Chapter 6
Interaction-Free Measurement

We discuss an experiment which provides an example of the unusual effects that may result from the superposition of states, and of the peculiarities that may be associated with the quantum-mechanical measurement process. In addition, we make the acquaintance of unitary operators.

Self-interference, i.e. the interference of a quantum object with itself, is a fascinating phenomenon of quantum mechanics, which we discuss below in terms of the *interaction-free quantum measurement*. The experiment is based on the principle of the Mach–Zehnder interferometer (MZI). It shows the existence of quantum superpositions as clearly as the famous double-slit experiment, but it is by comparison formally and experimentally much 'handier', so that it is increasingly finding its way into textbooks. At the same time, it also allows for the treatment of further-reaching questions. That is why we meet the MZI not only in many modern basic experiments, but also for example in the field of quantum information, where we can realize basic functions of the quantum computer by means of the MZI and its components (see the closing remarks to this chapter).

6.1 Experimental Results

6.1.1 Classical Light Rays and Particles in the Mach–Zehnder Interferometer

6.1.1.1 Light Rays

The experimental setup consists of a Mach–Zehnder interferometer and two photodetectors, which respond to incident light; see Fig. 6.1. Coherent light enters the apparatus at the lower left and is split by a *beam splitter* (or half-silvered mirror)

J. Pade, *Quantum Mechanics for Pedestrians 1*, Undergraduate Lecture Notes in Physics, https://doi.org/10.1007/978-3-030-00464-4_6

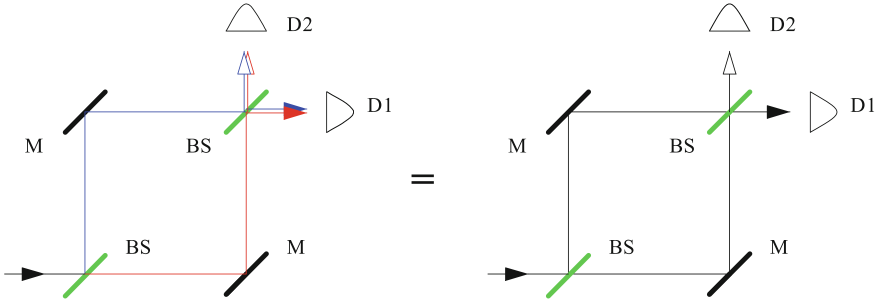

Fig. 6.1 Schematic of a MZI. BS = beam splitter, M = mirror, D = detector. Left: At the first beam splitter, the light is split into two sub-beams (blue and red do not signify the colors of the light beams, but serve only for better visualization), and these two sub-beams are split again at the second beam splitter, resulting in four sub-beams. The figure at the right is a compact description of these facts, which we will use in the following

into two beams.[1] These beams impinge, after reflection by a mirror, on a second beam splitter, so as to produce a total of four sub-beams, two each of which meet at one of the two detectors. The experimental finding is now that the upper detector D2 *never* responds and the lower detector D1 *always* responds. In other words, the relative intensity I on D1 is given by $I_1 = 1$, and on D2 by $I_2 = 0$. Here, we assume on the whole ideal conditions: the optical paths 'above' and 'below' have exactly the same length, there is no absorption by the mirrors, the efficiency of the detectors is 100%, and so on.

This different behavior of the two detectors may perhaps be surprising, since the experimental setup appears to be completely symmetrical at first glance. But in fact, its symmetry is broken, as long as the light enters the first beam splitter only in the horizontal and not also in the vertical direction (and with the same intensity).

The following consideration shows why the two detectors react differently: That part of the lower beam which after the second beam splitter enters D2 or D1 undergoes a reflection (1 × mirror) or two reflections (1 × mirror, 1 × beam splitter), while the part of the upper beam which after the second beam splitter enters D2 or D1 undergoes three reflections (1 × mirror, 2 × beam splitter) or two reflections (1 × mirror, 1 × beam splitter). In other words, the detector D1 sees two light beams with the same history (i.e. the same phase), which consequently interfere constructively. In contrast, the detector D2 sees two beams with different histories. We will show immediately that this indeed gives destructive interference.

