta}, \text{ that is: } F_M = \frac{W \cos \beta}{\sin(\beta - \theta)}$$

$$F_J = \frac{W \cos \theta}{\cos \theta \sin \beta - \sin \theta \cos \beta}, \text{ that is: } F_J = \frac{W \cos \theta}{\sin(\beta - \theta)}$$

For example, assume that $\theta = 45^\circ$ and $\beta = 60^\circ$. Then:

$$F_M = 1.93 W \quad F_J = 2.73 W$$

5.11 Exercise Problems

Problem 5.1 Consider a person holding an object in his hand with his elbow flexed at the right angle with respect to the upper arm (Fig. 5.4). The forces acting on the forearm and the mechanics model of the system are shown in Fig. 5.5a, b. As for this system assume that the biceps is the major flexor and the line of action of the muscle makes the right angle with the long axis of the forearm. Point O designates the axis of rotation at the elbow joint, A is the point of attachment of the biceps muscle to the radius, point B is the center of gravity of the forearm, and point C is the center of gravity of the object held in the hand. Furthermore, the distances between the axis of rotation of the elbow joint (point O) and points A, B, and C are $a = 4.5$ cm, $b = 16.5$ cm, and $c = 37$ cm. If the total weight of the forearm is $W = 83$ N, and the magnitude of the muscle force is $F_M = 780$ N:

- (a) Determine the weight (W_0) of the object held in the hand.
- (b) Determine the magnitude of the reaction force (F_J) at the elbow joint.

- (c) Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the object held in the hand is increased by 5 N.

Answers: (a) $W_0 = 57.8$ N; (b) $F_J = 639.2$ N; (c) $F_{M1} = 820$ N, $F_{J1} = 674.2$ N

Problem 5.2 Consider a person performing shoulder exercises by using a dumbbell (Fig. 5.11). The forces acting on the arm and the mechanical model of the system are shown in Fig. 5.12. For this system assume that the arm of the person is fully extended to the horizontal. Point O designates the axis of rotation of the shoulder joint, A is the point of attachment of the deltoid muscle to the humerus, point B is the center of gravity of the entire arm, and point C is the center of gravity of the dumbbell. The distances between the axis of rotation of the shoulder joint (point O) and points A, B, and C are $a = 17$ cm, $b = 33$ cm, and $c = 63$ cm. The dumbbell weighs $W_0 = 64$ N and for this position of the arm it is estimated that the magnitude of the muscle force is $F_M = 1051$ N. If the lines of action of the muscle (F_M) and the joint reaction forces (F_J) make an angle $\theta = 18^\circ$ and $\beta = 12^\circ$ with the horizontal, respectively:

- (a) Determine the magnitude of reaction force (F_J) at the shoulder joint.
 (b) Determine the total weight (W) of the arm.
 (c) Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the dumbbell is increased by 5 N.

Answers: (a) $F_J = 1021.9$ N; (b) $W = 47.3$ N; (c) $F_{M1} = 1136.5$ N, $F_{J1} = 1105$ N

Problem 5.3 Consider the position of the head and neck as well as forces acting on the head shown in Fig. 5.15. For this equilibrium condition assume that the forces involved form a concurrent force system. Point C is the center of gravity of the head, A is the point of application of force (F_M) exerted by the neck extensor muscles on the head, and point B is the center of rotation of the atlantooccipital joint. For this position of the head, it is estimated that the magnitude of the resultant force exerted by the neck extensor muscles is $F_M = 57$ N, and the lines of action of the muscles and the joint reaction forces make an angle $\theta = 36^\circ$ and $\beta = 63^\circ$ with the horizontal, respectively. Determine the magnitude of the gravitational force acting on the head.

