23 Dynamic Method

In Chap. 5, we introduced the concept of response spectrum as a plot of the maximum response (spectral displacement, spectral velocity, or spectral acceleration) versus the natural frequency or natural period of a single degree-of-freedom system subjected to a specific excitation. In the present chapter, we will use seismic response spectra for earthquake resistant design of buildings modeled as discrete systems with concentrated masses at each level of the building.

Since response spectral charts are prepared for single degree-of-freedom system, it is necessary to perform a transformation of coordinates to obtain the modal equations of motion, and then combine their spectral responses to obtain the maximum response of the structure. In earthquake resistant design of buildings, the maximum responses include displacements, accelerations, shear forces, overturning moments, and torsional moments.

23.1 Modal Seismic Response of Building

The equations of motion of a shear building modeled with lateral displacement coordinates at the N levels and subjected to seismic excitation at the base in the same direction of the lateral displacement, may be written, neglecting damping, from Eq. (8.35) as

$$[M]\{\ddot{u}_r\} + [K]\{u_r\} = -[M]\{1\}\ddot{u}_s(t) \tag{23.1}$$

In Eq. (23.1), $[M]$ and $[K]$ are respectively the mass and the stiffness matrices of the system, $\{u_r\}$ and $\{\ddot{u}_r\}$ are respectively the displacement and acceleration vectors (relative to the base), $\ddot{u}_s(t)$ is the function of the seismic acceleration at the base of the building, and $\{1\}$ a vector with all its elements equal to 1.

23.1.1 Modal Equation and Participation Factor

As presented in Chap. 8, the solution of Eq. (23.1) may be found by solving the corresponding eigenproblem

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$$\left[[K] \right] - \omega^2 [M] \big] \{\Phi\} = \{0\} \tag{23.2}$$

to determine the natural frequencies ω_1, ω_2, . . ., ω_N (or natural periods T_1, T_2,. . ., T_N) and the modal matrix $[\Phi]$ with its columns containing the normalized modal shapes. Then the linear transformation

$$\{u_r\} = [\Phi]\{z\} \tag{23.3}$$

introduced in Eq. (23.1) yields the modal equations;

$$\ddot{z}_m + \omega_m^2 z_m = -\Gamma_m \ddot{u}_s(t) \qquad (m = 1, 2, \ldots, N) \tag{23.4}$$

in which Γ_m is the participation factor given by Eq. (8.39) as

$$\Gamma_m = \frac{\sum_{i=1}^{N} W_i \phi_{im}}{\sum_{i=1}^{N} W_i \phi_{im}^2} \tag{23.5}$$

For normalized eigenvectors, the participation factor reduces to

$$\Gamma_m = \frac{1}{g} \sum_{i=1}^{N} \phi_{im} W_i \tag{23.6}$$

because for normalized eigenvectors,

$$\sum_{i=1}^{N} \frac{W_i}{g} \phi_{im}^2 = 1$$

where g is the acceleration due to gravity. Damping may be introduced in the modal Eq. (23.4) by simply adding the damping term to this equation, namely,

$$\ddot{z}_m + 2\xi_m \omega_m \dot{z}_m + \omega_m^2 z_m = -\Gamma_m \ddot{u}_s(t) \tag{23.7}$$

where ξ_m is the modal damping ratio. For convenience Eq. (23.7), can be written with omission of the participation factor as

$$\ddot{q}_m + 2\xi_m \omega_m \dot{q}_m + \omega_m^2 q_m = \ddot{u}_s(t) \tag{23.8}$$

with the substitution

$$z_m = -\Gamma_m q_m \tag{23.9}$$

23.1.2 Modal Shear Force

The value of the maximum response in Eq. (23.8) for the modal spectral acceleration, $S_{am} = (\ddot{q}_m)_{\max}$, is found from an appropriate response spectral chart.

From Eqs. (23.3) and (23.9), the maximum acceleration a_{xm} of the mth mode at the level x of the building is given by

$$a_{xm} = \Gamma_m \phi_{xm} S_{am} \tag{23.10}$$

in which S_{am} and a_{xm} are usually expressed in units of the gravitational acceleration g.