In a variant of the experimental setup, we use a blocker that absorbs the upper beam or scatters it out of the MZI; see Fig. 6.2. Obviously, the upper and lower sub-beams now cannot interfere and the intensities at the detectors are given by $I_1 = I_2 = 1/4$.

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[1] The two beams can in principle be separated quite far apart. In this way, the non-classical effects of certain quantum-mechanical setups can be demonstrated more impressively than in the double-slit experiment.

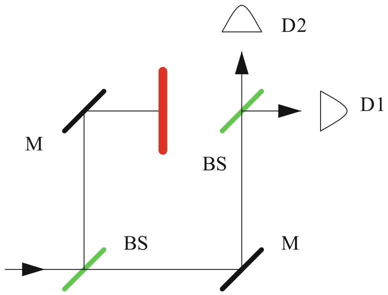

Fig. 6.2 MZI with a blocker in the upper beam

6.1.1.2 Particles in the MZI

What happens if we send *particles* ($m \neq 0$) instead of light waves through the apparatus? Of course we have to replace the beam splitters by devices which let the particles pass or reflect them with probabilities of 1/2, but otherwise the experimental setup remains the same. If we now interpret the number of particles per detector as intensities, it follows directly that for the case without a blocker, $I_1 = I_2 = 1/2$ holds, and for the case with a blocker, $I_1 = I_2 = 1/4$.

6.1.1.3 Comparison: Light–Particles

It follows that with a blocker, the intensities are given by $I_1 = I_2 = 1/4$, regardless of whether we use waves or particles. For the case without a blocker, however, there is a distinctive difference, since for waves we have $I_1 = 1$ and $I_2 = 0$, while for particles, $I_1 = I_2 = 1/2$. So we can conclude that if we perform an experiment (in the sense of a black-box setup) without the blocker and measure $I_2 = 0$, then we know that a wave and not a particle has passed through the apparatus. These results are summarized in the Table 6.1.

6.1.2 Photons in the Mach–Zehnder Interferometer

6.1.2.1 Single-Photon Experiments (MZI Without Blocker)

We let light enter the MZI and reduce its intensity (similar to the polarization experiments of Chap. 2). Since our previous considerations do not rely on the intensity of the incident light, they should also apply to the limit of vanishing light intensity. This

Table 6.1 Intensities at the two detectors

	Without blocker	With blocker
Wave	$I_1 = 1;\quad I_2 = 0$	$I_1 = \frac{1}{4};\quad I_2 = \frac{1}{4}$
Particle	$I_1 = \frac{1}{2};\quad I_2 = \frac{1}{2}$	$I_1 = \frac{1}{4};\quad I_2 = \frac{1}{4}$

means that eventually there is only one photon in the MZI at a given time. In fact, the experimental findings are: even if we operate with single photons, only detector D1 responds, while D2 remains silent, or $I_1 = 1$ and $I_2 = 0$.

So we must conclude that a single photon is a wave and not a particle. On the other hand, a photon is a point object as far as we know. Our everyday understanding perceives the situation as contradictory: An object can be both point-like and wave-like. But our cognitive abilities are, as we have mentioned before, formed and trained by evolution in our macro-physical environment and not under quantum-mechanical conditions.

In addition, we have to conclude that due to the interference effect, the photon 'somehow' interacts with itself. It is not intuitively obvious how this takes place. Certainly, it is not the case that the photon splits into two smaller fragments. We have here the same problem as in the double slit experiment—if there are two possibilities which can be realized by a quantum-mechanical system, then certain interference phenomena will appear which have no classical analogues (self-interference).

As we said previously, it is perhaps best to imagine a quantum-mechanical *possibility landscape*, in which the quantum object (photon, electron, ...) moves. A superposition of possibilities yields a new landscape with new features, in which the object moves differently than in the landscape of only one possibility.

