



# Evaluation of Absolute Damping from Modal Damping Ratios

# 20

In the previous chapters, we determined the natural frequencies and modal shapes for undamped structures when modeled as shear buildings. We also determined the response of these structures using the modal superposition method. In this method, as we have seen, the differential equations of motion are uncoupled by means of a transformation of coordinates that incorporates the orthogonality property of the modal shapes.

The consideration of damping in the dynamic analysis of structures complicates the problem. Not only will the differential equations of motion have additional terms due to damping forces, but the uncoupling of the equations will be possible only by imposing some restrictions or conditions on the functional expression for the damping coefficients.

The damping normally present in structures is relatively small and practically does not affect “the calculation of natural frequencies and modal shapes of the system. Hence, the effect of damping is neglected in determining the natural frequencies and” modal shapes of the structural systems. Therefore, in practice, the eigenproblem for the damped structure is solved by using the same methods employed for undamped structures.

## 20.1 Equations for Damped Shear Building

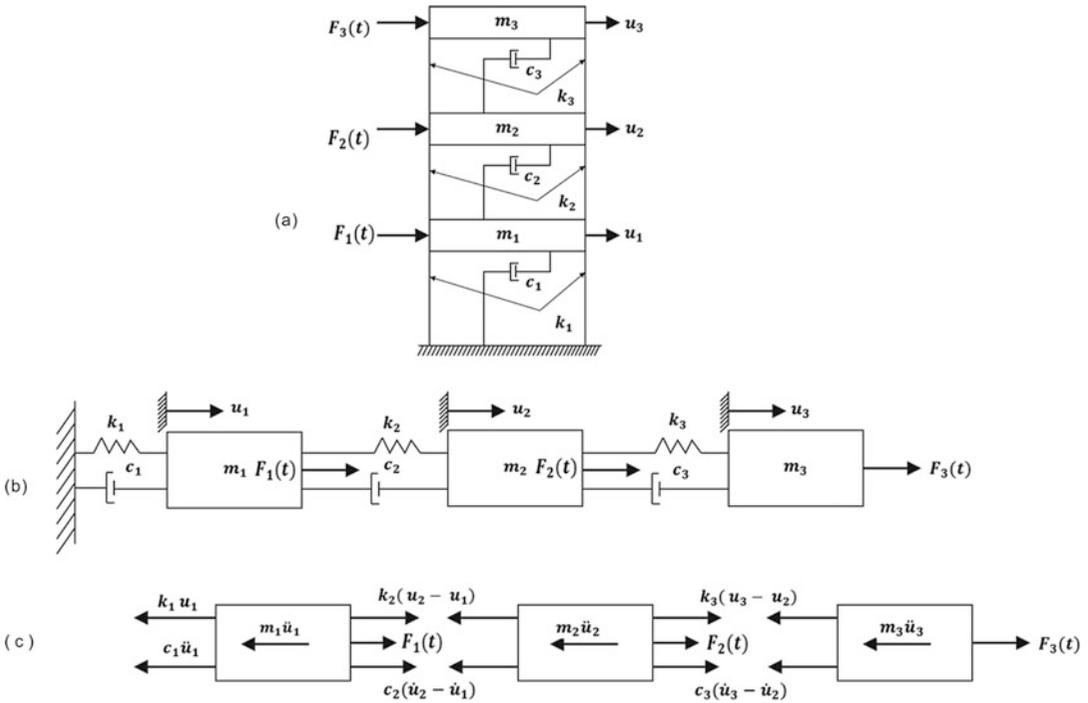
For a viscously damped shear building, such as the three-story building shown in Fig. 20.1, the equations of motion obtained by summing forces in the corresponding free body diagrams are

$$\begin{aligned} m_1 \ddot{u}_1 + c_1 \dot{u}_1 + k_1 u_1 - c_2 (\dot{u}_2 - \dot{u}_1) - k_2 (u_2 - u_1) &= F_1(t) \\ m_2 \ddot{u}_2 + c_2 (\dot{u}_2 - \dot{u}_1) + k_2 (u_2 - u_1) - c_3 (\dot{u}_3 - \dot{u}_2) - k_3 (u_3 - u_2) &= F_2(t) \\ m_3 \ddot{u}_3 + c_3 (\dot{u}_3 - \dot{u}_2) + k_3 (u_3 - u_2) &= F_3(t) \end{aligned} \quad (20.1)$$