As stated in Chap. 5, the modal values of the spectral acceleration S_{am}, the spectral velocity S_{vm}, and the spectral displacement S_{dm} are related by an apparent harmonic relationship:

$$S_{am} = \omega_m S_{\text{vm}} = \omega_m^2 S_{dm}$$

or in terms of the modal period $T_m = 2\pi/\omega_m$ by

$$S_{am} = \frac{2\pi}{T_m} S_{vm} = \left(\frac{2\pi}{T_m}\right)^2 S_{dm}$$

On the basis of these relations, the modal spectral acceleration S_{am} in Eq. (23.10) may be replaced by the spectral displacement S_{dm} times ω_m^2 or by the spectral velocity S_{vm} times ω_m.

The modal lateral force F_{xm} at the level x of the building is then given by Newton's Law as

$$F_{xm} = a_{xm} W_x$$

or by Eq. (23.10) as

$$F_{xm} = \Gamma_m \phi_{xm} S_{am} W_x \tag{23.11}$$

in which S_{am} is the modal spectral acceleration in g units (Note: ASCE 7-16 requires to scale S_am multiplied by the importance factor, I_e (Table 23.1) and divided by response modifications coefficient, R (Table 23.2)) and W_x is the weight attributed to the level x of the building.

Table 23.1 Modal seismic force F_{xm} (Kip)

Level	Model 1	Model 2	Model 3	Model 4	Design F_x Values
4	46	−29	11	−6	56
3	47	5	−20	10	52
2	30	36	4	−19	51
1	12	24	14	23	38

Table 23.2 Modal torsional moment M_{txm} (Kip-ft)

Level x	Model 1	Model 2	Model 3	Model 4	Design M_{tx} values
4	221	−139	53	−31	268
3	448	−113	−41	14	464
2	590	62	−20	−77	598
1	649	178	46	31	675

The modal shear force V_{xm} at the level x of the building is equal to the sum of the seismic forces F_{xm} above that level, namely,

$$V_{xm} = \sum_{i=x}^{N} F_{im} \tag{23.12}$$

The total modal shear force V_m at the base of the building is then calculated as

$$V_m = \sum_{i=1}^{N} F_{im} \tag{23.13}$$

or using Eq. (23.11)

$$V_m = \sum_{i=1}^{N} \Gamma_m \phi_{im} W_i S_{am} \tag{23.14}$$

23.1.3 Effective Modal Weight

The effective modal weight W_m is defined by the equation

$$V_m = W_m S_{am} \tag{23.15}$$

Then, from Eq. (23.14), the modal weight is

$$W_m = \Gamma_m \sum_{i=1}^{N} \phi_{im} W_i \tag{23.16}$$

Combining Eqs. (23.5) and (23.16) results in the following important expression for the effective modal weight:

$$W_m = \frac{\left[\sum_{i=1}^{N} \phi_{im} W_i\right]^2}{\sum_{i=1}^{N} \phi_{im}^2 W_i} \tag{23.17}$$

It can be proven (Clough and Penzien 1975, pp. 559–560) analytically that the sum of the effective modal weights for all the modes of the building is equal to the total design weight of the building, that is,

$$\sum_{m=1}^{N} W_m = \sum_{i=1}^{N} W_i \tag{23.18}$$

Equation (23.18) is most convenient in assessing the number of significant modes of vibration to consider in the design. Specifically, the ASCE 7-16 (12.9.1.1) requires that, in applying the dynamic method of analysis, a sufficient number of modes are needed to estimate a combined modal mass participation of 100% of the structure's mass. Alternatively, this requirement can be satisfied by including a sufficient number of modes such that their total effective modal weight is at least 90% of the total design weight of the building. Thus, this requirement can be satisfied by simply adding a sufficient number of effective modal weights [Eq. (23.17)] until their total weight is 90% or more of the seismic design weight of the building.

23.1.4 Modal Lateral Forces

By combining Eq. (23.11) with Eqs. (23.15) and (23.16), we may express the modal lateral force F_{xm} as

$$F_{xm} = C_{xm} V_m \tag{23.19}$$

where the modal seismic coefficient C_{xm} at level x is given by

$$C_{xm} = \frac{\phi_{xm} W_x}{\sum_{i=1}^{N} \phi_{im} W_i} \tag{23.20}$$

23.1.5 Modal Displacements

The modal displacement δ_{xm} at the level x of the building may be expressed, in view of Eqs. (23.3) and (23.9), as

$$\delta_{xm} = \Gamma_m \phi_{xm} S_{dm} \tag{23.21}$$

where Γ_m is the participation factor for the mth mode, ϕ_{xm} is the component of the modal shape at level x of the building, and S_{dm} is the spectral displacement for that mode.