6.1.2.2 Interaction-Free Measurement (MZI With Blocker)

With a beam blocker, we have $I_1 = I_2 = 1/4$, and that means that in 25% of all cases, detector 2 responds. This in turn implies that we know in these cases that there is a blocker in the apparatus without the photon having interacted directly with the blocker (otherwise it would have disappeared from the apparatus and could not be detected in either detector).[2] This situation is called an *interaction-free quantum measurement*. Below, we make some critical remarks about this terminology.

The whole issue can be formulated more sensationally[3] by choosing a bomb[4] as the blocker. The bomb is so sensitive that a single photon is enough to cause it to detonate[5]—so to speak, just seeing the bomb means that it explodes.[6] We can use

[2]Thus, there are apparently physical effects influenced by *potential but unrealized* events, that is, events that *could* have happened, but did not actually occur. Such events are called *counterfactual* (not corresponding to the facts).

[3]A.C. Elitzur and L. Vaidman, "Quantum Mechanical Interaction-Free Measurements", Foundations of Physics 23, 987 (1993).

[4]In order to avoid the militaristic note, some textbooks use 'cracker test' instead of 'bomb test', but this sounds a bit whimsical.

[5]"A physical experiment which makes a bang is always worth more than a quiet one. Therefore a man cannot strongly enough ask of Heaven: If it wants to let him discover something, may it be something that makes a bang. It will resound into eternity." Georg Christoph Lichtenberg, *Scrap Books*, Vol. F (1147).

[6]This remark seems a bit exaggerated, but in fact the rods of the human eye can apparently react to even a single photon. The cones, responsible for color vision, need about 100 times stronger

this fact in the following setup: Assume that we have a black-box MZI, and we do not know whether it contains a bomb or not. The task now is to clarify this issue. It cannot be solved by means of classical physics. Quantum mechanics helps us—at least, we know in a quarter of the cases that a bomb is hidden in the apparatus without its blowing up in our faces. In fact, one can increase the 'efficiency' in a somewhat modified apparatus to virtually 100% by exploiting the so-called quantum Zeno effect. More about this in Appendix L, Vol. 1.

We have here again - as a purely quantum-mechanical effect - the *superposition of possibilities* (self-interference) that makes possible this surprising result. Of course, it is again not the case that the photon 'splits' up and, quasi by way of trial and error, passes at the same time through both arms of the MZI. The superposition of the possibilities provides precisely the different landscape mentioned above, in which the photon propagates in a different way. We can most easily describe this propagation by means of probabilities—if we let a photon start through the apparatus, it will end up with a probability of 1/4 in detector 2, and then we know that a bomb is in the beam path. But if we (in whatever way) know which arm the photon has passed through (*which-way information*), the landscape of possibilities or probabilities changes dramatically: in 50% of the cases the bomb explodes, in the other 50% nothing exciting happens. Formulated as a 'standard rule': If the path is known/unknown, then the probabilities/amplitudes are added:

$$\text{path is } \begin{matrix}\text{known}\\ \text{unknown}\end{matrix} \quad \rightarrow \quad \text{add } \begin{matrix}\text{probabilities}\\ \text{amplitudes}\end{matrix}. \tag{6.1}$$

Further comments on which-way experiments (or delayed-choice experiments) are found in Appendix M, Vol. 1.

In this context, the term *wave-particle duality* occasionally crops up. What is meant is this: Depending on the experimental situation, a quantum system shows either particle-like or wave-like features. We take as an example electrons in the double slit experiment. If we allow for interference, then the electrons show their 'wave nature'; if we want to see them as particles, e.g. by following their path through the slits, they show their 'particle nature'. Dualism in this context means that these properties are complementary—either particle or wave, but we can never measure both at the same time. We can state briefly and quite generally that asking for a particular property of an object leads to an answer that puts that property in the foreground and suppresses the other (complementary) property.

