Chapter 12

Introduction to Deformable Body Mechanics

N. Özkaya et al., *Fundamentals of Biomechanics*, DOI 10.1007/978-3-319-44738-4_12

12.1 Overview

Basic concepts of statics were introduced in Chaps. 4 and 5 along with some of their applications. The field of statics is based on Newton's laws (Newtonian mechanics). It constitutes one of the two main branches of the more general field of rigid body mechanics, dynamics being the other branch. The basic assumption in rigid body mechanics is that the bodies involved do not deform under applied loads. This idealization is necessary to simplify the problem under investigation for the sake of analyzing external forces and moments. The field of deformable body mechanics, on the other hand, does not treat the body as rigid, but incorporates the deformability (ability to undergo shape change) and the material properties of the body into the analyses. This field of applied mechanics utilizes the experimentally determined and/or verified relationships between applied forces and corresponding deformations.

Rigid body mechanics has its limitations. One of these limitations was discussed in Sect. 4.6 where the concept of statically indeterminant systems was introduced. A system for which the equations of equilibrium are not sufficient to determine the unknown forces is called *statically indeterminate*. For the analyses of such systems, there is a need for equations in addition to those provided by the conditions of static equilibrium. These additional equations can be derived by considering the material properties of the parts constituting a system and by relating forces to deformations, which is the focus of deformable body mechanics.

The desire to analyze statically determinate systems is only one of the reasons why deformable body mechanics is important. The applications of this field extend to almost all branches of engineering by providing essential design and analysis tools. The task of an engineer—mechanical, civil, electrical, or biomedical—is to determine the safest and most efficient operating condition for a machine, a structure, a piece of equipment, or a prosthetic device. A design engineer can accomplish this task by first assessing the proper operational environment through force analyses, making the correct structural design, and choosing the material that can sustain the forces involved in that environment. The primary concern of a design engineer is to make sure that when loaded, a machine part, a structure, a piece of equipment, or a device will not break or deform excessively.

12.2 Applied Forces and Deformations

Mechanics is concerned with forces and motions. It is possible to distinguish two types of motions. If the resultant of external forces or moments applied on a body is not zero, then the body will undergo gross overall motion (translation and/or rotation). In other words, the position of the body as a whole will change over time. Such movements are studied within the field of dynamics. The second type of motion involves local changes of shape within a body, called *deformations*, which are the primary concern of the field of deformable body mechanics. If a body is subjected to externally applied forces and moments but remains in static equilibrium, then it is most likely that there is some local shape change within the body. The extent of the shape change may depend upon the magnitude, direction, and duration of the applied forces, material properties of the body, and environmental conditions such as heat and humidity.

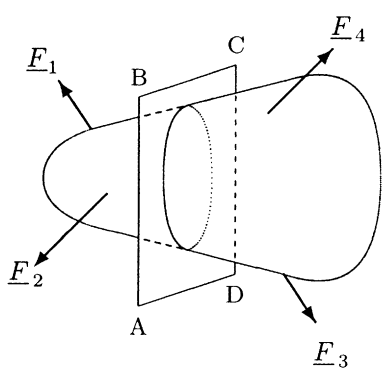

Fig. 12.1 *An object subjected to externally applied forces*

12.3 Internal Forces and Moments

Consider the arbitrarily shaped object illustrated in Fig. 12.1, which is subjected to a number of externally applied forces. Assume that the resultant of these forces and the net moment acting on the object are equal to zero. That is, the object is in static equilibrium. Also assume that the object is fictitiously separated into two parts by an arbitrary plane ABCD passing through the object. If the object as a whole is in equilibrium, then its individual parts must be in equilibrium as well. If one of these two parts is considered, then the equilibrium condition requires that there is a force vector and/or a moment vector acting on the cut section to counterbalance the effects of the external forces and moments applied on that part. These are called the *internal force* and *internal moment* vectors. Of course, the same argument is true for the other part of the object. Furthermore, for the overall equilibrium of the object, the force vectors and moment vectors on either surface of the cut section must have equal magnitudes and opposite directions (Fig. 12.2).