Alternatively, the modal displacement δ_{xm} may be calculated from Newton's Law of Motion in the form

$$F_{xm} = \frac{W_x}{g} \omega_m^2 \delta_{xm} \tag{23.22}$$

because the magnitude of the modal acceleration corresponding to the modal displacement δ_{xm} is ω_m^2 δ_{xm}. Hence, from Eq. (23.22)

$$\delta_{xm} = \frac{g}{\omega_m^2} \cdot \frac{F_{xm}}{W_x} \tag{23.23}$$

or substituting $\omega_m = 2\pi/T_m$

$$\delta_{xm} = \frac{g}{4\pi^2} \cdot \frac{T_m^2 F_{xm}}{W_x} \tag{23.24}$$

where T_m is the mth natural period. In accordance with ASCE 7-16, the value for displacement and drift quantities shall be multiplied by the quantity of C_d/I_e. The importance factor, Ie and the deflection amplification factor, C_d can be determined by Table 23.1 and 23.2, respectively.

23.1.6 Modal Drift

The modal drift $\boldsymbol{\Delta}_{xm}$ for the xth story of the building, defined as the relative displacement of two consecutive levels, is given by

$$\Delta_{xm} = \delta_{xm} - \delta_{(x-1)m} = < \Delta a \tag{23.25}$$

with $\delta_{0m} = 0$.

Allowable story drift is presented in Table 23.3 (ASCE 7-16: Table 12.12-1).

Table 23.3 Calculation of ratio of secondary to primary moment

Level x	Story weight W_x(Kip)	Above weight P_x (Kip)	Story drift Δ_x (in.)	Story shear V_x (Kip)	Story height H_x (in.)	$Ms/Mp = P_x\Delta_x/V_xH_x$
4	645.1	645.1	0.362	56	144	0.005
3	789.1	1434.2	0.625	97	144	0.012
2	789.1	2223.3	0.605	125	144	0.014
1	789.1	3012.4	0.441	141	144	0.012

23.1.7 Modal Overturning Moment

The modal overturning moment M_{xm} at the level x of the building which is calculated as the sum of the moments of the seismic forces F_{xm} above that level is given by

$$M_{xm} = \sum_{x+1}^{N} F_{im}(h_i - h_x) \tag{23.26}$$

where h_i and h_x are, respectively, the height of levels i and x.

The modal overturning moment M_m at the base of the building then is given by

$$M_m = \sum_{i=1}^{N} F_{im} h_i \tag{23.27}$$

23.1.8 Modal Torsional Moment

The modal torsional moment M_{tmx} at level x, which is due to eccentricity e_x between the center of the above mass and the center of stiffness at that level (measured normal to the direction considered), is calculated as

$$M_{tmx} = e_x V_{xm} \tag{23.28}$$

where V_{xm} is the modal shear force at level x.

As mentioned in Chap. 23, the ASCE 7-16 requires that an accidental torsional moment be added to the torsional moment existent at each level. The recommended way to add the accidental torsion is to offset the center of mass at each level by 5% of the dimension of the building normal to the direction under consideration.

23.2 Total Design Values

The design values for the base shear, story shear, lateral deflection, story drift, overturning moment and torsional moment are obtained by combining corresponding modal responses. Such combination has been performed by application of the technique SRSS. This technique, as mentioned in Chap. 8, estimates the maximum modal response by calculating the square root of the sum of the squared values of the modal contributions. However, as discussed in Sect. 8.6 of Chap. 8, the SRSS technique may result in relatively large errors when some of the natural frequencies are closely spaced. This situation generally occurs in the analysis of three-dimensional structures. At the present, the more refined technique described in that section, CQC (complete quadratic contribution), and the complete quadratic combination method as modified by ASCE 4 (CQC-4), or an approved equivalent approach (ASCE 7-18 Sect. 12.9.1.3), is becoming the technique of choice for implementation in computer programs. For preliminary or hand calculation, however, the simpler technique SRSS is commonly used. The following formulas may be used to estimate maximum design values by application of the SRSS technique:

1. Design Base Shear:

$$V = \sqrt{\sum_{m=1}^{N} V_m^2} \tag{23.29}$$

   where the modal base shear V_m is given by Eq. (23.15).

2. Design Lateral Seismic Force at Level x:

$$F_x = \sqrt{\sum_{m=1}^{N} F_{xm}^2} \tag{23.30}$$

   where the seismic modal force F_{xm} is given by Eq. (23.19).