On closer inspection, the term wave-particle duality seems, however, to be redundant, or to favor misunderstandings, since it supports the widespread but erroneous notion that, before a measurement, a quantum object is actually a particle or a wave. That is a misinterpretation which can cloud the mind in the process of learning quantum mechanics. Indeed, before a measurement, quantum objects in general do *not* have well-defined properties. It is therefore understandable that one is often advised

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excitation. See e.g. Davide Castelvecchi, People can sense single photons, *Nature*, https://doi.org/10.1038/nature.2016.20282 (Jul 2016).

to omit completely the terms ‘wave-particle duality’ or ‘complementarity’; indeed, doing so does not cause a noticeable loss in understanding.[7]

A quantum object is simply something for which we have no detailed everyday terms, and depending on how we look at it, it seems to be more like a particle or more like a wave (but in fact it is neither)—it is simply a quantum object.[8] We could call it informally a ‘quob,’ but would it then be more familiar or intuitive?

6.2 Formal Description, Unitary Operators

To arrive at a simple, clear-cut description of states, we choose as the only distinctive criterion their direction of motion—either horizontal or vertical. Thus, we neglect polarization, beam profile, explicit time behavior and so on. We describe the conditions with and without a blocker under the assumption that we have two identical beam splitters.

6.2.1 First Approach

We divide the setup into four regions, as shown in Fig. 6.3. We denote the state in the region i by $|z_i\rangle$. With regard to the simplest possible description as just mentioned, we represent $|z_i\rangle$ as a superposition of horizontal $|H\rangle$ and vertical $|V\rangle$ propagation directions,[9] where these states constitute a CONS in a two-dimensional vector space. One can see that the propagation is horizontal in region 1 and both vertical and horizontal in region 2. Accordingly, we can write $|z_1\rangle = |H\rangle$ and $|z_2\rangle = c_1 |H\rangle + c_2 |V\rangle$. To determine the numbers c_1 and c_2, we take into account that (i) the relative phase shift is $90° \hat{=} \frac{\pi}{2}$ (see Appendix K, Vol. 1), which corresponds to $e^{i\pi/2} = i$, and (ii) that the intensity in a *half-silvered mirror*[10] is equal for ‘horizontal’ and

[7]The Feynman Lectures on Physics, 5th Edition, 1970, Vol II, p. 37-1: “Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behaves like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaved like a wave. So it really behaves like neither. Now we have given up. We say: ‘It is like *neither*.’” Richard P. Feynman, S. Tomonaga and J. Schwinger were awarded the Nobel Prize in Physics 1965 for their fundamental work in quantum electrodynamics.

[8]We note at this point, more generally, that the practice of declaring all things perceived to simply ‘exist’ may be inadequate. Instead, one should first look at perception itself and examine its predictability. Therefore, in quantum mechanics we need advanced methods, because we cannot come to grips with the ‘perceptions’ (observations, measurements) by simply using intuitive, classical instruments. To obtain the information relevant to quantum mechanics, we have to think and act in a largely formal manner.

[9]Not to be confused with the polarization states $|h\rangle$ and $|v\rangle$.

[10]For asymmetrical beam splitters (reflectance $\neq$ transmittance), see the exercises.

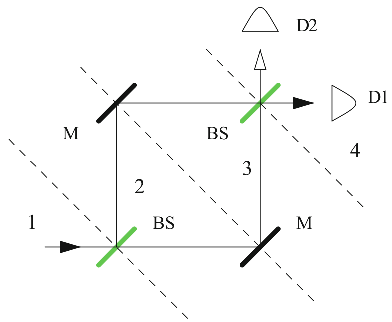

Fig. 6.3 Division of the MZI into four regions

'vertical', i.e. $|c_1|^2 = |c_2|^2$. Thus, it follows that $|z_2\rangle = c\,[|H\rangle + i\,|V\rangle]$. We postpone the determination of the constant c and summarize:

$$|H\rangle \underset{\text{beam splitter}}{\rightarrow} c\,[|H\rangle + i\,|V\rangle] \tag{6.2}$$

and analogously

$$|V\rangle \underset{\text{beam splitter}}{\rightarrow} c\,[|V\rangle + i\,|H\rangle]. \tag{6.3}$$