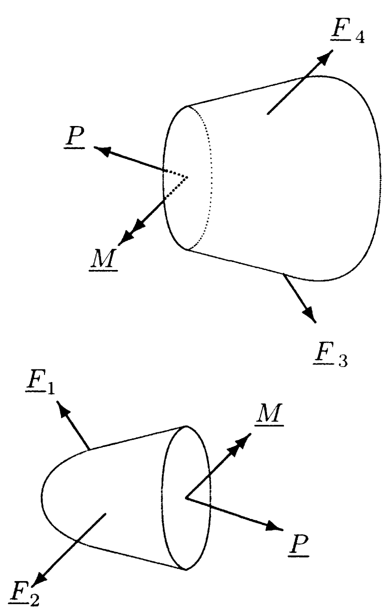

Fig. 12.2 *Method of sections*

For a three-dimensional object, the internal forces and moments can be resolved into their components along three mutually perpendicular directions, as illustrated in Fig. 12.3. The force and moment vector components measured at the cut sections take special names reflecting their orientation and effects on the cut sections. Assuming that x is the direction normal (perpendicular) to the cut section, the force component P_x in Fig. 12.3 is called the *axial* or *normal force*, and it is a measure of the pulling or pushing action of the externally applied forces in a direction perpendicular to the cut section. It is called a *tensile force* if it has a pulling action trying to elongate the part, or a *compressive force* if it has a pushing action tending to shorten the part. The force components P_y and P_z are called *shear forces*, and they are measures of resistance to the sliding action of one cut section over the other. Their subscripts indicate their lines of action. The moment component M_x is also called *twisting torque*, and it is a measure of the twisting action of the externally applied forces along an axis normal to the plane of the cut section (in this case, in the x direction). The components M_y and M_z of the moment vector are called the *bending moments*, and they respectively indicate the extent of bending action to which the cut part is subjected in the y and z directions.

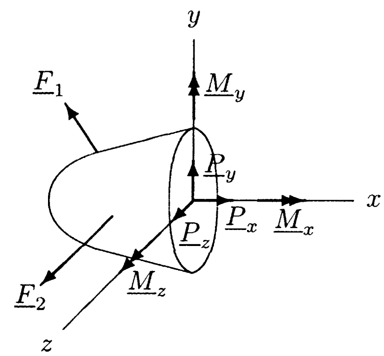

Fig. 12.3 *Internal forces and moments*

Note here that it may be more informative to refer to forces and moments with double subscripted symbols. For example, using P_{xy} instead of P_y would indicate that the force component is acting in the y direction (second subscript) on a section whose normal is in the x direction (first subscript). Similarly, M_{xz} would refer to the component of the moment vector in the z direction measured on the same section.

12.4 Stress and Strain

The purpose of studying the mechanics of deformable bodies or strength of materials is to make sure that the design of a structure is safe against the combined effects of applied forces and moments. The idea is to select the proper material for the structure, or if there is an existing structure, to determine the loading conditions under which the structure can operate safely and efficiently. To make a selection, however, one needs to know the mechanical properties of materials under different loading conditions.

Consider the two bars shown in Fig. 12.4, which are made of the same material, and have the same length but different sizes. The cross-sectional area A_1 of bar 1 is less than the cross-sectional area A_2 of bar 2. Assume that these bars are subjected to successively increasing forces until they break. If the forces F_1 and F_2 at which the bars 1 and 2 break were recorded, it would be observed that the force F_2 required to break bar 2 is greater than the force F_1 required to break bar 1 because bar 2 has a larger cross-sectional area and volume than bar 1. These forces might be an indication of the strength of the bars. However, the fact that the force-to-failure depends on the cross-sectional area of the specimen (in addition to some other factors) makes force an impractical measure of the strength of a material. To eliminate this inconvenience, a concept called *stress* is defined by dividing force with the cross-sectional area:

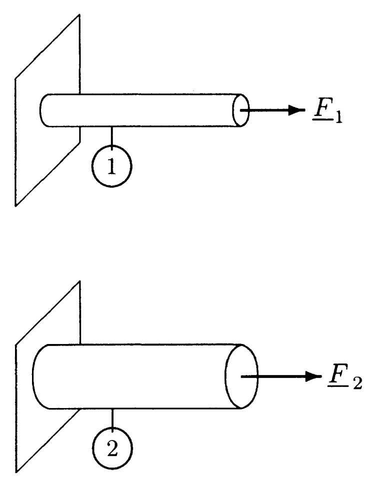

Fig. 12.4 *Two bars made of the same material, have the same length, but different cross-sectional areas*

$$\text{STRESS} = \frac{\text{FORCE}}{\text{AREA}}$$

Although the bars in Fig. 12.4 have different cross-sectional areas and require different forces-to-failure, since they are made of the same material, their stress measurement at failure would be equal.

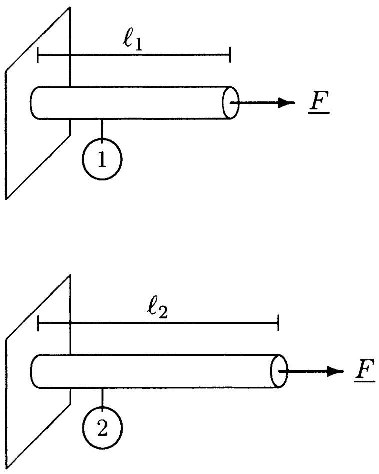

Fig. 12.5 *Two bars made of the same material, have the same cross-sectional area, but different lengths*

As stated earlier, the mechanics of deformable bodies is concerned with applied forces and their internal effects on bodies. One of these effects is shape change or deformation. The amount of deformation an object will undergo depends on its size, material properties, and the magnitude and duration of applied forces. Consider the two bars shown in Fig. 12.5, which are made of the same material and have the same cross-sectional area, but have different lengths. The length l_1 of bar 1 is less than the length l_2 of bar 2. Assume that the same force F is applied to both bars, and the elongation of each bar is measured. It would be observed that the increase of length in bar 2 is greater than the increase of length in bar 1, indicating that the amount of elongation depends on the original length of the specimen. To eliminate the size dependence of deformation measurements, another concept called *strain* is defined by dividing the amount of elongation with the original length of the specimen in the direction of elongation:

$$\text{STRESS} = \frac{\text{AMOUNT OF ELONGATION}}{\text{ORIGINAL LENGTH}}$$

Broad definitions of stress and strain are introduced here. More detailed descriptions of these concepts will be provided in the following sections.