These equations may be conveniently written in matrix notation as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \quad (20.2)$$

where the matrices and vectors are as previously defined, except for the damping matrix  $[C]$ , which is given by



**Fig. 20.1** (a) Damped shear building. (b) Mathematical model. (c) Free body diagram

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \tag{20.3}$$

In the next section, we shall establish the conditions under the damped equations of motion may be transformed to an uncoupled set of independent equations.

## 20.2 Uncoupled Damped Equations

To solve the differential equations of motion, Eq. (20.2), we seek to uncouple these equations. We, therefore, introduce the transformation of coordinates

$$\{u\} = [\Phi]\{z\} \tag{20.4}$$

where  $[\Phi]$  is the modal matrix obtained in the solution of the undamped free-vibration system. The substitution of Eq. (20.4) and its derivatives into Eq. (20.2) leads to

$$[M][\Phi]\{\ddot{z}\} + [C][\Phi]\{\dot{z}\} + [K][\Phi]\{z\} = \{F(t)\} \tag{20.5}$$

Premultiplying Eq. (20.5) by the transpose of the  $n$ th modal vector  $\{\phi\}_n^T$  yield

$$\{\phi\}_n^T [M][\Phi]\{\ddot{z}\} + \{\phi\}_n^T [C][\Phi]\{\dot{z}\} + \{\phi\}_n^T [K][\Phi]\{z\} = \{\phi\}_n^T \{F(t)\} \tag{20.6}$$

It is noted that the orthogonality property of the modal shapes,

$$\begin{aligned}\{\phi\}_n^T[M]\{\phi\}_m &= 0, m \neq n \\ \{\phi\}_n^T[K]\{\phi\}_m &= 0\end{aligned}\quad (20.7)$$

causes all components except the  $n$ th mode in the first and third terms of Eq. (20.6) to vanish. A similar reduction is *assumed* to apply to the damping term in Eq. (20.6), that is, if it is assumed that

$$\{\phi\}_n^T[C]\{\phi\}_m = 0, \quad n \neq m \quad (20.8)$$

then the coefficient of the damping term in Eq. (20.6) will reduce to

$$\{\phi\}_n^T[C]\{\phi\}_n$$

In this case Eq. (20.6) may be written as

$$M_n \ddot{z}_n + C_n \dot{z}_n + K_n z_n = F_n(t)$$

or alternatively as

$$\ddot{z}_n + 2\xi_n \omega_n \dot{z}_n + \omega_n^2 z_n = F_n(t)/M_n \quad (20.9)$$

in which case

$$M_n = \{\phi\}_n^T[M]\{\phi\}_n \quad (20.10a)$$

$$K_n = \{\phi\}_n^T[K]\{\phi\}_n = \omega_n^2 M_n \quad (20.10b)$$

$$C_n = \{\phi\}_n^T[C]\{\phi\}_n = 2\xi_n \omega_n M_n \quad (20.10c)$$

$$F_n(t) = \{\phi\}_n^T\{F(t)\} \quad (20.10d)$$

The normalization discussed previously (Sect. 7.2)

$$\{\phi\}_n^T[M]\{\phi\}_n = 1 \quad (20.11)$$

will result in

$$M_n = 1$$

so that Eq. (20.9) reduces to

$$\ddot{z}_n + 2\xi_n \omega_n \dot{z}_n + \omega_n^2 z_n = F_n(t) \quad (20.12)$$

which is a set of  $N$  uncoupled differential equations ( $n = 1, 2, \dots, N$ ).