At a mirror, we have a phase shift of $180° \mathrel{\hat{=}} \pi$ or $e^{i\pi} = -1$ and therefore

$$|H\rangle \underset{\text{mirror}}{\rightarrow} -|V\rangle; \quad |V\rangle \underset{\text{mirror}}{\rightarrow} -|H\rangle. \tag{6.4}$$

All in all, we have

$$\begin{aligned} |z_1\rangle &= |H\rangle \\ |z_2\rangle &= c\,[|H\rangle + i\,|V\rangle] \\ |z_3\rangle &= -c\,[|V\rangle + i\,|H\rangle] \\ |z_4\rangle &= -c^2\,[|V\rangle + i\,|H\rangle] - ic^2\,[|H\rangle + i\,|V\rangle] = -2ic^2\,|H\rangle. \end{aligned} \tag{6.5}$$

It follows immediately that only detector 1 responds, while detector 2 remains dark, as is indeed observed experimentally.

We can define the constant c as follows: We assume that the setup operates without losses—what goes in, comes out. This manifests itself in the fact that the norms are equal, or more precisely must be equal, $\langle z_i|\, z_i\rangle = \langle z_j|\, z_j\rangle$. The simplest choice is $-2ic^2 = 1$ or $c = \pm e^{i\pi/4}/\sqrt{2}$. We choose the upper sign and find $c = \frac{1+i}{2}$.

For the case with a blocker, we have analogously

$$\begin{aligned} |z_1\rangle &= |H\rangle \\ |z_2\rangle &= c\,[|H\rangle + i\,|V\rangle] \end{aligned}$$

$$|z_3\rangle = -c\,|V\rangle$$
$$|z_4\rangle = -c^2\,[|V\rangle + i\,|H\rangle] = \frac{1}{2i}\,[|V\rangle + i\,|H\rangle]\,. \tag{6.6}$$

We see that also in this case, the intensities measured by the two detectors are displayed correctly. We note that the transition $|z_2\rangle \rightarrow |z_3\rangle$ does not conserve the norm: $\langle z_2|\, z_2\rangle = 2\,|c|^2 \neq \langle z_3|\, z_3\rangle = |c|^2$. It is the absorbing effect of the blocker which leads to this inequality.

6.2.2 Second Approach (Operators)

We have just described the experiment with 'states and arrows'. A more compact approach is permitted by using operators. We can describe the effect of a beam splitter by an operator T, and the effect of a mirror without or with a blocker by S and S'. Without the blocker, this leads to:

$$\begin{aligned}
|z_1\rangle &= \text{initial state} \\
|z_2\rangle &= T\,|z_1\rangle \\
|z_3\rangle &= S\,|z_2\rangle = ST\,|z_1\rangle \\
|z_4\rangle &= T\,|z_3\rangle = TS\,|z_2\rangle = TST\,|z_1\rangle = \text{final state}
\end{aligned} \tag{6.7}$$

and with the blocker, to:

$$|z_1\rangle = \text{initial state};\;\; |z_4\rangle = TS'T\,|z_1\rangle = \text{final state}. \tag{6.8}$$

The operators are applied in sequence from right to left: $TST\,|z_1\rangle = T\,(S\,(T\,|z_1\rangle))$.