3. Design Shear Force at Story x:

$$V_x = \sqrt{\sum_{m=1}^{N} V_{xm}^2} \tag{23.31}$$

   where the modal shear force V_{xm} at level x is given by Eq. (23.12).

4. Design Lateral Deflection at Level x:

$$\delta_x = \sqrt{\sum_{m=1}^{N} \delta_{xm}^2} \tag{23.32}$$

   where the modal displacement δ_{xm} at level x is given by Eq. (23.23) or Eq. (23.24).

5. Design Drift for Story x:

$$\Delta_x = \sqrt{\sum_{m=1}^{N} \Delta_{xm}^2} \tag{23.33}$$

where the modal drift Δ_{xm} at story x is given by Eq. (23.25).

6. Design Overturning Moment at Level x:

$$M_x = \sqrt{\sum_{m=1}^{N} M_{xm}^2} \tag{23.34}$$

where the modal overturning moment M_{xm} at level x is given by Eq. (23.26).

7. Design Torsional Moment at Level x:

$$M_{tx} = \sqrt{\sum_{m=1}^{N} M_{txm}^2} \tag{23.35}$$

where the modal torsional moment M_{tum} at level x is given by Eq. (23.28).

23.3 Scaling of Results

When the base shear force calculated by the dynamic method is less than that determined by the equivalent lateral force method, then for irregular buildings, the base shear calculated by dynamic analysis shall be scaled up to match 100% of the base shear determined by the static lateral force procedure. All corresponding response parameters, including member forces and moments, shall be adjusted proportionally.

The code also stipulates that the base shear for a given direction, determined using dynamic analysis, need not exceed the value obtained by the equivalent lateral force method. In this case, all corresponding response parameters are adjusted proportionately.

Illustrative Example 23.1
Model as a plane orthogonal frame the four-story reinforced building.

Solution:

The stiffness method and static condensation method to determine the reduce stiffness matrix that correponds to the lateral coordinate at the various levels of the building. This problem requires numbering the joints of the frame consecutively excluding the fixed joint at the foundation (labeled zero) as shown in Fig. 23.1. In this model each vertical or horizontal member represents all nine elements or members along the building. The flexural stiffness values EI for this model are calculated as follows:

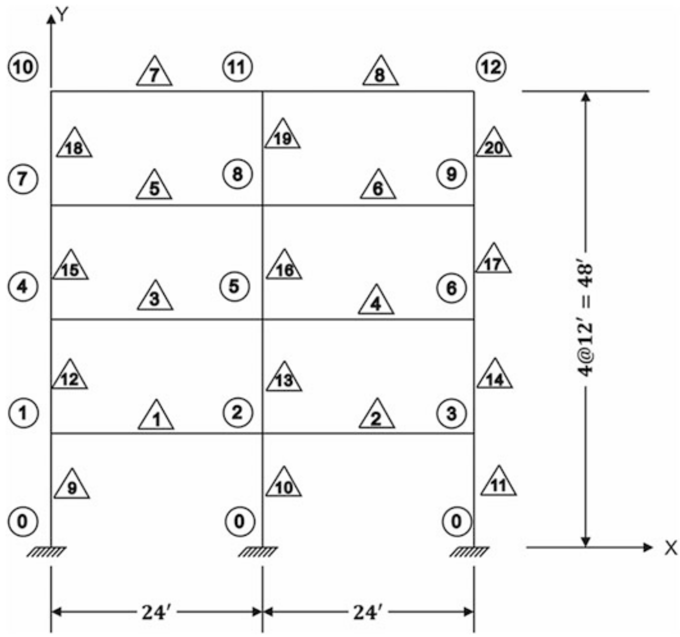

Fig. 23.1 Normalized response spectra shapes. (Reproduced from 1997 Uniform Building Code. © 1997, with permission of the publishers, the International Conference of Building Officials.)