### 20.3 Conditions for Damping Uncoupling

In the derivation of the uncoupled damped Eq. (20.12), it has been assumed that the normal coordinate transformation, Eq. (20.4), that serves to uncouple the inertial and elastic forces also uncouples the damping forces. It is of interest to consider the conditions under which this uncoupling will occur, that is, the form of the damping matrix  $[C]$  to which Eq. (20.8) applies.

When the damping matrix is of the form

$$[C] = a_0[M] + a_1[K] \quad (20.13)$$

in which  $a_0$  and  $a_1$  are arbitrary proportionality factors, the orthogonality condition will be satisfied. This may be demonstrated by applying the orthogonality condition to Eq. (20.13), that is, premultiplying both sides of this equation by the transpose of the  $n$ th mode  $\{\phi\}_n^T$  and postmultiplying by the modal matrix  $[\Phi]$ . We obtain

$$\{\phi\}_n^T [C] [\Phi] = a_0 \{\phi\}_n^T [M] [\Phi] + a_1 \{\phi\}_n^T [K] [\Phi] \quad (20.14)$$

The orthogonality conditions, Eq. (20.7), then reduce Eq. (20.14) to

$$\{\phi\}_n^T [C] [\Phi] = a_0 \{\phi\}_n^T [M] \{\phi\}_n + a_1 \{\phi\}_n^T [K] \{\phi\}_n$$

or, by Eqs. (20.10), to

$$\{\phi\}_n^T [C] [\Phi]_n = a_0 M_n + a_1 M_n \omega_n^2$$

$$\{\phi\}_n^T [C] [\Phi]_n = (a_0 + a_1 \omega_n^2) M_n$$

which shows that, when the damping matrix is of the form of Eq. (20.13), the damping forces are also uncoupled with the transformation Eq. (20.4). However, it can be shown that there are other matrices formed from the mass and stiffness matrices which also satisfy the orthogonality condition. In general, damping matrix may be of the form

$$[C] = [M] \sum_i a_i ([M])^{-1} [K]^i \quad (20.15)$$

where  $i$  can be anywhere in the range  $-\infty < i < \infty$  and the summation may include as many terms as desired. The damping matrix, Eq. (20.13), can obviously be obtained as a special case of Eq. (20.15). By taking two terms corresponding to  $i = 0$  and  $i = 1$  in Eq. (20.15), we obtain the damping matrix expressed by Eq. (20.13). With this form of the damping matrix it is possible to compute the damping coefficients necessary to provide uncoupling of a system having any desired damping ratios in any specified number of modes. For any mode  $n$ , the modal damping is given by Eq. (20.10c), that is

$$C_n = \{\phi\}_n^T [C] \{\phi\}_n = 2\xi_n \omega_n M_n$$

If  $[C]$  as given by Eq. (20.15) is substituted in the expression for  $C_n$ , we obtain

$$C_n = \{\phi\}_n^T [M] \sum_i a_i ([M]^{-1} [K])^i \{\phi\}_n \quad (20.16)$$

Now, using Eq. (7.24) ( $[K] \{\phi\}_n = \omega_n^2 [M] \{\phi\}_n$ ) and performing several algebraic operations, we can show (Clough and Penzien 1975, p. 195) that the damping coefficient associated with any mode  $n$  may be written as

$$C_n = \sum_i a_i \omega_n^{2i} M_n = 2\xi \omega_n M_n \quad (20.17)$$

from which

$$\xi_n = \frac{1}{2\omega_n} \sum_i a_i \omega_n^{2i} \quad (20.18)$$

Equation (20.18) may be used to determine the constants  $a_i$ , for any desired values of modal damping ratios corresponding to any specified numbers of modes. For example, to evaluate these constants specifying the first four modal damping ratios  $\xi_1, \xi_2, \xi_3, \xi_4$  we may choose  $i = 1, 2, 3, 4$ . In this case Eq. (20.18) gives the following system of equations:

$$\begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \omega_1 & \omega_1^3 & \omega_1^5 & \omega_1^7 \\ \omega_2 & \omega_2^3 & \omega_2^5 & \omega_2^7 \\ \omega_3 & \omega_3^3 & \omega_3^5 & \omega_3^7 \\ \omega_4 & \omega_4^3 & \omega_4^5 & \omega_4^7 \end{Bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \quad (20.19)$$

