7
Hubble's Law and the Expanding Universe

In the early 1900s, Vesto M. Slipher of the Lowell observatory in Arizona analyzed the spectra of many spiral nebulae. He found that most of them mysteriously had spectral lines that were red-shifted, indicating that they were moving away from the Earth, some at speeds of up to 1000 km/s. Motion at high speed is not uncommon in the cosmos—the Sun, for example, orbits around the center of our Galaxy at 300 km/s. The puzzling thing about Slipher's result was that the nebulae conspired to move predominantly away from us, as if in a display of some cosmic rejection (Fig. 7.1).

Hubble set out to investigate Slipher's curious findings. He started by measuring distances to an extended sample of nebulae, now recognized as galaxies. Unfortunately, Cepheids were too faint to be observed in all but the nearest galaxies, so Hubble had to find a new standard candle. He noticed that the brightest stars, in those galaxies whose distances he could measure (using Cepheids), had about the same luminosity, so he used them as standard candles, extending the cosmic distance ladder. In the meantime, Hubble's assistant Milton Humason extended Slipher's redshift measurements to a larger set of galaxies. Hubble then plotted the redshifts obtained by Slipher and Humason versus his distance estimates. He published his findings in 1929—and our view of the universe has never been the same (Fig. 7.2).

D. Perlov and A. Vilenkin, *Cosmology for the Curious,*
DOI 10.1007/978-3-319-57040-2_7

Fig. 7.1 Vesto Slipher undertook the painstaking task of obtaining spectra for various spiral nebulae because he wanted to understand the origin of the Sun and planets. At the time, it was commonly thought that spiral nebulae might be other Solar systems in the process of forming

7.1 An Expanding Universe

Hubble uncovered a very simple relation between the speed at which a galaxy moves away from us and the distance to the galaxy[1]: the speed grows proportionally to the distance. The further away the galaxy is, the greater is its speed. If you double the distance, the speed is also doubled. This is the celebrated Hubble law (Fig. 7.3).

Mathematically, the Hubble law can be stated as follows

$$v = H_0 d \tag{7.1}$$

[1]More precisely, Hubble uncovered a linear relation between the redshift of a galaxy and its distance. The redshift is then converted to a recession velocity.

Fig. 7.2 Edwin Hubble (1889–1953) made some of the most important discoveries in modern astronomy. He showed that our Milky Way is only one of a multitude of galaxies scattered throughout the cosmos. His greatest achievement though was the discovery of the expansion of the universe. After graduating from Law school at Oxford University, and a short stint practicing Law and then teaching at a high school, Hubble obtained a Ph.D. in Astronomy at the University of Chicago in 1917. He was offered a job at Mt Wilson Observatory, which he took up only after first enlisting in the US Army to fight Germany. Hubble was a talented athlete, excelling in track and boxing amongst other sports. Had he not died suddenly from a stroke at the age of 63, he most probably would have been awarded a Nobel Prize, something that was impossible earlier in his career, as astronomers were then not eligible. *Credit* Hale Observatories, courtesy AIP Emilio Segre Visual Archives

where v is the galaxy's velocity, and d is its distance. The constant of proportionality is called the Hubble parameter; its numerical value is[2]

$$H_0 = 2.2 \times 10^{-18} \mathrm{s}^{-1}. \tag{7.2}$$

[2]Because of uncertainties in distance measurements, it took scientists more than half a century to converge on this value: Hubble's original estimate was 4 times higher.

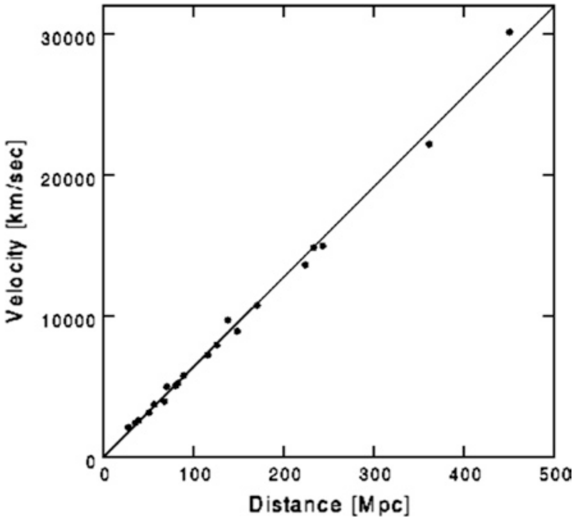

Fig. 7.3 Hubble's law with data from the High Redshift Supernova team (1996). Recession velocity is plotted versus the distance, measured in megaparsecs (Mpc) (1 Mpc $\approx 3 \times 10^6$ light years). *Credit* Ned Wright (UCLA) using data from Riess, Press & Kirshner (1996, astro-ph/9604143)