Horizontal Members:

$$\begin{aligned} EI &= 9 \times 3000 \times 12 \times 24^3/12 \\ &= 0.37325 \times 10^9 \left(\text{kip-in}^2\right) \end{aligned}$$

Exterior Columns:

First and Second stories:

$$\begin{aligned} EI &= 9 \times 3000 \times 8000 \\ &= 0.216 \times 10^9 \left(\text{kip} - \text{in}^2\right) \end{aligned}$$

Third and Fourth stories:

$$\begin{aligned} EI &= 9 \times 3000 \times 4096 \\ &= 0.111 \times 10^9 \left(\text{kip} - \text{in}^2\right) \end{aligned}$$

Interior Columns:

First and Second stories:

$$\begin{aligned} EI &= 9 \times 3000 \times 13,824 \\ &= 0.373 \times 10^9 \left(\text{kip} - \text{in}^2\right) \end{aligned}$$

Third and Fourth stories:

$$\begin{aligned} EI &= 9 \times 3000 \times 8000 \\ &= 0.216 \times 10^9 \left(\text{kip} - \text{in}^2\right) \end{aligned}$$

Illustrative Example 23.2

Consider again the four-story reinforced concrete building. Model this building as a plane orthogonal frame and perform the seismic analysis: dynamic method.

Solution:
Seismic weights:

$$W_1 = W_2 = W_3 = 789.1 \text{ Kip}, \text{W}_4 = 645.1 \text{ Kip}$$

1. Modeling the Structure: This structure has been modeled as a plane orthogonal frame (using Program in Chaps. 13 and 9) in the solution of Illustrative Example 23.1. The reduced stiffness matrix in reference to the four lateral displacement coordinates of the building, from Example 23.1, is given by

$$[K] = \begin{bmatrix} 5611 & -2916 & 458 & -45 \\ -2916 & 3863 & -1885 & 269 \\ 458 & -1885 & 2893 & -1398 \\ -45 & 269 & -1398 & 1168 \end{bmatrix} (\text{Kip/in})$$

and the mass matrix (W_i/g) by

$$[M] = \begin{bmatrix} 2.036 & 0 & 0 & 0 \\ 0 & 2.036 & 0 & 0 \\ 0 & 0 & 2.036 & 0 \\ 0 & 0 & 0 & 1.671 \end{bmatrix} \left(\text{Kip} \cdot \sec^2/\text{in}\right)$$

2. Natural Periods and Modal Shapes: The natural frequencies and the normalized modal shapes are obtained by solving the eigenproblem

$$\left[[K] - \omega^2[M]\right]\{\phi\} = \{0\} \tag{e}$$

The roots of the corresponding characteristic equation

$$\left|[K] - \omega^2[M]\right| = \{0\} \tag{f}$$

are

$$\begin{aligned} \omega_1^2 &= 77.22, \omega_3^2 = 1939.75 \\ \omega_2^2 &= 678.60, \omega_4^2 = 4052.78 \end{aligned} \tag{g}$$

resulting in the natural frequencies ($f = \omega/2\pi$)

$$\begin{aligned} f_1 &= 1.40 \text{ cps}, \quad f_3 = 7.01 \text{ cps} \\ f_2 &= 4.14 \text{ cps}, \quad f_4 = 10.13 \text{ cps} \end{aligned} \tag{h}$$

or natural periods ($T = 1/f$)

$$\begin{aligned} T_1 &= 0.715 \sec, \quad T_3 = 0.143 \sec \\ T_2 &= 0.241 \sec, \quad T_4 = 0.099 \sec \end{aligned} \tag{i}$$

and the corresponding modal shapes arranged in the columns of the modal matrix are

$$[\Phi] = \begin{bmatrix} 0.11277 & -0.31074 & -0.35308 & 0.50518 \\ 0.27075 & -0.46790 & -0.11497 & -0.42860 \\ 0.43123 & -0.06907 & 0.50310 & 0.21286 \\ 0.51540 & 0.45452 & -0.34652 & -0.17766 \end{bmatrix} \tag{j}$$

3. Spectral Accelerations: The spectral accelerations (multiplied by the importance factor, Ie (Table 23.1) and divided by Response modifications coefficient, R (Table 23.2)) for the natural periods in Eq. (i) obtained from the spectral chart (Note: the procedure of estimations is not presented in this example. More detailed procedure is presented in the illustrative example 23.1)

$$\begin{aligned} S_{a1} &= 0.034\ g, S_{a3} = 0.061\ g \\ S_{a2} &= 0.063\ g, S_{a4} = 0.050\ g \end{aligned} \tag{k}$$

4. Effective Modal Weights: The effective modal weight is given by Eq. (23.17) as

$$W_m = \frac{\left[\sum_{i=1}^{N} \phi_{im} W_i\right]^2}{\sum_{i=1}^{N} \phi_{im}^2 W_i}$$

Values obtained for W_m ($m = 1, 2, 3, 4$) are shown in Table 23.4. This table also shows the effective modal weight as a percentage of the total seismic weight of the building.