In general, Eq. (20.19) may be written symbolically as

$$\{\xi\} = \frac{1}{2}[Q]\{a\} \quad (20.20)$$

where  $[Q]$  is a square matrix having different powers of the natural frequencies. The solution of Eq. (20.20) gives the constants  $\{a\}$  as

$$\{a\} = 2[Q]^{-1}\{\xi\} \quad (20.21)$$

Finally the damping matrix is obtained after the substitution of Eqs. (20.21) into (20.15).

It is interesting to observe from Eq. (20.18) that in the special case when the damping matrix is proportional to the mass  $\{C\} = a_0 [M]$  ( $i = 0$ ), the damping ratios are inversely proportional to the natural frequencies; thus the higher modes of the structures will be given very little damping. Analogously, when the damping is proportional to the stiffness matrix ( $\{C\} = a_1 [K]$ ), the damping ratios are directly proportional to the corresponding natural frequencies, as can be seen from Eq. (20.18) evaluated for  $i = 1$ ; and in this case the higher modes of the structure will be very heavily damped.

### Illustrative Example 20.1

Determine the absolute damping coefficients for the structure presented in Example 7.1. Assume 10% of the critical damping for each mode.

Solution:

From Example 7.1, we have the following information.

Natural frequencies:

$$\begin{aligned} \omega_1 &= 11.83 \text{ rad/sec} \\ \omega_2 &= 32.89 \text{ rad/sec} \end{aligned} \quad (a)$$

Modal matrix:

$$[\Phi] = \begin{bmatrix} 1.00 & 1.00 \\ 1.26 & -1.63 \end{bmatrix} \quad (b)$$

Mass matrix:

$$[M] = \begin{bmatrix} 136 & 0 \\ 0 & 66 \end{bmatrix}$$

Stiffness matrix:

$$[K] = \begin{bmatrix} 75,000 & -44,300 \\ -44,300 & 44,300 \end{bmatrix}$$

Using Eqs. (20.18) and (20.19) with  $i = 0, 1$  to calculate the constants  $a_i$  needed in Eq. (20.15), we obtain the following system of equations:

$$\begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 11.83 & (11.83)^3 \\ 32.89 & (32.89)^3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

Solving this system of equations gives

$$\begin{aligned} a_1 &= 0.01851 \\ a_2 &= -0.00001146 \end{aligned}$$

We also calculate

$$[M]^{-1} = \begin{bmatrix} 0.007353 & 0 \\ 0 & 0.01515 \end{bmatrix}$$

and

$$[M]^{-1}[K] = \begin{bmatrix} 551475 & -325738 \\ -671145 & 671145 \end{bmatrix}$$

Then

$$\begin{aligned} \sum_{i=1}^2 a_i \left( [M]^{-1}[K] \right)^i &= 0.0185 \begin{bmatrix} 551475 & -325738 \\ -671145 & 671145 \end{bmatrix} \\ &\quad - 0.00001146 \begin{bmatrix} 551.475 & -325.738 \\ -671.145 & 671.145 \end{bmatrix}^2 \\ &= \begin{bmatrix} 4.2172 & -1.4654 \\ -3.0193 & 4.7556 \end{bmatrix} \end{aligned}$$

Finally, substituting this matrix into Eq. (20.15) yields the damping matrix as

$$[C] = \begin{bmatrix} 136 & 0 \\ 0 & 66 \end{bmatrix} \begin{bmatrix} 4.2172 & -1.4654 \\ -3.0193 & 4.7556 \end{bmatrix} = \begin{bmatrix} 573.5 & -199.3 \\ -199.3 & 313.9 \end{bmatrix}$$

