

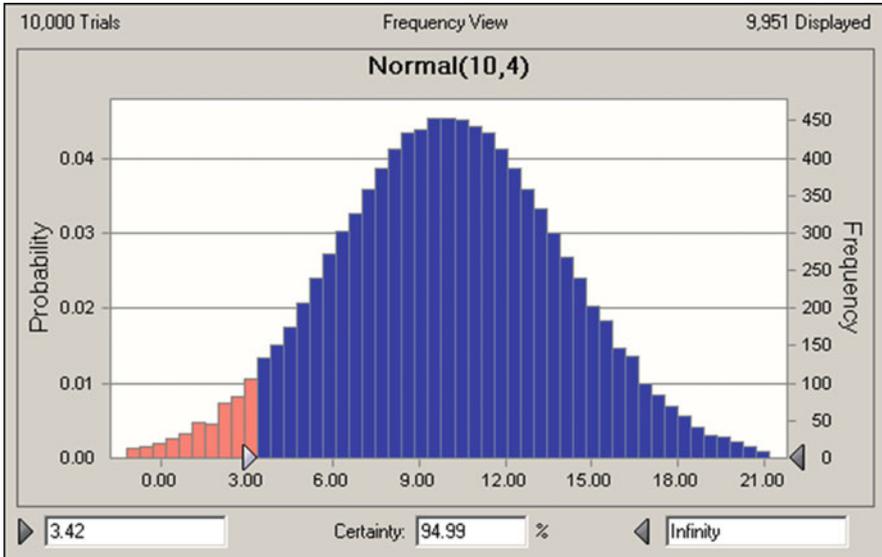
Value at risk (VaR) is one of the most widely used models in risk management. It is based on probability and statistics.<sup>1</sup> VaR can be characterized as a maximum expected loss, given some time horizon and within a given confidence interval. Its utility is in providing a measure of risk that illustrates the risk inherent in a portfolio with multiple risk factors, such as portfolios held by large banks, which are diversified across many risk factors and product types. VaR is used to estimate the boundaries of risk for a portfolio over a given time period, for an assumed probability distribution of market performance. The purpose is to diagnose risk exposure.

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### Definition

Value at risk describes the probability distribution for the value (earnings or losses) of an investment (firm, portfolio, etc.). The mean is a point estimate of a statistic, showing historical central tendency. Value at risk is also a point estimate, but offset from the mean. It requires specification of a given probability level, and then provides the point estimate of the return or better expected to occur at the prescribed probability. For instance, Fig. 6.1 gives the normal distribution for a statistic with a mean of 10 and a standard deviation of 4 (Crystal Ball was used, with 10,000 replications).

This indicates a 0.95 probability (for all practical purposes) of a return of at least 3.42. The precise calculation can be made in Excel, using the NormInv function for a probability of 0.05, a mean of 10, and a standard deviation of 4, yielding a return of 3.420585, which is practically the same as the simulation result shown in Fig. 6.1. Thus the value of the investment at the specified risk level of 0.05 is 3.42. The interpretation is that there is a 0.05 probability that things would be worse than the value at this risk level. Thus the greater the degree of assurance, the lower the value at risk return. The value at the risk level of 0.01 would only be 0.694609.



**Fig. 6.1** Normal distribution (10,4). ©Oracle. used with permission

## The Basel Accords

VaR is globally accepted by regulatory bodies responsible for supervision of banking activities. These regulatory bodies, in broad terms, enforce regulatory practices as outlined by the Basel Committee on Banking Supervision of the Bank for International Settlements (BIS). The regulator that has responsibility for financial institutions in Canada is the Office of the Superintendent of Financial Institutions (OSFI), and OSFI typically follows practices and criteria as proposed by the Basel Committee.

### Basel I

Basel I was promulgated in 1988, focusing on credit risk. A key agreement of the Basel Committee is the Basel Capital Accord (generally referred to as “Basel” or the “Basel Accord”), which has been updated several times since 1988. In the 1996 (updated, 1998) Amendment to the Basel Accord, banks were encouraged to use internal models to measure Value at Risk, and the numbers produced by these internal models support capital charges to ensure the capital adequacy, or liquidity, of the bank. Some elements of the minimum standard established by Basel are:

- VaR should be computed daily, using a 99th percentile, one-tailed confidence interval.
- A minimum price shock equivalent to ten trading days be used. This is called the “holding period” and simulates a 10-day period of liquidating assets in a period of market crisis.
- The model should incorporate a historical observation period of at least 1 year.
- The capital charge is set at a minimum of three times the average of the daily value-at-risk of the preceding 60 business days.