At first sight it may appear that the Hubble law implies that we are located right at the center of some gigantic explosion. But the work of Friedmann and Lemaitre demonstrated that cosmic expansion need not have a center. In a homogeneous and isotropic expanding universe, all observers see the surrounding galaxies recede. Moreover, it is not difficult to understand that they must recede according to the Hubble law.

Once again, we can picture an expanding universe using the rubber sheet analogy (see Fig. 7.4). The sheet is uniformly stretched in both directions, and the dots on the surface of the sheet represent galaxies. Suppose, for the sake of argument, that the sheet has been stretched to twice its original size in one second. The dots that were initially 1 cm apart are now 2 cm apart, so they separated at the speed of 1 cm/s. At the same time, the dots that were 2 cm apart are now 4 cm apart and therefore separated at 2 cm/s—twice the speed of the first pair of dots. You can easily convince yourself that the speed at which any two dots separate is proportional to the distance between them. But this is precisely the Hubble law. It thus appears that we live in an expanding universe.

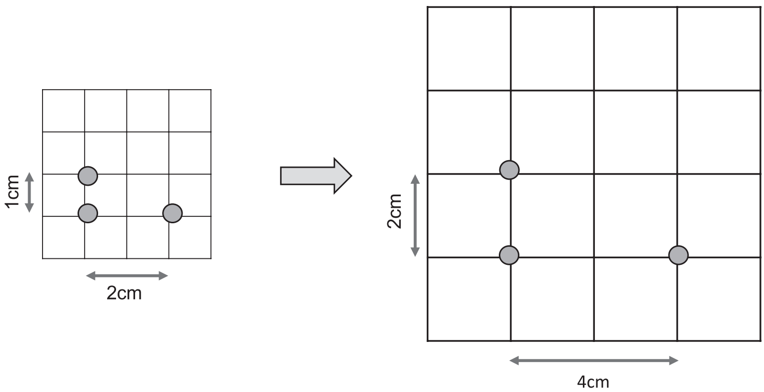

Fig. 7.4 Expanding "rubber sheet" universe. In 1 s the size doubles

7.2 A Beginning of the Universe?

The implications of Hubble's discovery were truly mind-boggling. If the distances between galaxies are getting larger, they must have been smaller at earlier times. As we follow the motion of galaxies back in time, they get closer and closer together, until they all merge at some moment of time in the past. This seems to imply that the expansion of the universe must have had a beginning. Was that the beginning of our world?

The problem of the origin of the universe, which had for centuries been the province of philosophers and theologians, had thus invaded the world of physicists and astronomers. Friedmann's models suggested that the whole universe began at a singular event a finite time ago. But for many this was too much to take. "Philosophically, the notion of a beginning of the present order of Nature is repugnant to me," wrote Sir Arthur Eddington, a prominent British astronomer. "As a scientist I simply do not believe that the Universe began with a bang." Einstein was equally disturbed. In a letter to the Dutch astronomer Willem de Sitter he wrote: "To admit such possibilities seems senseless."

And indeed, a cosmic beginning a finite time ago appeared to raise a host of perplexing problems. What actually happened at the beginning? And what caused it to happen? What determined the initial state of the universe? In the wake of Hubble's discovery, no obvious answers presented themselves. But once these problems came into focus, much of the further progress in cosmology was driven by attempts to understand the early stages of the expansion, what caused the expansion to begin, and ultimately how—and whether—the universe came into being.

7.3 The Steady State Theory

Most physicists hoped that Hubble's discovery would somehow be explained without having to postulate that the universe had a beginning. The most notorious attempt of this kind was the "steady state theory", proposed in 1948 by Fred Hoyle, Hermann Bondi and Thomas Gold, all at Cambridge University. This theory was based on the so-called "perfect cosmological principle", which asserts that the universe looks more or less the same at all times, at all places, and in all directions. An obvious implication is that the universe had no beginning in time. But how could this picture be reconciled with the fact that the universe was known to be expanding? Surely the distances between galaxies would grow and the average matter density would dilute?