There is yet a second method for evaluating the damping matrix corresponding to any set of specified modal damping ratio. The method may be explained starting with the relationship

$$[A] = [\Phi]^T [C] [\Phi] \quad (20.22)$$

where the matrix  $[A]$  is defined as

$$[A] = \begin{bmatrix} 2\xi_1\omega_1 M_1 & 0 & 0 & - \\ 0 & 2\xi_2\omega_2 M_2 & 0 & - \\ 0 & 0 & 2\xi_3\omega_3 M_3 & - \\ - & - & - & - \end{bmatrix} \quad (20.23)$$

in which the modal masses  $M_1, M_2, M_3, \dots$  are equal to one if the modal matrix  $\{\Phi\}$  has been normalized. It is evident that the damping matrix  $[C]$  may be evaluated by postmultiplying and premultiplying Eq. (20.22) by the inverse of the modal matrix  $[\Phi]^{-1}$  and its inverse transpose  $[\Phi]^{-T}$ , such that

$$[C] = [\Phi]^{-T} [A] [\Phi]^{-1} \quad (20.24)$$

Therefore, for any specified set of modal damping ratios  $\{\xi\}$ , matrix  $[A]$  can be evaluated from Eq. (20.23) and the damping matrix  $[C]$  from Eq. (20.24). However, in practice, the inversion of the modal matrix is a large computational effort. Instead, taking advantage of orthogonality properties of the mode shapes, we can deduce the following expression for the system damping matrix:

$$[C] = [M] \left( \sum_{n=1}^N \frac{2\xi_n \omega_n}{M_n} \{\Phi\}_n \{\Phi\}_n^T \right) [M] \quad (20.25)$$

Equation (20.25) may be obtained from the condition of orthogonality of the normal modes given by Eq. (7.35) as

$$[I] = [\Phi]^{-T} [M] [\Phi] \quad (20.26)$$

Postmultiplying Eq. (20.26) by  $[\Phi]^{-1}$  we obtain

$$[\Phi]^{-1} = [\Phi]^{-T} [M] \quad (20.27)$$

Applying the transpose operation to Eq. (20.27) results in

$$[\Phi]^{-T} = [M] [\Phi] \quad (20.28)$$

in which  $[M] = [M]^T$  since the mass matrix  $[M]$  is a symmetric matrix.

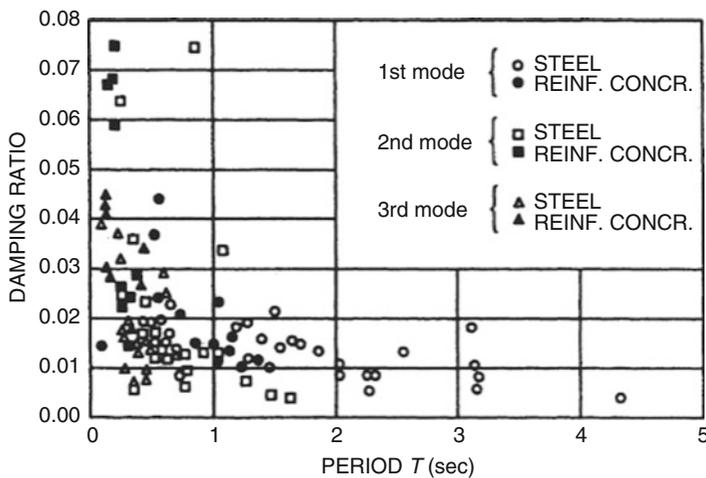
Finally, the substitution into Eq. (20.24) of Eqs. (20.27) and (20.28) gives

$$[C] = [M] [\Phi] [A] [\Phi]^T [M]$$

which results in Eq. (20.25) after substituting matrix  $[A]$  from Eq. (20.23). The damping matrix  $[C]$  obtained from Eq. (20.25) will satisfy the orthogonality property and, therefore, the damping term in the differential Eq. (20.2) will be uncoupled with the same transformation, Eq. (20.4), which serves to uncouple the inertial and elastic forces.