Table 23.4 Modal effective weight and modal base shear

Mode m	Modal effective W_m (Kip)	Weight (%)	Modal base shear V_m (Kip)
1	2465	82	84
2	366	12	23
3	99	3	6
4	82	3	4
Total Weight = 3012 Kips			

5. Modal Base Shear: The modal base shear is given by Eq. (23.15) as

$$V_m = W_m S_{am}$$

Numerical values of V_m also are given in Table 23.4. The total base shear force given by Eq. (23.29) then is calculated from values in the last column of Table 23.4 as

$$V = \sqrt{(84)^2 + (23)^2 + (6)^2 + (4)^2} = 87.4 \quad \text{(Kip)}$$

6. Scaling Modal Effective Weight and Modal Base Shear: The modal values for the effective weight and for the base shear force in Table 23.4 are scaled up by the ratio r equal to 100% of the base shear determined by the equivalent lateral force method and the value for the base shear force calculated by the dynamic method. The base shear determined in Illustrative Example 23.2 using the equivalent lateral force method was equal to 140.75 Kip, while the value calculated for this example using the dynamic method is equal to 87.4 Kip. Therefore, the scaling ratio is

$$r = \frac{1.0 \times 140.75}{87.4} = 1.61$$

Table 23.5 shows the result of scaling (by the factor $r = 1.61$) the values in Table 23.4 for the modal effective weight W_m and the modal base shear V_m.

Table 23.5 Scaled values for modal effective weight and modal base shear

Mode m	Modal effective W_m (Kip)	Weight (%)	Modal base shear V_m (Kip)
1	3969	82	135
2	589	12	37
3	159	3	10
4	132	3	6
$\Sigma = 4849$ Kips			

7. Modal Seismic Force: The numerical values for the seismic coefficients C_{xm} and for the seismic lateral forces F_{xm} calculated from Eqs. (23.20) and (23.19) are shown respectively in Tables 23.1 and 23.6. The design seismic forces calculated by Eq. (23.30) are shown in the last column of Table 23.1.

Table 23.6 Modal seismic coefficient C_{xm}

Level	Model 1	Model 2	Model 3	Model 4
4	0.341	−0.780	1.141	−1.007
3	0.349	0.145	−2.027	1.476
2	0.219	0.983	0.463	−2.972
1	0.091	0.653	1.422	3.503

8. Model Shear Force: Values for the model shear force V_{xm} at level x calculated using Eq. (23.12) and results from Table 23.1 are given in Table 23.7. Design values for the story shear forces V_x calculated by Eq. (23.31) are given in the last column of Table 23.7. It should be noticed that the values shown in Table 23.7 for the modal shear force at level $x = 1$ are precisely the values for the base shear force calculated by Eq. (23.15) and shown in Table 23.5.

Table 23.7 Modal shear force V_{xm} (Kip)

Level x	Model 1	Model 2	Model 3	Model 4	Design V_x values
4	46	−29	11	−6	56
3	93	−24	−9	3	97
2	123	13	−4	−16	125
1	135	37	10	6	141

9. Modal Lateral Displacement: Values for the modal lateral displacement δ_{xm} at level x calculated from Eq. (23.24) are shown in Table 23.8. This table also shows in the last column design values for lateral displacements δ_x calculated by Eq. (23.32). In accordance with ASCE 7-16, the value for displacement and drift quantities shall be multiplied by the quantity of C_d/I_e. For this problem, the importance factor, I_e= 1 and the deflection amplification factor, C_d = 5.5.

Table 23.8 Modal lateral displacement δ_{xm} (in.)

Level	Model 1	Model 2	Model 3	Model 4	Design δ_x values
4	1.965	−0.140	0.019	−0.005	1.970
3	1.644	0.021	−0.027	0.006	1.644
2	1.032	0.144	0.006	−0.013	1.042
1	0.430	0.096	0.019	0.015	0.441

Note: In accordance with ASCE 7-16 Section 12.8.6, the deflection shall be adjusted by the deflection amplification factor and the importance factor. Herein, Cd =5.5 and Ie =1 are used in the calculation of design dx values.