To compensate for the expansion, Hoyle, Bondi and Gold proposed that matter is continuously created out of the vacuum, so that the average mat-

Fig. 7.5 Fred Hoyle (1915–2001) is best known for his contribution to the theory of stellar nucleosynthesis, explaining how heavy elements were formed in the interiors of stars. He was also the main proponent of the steady state theory and an ardent opponent of the big bang model. Yet ironically he coined the term "big bang" (in derision) during a radio broadcast for the BBC in 1949. *Credit* Photo by Ramsey and Muspratt, courtesy AIP Emilio Segre Visual Archives, Physics Today Collection

ter density remains constant. To achieve this, only a few atoms per cubic kilometer per century would need to materialize. So instead of one sudden creation of all matter, a very small amount of matter would need to be continuously created (Fig. 7.5).

Many physicists supported the steady state model on philosophical grounds. But ultimately, it was proven wrong. One steady state prediction was that distant galaxies, which we see as they were billions of years ago, should look more or less the same as galaxies in our neighborhood. We now know that distant galaxies are smaller, have irregular shapes and are populated by very bright, short-lived stars. Unlike nearby galaxies, many of them are powerful sources of radio waves.

The final demise of the steady state theory came with the discovery of the Cosmic Microwave Background (CMB) radiation in the mid-1960s. The detection of the CMB proved that the early universe was very hot, and the hot big bang model emerged as the standard cosmological paradigm. Cosmologists had to accept that dealing with the beginning of the universe was an unavoidable workplace hazard!

7.4 The Scale Factor

As the universe expands, the distances between all galaxies are stretched by the same factor. Similarly, if we go back in time, all distances are contracted by the same factor. The factor by which the distances change as we go from the present cosmic time t_0 to some future or past time t is called the *scale factor*; it is denoted by $a(t)$.

If two galaxies are currently separated by a distance d_0 then their separation at any other time t is

$$d = a(t)d_0. \tag{7.3}$$

At the present time t_0, the scale factor is conventionally defined to be $a(t_0) = 1$; at earlier times $a(t) < 1$, at later times $a(t) > 1$, and at the big bang $a(t = 0) = 0$. The relative velocity of a pair of galaxies is given by the rate of change of their distance. We write $v = \dot{d}$, where an overdot is the standard physics notation for "the rate of change". (If you are familiar with calculus, you will recognize that $\dot{d}$ is the derivative of the distance with respect to time.) From Eq. (7.3) we find

$$v = \dot{a}(t)d_0 \tag{7.4}$$

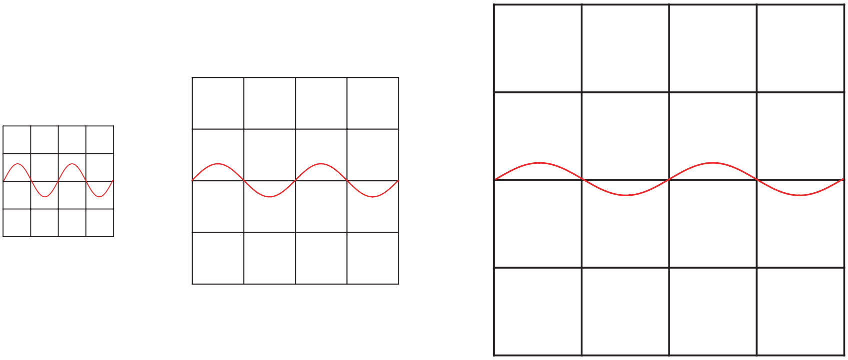

Fig. 7.6 Cosmological redshift. The wavelength of light gets stretched because space itself stretches

where $\dot{a}$ is the rate of change of the scale factor.[3] Thus, the relative velocity of the galaxies depends on how fast the scale factor changes with time. The Hubble parameter can now be found from $H = v/d$, which gives [using Eqs. (7.3) and (7.4)]

$$H = \frac{\dot{a}(t)}{a(t)} \tag{7.5}$$

Thus the Hubble parameter at any time is equal to the rate at which the scale factor is changing divided by the scale factor at that time. It is important to note that the Hubble parameter (or Hubble constant, as it is sometimes called) is constant in space, but it can change with time.

7.5 Cosmological Redshift

So far we have explained the observed redshift of light by the Doppler effect, due to the motion of galaxies away from us. We can now give an alternative interpretation, which is simpler: as the light waves travel to us, their wavelength is stretched by cosmic expansion (see Fig. 7.6).