In 2001 the Basel Committee on Banking Supervision published principles for management and supervision of operational risks for banks and domestic authorities supervising them.

## Basel II

Basel II was published in 2009 to deal with operational risk management of banking. Banks and financial institutions were bound to use internal and external data, scenario analysis, and qualitative criteria. Banks were required to compute capital charges on a yearly basis and to calculate 99.9 % confidence levels (one in one thousand events as opposed to the earlier one in one hundred events). Basel II included standards in the form of three pillars:

1. **Minimum capital requirements.**
2. **Supervisory review**, to include categorization of risks as systemic, pension related, concentration, strategic, reputation, liquidity, and legal.
3. **Market discipline**, to include enhancements to strengthen disclosure requirements for securitizations, off-balance sheet exposures, and trading activities.

## Basel III

Basel III was a comprehensive set of reform measures published in 2011 with phased implementation dates. The aim was to strengthen regulation, supervision, and risk management of the banking sectors.

Pillar 1 dealt with capital, risk coverage, and containing leverage:

- **Capital requirements** to improve bank ability to absorb shocks from financial and economic stress:  
Common equity  $\geq 0.045$  of risk-weighted assets
- **Leverage requirements** to improve risk management and governance:  
Tier1 capital  $\geq 0.03$  of total exposure
- **Liquidity requirements** to strengthen bank transparency and disclosure:  
High quality liquid assets  $\geq$  total net liquidity outflows over 30 days

Pillar 2 dealt with risk management and supervision.

Pillar 3 dealt with market discipline through disclosure requirements.

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## The Use of Value at Risk

In practice, these minimum standards mean that the VaR that is produced by the Market Risk Operations area is multiplied first by the square root of 10 (to simulate 10 days holding) and then multiplied by a minimum capital multiplier of 3 to establish capital held against regulatory requirements.

In summary, VaR provides the worst expected loss at the 99 % confidence level. That is, a 99 % confidence interval produces a measure of loss that will be exceeded only 1 % of the time. But this does mean there will likely be a larger loss than the VaR calculation two or three times in a year. This is compensated for by the inclusion of the multiplicative factors, above, and the implementation of Stress Testing, which falls outside the scope of the activities of Market Risk Operations.

Various approaches can be used to compute VaR, of which three are widely used: Historical Simulation, Variance-covariance approach, and Monte Carlo simulation. Variance-covariance approach is used for investment portfolios, but it does not usually work well for portfolios involving options that are close to delta neutral. Monte Carlo simulation solves the problem of non-linearity approximation if model error is not significant, but it suffers some technical difficulties such as how to deal with time-varying parameters and how to generate maturation values for instruments that mature before the VaR horizon. We present Historical Simulation and Variance-covariance approach in the following two sections. We will demonstrate Monte Carlo Simulation in a later section of this chapter.

### Historical Simulation

Historical simulation is a good tool to estimate VAR in most banks. Observations of day-over-day changes in market conditions are captured. These market conditions are represented using upwards of 100,000 points daily of observed and implied Market Data. This historical market data is captured and used to generate historical 'shocks' to current spot market data. This shocked market data is used to price the Bank's trading positions as against changing market conditions, and these revalued positions then are compared against the base case (using spot data). This simulates a theoretical profit or loss. Each day of historically observed data produces a theoretical profit/loss number in this way, and all of these theoretical P&L numbers produce a distribution of theoretical profits/losses. The (1-day) VaR can then be read as the 99th percentile of this distribution.

The primary advantage of historical simulation is ease of use and implementation. In Market Risk Operations, historical data is collected and reviewed on a regular basis, before it is added to the historical data set. Since this data corresponds to historical events, it can be reviewed in a straightforward manner. Also, the

historical nature of the data allows for some clarity of explanation of VaR numbers. For instance, the Bank's VaR may be driven by widening credit spreads, or by decreasing equity volatilities, or both, and this will be visible in actual historical data. Additionally, historical data implicitly contains correlations and non-linear effects (e.g. gamma, vega and cross-effects).