10. Modal Story Drift: Table 23.9 shows the values for modal story drift Δ_{xm} calculated by Eq. (23.25). Design values for story drift obtained from Eq. (23.33) are given in the last column of this table. The maximum story drift permitted by the ASCE 7-16 (Table 23.3) should not exceed 3.6 in. (0.025 H_x), where H_x is the story height. The design values Δ_x calculated in Table 23.9 for this example are well below these limits for story drift. It should be noted that the scaling of drifts are not included in this problem. If the modal base shear (V_t) is less than C_sW, where C_s is determined in accordance with Eq. 23.16, the deflection shall be adjusted by the factor of C_sW/V_t.

Table 23.9 Modal story drift Δ_{xm} (in.)

Story	Model 1	Model 2	Model 3	Model 4	Design Δ_x values
4	0.321	−0.161	0.046	−0.012	0.362
3	0.612	−0.123	−0.033	0.019	0.625
2	0.602	0.048	−0.013	−0.028	0.605
1	0.430	0.096	0.019	0.015	0.441

11. Modal Overturning Moments: Table 23.10 shows the values for modal overturning moment calculated using Eq. (23.26). The last column of this table gives the design values for overturning moments M_x calculated by Eq. (23.34).

Table 23.10 Modal overturning moment M_{xm} (Kip-ft)

Level x	Model 1	Model 2	Model 3	Model 4	Design M_x values
4	–	–	–	–	–
3	553	−347	132	−78	671
2	1673	−629	30	−42	1788
1	3147	−475	−19	−235	3192
Base	4770	−30	97	−158	4774

12. Modal Torsional Moments: Table 23.2 shows the values calculated from Eq. (23.28) for the modal torsional moments assuming only accidental eccentricity e_x of 5% at each level x of the building (for this example $e_x = 0.05 \times 96$ ft $= 4.8$ ft). Design values for torsional moments M_{tx} calculated from Eq. (23.35) are shown in the last column of Table 23.2.

13. The P-Δ Effect: The ASCE 7-16 specifies that the P-Δ effect does not need to be considered when the ratio of the secondary moment $M_s = P_x\,\Delta_x$ to the overturning or primary moment M_p is less than 0.10 calculated for each level x of the building. If the modal lateral displacement in Table 23.8 is adjusted by the deflection amplification factor and the importance factor, the adjustment should be canceled out in this calculation. Therefore, the quantities shall be multiplied by the quantity of I_e/C_d. The results of the necessary calculations to evaluate this ratio are shown in Table 23.3. The largest moment ratio in Table 23.3 is 0.014, which is well below the code limit of 0.1. Consequently, there is no need to account for the P-Δ effect. In addition, the ratio shall not exceed the maximum value defined in ASCE 7-16 Sect. 12.8.7 (Eq. 12.8-17). This is not necessary in this problem. It is permitted to be conservatively 1.0.

23.4 Summary

The provisions of the ASCE 7-16 for earthquake-resistant design by the dynamic method require the modeling of the building as a discrete system with one coordinate (horizontal displacement) at each level of the building. Possible models include: (1) the shear building in which the horizontal diaphragms are assumed absolutely rigid. (2) the cantilever building in which the horizontal diaphragms are considered absolutely flexible, and (3) the plane orthogonal frame in which the stiffness of horizontal diaphragms is considered as part of the effect of horizontal members of the frame. The implementation of the dynamic method also requires the solution of the corresponding eigenproblem for the modeled structure to determine its natural periods and modal shapes.

The maximum response of the modal equations is then obtained from a spectral chart such as the one provided by the code. Modal seismic forces and modal response in terms of shear forces, overturning moments, torsional moments, lateral displacements, and story drifts are determined at each level of the building. The final design values are calculated using the SRSS technique or the CQC technique to combine the modal contributions, and the complete quadratic combination method as modified by ASCE 4 (CQC-4), or an approved equivalent approach.

The modal method in which the modal responses are combined is valid while the structure remains in linear elastic behavior as expected when subjected to an earthquake of moderate intensity. When subjected to a strong earthquake, the structure will deform in the inelastic range producing plastic deformations and structural damage. However, its ductility will provide a mechanism to absorb energy and structural stability will be maintained, although the structure may continue to undergo large deflections which may create further damage.

23.5 Problems

Problem 23.1
Perform the seismic resistant design of the 10-story building of Problem 23.1 using the dynamic method.

Problem 23.2
Solve Problem 23.2 using the dynamic method of the ASCE 7-16. The total length of the building is 120 ft and its width 24 ft. Consider only the accidental torsional eccentricity.