When light leaves a distant galaxy at some time t, it starts off with a certain wavelength λ. By the time it reaches us, the universe has increased in

[3] If we multiply some variable by a constant, its rate of change gets multiplied by the same constant.

size by the factor $1/a(t)$, and the wavelength of light is stretched by the same factor. The wavelength λ_0 observed at present on Earth can be found from

$$\frac{\lambda_0}{\lambda} = \frac{1}{a(t)}. \tag{7.6}$$

The cosmological redshift z is defined as the fractional change in the wavelength,

$$z = \frac{\lambda_0 - \lambda}{\lambda} \tag{7.7}$$

and we have a relation between redshift and scale factor,[4]

$$z + 1 = \frac{\lambda_0}{\lambda} = \frac{1}{a(t)}. \tag{7.8}$$

Thus, by measuring the redshift of light coming from a distant galaxy, we know immediately by how much the universe has expanded since the light was emitted.

7.6 The Age of the Universe

If the universe began a finite time ago, then how old is it? To find out, we can follow the motion of galaxies back in time and evaluate how long it takes until they merge at the big bang. For a rough estimate, we shall first neglect the effect of gravity. Under this condition, any given pair of galaxies moves at a constant relative speed. Consider two galaxies at a distance d from one another. According to Hubble's law, they move apart at speed $v = H_0 d$. If they have always moved at this speed, then the time elapsed since the big bang is

$$t_0 = d/v = d/H_0 d = 1/H_0 = 4.5 \times 10^{17} s \approx 14.4 \times 10^9 \text{ yrs}. \tag{7.9}$$

[4]For light emitted at an early epoch, when the universe was much smaller than it is today, we have $a(t) \ll 1$ and $z \gg 1$. Note that in this regime Eq. (6.1) for the Doppler shift cannot be used. It applies only to light sources moving at speeds small compared to the speed of light, that is, only to $z \ll 1$. (The symbols "$\ll$" and "$\gg$" mean "much less than" and "much greater than" respectively).

Note that this time is independent of the distance d, so it is the same for all pairs of galaxies. Cosmologists call $1/H_0$ the "Hubble time".

We can improve this estimate, by taking into account that the universe actually has a time-varying expansion rate. Early in its history the universe decelerated due to gravity, while later on it began a period of accelerated expansion (for reasons to be discussed later). It turns out that these two effects almost cancel one another. The best modern estimates taking all the details into account give an age of 13.77 billion years. It is quite remarkable that less than 100 years ago, we did not even know that the universe contained other galaxies, yet today we can calculate the age of the universe to within a half of a percent.

7.7 The Hubble Distance and the Cosmic Horizon

Hubble's law tells us that the velocities of galaxies grow in proportion to their distance. It follows that the velocities can get arbitrarily large for galaxies sufficiently far away. This may sound alarming, since motion faster than light appears to contradict special relativity. But in fact there is no contradiction. It is important to realize that the expansion of the universe is an expansion of space, not an expansion of galaxies into some pre-existing space. The theory of relativity requires that objects cannot move past one another faster than the speed of light, but there is no limit to how fast the space between objects can expand. The distance beyond which galaxies recede faster than the speed of light is called the *Hubble distance*. We can find it by setting $v = c$ in Eq. (7.1) and solving for d; this gives

$$d_H = \frac{c}{H_0} = 14.4 \times 10^9 \,\text{ly}. \tag{7.10}$$

Another important distance scale is set by our cosmic horizon. In a universe of a finite age, there is a limit to how far we can see into space. The distance that light has traveled since the big bang is finite, and light sources that are too far away cannot be seen, simply because their light has not yet reached the Earth. We can imagine ourselves at the center of a gigantic sphere—the observable part of the universe. The boundary of this sphere is called the *particle horizon*; its radius d_{hor} is the distance to the most remote objects ("particles") that we can possibly observe. We shall refer to the particle horizon as simply "the horizon" and will use the term "particle horizon" only where it can be confused with another kind of horizon—the *event horizon*, which we shall later encounter.