To obtain an explicit formulation for T, we consider the effect of this operator on the basis vectors. According to (6.2) and (6.3), we have

$$T\,|H\rangle = \frac{1+i}{2}\,[|H\rangle + i\,|V\rangle]\,;\;\; T\,|V\rangle = \frac{1+i}{2}\,[i\,|H\rangle + |V\rangle]\,. \tag{6.9}$$

Using the completeness relation $|H\rangle\,\langle H| + |V\rangle\,\langle V| = 1$ leads to

$$T\,|H\rangle\,\langle H| + T\,|V\rangle\,\langle V| = T = \frac{1+i}{2}\,[|H\rangle + i\,|V\rangle]\,\langle H| + \frac{1+i}{2}\,[i\,|H\rangle + |V\rangle]\,\langle V|\,, \tag{6.10}$$

or compactly,

$$T = \frac{1+i}{2}\,[1 + i\,|H\rangle\,\langle V| + i\,|V\rangle\,\langle H|]\,. \tag{6.11}$$

Analogously, the 'mirror-operator' without a blocker is given by:

$$S = -\left|H\right\rangle\left\langle V\right| - \left|V\right\rangle\left\langle H\right| \tag{6.12}$$

and with the blocker by

$$S' = -\left|V\right\rangle\left\langle H\right|. \tag{6.13}$$

We learn from this that operators can generally be represented as linear combinations of dyadic products of the basis vectors.

It is easily verified that with (6.11)–(6.13), we have

$$TST = 1 \tag{6.14}$$

and

$$TS'T = \frac{1}{2}\left[1 + i\left|H\right\rangle\left\langle V\right| - i\left|V\right\rangle\left\langle H\right|\right], \tag{6.15}$$

so that we obtain again from the initial state $\left|z_1\right\rangle = \left|H\right\rangle$ the final states $\left|z_4\right\rangle = \left|H\right\rangle$ and $\left|z_4\right\rangle = \frac{1}{2}\left[\left|H\right\rangle - i\left|V\right\rangle\right]$ for the case with and without the blocker, respectively. For the explicit representation of the operators and their products as matrices, see the exercises.

The adjoint of the operator T is

$$T^\dagger = \frac{1-i}{2}\left[1 - i\left|H\right\rangle\left\langle V\right| - i\left|V\right\rangle\left\langle H\right|\right] \tag{6.16}$$

and it follows that

$$T^\dagger T = TT^\dagger = 1. \tag{6.17}$$

Analogously, the same holds true for S, but not for S', since here an (irreversible) absorption is included:

$$SS^\dagger = S^\dagger S = 1;\ \ S'S'^\dagger = \left|V\right\rangle\left\langle V\right|;\ \ S'^\dagger S' = \left|H\right\rangle\left\langle H\right|. \tag{6.18}$$

In fact, the operators T and S share an important property—they are *unitary*. As a generalization of (6.17), an operator (or matrix) U is unitary if

$$U^\dagger U = UU^\dagger = 1 \text{ or } U^\dagger = U^{-1}. \tag{6.19}$$

The name 'unitary' stems from the fact that certain expressions are left unchanged under the transformation performed by the operator—in a way, it acts similarly to multiplication by 1. For example, the scalar product and thus also the norm are invariant. To show this, we start with two states $\left|\varphi\right\rangle$ and $\left|\psi\right\rangle$, and the unitary transformed states $\left|\varphi'\right\rangle = U\left|\varphi\right\rangle$ and $\left|\psi'\right\rangle = U\left|\psi\right\rangle$. Remember that a product of operators is reversed[11] in the adjoint, $(AB)^\dagger = B^\dagger A^\dagger$. This means that

[11] This is well known from linear algebra, e.g. when transposing or inverting matrices.

$$\left(\left|\psi'\right\rangle\right)^{\dagger} = \left(U\left|\psi\right\rangle\right)^{\dagger} \rightarrow \left\langle\psi'\right| = \left\langle\psi\right| U^{\dagger}. \tag{6.20}$$

It follows that

$$\left\langle\psi'\right|\left.\varphi'\right\rangle = \left\langle\psi\right| U^{\dagger}U\left|\varphi\right\rangle = \left\langle\psi\right|\left.\varphi\right\rangle, \tag{6.21}$$

i.e. the scalar product is conserved. Unitary transformations can always be understood in the end as a coordinate or basis transformation, even if the corresponding space is more elaborate than our two-dimensional space. These transformations conserve in particular scalar products, hence also lengths and angles, and they are reversible (because $U^{-1} = U^{\dagger}$ exists). Irreversible processes (e.g. measurements) can therefore not be represented by unitary transformations.