The most obvious disadvantage of historical simulation is the assumption that the past presents a reasonable simulation of future events. Additionally, a large bank usually holds a large portfolio, and there can be considerable operational overhead involved in producing a VaR against a large portfolio with dependencies on a large and varied number of model inputs. All the same, other VaR methods, such as variance-covariance (VCV) and Monte Carlo simulation, produce essentially the same objections. The main alternative to historical simulation is to make assumptions about the probability distributions of the returns on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically. This is known as the variance-covariance approach. VCV is a parametric approach and contains the assumption of normality, and the assumption of the stability of correlation and at the same time. Monte Carlo simulation provides another tool to these two methods. Monte Carlo methods are dependent on decisions regarding model calibration, which have effectively the same problems. No VaR methodology is without simplifying assumptions, and several different methods are in use at institutions worldwide. The literature on volatility estimation is large and seemingly subject to unending growth, especially in acronyms.<sup>2</sup>

## Variance-Covariance Approach

VCV Models portfolio returns as a multivariate normal distribution. We can use a position vector containing cash flow present values to represent all components of the portfolio and describe the portfolio. VCV approach concerns most the return and covariance matrix ( $Q$ ) representing the risk attributes of the portfolio over the chosen horizon. The standard deviation of portfolio value ( $\sigma$ ), also called volatility, is computed:

$$\sigma = \sqrt{h^T Q h} \quad (1)$$

The volatility ( $\sigma$ ) is then scaled to find the desired centile of portfolio value that is the predicted maximum loss for the portfolio or VaR:

$$\begin{aligned} VaR &= \sigma f(Y) \\ \text{where : } & f(Y) \text{ is the scale factor for centile } Y. \end{aligned} \quad (2)$$

For example, for a multivariate normal return distribution,  $f(Y) = 2.33$  for  $Y = 1\%$ .

It is then easy to calculate VaR from the standard deviation (1-day VaR = 2.33 s). The simplest assumption is that daily gains/losses are normally distributed and independent. The  $N$ -day VaR equals  $\sqrt{N}$  times the one-day VaR. When there is autocorrelation equal to  $r$  the multiplier is increased from  $N$  to

$$N + 2(N - 1)\rho + 2(N - 2)\rho^2 + 2(N - 3)\rho^3 + \dots + 2\rho^{n-1}$$

Besides being easy to compute, VCV also lends itself readily to the calculation of the calculation of the marginal risk (Marginal VaR), Incremental VaR and Component VaR of candidate trades. For a Portfolio where an amount  $x_i$  is invested in the  $i$ th component of the portfolio, these three VaR measures are computed as:

- Marginal VaR:  $\frac{\partial \text{VaR}}{\partial x_i}$
- Incremental VaR: Incremental effect of  $i$ th component on VaR
- Component VaR  $x_i \frac{\partial \text{VaR}}{\partial x_i}$

VCV uses delta-approximation, which means the representative cash flow vector is a linear approximation of positions. In some cases, a second-order term in the cash flow representation is included to improve this approximation.<sup>3</sup> However, this does not always improve the risk estimate and can only be done with the sacrifice of some of the computational efficiency. In general, VCV works well in calculating linear instruments such as forward, interest rate SWAP, but works quite badly in non-linear instruments such as various options.

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## Monte Carlo Simulation of VaR

Simulation models are sets of assumptions concerning the relationship among model components. Simulations can be time-oriented (for instance, involving the number of events such as demands in a day) or process-oriented (for instance, involving queuing systems of arrivals and services). Uncertainty can be included by using probabilistic inputs for elements such as demands, inter-arrival times, or service times. These probabilistic inputs need to be described by probability distributions with specified parameters. Probability distributions can include normal distributions (with parameters for mean and variance), exponential distributions (with parameter for a mean), lognormal (parameters mean and variance), or any of a number of other distributions. A simulation run is a sample from an infinite population of possible results for a given model. After a simulation model is built, a selected number of trials is established. Statistical methods are used to validate simulation models and design simulation experiments.

Many financial simulation models can be accomplished on spreadsheets, such as Excel. There are a number of commercial add-on products that can be added to Excel, such as @Risk or Crystal Ball, that vastly extend the simulation power of spreadsheet models.<sup>4</sup> These add-ons make it very easy to replicate simulation runs,

and include the ability to correlate variables, expeditiously select from standard distributions, aggregate and display output, and other useful functions.

## The Simulation Process

Using simulation effectively requires careful attention to the modeling and implementation process. The simulation process consists of five essential steps:

**Develop a conceptual model of the system or problem under study.** This step begins with understanding and defining the problem, identifying the goals and objectives of the study, determining the important input variables, and defining output measures. It might also include a detailed logical description of the system that is being studied. Simulation models should be made as simple as possible to focus on critical factors that make a difference in the decision. The cardinal rule of modeling is to build simple models first, then embellish and enrich them as necessary.