It is of interest to note in Eq. (20.25) that the contribution to the damping matrix of each mode is proportional to the modal damping ratio; thus any undamped mode will contribute nothing to the damping matrix.

We should mention at this point the circumstances under which it will be desirable to evaluate the elements of the damping matrix, as Eq. (20.15) or Eq. (20.25). It has been stated that absolute structural damping is a rather difficult quantity to determine or even to estimate. However, modal damping ratios may be estimated on the basis of past experience. This past experience includes laboratory determination of damping in different materials, as well as damping values obtained from vibration tests in existing buildings and other structures. Numerical values for damping ratios in structures are generally in the range of 1–10%. These values depend on the type of structure, materials utilized, nonstructural elements, etc. They also depend on the period and the magnitude of vibration. It has also been observed that damping ratios corresponding to higher modes have increasing values. Figure 20.2 shows the values of damping ratios measured in existing buildings as reported by H. Aoyama (1980) [in Wakabayashi (1986)]. It may be observed from this figure that experimental values for damping scatter over a wide range, and that it is difficult to give definite recommendations. The scatter observed in Fig. 20.2 is typical for experiments conducted to determine damping. On this basis, the obvious conclusion should be that the assumption of viscous damping to represent damping does not describe the real mechanism of energy dissipation in structural dynamics. However, for analytical expediency and also because of the uncertainties involved in attempting other formulation of damping, we still accept the assumption of viscous damping. At this time, the best recommendation that can be given in regard to damping is to use conservative values, 1–2% for steel buildings and 3–5% for reinforced concrete buildings for the fundamental frequency, and to assume damping ratios for the higher modes to increase in proportion to the natural frequencies. Thus, on the basis of giving some consideration to the type of structure, we assign numerical values to the damping ratios in all the modes of interest. These values are then used directly in the modal equations or they are used to determine the damping matrix that is needed when the dynamic response is obtained by some analytical method other than modal superposition, e.g., the time history response of a linear or nonlinear system.



**Fig. 20.2** Damping ratios measured in existing buildings. [H. Aoyama in Wakabayashi (1986)]

**Illustrative Example 20.2**

Determine the damping matrix of Example 20.1 using the method based on Eq. (20.22).

Solution:

From Example 7.2 we have the natural frequencies

$$\omega_1 = 11.83 \text{ rad/sec}$$

$$\omega_2 = 32.89 \text{ rad/sec}$$

and the mass matrix

$$[M] = \begin{bmatrix} 136 & 0 \\ 0 & 66 \end{bmatrix}$$

To determine  $[C]$ , we could use either Eq. (20.24) or (20.25). From Example 7.2, the normalized modal matrix is

$$[\Phi] = \begin{bmatrix} 0.06437 & 0.567 \\ 0.0813 & -0.0924 \end{bmatrix}$$

and its inverse by Eq. (20.27) is

$$[\Phi]^{-1} = \begin{bmatrix} 8.752 & 5.370 \\ 7.700 & -6.097 \end{bmatrix}$$

Substituting into Eq. (20.23), we obtain

$$2\xi_1\omega_1M_1 = (2)(0.1)(11.83)(1) = 2.366$$

$$2\xi_2\omega_2M_2 = (2)(0.1)(32.89)(1) = 6.578$$

Then by Eq. (20.24)

$$[C] = \begin{bmatrix} 8.752 & 7.700 \\ 5.370 & -6.097 \end{bmatrix} \begin{bmatrix} 2.366 & 0 \\ 0 & 6.578 \end{bmatrix} \begin{bmatrix} 8.752 & 7.700 \\ 5.370 & -6.097 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 572 & -198 \\ -198 & 313 \end{bmatrix}$$

which in this case of equal damping ratios in all the models checks with the damping matrix obtained in Example 20.1 for the same structure using Eq. (20.15).