6.3 Concluding Remarks

As mentioned earlier in this chapter, the MZI is an essential tool for many modern fundamental experiments, both theoretically and experimentally. Due to lack of space, we can only outline some of them here, but a more detailed discussion is found in the Appendices (L and M, Vol.1; J, P and Q, Vol. 2). At the end of this chapter, we will take a closer look at the term ‘interaction-free’.

6.3.1 Extensions

Out of a great number of applications of the MZI, we have selected those which are understandable with our present knowledge and which do not require additional concepts such as the Aharonov–Bohm effect.

6.3.2 Quantum Zeno Effect

There is an extension of the ‘bomb test’ which uses the *quantum Zeno effect*. This effect essentially implies that one can prevent the change of a system under appropriate circumstances by frequently-repeated measurements (‘a watched pot never boils’). The experiment uses a modified MZI setup and is based on the observation of the polarization state of photons. In principle, one can achieve an efficiency of up to 100% (see Appendix L, Vol. 1).

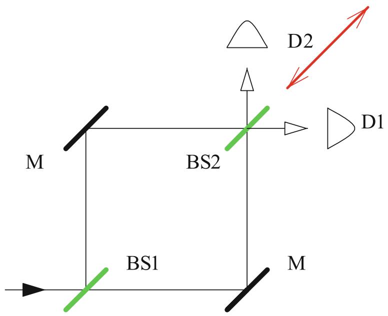

Fig. 6.4 Delayed-choice experiment. The second beam splitter can be removed or inserted

6.3.3 Delayed-Choice Experiments

Here, we use the familiar MZI setup, but the second beam splitter BS2 can be removed or inserted *after* the photon has passed the first beam splitter (hence the name 'delayed decision or choice'); see Fig. 6.4. The operation can be executed so quickly that a 'notification' of the photon would have to be superluminal.

Thus, the photon has to 'decide' whether it passes through the MZI as a coherent superposition (BS2 is inserted and only D1 responds) or whether it passes through only one of the two arms (BS2 is removed, the respective detector responds). The salient point is that the photon must take the decision after it has passed the first beam splitter (and possibly the mirror) but *before* we decide whether BS2 will be left in the path or not. That would mean (at least in a classical argument) that the photon must know before entering BS1 whether BS2 will be left or removed. In other words, the photon had to know about our future decision. Does that mean that delayed-choice experiments prove that certain events may have a retroactive action with respect to time?[12]

With a similar setup, one can produce a *quantum eraser*, with which one can subsequently delete ('erase') which-way information in certain experiments and thus restore their interference effects (see Appendix M, Vol. 1).

6.3.4 The Hadamard Transformation

The *Hadamard transformation* plays an important role in quantum information. It can be carried out by means of the MZI. Another method that can be experimentally realized uses the combination of a beam splitter and a phase shifter. Written as a

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[12]Experiments are not confined to small distances. See e.g. F. Vedovato et al., Extending Wheeler's delayed-choice experiment to space, *Science Advances* Vol. 3, no.10, https://doi.org/10.1126/sciadv.1701180 (Oct 2017), where a delayed-choice experiment is reported with a propagation distance of up to 3500 km.

2×2 matrix, the Hadamard transformation H is (see Appendix P, Vol. 2 for the case $n \times n$):

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{6.22}$$

6.3.5 From the MZI to the Quantum Computer

The MZI, with additional phase shifts, can be described as a network consisting of three simple *quantum logic gates*, namely as a combination of two Hadamard gates and a phase shifter. On this basis, other building blocks of quantum information such as the CNOT gate can be constructed (see Chap. 26, Vol. 2, and Appendix Q, Vol. 2).