1. **Build the simulation model.** This includes developing appropriate formulas or equations, collecting any necessary data, determining the probability distributions of uncertain variables, and constructing a format for recording the results. This might entail designing a spreadsheet, developing a computer program, or formulating the model according to the syntax of a special computer simulation language (which we discuss further in Chap. 7).
2. **Verify and validate the model.** Verification refers to the process of ensuring that the model is free from logical errors; that is, that it does what it is intended to do. Validation ensures that it is a reasonable representation of the actual system or problem. These are important steps to lend credibility to simulation models and gain acceptance from managers and other users. These approaches are described further in the next section.
3. **Design experiments using the model.** This step entails determining the values of the controllable variables to be studied or the questions to be answered in order to address the decision maker's objectives.
4. **Perform the experiments and analyze the results.** Run the appropriate simulations to obtain the information required to make an informed decision.

As with any modeling effort, this approach is not necessarily serial. Often, you must return to previous steps as new information arises or as results suggest modifications to the model. Therefore, simulation is an evolutionary process that must involve not only analysts and model developers, but also the users of the results.

## Demonstration of VaR Simulation

We use an example Monte Carlo simulation model published by Beneda<sup>5</sup> to demonstrate simulation of VaR and other forms of risk. Beneda considered four risk categories, each with different characteristics of data availability:

- Financial risk—controllable (interest rates, commodity prices, currency exchange)
- Pure risk—controllable (property loss and liability)
- Operational—uncontrollable (costs, input shortages)
- Strategic—uncontrollable (product obsolescence, competition)

Beneda's model involved forward sale (45 days forward) of an investment (CD) with a price that was expected to follow the uniform distribution ranging from 90 to 110. Half of these sales (20,000 units) were in Canada, which involved an exchange rate variation that was probabilistic (uniformly distributed from  $-0.008$  to  $-0.004$ ). The expected price of the CD was normally distributed with mean 0.8139, standard deviation 0.13139. Operating expenses associated with the Canadian operation were normally distributed with mean \$1,925,000 and standard deviation \$192,500. The other half of sales were in the US. There was risk of customer liability lawsuits (2, Poisson distribution), with expected severity per lawsuit that was lognormally distributed with mean \$320,000, standard deviation \$700,000. Operational risks associated with US operations were normally distributed with mean \$1,275,000, standard deviation \$127,500. The Excel spreadsheet model for this is given in Table 6.1.

In Crystal Ball, entries in cells B2, B3, B7, B10, B21, B22 and B23 were entered as assumptions with the parameters given in column C. Prediction cells were defined for cells B17 (Canadian net income) and B29 (Total net income after tax). Results for cell B17 are given in Fig. 6.2, with a probability of 0.9 prescribed in Crystal Ball so that we can identify the VaR at the 0.05 level.

Statistics are given in Table 6.2.

The value at risk at the 0.95 level for this investment was  $-540,245.40$ , meaning that there was a 0.05 probability of doing worse than losing \$540,245.50 in US dollars. The overall investment outcome is shown in Fig. 6.3.

Statistics are given in Table 6.3.

On average, the investment paid off, with a positive value of \$96,022.98. However, the worst case of 500 was a loss of over \$14 million. (The best was a gain of over \$1.265 million.) The value at risk shows a loss of \$1.14 million, and Fig. 6.3 shows that the distribution of this result is highly skewed (note the skewness measures for Figs. 6.2 and 6.3).

Beneda proposed a model reflecting hedging with futures contracts, and insurance for customer liability lawsuits. Using the hedged price in cell B4, and insurance against customer suits of \$640,000, the after-tax profit is shown in Fig. 6.4.

Mean profit dropped to \$84,656 (standard deviation \$170,720), with minimum  $-\$393,977$  (maximum gain \$582,837). The value at risk at the 0.05 level was a loss