12.5 General Procedure

A general procedure for analyzing problems in deformable body mechanics, including the purpose of these analyses, is provided below.

- **Static analyses**. At this first stage, the analytical methods of statics are employed to determine the external reaction forces and moments. This stage involves drawing free-body diagrams and applying the conditions of equilibrium to determine the unknown reaction forces and moments by utilizing concepts such as equivalent force systems.

- **Analyses of internal forces and moments**. The internal forces and moments can be determined by the *method of sections*. As discussed briefly in Sect. 12.3, this can be done by separating the body into two sections at the location where the forces and moments need to be calculated. Here, the concern is to determine critical load conditions that correspond to maximum stress levels. These critical loads can be determined by drawing the shear and bending diagrams of the body, which essentially requires the application of the method of sections throughout the body.

• **Stress analyses**. This stage involves the conversion of internal forces and moments, in particular the critical forces and moments, into corresponding stresses by using formulas that also incorporate the material and geometric properties of the problem into the analyses.

• **Material selection**. Materials can be distinguished by their physical and mechanical properties. At this final stage of analysis, a material must be selected for the safe operation of the structure based on the maximum stresses calculated. If the material is already selected and the design is already made, then the maximum stresses calculated are used to set the allowable load conditions.

Note that the prerequisite for analyses in deformable body mechanics is statics. However, the procedure outlined above is not limited to analyzing systems in equilibrium. Under the effect of externally applied forces, a body may deform and undergo overall motion simultaneously. Such a problem can also be analyzed with the procedure outlined above by utilizing the *d'Alembert principle*. This principle is applied by treating the inertial effects due to the acceleration of the body as another external force acting at the center of gravity of the body in a direction opposite to the direction of acceleration.

12.6 Mathematics Involved

The analyses in this part of the text (Chaps. 12–15) will utilize vector algebra and differential and integral calculus as computational tools. Therefore, the reader is advised to review Appendices A through C. Some of the analyses may also require familiarity with ordinary differential equations.

12.7 Topics to Be Covered

At the beginning of Chap. 13, detailed definitions of stress and strain will be provided. Based on the stress–strain diagrams, material properties such as ductility, stiffness, and brittleness will be discussed. Elastic and plastic deformations, Hooke's law, the necking phenomenon, and the concepts of work and strain energy will be explained. The analyses in Chap. 13 will be limited to uniaxial deformations. The concepts introduced will be applied to analyze relatively simple systems.

In Chap. 14, more advanced topics in stress–strain analyses will be introduced. Two- and three-dimensional stress analyses, techniques of transforming stresses from one plane to another, methods for finding critical stresses, reasons why stress analyses are important in the design of structures, failure theories, concepts such as fatigue, endurance, and stress concentration will also be discussed in Chap. 14. Also in Chap. 14, analyses of bodies subjected to torsion, bending, and combined loading will be explained.

In Chap. 15, the viscoelastic behavior of materials and empirical models of viscoelasticity will be reviewed, and elasticity and viscoelasticity will be compared. Also in Chap. 15, the mechanical properties of biological tissues including bone, tendons, ligaments, muscles, and articular cartilage will be discussed and the relevance of mechanical concepts introduced earlier to orthopaedics will be demonstrated.

Suggested Reading[1]

Beer, F.P., Johnston, R.E., DeWolf, J.T. and Mazurek, D.F., 2015. Mechanics of Materials, 7th Edition. New York: McGraw-Hill Education.
Boresi, A.P. and Schmidt, R.J., 2002. Advanced Mechanics of Materials, 6th Edition. New York: Wiley.
deSilva, C.W., 2013. Mechanics of Materials. Boca Raton: CRC Press.
Gere, J.M. and Goodno, B.J., 2012. Mechanics of Materials. Stamford: CENGAGE Learning.
Hibbeler, R.C., 2013. Mechanics of Materials, 9th Edition. Upper Saddle River: Prentice Hall.
Phipot, T.A., 2012. Mechanics of Materials: An Integrated Learning System, 3rd Edition. Somerset: Wiley.
Popov, E.P. 1999. Engineering Mechanics of Solids. Upper Saddle River, NJ: Prentice-Hall.
Shames, I.H. 1989. Introduction to Solid Mechanics. Englewood Cliffs, NJ: Prentice-Hall.
Timoshekno, S.P., 1969. Strength of Materials: Elementary Theory and Problems. New York: Van Nostrand Reinhold, Inc.

[1] The field of deformable body mechanics has been studied under various titles such as solid mechanics, mechanics of materials, and strength of materials. The subjects covered within deformable body mechanics form the basis for the study of more advanced topics in elasticity, inelasticity, and continuum mechanics. The following books can be reviewed to gain more detailed information on the principles of deformable body mechanics.