6.3.6 Hardy's Experiment

This experiment combines an interaction-free measurement with quantum entanglement. This concept (which we will learn about in Chap. 20, Vol. 2) is another key aspect of quantum mechanics which has no classical counterpart. The experiment consists essentially of two superimposed MZI's (see Appendix J, Vol. 2).

6.3.7 How Interaction-Free is the 'Interaction-Free' Quantum Measurement?

Finally, a few words about the adjective 'interaction-free'. Indeed, we should always put it in quotation marks. This is due to the fact that, strictly speaking, this experiment can never be completely 'interaction-free'; there is an operator that describes the behavior of the photon in the interferometer, and this operator takes on a different form depending on whether a bomb is placed in the light path or not. Taking this into account, the term 'measurement with minimal interaction' is more correct and insofar preferable.

This is because there is a fundamental limit to the attainable sensitivity of the detonator of the bomb, and the measurement can be called interaction-free at most within the limits of this sensitivity. The reason for this limitation is the *uncertainty principle* $\Delta x \Delta p \geq \hbar/2$. It is the basis for the following argument: If the bomb (the detonator) is located with an uncertainty Δx, then a given momentum uncertainty Δp results (for $\Delta x \to 0$, we would have $\Delta p = \infty$). To prevent the bomb from going off 'by itself', the detonator must not respond to momentum transfers smaller than Δp. In other words, the uncertainty principle necessarily requires that the bomb have an 'ignition threshold'. Under such circumstances one cannot speak of 'interaction-free'; a more appropriate term is *measurement with minimal interaction*. Along with a momentum transfer, there is also a possible energy transfer. The fact that this transfer can be very small in macroscopic objects ($\sim$1/M) and vanishes in the limit $M \to \infty$ does not fundamentally alter the situation.

Conclusion: There is no 'interaction-free' quantum measurement, i.e. a measurement without interaction, but at most a measurement with minimal interaction. It is perhaps surprising that the term 'interaction-free' has established itself in the physics community (almost) without difficulty. On the other hand, one must admit that this term is very striking and much more effective in catching public attention than the more correct expressions (just as the term 'ozone hole' is in use rather than the more correct 'stratospheric region of low ozone concentration'). Thus, the interaction-free quantum measurement is another example of the fact that physics operates not only as pure science, but also through its perceptions by the larger society.

6.4 Exercises

1. Show that for all $|z_i\rangle$ in (6.5), $\| |z_i\rangle \|^2 = 1$ holds.
2. Given a MZI with symmetrical beam splitters, calculate the final state with and without a blocker if the initial state is given by $\alpha |H\rangle + \beta |V\rangle$.
3. Given an operator A with

$$A\,|H\rangle = a\,|H\rangle;\; A\,|V\rangle = b\,|V\rangle, \tag{6.23}$$

   determine the explicit form of A.
4. Which eigenvalues can a unitary operator have?
5. Circularly- and linearly-polarized states are connected by $|r\rangle = \frac{1}{\sqrt{2}}\,|h\rangle + \frac{i}{\sqrt{2}}\,|v\rangle$ and $|l\rangle = \frac{1}{\sqrt{2}}\,|h\rangle - \frac{i}{\sqrt{2}}\,|v\rangle$. Show that this basis transformation is unitary (or that the transformation matrix is unitary).
6. Give the matrix representations of the operators T, S and S' from (6.11)–(6.13) and their combinations TST and $TS'T$.
7. Given the operator

$$U = a\,|H\rangle\langle H| + b\,|H\rangle\langle V| + c\,|V\rangle\langle H| + d\,|V\rangle\langle V| \cong \begin{pmatrix} a & b \\ c & d \end{pmatrix}; \tag{6.24}$$

   for which values of the coefficients is U a unitary operator? In other words: How is the general two-dimensional unitary transformation formulated?
8. Given a MZI without a blocker and with asymmetrical beam splitters (transmittance $\neq$ reflectance), determine the properties required of the beam splitters in order that a beam entering horizontally activates only detector 1, while detector 2 remains dark.