The MATLAB program can be used to determine damping matrix.

```

clc
clear all
close all

%
% Inputs:
% M, K
% F = forcing function
% t = Time period
% u0 = initial displacement
% v0 = initial velocity
%

%%%-GIVEN VALUES-%%

%%Time for Response
t = 0:0.01:10;

%%Define Mass Matrix
M = [136 0; 0 66]

%%Define Stiffness Matrix
k1=30700;
k2=44300;

K = [k1+k2 -k2;
     -k2 k2];

%%Determine #s of DOFs
[n,n]= size(M);

%
% Solve the eigenvalue problem and normalized eigenvectors
%

%%Solve for eigenvalues (D) and eigenvectors (a)
[a, D] = eig(K, M)

%%Natural Frequencies
[omegas,k] = sort(sqrt(diag(D)));

%%Eigenvectors
a = a(:,k)

%%Natural Periods
T = 2*pi./omegas;

%%aMa = {a}'*[M]*(a)
aMa = diag(a'*M*a)

%%Normalized modal matrix
nom_phi = (a)*inv(sqrt(diag(aMa)))

%
% Damping matrix using the proportional damping matrix
% [C] = a0[M]+a1[K] (Eq. 8.55)
% zetas = damping ratios
%

%%Define damping ratio for each mode
zetas = zeros(n,1);
zetas(1)=0.1;
zetas(2)=0.1;

Q= [omegas(1) omegas(1)^3;
     omegas(2) omegas(2)^3]; %Eq.20.20

a =2*inv(Q)*zetas; %Eq.20.21

aMK = a(1)*(inv(M)*K)+a(2)*(inv(M)*K)^2; %Sum(a(M^-1)(K))^i

C = M*aMK %Eq.20.15

nom_C = nom_phi'*(C)*nom_phi; %Eq.20.25
zetas = diag((1/2)*nom_C*inv(diag(omegas)));

```

The MATLAB program prints out the damping matrix of the illustrative Example 20.2.

$$[C] = \begin{bmatrix} 573.5 & -199.3 \\ -199.3 & 313.9 \end{bmatrix}$$

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## 20.4 Summary

The most common method of taking into account the dissipation of energy in structural dynamics is to assume in the mathematical model the presence of damping forces of magnitudes that are proportional to the relative velocity and of directions opposite to the motion. This type of damping is known as viscous damping because it is the kind of damping that will be developed by motion in an ideal viscous fluid. The inclusion of this type of damping in the equations does not alter the linearity of the differential equations of motion. Since the amount of damping commonly present in structural systems is relatively small, its effect is neglected in the calculation of natural frequencies and mode shapes. However, to uncouple the damped differential equations of motion, it is necessary to impose some restrictions on the values of damping coefficients in the system. These restrictions are of no consequence owing to the fact that in practice it is easier to determine or to estimate modal damping ratios rather than absolute damping coefficients. In addition, when solving the equations of motion by the modal superposition method, only damping ratios are required. When the solution is sought by other methods, the absolute value of the damping coefficients may be calculated from modal damping ratios by one of the two methods presented in this chapter.

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## 20.5 Problems

### Problem 20.1

The stiffness and mass matrices for a certain two-degree-of-freedom structure are

$$[K] = \begin{bmatrix} 400 & -200 \\ -200 & 200 \end{bmatrix}, [M] = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Determine the damping matrix for this system corresponding to 20% of the critical damping for the first mode and 10% for the second mode. Use the method based on Eqs. (20.16) and (20.17).

### Problem 20.2

Repeat Problem 20.1 using the method based on Eqs. (20.22) and (20.25).