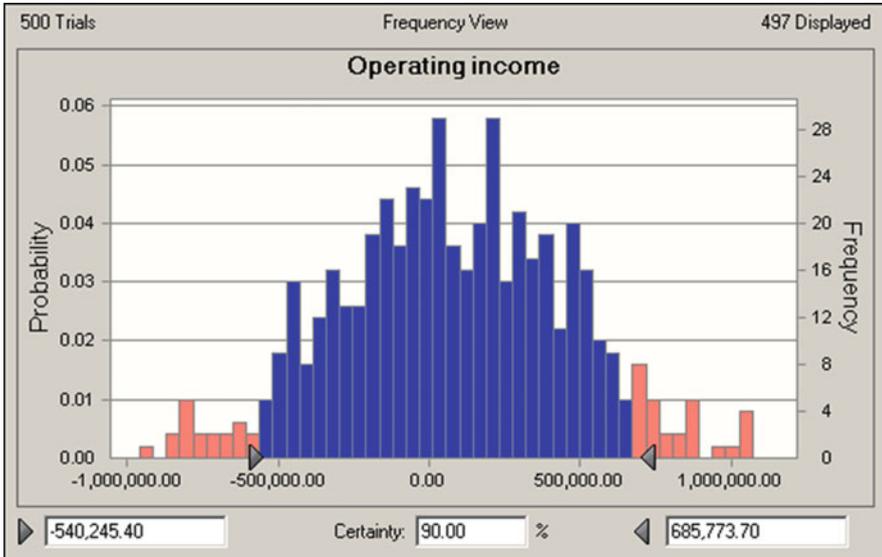
**Table 6.1** Excel model of investment

	A	B	C
1	Financial risk	Formulas	Distribution
2	Expected basis	-0.006	Uniform(-0.008,-0.004)
3	Expected price per CD	0.8139	Normal(0.8139,0.13139)
4	March futures price	0.8149	
5	Expected basis 45 days	=B2	
6	Expected CD futures	0.8125	
7	Operating expenses	1,925	Normal(1,925,000,192,500)
8	Sales	20,000	
9			
10	Price \$US	100	Uniform(90,110)
11	Sales	20,000	
12	Current	0.8121	
13	Receipts	=B10 * B11/B12	
14	Expected exchange rate	=B3	
15	Revenues	=B13 * B14	
16	COGS	=B7 * 1,000,000	
17	Operating income	=B15 - B16	
18			
19	Local sales	20,000	
20	Local revenues	=B10 * B19	
21	Lawsuit frequency	2	Poisson(2)
22	Lawsuit severity	320,000	Lognormal(320,000,700,000)
23	Operational risk	1,275,000	Normal(1,275,000,127,500)
24	Losses	=B21 * B22 + B23	
25	Local income	=B20 - B24	
26			
27	Total income	=B17 + B25	
28	Taxes	=0.35 * B27	
29	After Tax Income	=B27 - B28	

of \$205,301. Thus there was an expected cost of hedging (mean profit dropped from \$96,022 to \$84,656), but the worst case was much improved (loss of over \$14 million to loss of \$393,977) and value at risk improved from a loss of over \$1.14 million to a loss of \$205 thousand.

## Conclusions

Value at risk is a useful concept in terms of assessing probabilities of investment alternatives. It is a point estimator, like the mean (which could be viewed as the value at risk for a probability of 0.5). It is only as valid as the assumptions made, which include the distributions used in the model and the parameter estimates. This



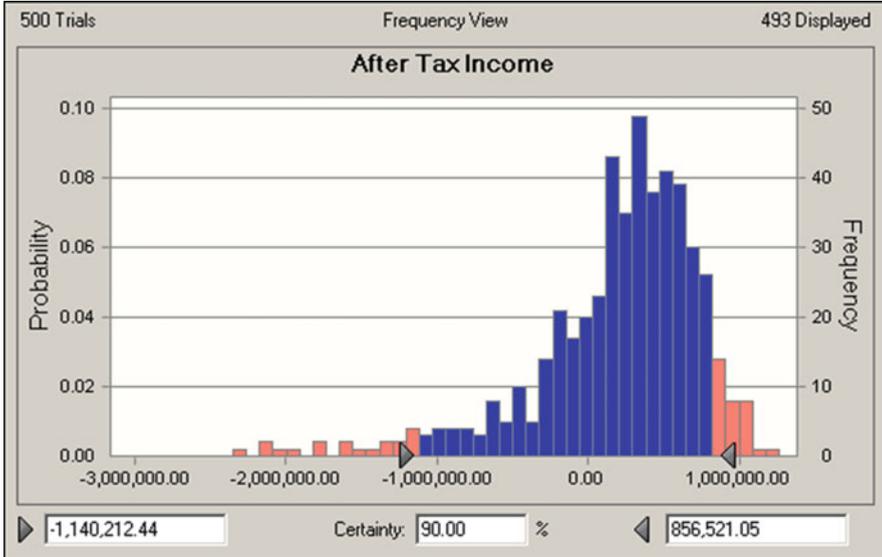
**Fig. 6.2** Output for Canadian investment. ©Oracle. used with permission

**Table 6.2** Output statistics for operating income

Forecast	Operating income
Statistic	Forecast values
Trials	500
Mean	78,413.99
Median	67,861.89
Mode	–
Standard Deviation	385,962.44
Variance	148,967,005,823.21
Skewness	–0.0627
Kurtosis	2.99
Coefficient of variability	4.92
Minimum	–1,183,572.09
Maximum	1,286,217.07
Mean standard error	17,260.77

is true of any simulation. However, value at risk provides a useful tool for financial investment. Monte Carlo simulation provides a flexible mechanism to measure it, for any given assumption.