Since the age of the universe is $t_0 \approx 14 \times 10^9$y, you might think that the horizon distance is simply $ct_0 \approx 14 \times 10^9$ly. You would be right if the distance from us to cosmological light sources did not change with time. But in an expanding universe a given source moves away from us while its light travels towards the Earth. Thus, by the time we detect the source, it is at a greater distance than when its light was emitted. For the most remote observable sources, the emission time is close to the big bang. The source was then much closer to us than it is now, and its present distance depends on the entire expansion history of the universe from the big bang to the present time. Calculations based on our current understanding of this history give the horizon distance

$$d_{hor} \approx 46 \times 10^9 \text{ly}, \tag{7.11}$$

about 3 times larger than the naïve estimate. In upcoming chapters we will learn that the evolution of the early universe was first dominated by radiation, then matter and finally by "dark energy". All of these components cause the horizon to grow in different ways (when radiation dominated, the horizon grew the slowest, and when dark energy becomes dominant, the horizon grows the fastest.). For a matter dominated universe with a flat geometry, $d_{hor}(t) = 3\,ct$. This gives $d_{hor} \approx 42 \times 10^9$ly, slightly less than the actual horizon distance given in Eq. (7.11). For our purposes, an order of magnitude estimate for the horizon and Hubble distances at any cosmic time t can be found from the relation

$$d_{hor}(t) \sim d_H(t) \sim ct. \tag{7.12}$$

Light propagation and the horizon in an expanding universe are illustrated in a spacetime diagram in Fig. 7.7. The worldline of our galaxy is along the vertical axis, and the worldlines of a few other galaxies are plotted as blue curves. The galaxies get closer together as we go back in time and merge at the big bang. The slope of the curves tells us how fast the galaxies are moving away: the more steeply the curve slopes upward, the slower is the recession speed. We see from the figure that the speed grows with the distance, as required by the Hubble law. We can also tell how the recession speed changes with time for a given galaxy. The galaxies initially move apart at very high speeds. Then, as one might expect, they are slowed down by gravity. But about 5 billion years ago their motion begins to accelerate. We shall discuss the reason for this unexpected phenomenon in Chap. 9.

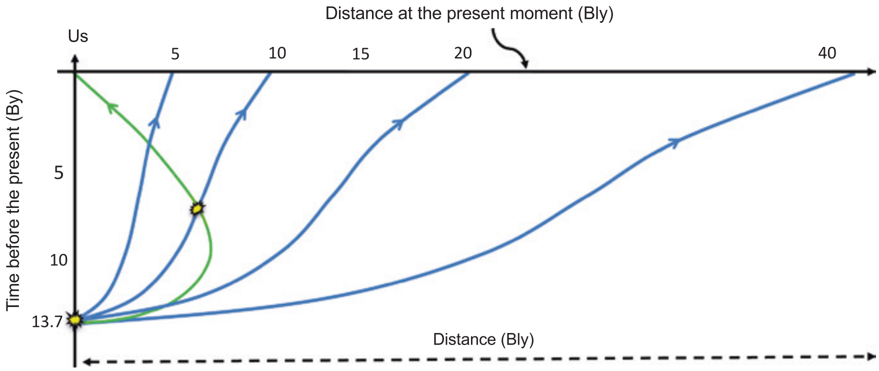

Fig. 7.7 Worldlines of galaxies (*blue*) and light propagation (*green*) in an expanding universe

Light propagation is indicated by a green line in the diagram. This line marks our past light cone. Note that it is rather different from straight-line propagation at a 45° angle that we would have in flat space. At early times, close to the big bang, light is dragged along by the expanding space, so light emitted in our direction initially moves away from us. It later turns around and, as it propagates through slower expanding space, finally approaches our galaxy along a 45° line.

Distant galaxies are now observed as they were at earlier times; these times can be found by looking at intersections of our past light cone (the green line) with worldlines of the galaxies. For example, the supernova marked in the figure occurred about 7.5 billion years ago in a galaxy that was about 7 Bly(billion light years) away at that time. The present distance to that galaxy is 10 Bly. As we look at more remote galaxies, the intersection occurs at earlier times, until we reach the galaxy whose worldline just touches our past light cone at the big bang. This galaxy is now about 46 Bly away. There are certainly more distant galaxies, but they cannot be observed, since their worldlines do not cross our past light cone. Thus, $d_{hor} \approx 46\mathrm{Bly}$ is the cosmic horizon distance.

7.8 Not Everything is Expanding

Since the universe is expanding, you may be wondering whether or not the Solar System, the Earth, or perhaps even you yourself are expanding as well. Don't worry, you are not expanding! Objects that are bound together by forces, like atoms, planets, stars, galaxies, and even groups of galaxies, are not undergoing Hubble expansion.