Answer: $W = 47$ N

Problem 5.4 Consider a weight lifter who is trying to lift a barbell. The forces acting on the lower part of the athlete's body and the mechanical model of the system are shown in Figs. 5.19 and 5.20, respectively. Point O designates the center of rotation at the joint formed by the sacrum and the fifth lumbar vertebra. A is the point of application of force exerted by the back muscles, point B is the center of gravity of the lower body, and C is the point of application of the ground reaction force. With respect to point O, $a = 3.6$ cm, $b = 14.6$ cm, and $c = 22$ cm, are the shortest distances between the lines of action of the back muscles' force, the lower body's gravitational force, and the ground reaction force with the center of rotation of the joint. For a weight lifter in this position, it is estimated that the force exerted by the back muscles is $F_M = 6856$ N and the line of action of this force makes an angle $\theta = 43^\circ$ with the vertical. If the barbell weighs $W_0 = 637$ N and the magnitude of the gravitational force acting on the lower body is $W_1 = 333$ N:

- Determine the weight (W) of the athlete.
- Determine the magnitude of the reaction force (F_J) acting at the joint.
- Determine an angle α that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 705.9$ N; (b) $F_J = 7625.8$ N; (c) $\alpha = 52^\circ$

Problem 5.5 Consider a person that momentarily put the entire weight of his body on one leg when walking or running. The forces acting on the leg and the mechanical model of the system are shown in Figs. 5.24 and 5.25, respectively. Point O designates the center of rotation of the hip joint. A is the point of attachment of the hip abductor muscles to the femur, point B is the center of gravity of the leg, and C is the point of application of the ground reaction force. The distances between point A and points O, B, and C are specified as $a = 8.6$ cm, $b = 34.3$ cm, and $c = 89.4$ cm. The angles that the femoral neck and the long axis of the femoral shaft make with the horizontal are specified as $\alpha = 43^\circ$ and $\beta = 79^\circ$, respectively. Furthermore, for this single-leg stance, it is estimated that the magnitude of force exerted by the hip abductor muscles is $F_M = 2062.6$ N and its line of action makes an angle $\theta = 69^\circ$ with the horizontal. If the magnitude of gravitational force acting on the leg is $W_1 = 125$ N:

- Determine the total weight (W) of the person.
- Determine the magnitude of the reaction force (F_J) acting at the hip joint.

- (c) Determine an angle γ that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 729.7$ N; (b) $F_J = 2636.1$ N; (c) $\gamma = 73.7^\circ$

Problem 5.6 Consider a person performing lower leg flexion–extension exercises from a sitting position while wearing a weight boot. Forces acting on the leg and the mechanical model of the system are shown in Fig. 5.38. Point O designates the center of rotation of the tibiofemoral joint. A is the point of attachment of the patellar tendon to the tibia, point B is the center of gravity of the lower leg, and point C is the center of gravity of the weight boot. For this system assume that the points O, A, B, and C all lie along a straight line. The distances between point O and points A, B, and C are measured as $a = 13$ cm, $b = 23.5$ cm, and $c = 53$ cm, respectively. For this position of the leg, the long axis of the tibia makes an angle $\beta = 47^\circ$ with the horizontal, and the line of action of the quadriceps muscle force makes an angle $\theta = 17^\circ$ with the long axis of the tibia. Furthermore, for this position of the leg, it is estimated that the force exerted by the quadriceps muscle is $F_M = 1.940$ N.

If the weight of the lower leg is $W_1 = 163$ N:

- Determine the weight (W_0) of the weight boot.
- Determine the magnitude of the reaction force (F_J) of the tibiofemoral joint.
- Determine an angle φ that the line of action the joint reaction force makes with the horizontal.

Answers: (a) $W_0 = 98.4$ N; (b) $F_J = 1707.5$ N; (c) $\varphi = 60.2^\circ$

Problem 5.7 Consider a person standing on tiptoe on one foot. For this position, the forces acting on the foot are shown in Fig. 5.45. Point A is the point of attachment of the Achilles tendon through which a force is exerted by the gastrocnemius and soleus muscles on the calcaneus. Point B designates the center of the ankle joint and C is the point of application of the ground reaction force. For this system assume that the weight of the foot can be ignored as it is relatively small when compared to the weight of the entire body of the person. For this position of the foot, it is estimated that the lines of action of the tensile force in the Achilles tendon and the reaction force (F_J) of the ankle joint make an angle $\theta = 49^\circ$ and $\beta = 65^\circ$ with the horizontal, respectively. Furthermore, for this position of the foot, it is also estimated that the magnitude of force exerted by

the gastrocnemius and soleus muscles on the calcaneus is $F_M = 1275.4 \text{ N}$.

- (a) Determine the entire weight (W) of the person.
- (b) Determine the magnitude of the reaction force (F_J) of the ankle joint.

Answers: (a) $W = 831.8 \text{ N}$; (b) $F_J = 1980.3 \text{ N}$

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