### Problem 20.3

The natural frequencies and corresponding normal modes (arranged in the modal matrix) for the three-story shear building shown in Fig. P20.3 are  $\omega_1 = 9.31$  rad/sec,  $\omega_2 = 20.94$  rad/sec,  $\omega_3 = 29.00$  rad/sec, and

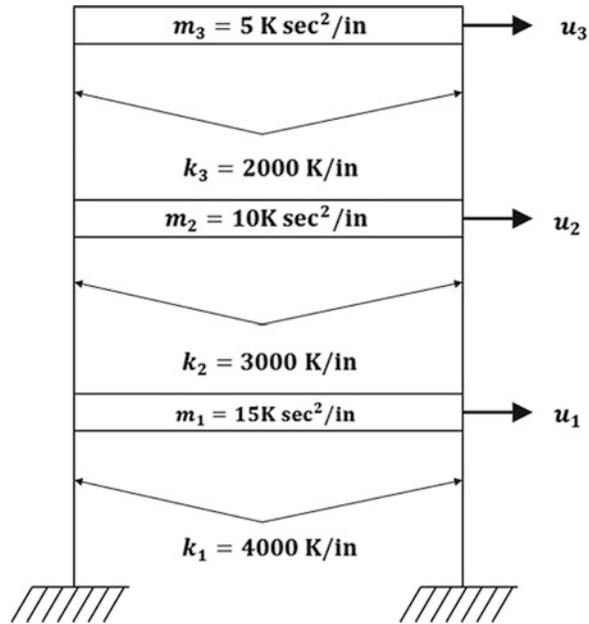


Fig. P20.3

$$[\Phi] = \begin{bmatrix} 0.1114 & -0.1968 & -0.1245 \\ 0.2117 & -0.0277 & 0.2333 \\ 0.2703 & 0.2868 & -0.2114 \end{bmatrix}$$

Determine the damping matrix for the system corresponding to damping ratios of 10% for all the modes.

#### Problem 20.4

Repeat Problem 20.3 for damping ratios of 20% for all the modes.

#### Problem 20.5

Repeat Problem 20.3 for the following value of modal damping ratios:

$$\xi_1 = 0.2, \quad \xi_2 = 0.1, \quad \xi_3 = 0.0$$

#### Problem 20.6

Use Program 7 and 8 to model and determine the natural frequencies and normal modes for the five-story shear building shown in Fig. P20.6; then use Program 11 to determine the damping matrix corresponding to an 8% damping ratio in all the modes.

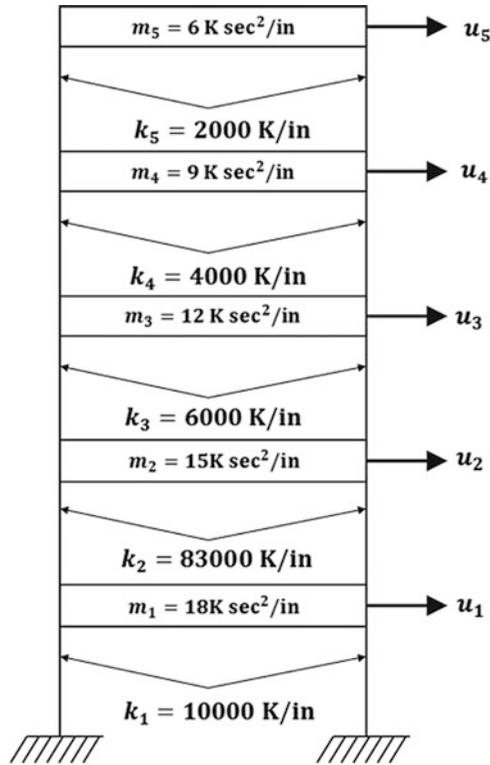


Fig. P20.6

**Problem 20.7**

Repeat Problem 20.6 for the following values of the modal damping ratios:

$$\xi_1 = 0.20, \quad \xi_2 = 0.15, \quad \xi_3 = 0.10, \quad \xi_4 = 0.05, \quad \xi_5 = 0,$$