Fig. 7.8 Hubble's law does not apply to galaxies bound together by gravitational forces, like the colliding galaxies in this photograph. *Credit* NASA, H. Ford (JHU), G. Illingworth (UCSC/LO), M.Clampin (STScI), G. Hartig (STScI), the ACS Science Team, and ESA—APOD 2004-06-12

As in the rubber sheet analogy, we can imagine some objects in an expanding universe which are fixed in space, while the space itself is being stretched by cosmic expansion. We shall refer to such objects as *comoving*. Galaxies are comoving, but only approximately: in addition to Hubble expansion, they move under the action of gravitational forces. There are many beautiful photographs of galaxies colliding (see Fig. 7.8); in fact, our Milky Way and Andromeda are falling toward one another and will collide in about 4 billion years. However, on the largest scales, when we ignore these relatively "local" motions, all matter obeys Hubble's Law. Note that electric and magnetic fields in electromagnetic waves are not bound together by any force; that is why the light waves do get stretched.

Summary

Distant galaxies are moving away from the Milky Way, indicating that the universe is expanding. A simple relation between the speed at which a galaxy recedes from us and the distance to the galaxy was discovered by Edwin Hubble in 1929: the speed grows proportionally to the distance. This is now known as Hubble's Law. It suggests that there is no preferred center to the expansion (all observers see galaxies receding away from their host galaxy), that the universe was much denser in the past than it is today, and that the universe had a beginning in time, the big bang, roughly 14 billion years ago.

Questions

1. State Hubble's law mathematically, and describe what it means.
2. According to Hubble's law, we should see distant galaxies receding away from us faster and faster the further out we look. Does this mean we are at the center of the expanding universe?
3. What is the universe expanding into?
4. According to special relativity the speed of light is the ultimate speed limit. Is there a limit to how fast the distances to remote galaxies can grow? Explain.
5. Does everything in the universe undergo "Hubble" expansion? For example is the distance between the Earth and Sun expanding? What about the distance between your head and toes?
6. The Andromeda galaxy is moving towards us. Does this fact falsify Hubble's Law? Explain.
7. A universe expanding according to the Hubble law, $v = Hd$, remains homogeneous and isotropic if it was homogeneous and isotropic to begin with. In such a universe, observers in any galaxy will see other galaxies receding according to the same law. Would these properties still hold if instead of the Hubble law the recession speeds of galaxies were proportional to the square of their distance, $v = Hd^2$?
8. Using Hubble's law and the nonrelativistic redshift formula $z = v/c$, calculate the distance to a galaxy that has a measured redshift of $z = 0.01$ (Assume $H_0 = 2.2 \times 10^{-18}\,\mathrm{s}^{-1}$ and $c = 3 \times 10^8\,\mathrm{ms}^{-1}$).
9. Astronomers identified carbon lines in the spectrum of a remote galaxy and determined that their wavelengths are 1.5 times greater than the corresponding wavelengths in the carbon spectrum on Earth. By how much has the universe expanded since the time this light was emitted?
10. If the expansion of the universe has always been decelerating since the big bang, is the Hubble time greater than or less than the age of the universe? (Hint: suppose you and your friend are running a race. At some point you catch up with each other and momentarily have the same velocity. If your friend has always run with this constant velocity, and if you started out faster and have been decelerating, which one of you must have started the race first?)
11. A pulse of light is emitted from a source towards an observer, who is initially at rest with respect to the source. Consider the following two scenarios:
    (a) After the pulse is emitted, the observer starts moving rapidly away from the source. He stops when the distance between him and the source doubles; soon after that the pulse reaches the observer. The universe does not expand in this scenario.

(b) After the pulse is emitted, the universe starts expanding and expands by a factor of 2, so the distance between the observer and the source is stretched by the same factor. After expansion stops, the light reaches the observer. Will the observer detect any redshift in either of these situations?

12. Eternal, static models of the universe are in conflict with the second law of thermodynamics. Explain why an expanding universe can avoid this conflict.
13. Models assuming that the universe is static and infinite suffer from Olbers' paradox: each line of sight encounters a star, so the entire sky should be shining like the surface of the Sun. Explain why an expanding universe of a finite age does not have this problem.
14. The steady state theory is based on the "perfect cosmological principle" which states that on average the universe looks the same at all places, in all directions, and at all times. What observations cannot be explained by the steady state theory and why?
15. What is the cosmic horizon? If t_0 is the age of the universe, why is the horizon distance greater than ct_0?