However, Value at risk has undesirable properties, especially for gain and loss data with non-elliptical distributions. It satisfies the well-accepted principle of diversification under assumption of normally distributed data. However, it violates the widely accepted subadditive rule; i.e., the portfolio VaR is not smaller than the sum of component VaR. The reason is that VaR only considers the extreme

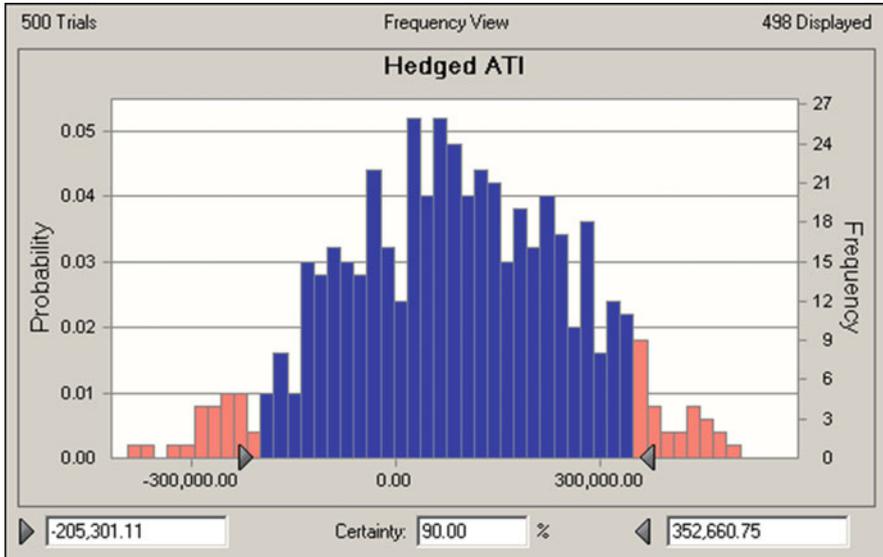


**Fig. 6.3** Output for after tax income. ©Oracle. used with permission

**Table 6.3** Output statistics for after tax income

Forecast	Operating income
Statistic	Forecast values
Trials	500
Mean	96,022.98
Median	304,091.58
Mode	–
Standard Deviation	1,124,864.11
Variance	1,265,319,275,756.19
Skewness	–7.92
Kurtosis	90.69
Coefficient of variability	11.71
Minimum	–14,706,919.79
Maximum	1,265,421.71
Mean standard error	50,305.45

percentile of a gain/loss distribution without considering the magnitude of the loss. As a consequence, a variant of VaR, usually labeled *Conditional-Value-at-Risk* (or CVaR), has been used. With respect to computational issues, optimization CVaR can be very simple, which is another reason for adoption of CVaR. This pioneer work was initiated by Rockafellar and Uryasev,<sup>6</sup> where CVaR constraints in optimization problems can be formulated as linear constraints. CVaR represents a weighted average between the value at risk and losses exceeding the value at risk. CVaR is a risk assessment approach used to reduce the probability a portfolio will



**Fig. 6.4** After-tax profit with hedging and insurance. ©Oracle. used with permission

incur large losses assuming a specified confidence level. CVaR has been applied to financial trading portfolios,<sup>7</sup> implemented through scenario analysis,<sup>8</sup> and applied via system dynamics.<sup>9</sup> A popular refinement is to use copulas, multivariate distributions permitting the linkage of a huge number of distributions.<sup>10</sup> Copulas have been implemented through simulation modeling<sup>11</sup> as well as through analytic modeling.<sup>12</sup>

We will show how specified confidence levels can be modeled through chance constraints in the next chapter. It is possible to maximize portfolio return subject to constraints including Conditional Value-at-Risk (CVaR) and other downside risk measures, both absolute and relative to a benchmark (market and liability-based). Simulation CVaR based optimization models can also be developed.

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